

Title: 12/13 PSI - Explorations in Cosmology Lecture 3

Date: Apr 10, 2013 10:15 AM

URL: <http://pirsa.org/13040064>

Abstract:

Recap

Most general metric w/ scalar fluctuations:

$$ds^2 = a^2(\eta) \left[-(1+2\phi) d\eta^2 + 2\partial_i B \partial X^i d\eta + \right.$$

$$\left. \left[(1+2\psi) \delta_{ij} - 2\partial_i \partial_j E \right] dx^i dx^j \right]$$

Only 2 independent - can change coordinates

$$\tilde{X}^0 = X^0 + \xi$$

$$\tilde{A} = A$$

$$\tilde{B} = B$$

$$\tilde{C} = C$$

$$= E + \xi$$

Gauge fix or use

$$\Phi_A = A + \frac{1}{a} \frac{\partial}{\partial \eta}$$

$$\Phi_H = -C + \dot{a} \left(\frac{\partial}{\partial \eta} \right)$$

Trick: Work in Newtonian

$$A = \psi \quad C = \dots$$

$$\Phi_A = \psi$$

$$\tilde{g}_{\alpha\beta} \frac{\partial X^\alpha}{\partial x^\mu} \frac{\partial X^\beta}{\partial x^\nu} = g_{\mu\nu}$$

Recap

Most general metric w/ scalar fluctuations:

$$ds^2 = a^2(\tau) \left[-(1+2A)d\tau^2 + 2\partial_i B \partial X^i d\tau + \left[(1+2C)\delta_{ij} - 2\partial_i \partial_j E \right] \partial X^i \partial X^j \right]$$

Only 2 independent DOF - can change coordinates

$$\tilde{X}^0 = X^0 + \xi^0(\tau), \quad \tilde{X}^i = X^i + \delta^{ij} \partial_j \xi(x, \tau)$$

$$\tilde{A} = A - \frac{1}{a} \dot{\xi}^0$$

$$\tilde{B} = B - \frac{\xi^0}{a} + \dot{\xi}^i$$

$$\tilde{C} = C - \frac{\dot{\xi}^i}{a} \xi^0$$

$$\tilde{E} = E + \xi$$

Gauge Fix or use

$$\Phi_A = A + \frac{1}{a} \frac{\partial}{\partial \tau} \xi$$

$$\Phi_H = -C + \dot{\xi}$$

Trick: Work in Newtonian

$$A = \psi \quad C = \psi$$

$$\Phi_A = \psi$$

Recap

Most general metric w/ scalar fluctuations:

$$\partial S^2 = \alpha^2(\tau) \left[-(1+2A)\partial\tau^2 + 2\partial_i B \partial\tau \partial x^i + \left((1+2C)S_{ij} - 2\partial_i \partial_j E \right) \partial x^i \partial x^j \right]$$

Only 2 independent DOF - can change coordinates

$$\tilde{X}^0 = X^0 + \xi^0(\tau), \quad \tilde{X}^i = X^i + S^{ij} \partial_j \xi^i(\tau)$$

$$\tilde{A} = A - \frac{1}{\alpha} \dot{\xi}^0, \quad \tilde{E} = E + \xi^0$$

$$\tilde{B} = B - \frac{\dot{\xi}^0}{\alpha} + \dot{\xi}^i$$

$$\tilde{C} = C - \frac{\dot{\xi}^i}{\alpha} \xi^0$$

Gauge Fix or use GI vars:

$$\bar{\Phi}_A = \tilde{A} + \frac{1}{\alpha} \frac{\partial}{\partial \tau} (\alpha (\tilde{E} - \tilde{B}))$$

$$\bar{\Phi}_H = -\tilde{C} + \alpha (\tilde{B} - \tilde{E})$$

Trick: Work in Conformal
Newtonian Gauge:

$$A = \psi, \quad C = \Phi, \quad B = E = 0$$

$$\bar{\Phi}_A = \psi, \quad \bar{\Phi}_H = -\Phi$$

Einstein's Equations & Choose $c = 1$, G .

$$G^{\mu}_{\nu} = 8\pi G T^{\mu}_{\nu}$$

Einstein's Equations & Choose $c = 1$

$$G^{\mu}_{\nu} = 8\pi G T^{\mu}_{\nu}$$

$$G^{\mu}_{\nu}^{(0)} + \delta G^{\mu}_{\nu} = 8\pi G T^{\mu}_{\nu}^{(0)} + 8\pi G \delta T^{\mu}_{\nu}$$

Friedmann eqns

Eqs for Φ, ψ

Recap

Most general metric w/ scalar fluctuations:

$$ds^2 = \alpha^2(\tau) \left[-(1+2A) d\tau^2 + 2\partial_i B \partial x^i d\tau + \left[(1+2C) \delta_{ij} - 2\partial_i \partial_j E \right] \partial x^i \partial x^j \right]$$

Only 2 independent DOF - can change coordinates

$$\tilde{X}^0 = X^0 + \xi^0(\tau), \quad \tilde{X}^i = X^i + \delta^{ij} \partial_j \xi(\tau, x)$$

$$\tilde{A} = A - \frac{1}{\alpha} \dot{\xi}^0, \quad \tilde{E} = E + \xi$$

$$\tilde{B} = B - \frac{\xi^0}{\alpha} + \dot{\xi}$$

$$\tilde{C} = C - \frac{\dot{\xi}}{\alpha} + \frac{\xi^0}{\alpha^2}$$

Gauge Fix or use GI vars:

$$\bar{\Phi}_A = \tilde{A} + \frac{1}{\alpha} \frac{\partial}{\partial \tau} (\alpha (\tilde{E} - \tilde{B}))$$

$$\bar{\Phi}_H = -\tilde{C} (\tilde{B} - \tilde{E})$$

Trick: Work in Conformal

Gauge =

$$B = E = 0$$

$$\bar{\Phi}$$

Einstein's Equations & Choose $c = 1$

$$G^{\mu}_{\nu} = T^{\mu}_{\nu}$$

$$G^{\mu}_{\nu} + S^{\mu}_{\nu} = 8\pi G T^{\mu}_{\nu} + 8\pi G \delta T^{\mu}_{\nu}$$

$\uparrow$ $\uparrow$
 terms
 for Φ, ψ

$c \sim G$

$$\delta L^{\mu}_{\nu} = 8\pi G \delta T^{\mu}_{\nu}$$

$$\cdot = \frac{\partial}{\partial x}$$

$$0-0: \quad \nabla^2 \Phi + 3\frac{\dot{a}}{a} \left(-\dot{\Phi} + \frac{\dot{a}}{a} \Psi \right) = 4\pi G a^2 \delta T^0_0$$

$$0-i, i-j$$

$$\delta T^{\mu}_{\nu}$$

$c \sim G$

$$\delta L^{\mu}_{\nu} = 8\pi G \delta T^{\mu}_{\nu} \quad \bullet = \frac{\partial}{\partial x}$$

$$0-0: \quad \nabla^2 \Phi + 3\frac{\dot{a}}{a}(-\dot{\Phi} + \frac{\dot{a}}{a}\Psi) = 4\pi G a^2 \delta T^0_0$$

$$0-i, i-j$$

$$i \neq j \quad \partial_i \partial_j (\Psi + \Phi) = 8\pi G a^2 \delta T^i_j$$

$$\delta L_{\text{eff}} = 8\pi G \delta T^{\mu}_{\nu} \quad \bullet = \frac{d}{dt}$$

$$0-0: \quad \nabla^2 \Phi + 3\frac{\dot{a}}{a} \left(-\dot{\Phi} + \frac{\dot{a}}{a} \Psi \right) = 4\pi G a^2 \delta T^0_0$$

$$0-i, i-j$$

$$i \neq j \quad \partial_i \partial_j (\Psi + \Phi) = 8\pi G a^2 \delta T^i_j$$

- Spatial constraint eqns

$$T^{\mu}_{\nu} = (\rho + p) u^{\mu} u_{\nu} + p S^{\mu}_{\nu}$$

$$u^{\mu^{(0)}} = \left(\frac{1}{a}, 0, 0, 0 \right)$$

$$g_{\mu\nu} u^{\mu^{(0)}} u^{\nu^{(0)}} = -1$$

$$w/f \quad (1 + S u^0, v^i)$$

$$g_{\mu\nu} u^{\mu} u^{\nu} = g_{00} u^0{}^2 + g_{ij} u^i u^j$$

Recap

Most general metric w/ scalar fluctuations:

$$g_{\mu\nu} \partial S^2 = a^2(\eta) \left[-(1+2A) d\eta^2 + 2\partial_i B \partial X^i \partial \eta + (1+2C) \delta_{ij} - 2\partial_i \partial_j E \partial X^i \partial X^j \right]$$

Only 2 independent DOF - can change coordinates

$$\tilde{X}^0 = X^0 + \xi^0(\vec{x}), \quad \tilde{X}^i = X^i + \delta^{ij} \partial_j \xi^0(\vec{x})$$

$$\tilde{A} = A - \frac{1}{a} \dot{\xi}^0 \quad \tilde{E} = E + \xi$$

$$\tilde{B} = B - \frac{\xi^0}{a} + \dot{\xi}$$

$$\tilde{C} = C - \frac{\dot{\xi}}{a}$$

Gauge Fix or use GI vars:

$$\bar{\Phi}_A = \tilde{A} + \frac{1}{a} \frac{\partial}{\partial \eta} (a(\tilde{E} - \tilde{B}))$$

$$\bar{\Phi}_H = -\tilde{C} + \dot{a}(\tilde{B} - \tilde{E})$$

Trick: Work in Conformal
Newtonian Gauge:

$$A = \psi \quad C = \Phi, \quad B = E = 0$$

$$\bar{\Phi}_A = \psi, \quad \bar{\Phi}_H = -\Phi$$

$$\partial S^2 = a^2 \left[-(1+2\psi) d\eta^2 + (1+2\Phi) \partial \vec{x}^2 \right]$$

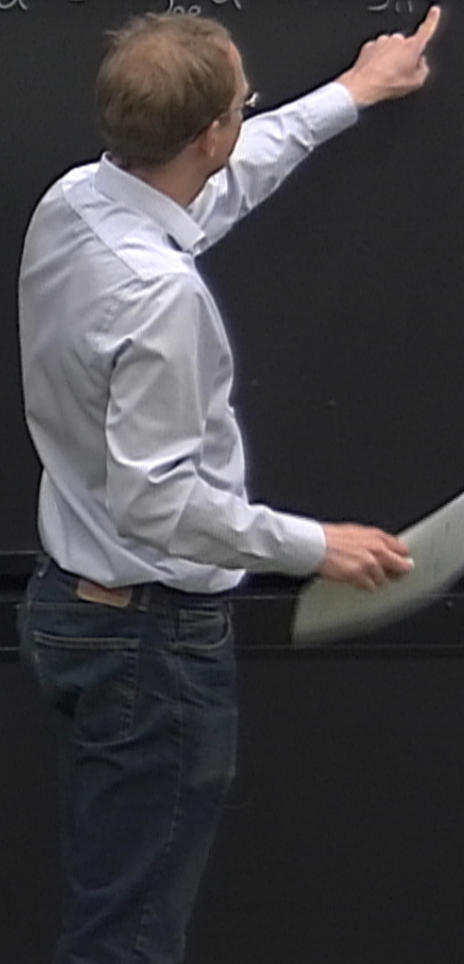
$$T^{\mu}_{\nu} = (\rho + p) u^{\mu} u_{\nu} + p S^{\mu}_{\nu}$$

Velocities: $u^{\mu^{(0)}} = \left(\frac{1}{a}, 0, 0, 0\right)$

$$g_{\mu\nu} u^{\mu^{(0)}} u^{\nu^{(0)}} = -1$$

w/fluct. $u^{\mu} = \frac{1}{a} (1 + \delta u^0, v^i)$

$$g_{\mu\nu} u^{\mu} u^{\nu} = g_{00} u^0{}^2 + g_{ij} v^i v^j$$



$$T^{\mu}_{\nu} = (\rho + p) u^{\mu} u_{\nu} + p S^{\mu}_{\nu}$$

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$$T^{\mu}_{\nu} = (\rho + p) u^{\mu} u_{\nu} + p S^{\mu}_{\nu}$$

Veloc. + cs^2 $u^{\mu^{(0)}} = \left(\frac{1}{a}, 0 \right)$

$$g_{\mu\nu} u^{\mu^{(0)}} u^{\nu^{(0)}} = -1$$

w/fluct. $u^{\mu} = \frac{1}{a} (1 + \delta u^0, \delta u^i)$

$$g_{\mu\nu} u^{\mu} u^{\nu} = g_{00} u^0{}^2 + g_{ij} u^i u^j \xrightarrow{\mathcal{O}(2)}$$

$$-1 = -a^2(1+2\psi)(1+\delta u^0{}^2)$$

$$1 = a^2(1+2\psi+2\delta u^0)$$

$$T^{\mu}_{\nu} = (\rho + p) u^{\mu} u_{\nu} + p \delta^{\mu}_{\nu}$$

Velocities: $u^{(\eta)} = \left(\frac{1}{a}, 0, 0, 0 \right)$

$$g_{\mu\nu} u^{(\eta)\mu} u^{(\eta)\nu} = -1$$

Fluct. $u^{\mu} = \frac{1}{a} (1 + \delta u^0, v^i)$

$$g_{\mu\nu} u^{\mu} u^{\nu} = g_{00} u^0{}^2 + g_{ij} u^i u^j \xrightarrow{O(2)}$$

$$-1 = -\frac{a^2}{\bar{a}^2} (1 + 2\psi) (1 + \delta u^0{}^2)$$

$$1 = \frac{a^2}{\bar{a}^2} (1 + 2\psi + 2\delta u^0)$$

$$\Rightarrow \delta u^0 = -A$$

$$u^{\mu} = \frac{1}{a} (1 - A, v^i)$$

$$g_{\mu\nu} u^\mu u^\nu = g_{00} u^0{}^2 + g_{ij} u^i u^j \quad \text{with } g_{ij} \propto \delta_{ij} \quad \text{and } \mathcal{O}(2)$$

$$-1 = -\frac{\lambda^2}{\kappa} (1+2\psi)(1+8u^0{}^2)$$

$$1 = \frac{\lambda^2}{\kappa} (1+2\psi+28u^0{}^2)$$

$$\Rightarrow 8u^0{}^2 = -A$$

$$u^\mu = \frac{1}{\alpha} (1-A, v^i)$$

$$P = P^{(0)} + \delta P$$

$$P = P^{(0)} + \delta P$$

$$P = wP$$

$$g_{\mu\nu} u^\mu u^\nu = g_{00} u^0{}^2 + g_{ij} u^i u^j \quad \text{with } g_{ij} \propto \delta_{ij} \quad \mathcal{O}(2)$$

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$$u^\mu = \frac{1}{\alpha} (1-A, v^i)$$

$$P = P^{(0)} + \delta P$$

$$P = P^{(0)} + \delta P$$

$$P = w\rho$$

$$S^{\mu}_{\nu} = \begin{pmatrix} -\delta P & (P^{(0)} + P^{(0)}) v_i \\ -(P^{(0)} + P^{(0)}) v_i & \delta_{ij} \delta P \end{pmatrix}$$

$$g_{\mu\nu} u^\mu u^\nu = g_{00} u^0{}^2 + g_{ij} u^i u^j \quad \text{with } g_{ij} \propto \delta_{ij} \quad \mathcal{O}(2)$$

$$-1 = -\frac{\lambda^2}{\kappa} (1+2\psi)(1+8u^0{}^2)$$

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$$u^\mu = \frac{1}{\alpha} (1-A, v^i)$$

$$P = P^{(0)} + \delta P$$

$$P = P^{(0)} + \delta P$$

$$P = w\rho$$

$$ST^\mu{}_\nu = \begin{pmatrix} -\delta P & (P^{(0)} + P^{(0)}) v_i \\ -(P^{(0)} + P^{(0)}) v_i & \delta_{ij} \delta P \end{pmatrix}$$

$$\nabla_\mu ST^{\mu\nu} = 0$$

$$\delta \mathcal{L}_0 = 8\pi G \delta T^{\mu}_{\nu} \quad \bullet = \frac{\partial}{\partial t}$$

$$\nabla^2 \Phi + \frac{1}{a} \left(-\dot{\Phi} + \frac{\dot{a}}{a} \Psi \right) = 4\pi G a^2 \delta T^0_0 = -4\pi G a^2 \delta \rho$$

$$\partial_i \partial_j (\Psi + \Phi) = 8\pi G a^2 \delta T^i_j$$

spatial constraint eqns

$$\delta L^4 = 8\pi G \delta T^4 \quad \bullet = \frac{\partial}{\partial t}$$

$$0-0: \quad \nabla^2 \Phi + 3\frac{\dot{a}}{a} \left(-\frac{\dot{a}}{a} \psi \right) = 4\pi G a^2 \delta T^0_0 = -4\pi G a^2 \delta \rho$$

$$0-i, i-j$$

$$i \neq j$$

$$\partial_i \partial_j (\psi)$$

- Spatial

$$\text{If } \dot{a} = 0$$

$$\nabla^2 \Phi = -4\pi G \delta \rho$$

Poisson Eqn

$$\delta L_{\text{eff}} = 8\pi G \delta T^{\mu}_{\mu} \quad \bullet = \frac{d}{dt}$$

$$0-0: \quad \nabla^2 \Phi + 3\frac{\dot{a}}{a}(-\dot{\Phi} + \frac{\dot{a}}{a}\Psi) - 6a^2 \delta T^0_0 = -4\pi G a^2 \delta \rho$$

If $\dot{a} = 0$

0-i, i-j

i ≠ j

$$\partial_i \partial_j (\Psi + \Phi) = 8\pi G$$

$$\nabla^2 \Phi = -4\pi G \delta \rho$$

Poisson Eqn

$\Phi = -\text{Newtonian Potential}$

- Spatial constraint eqns

$$\delta \mathcal{L}_m = 8\pi G \delta T^{\mu}_{\nu} \quad \bullet = \frac{d}{dt}$$

$$0-0: \quad \nabla^2 \Phi + 3\frac{\dot{a}}{a}(-\dot{\Phi} + \frac{\dot{a}}{a}\Psi) = 4\pi G a^2 \delta T^0_0 = -4\pi G a^2 \rho$$

0-i, i-j

i ≠ j

$$\partial_i \partial_j (\Psi + \Phi) = 8\pi G a^2 \delta T^i_j$$

- Spatial constraint eqns

If $\dot{a} = 0$

Perfect fluid
 $\Phi = \Psi$

$$\nabla^2 \Phi = -4\pi G \rho$$

Poisson Eqn

$\Phi = -\text{Newtonian Potential}$

Work in Phase space!

$$dN = f(\vec{x}, \vec{p}) \frac{d^3x d^3p}{(2\pi)^3}$$

Work in Phase space!

$$dN = f(\vec{x}, t, \vec{p}) \frac{d^3x d^3p}{(2\pi)^3}$$

Bose-Einstein $\circ$

$$f = \frac{1}{e^{(\epsilon - \mu)/T} - 1}$$

Work in Phase space!

$$dN = g f(\vec{x}, t, \vec{p}) \frac{d^3x d^3p}{(2\pi)^3}$$

Bose-Einstein $\circ$

$$f = \frac{1}{e^{(\epsilon - \mu)/T} - 1}$$

In homogeneous case

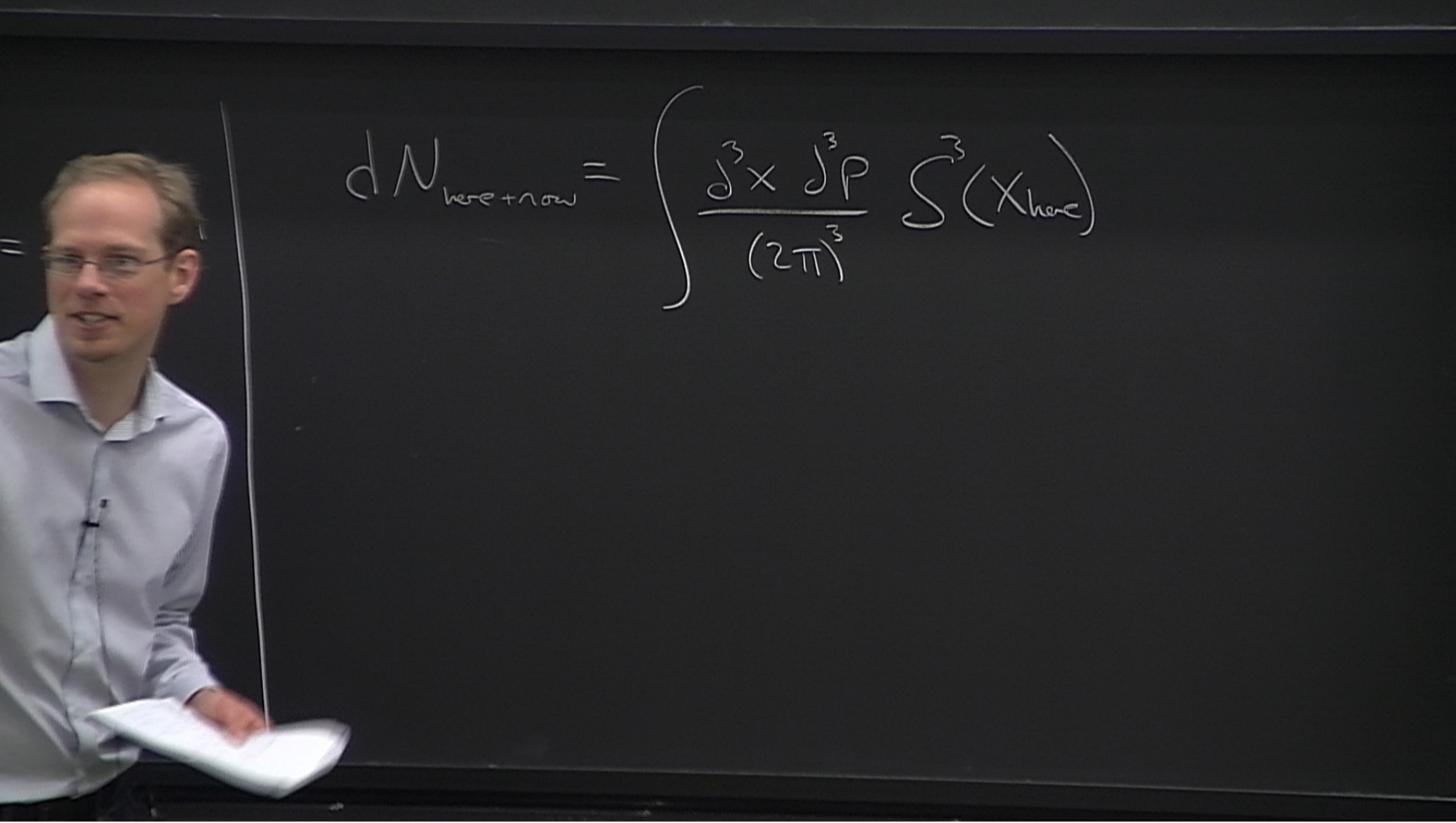
$$P \equiv \frac{E}{V} = g \int \frac{d^3p}{(2\pi)^3} f E, \quad P_x =$$

$$P = g \int \frac{d^3p}{(2\pi)^3} f \frac{p^2}{E}$$

In homogeneous case

$$P = \frac{E}{V} = g \int \frac{\delta^3 P}{(2\pi)^3} f E, \quad P_\gamma = \frac{\pi^2}{15} T^4$$

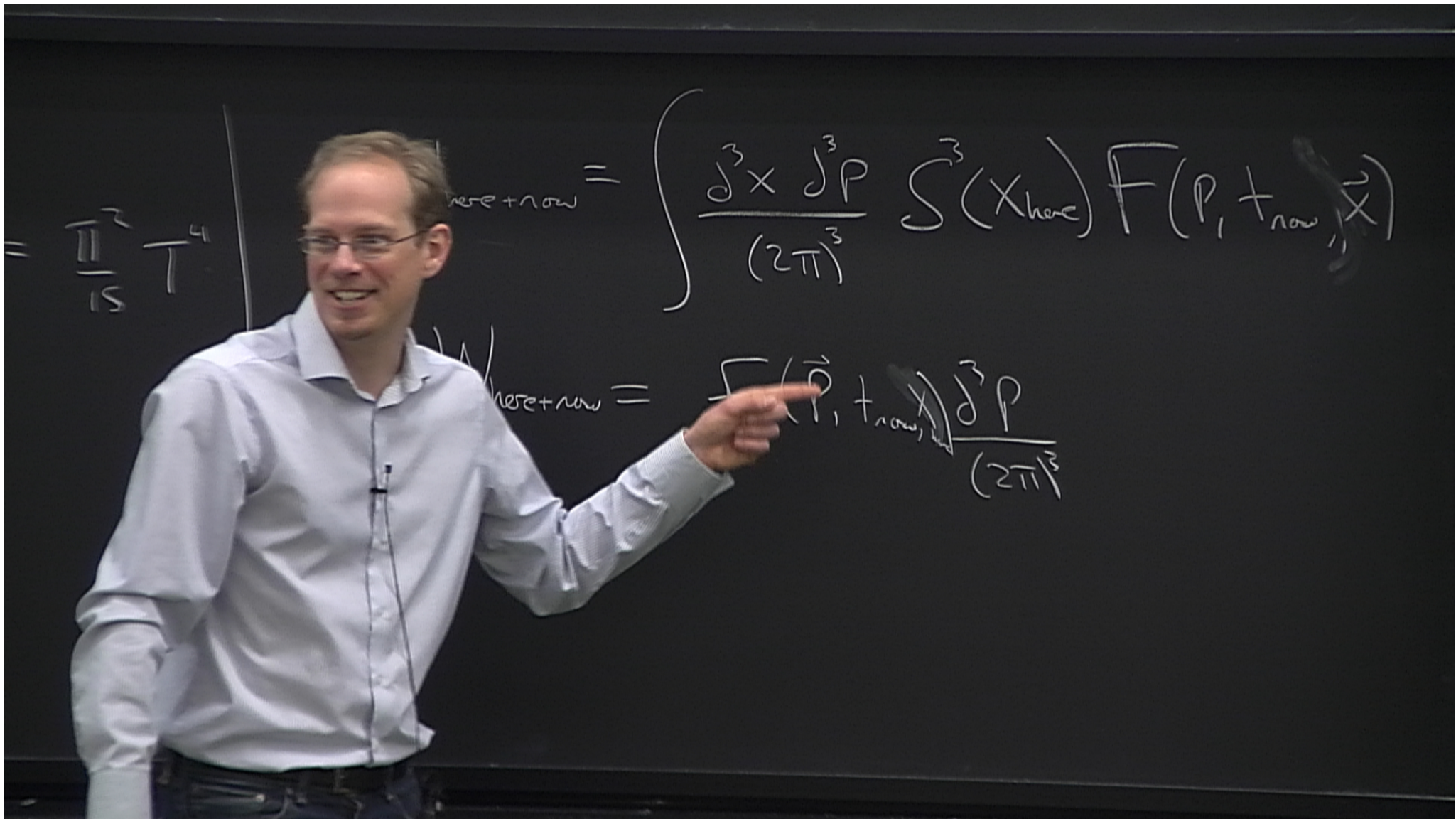
$$P = g \int \frac{\delta^3 P}{(2\pi)^3} f \frac{P^2}{E}$$


$$dN_{\text{here+now}} = \int \frac{d^3x \, d^3p}{(2\pi)^3} \delta^3(x_{\text{here}})$$

$$= \frac{\pi^2}{15} T^4$$

$$dN_{\text{here+now}} = \int \frac{\delta^3 x \delta^3 p}{(2\pi)^3} S^3(x_{\text{here}}) F(p, t_{\text{now}})$$

$$dN_{\text{here+now}} = F(p, t_{\text{now}}) \frac{\delta^3 p}{(2\pi)^3}$$



$$= \frac{\pi^2}{15} T^4$$

$$dN_{\text{here+now}} = \int \frac{d^3x d^3p}{(2\pi)^3} S^3(x_{\text{here}}) F(p, t_{\text{now}}, \vec{x})$$

$$dN_{\text{here+now}} = F(\vec{p}, t_{\text{now}}, \vec{x}) \frac{d^3p}{(2\pi)^3}$$

$$= \frac{\pi^2}{15} T^4$$

$$dN_{\text{here+now}} = \int \frac{d^3x d^3p}{(2\pi)^3} S^3(x_{\text{here}}) F(p, t_{\text{now}}, \vec{x})$$

$$dN_{\text{here+now}} = F(\vec{p}, t_{\text{now}}, x_{\text{here}}) \frac{d^3p}{(2\pi)^3}$$

$$= F(\vec{p}, t_{\text{now}}, x_{\text{here}}) p^2 dp d\Omega \frac{1}{(2\pi)^3}$$

$$= \frac{\pi^2}{15} T^4$$

$$dN_{\text{here+now}} = \int \frac{d^3x d^3p}{(2\pi)^3} S^3(x_{\text{here}}) F(p, t_{\text{now}}, \vec{x})$$

$$dN_{\text{here+now}} = F(\vec{p}, t_{\text{now}}, x_{\text{here}}) \frac{d^3p}{(2\pi)^3}$$

$$\frac{dN_{\text{here+now}}}{dp d\Omega} = F(\vec{p}, t_{\text{now}}, x_{\text{here}}) \frac{p^2}{(2\pi)^3}$$

$$= \frac{\pi^2}{15} T^4$$

$$dN_{\text{here+now}} = \int \frac{d^3x d^3p}{(2\pi)^3} S^3(x_{\text{here}}) F(p, t_{\text{now}}, \vec{x})$$

$$dN_{\text{here+now}} = F(\vec{p}, t_{\text{now}}, x_{\text{here}}) \frac{d^3p}{(2\pi)^3}$$

$$\frac{dN_{\text{here+now}}}{d^3p d\Omega} = F(\vec{p}, t_{\text{now}}, x_{\text{here}}) \frac{p^2}{(2\pi)^3}$$

Homogeneous

$$\frac{dN_{\text{here+now}}}{dP} = \frac{F P^2}{(2\pi)^3} 4\pi$$



Homogeneous

$$\frac{dE_{\text{wave+row}}}{dP} = \frac{F p^3}{(2\pi)^3} 4\pi$$

Blackbody Radiation

Homogeneous

$$\frac{dE_{\text{homogeneous}}}{dP} = \frac{F p^3}{(2\pi)^3} 4\pi$$

Blackbody Radiation

Inhomogeneous

Inhomogeneous

$$f = \frac{1}{e^{\frac{p}{T + (-)}} - 1}$$

$$(-) (\vec{x}, t, \vec{p}) \ll 1$$

Inhomogeneous

$$f = \frac{1}{e^{\frac{p}{T + (-)}} - 1}$$

$$\textcircled{-1} \quad \vec{p} \ll 1$$

$$\approx f^{(0)} - p \frac{\partial f^{(0)}}{\partial p} \textcircled{-1}$$

Inhomogeneous

$$f = \frac{1}{e^{\frac{p}{T + \langle \epsilon \rangle}} - 1}$$

$$\langle \epsilon \rangle (\vec{x}, t, \vec{p}) \ll 1$$

$$\approx f^{(0)} - p \frac{\partial f^{(0)}}{\partial p} \langle \epsilon \rangle$$

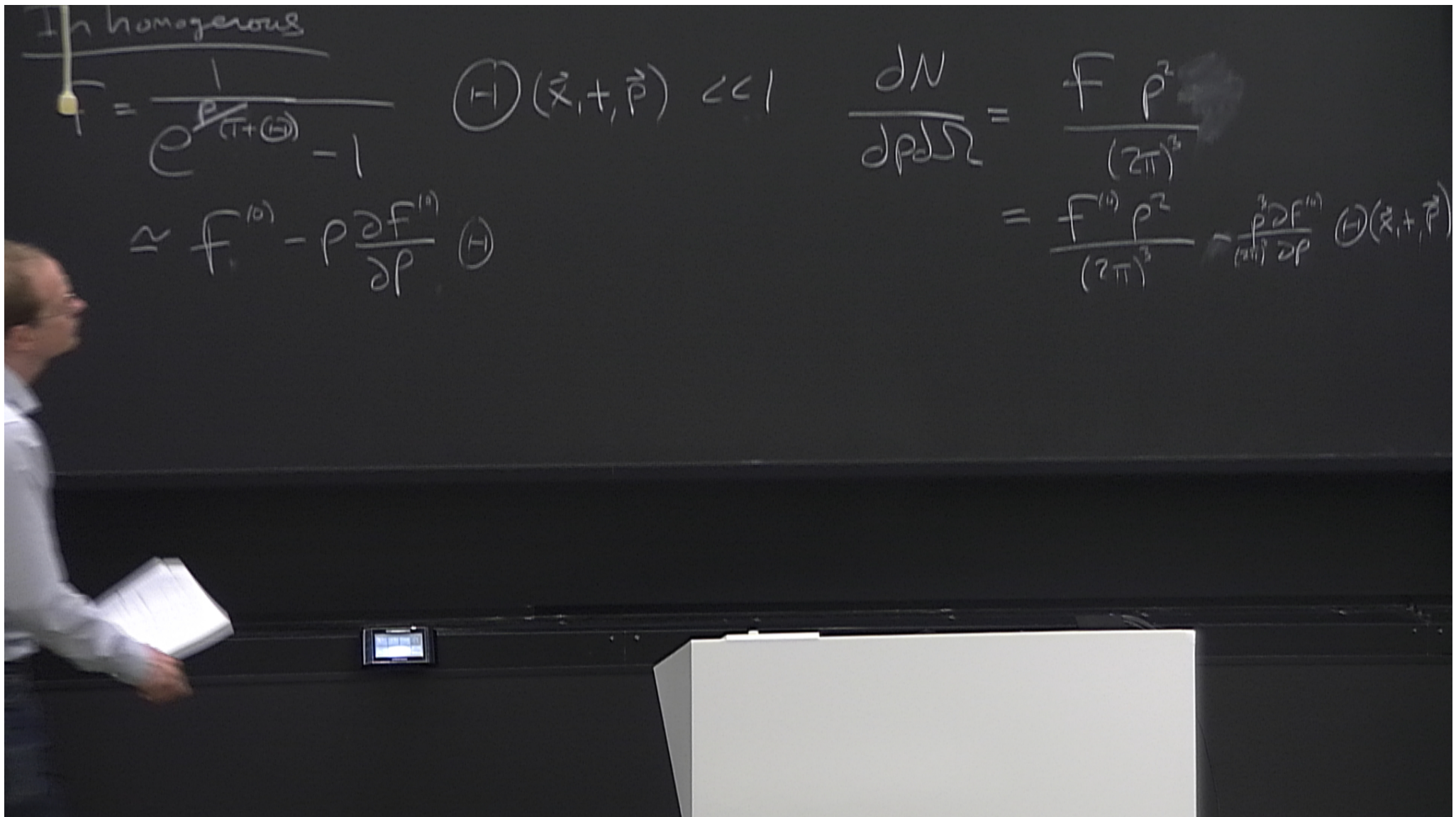
Inhomogeneous

$$f = \frac{1}{e^{\frac{p}{T + \frac{1}{2} \hbar^2}} - 1}$$

$$\textcircled{+} (\vec{x}, t, \vec{p}) \ll 1$$

$$\frac{dN}{dp d\Omega} = \frac{p^2}{(\pi)^3}$$

$$\approx f^{(0)} - p \frac{\partial f^{(0)}}{\partial p} \textcircled{+}$$



In homogenous

$$f = \frac{1}{e^{\frac{p}{T + \mu}} - 1}$$

$$\textcircled{+} (\vec{x}, t, \vec{p}) \ll 1$$

$$\approx f^{(0)} - p \frac{\partial f^{(0)}}{\partial p} \textcircled{+}$$

$$\frac{dN}{dp d\Omega} =$$

$$\frac{f p^2}{(2\pi)^3}$$

$$= \frac{f^{(0)} p^2}{(2\pi)^3} - \frac{p^2 \partial f^{(0)}}{(2\pi)^3 \partial p} \textcircled{+} (\vec{x}, t, \vec{p})$$

$$\text{If } \mathcal{L} = \mathcal{L}(\hat{p}) \rightarrow T = T(\theta, \varphi)$$

$$\text{If } \mathbb{H} = \mathbb{H}(\hat{p}) \rightarrow T = T(\theta, \varphi)$$

If $\mathbb{H} = \mathbb{H}(\vec{p}) \rightarrow \text{Spectral distortions}$

$$\text{If } \mathcal{L} = \mathcal{L}(\hat{p}) \rightarrow T = T(\theta, \varphi)$$

If $\mathcal{L} = \mathcal{L}(\vec{p}) \rightarrow$ Spectral distortions

$$\text{Goal} \rightarrow \mathcal{L}(t_{\text{now}}, \vec{x}_{\text{now}}, \hat{p})$$

$$\text{If } \mathcal{L} = \mathcal{L}(\hat{p}) \rightarrow T = T(\theta, \varphi)$$

If $\mathcal{L} = \mathcal{L}(\vec{p}) \rightarrow$ Spectral distortions

$$\text{Goal} \rightarrow \mathcal{L}(t_{\text{now}}, \vec{x}_{\text{now}}, \hat{p})$$

Boltzmann Equations

$$\frac{dF}{dt} = 0$$

(Collisionless
B-E)

$$\neq 0$$

(Collisional
B-E)

Boltzmann Equations

$$\frac{dF(\mathbf{x}, \mathbf{p})}{dt} = 0$$

(Collisionless
B-E)

$$\neq 0$$

(Collisional
B-E)

Boltzmann Equations

$$\frac{dF(t, \vec{x}, \vec{p})}{dt} = 0$$

$$\neq 0$$

(unless F)

$$dS^2 = -(1+2\psi)dt^2 + a^2(t)(1+2\Phi)d\vec{x}^2$$

Boltzmann Equations

$$\frac{dF(\vec{x}, \vec{p})}{dt} = 0$$

(Collisionless
B-E)

$$\neq 0$$

(Collisional
B-E)

$$dS^2 = -(1+2\psi)dt^2 + a^2(t)(1+2\Phi)d\vec{x}^2$$



Boltzmann Equations

$$\frac{dF(t, \vec{x}, \vec{p})}{dt} = 0$$

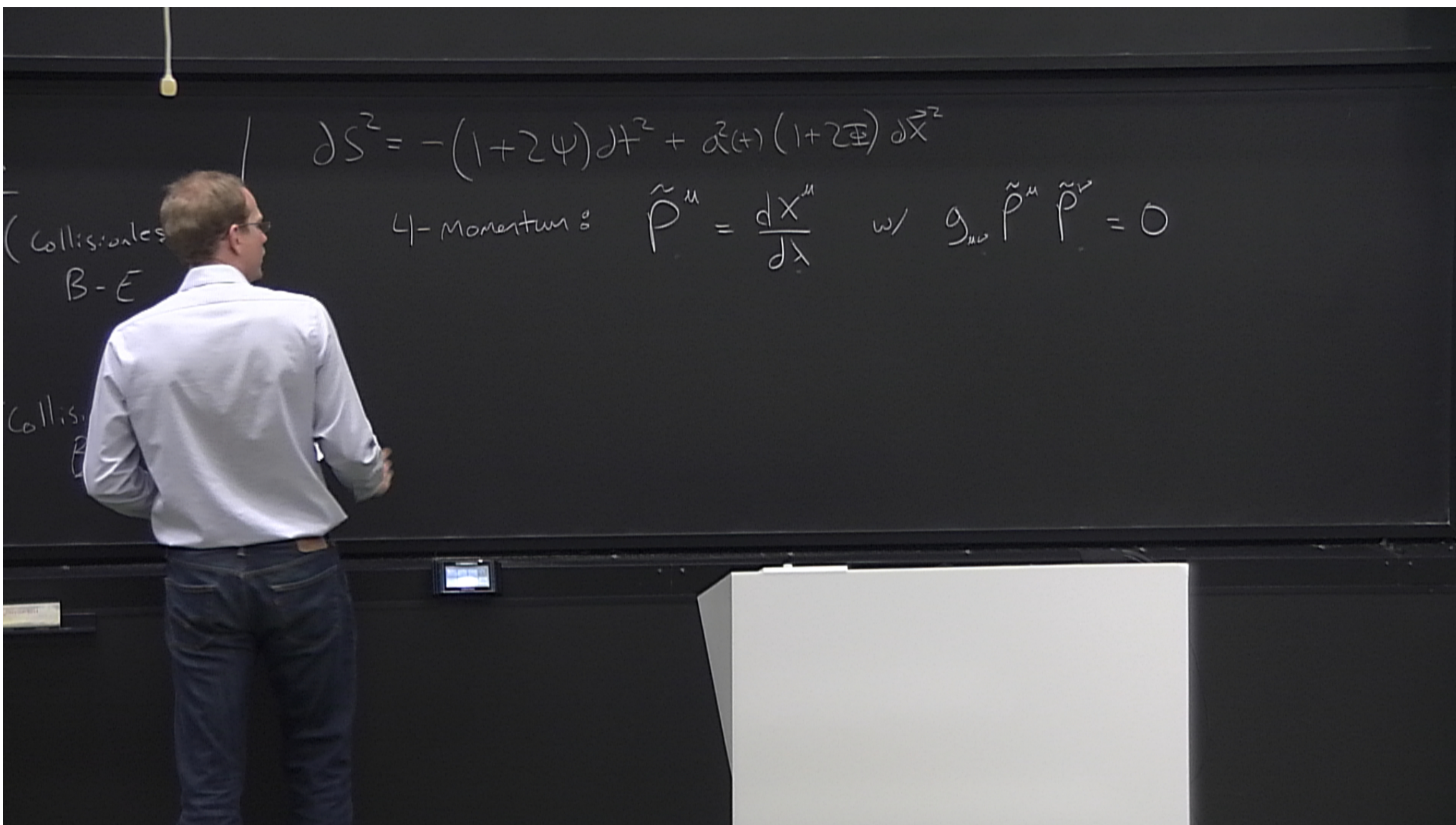
(Collisionless
B-E)

$$\neq 0 \quad (\text{Collisional})$$

B-E

$$dS^2 = -(1+2\Phi)dt^2 + a^2(t)(1+2\Psi)d\vec{x}^2$$

4-momentum: $\tilde{p}^\mu = \frac{dx^\mu}{d\lambda}$



$$dS^2 = -(1+2\psi)dt^2 + a^2(t)(1+2\Phi)d\vec{x}^2$$

4-momentum: $\tilde{p}^\mu = \frac{dx^\mu}{d\lambda}$ w/ $g_{\mu\nu} \tilde{p}^\mu \tilde{p}^\nu = 0$

(Collisionless)
B-E

(Collisional)
B-E

$$dS^2 = -(1+2\psi)dt^2 + a^2(t)(1+2\Phi)d\vec{x}^2$$

4-momentum: $\tilde{p}^\mu = \frac{dx^\mu}{d\lambda}$ w/ $g_{\mu\nu} \tilde{p}^\mu \tilde{p}^\nu$

$$-(1+2\psi)\tilde{p}^0{}^2 + \underbrace{a^2(1+2\Phi)}_{\text{metric}} \tilde{p}^i{}^2 = 0$$

(Collisionless)
B-E

(Collisional)
B-E

$$dS^2 = -(1+2\psi)dt^2 + a^2(t)(1+2\Phi)d\vec{x}^2$$

4-momentum: $\tilde{p}^\mu = \frac{dx^\mu}{d\lambda}$ w/ $g_{\mu\nu} \tilde{p}^\mu \tilde{p}^\nu = 0$

$$-(1+2\psi)\tilde{p}^0{}^2 + \underbrace{a^2(1+2\Phi)\tilde{p}^i{}^2}_{p^2} = 0 \rightarrow \tilde{p}^0 = p(1-\psi) + \mathcal{O}(1)$$

(Collisionless)
B-E

(Collisional)
B-E

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$$\tilde{p}^i = \frac{p\hat{p}^i}{a}(1-\Phi) \quad p^2$$


$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} \frac{dx^i}{dt} + \frac{\partial f}{\partial p} \frac{dp}{dt} + \frac{\partial f}{\partial \hat{p}^i} \frac{d\hat{p}^i}{dt}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x^i} \frac{dx^i}{dt} + \frac{\partial f}{\partial p} \frac{dp}{dt} + \frac{\partial f}{\partial \hat{p}^i} \frac{d\hat{p}^i}{dt}$$

$$+ \frac{\partial f}{\partial x^i} \frac{dx^i}{dt} \quad (1)$$

$$- \frac{dP}{dt} + \frac{\partial F}{\partial \hat{p}^i} \frac{d\hat{p}^i}{dt} \quad (2) \quad (3)$$

$$(1) \quad \frac{dx^i}{dt} = \frac{dt}{d\lambda} \frac{dx^i}{d\lambda} = \frac{\tilde{p}^i}{\tilde{p}^0}$$

$$+ \frac{\partial f}{\partial x'} \frac{dx'}{dt} + \frac{\partial f}{\partial p} \frac{dp}{dt} + \frac{\partial f}{\partial \hat{p}'} \frac{d\hat{p}'}{dt}$$

(2) (3)

$$\textcircled{1} \quad \frac{dx'}{dt} = \frac{dx}{dt} \frac{dx'}{dx} = \frac{\tilde{p}}{\tilde{p}'}$$

$$+ \frac{\partial f}{\partial x'} \frac{dx'}{dt} + \frac{\partial f}{\partial \hat{p}'} \frac{d\hat{p}'}{dt}$$

① ② ③

$$\textcircled{1} \quad \frac{dx'}{dt} = \frac{dx}{dt} \frac{dx'}{dx} = \frac{\tilde{p}'}{\tilde{p}^0} = \frac{\hat{p}'}{a} (1 - \Phi + \Psi)$$

$$\begin{aligned}
 & + \frac{\partial f}{\partial x'} \frac{dx'}{dt} + \frac{\partial f}{\partial p} \frac{dp}{dt} + \frac{\partial f}{\partial \hat{p}'} \frac{d\hat{p}'}{dt} \\
 & \quad \textcircled{1} \qquad \qquad \textcircled{2} \qquad \qquad \textcircled{3}
 \end{aligned}$$

$$\frac{dx'}{dt} = \frac{dx}{dt} \frac{dx'}{dx} = \frac{\tilde{p}}{\tilde{p}'} = \frac{\hat{p}'}{a} (1 - \Phi + \Psi)$$

$$\frac{d\tilde{p}'}{dx}$$

$$+ \frac{\partial f}{\partial x^i} \frac{dx^i}{dt} + \frac{\partial f}{\partial p} \frac{dp}{dt} + \frac{\partial f}{\partial \hat{p}^i} \frac{d\hat{p}^i}{dt}$$

① ② ③

$$\textcircled{1} \quad \frac{dx^i}{dt} = \frac{dx^i}{d\lambda} \frac{d\lambda}{dt} = \frac{\tilde{p}^i}{\tilde{p}^0} = \frac{\hat{p}^i}{a} (1 - \Phi + \Psi)$$

$$\textcircled{2} \quad \frac{d\tilde{p}^0}{d\lambda} = - \Gamma_{\alpha\beta}^0 \tilde{p}^\alpha \tilde{p}^\beta = \frac{d\tilde{p}^0}{dt} \tilde{p}^0$$

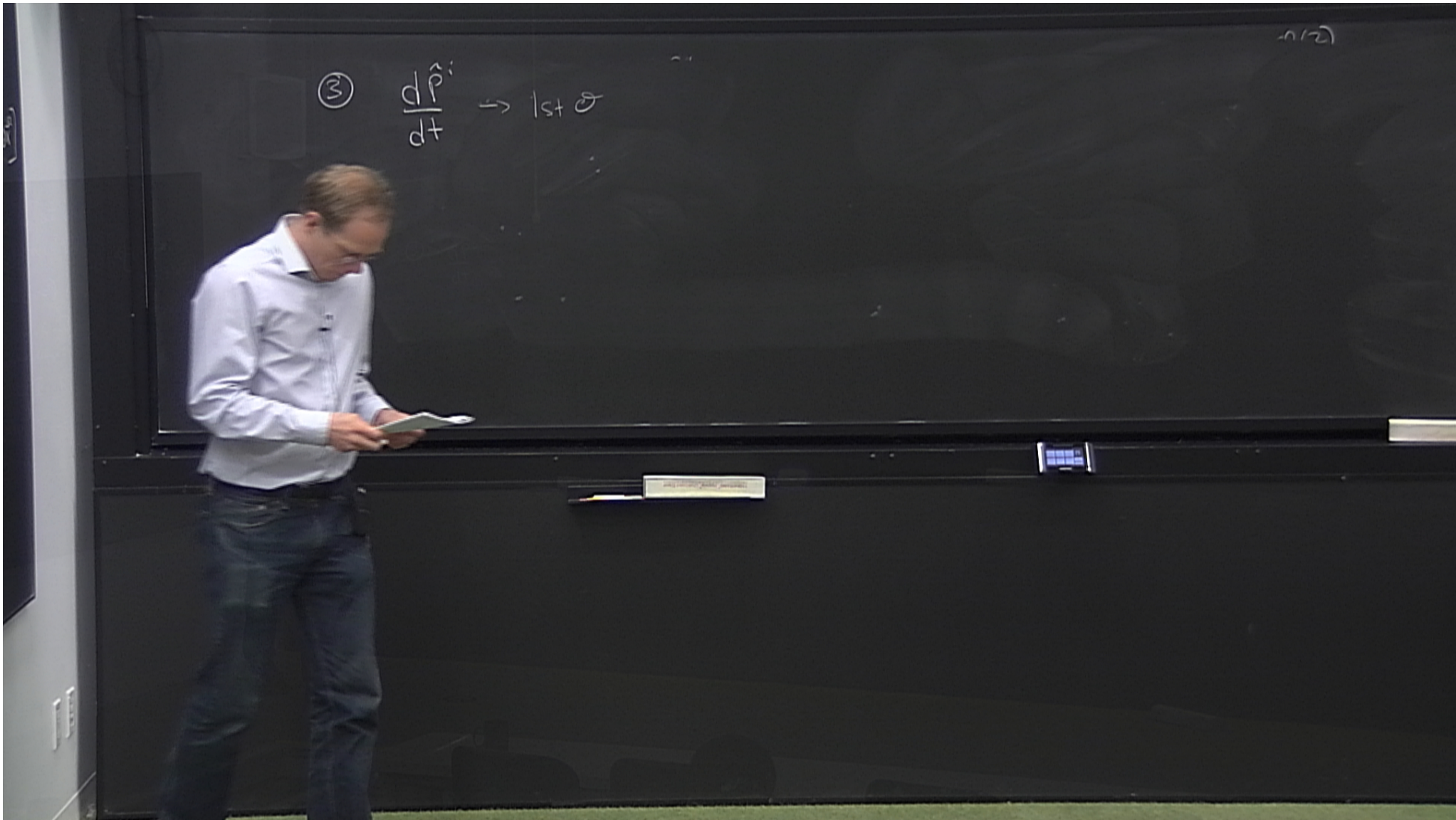
$$+ \frac{\partial f}{\partial x^i} \frac{dx^i}{dt} + \frac{\partial f}{\partial p} \frac{dp}{dt} + \frac{\partial f}{\partial \hat{p}}$$

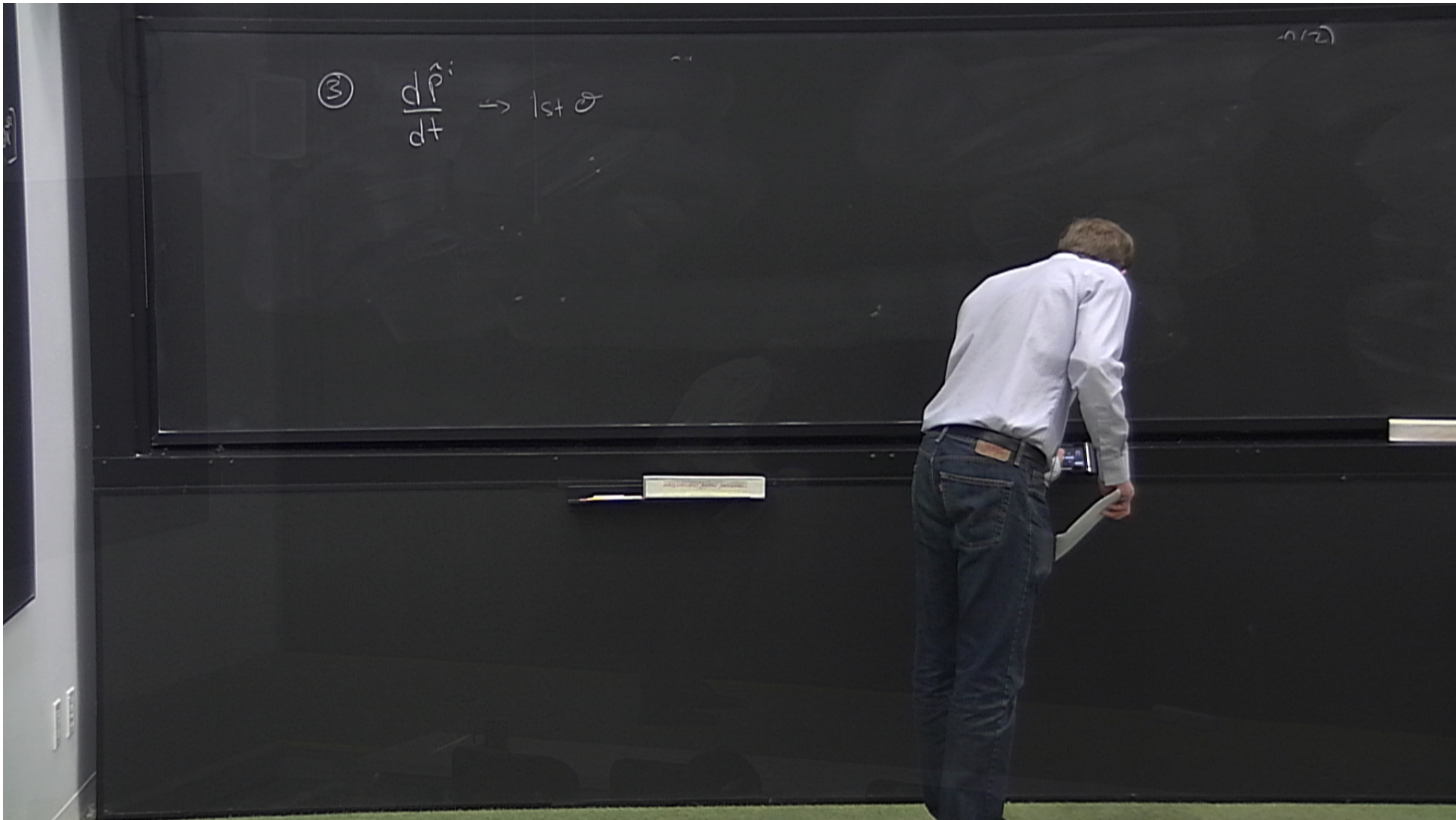
① ②

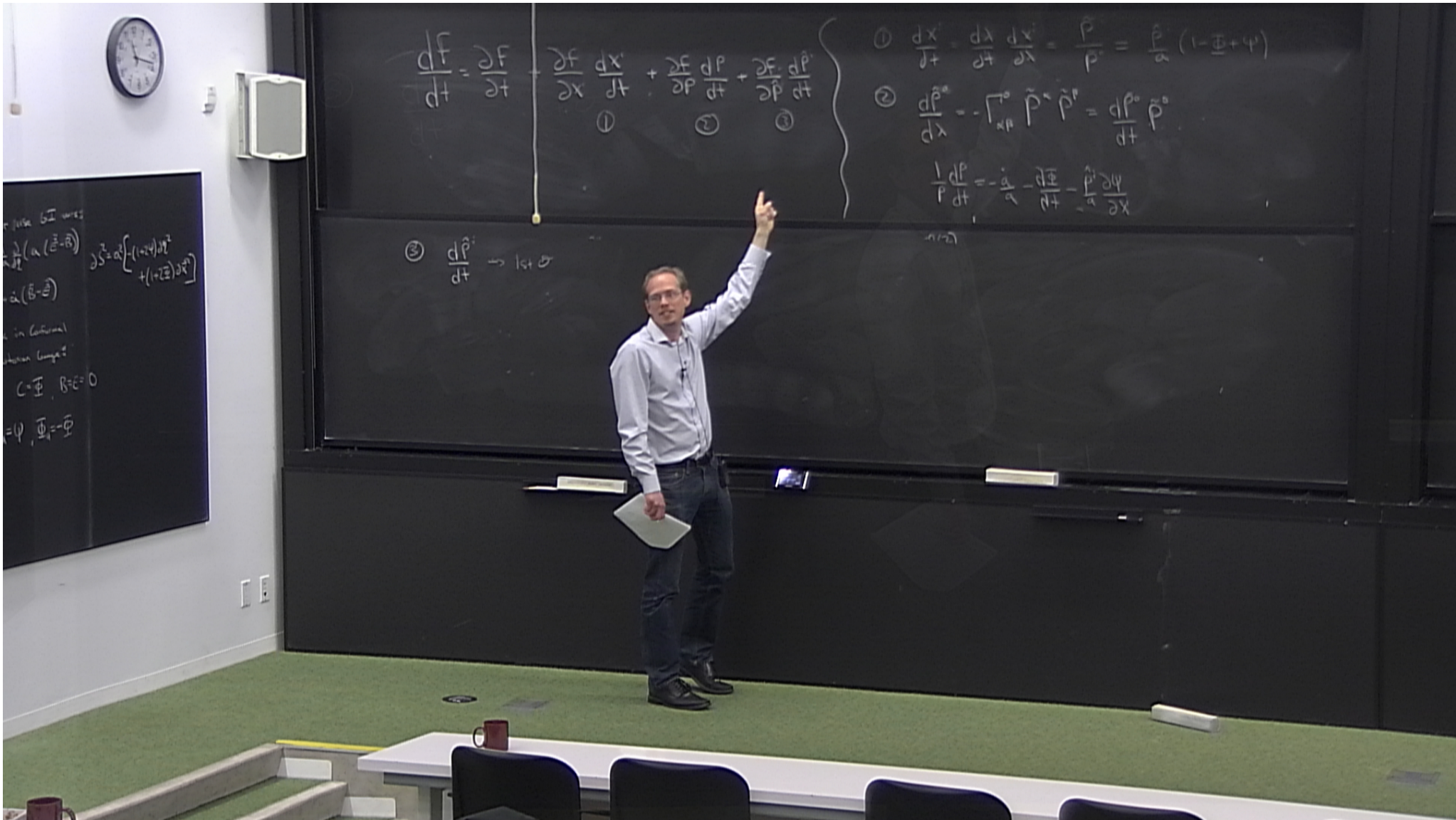
$$\textcircled{1} \quad \frac{dx^i}{dt} = \frac{dx^i}{d\lambda} \frac{d\lambda}{dt} = \frac{\tilde{p}^i}{\tilde{p}^0} = \frac{\hat{p}^i}{a} (1 - \Phi + \Psi)$$

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$$\frac{1}{p} \frac{dp}{dt} = - \frac{\dot{a}}{a} - \frac{\partial \Phi}{\partial t} - \frac{\hat{p}^i}{a} \frac{\partial \Psi}{\partial x^i}$$







③ $\frac{d\hat{p}^i}{dt} \rightarrow 1st \vartheta$

$\Rightarrow 2nd \vartheta \rightarrow \text{Forget}$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial f}{\partial x^i} - p \frac{\partial f}{\partial p} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \Psi}{\partial x^i} \right]$$

$$\textcircled{3} \quad \frac{d\hat{p}^i}{dt} \rightarrow \text{1st } \theta$$

$\Rightarrow \text{2nd } \theta \rightarrow \text{Forget}$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial F}{\partial x^i} - p \frac{\partial F}{\partial p} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \Psi}{\partial x^i} \right]$$

③ $\frac{d\hat{p}^i}{dt} \rightarrow 1st\ \theta$

$\Rightarrow 2nd\ \theta \rightarrow \text{Forget}$

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial F}{\partial x^i} - P \frac{\partial F}{\partial P} \left[H + \frac{\partial \Phi}{\partial t} + \frac{\hat{p}^i}{a} \frac{\partial \Psi}{\partial x^i} \right]$$