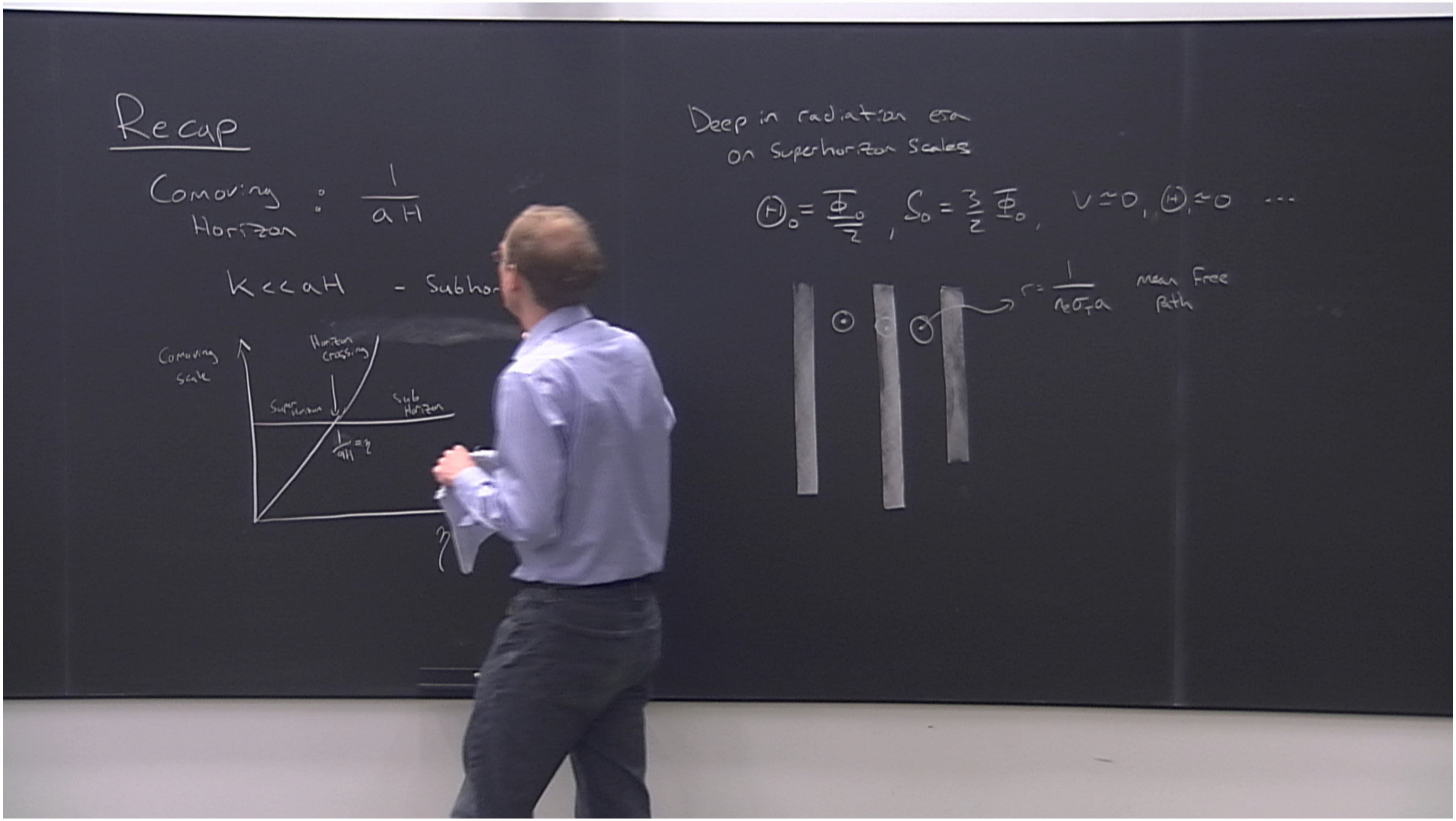
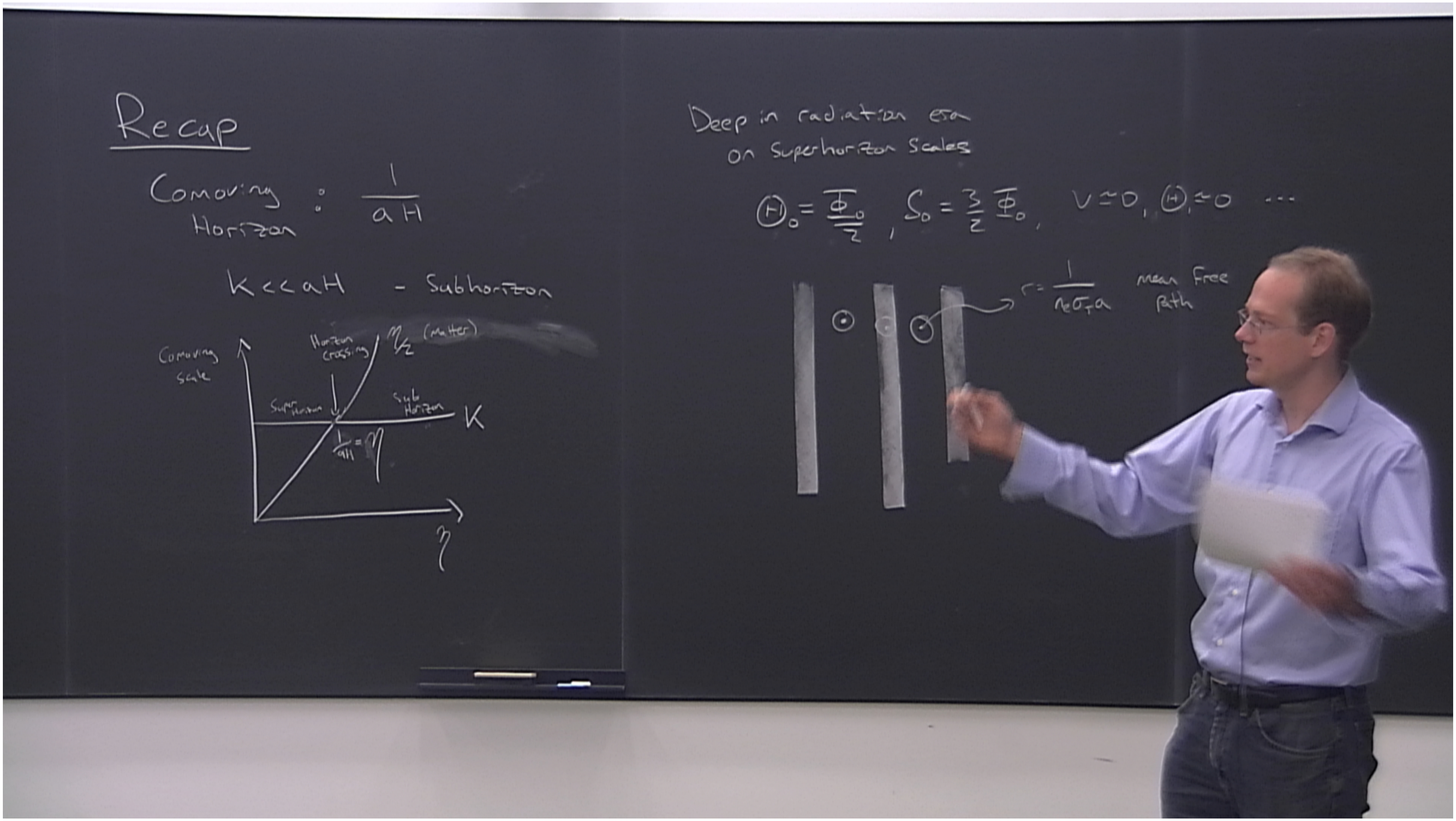
Title: 12/13 PSI - Explorations in Cosmology Lecture 6

Date: Apr 15, 2013 10:15 AM

URL: http://pirsa.org/13040069

Abstract:


Recap
Comoving Horizon : 1/aH
k << aH - Subhor
Comoving scale
Horizon crossing
Super horizon
Sub Horizon
1/aH = η
Deep in radiation era
on Superhorizon Scales
Θ₀ = Φ₀/2 , δ₀ = 3/2 Φ₀ , v ≃ 0 , Θ₁ ≃ 0 ...
mean free path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Comoving Horizon : 1/aH
K<<aH - Subhorizon
Comoving scale
Horizon crossing
(Matter)
Super horizon
Sub Horizon
K
Deep in radiation era
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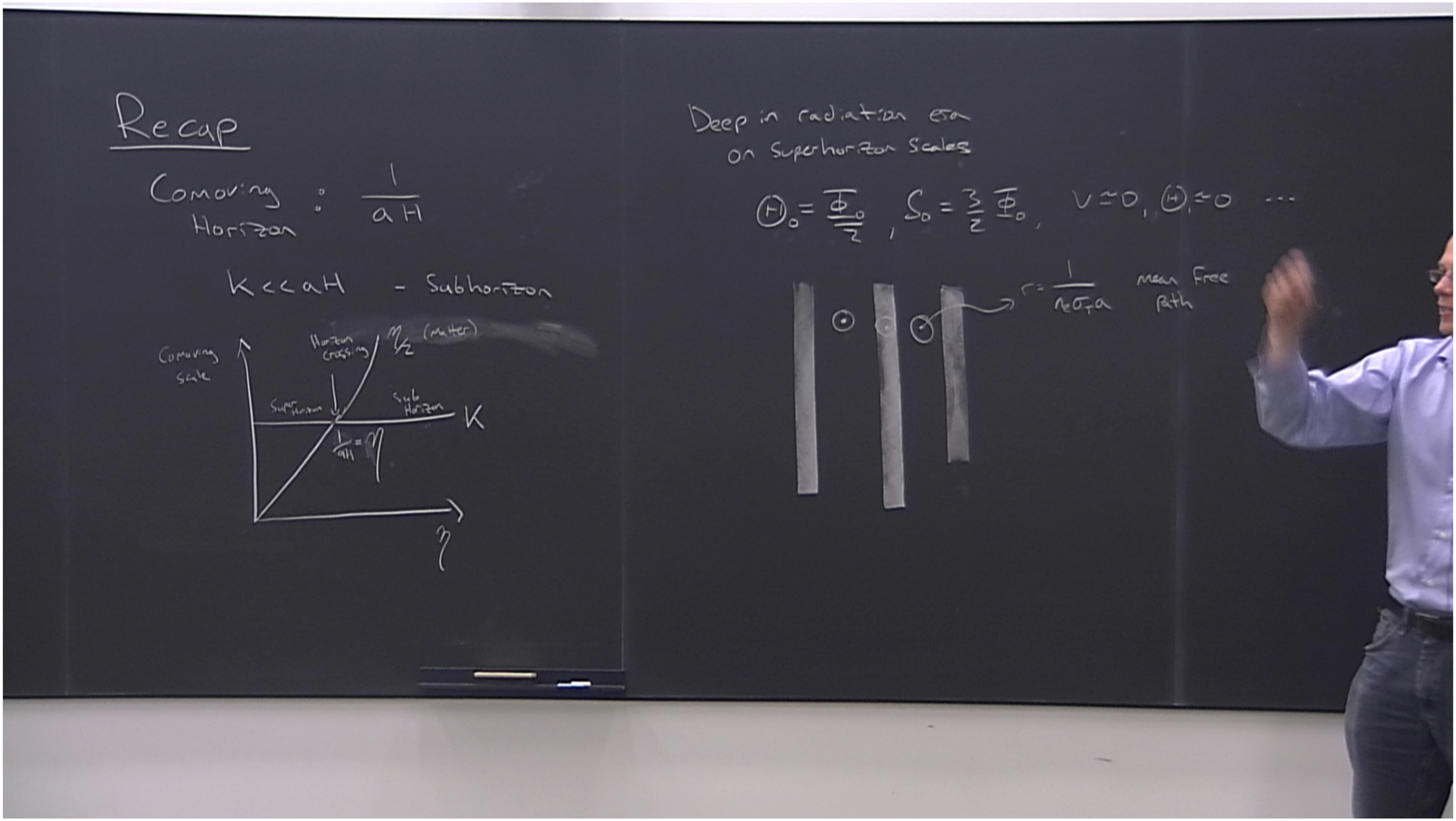
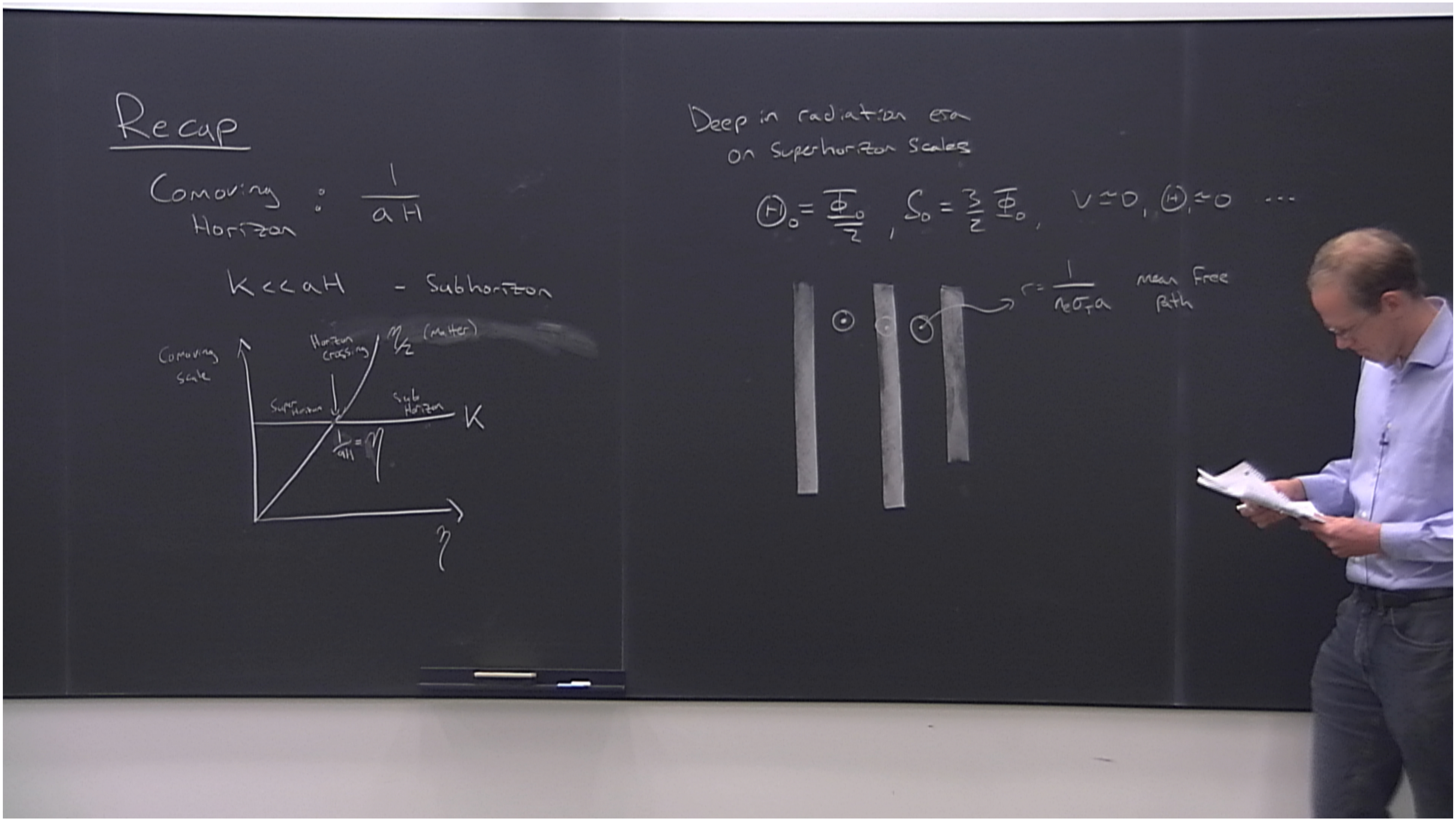
mean free path


Recap
Comoving Horizon : $\frac{1}{aH}$
$k << aH$ - Subhorizon
Comoving scale
Horizon crossing
η^2 (matter)
Super horizon
Sub Horizon
k
$\frac{1}{aH} = \eta$
η
Deep in radiation era
on Superhorizon Scales
$\Theta_0 = \frac{\Phi_0}{2}$, $\delta_0 = \frac{3}{2}\Phi_0$, $v \simeq 0$, $\Theta_1 \simeq 0$ ...
$r = \frac{1}{n_e \sigma_T a}$ mean free path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1/aH = η
η
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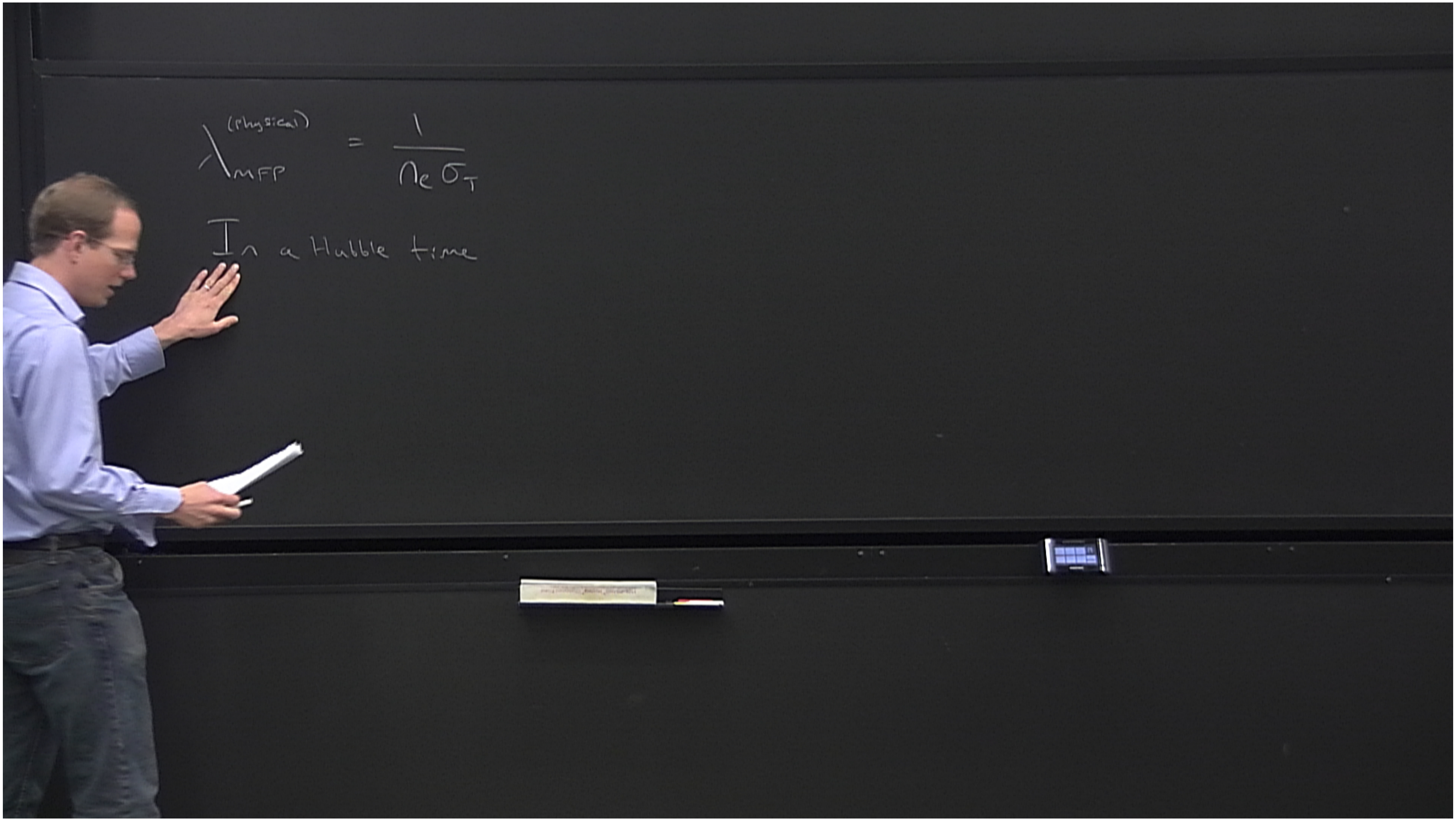
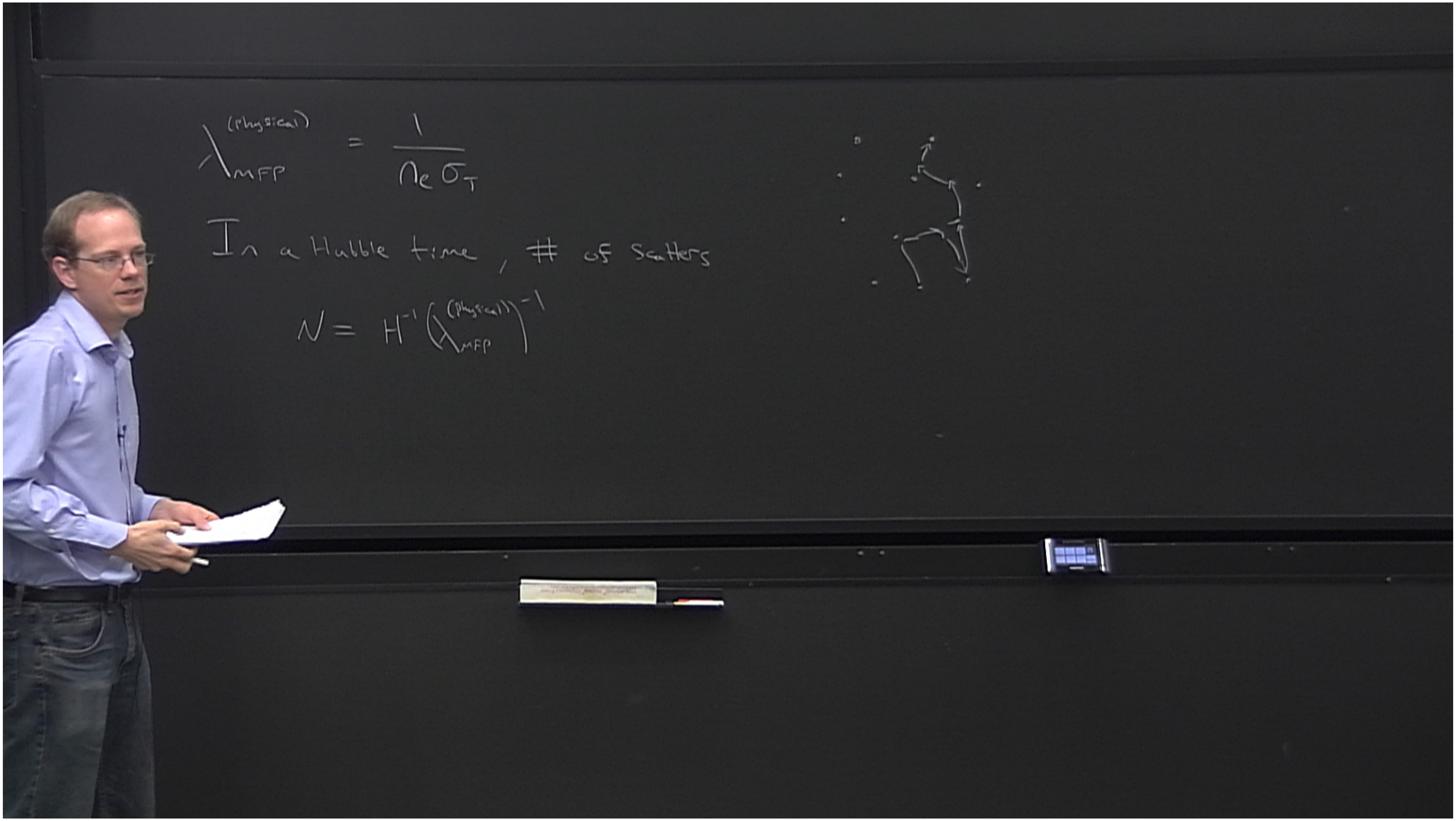
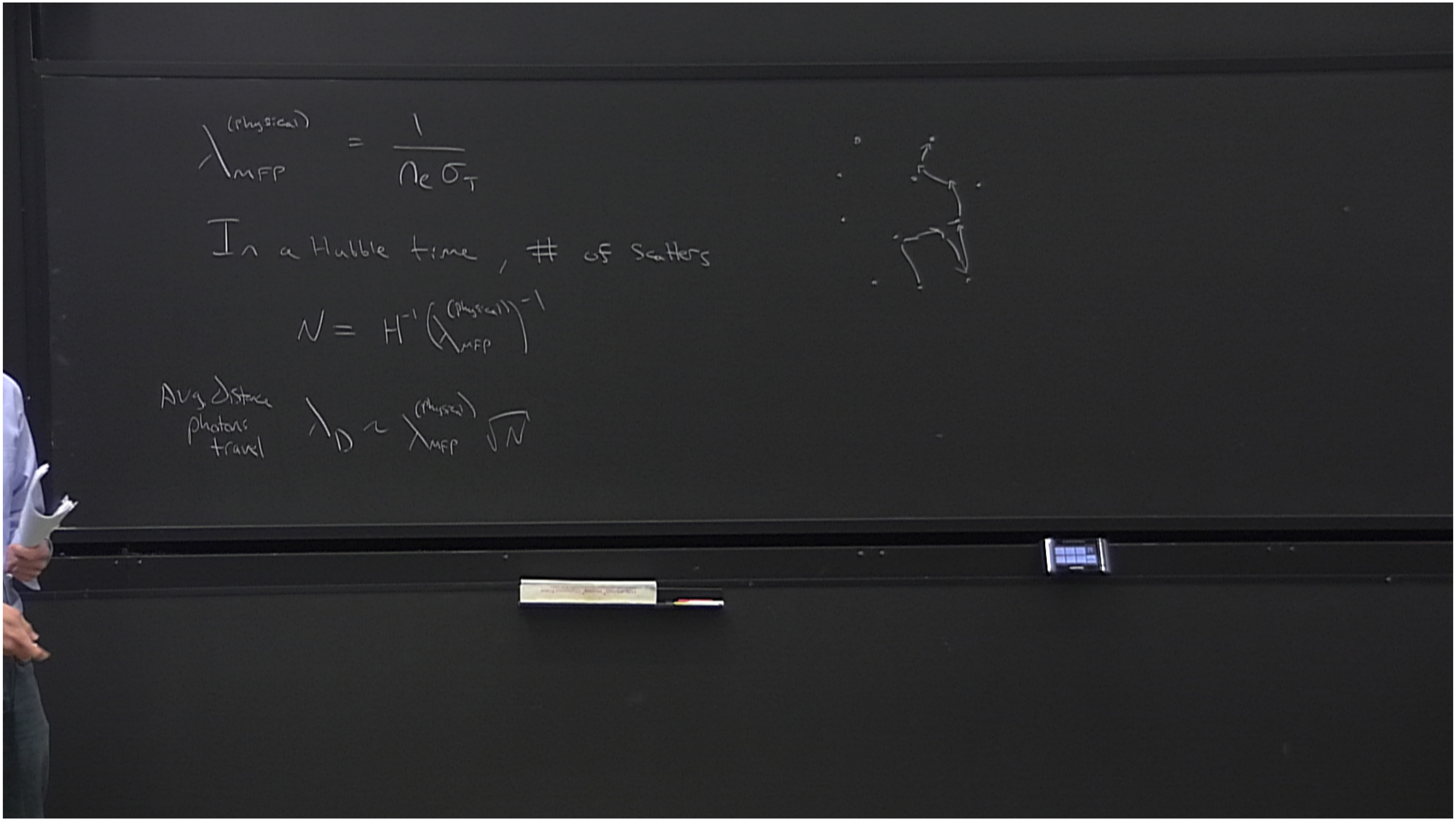
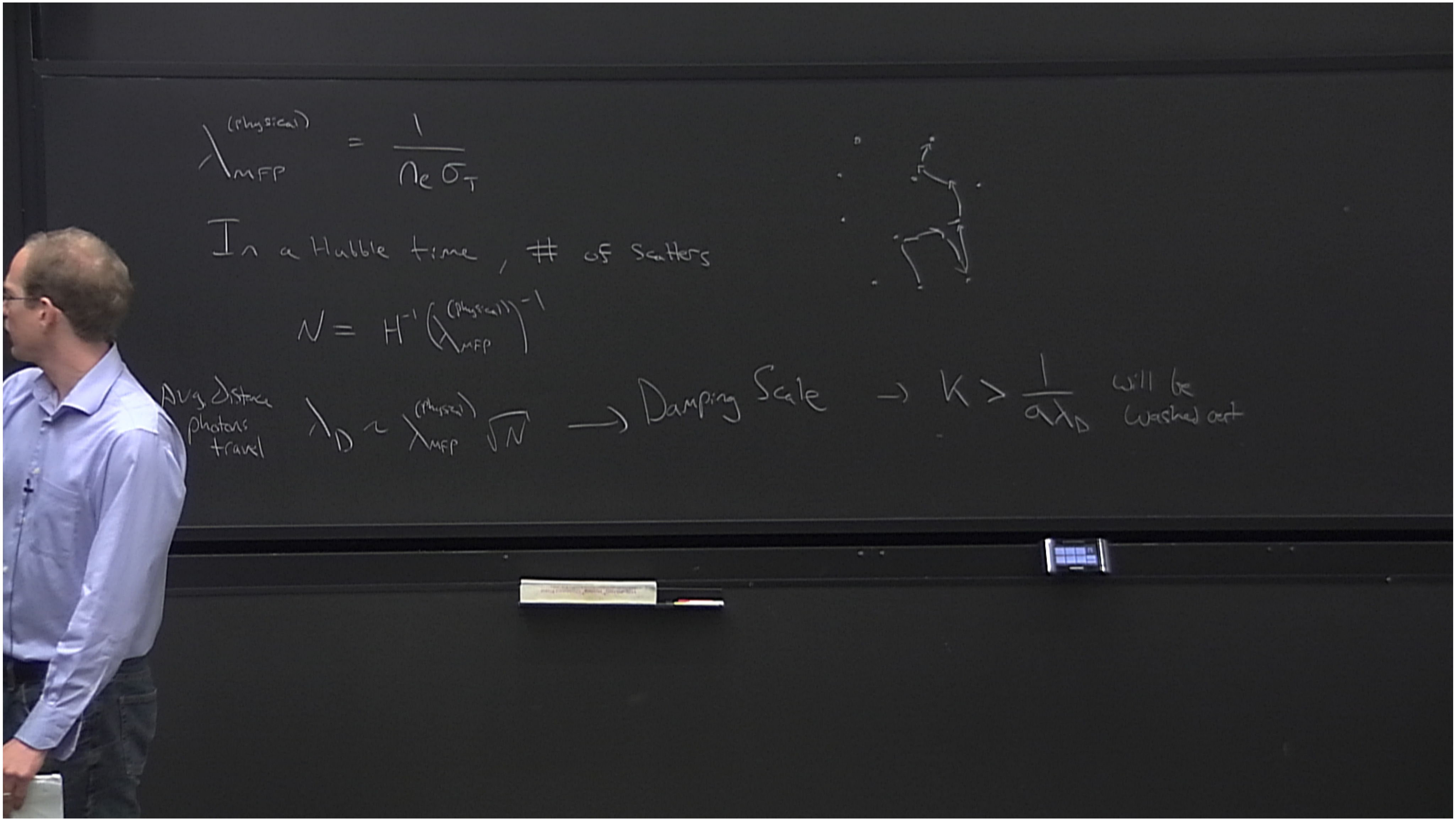
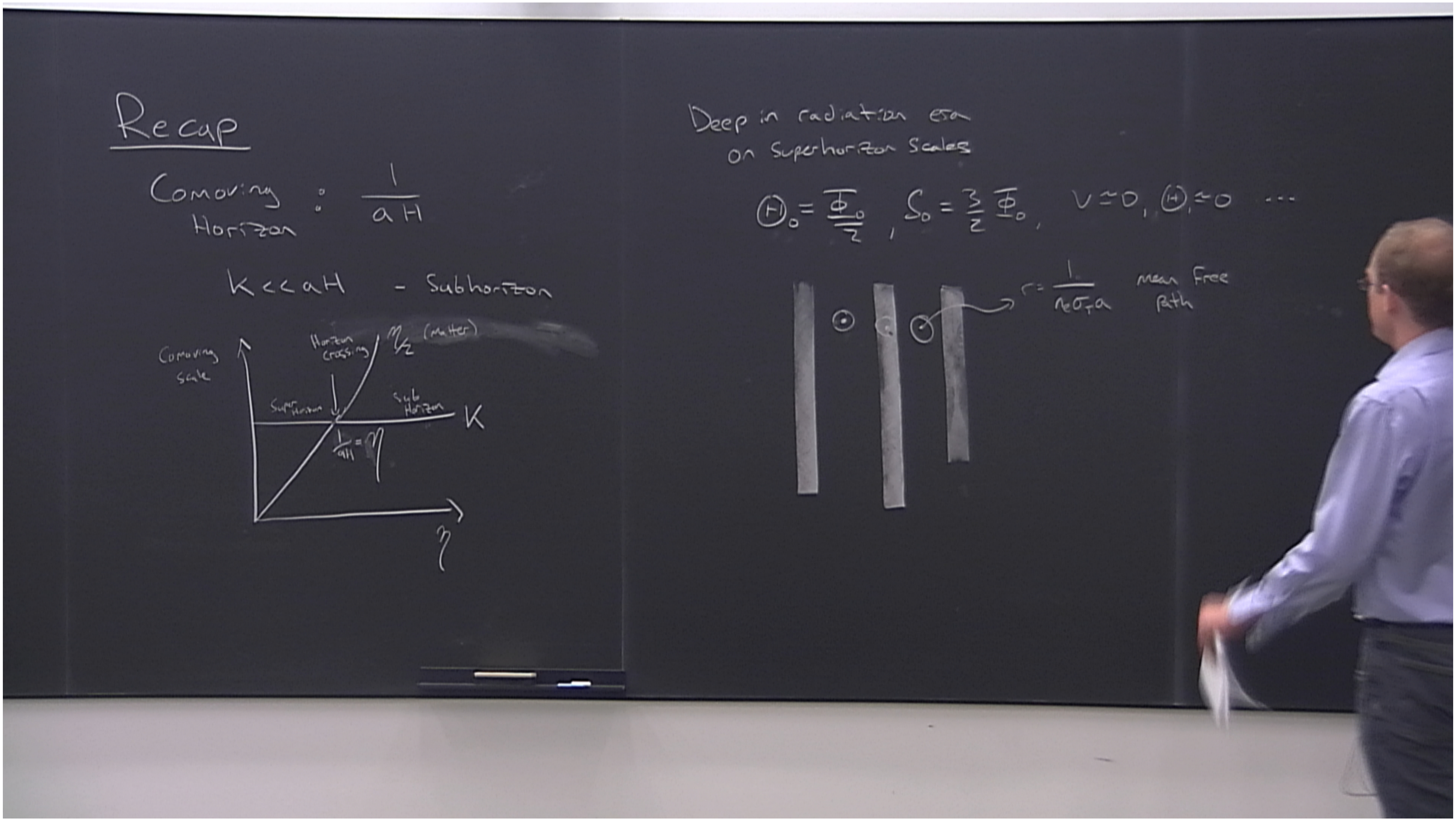
Θ0 = Φ0/2 , δ0 = 3/2 Φ0 , v ≃ 0 , Θ1 ≃ 0 . . .
= 1/(ne σT a)
mean free path


$\lambda_{MFP}^{(physical)} = \frac{1}{n_e \sigma_T}$
In a Hubble time


$\lambda_{MFP}^{(physical)} = \frac{1}{n_e \sigma_T}$
In a Hubble time, # of scatters
$N = H^{-1} \left(\lambda_{MFP}^{(physical)}\right)^{-1}$


$\lambda_{MFP}^{(physical)} = \frac{1}{n_e \sigma_T}$
In a Hubble time, # of scatters
$N = H^{-1} \left(\lambda_{MFP}^{(physical)}\right)^{-1}$
Avg. Distance photons travel
$\lambda_D \sim \lambda_{MFP}^{(physical)} \sqrt{N}$


$\lambda_{MFP}^{(physical)} = \frac{1}{n_e \sigma_T}$
In a Hubble time, # of scatters
$N = H^{-1} \left(\lambda_{MFP}^{(physical)}\right)^{-1}$
Avg. Distance photons travel
$\lambda_D \sim \lambda_{MFP}^{(physical)} \sqrt{N}$
$\longrightarrow$ Damping Scale $\rightarrow$ $K > \frac{1}{a \lambda_D}$ will be washed out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η/2 (Matter)
Super Horizon
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1/aH = η
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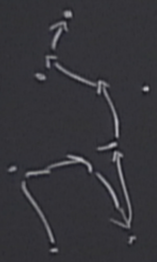
r = 1/(nₑσ_T a)
mean free path



$$\int \frac{d\mu}{2} P_\ell(\mu) \left[\dot{\Theta} + ik\mu\,\Theta = -\dot{\Phi} - ik\mu\,\Psi - \dot{\tau}\left(\Theta_0 - \Theta + \mu v_b\right) \right]$$

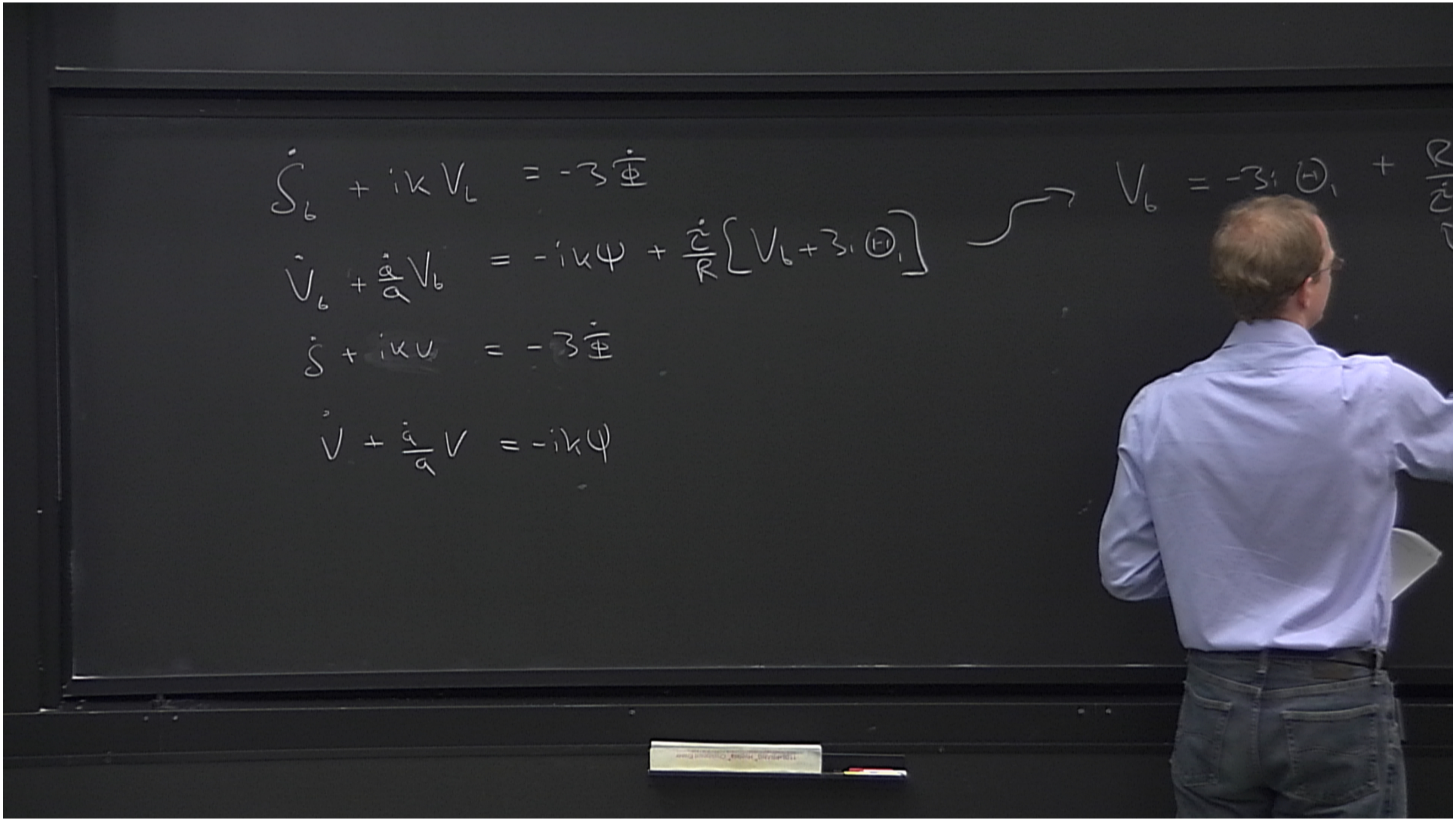
$$P_1 = \frac{3\mu \cdot 1}{2}$$

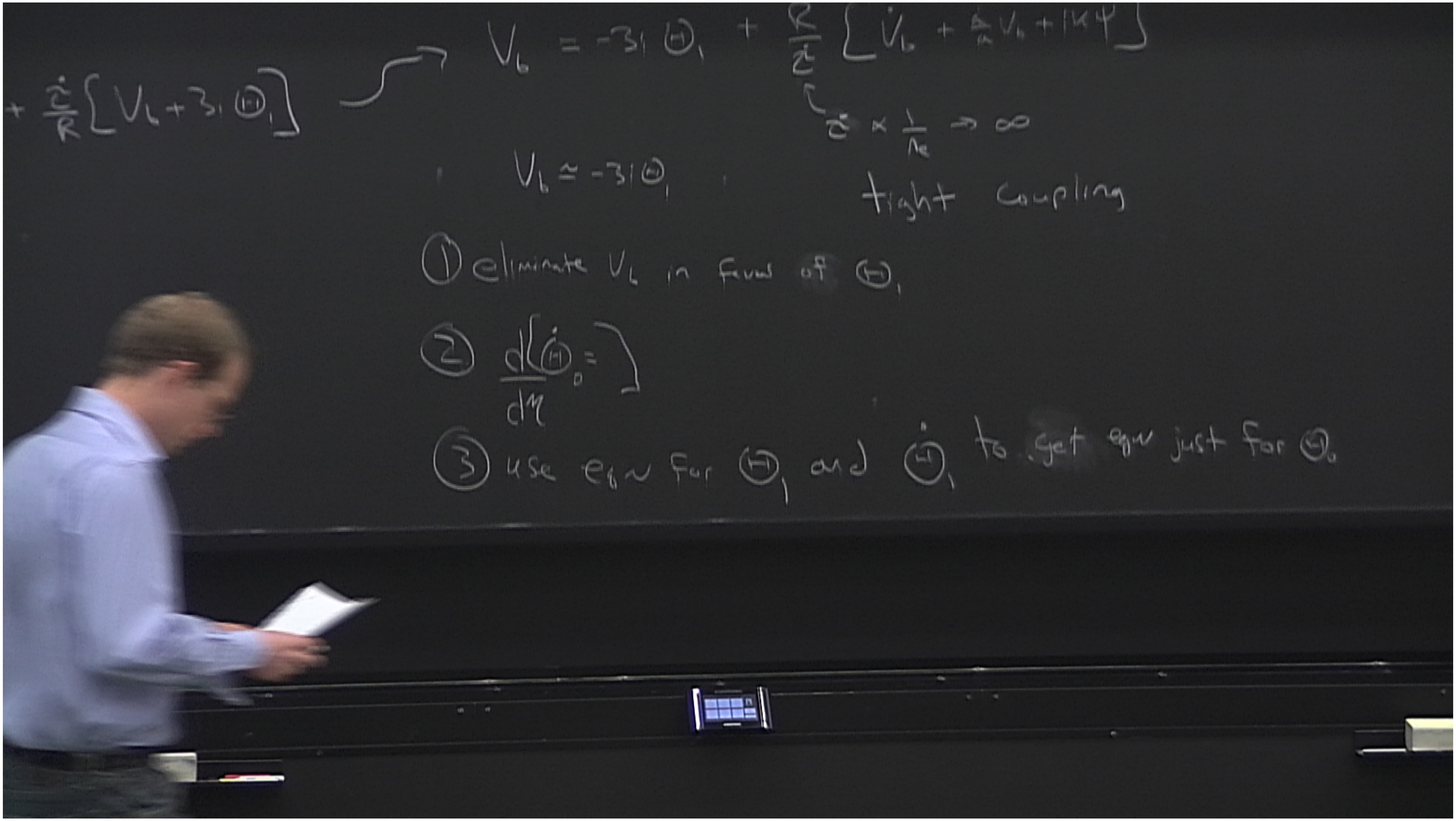
$$\Theta_\ell \propto \int \frac{d\mu}{2} P_\ell\, \Theta$$

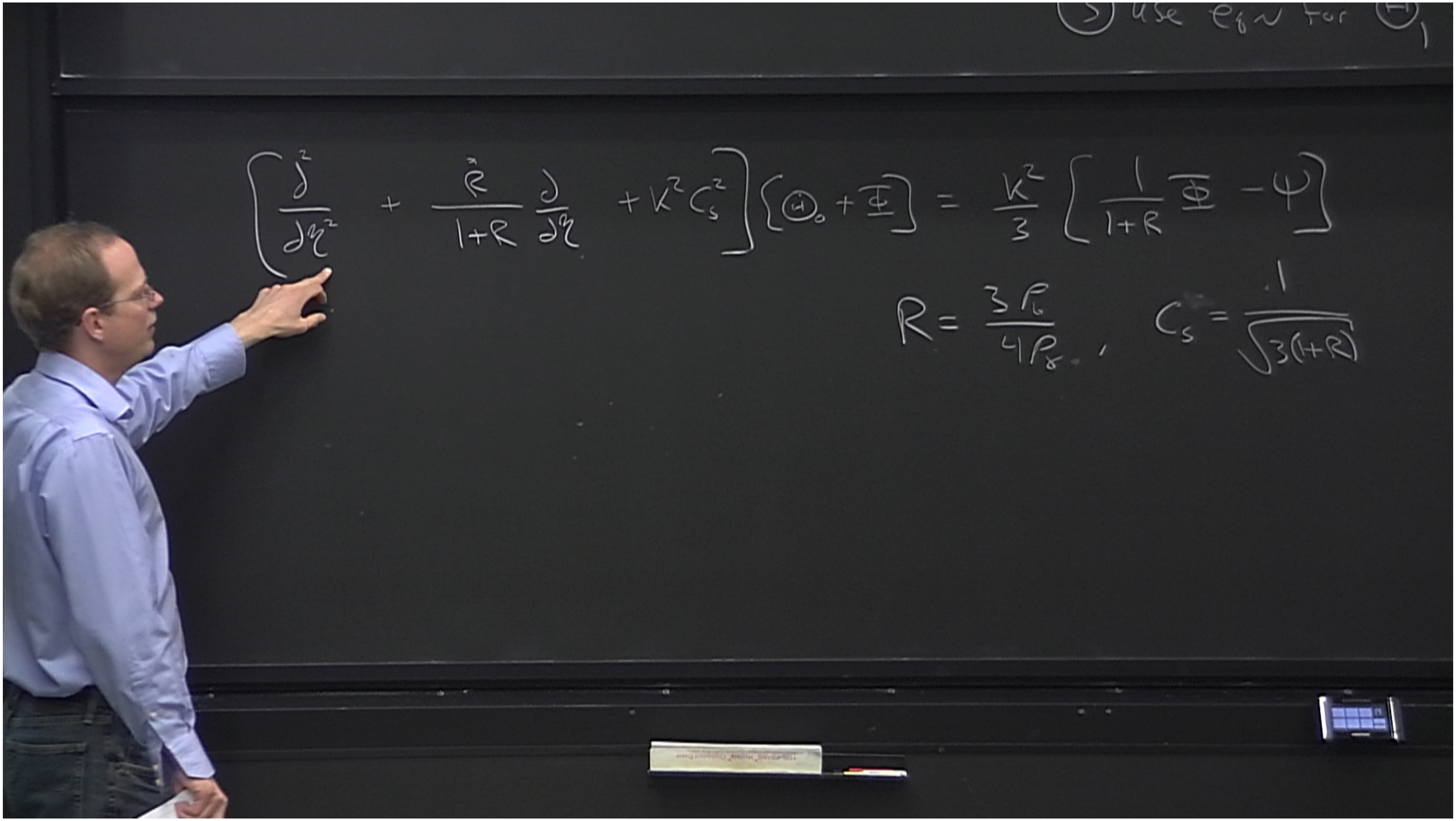
$\rightarrow k > \frac{1}{a\lambda_D}$ will be washed out

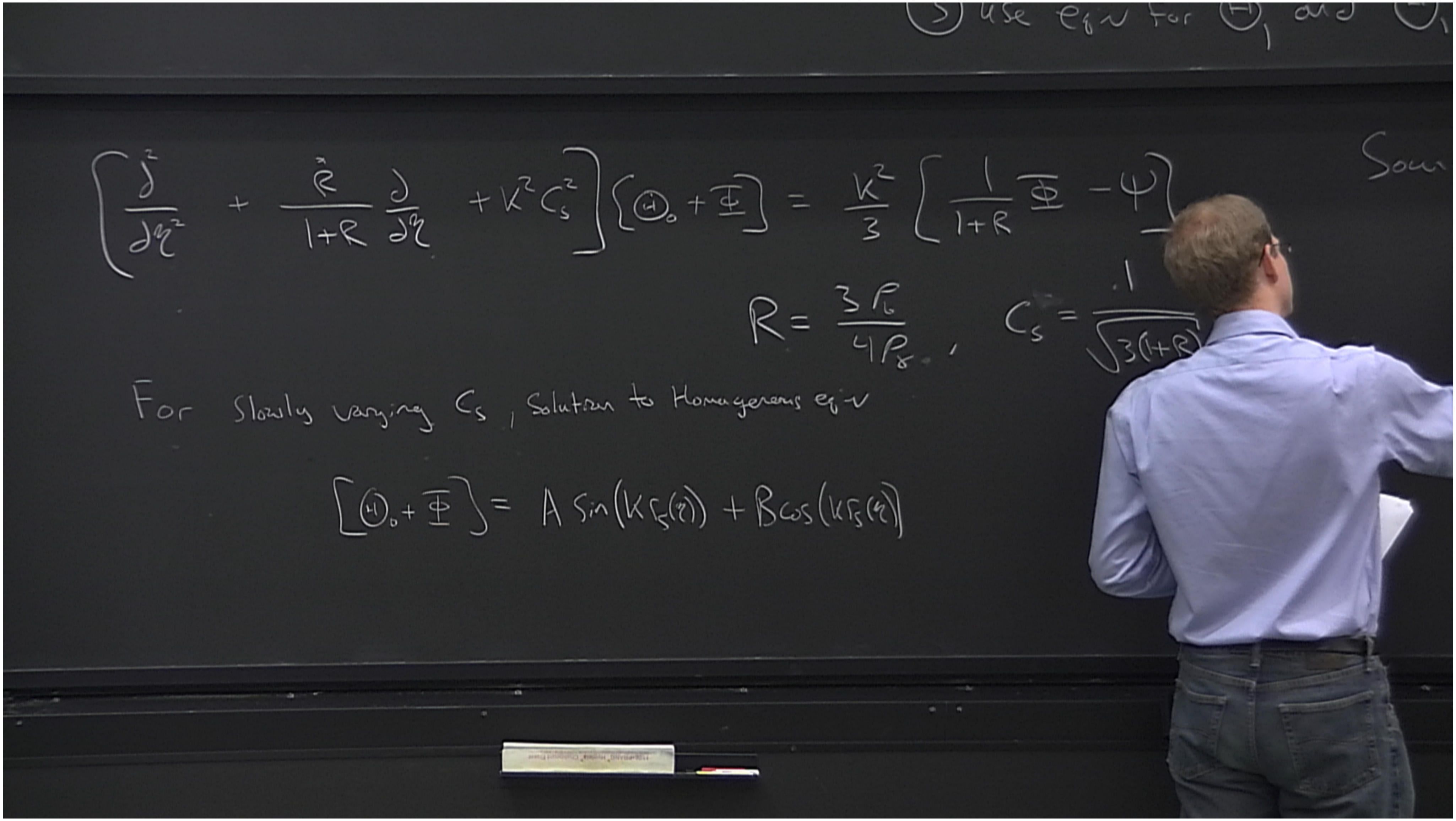
$$\dot{\Theta}_0 + k\,\Theta_1 = -\dot{\Phi}$$

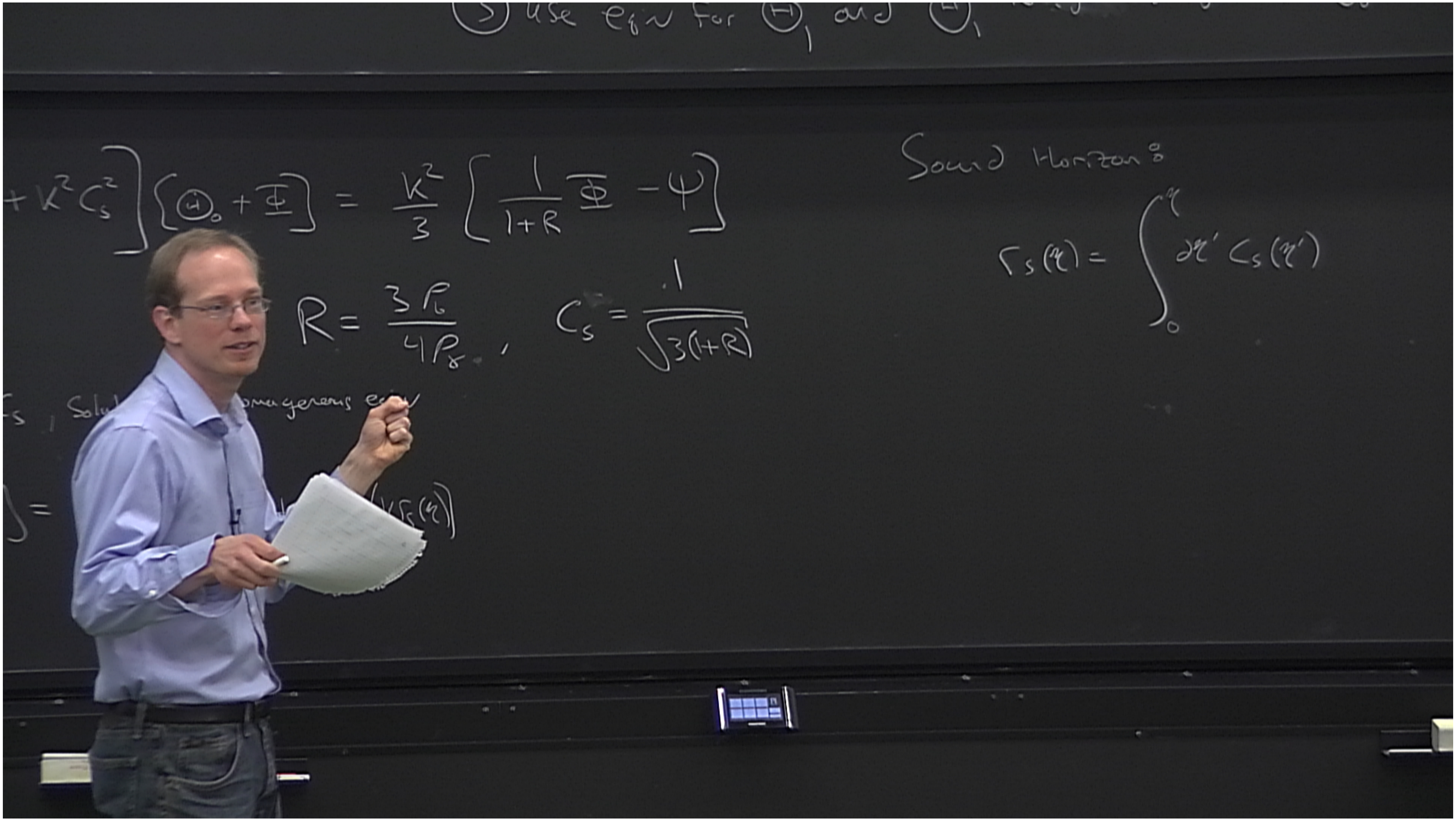
$$\dot{\Theta}_1 - \frac{k}{3}\Theta_0 = \frac{k\Psi}{3} + \dot{\tau}\left[\Theta_1 - \frac{i v_b}{3}\right]$$


$\dot{\delta}_b + ikV_b = -3\dot{\Phi}$
$\dot{V}_b + \frac{\dot{a}}{a}V_b = -ik\Psi + \frac{\dot{\tau}}{R}[V_b + 3i\Theta_1]$
$\dot{\delta} + ikv = -3\dot{\Phi}$
$\dot{v} + \frac{\dot{a}}{a}v = -ik\Psi$
$V_b = -3i\Theta_1 +$


$V_b = -3i\Theta_1 + \frac{R}{\dot\tau}\left[\dot V_b + \frac{\dot a}{a}V_b + ik\Psi\right]$
$+ \frac{\dot\tau}{R}\left[V_b + 3i\Theta_1\right]$
$\dot\tau \propto \frac{1}{n_e} \to \infty$
$V_b \simeq -3i\Theta_1$
tight coupling
1 eliminate V_b in favor of Θ_1
2 $\frac{d[\dot\Theta_0 =]}{d\eta}$
3 use eqn for Θ_1 and $\dot\Theta_1$ to get eqn just for Θ_0


5 use eqn for Θ_1
$\left[\frac{\partial^2}{\partial\eta^2} + \frac{\dot{R}}{1+R}\frac{\partial}{\partial\eta} + k^2 c_s^2\right]\left[\Theta_0 + \Phi\right] = \frac{k^2}{3}\left[\frac{1}{1+R}\Phi - \Psi\right]$
$R = \frac{3\rho_b}{4\rho_\gamma}$, $c_s = \frac{1}{\sqrt{3(1+R)}}$


$\left[\frac{\partial^2}{\partial\eta^2} + \frac{\dot{R}}{1+R}\frac{\partial}{\partial\eta} + k^2 c_s^2\right]\left[\Theta_0 + \Phi\right] = \frac{k^2}{3}\left[\frac{1}{1+R}\Phi - \Psi\right]$
$R = \frac{3\rho_b}{4\rho_\gamma}$, $c_s = \frac{1}{\sqrt{3(1+R)}}$
For slowly varying c_s, Solution to Homogeneous eq'n
$\left[\Theta_0 + \Phi\right] = A\sin(k r_s(\eta)) + B\cos(k r_s(\eta))$



Sound Horizon:
$r_s(\eta) = \int_0^\eta d\eta' \, c_s(\eta')$
$R = \frac{3\rho_b}{4\rho_\gamma}$
$c_s = \frac{1}{\sqrt{3(1+R)}}$
$= \frac{k^2}{3}\left[\frac{1}{1+R}\Phi - \Psi\right]$

Eqn for Θ_1 and Θ_1

$\Phi - \Psi]$

$c_s = \frac{1}{\sqrt{3(1+R)}}$

$\int_0^\eta d\eta' [\Phi(\eta') - \Psi(\eta')] \cos[k(r_s(\eta) - r_s(\eta'))]$

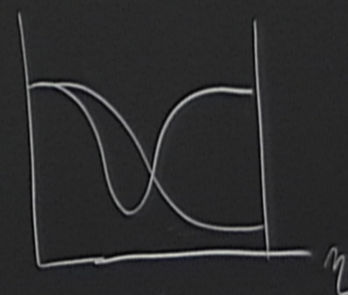
Sound Horizon:

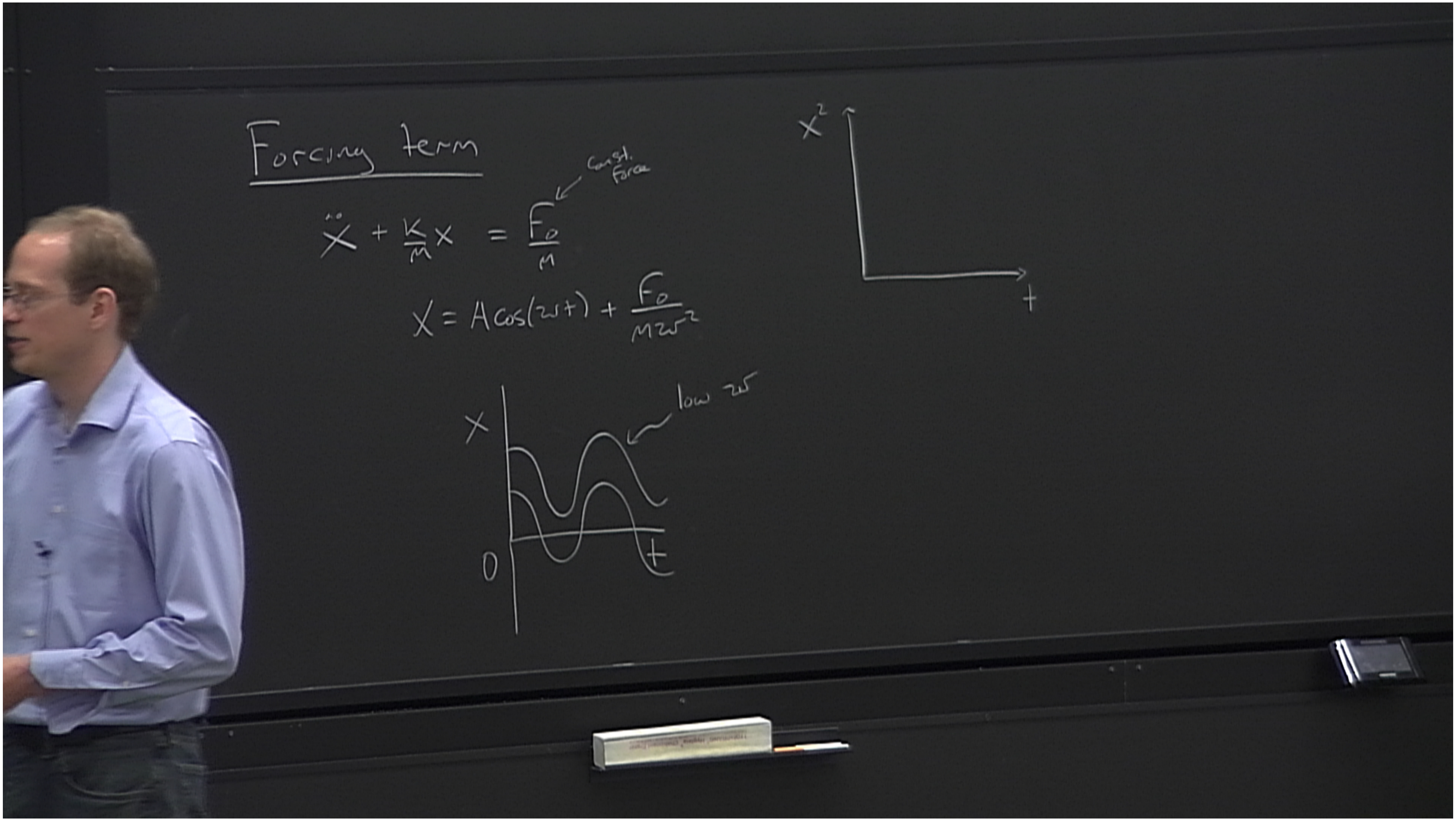
$$r_s(\eta) = \int_0^\eta d\eta' \, c_s(\eta')$$

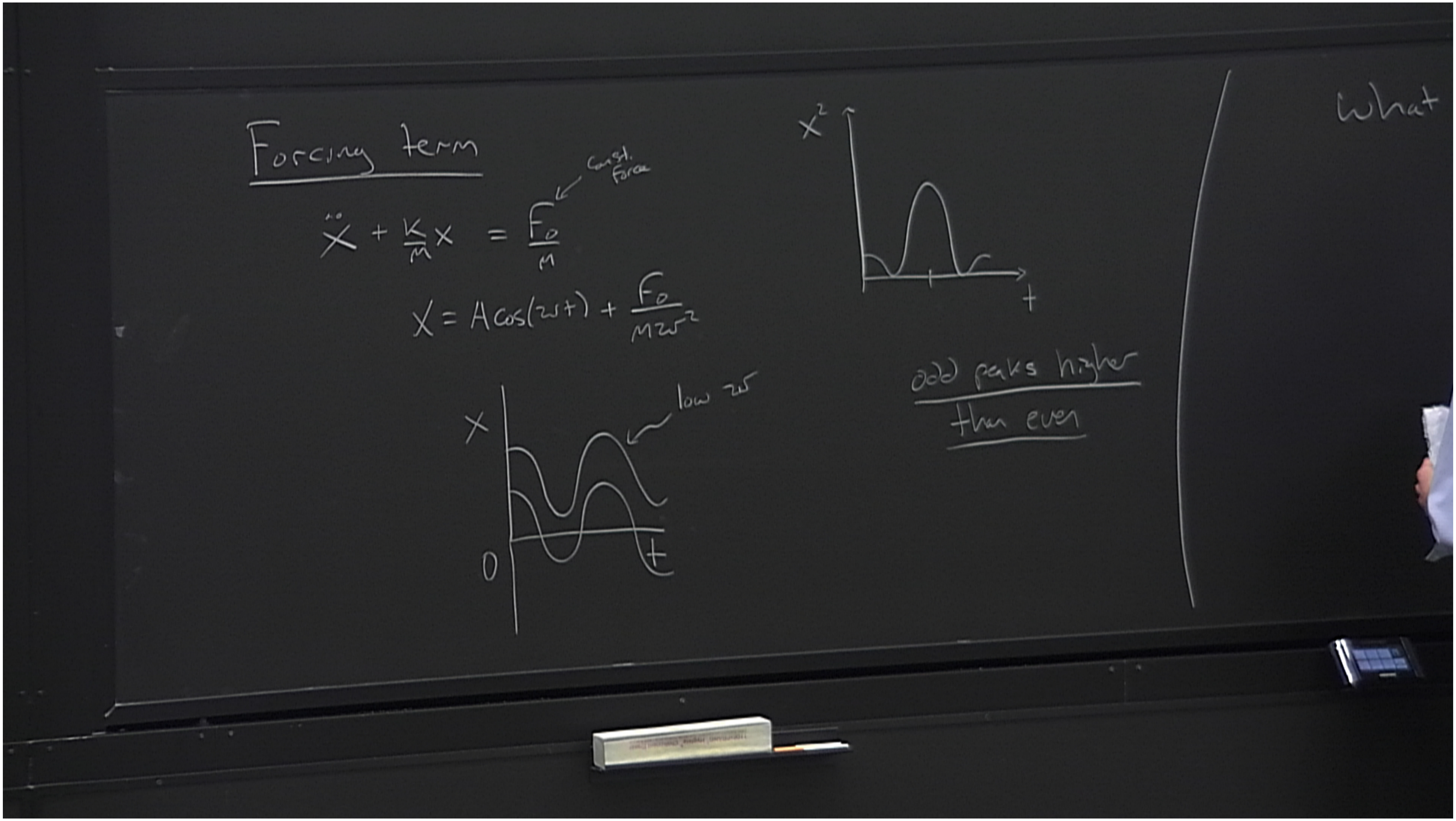
Fix η : Peaks

$$\boxed{k_p = \frac{n\pi}{r_s} \qquad n = 1, 2, 3, \ldots}$$

Baryon Acoustic Oscillations




Forcing term
const. force
$\ddot{x} + \frac{k}{m}x = \frac{F_0}{m}$
$x = A\cos(\omega t) + \frac{F_0}{m\omega^2}$
low ω
x
0
t
x^2
t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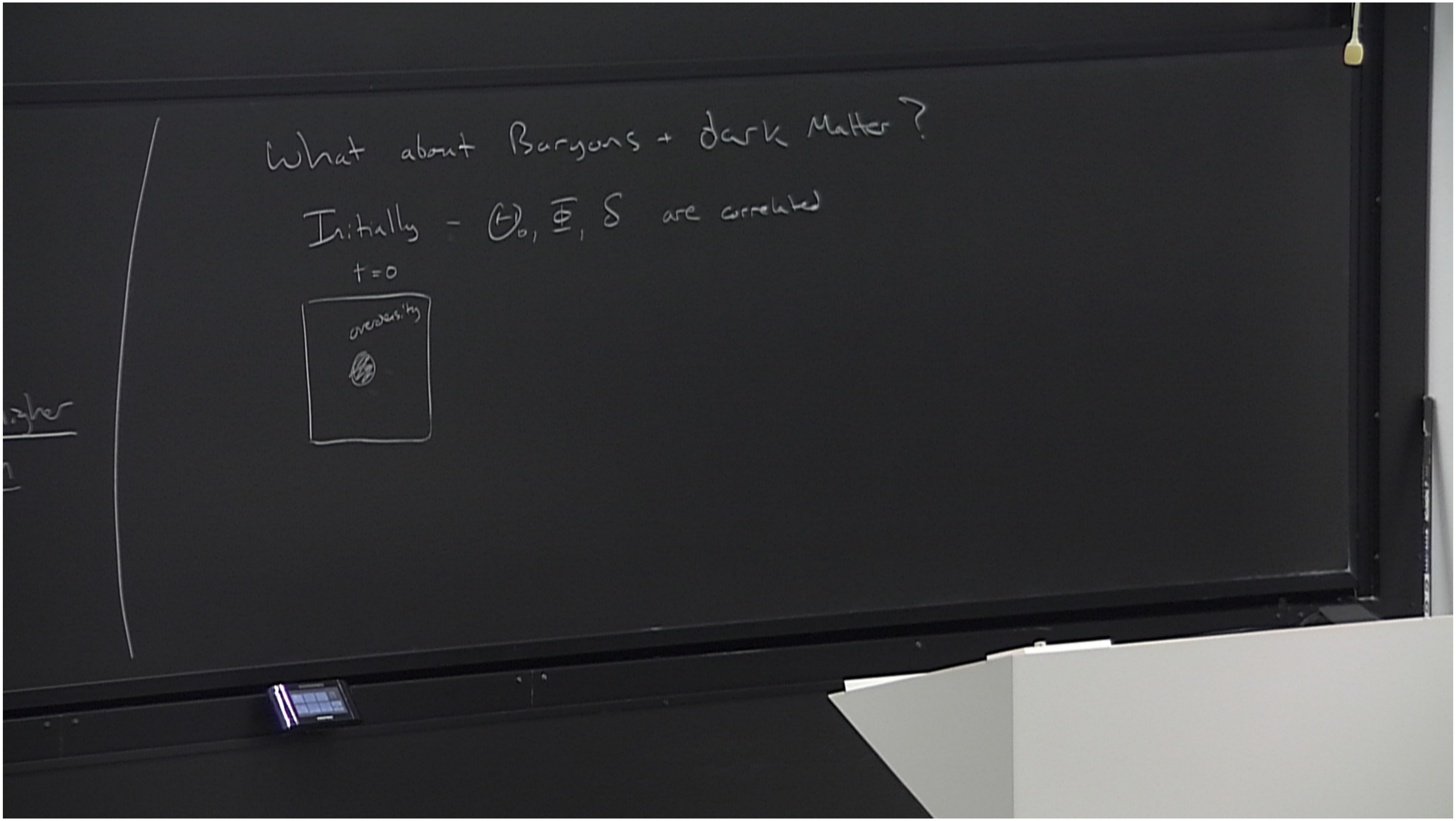
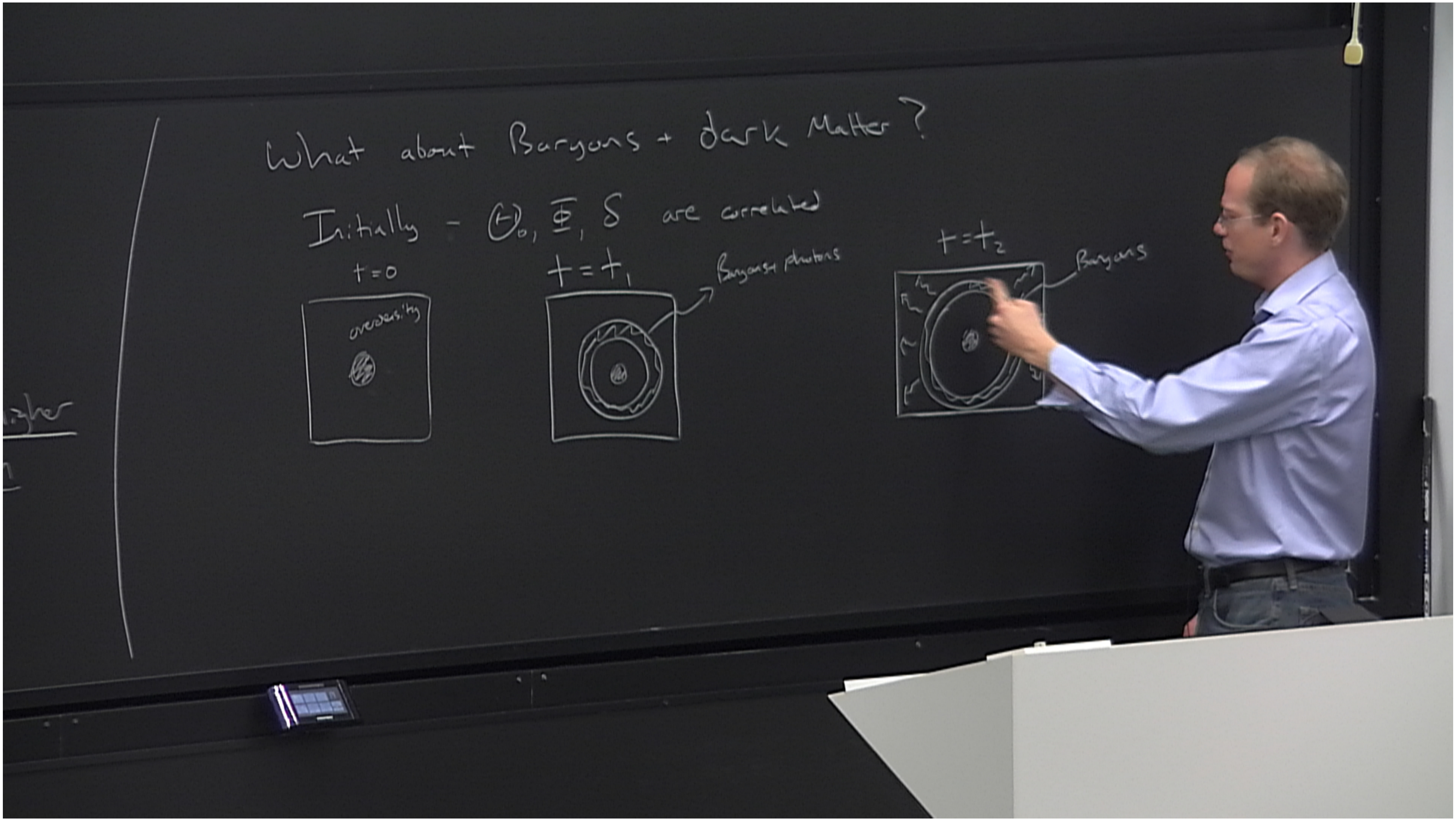
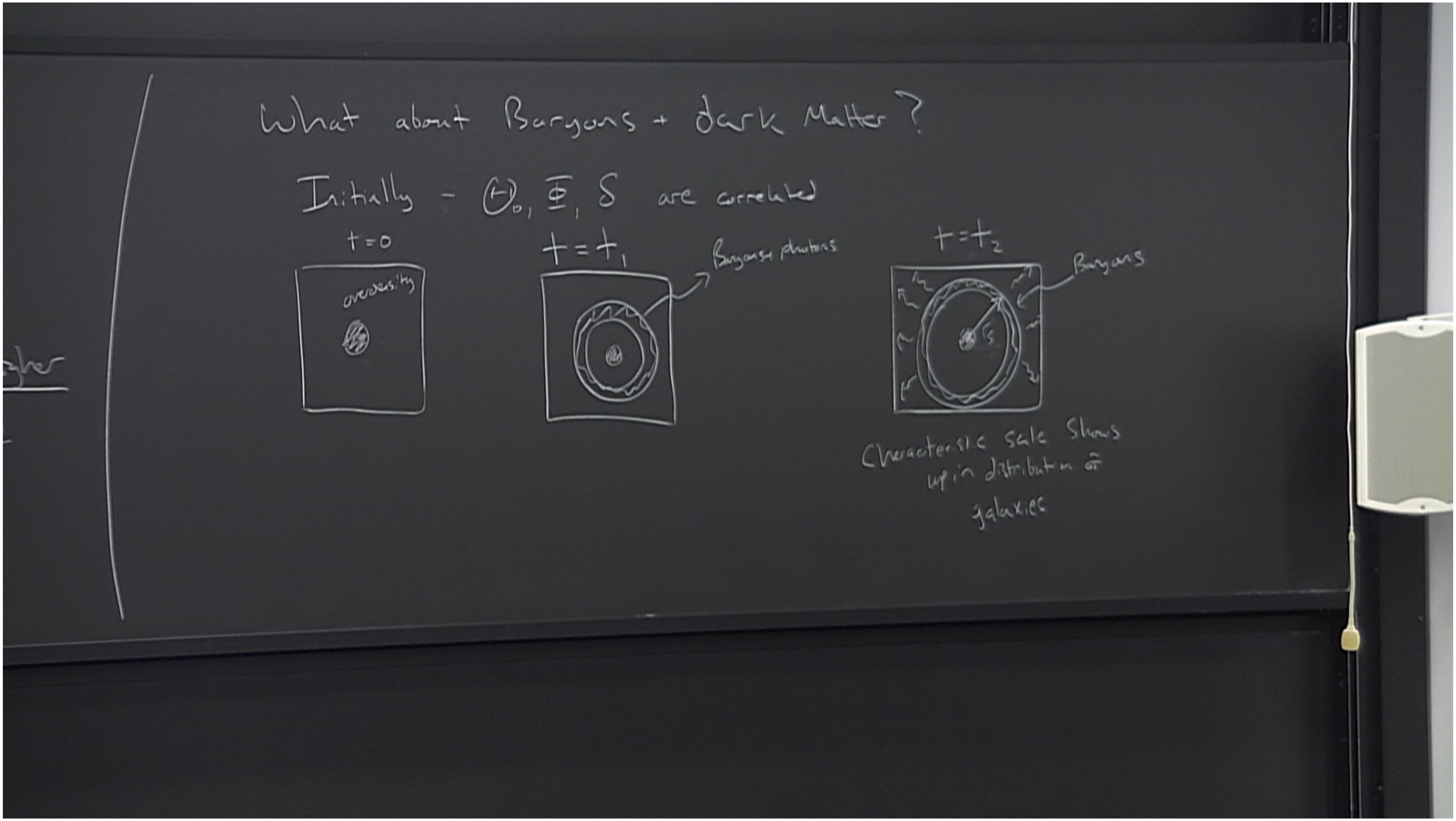
$x = A\cos(\omega t) + \frac{F_0}{m\omega^2}$
low ω
x
0
t
x^2
t
odd peaks higher
than even
What

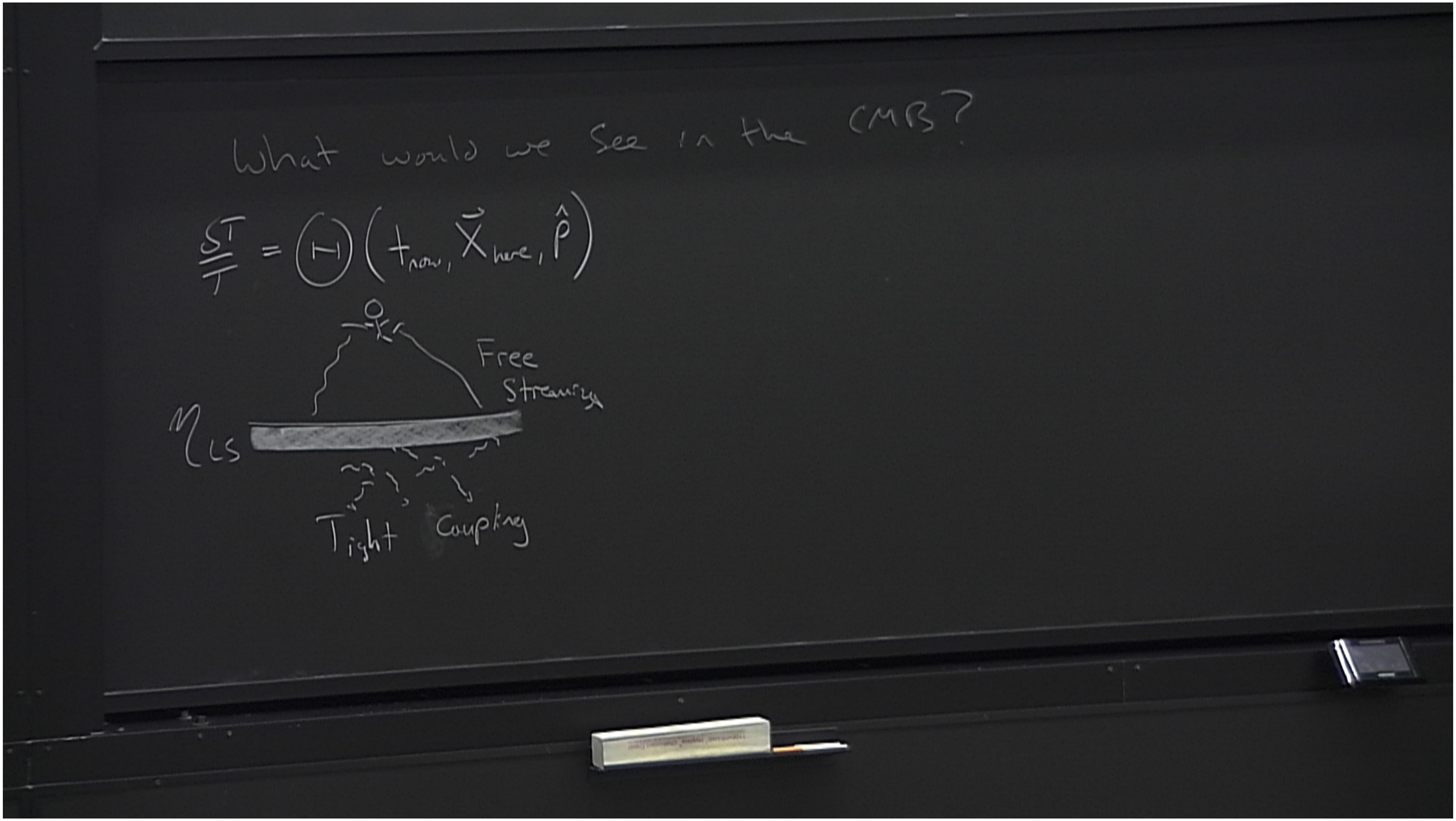

What about Baryons + dark Matter?
Initially – Θ_0, Φ, δ are correlated
$t=0$
overdensity

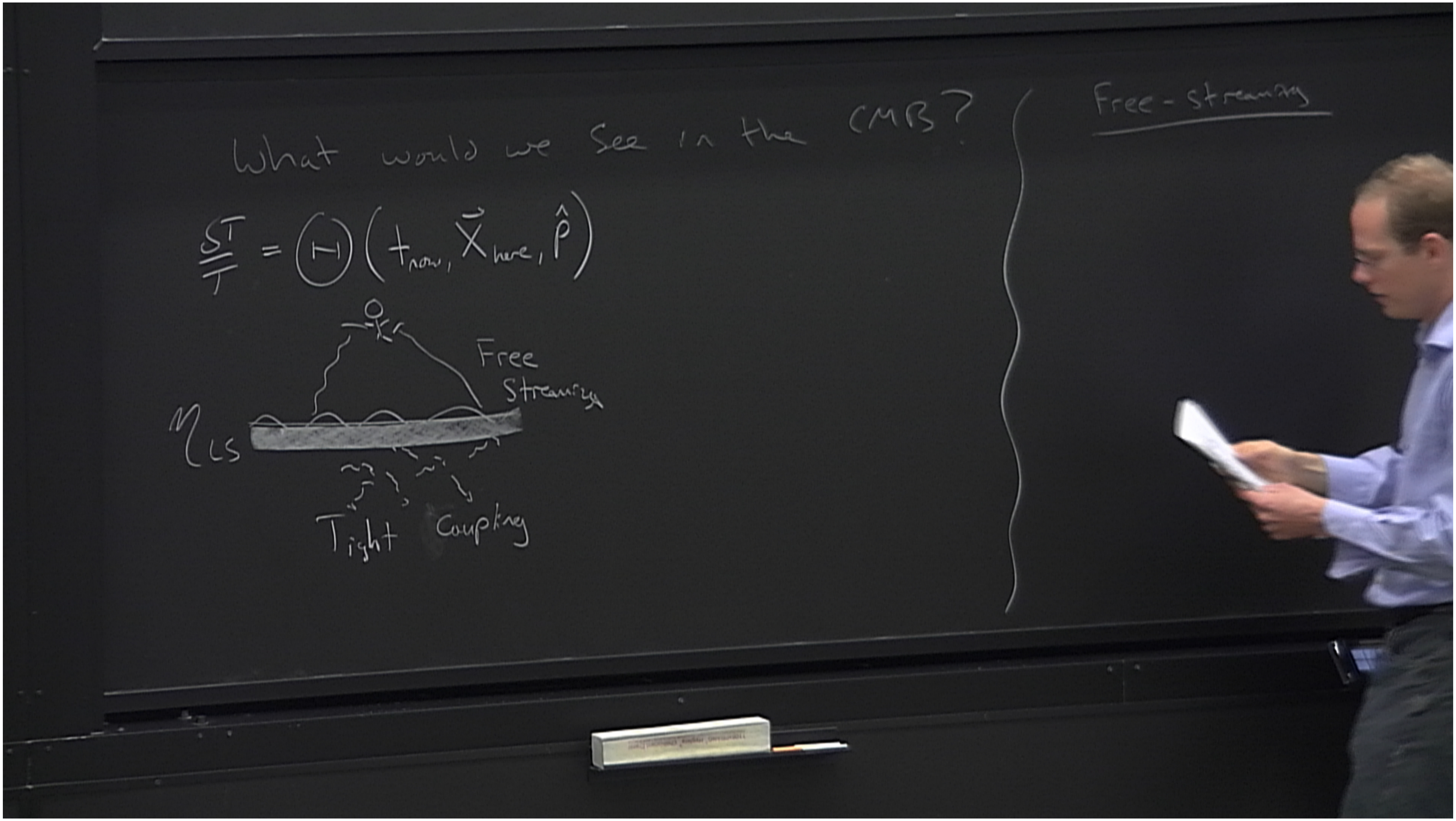

What about Baryons + dark Matter?
Initially - Θ_0, Φ, δ are correlated
t=0
overdensity
t=t1
Baryons+ photons
t=t2
Baryons

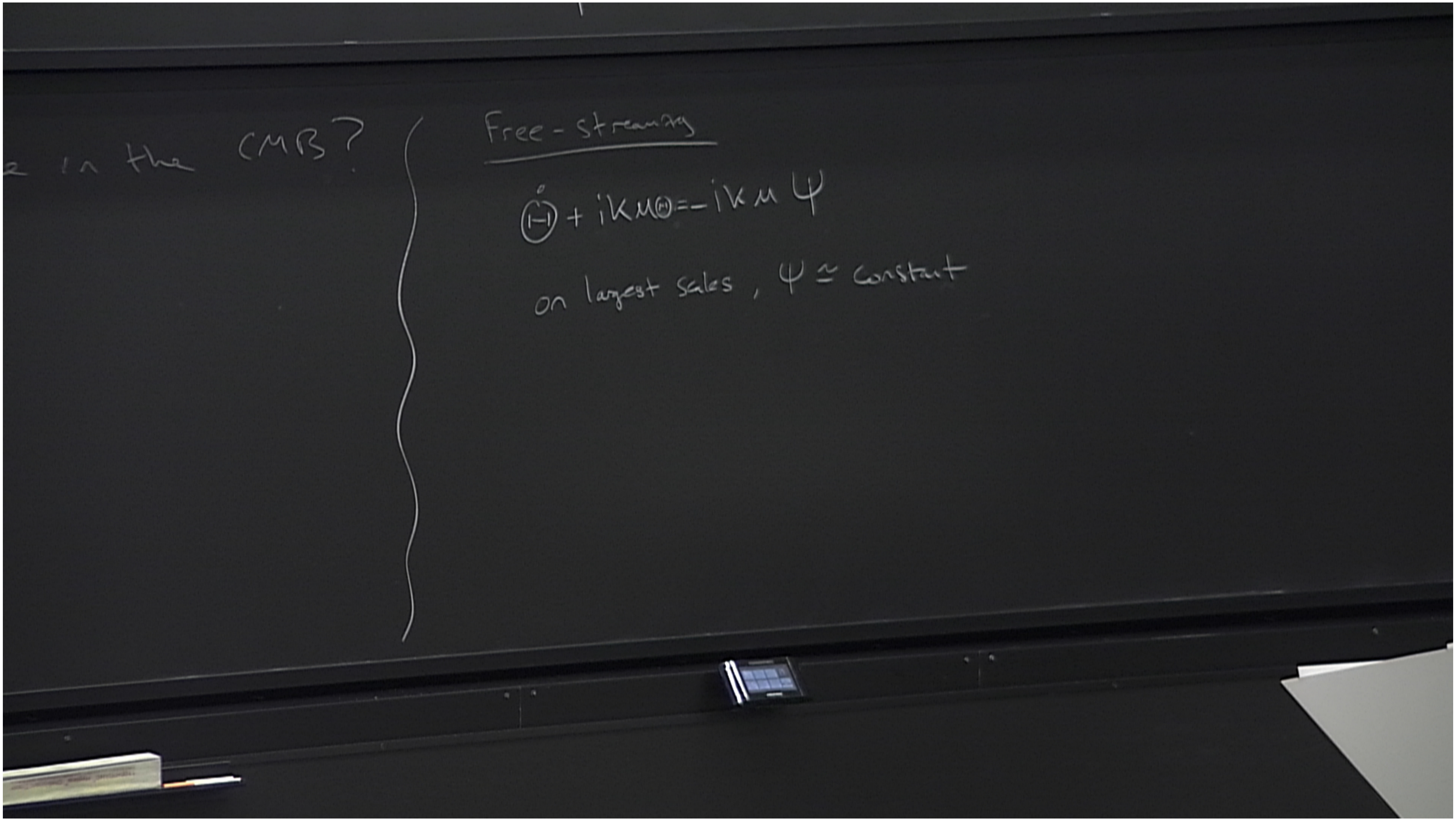

What about Baryons + dark Matter ?
Initially - Θ_0, Φ_1, δ are correlated
t = 0
overdensity
t = t1
Baryons + photons
t = t2
Baryons
Characteristic scale shows up in distribution of galaxies

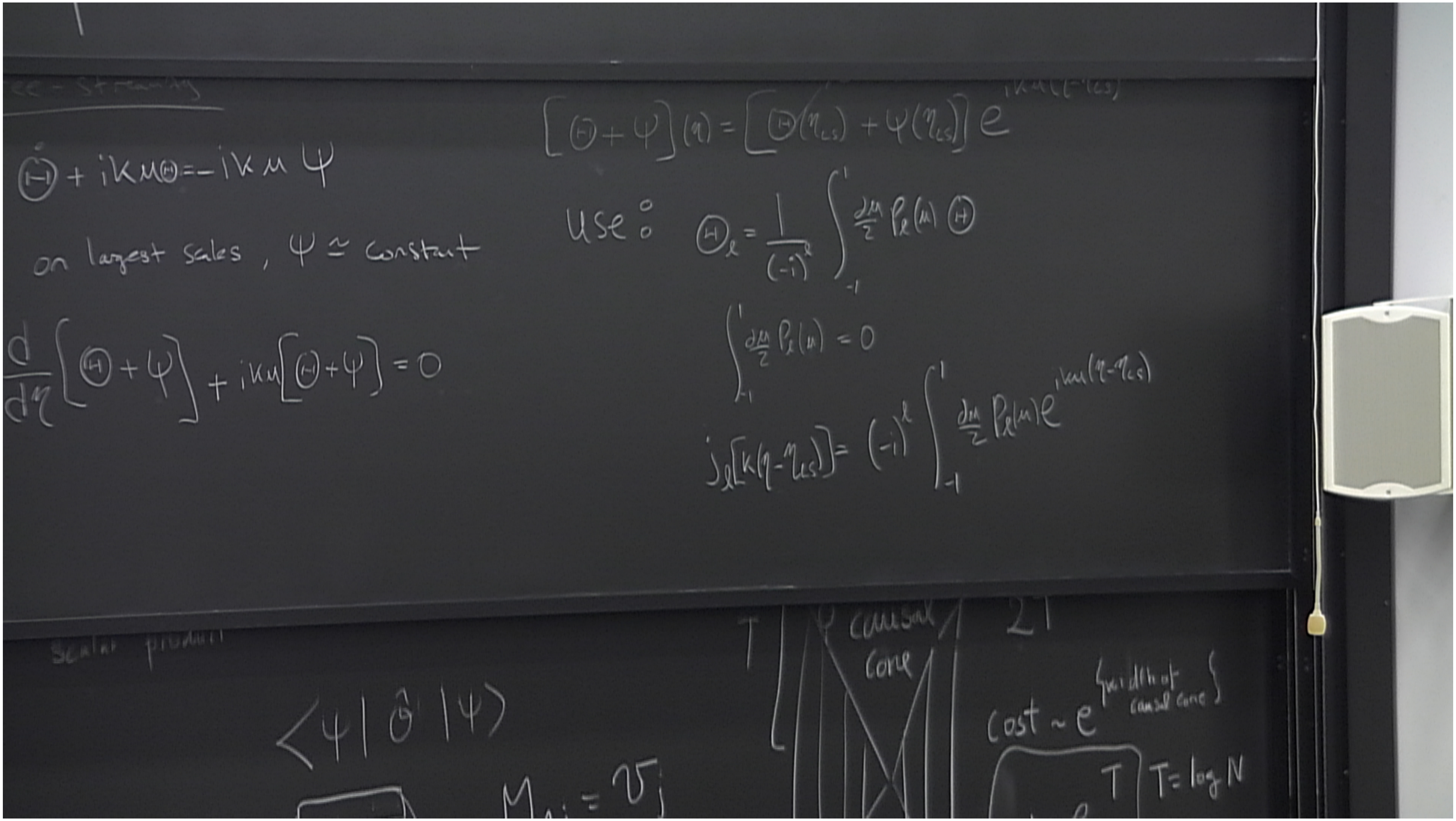
What would we see in the CMB?

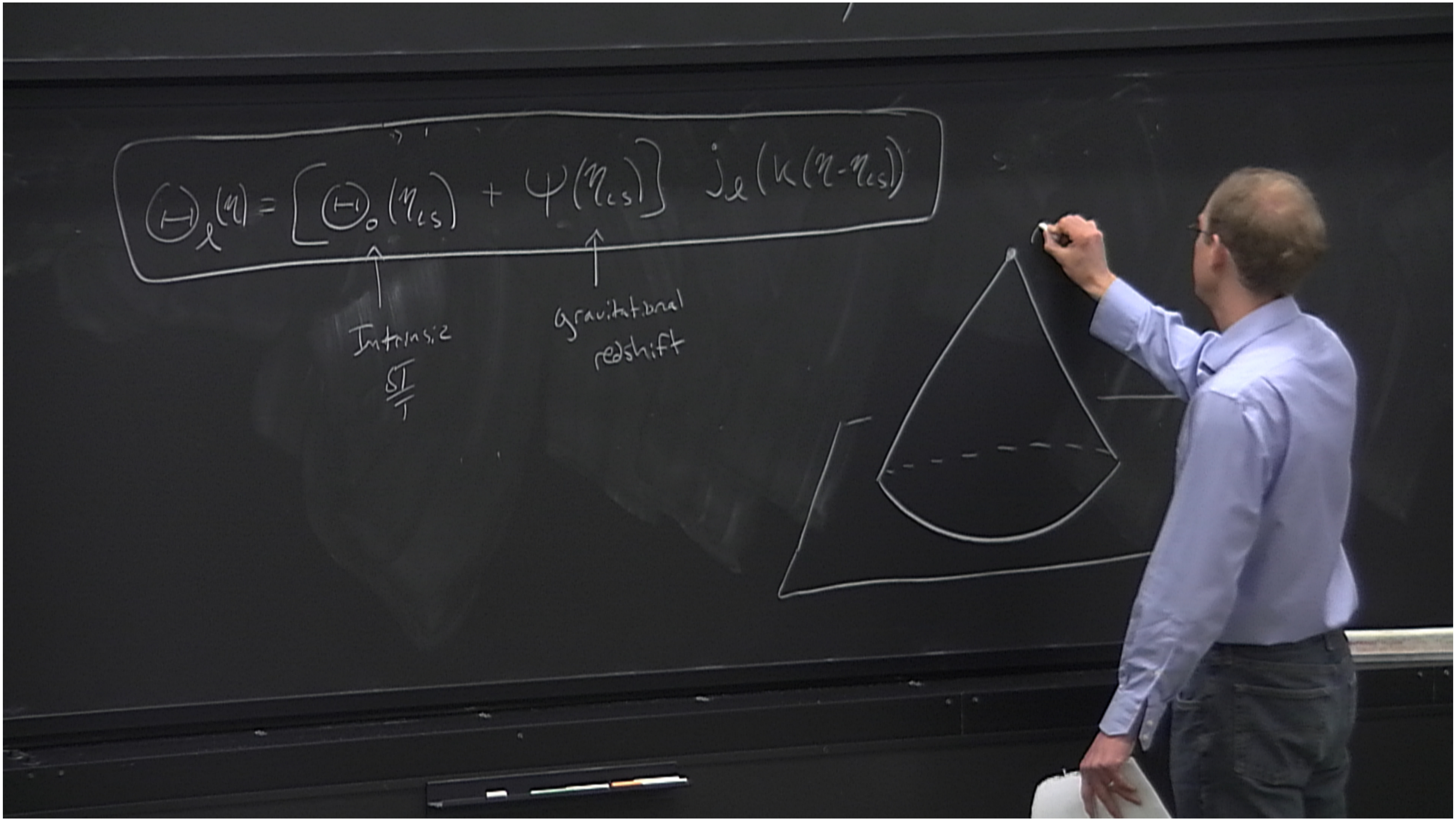
$$\frac{\delta T}{T} = \Theta\left(t_{now}, \vec{X}_{here}, \hat{p}\right)$$

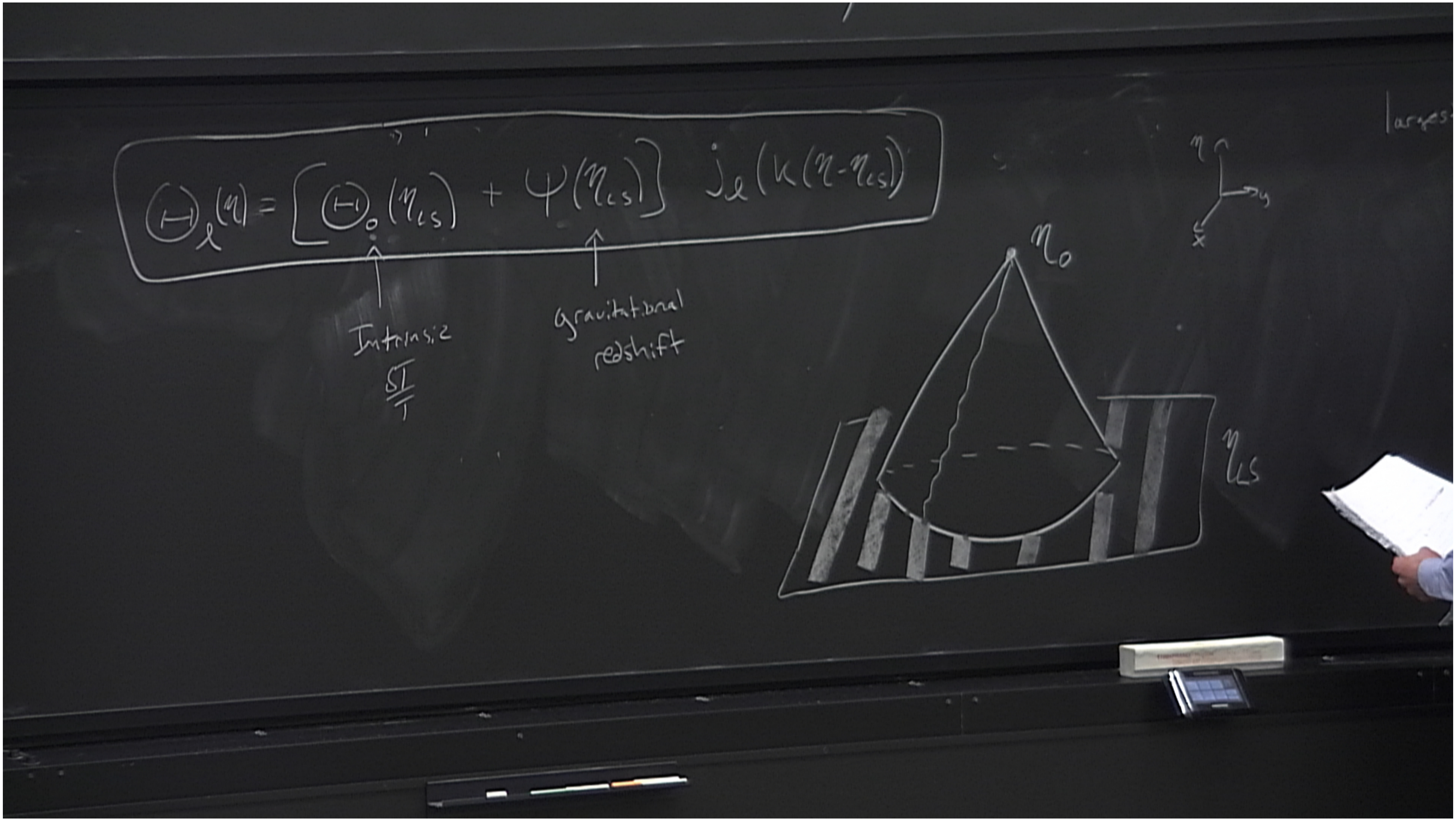

What would we see in the CMB?
$\frac{\delta T}{T} = \Theta(t_{now}, \vec{X}_{here}, \hat{p})$
Free Streaming
η_{LS}
Tight Coupling


What would we see in the CMB?
δT/T = Θ(t_now, X_here, p̂)
Free Streaming
η_LS
Tight Coupling
Free-streaming


e in the CMB?
Free-streaming
$\dot{\Theta} + ik\mu\Theta = -ik\mu\Psi$
on largest sales, $\Psi \simeq$ constant

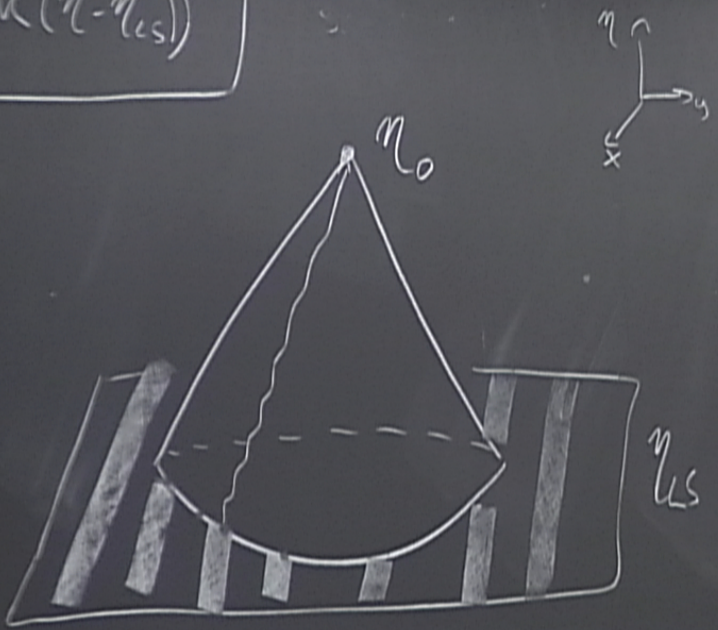

$[\Theta + \Psi](\eta) = [\Theta(\eta_{LS}) + \Psi(\eta_{LS})] e^{ik\mu(\eta-\eta_{LS})}$
$\dot{\Theta} + ik\mu\Theta = -ik\mu\Psi$
use:
$\Theta_\ell = \frac{1}{(-i)^\ell}\int_{-1}^{1}\frac{d\mu}{2}P_\ell(\mu)\Theta$
on largest scales, $\Psi \simeq$ constant
$\int_{-1}^{1}\frac{d\mu}{2}P_\ell(\mu) = 0$
$\frac{d}{d\eta}[\Theta + \Psi] + ik\mu[\Theta + \Psi] = 0$
$j_\ell[k(\eta-\eta_{LS})] = (-i)^\ell\int_{-1}^{1}\frac{d\mu}{2}P_\ell(\mu)e^{ik\mu(\eta-\eta_{LS})}$
causal cone
$\langle\psi|\hat{O}|\psi\rangle$
cost ~ e^{width of causal cone}
T = log N


$\Theta_\ell(\eta) = [\Theta_0(\eta_{cs}) + \Psi(\eta_{cs})] \, j_\ell(k(\eta-\eta_{cs}))$
Intrinsic
$\frac{\delta T}{T}$
gravitational
redshift

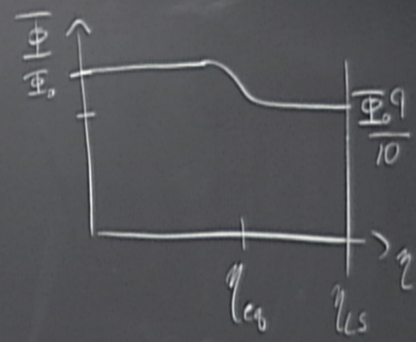


Θ_ℓ(η) = [Θ_0(η_LS) + Ψ(η_LS)] j_ℓ(k(η - η_LS))
Intrinsic
δT/T
gravitational
redshift
η_0
η_LS
η
y
x
largest

$\boxed{(k(\eta-\eta_{LS}))}$



Largest Scales (Superhorizon @ LS) $(\Phi = -\Psi)$



$$\dot{\Theta}_0 = -\dot{\Phi}$$

$$\Rightarrow \Theta_0(\eta) = -\Phi(\eta) + \text{const}$$

$$\Theta_0(\eta \to 0) = \frac{\Phi_0}{2} \Rightarrow \text{const} = \frac{3\Phi_0}{2}$$

What about Baryons + dark Matter?

Initially - Θ_0, Φ, δ are correlated

$t=0$ $\quad$ $t=t_1$ $\quad$ Baryons+photons $\quad$ $t=t_2$

$$\dot{\delta} + ikv = -3\dot{\Phi}$$

$$\dot{v} + \frac{\dot{a}}{a} v = -ik\Psi$$

$$3\frac{\dot{a}}{a}\left(\dot{\Phi} - \frac{\dot{a}}{a}\Psi\right) = 4\pi G a^2(\rho_m + \rho_r \cdots)$$

$V_b \simeq -3i\Theta_1$

① eliminate V_b in favor o

② $\frac{d(\dot{\Theta}_0 = }{d\eta}$

③ use eq-n for Θ_1

$$\left[\frac{d^2}{d\eta^2} + \frac{\dot{R}}{1+R}\frac{d}{d\eta} + k^2 c_s^2\right]\left[\Theta_0 + \Phi\right] = \frac{k^2}{3}\left[\frac{1}{1+R}\Phi - \Psi\right]$$

$$R = \frac{3\rho_b}{4\rho_\gamma}, \quad c_s = \frac{1}{\sqrt{3(1+R)}}$$

For slowly varying c_s, solution to homogeneous eq-n

CMB:

$$\Theta_0(\eta_{ls}) = -\Phi(\eta_{ls}) + \frac{3}{2}\Phi_0$$

$$= -\Phi(\eta_{ls}) + \frac{3}{2}\frac{10}{9}\Phi(\eta_{ls})$$

$$= \frac{2}{3}\Phi(\eta_{ls})$$

$$\boxed{\Theta_\ell = \left[\underbrace{\cancel{\tfrac{2}{3}\Phi(\eta_{ls})} - \Phi(\eta_{ls})}_{-\frac{1}{3}\Phi(\eta_{ls})}\right] j_\ell\left[k(\eta-\eta_{ls})\right]}$$

Sachs-Wolfe

Free-streaming

$$\dot{\Theta} + ik\mu\Theta = -ik\mu\Psi$$

on largest scales, $\Psi \simeq$

$$\Rightarrow \frac{d}{d\eta}\left[\Theta + \Psi\right] + ik\mu\left[\Theta + \Psi\right]$$

$$\Theta_0(\eta_{ls}) = -\Phi(\eta_{ls}) + \frac{3}{2}\Phi_0$$

$$= -\Phi(\eta_{ls}) + \frac{3}{2}\frac{10}{9}\Phi(\eta_{ls})$$

$$= \frac{2}{3}\Phi(\eta_{ls})$$

$$\Theta_\ell = \Big[\underbrace{\tfrac{2}{3}\Phi(\eta_{ls}) - \Phi(\eta_{ls})}_{-\frac{1}{3}\Phi(\eta_{ls})}\Big]\, j_\ell\big[k(\eta-\eta_{ls})\big]$$

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