

Title: 12/13 PSI - Explorations in Cosmology Lecture 4

Date: Apr 11, 2013 10:15 AM

URL: <http://pirsa.org/13040065>

Abstract:

$$F(\vec{x}, t, \vec{p}) = \frac{1}{e^{-\frac{p}{T(1+\frac{v}{c})}} - 1}$$

$$\textcircled{H} = \textcircled{H}(\vec{x}, t, \hat{p}^i)$$

$$\text{Goal: } \textcircled{H}(\vec{x}_{\text{here}}, t_{\text{now}}, \hat{p}^i) = S$$

Boltzmann eq'n

$$\frac{dF}{dt} = C[F]$$

$$F(\vec{x}, t, \vec{p}) = \frac{1}{e^{\frac{p}{T(1+\Theta)}} - 1}$$

$$\textcircled{H} = \textcircled{H}(\vec{x}, t, \hat{p}^i)$$

$$\text{Goal: } \textcircled{H}(\vec{x}_{\text{new}}, t_{\text{new}}, \hat{p}^i) = \frac{S_T}{T}(\Theta, \varphi)$$

Boltzmann eq'n

$$\frac{dF}{dt} = C[F]$$

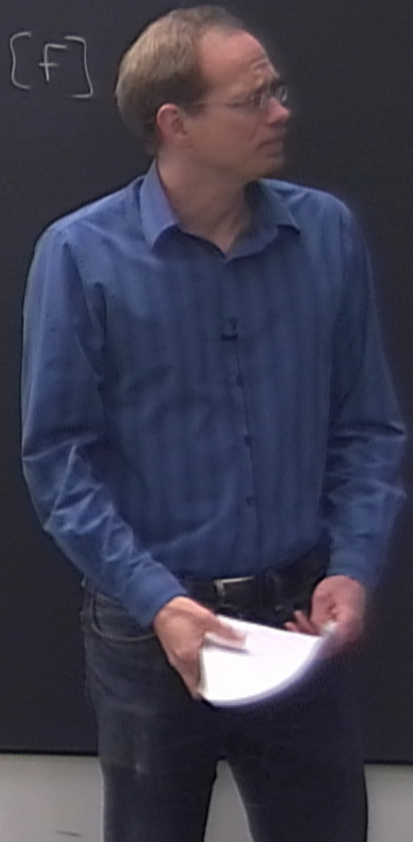
$$F(\vec{x}, t, \vec{p}) = \frac{1}{e^{\frac{p}{T(1+\hat{H})}} - 1}$$

$$\hat{H} = \hat{H}(\vec{x}, t, \hat{p})$$

$$\text{Goal: } \hat{H}(\vec{x}_{\text{new}}, t_{\text{new}}, \hat{p}) = \frac{S_T}{T}(\theta, \varphi)$$

Boltzmann eq'n

$$\frac{dF}{dt} = C[F]$$



$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial F}{\partial x^i} - P \frac{\partial F}{\partial P} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \right]$$

$$F = F^{(0)} - P \frac{\partial F^{(0)}}{\partial P} \quad (1-1)$$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial F}{\partial x^i} - P \frac{\partial F}{\partial P} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \right]$$

$\frac{\partial F}{\partial t}$ $\frac{\partial F}{\partial P}$ $\frac{1}{P} \frac{dP}{dt}$

$$F = F^{(0)} - P \frac{\partial F^{(0)}}{\partial P} \quad (1)$$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial F}{\partial x^i} - P \frac{\partial F}{\partial P} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \right]$$

$$F = F^{(0)} - P \frac{\partial F^{(0)}}{\partial P} \quad (1-1)$$

$$F(\vec{x}, t, \vec{p}) = \frac{1}{e^{\frac{p}{T(1+\Theta)}} - 1}$$

$$\textcircled{H} = \textcircled{H}(\vec{x}, t, \hat{p}^i)$$

$$\text{Goal: } \textcircled{H}(\vec{x}_{\text{new}}, t_{\text{new}}, \hat{p}^i) = \frac{S_T}{T}(\Theta, \Phi)$$

Boltzmann eq'n

$$\frac{dF}{dt} = C[F]$$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial F}{\partial x^i} - P \frac{\partial F}{\partial P} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \right]$$

$$F = F_{(T_M, P)}^{(0)} - P \frac{\partial F^{(0)}}{\partial P} \quad (1-1)$$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \hat{p}_i \frac{\partial F}{\partial x_i} - P \frac{\partial F}{\partial P} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x_i} \right] + \frac{1}{P} \frac{dP}{dt}$$

$$F = F^{(0)}_{(T, P)} \quad (1-1)$$

0th order

$$\rightarrow \frac{dT}{dt} \frac{dF^{(0)}}{dt} - P \frac{\partial F^{(0)}}{\partial P} H$$

$$dF = \frac{\partial F}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial F}{\partial x^i} - P \frac{\partial F}{\partial P} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \right]$$

$$F^{(0)}_{(T, P)} - P \frac{\partial F^{(0)}}{\partial P} \quad (1-1)$$

$$\frac{\partial F}{\partial t} = \frac{\partial T}{\partial t} \frac{\partial F}{\partial T}$$

$$, H P \frac{\partial F}{\partial P}$$

$$\rightarrow \frac{\partial T}{\partial t} \frac{\partial F^{(0)}}{\partial T} - P \frac{\partial F^{(0)}}{\partial P} H = 0$$

$$+ \frac{\partial \bar{\Phi}}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \Bigg]$$

$$\frac{1}{P} \frac{dP}{dt}$$

$$\frac{dT}{dt} \frac{df^{(a)}}{dT} - P \frac{\partial f^{(a)}}{\partial P} H = 0 = - \frac{1}{T} \frac{dT}{dt} P \frac{df^{(a)}}{dP} - P \frac{\partial f^{(a)}}{\partial P} H$$

$$\left[\frac{\partial \bar{\Phi}}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \right]$$

$$\frac{1}{P} \frac{dP}{dt}$$

$$\frac{dT}{dt} \frac{dF^a}{dT} - P \frac{dF^a}{dP} H = 0 = -\frac{1}{T} \frac{dT}{dt} P \frac{dF^a}{dP} H = 1 \frac{dF^a}{dt} \left(-\frac{1}{T} \frac{dT}{dt} - \frac{1}{a} \frac{da}{dt} \right) = 0$$

$$+ \frac{\partial \bar{\Phi}}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \Bigg]$$

$$\frac{1}{P} \frac{dP}{dt}$$

$$\frac{dT}{dt} \frac{df^{(a)}}{dT} - P \frac{\partial f^{(a)}}{\partial P} H = 0 = -\frac{1}{T} \frac{dT}{dt} P \frac{df^{(a)}}{dP} - P \frac{\partial f^{(a)}}{\partial P} H = P \frac{df^{(a)}}{dP} \left(-\frac{1}{T} \frac{dT}{dt} - \frac{1}{a} \frac{da}{dt} \right) = 0$$

$$\frac{\partial F}{\partial x^i} - \rho \frac{\partial F}{\partial \rho} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \right] = 0$$

$\uparrow$
 $\frac{1}{\rho} \frac{d\rho}{dt}$

$\frac{\partial F}{\partial \rho}$ (1-1)

$$\frac{\partial F}{\partial t}, H \rho \frac{\partial F}{\partial \rho} \rightarrow \frac{dT}{dt} \frac{\partial F^{(a)}}{\partial T} - \rho \frac{\partial F^{(a)}}{\partial \rho} H = 0 = -\frac{1}{T} \frac{dT}{dt} \rho \frac{\partial F^{(a)}}{\partial \rho} - \rho \frac{\partial F^{(a)}}{\partial \rho} H = \rho \frac{\partial F^{(a)}}{\partial \rho} \left(-\frac{1}{T} \frac{dT}{dt} - \frac{1}{a} \frac{da}{dt} \right) = 0$$

$$\left[\frac{\partial \Phi}{\partial t} + \frac{\hat{p}_i}{a} \frac{\partial \Psi}{\partial x^i} \right] = 0$$

$$\frac{1}{\rho} \frac{d\rho}{dt}$$

$$\frac{df^{(a)}}{dt} - \rho \frac{\partial f^{(a)}}{\partial \rho} H = 0 = - \left[\frac{df^{(a)}}{dt} - \rho \frac{\partial f^{(a)}}{\partial \rho} H \right] = \rho \frac{df^{(a)}}{d\rho} \left(-\frac{1}{T} \frac{dT}{dt} - \frac{1}{a} \frac{da}{dt} \right) = 0$$

$$T \propto \frac{1}{a}$$

$\frac{1}{P} \frac{dP}{dt}$

$$\frac{dF^{(n)}}{dt} - P \frac{dF^{(n)}}{dP} H = 0 = -\frac{1}{T} \frac{dT}{dt} P \frac{dF^{(n)}}{dF} - P \frac{dF^{(n)}}{dP} H = P \frac{dF^{(n)}}{dP} \left(-\frac{1}{T} \frac{dT}{dt} - \frac{1}{\alpha} \frac{d\alpha}{dt} \right) = 0$$

$$T \propto \frac{1}{\alpha}$$

1st Order

$$- \rho \frac{\partial F^{(1)}}{\partial p} \left[\frac{\partial \langle L \rangle}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \langle L \rangle}{\partial x^i} \right]$$

1st Order

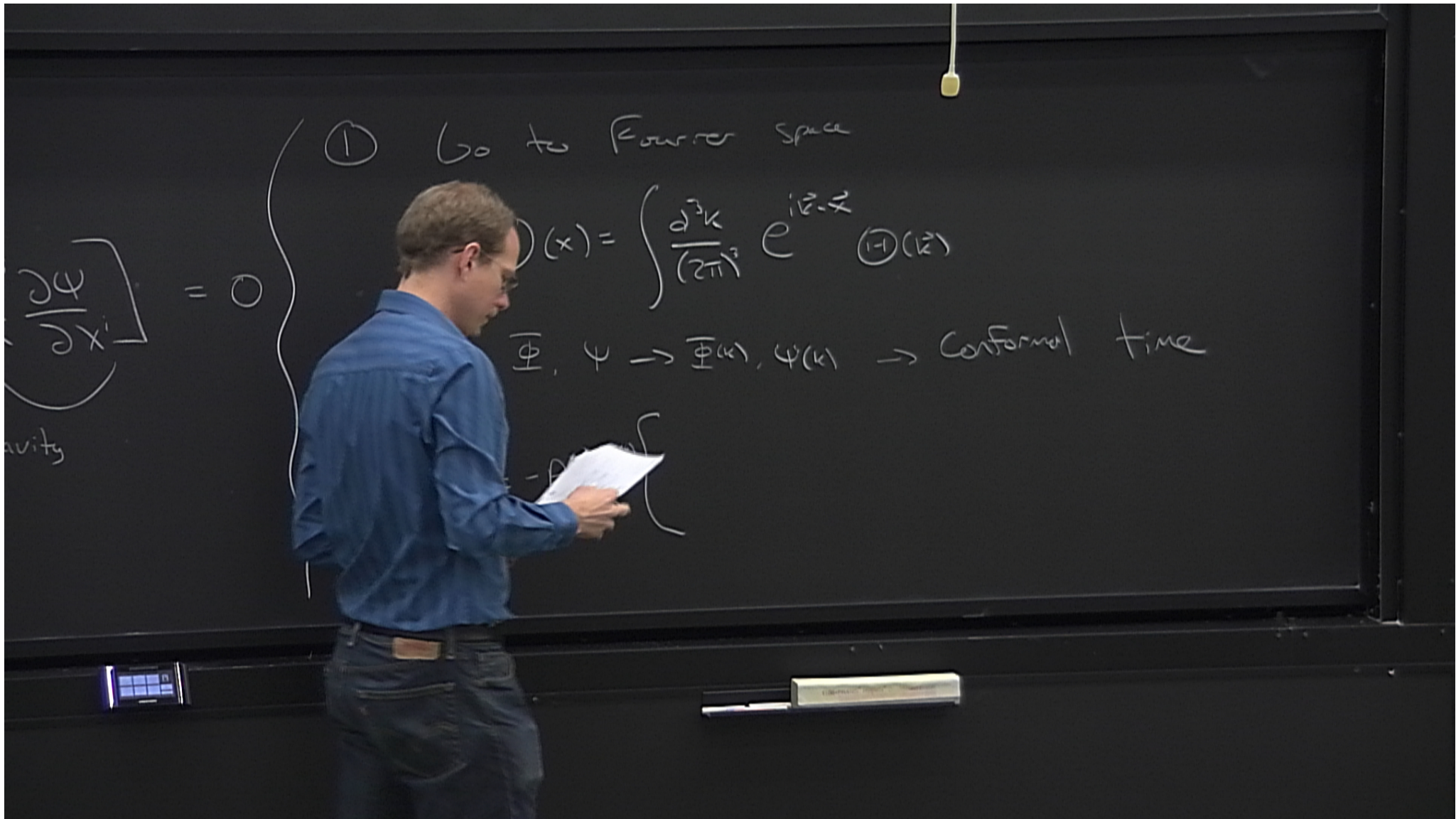
$$\frac{df}{dt} = -p \frac{\partial F^{(n)}}{\partial p} \left[\frac{\partial \mathcal{L}}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \mathcal{L}}{\partial y^i} + \frac{\hat{p}^i}{a} \frac{\partial \mathcal{L}}{\partial x^i} \right] = 0$$

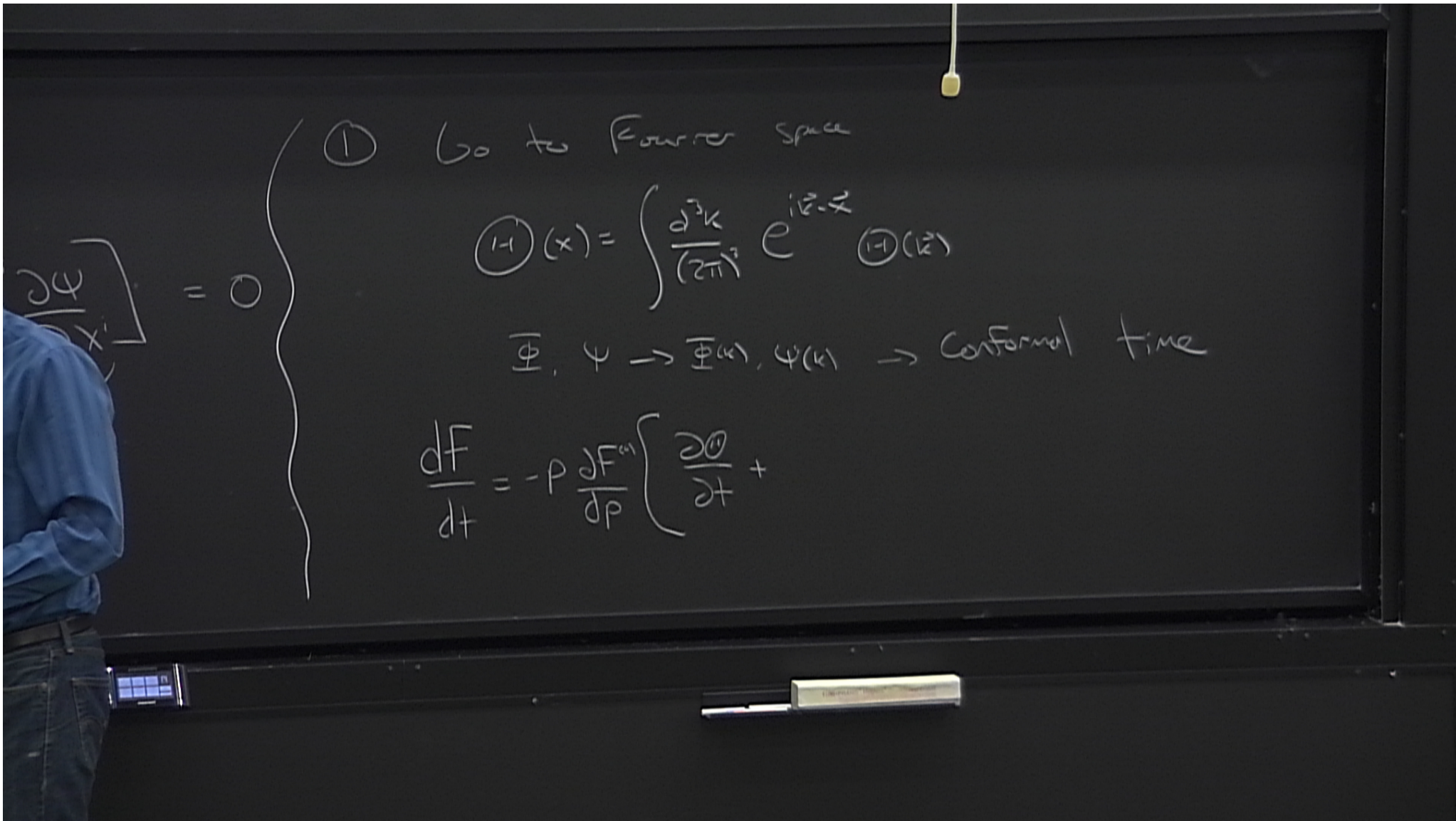
1st Order

$$\frac{df}{dt} = -\rho \frac{\partial f^{(0)}}{\partial p} \left[\underbrace{\frac{\partial \psi}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \psi}{\partial x^i}}_{\text{Free-Streaming}} + \underbrace{\frac{\partial \Phi}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \psi}{\partial x^i}}_{\text{gravity}} \right] = 0$$

1st Order

$$\frac{df}{dt} = -\rho \frac{\partial F^{(1)}}{\partial p} \left[\underbrace{\frac{\partial \Phi}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \Phi}{\partial x^i}}_{\text{Free-Streaming}} + \underbrace{\frac{\partial \Phi}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \Psi}{\partial x^i}}_{\text{gravity}} \right] = 0$$





$$\left[\frac{\partial \psi}{\partial x^i} \right] = 0$$

avity

① Go to Fourier space

$$\textcircled{1-1}(x) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \textcircled{1-1}(k)$$

$\bar{\Phi}, \psi \rightarrow \bar{\Phi}(k), \psi(k) \rightarrow$ conformal time

$$\frac{dF}{dt} = -P \frac{\partial F}{\partial P} \left[\frac{\partial \Theta}{\partial \eta} + i\vec{k} \cdot \hat{p} \Theta + \frac{\partial \bar{\Phi}}{\partial \eta} + \vec{k} \cdot \hat{p} \psi \right]$$

$$\left[\frac{\partial \psi}{\partial x^i} \right] = 0$$

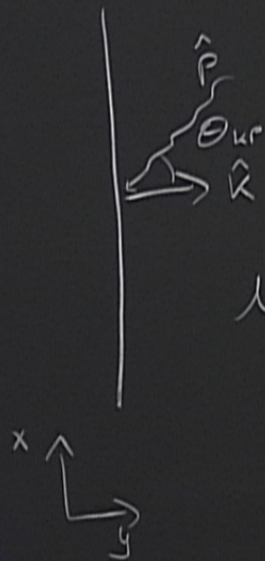
avity

① Go to Fourier space

$$\textcircled{1-1}(x) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \textcircled{1-1}(k)$$

$\bar{\Phi}, \psi \rightarrow \bar{\Phi}(k), \psi(k) \rightarrow$ conformal time

$$\frac{dF}{dt} = -P \frac{\partial F}{\partial P} \left[\frac{\partial \Theta}{\partial \eta} + i\vec{k} \cdot \hat{P} \Theta + \frac{\partial \bar{\Phi}}{\partial \eta} + \vec{k} \cdot \hat{P} \psi \right]$$

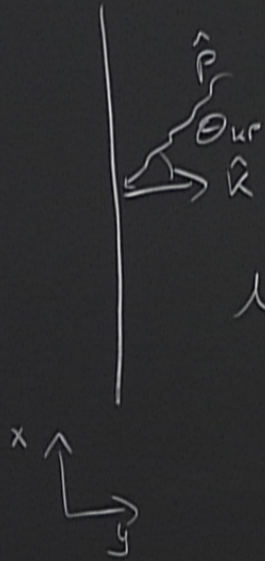


$$\mu = \frac{\vec{k} \cdot \hat{p}}{k}$$

1st Order

$$a \partial \eta = \partial t$$

$$\frac{1}{a} - \rho \frac{\partial F^{(0)}}{\partial \rho} \left[\underbrace{\frac{\partial \psi}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \psi}{\partial x^i}}_{\text{Free-Streaming}} + \underbrace{\frac{\partial \Phi}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \psi}{\partial x^i}}_{\text{gravity}} \right] =$$



$$\mu = \frac{\vec{k} \cdot \hat{p}}{k} = \cos(\theta_{kp})$$

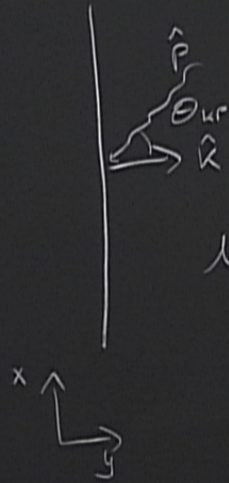
$$\left[\frac{\partial \psi}{\partial x^i} \right] = 0$$

① Go to Fourier space

$$\textcircled{1-1}(x) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \textcircled{1-1}(k)$$

$\bar{\Phi}, \psi \rightarrow \bar{\Phi}(k), \psi(k) \rightarrow$ Conformal time

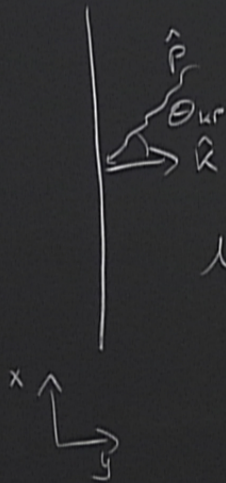
$$\frac{dF}{d\eta} = -P \frac{\partial F}{\partial P} \left[\frac{\partial \Theta}{\partial \eta} + i k \mu \Theta + \frac{\partial \bar{\Phi}}{\partial \eta} + k \mu \psi \right]$$



$$\mu = \frac{\vec{k} \cdot \hat{p}}{k} = \cos(\theta_{kp})$$

In Fourier Space
Azimuthal Symmetry

$$\Phi(k, \mu) = \sum_{\ell} (\ell+1) P_{\ell}(\mu) \Phi_{\ell}(k)$$



$$\mu = \frac{\vec{k} \cdot \hat{P}}{k} = \cos(\theta_{kp})$$

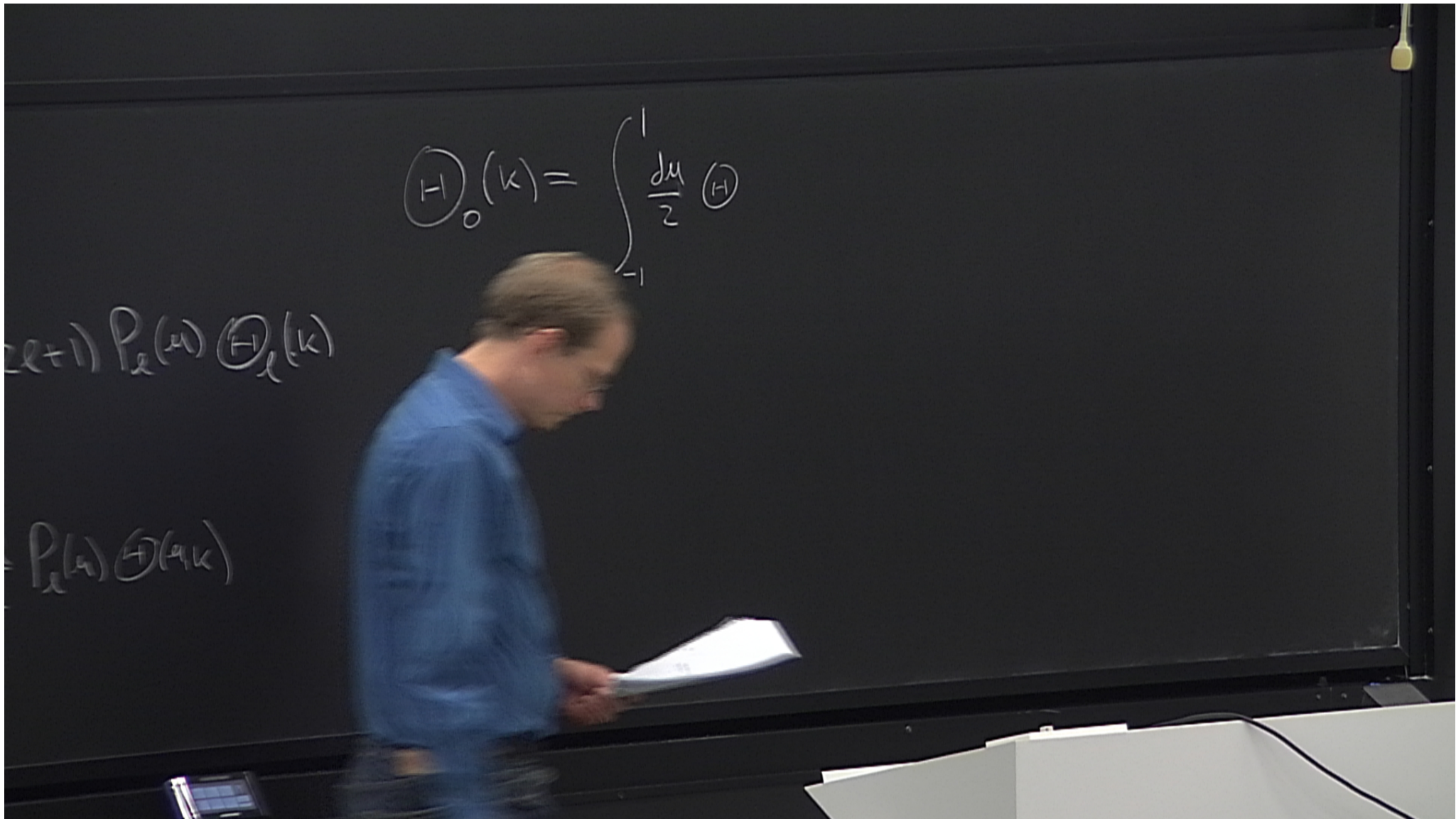
In Fourier Space

Azimuthal Symmetry

$$\textcircled{H}(k, \mu) = \sum_{\ell} (-i)^{\ell} (2\ell+1) P_{\ell}(\mu) \textcircled{H}_{\ell}(k)$$

$$\textcircled{H}_{\ell}(k) = \frac{1}{(-i)^{\ell}} \int_{-1}^1 \frac{d\mu}{2} P_{\ell}(\mu) \textcircled{H}(k, \mu)$$

Moments



$$\Theta_0(k) = \int_{-1}^1 \frac{d\mu}{2} \Theta(\mu) = \frac{1}{4\pi} \int d\Omega \Theta(\hat{p}, +, \vec{x})$$

$$(l+1) P_l(\mu) \Theta_l(k)$$

$$P_l(\mu) \Theta_l(k)$$

$$\left[\frac{\partial \psi}{\partial x^i} \right] = 0$$

① Go to Fourier space

$$\phi(x) = \int \frac{d^3 k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{x}} \phi(k)$$

$\Phi, \psi \rightarrow \Phi(k), \psi(k) \rightarrow$ conformal time

$$\frac{dF}{d\eta} = -\rho \frac{dF}{d\rho} \left[\frac{\partial \Phi}{\partial \eta} + i k \mu \Phi + \frac{\partial \psi}{\partial \eta} + k \mu \psi \right] = -[F]$$

* Compton Scattering

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

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$$e^- + \gamma \leftrightarrow e^- + \gamma$$

$$\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} \sim 7 \times 10^{-25} \text{ cm}^2$$

Mean-Free Path:

$$l = \frac{1}{n_e \sigma_T} \ll \frac{a^3}{\sigma_T}$$

* Compton Scattering

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

$$\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} \sim 7 \times 10^{-25} \text{ cm}^2$$

Mean-Free Path λ

$$\lambda = \frac{1}{n_e \sigma_T} \ll \frac{a^3}{\sigma_T}$$

* Compton Scattering

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

$$\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} \sim 7 \times 10^{-25} \text{ cm}^2$$

Mean-Free Path:

$$l = \frac{1}{n_e \sigma_T} \propto \frac{a^3}{\sigma} \rightarrow 0$$

* Compton Scattering

$$e^- + \gamma \leftrightarrow e^- + \gamma$$

$$\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} \sim 7 \times 10^{-25} \text{ cm}^2$$

Mean-Free Path:

$$l = \frac{1}{n_e \sigma_T} \propto \frac{a^3}{\sigma_T} \rightarrow 0 \text{ in early universe}$$

* Compton Scattering

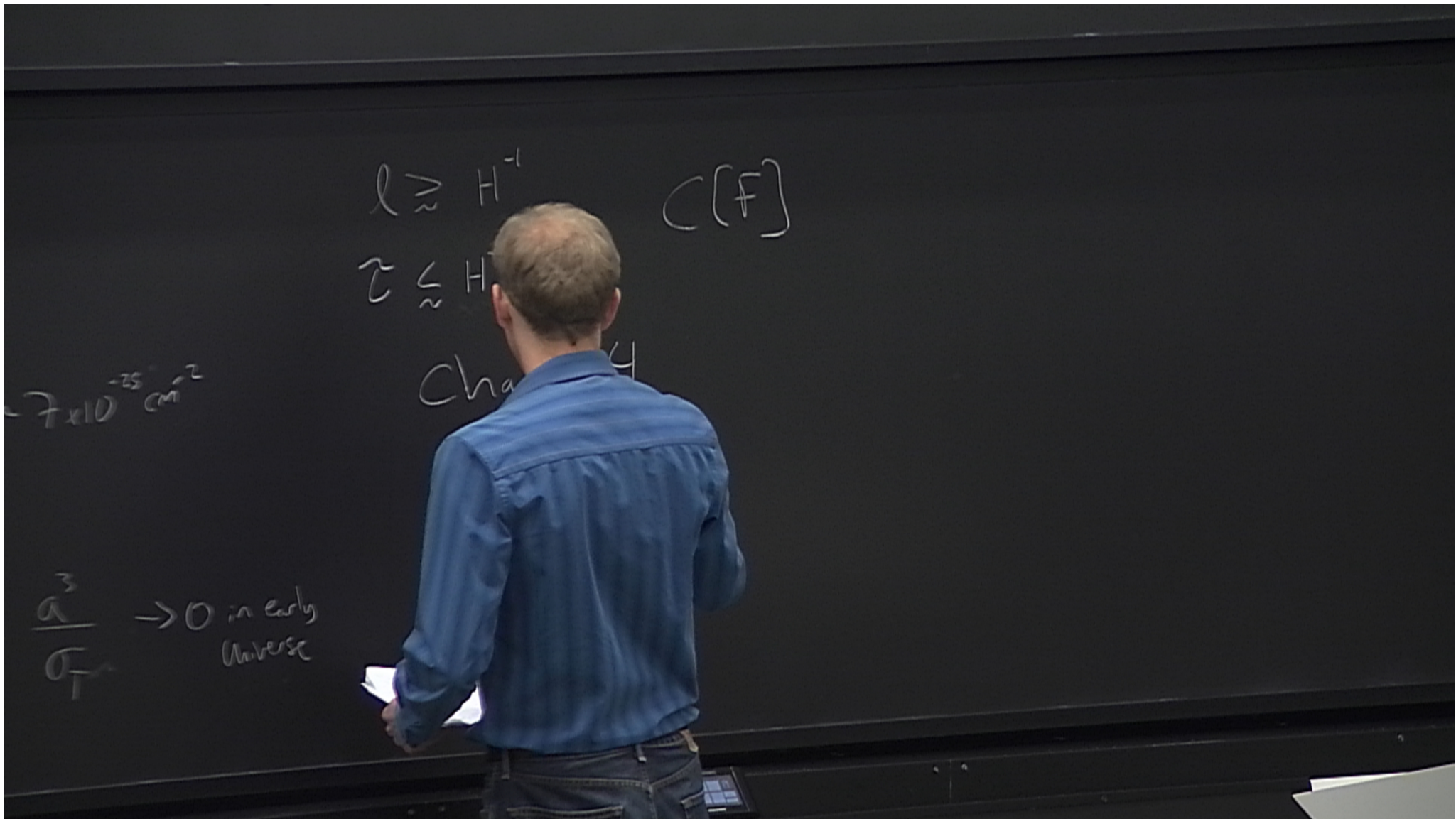
$$e^- + \gamma \leftrightarrow e^- + \gamma$$

$$\sigma_T = \frac{8\pi\alpha^2}{3m_e^2} \sim 7 \times 10^{-25} \text{ cm}^2$$

Mean-Free Path:

$$l = \frac{1}{n_e \sigma_T} < \frac{a^3}{\sigma_T} \rightarrow 0 \text{ in early Universe}$$

$$l \gtrsim H^{-1}$$
$$\tau \lesssim H^{-1}$$



$$l \gtrsim H^{-1}$$

$$\tau \lesssim H^{-1}$$

Chapter 4
Dodelson

$$C(F) = -P \frac{dF^{(0)}}{dP} n_e \sigma_T \left[\langle H_0 \rangle - \langle -1 \rangle + \hat{p} \cdot \vec{V}_b \right]$$

$$l \gtrsim H^{-1}$$

$$\tau \lesssim H^{-1}$$

Chapter 4
Dodelson

$$C(F) = -\rho \frac{\partial F^{(0)}}{\partial \rho} n_e \sigma_T \left[\Theta_0 - \langle \Theta \rangle + \hat{p} \cdot \vec{V}_b \right]$$

$$\frac{\partial \langle \Theta \rangle}{\partial \eta} + i k \mu \langle \Theta \rangle + \frac{\partial \bar{\Phi}}{\partial \eta} + i k \mu \psi = -n_e \sigma_T a \left[\Theta_0 - \langle \Theta \rangle + \hat{p} \cdot \vec{V}_b \right]$$

$$C[F] = -P \frac{\partial F^{(0)}}{\partial P} n_e \sigma_T \left[\langle H \rangle_0 - \langle H \rangle + \hat{p} \cdot \vec{V}_b \right] = \frac{\partial F}{\partial T}$$

$$\boxed{\frac{\partial \langle H \rangle}{\partial \eta} + i k u \langle H \rangle + \frac{\partial \Phi}{\partial \eta} + i k u \psi = -n_e \sigma_T a \left[\langle H \rangle_0 - \langle H \rangle + \hat{p} \cdot \vec{V}_b \right]}$$

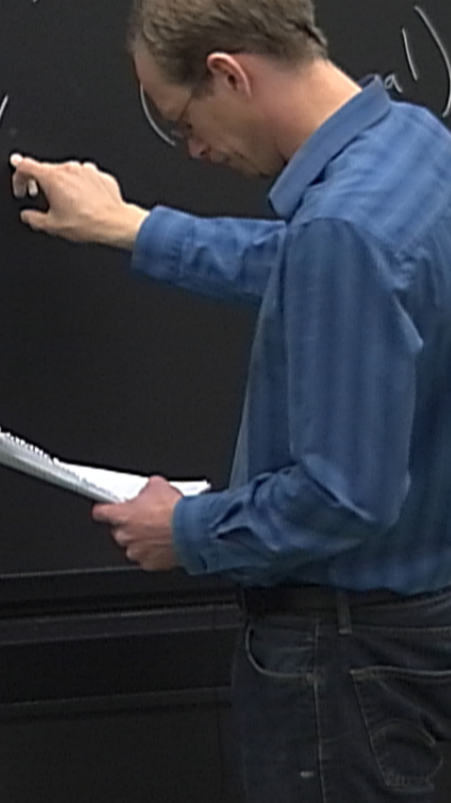
apter-4
Waldson

$$D_u T^u = 0$$
$$P = P(1 + \delta)$$

$$D_{uT}^m = 0$$

$$P = P(1 + \delta)$$

$$V^i = \frac{K^i}{K} V$$



$$\nabla_{\mu} T^{\mu\nu} = 0$$

$$P = P(1 + S)$$

$$V^i = \frac{k^i}{k} V \quad (\text{irrotational})$$

$$\dot{S} + i k V = -3 \dot{\Phi}$$

$$\dot{V} + \frac{\dot{a}}{a} V = -$$

$$\nabla_{\mu} T^{\mu\nu} = 0$$

$$\rho = \rho(1 + \delta)$$

$$V^i = \frac{k^i}{k} V \quad (\text{irrotational})$$

$$\dot{\delta} + ikV = -3\dot{\Phi}$$

$$\dot{V} + \frac{\dot{a}}{a} V = -ik\psi$$

$$\nabla_{\mu} T^{\mu\nu} = 0$$

$$\rho = \rho(1 + \delta)$$

$$V^i = \frac{k^i}{k} V \quad (\text{irrotational})$$

$$\dot{S}_m + i k V_m = -3 \dot{\Phi}$$

$$\dot{V}_m + \frac{\dot{a}}{a} V_m = -i k \Psi$$

$$\dot{S}_b + i k V_b = -3 \dot{\Phi}$$

$$\dot{V}_b + \frac{\dot{a}}{a} V_b = -i k \Psi$$

$$\dot{S}_m + i k V_m = -3 \dot{\Phi}$$

$$\dot{V}_m + \frac{\dot{a}}{a} V_m = -i k \psi$$

$$\dot{S}_b + i k V_b = -3 \dot{\Phi}$$

$$\dot{V}_b + \frac{\dot{a}}{a} V_b = -i k \psi + \frac{\dot{c}}{R} [V_b + 3i \psi]$$

$$R = \frac{3 P_b^{(1)}}{4 P_v^{(1)}}$$

$$k^2 \Phi + 3 \frac{\dot{a}}{a} (\dot{\Phi} - \psi)$$

$$3\dot{\Phi}$$

$$k\psi$$

$$\dot{\Phi}$$

$$k\psi + \frac{\dot{\Sigma}}{R} \left[V_b + 3i(\mu)_1 \right]$$

$$R = \frac{3\rho_b^{(u)}}{4\rho_s^{(v)}}$$

$$K^2 \Phi + 3 \frac{\dot{a}}{a} (\dot{\Phi} - \psi \frac{\dot{a}}{a}) = 4\pi G a^2 T^0_0$$

$$K_i K^i (\Phi + \psi) = 4\pi G a^2 T^i_i$$

$$3\dot{\Phi}$$

$$k^2 \Phi + 3 \frac{\dot{a}}{a} (\dot{\Phi} - \psi \frac{\dot{a}}{a}) = 4\pi G \bar{\rho} T^0_0$$

$$k_i k^i (\Phi + \psi) = 8\pi G \bar{\rho} T^i_i$$

$$\dot{\Phi}$$

$$T^0_0 = -\rho_m^{(i)}(1+\delta_m) - \rho_b^{(i)}(1+\delta_b) + T^{(r)}_0$$

$$k\psi + \frac{\dot{c}}{R} [V_b + 3i(\mu)_i]$$

$$T^0_0 = -2 \left(\frac{\partial^3 \rho}{(2\pi)^3} \rho f = -\rho_\gamma (1+4\theta) \right)$$

$$R = \frac{3\rho_b^{(i)}}{4\rho_\gamma^{(i)}}$$

Moments

$$l \approx H^{-1}$$

$$z \approx H^{-1}$$

$C(F)$

Chapter 4
Dodelson

$$\frac{\partial \langle \dots \rangle}{\partial \alpha}$$

$$i \neq j \quad T^i_j =$$

$$\left(\hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3} \right) \delta^i_j = \hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3}$$

Moments

$l \approx H^{-1}$
 $\tau \approx H^{-1}$
 Cha
 Doo
 $C(F)$
 $\frac{\partial \Theta}{\partial \theta}$

$$i \neq j \quad T^i_j =$$

$$\left(\hat{k}_i \hat{k}^j - \frac{\delta^j_i}{3} \right) \delta^i_j = \hat{k}_i \hat{k}^i - \frac{\delta^i_i}{3} = 0$$

$$\left(\right) T^i_j = 2 \left(\frac{\delta^3 P}{(2\pi)^3} \frac{P^i P_j}{P} f \left(\hat{k}_i \hat{k}^j - \frac{\delta^j_i}{3} \right) \right) = 2$$

Moments

$$l \approx H^{-1}$$

$$z \approx H^{-1}$$

$C[F]$

Chapter 4
Dodelson

$$\frac{\partial \langle \dots \rangle}{\partial \alpha}$$

$$i \neq j \quad T^i_j =$$

$$\left(\hat{k}_i \hat{k}^j - \frac{\delta^j_i}{3} \right) \delta^i_j = \hat{k}_i \hat{k}^i - \frac{\delta^i_i}{3} = 0$$

$$T^i_j = 2 \int \frac{d^3 p}{(2\pi)^3} \frac{p^i p_j}{p} f \left(\hat{k}_i \hat{k}^j - \frac{\delta^j_i}{3} \right) = 2 \int \frac{d^3 p}{(2\pi)^3} \left(\mu^2 - \frac{1}{3} \right) p F$$

Moments

$$l \approx H^{-1}$$

$$z \approx H^{-1}$$

$C[F]$

Chapter 4
Dodelson

$$\frac{\partial \langle \dots \rangle}{\partial \alpha}$$

$$i \neq j \quad T_{ij} =$$

$$\left(\hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3} \right) S^i_j = \hat{k}_i \hat{k}_j - \frac{S^i_i S^j_j}{3} = 0$$

$$T_{ij} = 2 \int \frac{d^3 p}{(2\pi)^3} \frac{p^i p^j}{p} f \left(\hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3} \right) = 2 \int \frac{d^3 p}{(2\pi)^3} \left(\mu^2 - \frac{1}{3} \right) p f$$

$\propto \rho_2 / \mu$

Moments

$$l \gtrsim H^{-1}$$

$$l \lesssim H^{-1}$$

Chapter 4
Dodelson

$$C(F) = -P \frac{\partial F}{\partial P} a n_e \sigma_T \left[\Theta_0 - \Theta + \dots \right]$$

$$\hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3} = 0$$

$$\frac{\partial \Theta}{\partial \eta} + i k \mu \Theta + \frac{\partial \Phi}{\partial \eta} + i k \mu \Psi = -n_e \sigma_T a \left[\Theta_0 - \Theta + \dots \right]$$

$$\dot{\hat{C}} = n_e \sigma_T a \quad \text{opti}$$

$$2 \int \frac{d^3 P}{(2\pi)^3} \frac{P^i P_j}{P} F \left(\hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3} \right) = 2 \int \frac{d^3 P}{(2\pi)^3} \left(\mu^2 - \frac{1}{3} \right) P F$$

$$\uparrow \propto P_2(\mu)$$

$$= -\frac{8}{3}$$

Moments

$$l \approx H^{-1}$$

$$z \approx H^{-1}$$

$$C[F] = -P \frac{\partial F}{\partial P} a n_e \sigma_T \left[\Theta_0 - \Theta + \right.$$

Chapter

Do

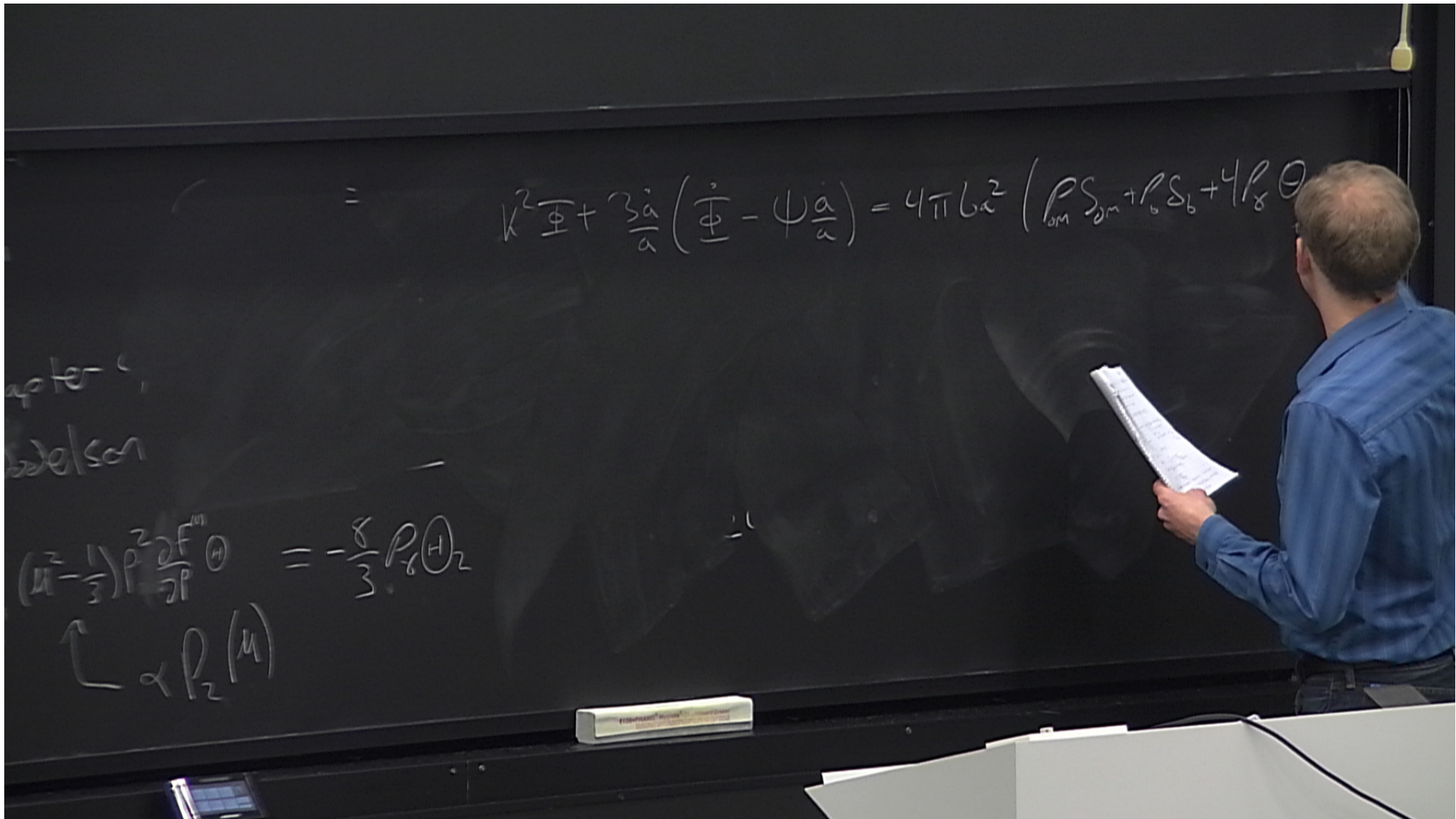
$$\frac{\partial \Theta}{\partial \eta} + i k u \Theta + \frac{\partial \Phi}{\partial \eta} + i k u \Psi = -n_e \sigma_T a \left[\Theta_0 - \right.$$

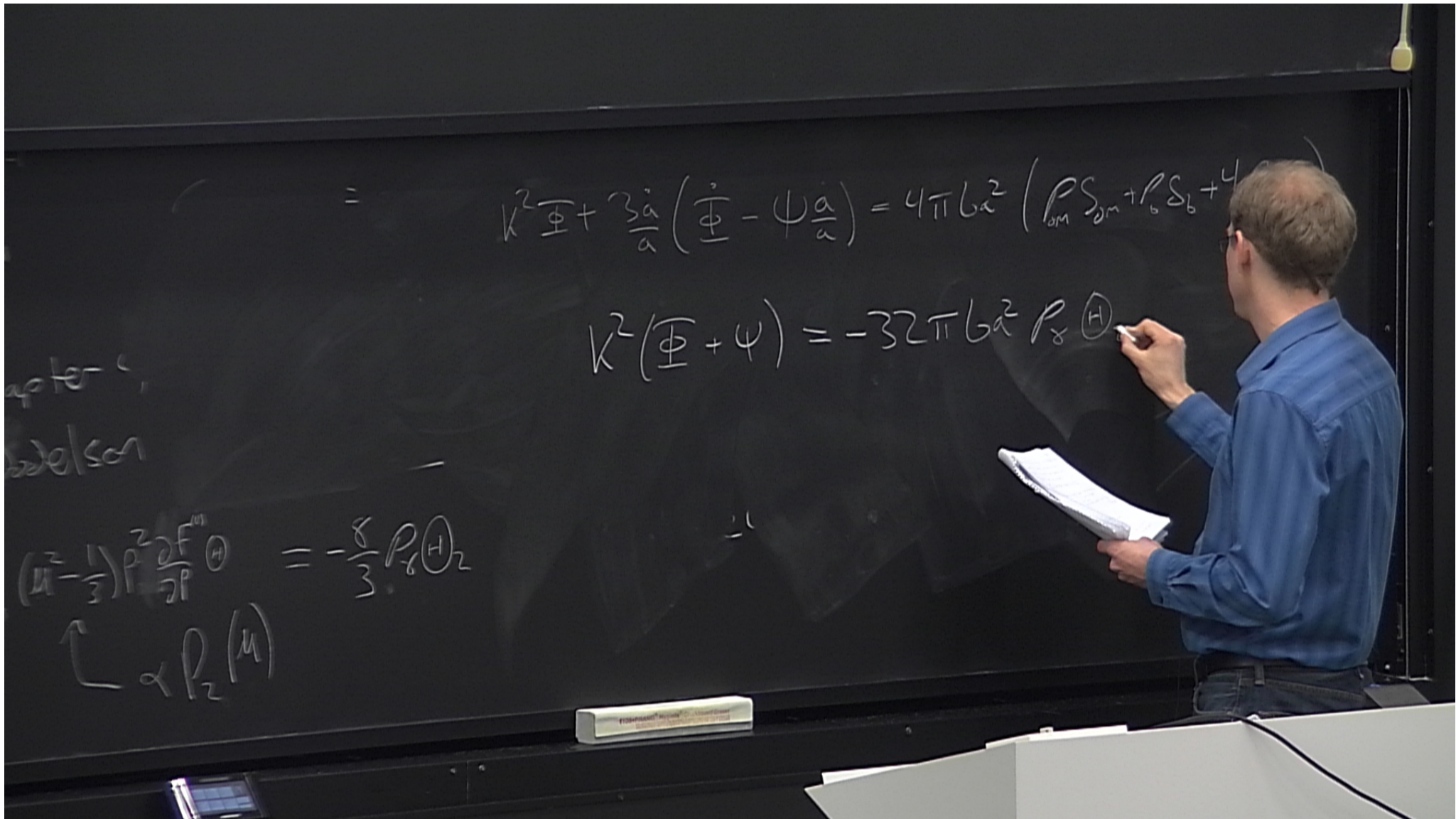
$$\hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3} = 0$$

$$2 \int \frac{d^3 P}{(2\pi)^3} \frac{P^i P_j}{P} f \left(\hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3} \right) = 2 \int \frac{d^3 P}{(2\pi)^3}$$

$$-\frac{8}{3} P_2 \Theta_2$$

$$\dot{C} = n_e \sigma_T a \quad \text{opti}$$





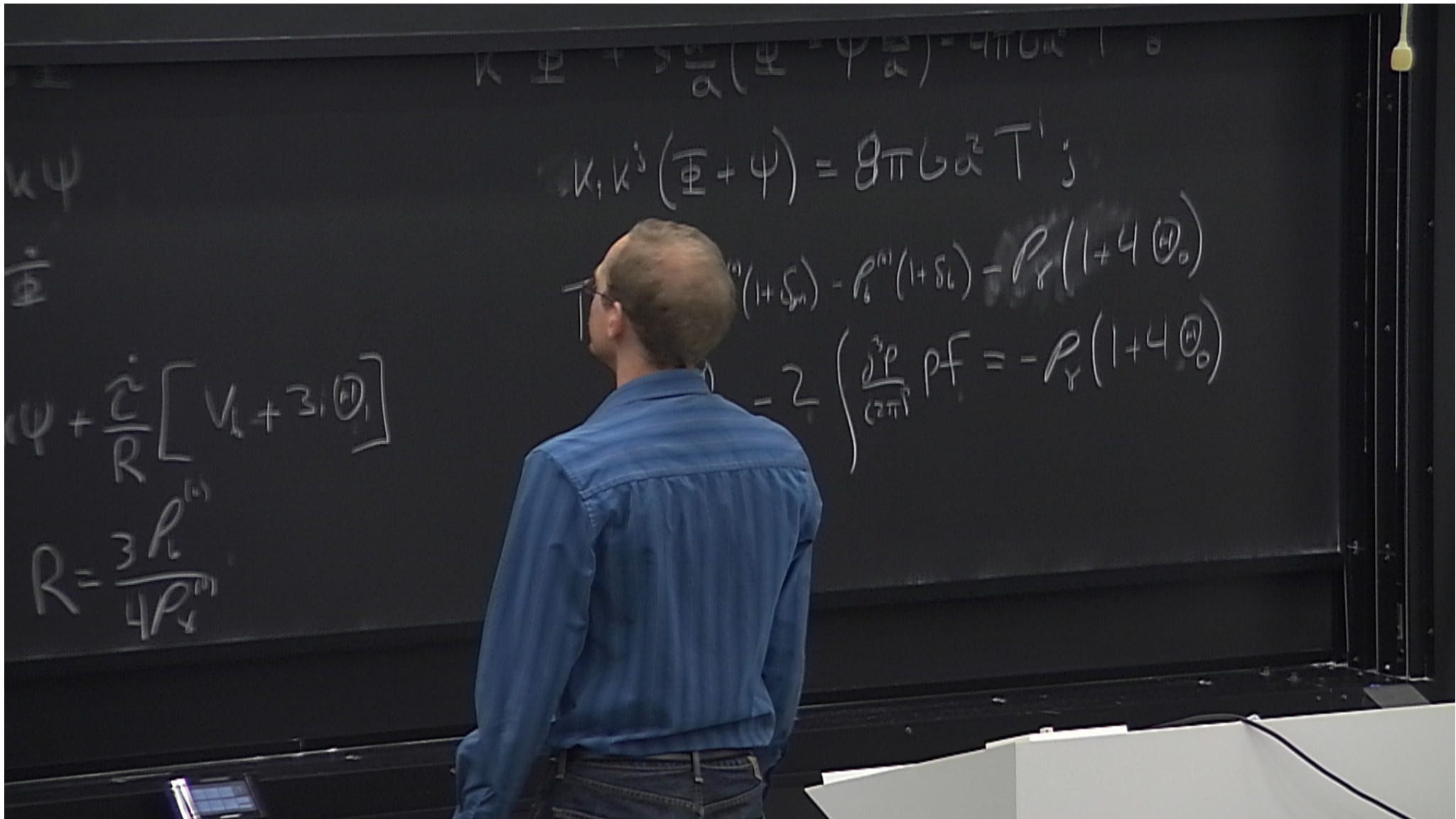
$$= k^2 \Phi + 3 \frac{\dot{a}}{a} \left(\dot{\Phi} - \psi \frac{\dot{a}}{a} \right) = 4\pi G a^2 \left(\rho_m \delta_m + \rho_b \delta_b + 4\rho_r \Theta_0 \right)$$

$$k^2 (\Phi + \psi) = -32\pi G a^2 \rho_r \Theta_2$$

apter 4,
Waldson

$$\left(\mu - \frac{1}{3} \right) \rho^2 \frac{\partial F^m}{\partial \rho} \Theta = -\frac{8}{3} \rho_r \Theta_2$$

$\uparrow$
 $\propto \rho_r(\mu)$



$$K^2 \Phi + 3 \frac{\dot{a}}{a} \left(\dot{\Phi} - \psi \frac{\dot{a}}{a} \right) = 4\pi G a^2 T^0_0$$

$$K_i K^j (\Phi + \psi) = 8\pi G a^2 T^i_j$$

$$T^0_0 = -\rho_m^{(0)}(1+\delta_m) - \rho_b^{(0)}(1+\delta_b) - \rho_r(1+4\Theta_0)$$

$$T^0_{(r)} = -2 \left(\frac{\partial^3 \rho}{(2\pi)^3} \right) p_F = -\rho_r(1+4\Theta_0)$$

$$\frac{P}{a^3} \frac{P_i P_j}{P} F(k_i k_j - \frac{\delta_{ij}}{3}) = 2 \int \frac{d^3 p}{(2\pi)^3} \left(\mu - \frac{1}{3} \right) P \frac{\partial P}{\partial P} = -\frac{1}{3} P_i P_i$$

$\uparrow \propto \rho_2(\mu)$

$$\dot{S}_m + i k V_m = -3 \dot{\Phi}$$

$$\dot{V}_m + \frac{\dot{a}}{a} V_m = -i k \Psi$$

$$\dot{S}_b + i k V_b = -3 \dot{\Phi}$$

$$\dot{V}_b + \frac{\dot{a}}{a} V_b = -i k \Psi + \frac{\dot{c}}{R} \left[V_b + 3 \Phi \right]$$

$$R = \frac{3 \rho_c^{(i)}}{4 \rho_v^{(i)}}$$

$$T^0_0 = -\rho^{(i)}$$

$$T^0_0 = -$$

$$\frac{P}{a^3} \frac{P_i P_j}{P} f(k_i k_j - \frac{\delta_{ij}}{3}) = 2 \left(\frac{\partial}{\partial t} \right)^3 \left(\mu - \frac{1}{3} \right) P \frac{\partial P}{\partial P} = -\frac{1}{3} P \frac{\partial^2 P}{\partial t^2}$$

$\uparrow \propto \rho_2(\mu)$

$$\dot{S}_m + i k V_m = -3 \dot{\Phi}$$

$$\dot{V}_m + \frac{\dot{a}}{a} V_m = -i k \Psi$$

$$\dot{S}_b + i k V_b = -3 \dot{\Phi}$$

$$\dot{V}_b + \frac{\dot{a}}{a} V_b = -i k \Psi + \frac{\dot{c}}{R} \left[V_b + 3 \mu^{(1)} \right]$$

$$R = \frac{3 \rho_c^{(1)}}{4 \rho_s^{(1)}}$$

$$\dot{\mu}^{(1)} + i k \mu^{(1)} + \frac{\partial \Phi}{\partial \eta} + i k \mu \Psi = \dot{c} \left[\mu^{(1)} - \mu^{(0)} + \mu V_b \right]$$

$$\frac{P}{\omega} \frac{P P_i}{P} F(k_i k^2 - \frac{\delta_i}{3}) = 2 \left(\frac{\partial}{\partial \mu} \right)^3 \left(\mu - \frac{1}{3} \right) P \frac{\partial}{\partial P} \psi = -\frac{1}{3} P \psi$$

$\uparrow \propto P_2(\mu)$

$$\dot{S}_m + i k V_m = -3 \dot{\Phi}$$

$$\dot{V}_m + \frac{\dot{a}}{a} V_m = -i k \psi$$

$$\dot{S}_b + i k V_b = -3 \dot{\Phi}$$

$$\dot{V}_b + \frac{\dot{a}}{a} V_b = -i k \psi + \frac{\dot{c}}{R} \left[V_b + 3 \psi \right]$$

$$R = \frac{3 P_c}{4 P_v^{(1)}}$$

$$\dot{\psi} + i k \mu \psi + \frac{\partial \Phi}{\partial \eta} + i k \mu \psi = \dot{c} \left[\psi - \psi + \mu V_b \right]$$

$$) (2\pi) \uparrow \propto P_2(\mu)$$

$$\dot{S}_x + i k V_x = -3 \dot{\Phi}$$

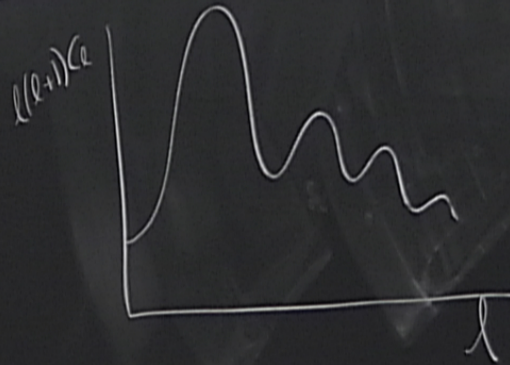
$$\dot{V}_x + \frac{a}{a} V_x = -i k \Psi$$

$$\dot{S}_y + i k V_y = -3 \dot{\Phi}$$

$$\dot{V}_y + \frac{a}{a} V_y = -i k \Psi + \frac{\dot{c}}{R} \left[V_y + 3i \Phi \right]$$

$$R = \frac{3 P_c^{(u)}}{4 P_v^{(u)}}$$

$$\dot{\Phi} + i k \mu \Phi + \frac{\partial \Phi}{\partial \eta} + i k \mu \Psi = \dot{c} \left[\Phi - \Phi + \mu V_y \right]$$



$$) (2\pi) \uparrow \propto P_2(\mu)$$

$$\dot{S}_x + i k V_x = -3 \dot{\Phi}$$

$$\dot{V}_x + \frac{\dot{a}}{a} V_x = -i k \Psi$$

$$\dot{S}_y + i k V_y = -3 \dot{\Phi}$$

$$\dot{V}_y + \frac{\dot{a}}{a} V_y = -i k \Psi + \frac{\dot{c}}{R} \left[V_y + 3i \Phi_y \right]$$

$$R = \frac{3 P_c^{(u)}}{4 P_v^{(u)}}$$

$$\dot{\Phi} + i k \mu \Phi + \frac{\partial \Phi}{\partial \eta} + i k \mu \Psi = \dot{c} \left[\Phi_y - \Phi + \mu V_y \right]$$

