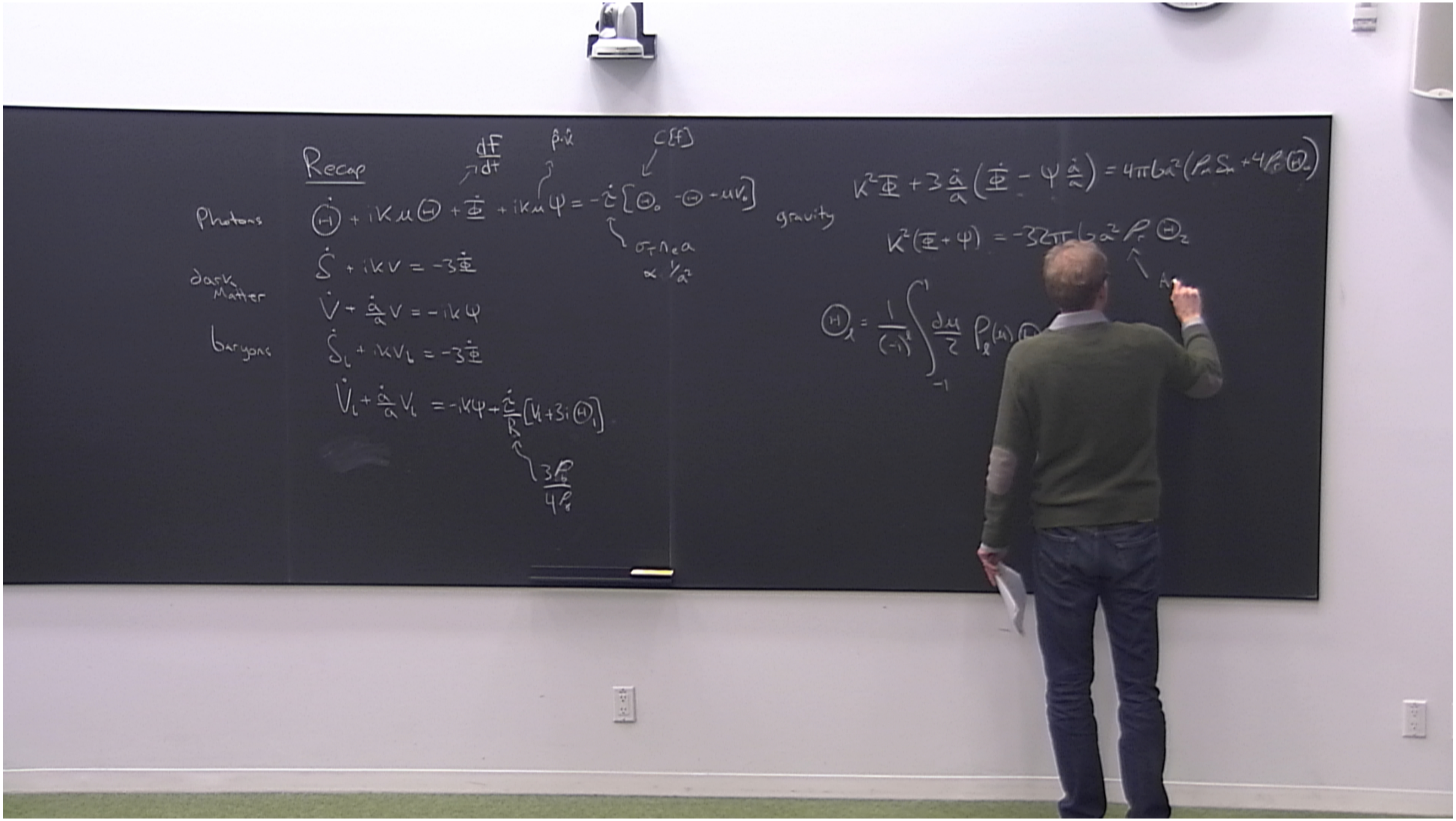
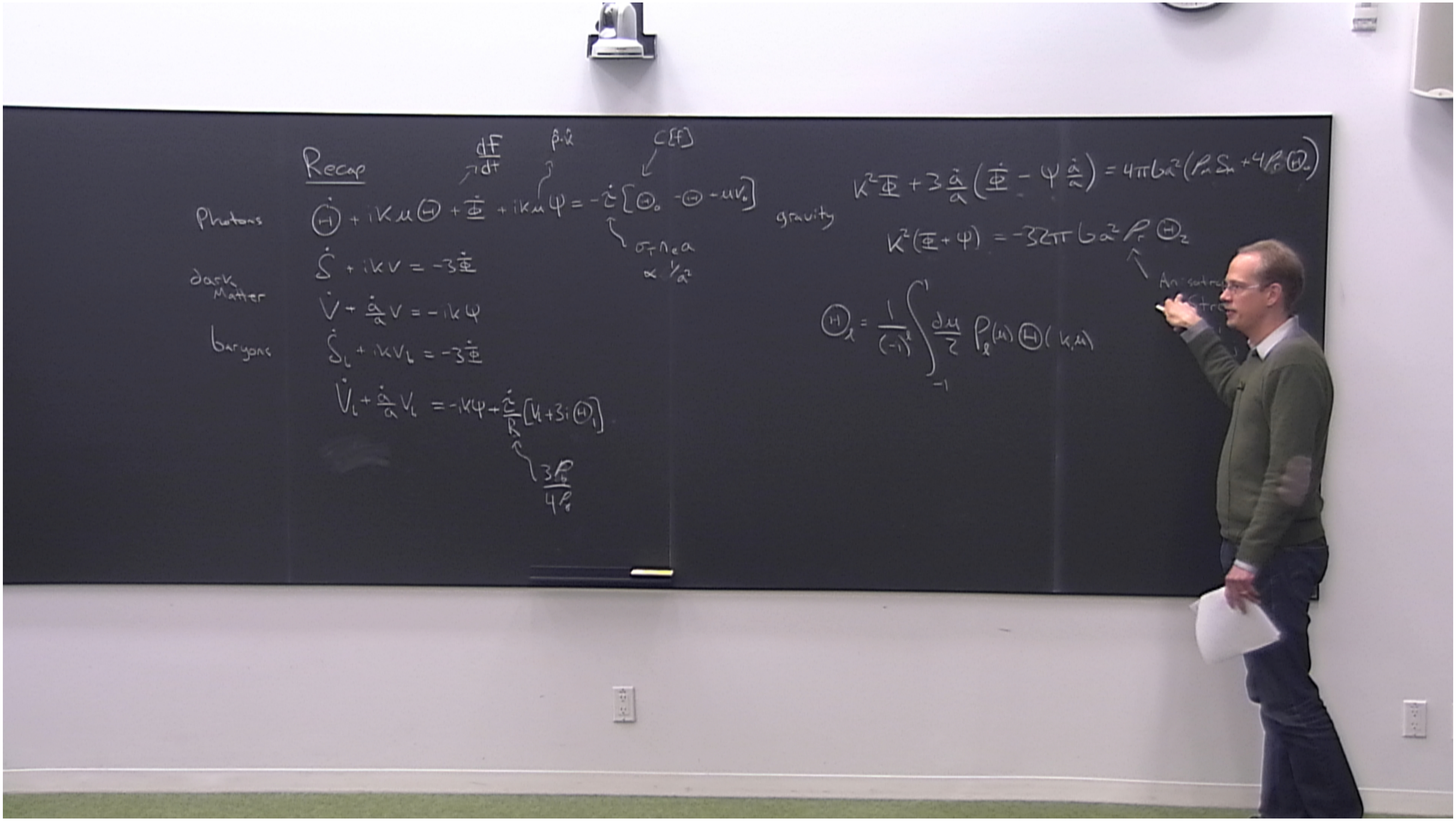
Title: 12/13 PSI - Explorations in Cosmology Lecture 5

Date: Apr 12, 2013 10:15 AM

URL: http://pirsa.org/13040066

Abstract:


Recap
Photons
dark Matter
baryons
gravity
$\dot{\Theta} + ik\mu\Theta + \dot{\Phi} + ik\mu\Psi = -\dot{\tau}[\Theta_0 - \Theta - \mu v_b]$
$\frac{df}{dt}$
$C[f]$
$\sigma_T n_e a$
$\propto 1/a^2$
$\dot{\delta} + ikv = -3\dot{\Phi}$
$\dot{v} + \frac{\dot{a}}{a}v = -ik\Psi$
$\dot{\delta}_b + ikv_b = -3\dot{\Phi}$
$\dot{v}_b + \frac{\dot{a}}{a}v_b = -ik\Psi + \frac{\dot{\tau}}{R}[v_b + 3i\Theta_1]$
$\frac{3\rho_b}{4\rho_\gamma}$
$k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G a^2(\rho_m \delta_m + 4\rho_r\Theta_0)$
$k^2(\Phi + \Psi) = -32\pi G a^2 \rho_r \Theta_2$
$\Theta_\ell = \frac{1}{(-i)^\ell}\int_{-1}^{1}\frac{d\mu}{2}P_\ell(\mu)\Theta$


Recap
$\frac{dF}{dt}$
$\hat{p}\cdot\hat{k}$
$C[f]$
Photons
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$\sigma_T n_e a$
$\propto 1/a^2$
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baryons
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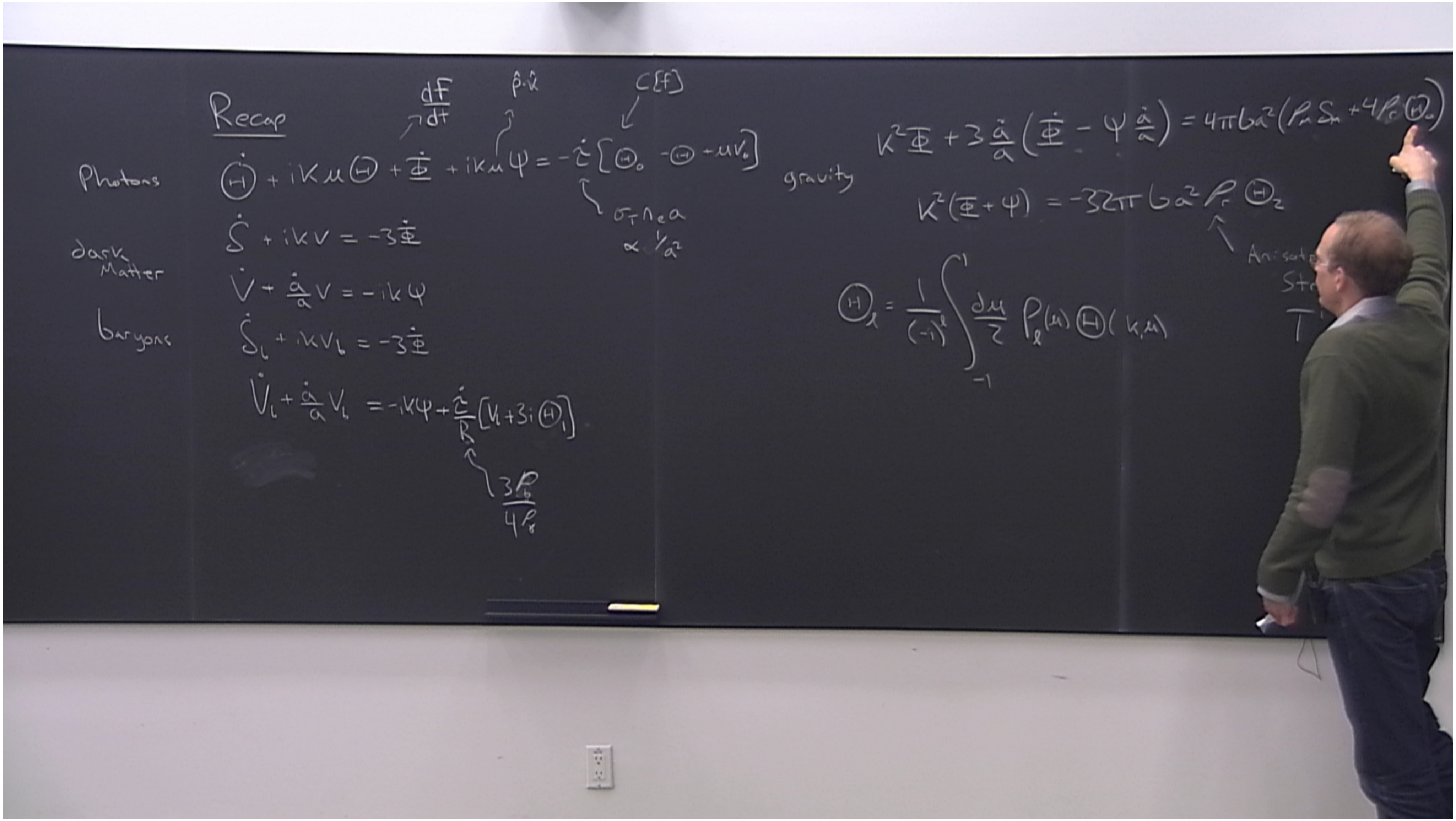
Recap
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$\propto 1/a^2$
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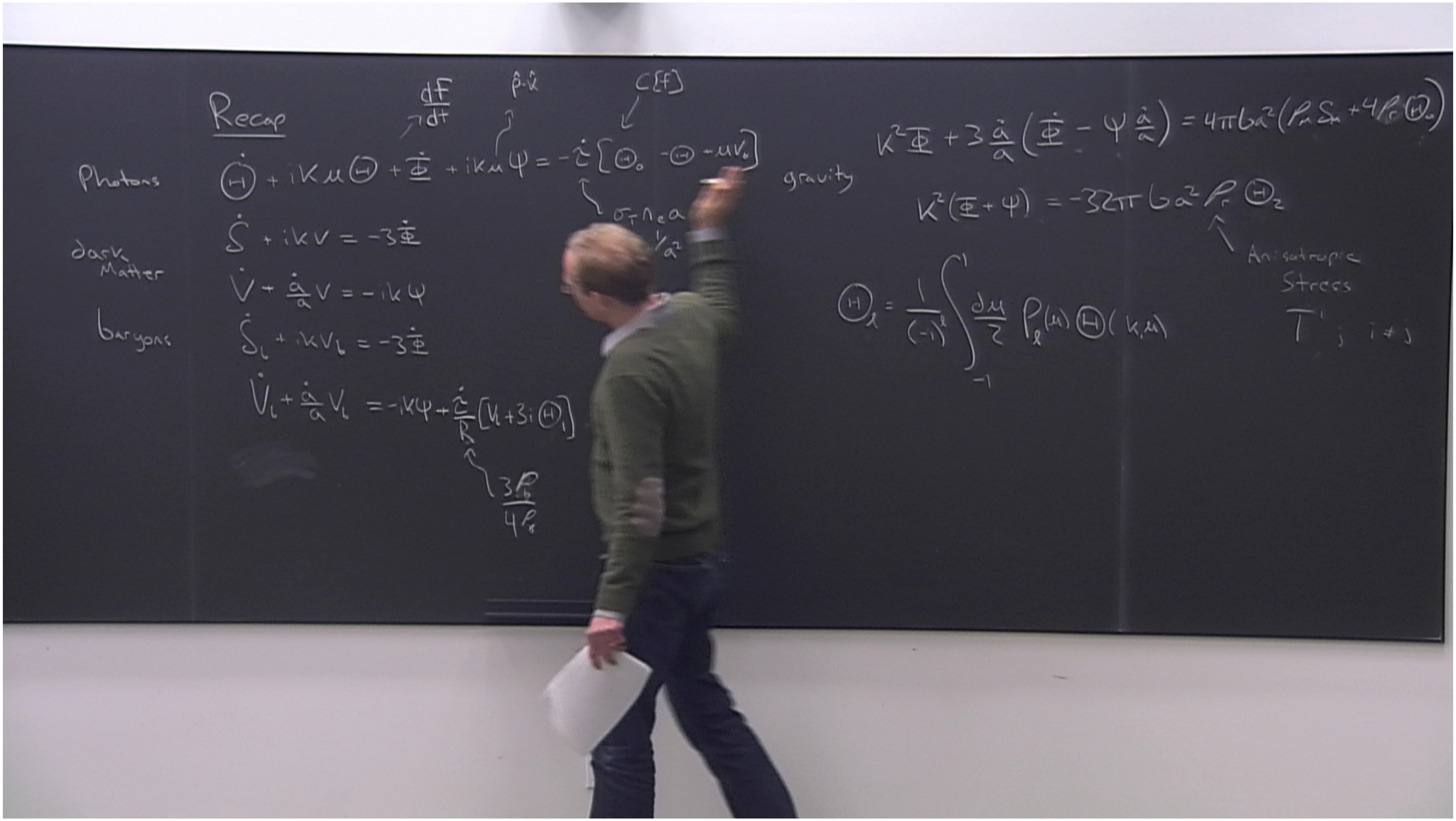
$k^2(\Phi + \Psi) = -32\pi G a^2\rho_r\Theta_2$
$\Theta_l = \frac{1}{(-i)^l}\int_{-1}^{1}\frac{d\mu}{2}P_l(\mu)\Theta(k,\mu)$


Recap
dF/dt
p̂·k̂
C[f]
Photons
Θ̇ + ikμΘ + Φ̇ + ikμΨ = −τ̇ [Θ₀ − Θ − μV_b]
σ_T n_e a
1/a²
gravity
dark Matter
δ̇ + ikV = −3Φ̇
V̇ + (ȧ/a)V = −ikΨ
baryons
δ̇_b + ikV_b = −3Φ̇
V̇_b + (ȧ/a)V_b = −ikΨ + (τ̇/R)[V_b + 3iΘ₁]
3ρ_b/4ρ_γ
k²Φ + 3(ȧ/a)(Φ̇ − Ψ ȧ/a) = 4πGa²(ρ_m δ_m + 4ρ_r Θ₀)
k²(Φ + Ψ) = −32πGa² ρ_r Θ₂
Anisotropic Stress
Θ_l = 1/(−i)^l ∫_{−1}^{1} dμ/2 P_l(μ) Θ(k,μ)
T^i_j i≠j

Recap

Photons

$$\dot{\Theta} + ik\mu\Theta + \dot{\Phi} + ik\mu\Psi = -\dot{\tau}\left[\Theta_0 - \Theta - \mu v_b\right]$$

$\frac{df}{dt}$ $\quad \hat{p}\cdot\hat{k}$ $\quad C[f]$

$\dot{\tau}$: $\sigma_T n_e a \propto 1/a^2$

dark Matter

$$\dot{\delta} + ikv = -3\dot{\Phi}$$

$$\dot{v} + \frac{\dot{a}}{a}v = -ik\Psi$$

baryons

$$\dot{\delta}_b + ikv_b = -3\dot{\Phi}$$

$$\dot{v}_b + \frac{\dot{a}}{a}v_b = -ik\Psi + \frac{\dot{\tau}}{R}\left[v_b + 3i\Theta_1\right]$$

$$R = \frac{3\rho_b}{4\rho_\gamma}$$

gravity

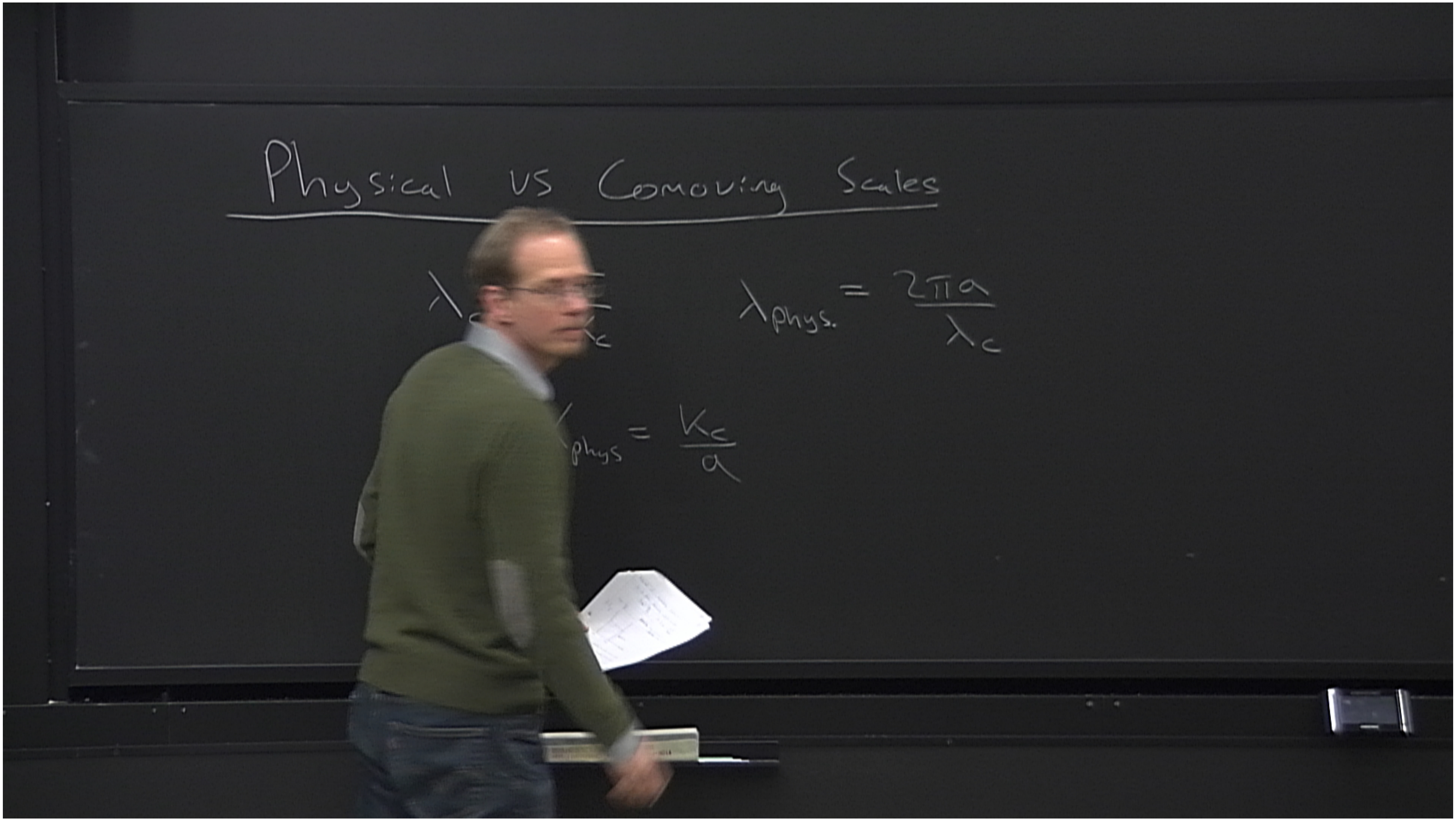
$$k^2\Phi + 3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G a^2\left(\rho_m\delta_m + 4\rho_r\Theta_0\right)$$

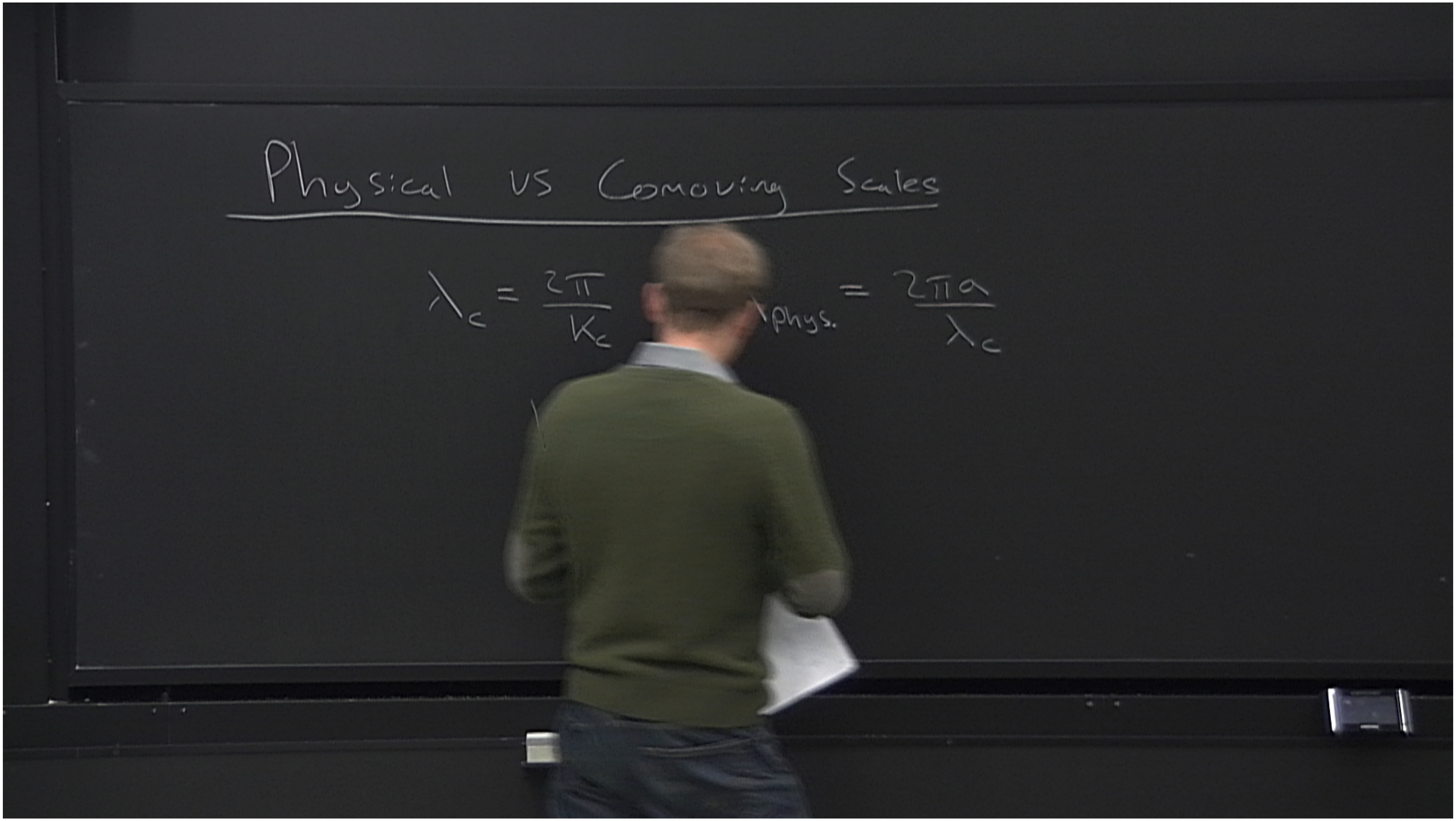
$$k^2(\Phi + \Psi) = -32\pi G a^2 \rho_r \Theta_2$$

Anisotropic Stress

$T^i{}_j \quad i \neq j$

$$\Theta_\ell = \frac{1}{(-i)^\ell}\int_{-1}^{1}\frac{d\mu}{2}P_\ell(\mu)\,\Theta(k,\mu)$$


Physical vs Comoving Scales
$\lambda_{phys.} = \frac{2\pi a}{\lambda_c}$
$k_{phys} = \frac{k_c}{a}$

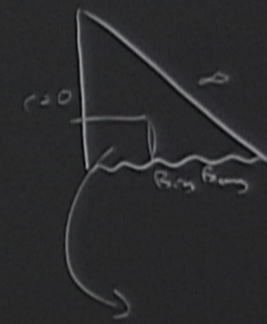

Physical vs Comoving Scales
$\lambda_c = \frac{2\pi}{K_c}$
phys. $= \frac{2\pi a}{\lambda_c}$

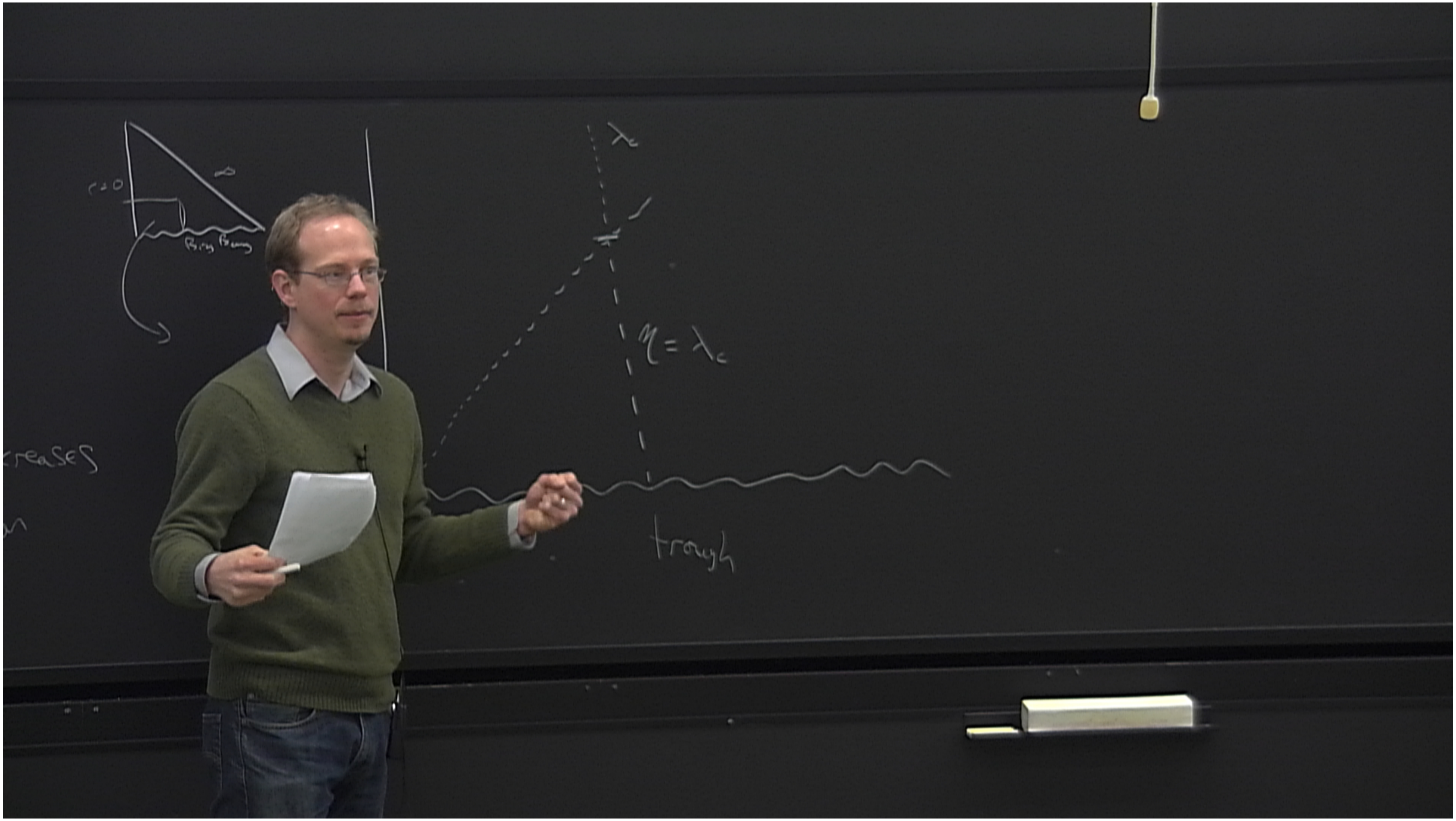
Physical vs Comoving Scales

$$\lambda_c = \frac{2\pi}{K_c} \qquad \lambda_{phys.} = \frac{2\pi a}{\lambda_c}$$

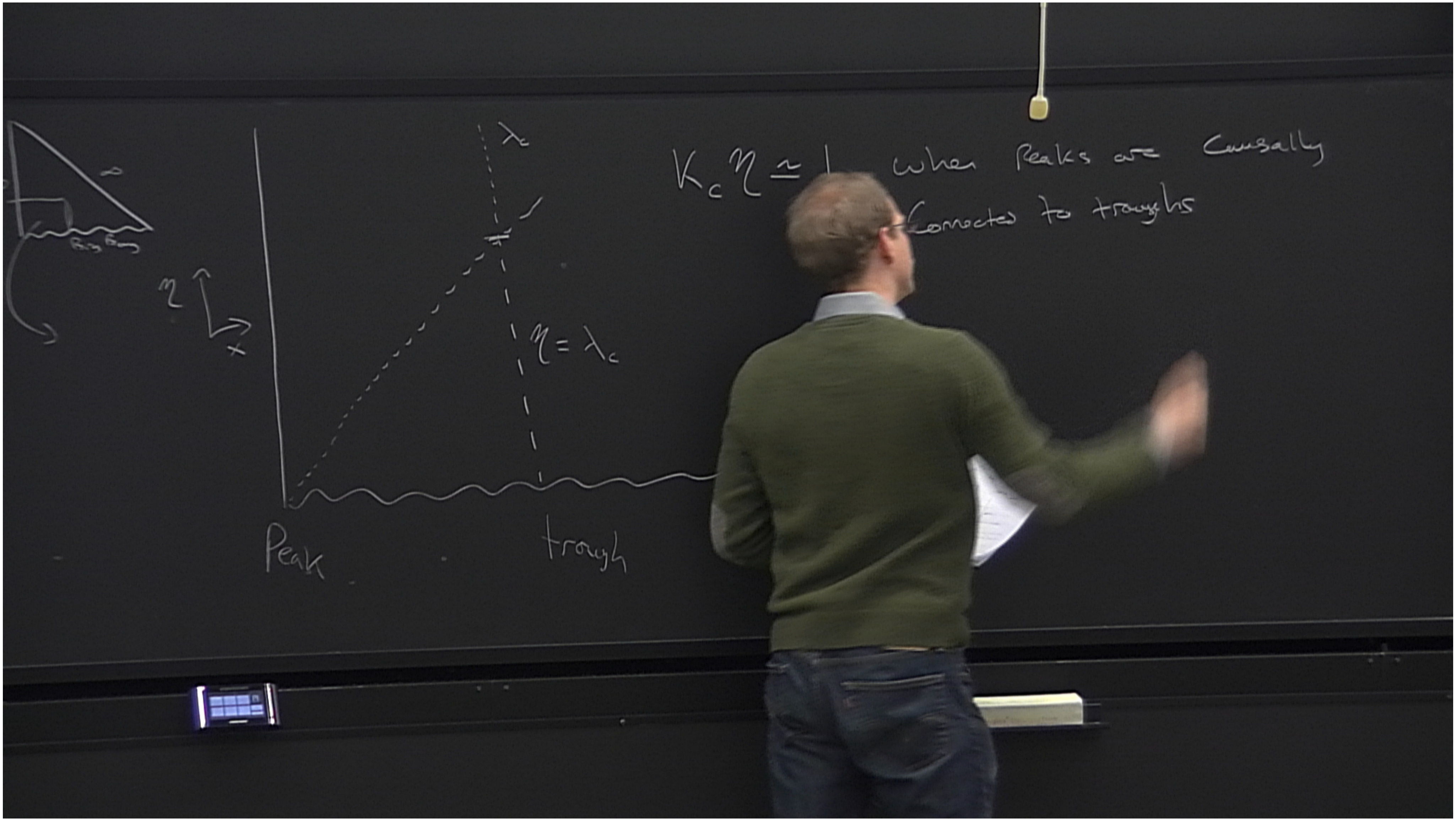
$$K_{phys} = \frac{K_c}{a}$$

fixed K_c decreases
w/ expansion

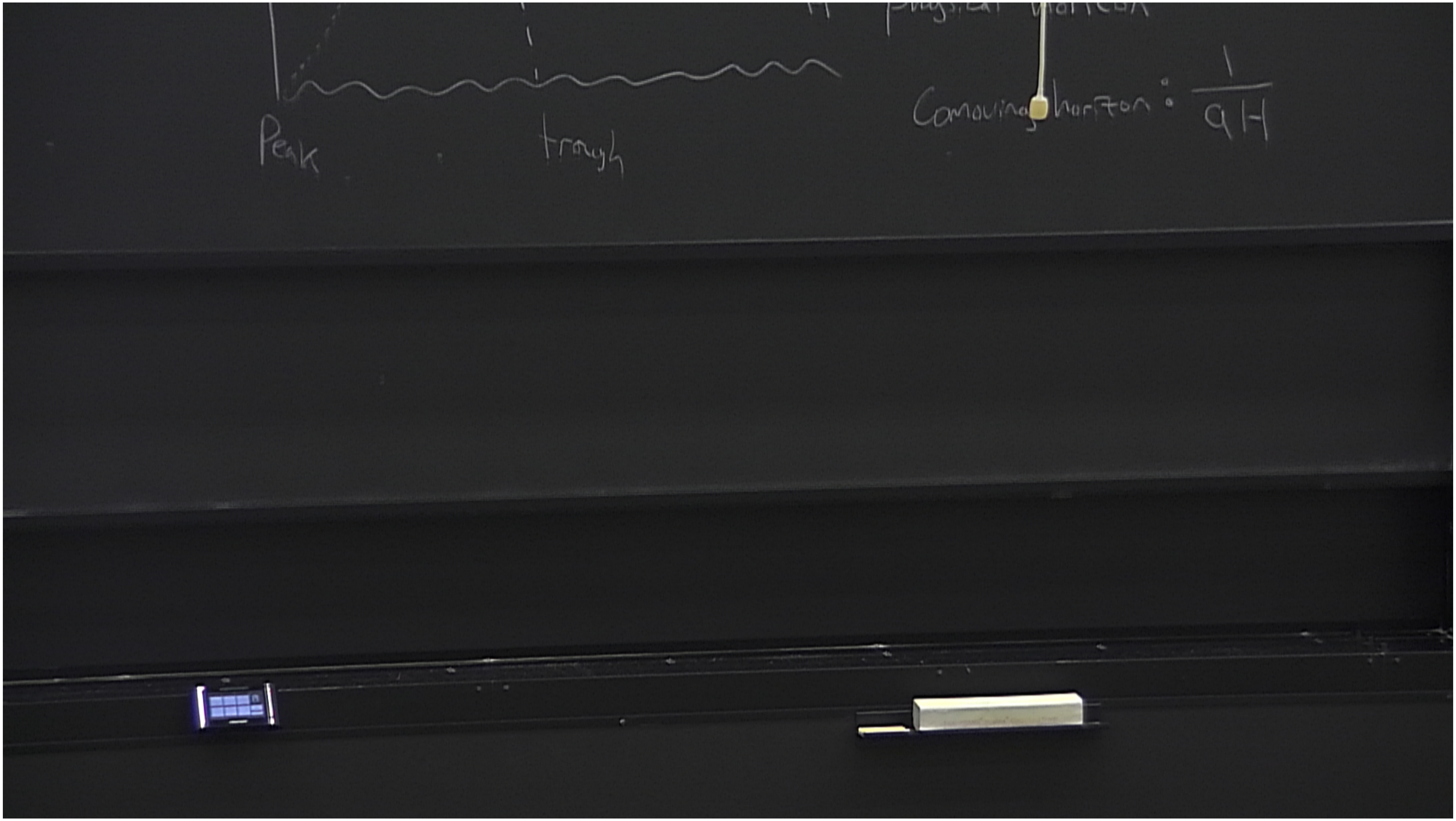


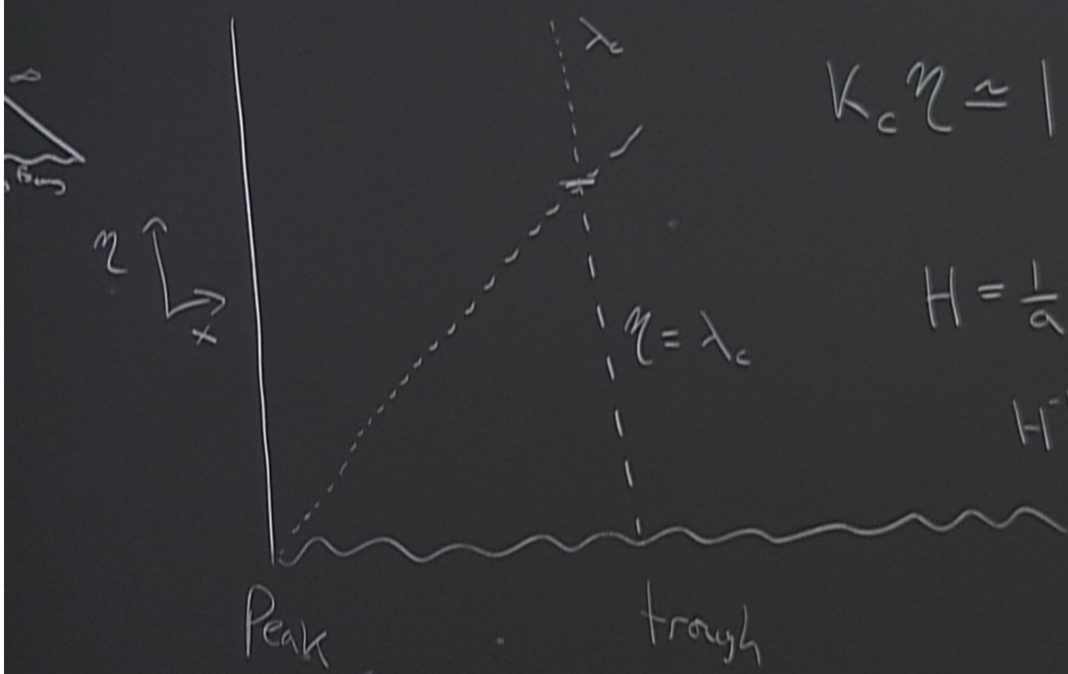


Big Bang
$\eta = \lambda_c$
λ_c
trough



Big Bang
η
x
λ_c
$\eta = \lambda_c$
Peak
trough
$K_c \eta \simeq 1$ when Peaks are Causally
Connected to troughs


Peak
trough
Comoving horizon : 1/aH



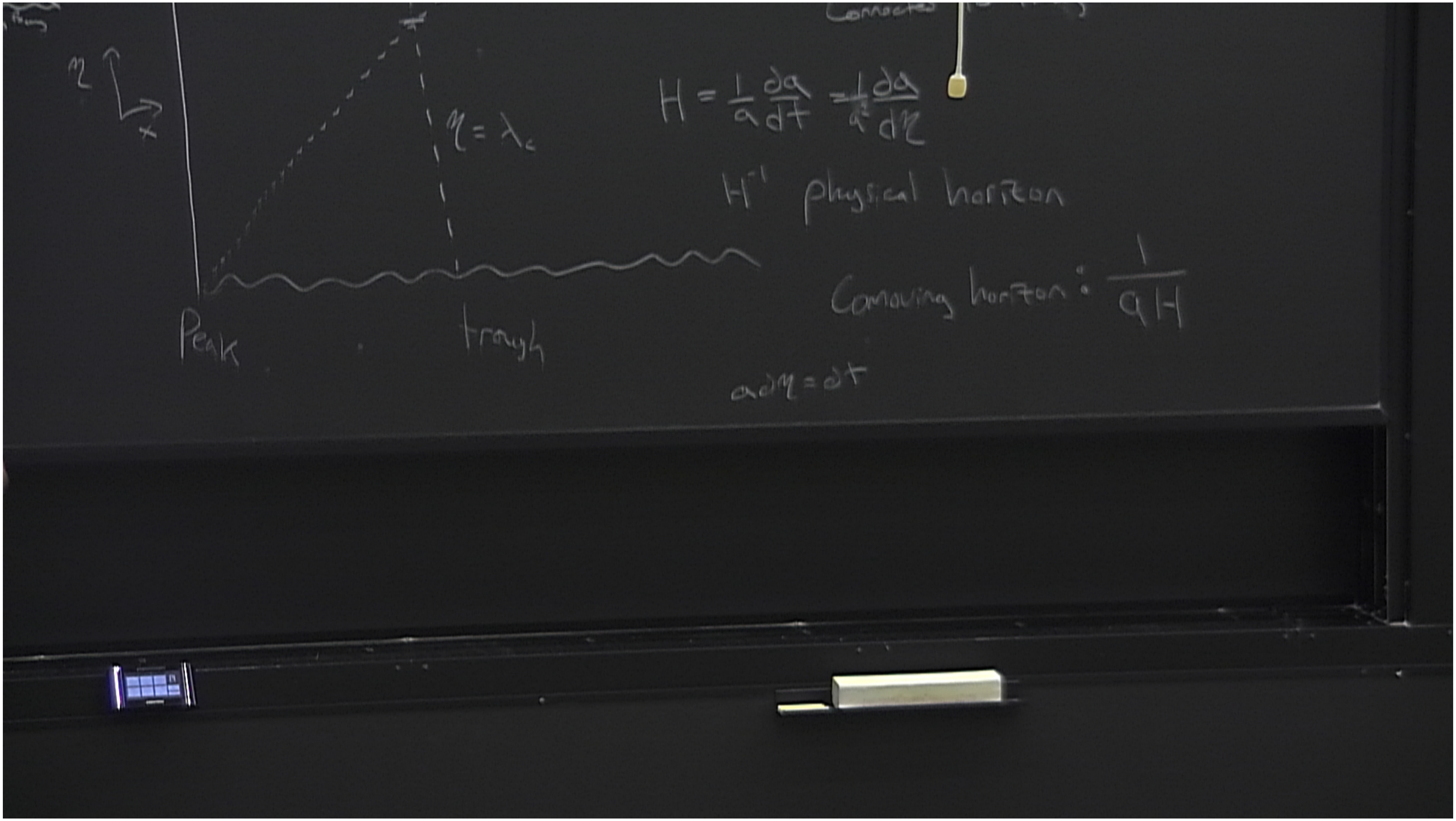
$K_c \eta \simeq 1$ when Peaks are Causally Connected to troughs

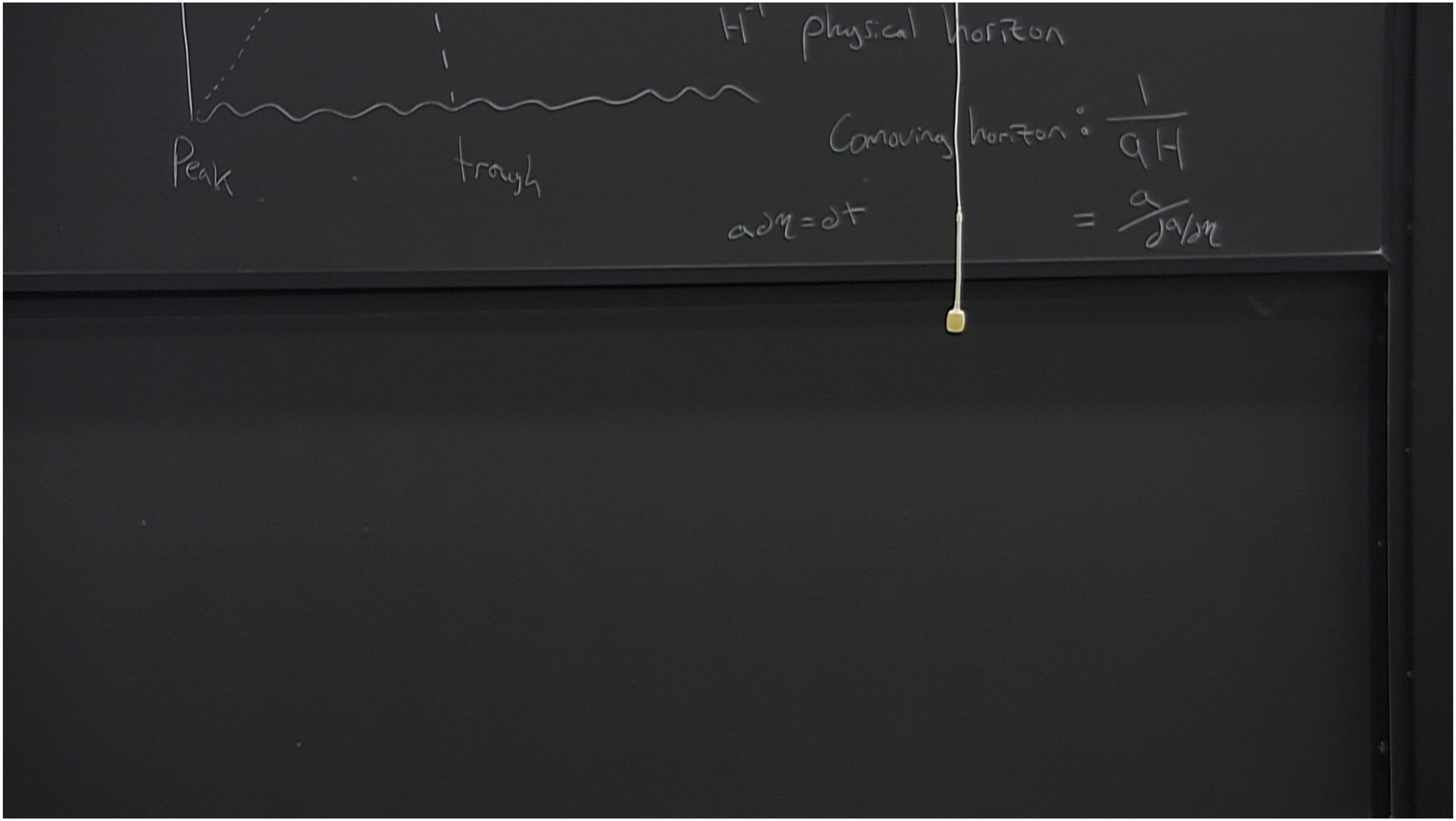
$$H = \frac{1}{a}\frac{da}{dt} = \frac{da}{d\eta}$$

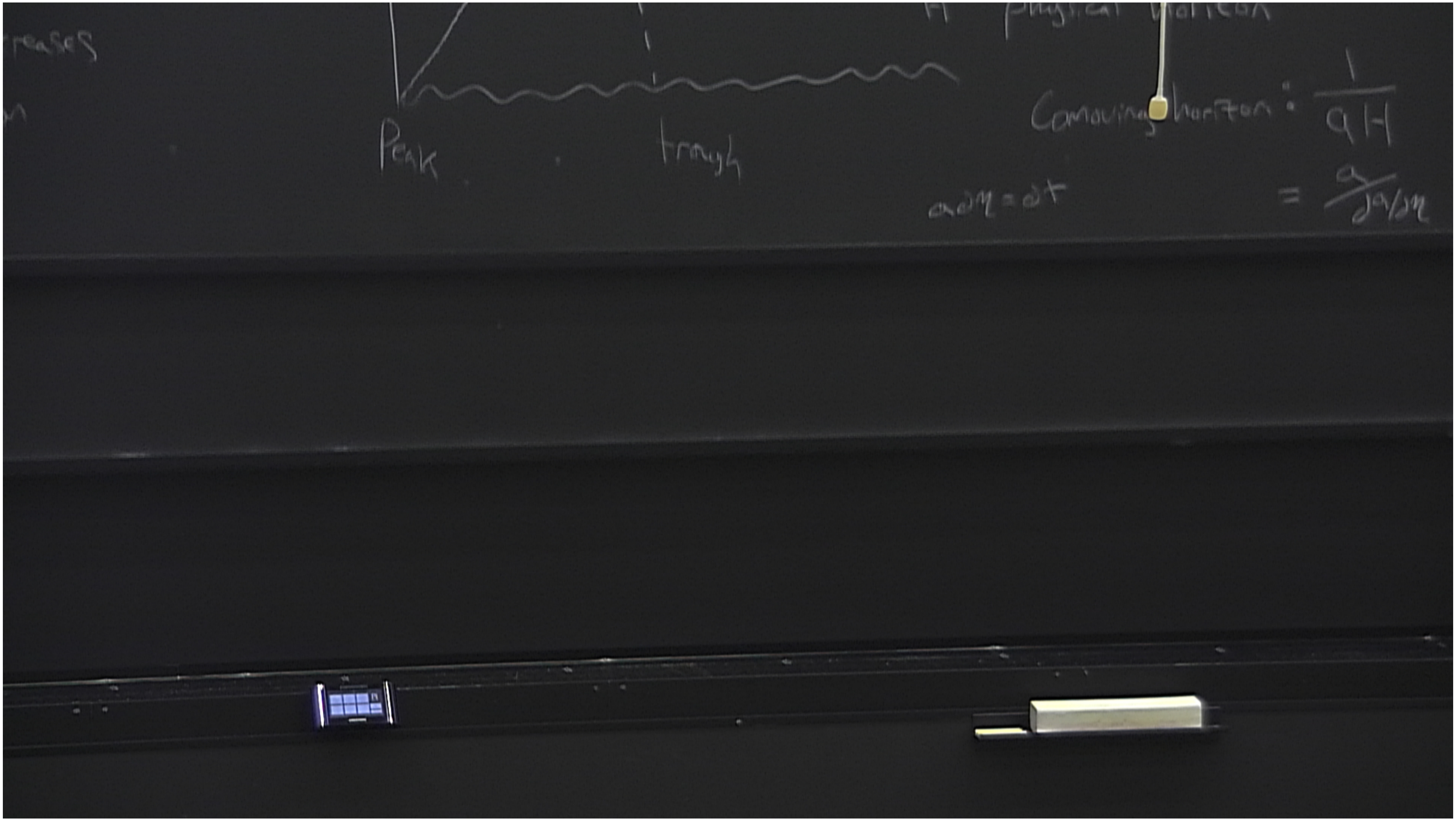
H^{-1} physical horizon

Comoving horizon: $\frac{1}{aH}$

$a\,d\eta = dt$


$H = \frac{1}{a}\frac{da}{dt} = \frac{1}{a^2}\frac{da}{d\eta}$
H^{-1} physical horizon
Comoving horizon: $\frac{1}{aH}$
$\eta = \lambda_c$
Peak
trough
$a\,d\eta = dt$


H^{-1} physical horizon
Comoving horizon: $\frac{1}{aH}$
$= \frac{a}{\partial a/\partial\eta}$
$a\,\partial\eta = \partial t$
Peak
trough


Peak
trough
Comoving horizon

$k = aH$ "horizon crossing"

$\eta \to 0 \quad (\eta \ll \eta_{eq})$

$\eta \to \infty \quad (\eta \gg \eta_{eq})$

$$\frac{k^2}{(\dot a/a)^2}\Phi + 3\left(\frac{a}{\dot a}\dot\Phi - \Psi\right) = \frac{4\pi G a^2}{(\dot a/a)^2}\left(\rho_m \delta_m + 4\rho_r \Theta_{r,0}\right)$$

$k \gg \frac{\dot a}{a} \longrightarrow$ Non-relativistic limit

$$k^2 \Phi = 4\pi G a^2 (\quad)$$

$a\,d\eta = dt$ $\quad = \ldots / da/d\eta$

$k = aH$ "horizon crossing"

$\eta \to 0$ $(\eta \ll \eta_{eq})$

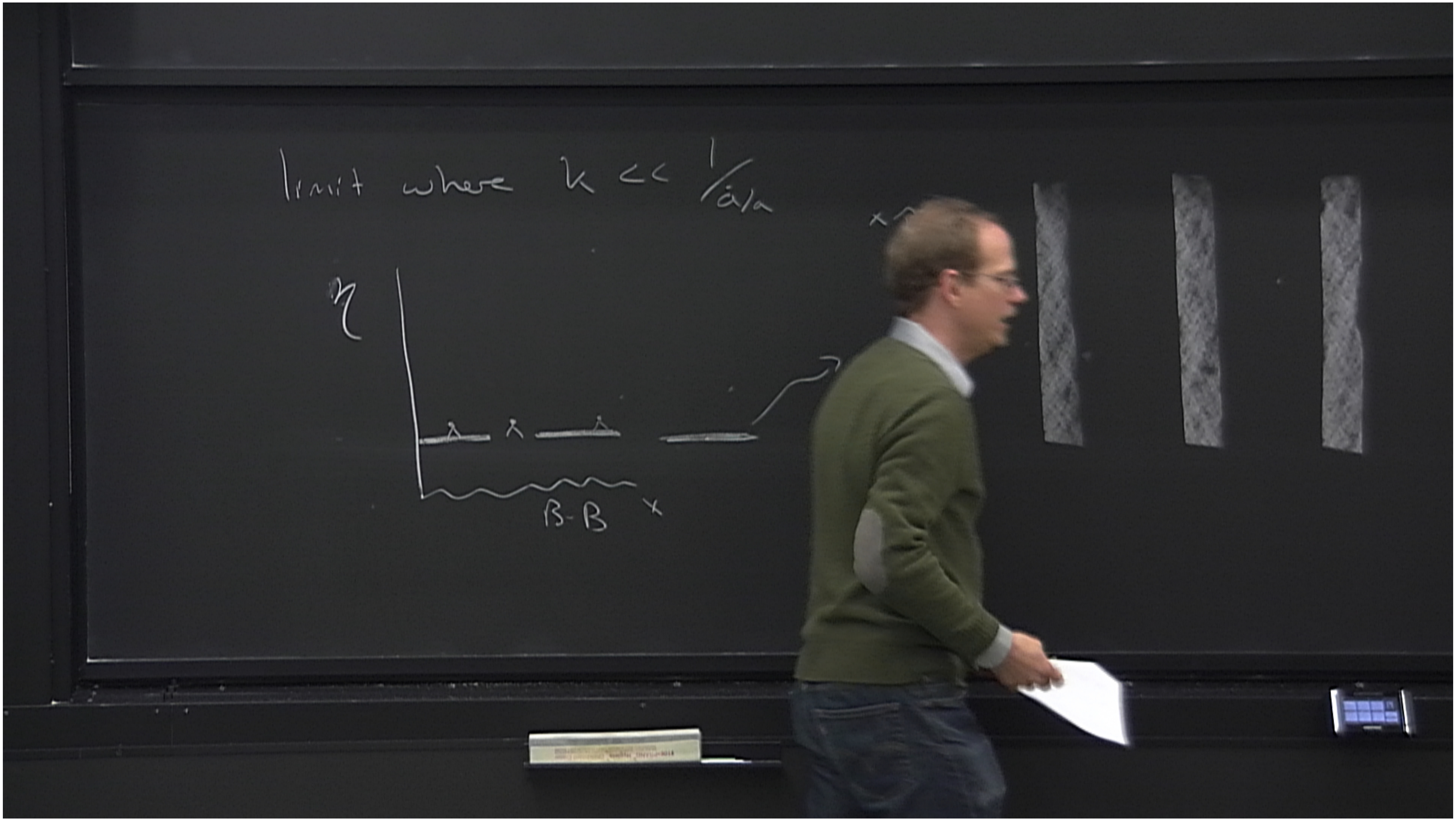
$\eta \to \infty$ $(\eta \gg \eta_{eq})$

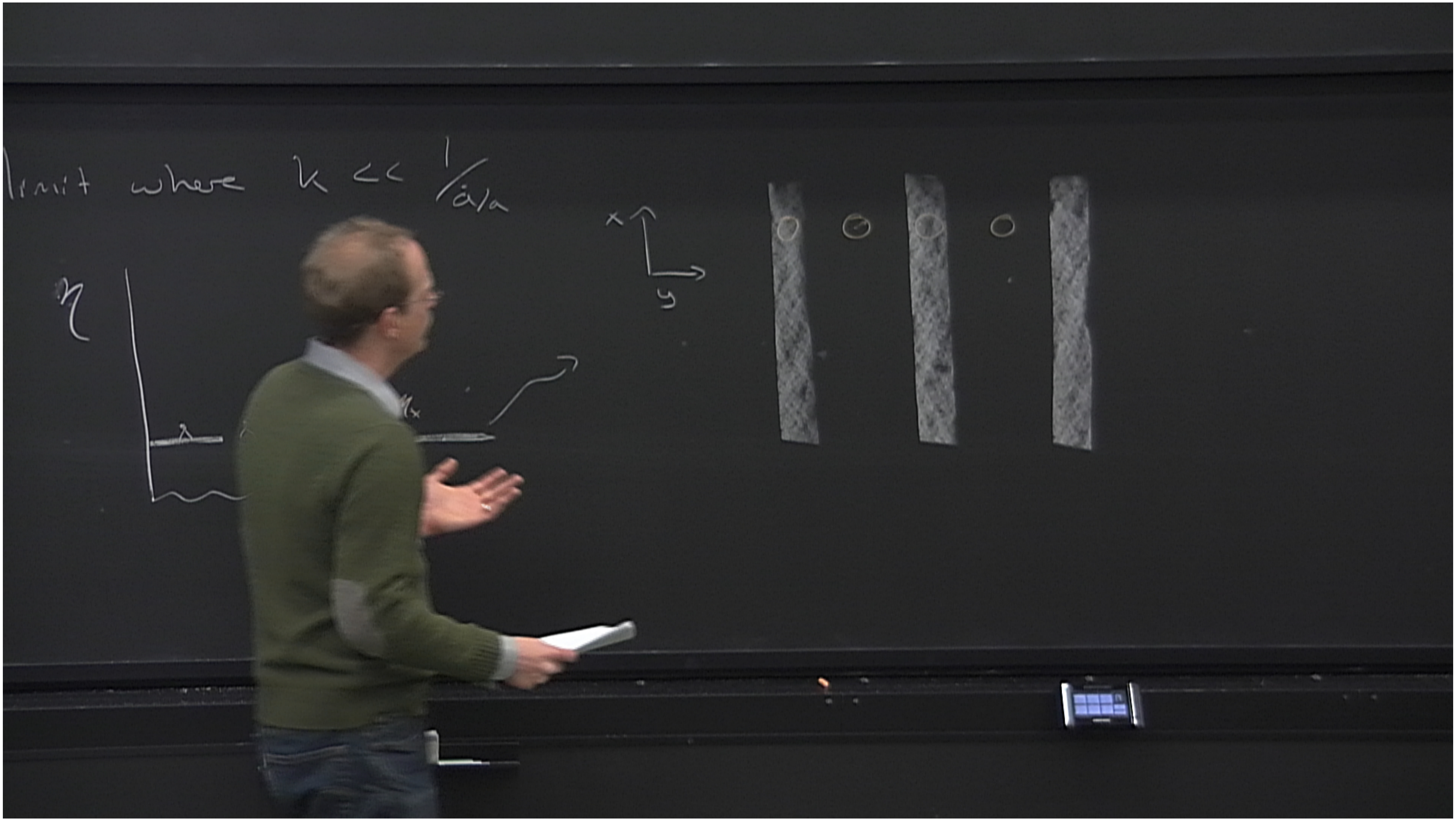
$$\frac{k^2}{(\dot a/a)^2}\Phi + 3\left(\frac{a}{\dot a}\dot\Phi - \Psi\right) = \frac{4\pi G a^2}{(\dot a/a)^2}\left(\rho_m \delta_m + 4\rho_r \Theta_0\right)$$

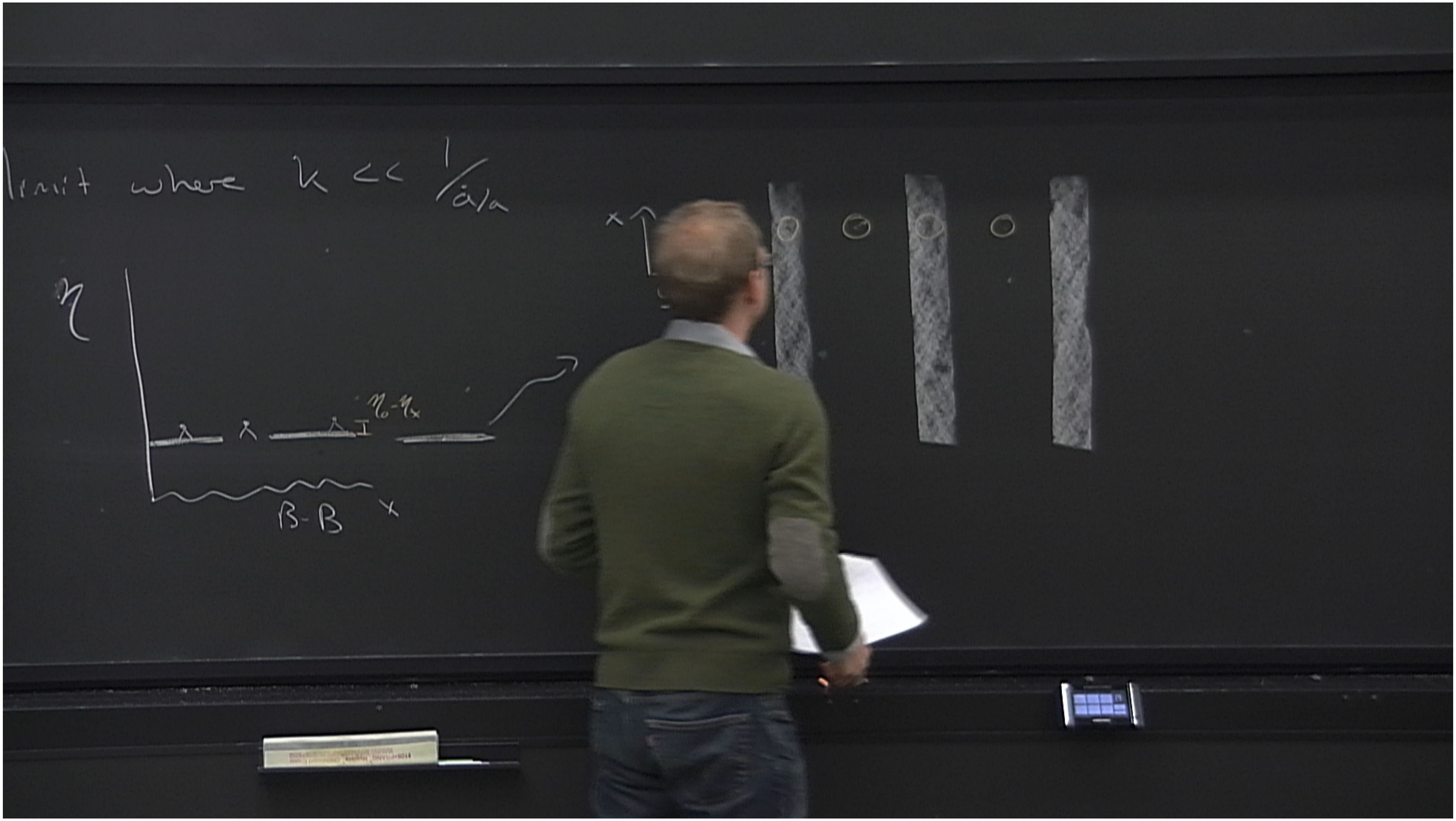
$k \gg \frac{\dot a}{a} \longrightarrow$ Non-relativistic limit

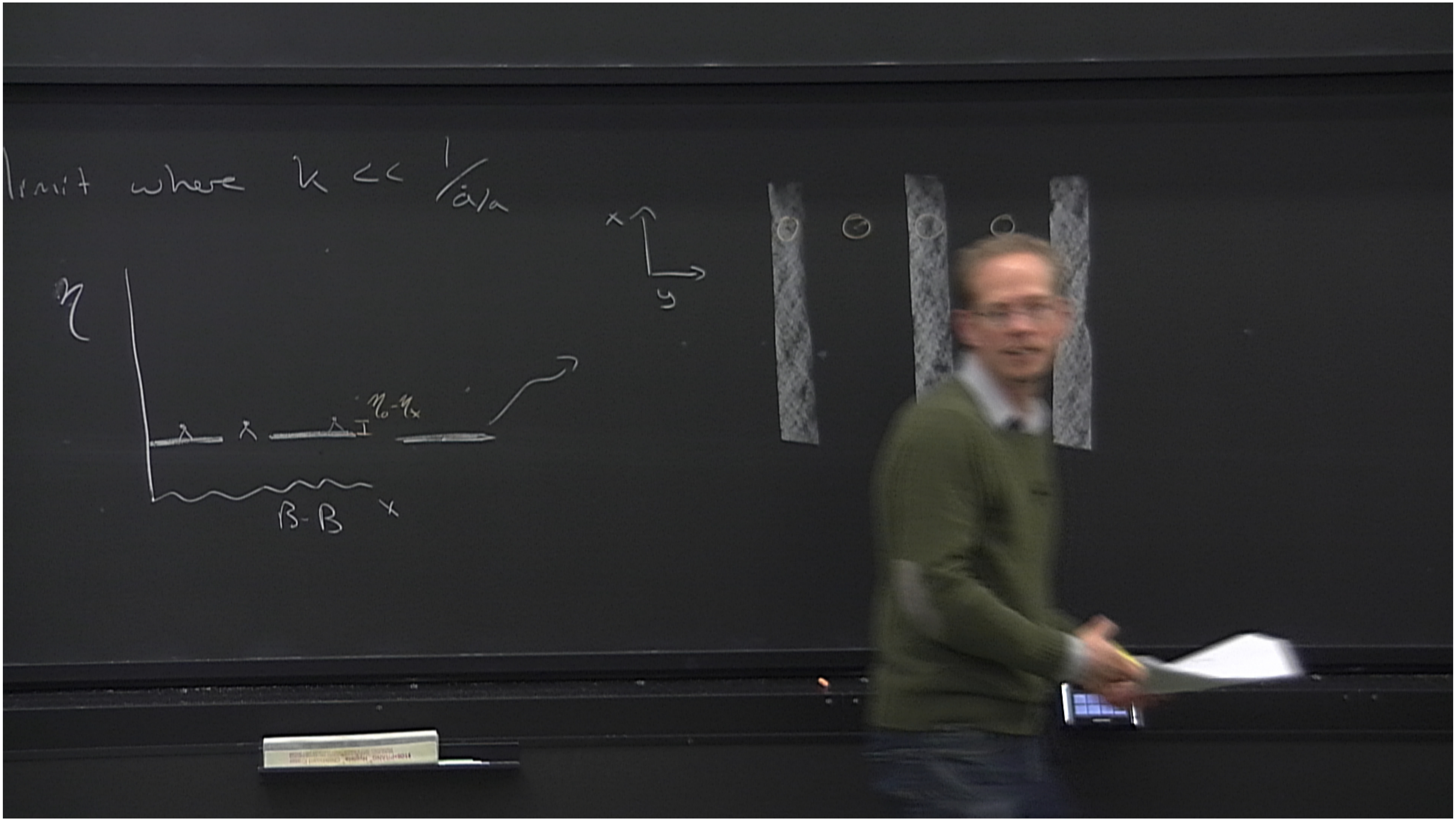
$$k^2\Phi = 4\pi G a^2 (\quad)$$

dm collapses $\to$ δ grows

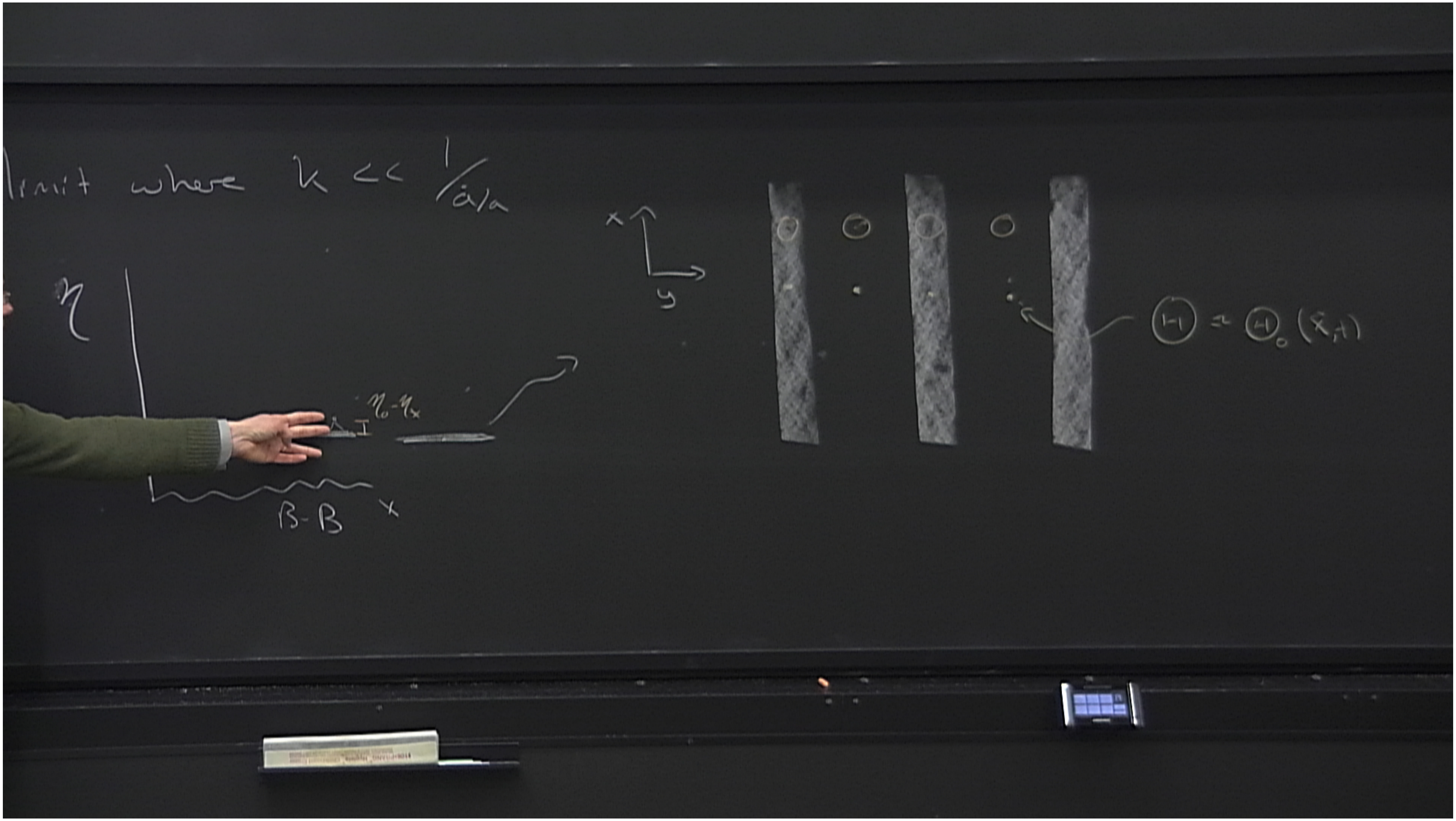

limit where k << 1/ȧ/a
η
B-B
x

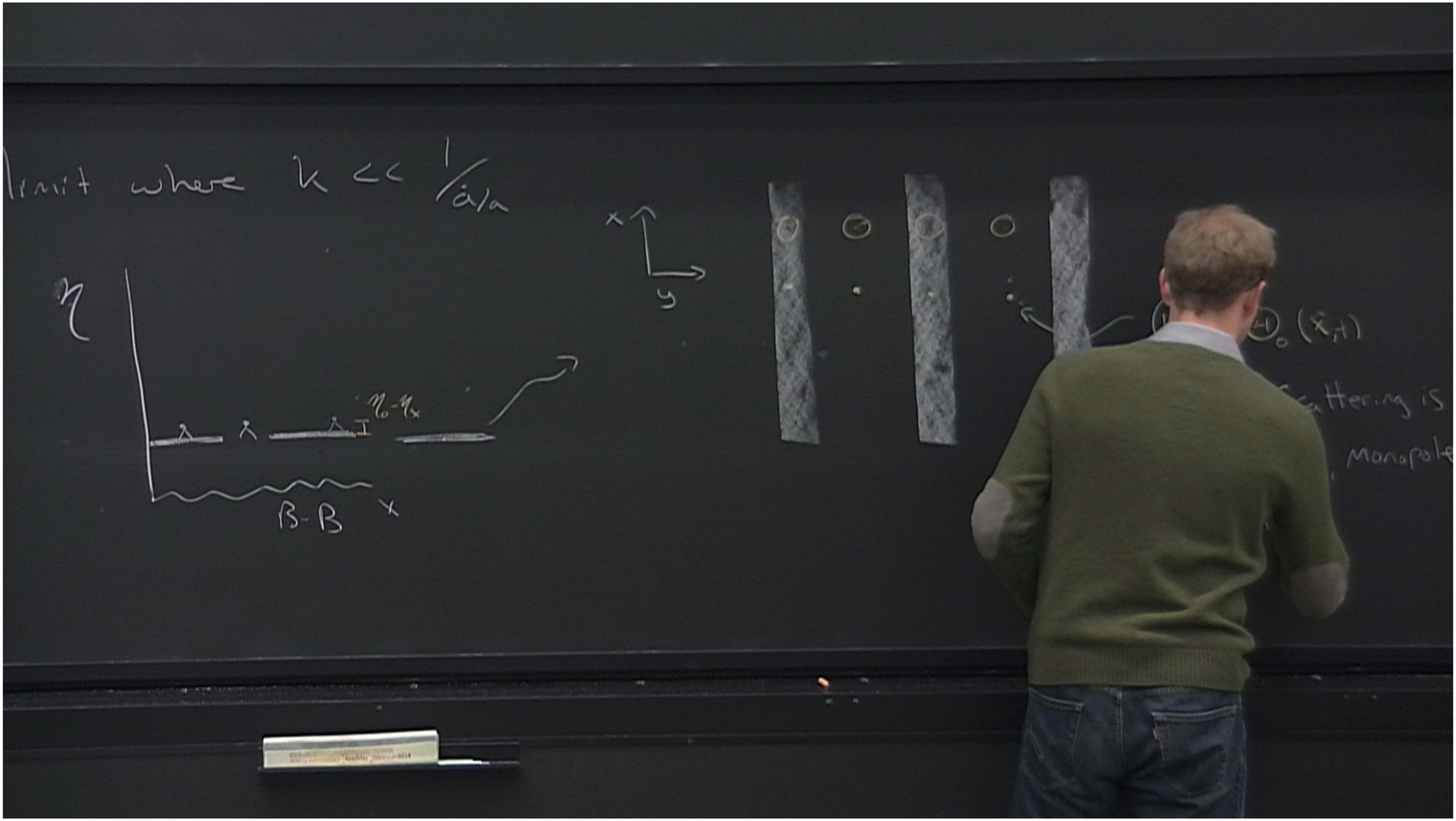

limit where $k \ll 1/\dot{a}/a$


limit where k << 1/(ȧ/a)
η
η₀-ηₓ
B-B
x

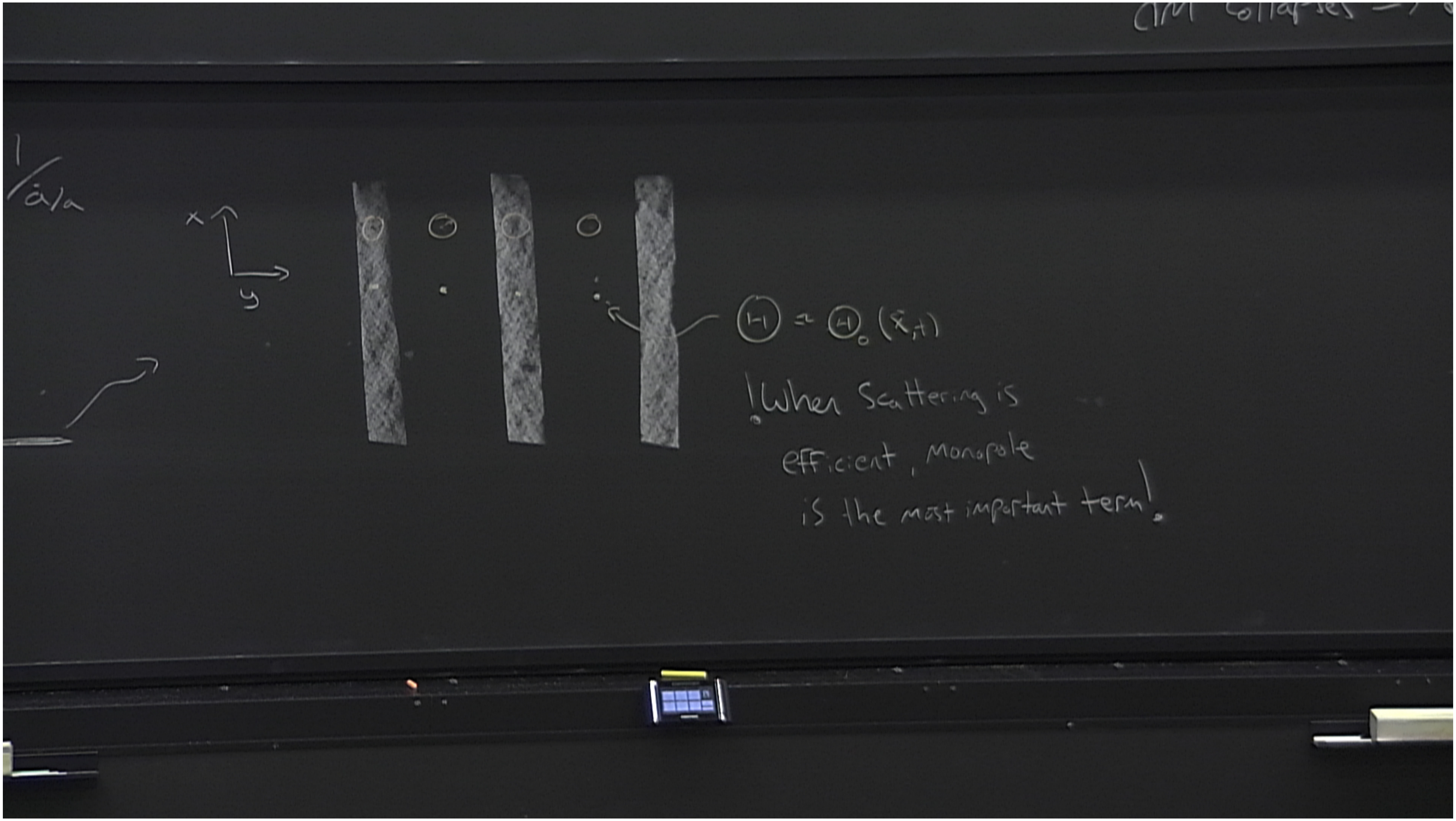


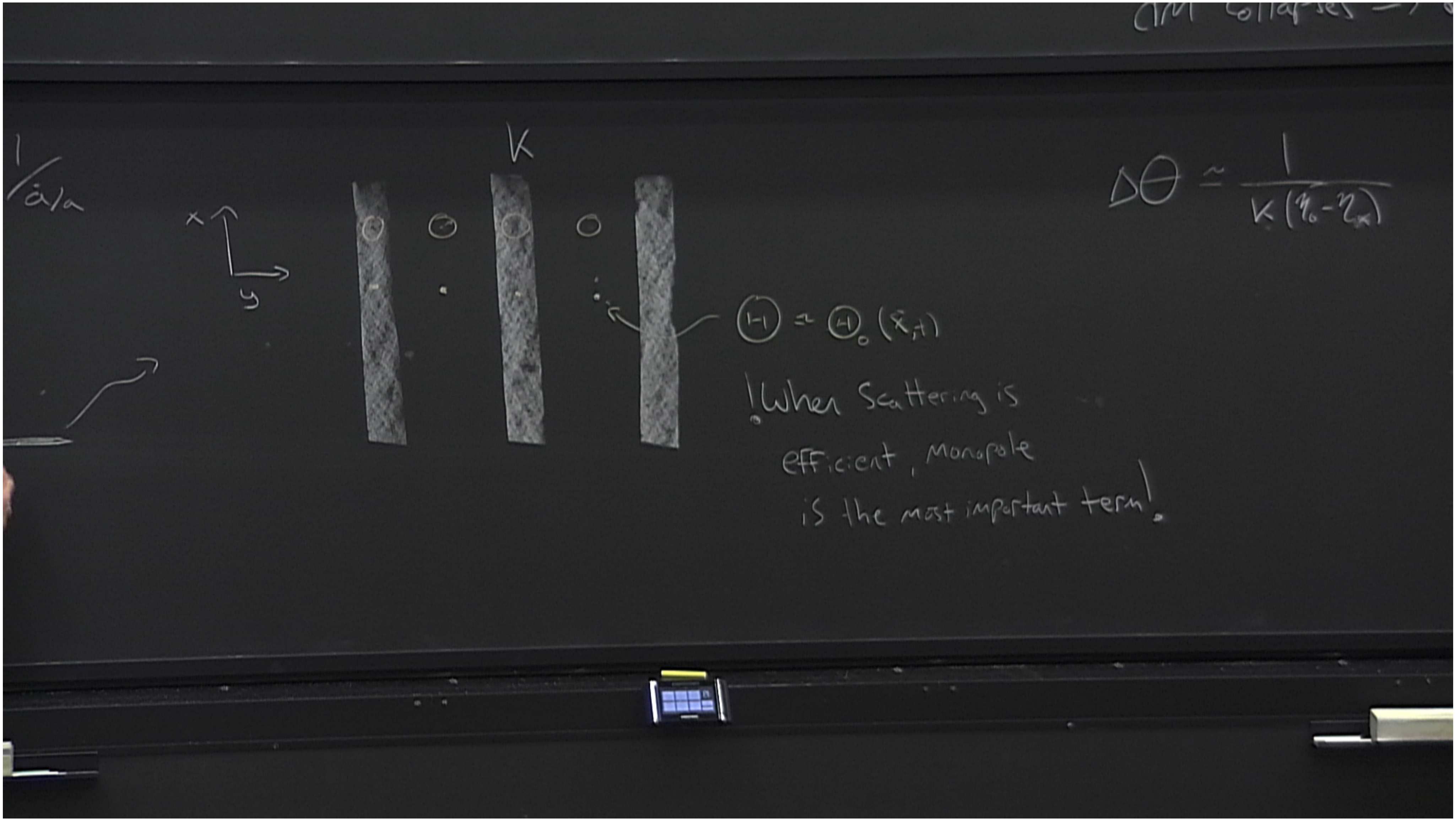
limit where k << 1/ȧ/a
η
η₀-ηₓ
B-B
x
y


limit where k << 1/ȧ/a
η
η₀-ηₓ
B-B
x
x
y
Θ = Θ₀(x,t)



where k << 1/ȧ/a
B-B
x
y
Monopole


1/ȧ/a
x
y
Θ = Θ₀(x,t)
!When Scattering is efficient, Monopole is the most important term!


K
ΔΘ ≃ 1/(k(η₀ − η*))
Θ = Θ₀(x, t)
!When Scattering is efficient, Monopole is the most important term!

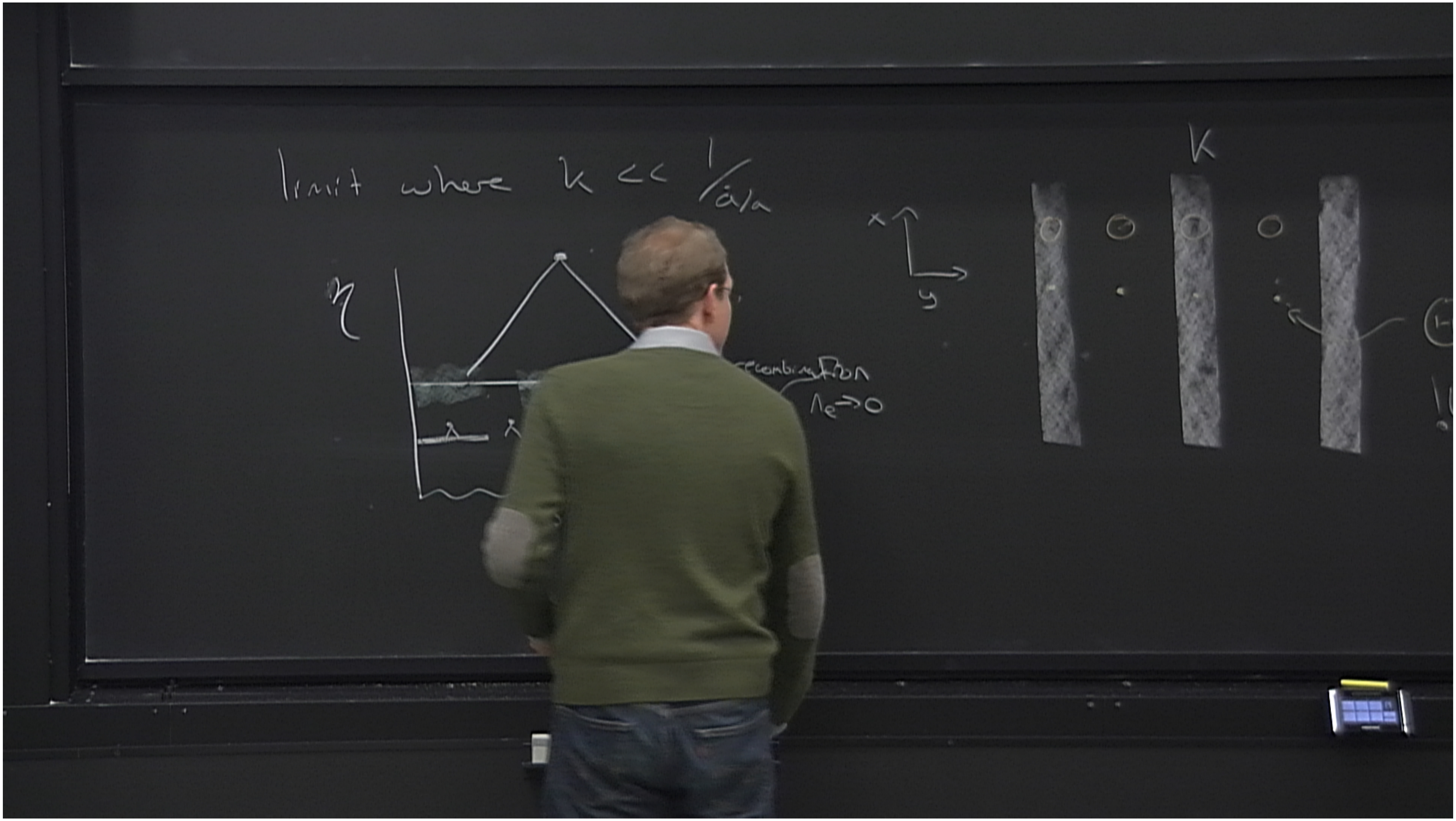
DM collapses → δ grows

$$\Delta\Theta \simeq \frac{1}{k(\eta_0-\eta_s)} \simeq 1/\ell$$

$$\ell \simeq k(\eta_0-\eta_s)$$

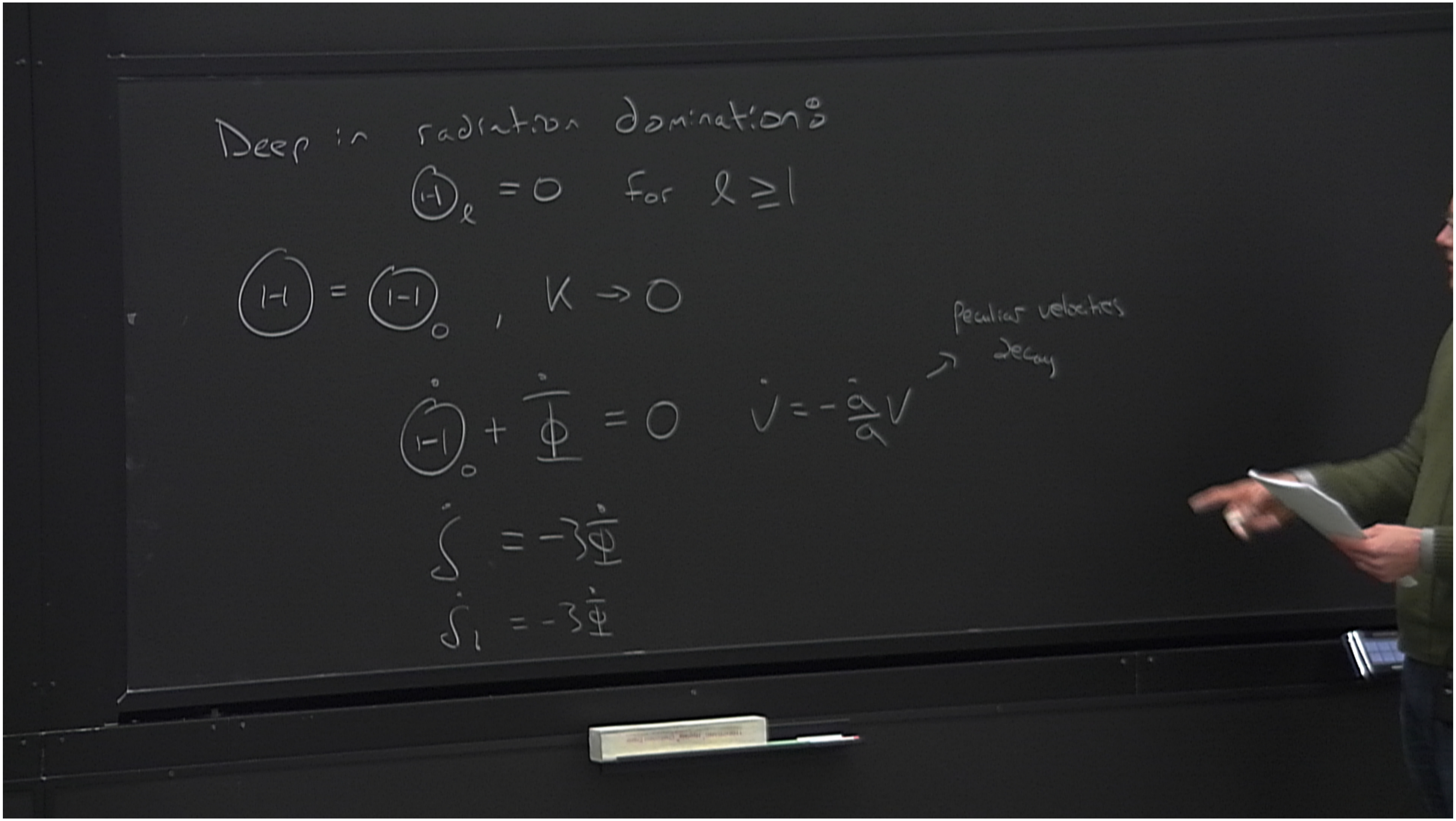
$\Theta \simeq \Theta_0(\vec{x},t)$

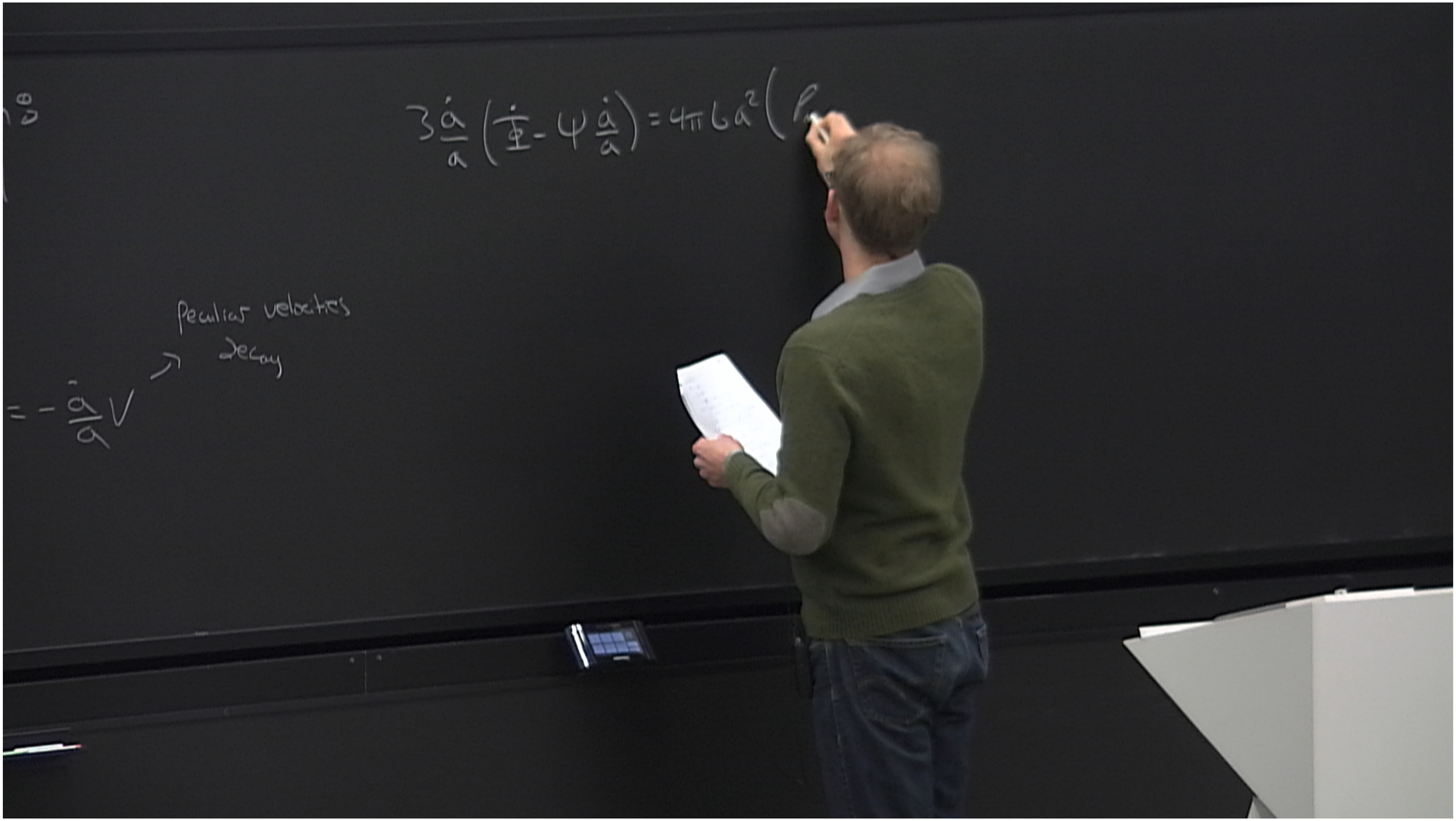
! When Scattering is efficient, Monopole is the most important term!

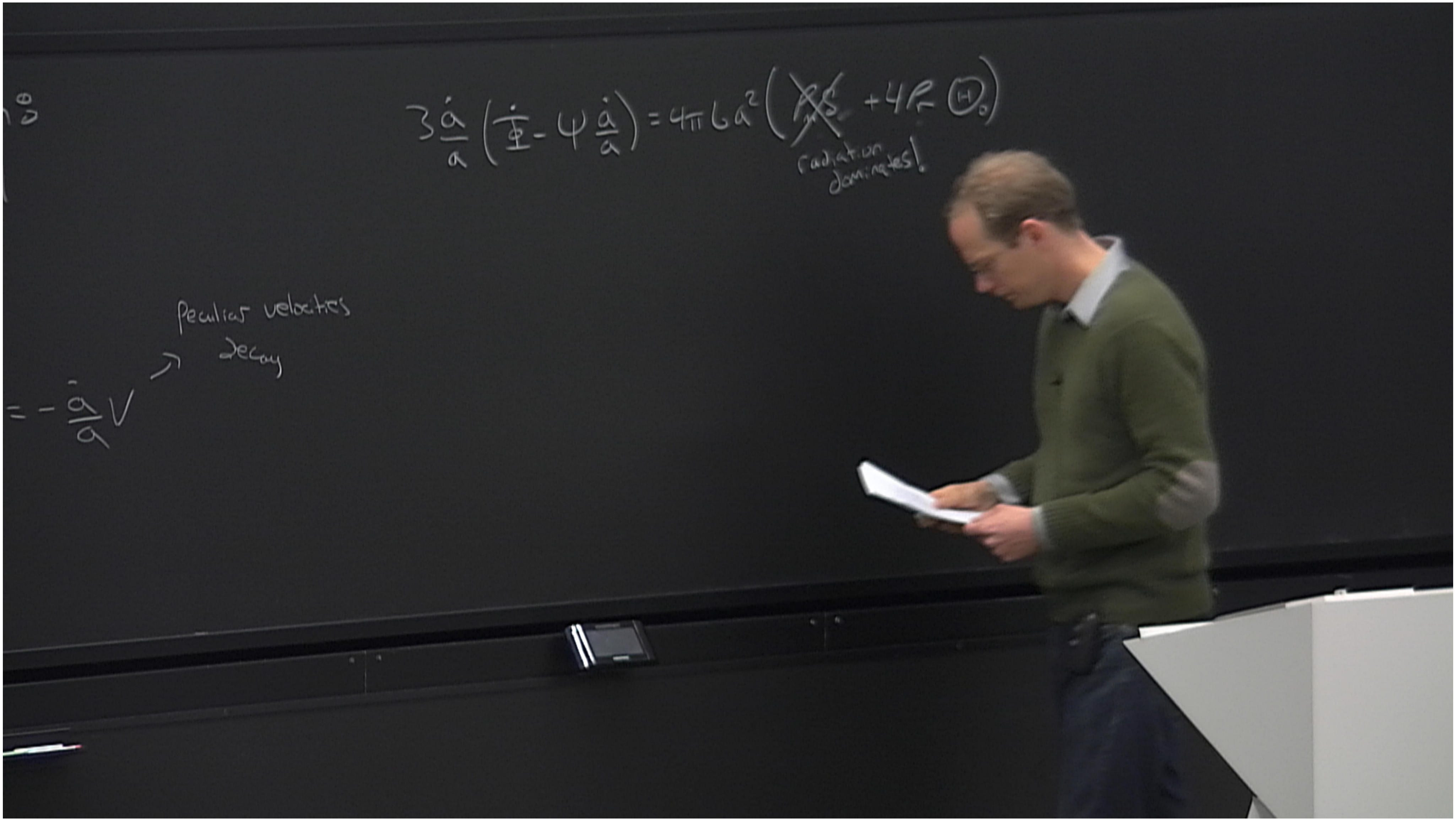

limit where k << 1/(ȧ/a)
η
recombination
Λe → 0
x
y
K

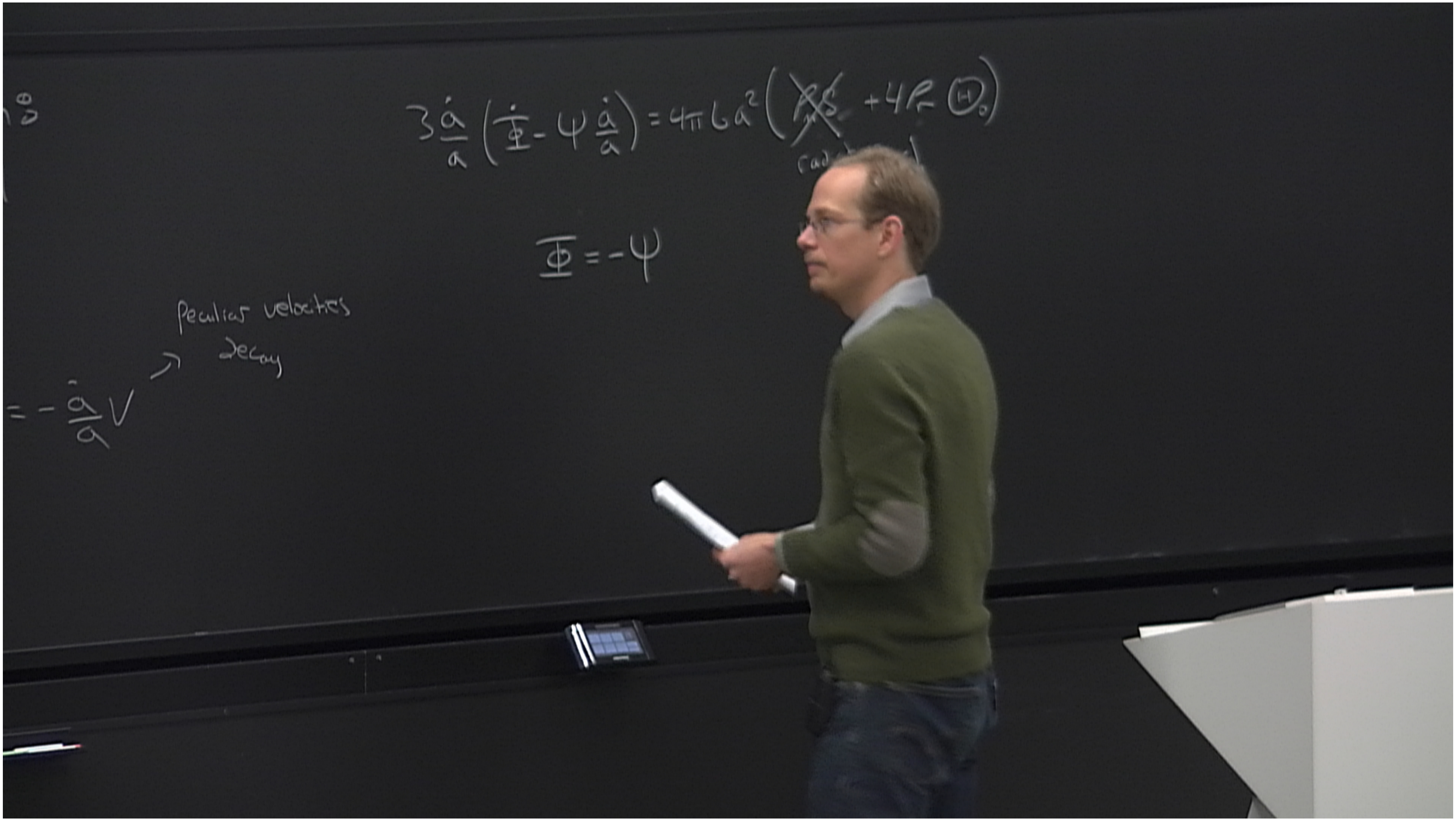
Deep in radiation domination:

$$\Theta_\ell = 0 \quad \text{for } \ell \geq 1$$


Deep in radiation domination:
$\Theta_\ell = 0$ for $\ell \geq 1$
$\Theta = \Theta_0$, $k \to 0$
$\dot{\Theta}_0 + \dot{\Phi} = 0$
$\dot{v} = -\frac{\dot{a}}{a} v$
peculiar velocities decay
$\dot{\delta} = -3\dot{\Phi}$
$\dot{\delta}_b = -3\dot{\Phi}$


$3\frac{\dot{a}}{a}\left(\dot{\Phi} - \Psi\frac{\dot{a}}{a}\right) = 4\pi G a^2 \left(\rho\right.$
Peculiar velocities
decay
$= -\frac{\dot{a}}{a}V$


$3\frac{\dot a}{a}\left(\dot\Phi - \psi\frac{\dot a}{a}\right) = 4\pi G a^2\left(\cancel{\rho_m \delta} + 4\rho_r \Theta_0\right)$
radiation dominates!
peculiar velocities decay
$= -\frac{\dot a}{a} V$


Φ = -Ψ
Peculiar velocities
decay

Peculiar velocities
Decay

$$3\frac{\dot a}{a}\left(\dot\Phi - \Psi\frac{\dot a}{a}\right) = 4\pi G a^2\left(\cancel{\rho_m\delta} + 4\rho_r\Theta_0\right)$$

radiation dominates!

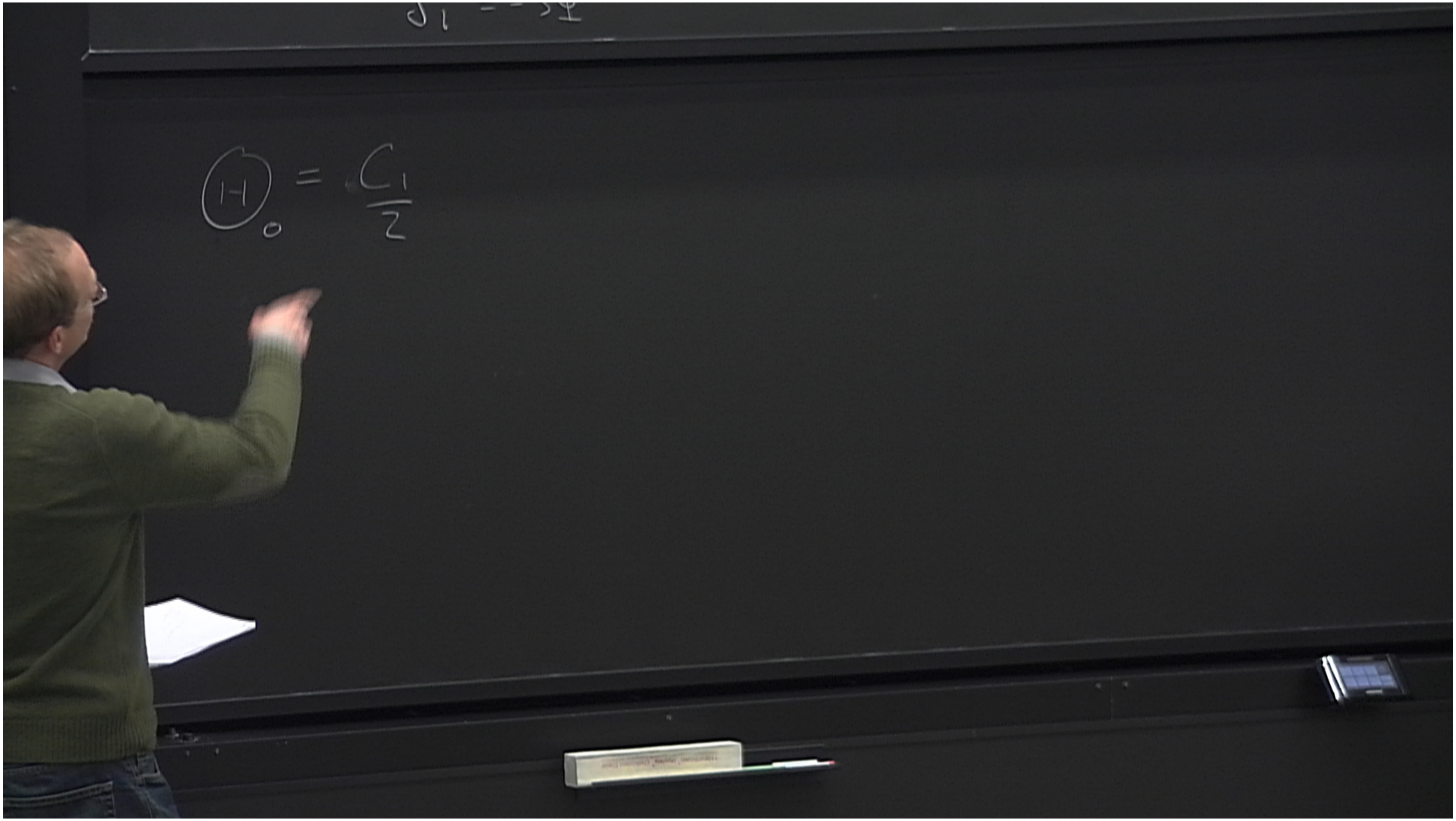
$$\Phi = -\Psi \quad , \quad a = \eta$$

$$\Rightarrow \frac{\dot\Phi}{\eta} + \frac{\Phi}{\eta^2} = \frac{2}{\eta^2}\Theta_0 \quad (1)$$

$\frac{d(1)}{d\eta}$ then Substitute for $\dot\Theta_0$

$$\ddot\Phi + \frac{4}{\eta}\dot\Phi = 0$$

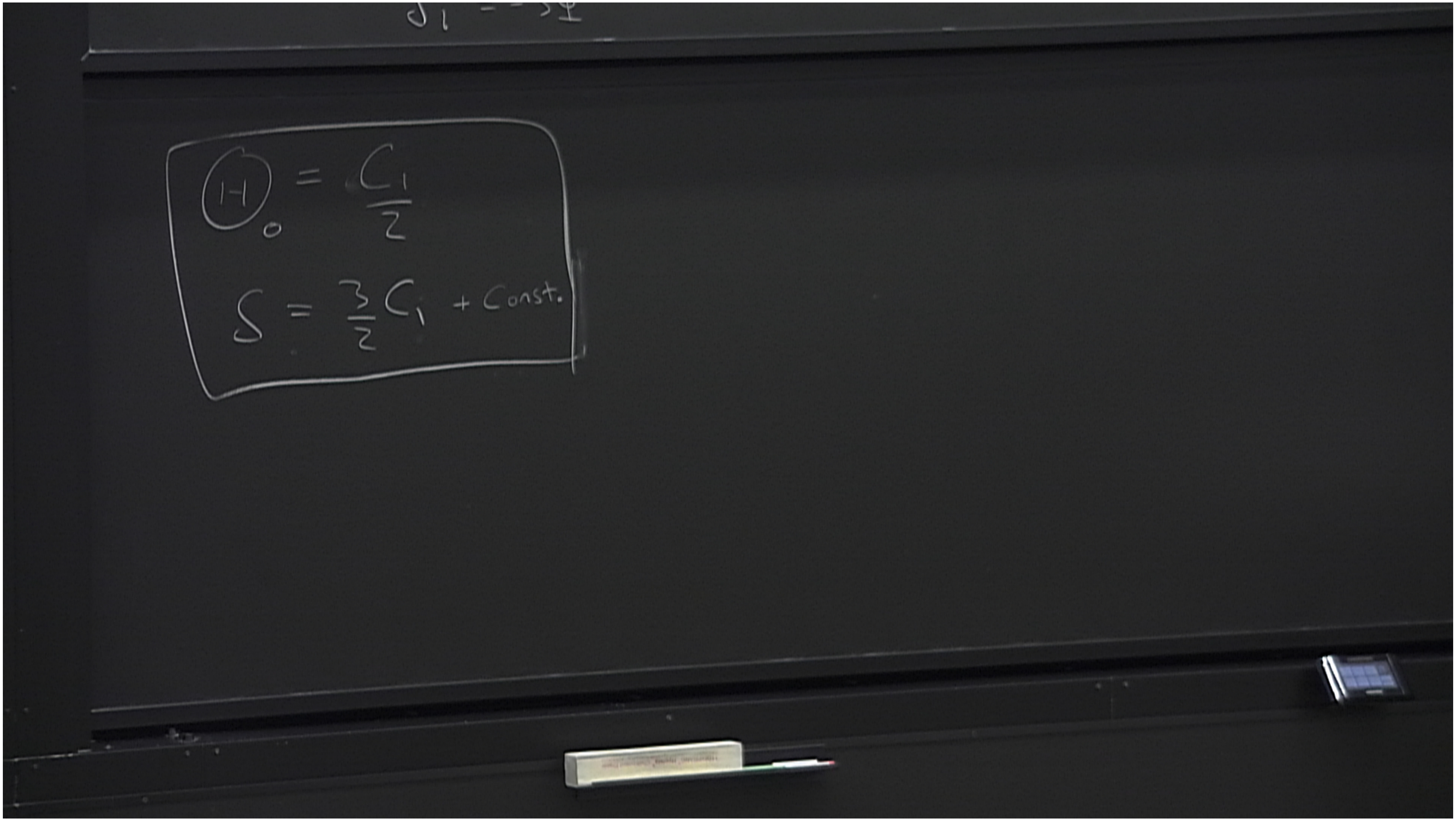
$$\boxed{\Phi(\eta) = C_1 + C_2\eta^{-3}}$$

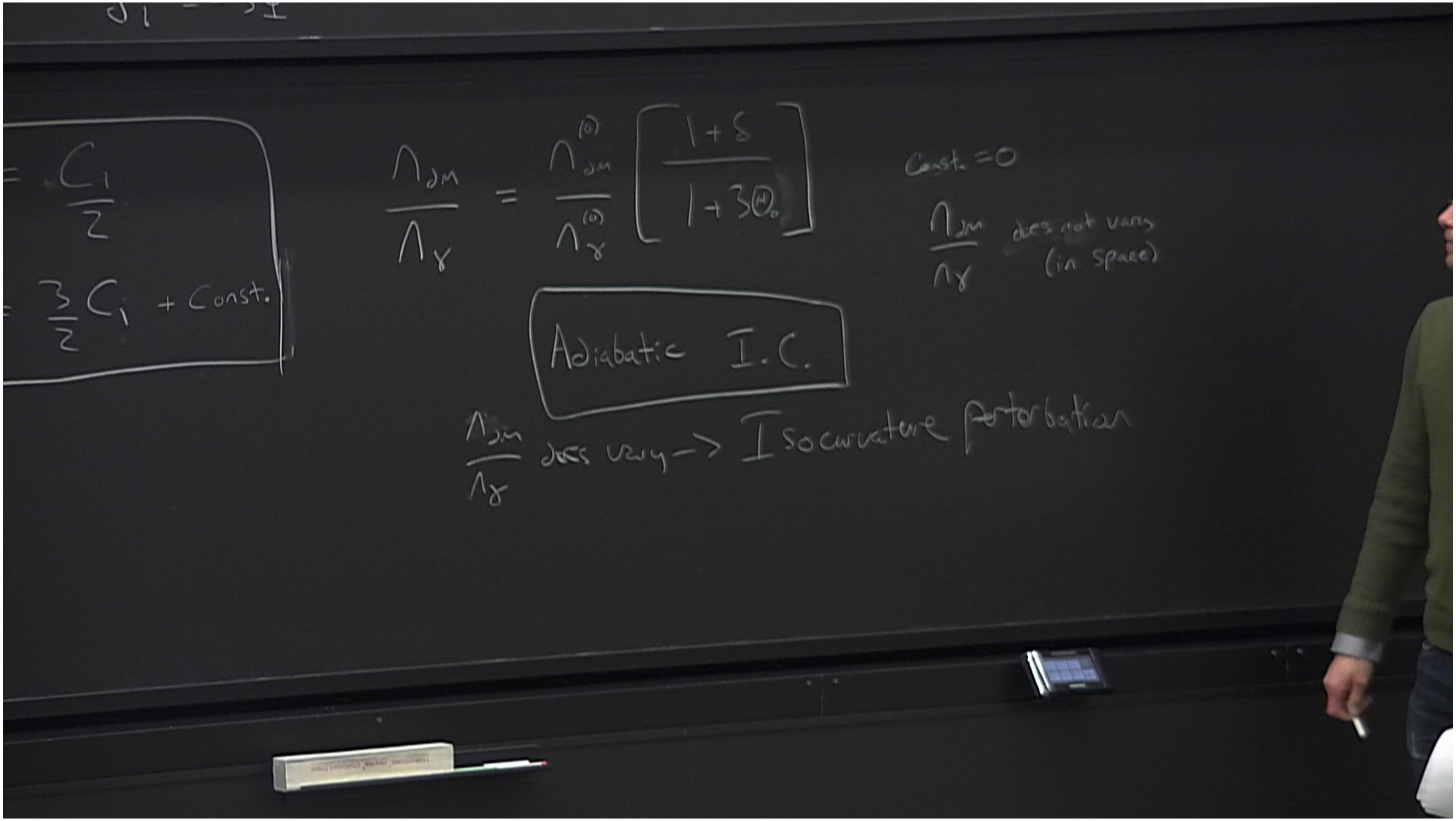

$\Theta_0 = \frac{C_1}{2}$

$$\delta_1 = -3\Phi$$

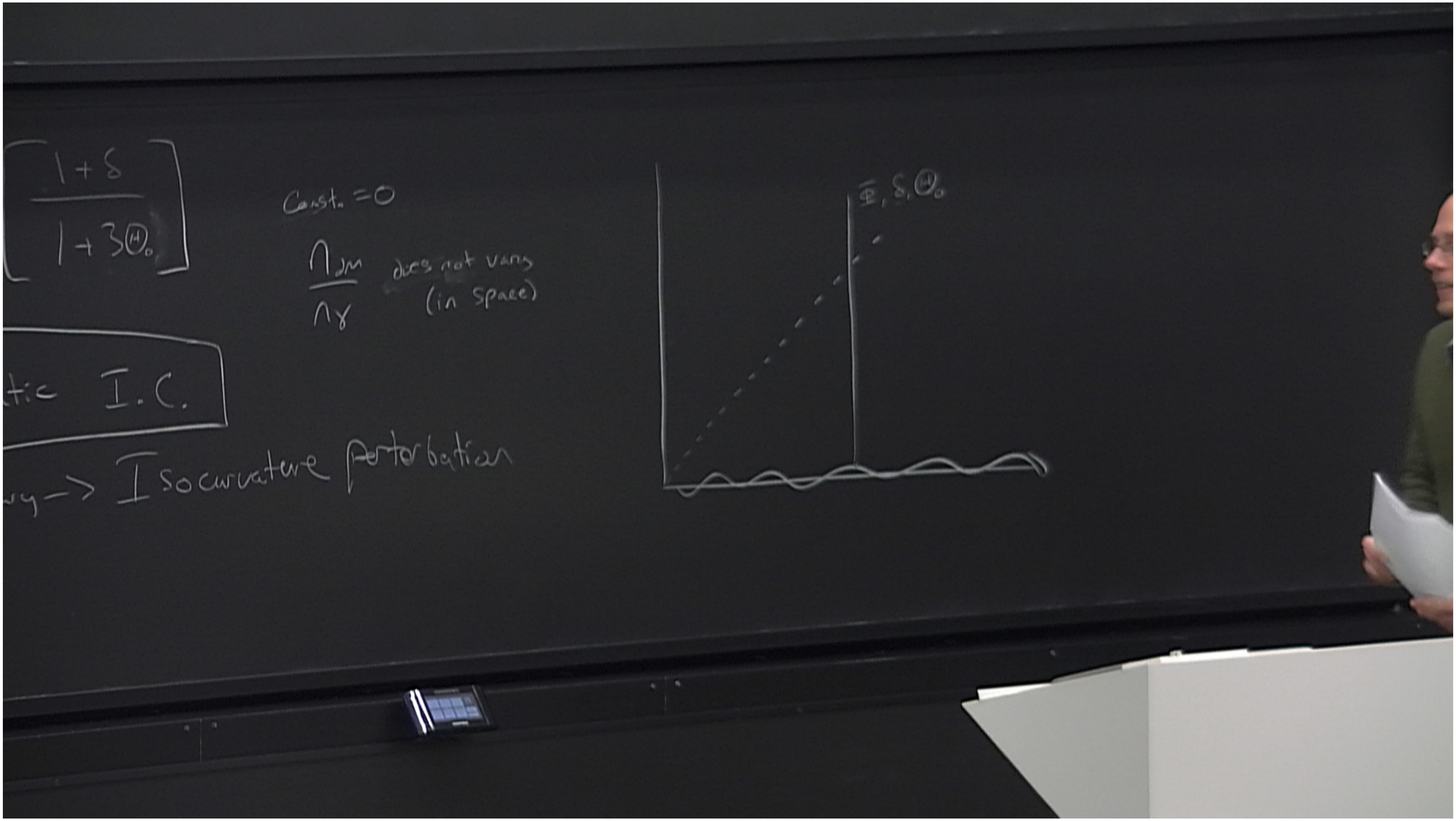
$$\Theta_0 = \frac{C_1}{2}$$

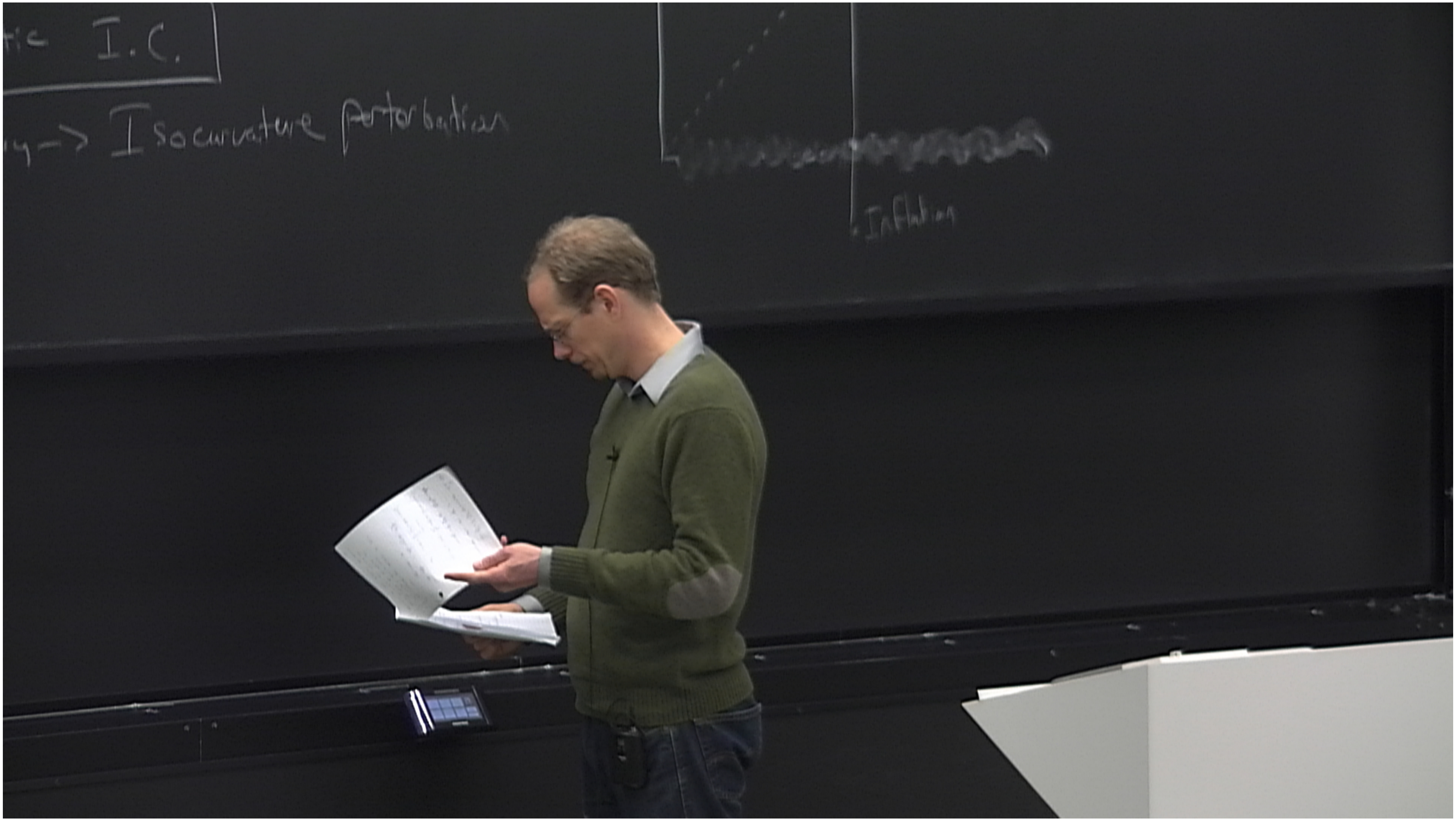
$$S = \frac{3}{2} C_1 + \text{Const.}$$

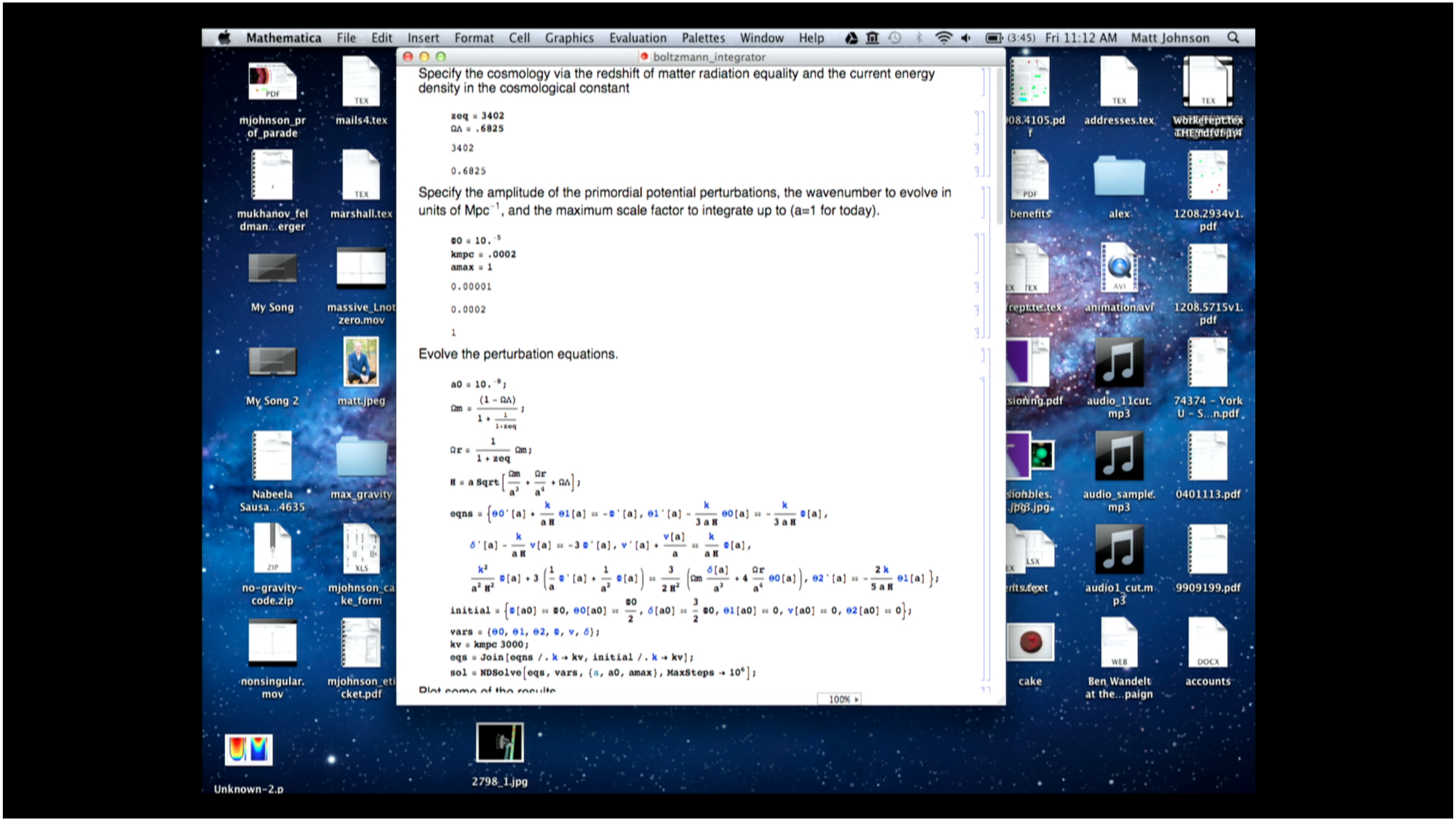

$\Theta_0 = \frac{C_1}{2}$
$S = \frac{3}{2} C_1 + \text{Const.}$



$= \frac{C_1}{2}$
$= \frac{3}{2} C_1 + \text{Const.}$
$\frac{\Lambda_{DM}}{\Lambda_\gamma} = \frac{\Lambda^{(0)}_{DM}}{\Lambda^{(0)}_\gamma} \left[\frac{1+\delta}{1+3\Theta_0} \right]$
Const. = 0
$\frac{\Lambda_{DM}}{\Lambda_\gamma}$ does not vary (in space)
Adiabatic I.C.
$\frac{\Lambda_{DM}}{\Lambda_\gamma}$ does vary -> Isocurvature perturbation


1 + δ
1 + 3Θ₀
Const. = 0
n_DM
n_γ
does not vary
(in space)
I.C.
Isocurvature perturbation
Φ, δ, Θ₀


tic I.C.
y -> Isocurvature perturbation
Inflation


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Fri 11:12 AM Matt Johnson
boltzmann_integrator
Specify the cosmology via the redshift of matter radiation equality and the current energy density in the cosmological constant
zeq = 3402
ΩΛ = .6825
3402
0.6825
Specify the amplitude of the primordial potential perturbations, the wavenumber to evolve in units of Mpc^-1, and the maximum scale factor to integrate up to (a=1 for today).
Φ0 = 10.^-5
kmpc = .0002
amax = 1
0.00001
0.0002
1
Evolve the perturbation equations.
a0 = 10.^-8;
$\Omega m = \frac{(1-\Omega\Lambda)}{1+\frac{1}{1+zeq}}$;
$\Omega r = \frac{1}{1+zeq}\,\Omega m$;
$H = a\,\mathrm{Sqrt}\left[\frac{\Omega m}{a^3}+\frac{\Omega r}{a^4}+\Omega\Lambda\right]$;
$eqns = \{\Theta 0'[a] + \frac{k}{a H}\Theta 1[a] == -\Phi'[a],\ \Theta 1'[a] - \frac{k}{3 a H}\Theta 0[a] == -\frac{k}{3 a H}\Phi[a],$
$\delta'[a] - \frac{k}{a H} v[a] == -3\Phi'[a],\ v'[a] + \frac{v[a]}{a} == \frac{k}{a H}\Phi[a],$
$\frac{k^2}{a^2 H^2}\Phi[a] + 3\left(\frac{1}{a}\Phi'[a] + \frac{1}{a^2}\Phi[a]\right) == \frac{3}{2 H^2}\left(\Omega m \frac{\delta[a]}{a^3} + 4\frac{\Omega r}{a^4}\Theta 0[a]\right),\ \Theta 2'[a] == -\frac{2k}{5 a H}\Theta 1[a]\}$;
$initial = \{\Phi[a0] == \Phi 0,\ \Theta 0[a0] == \frac{\Phi 0}{2},\ \delta[a0] == \frac{3}{2}\Phi 0,\ \Theta 1[a0] == 0,\ v[a0] == 0,\ \Theta 2[a0] == 0\}$;
vars = {Θ0, Θ1, Θ2, Φ, v, δ};
kv = kmpc 3000;
eqs = Join[eqns /. k → kv, initial /. k → kv];
sol = NDSolve[eqs, vars, {a, a0, amax}, MaxSteps → 10^6];
Plot some of the results
100%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (3:21) Fri 11:14 AM Matt Johnson

boltzmann_integrator

Specify the cosmology via the redshift of matter radiation equality and the current energy density in the cosmological constant

In[1672]:=
```
zeq = 3402
ΩΛ = .6825
```
Out[1672]=
```
3402
```
Out[1673]=
```
0.6825
```

Specify the amplitude of the primordial potential perturbations, the wavenumber to evolve in units of Mpc^{-1}, and the maximum scale factor to integrate up to (a=1 for today).

```
Φ0 = 10.^-5
kmpc = .0002
amax = 1
```
```
0.00001

0.0002

1
```

Evolve the perturbation equations.

200%

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boltzmann_integrator

Specify the amplitude of the primordial potential perturbations, the wavenumber to evolve in units of Mpc^{-1}, and the maximum scale factor to integrate up to (a=1 for today).

$$\Phi 0 = 10.^{-5}$$
$$kmpc = .0002$$
$$amax = 1$$

0.00001

0.0002

1

Evolve the perturbation equations.

$$a0 = 10.^{-8};$$

$$\Omega m = \frac{(1 - \Omega\Lambda)}{1 + \frac{1}{1+zeq}};$$

$$\Omega r = \frac{1}{1 + zeq}\,\Omega m;$$

$$H = a\,\mathrm{Sqrt}\left[\frac{\Omega m}{a^3} + \frac{\Omega r}{a^4} + \Omega\Lambda\right];$$

$$eqns = \left\{\Theta 0'[a] + \frac{k}{a\,H}\,\Theta 1[a] == -\Phi'[a],\ \Theta 1'[a] - \frac{k}{3\,a\,H}\,\Theta 0[a] == -\frac{k}{3\,a\,H}\,\Phi[a],\right.$$

200%

Specify the cosmology via the redshift of matter radiation equality and the current energy density in the cosmological constant

In[1672]:=

```
zeq = 3402
ΩΛ = .6825
```

Out[1672]=

```
3402
```

Out[1673]=

```
0.6825
```

Specify the amplitude of the primordial potential perturbations, the wavenumber to evolve in units of Mpc^{-1}, and the maximum scale factor to integrate up to (a=1 for today).

In[1674]:=

```
Φ0 = 10.^-5
kmpc = .0002
amax = 1
```

Out[1674]=

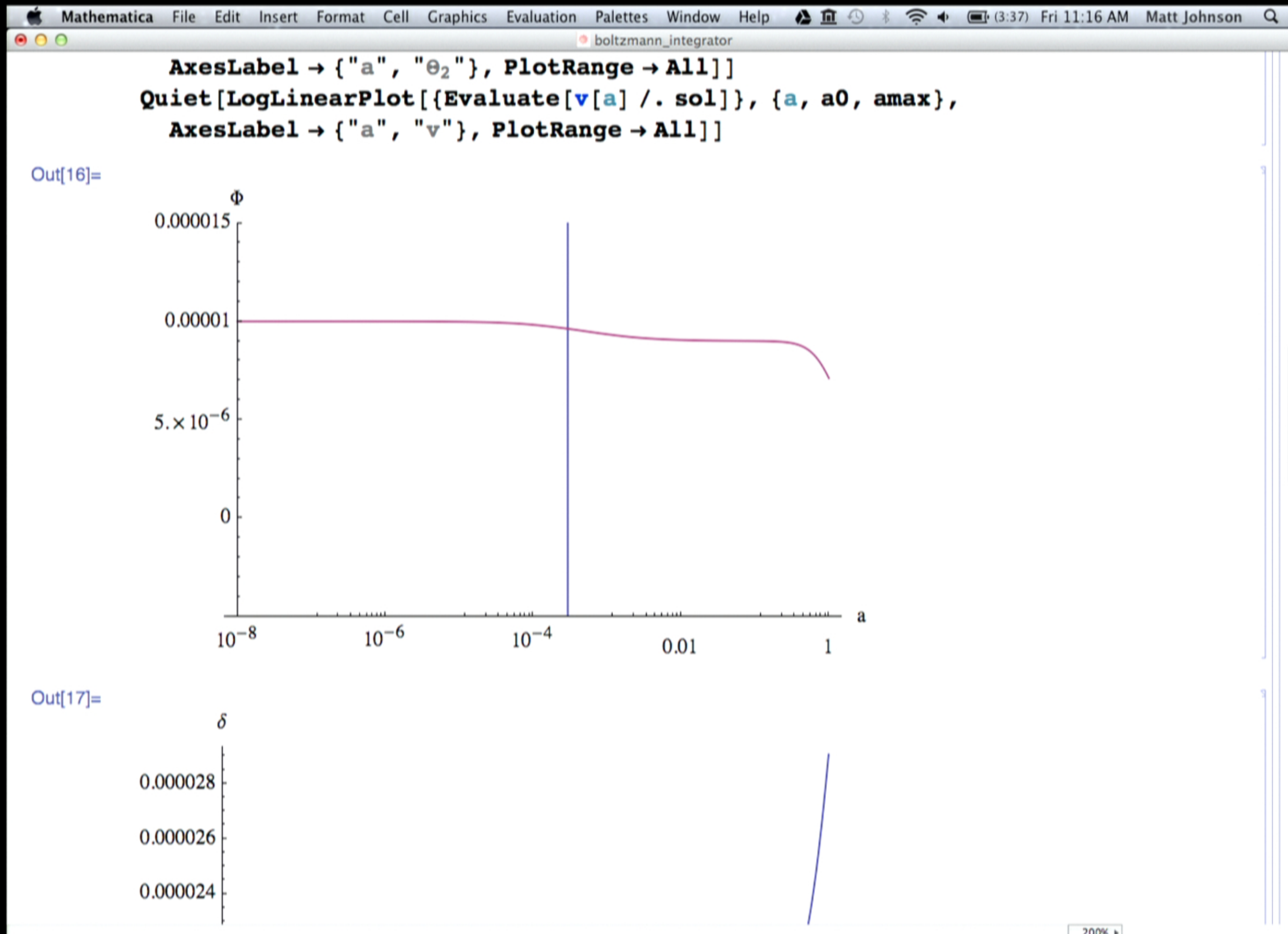
```
0.00001
```

Out[1675]=

```
0.0002
```

Out[1676]=

```
1
```


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(3:37) Fri 11:16 AM Matt Johnson
boltzmann_integrator
AxesLabel → {"a", "θ2"}, PlotRange → All]]
Quiet[LogLinearPlot[{Evaluate[v[a] /. sol]}, {a, a0, amax},
AxesLabel → {"a", "v"}, PlotRange → All]]
Out[16]=
Φ
0.000015
0.00001
5.×10^-6
0
a
10^-8
10^-6
10^-4
0.01
1
Out[17]=
δ
0.000028
0.000026
0.000024
200%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (3:27) Fri 11:19 AM Matt Johnson

boltzmann_integrator

$\Omega\Lambda = .6825$

Out[1]= 3402

Out[2]= 0.6825

Specify the amplitude of the primordial potential perturbations, the wavenumber to evolve in units of Mpc^{-1}, and the maximum scale factor to integrate up to (a=1 for today).

$\Phi0 = 10.^{-5}$

kmpc = 2

amax = 1

Out[3]= 0.00001

Out[4]= 0.0002

Out[5]= 1

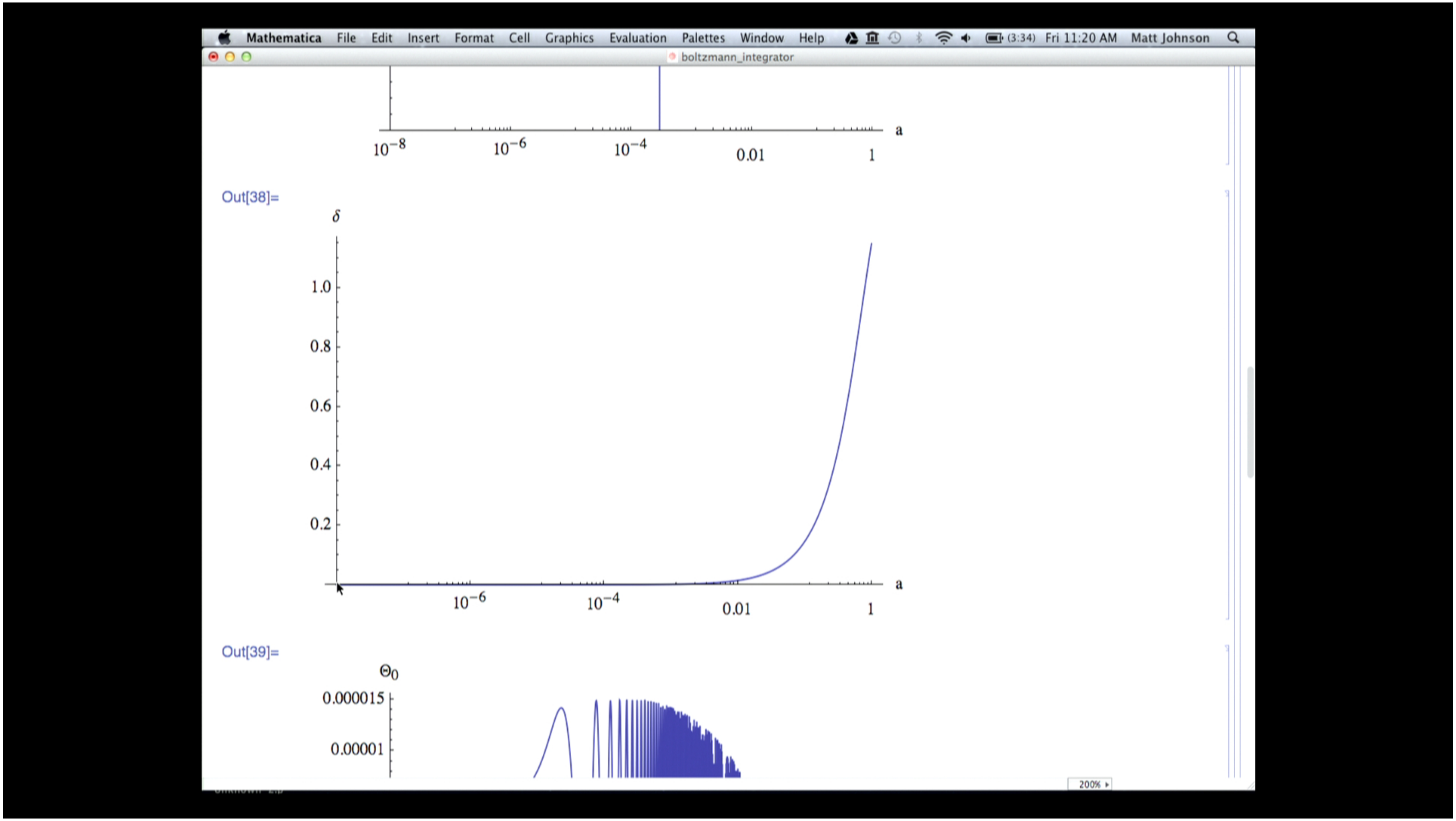
Evolve the perturbation equations.

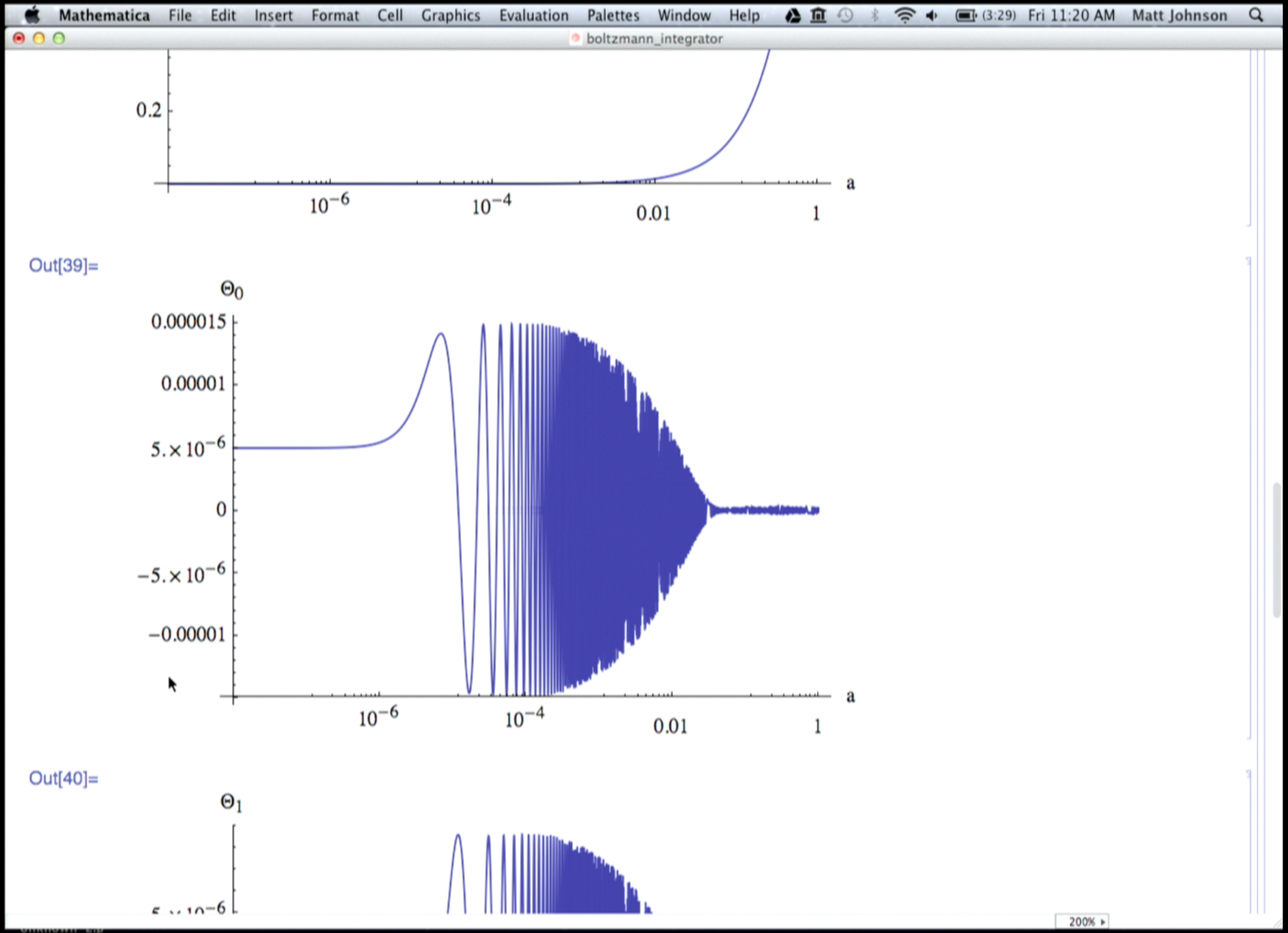
In[6]:= $a0 = 10.^{-8};$

$$\Omega m = \frac{(1 - \Omega\Lambda)}{1 + \frac{1}{1+zeq}};$$

$$\Omega r = \frac{1}{1 + zeq}\,\Omega m;$$

200%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
Fri 11:20 AM Matt Johnson
boltzmann_integrator
Out[38]=
Out[39]=
200%


Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(3:29) Fri 11:20 AM Matt Johnson
boltzmann_integrator
0.2
a
10^−6
10^−4
0.01
1
Out[39]=
Θ0
0.000015
0.00001
5.×10^−6
0
−5.×10^−6
−0.00001
a
10^−6
10^−4
0.01
1
Out[40]=
Θ1
200%

Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help (3:22) Fri 11:21 AM Matt Johnson

boltzmann_integrator

Specify the cosmology via the redshift of matter radiation equality and the current energy density in the cosmological constant

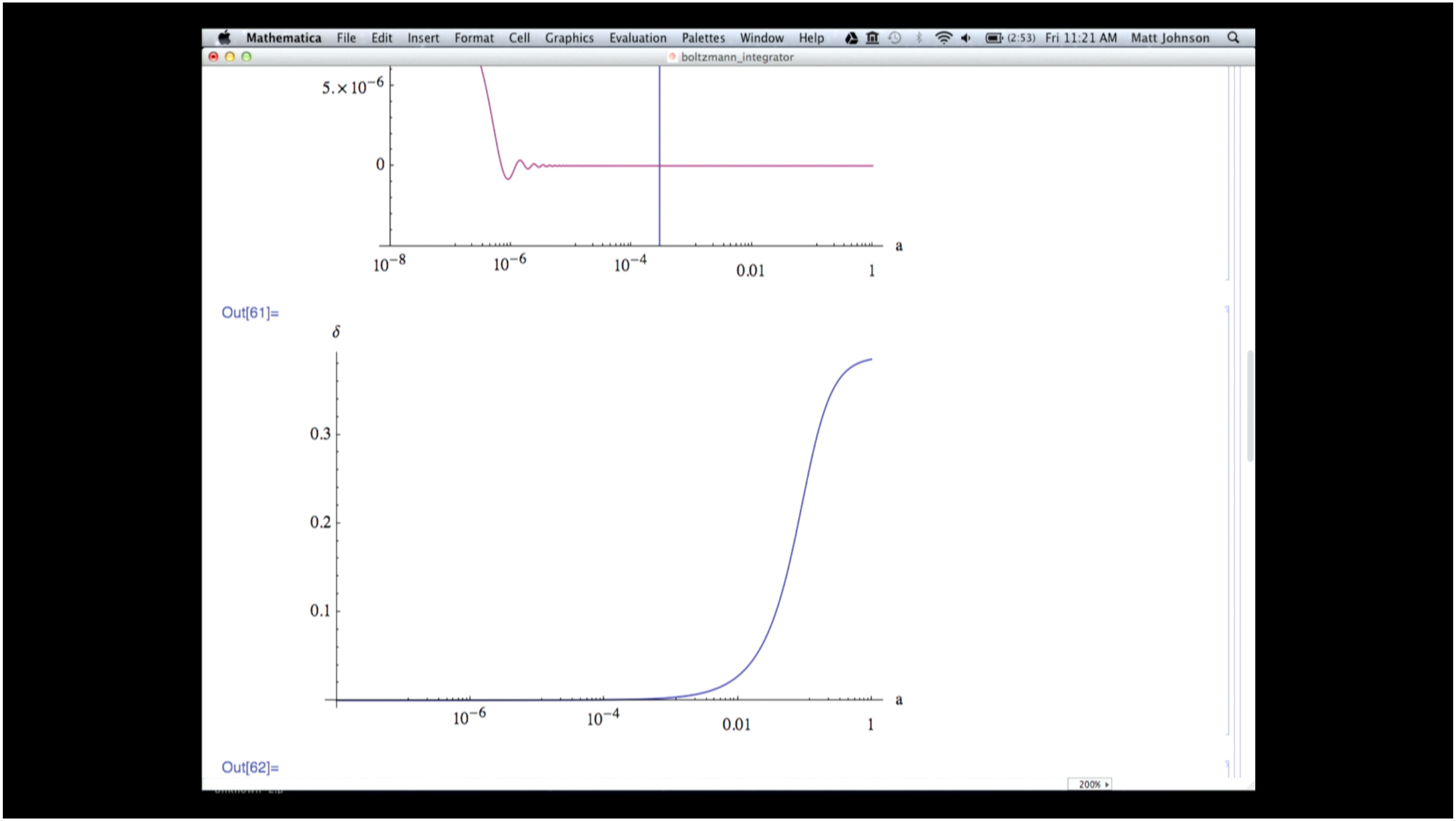
```
In[22]:= zeq = 3402
         ΩΛ = .6825

Out[22]= 3402

Out[23]= 0.6825
```

Specify the amplitude of the primordial potential perturbations, the wavenumber to evolve in units of Mpc^{-1}, and the maximum scale factor to integrate up to (a=1 for today).

```
In[24]:= Φ0 = 10.^-5
         kmpc = 2
         amax = 1

Out[24]= 0.00001

Out[25]= 2

Out[26]= 1
```

Evolve the perturbation equations.

200%



Mathematica File Edit Insert Format Cell Graphics Evaluation Palettes Window Help
(2:53) Fri 11:21 AM Matt Johnson
boltzmann_integrator
5.× 10^−6
0
a
10^−8
10^−6
10^−4
0.01
1
Out[61]=
δ
0.3
0.2
0.1
a
10^−6
10^−4
0.01
1
Out[62]=
200%