

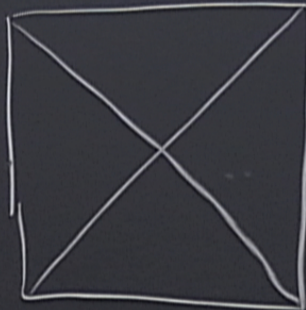
Title: 12/13 PSI - Explorations in Cosmology Lecture 2

Date: Apr 09, 2013 10:15 AM

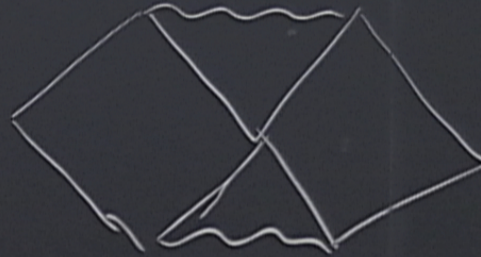
URL: <http://pirsa.org/13040063>

Abstract:

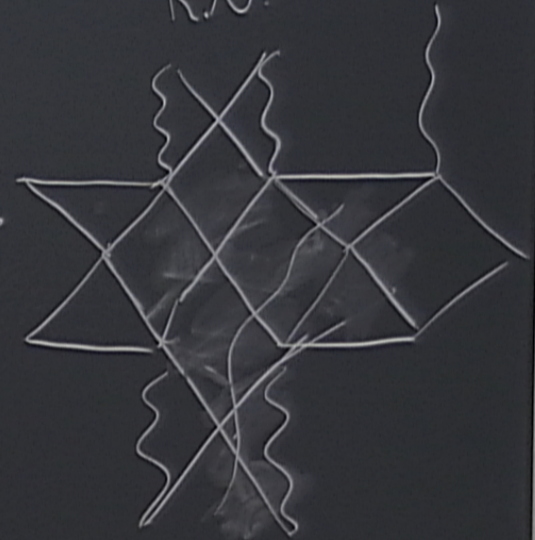
DS

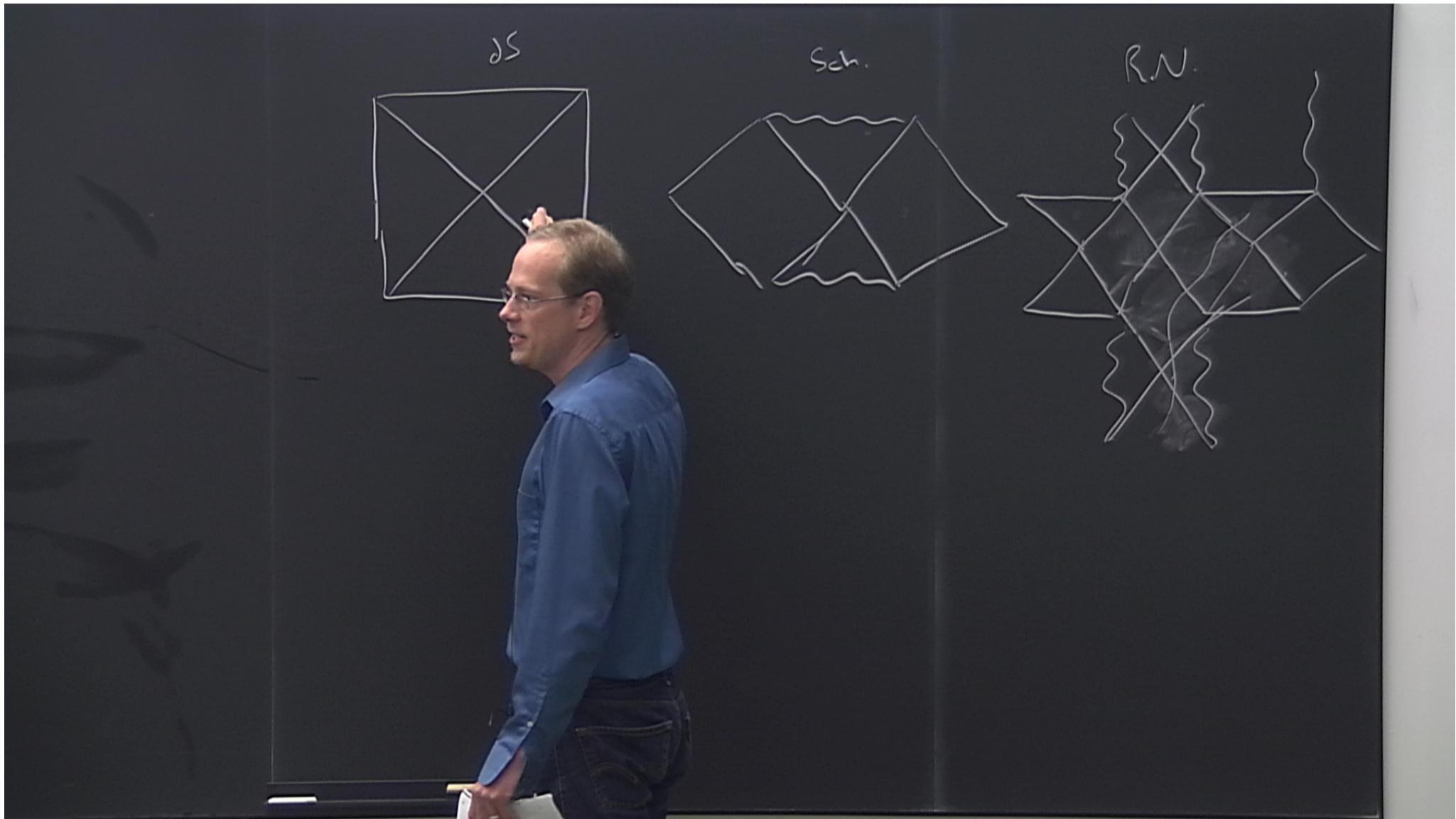


Sch.



R.N.

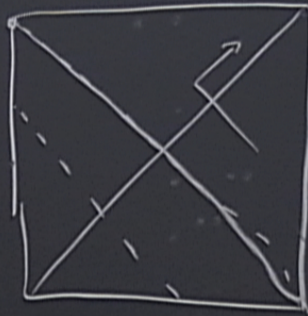




FRW



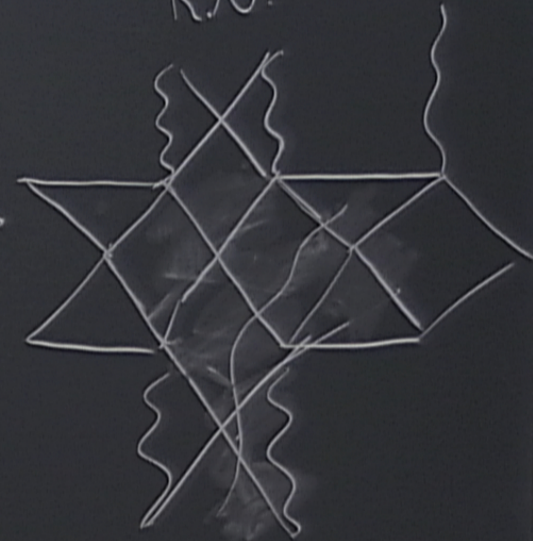
ds



Sch.



R.N.



Homogeneous Universe

$$ds^2 = -dt^2 + a^2(t) d\vec{x}^2$$
$$ds^2 = a(t)^2 \left[-dt^2 + d\vec{x}^2 \right]$$

$$\left(\frac{H}{H_0} \right)^2 = \sum_i \Omega_i a^{-3(1+w_i)}$$

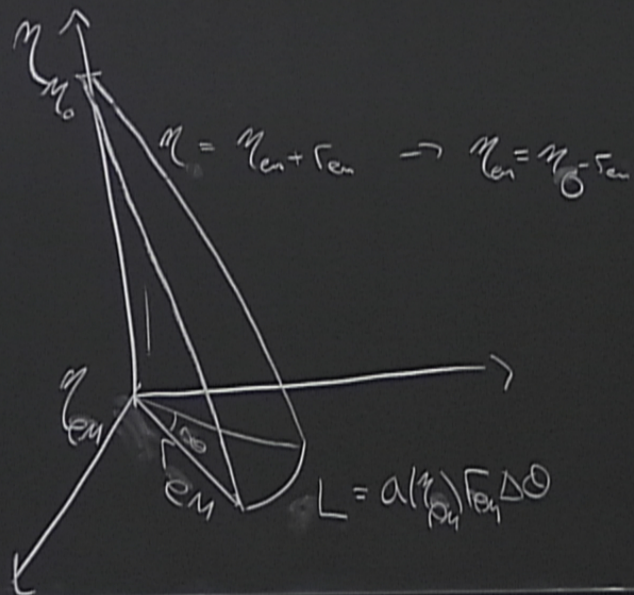
$$t_0 = \int_{t=0}^{t_0} dt = \int_{a=0}^{a_0=1} \frac{da}{aH}$$

Matter: $H = H_0$

$$t_0 = \int_{t=0}^{t_0} dt = \int_{a=0}^{a_0=1} \frac{da}{aH} = H_0^{-1} \int_{a=0}^1 \frac{da}{\sqrt{a}} = \frac{2}{3} H_0^{-1} \approx 10^9 \text{ yrs} \approx 13.7 \dots \text{ y}$$

Matter: $H = H_0 a^{-3/2}$

Angular Diameter Distance



$$\Delta\theta = \frac{L}{a(t_0 - t_{en}) r_{en}}$$

$$\Delta\theta = (1+z) \frac{L}{r_{en}}$$

$$r_{en} = \int_0^z \frac{dz}{H(z)} = \frac{z}{H_0} \left| 1 - \right.$$

$$\Delta\theta = \frac{L}{a(\eta_0 - \eta_{em}) r_{em}}$$

$$\Delta\theta = (1+z) \frac{L}{r_{em}}$$

$$r_{em} = \int_0^z \frac{dz}{H(z)} = \frac{z}{H_0} \left(1 - \frac{1}{\sqrt{1+z}} \right) \text{ Mpc}$$

Standard Ruler - know L , measure
infer

$$\Delta\theta = \frac{L}{a(\eta_0 - \eta_{\text{em}}) r_{\text{em}}}$$

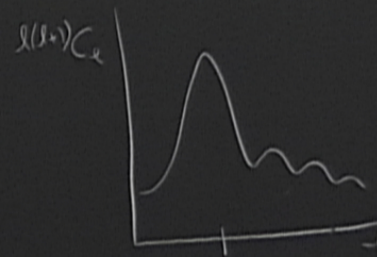
$$\Delta\theta = (1+z) \frac{L}{r_{\text{em}}}$$

$$r_{\text{em}} = \int_0^z \frac{dz}{H(z)} =$$

Matter

Standard Ruler - know L , measure
infer properties of
expansion $\rightarrow a(t)$

CMB: Sound horizon



Perturbed Universe

$$\partial_s^2 = a^2(\eta) \left[-(1+\Phi) \partial_t^2 + (1+2\Phi) \partial_x^2 \right]$$

$\Psi, \Phi \ll 1$

$\approx g_{\mu\nu}$

Perturbed Universe

$$ds^2 = a^2(\eta) \left[-(1+\Phi) dt^2 + (1+2\psi) d\vec{x}^2 \right]$$

$\psi, \Phi \ll 1$

More generally:

$$ds^2 = (g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}) dx^\mu dx^\nu$$

$$\delta g_{\mu\nu} = \delta g_{\mu\nu}$$

$Sg_{\mu\nu}$ - Symmetric, in 4-D 10 components

But Bianchi identities $\Rightarrow$ really, only 6 independent components

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But Bianchi identities $\Rightarrow$ really, only 6 independent components

Scalar, vectors, tensors

$g_{\mu\nu}$ - Symmetric, in 4-D 10 components

But Bianchi identities & really, only 6 independent components

vectors, tensors under spatial rotations

$$x^{\mu'} = X^{\mu'}_{\rho} x^{\rho}$$

$$X^{\mu'}_{\rho} = \begin{pmatrix} 1 & 0 \\ 0 & R^i_j \end{pmatrix}$$

$$R^T R = 1$$

$$g_{\rho'\sigma'} = X^{\mu}_{\rho'} X^{\nu}_{\sigma'} g_{\mu\nu}$$

$g_{\mu\nu}$ - Symmetric, in 4-D 10 components

But Bianchi identities & 4 really, only 6 independent components

Scalar, vectors, tensors under spatial rotations

$$x^{\mu'} = X^{\mu'}_{\rho} x^{\rho}$$

$$X^{\mu'}_{\rho} = \begin{pmatrix} 1 & 0 \\ 0 & R^i_k \end{pmatrix} \quad R^T R = 1$$

$$g_{\rho'\sigma'} = X^{\mu}_{\rho'} X^{\nu}_{\sigma'} g_{\mu\nu}$$

Scalars : $A'(x') = A(x)$

Vectors : $B_{x'} = R^i_{x'} B_i$

Tensors : $E_{x'x'} = R^i_{x'} R^j_{x'} E_{ij}$

Scalars : $A'(x') = A(x)$

Vectors : $B_{x'} = R^j_{x'} B_j$

Tensors : $E_{\kappa\lambda} = R^i_{\kappa} R^j_{\lambda} E_{ij}$

$$\vec{B} = \vec{B}_s + \vec{B}_v, \quad \vec{\nabla} \times \vec{B}_s = 0$$

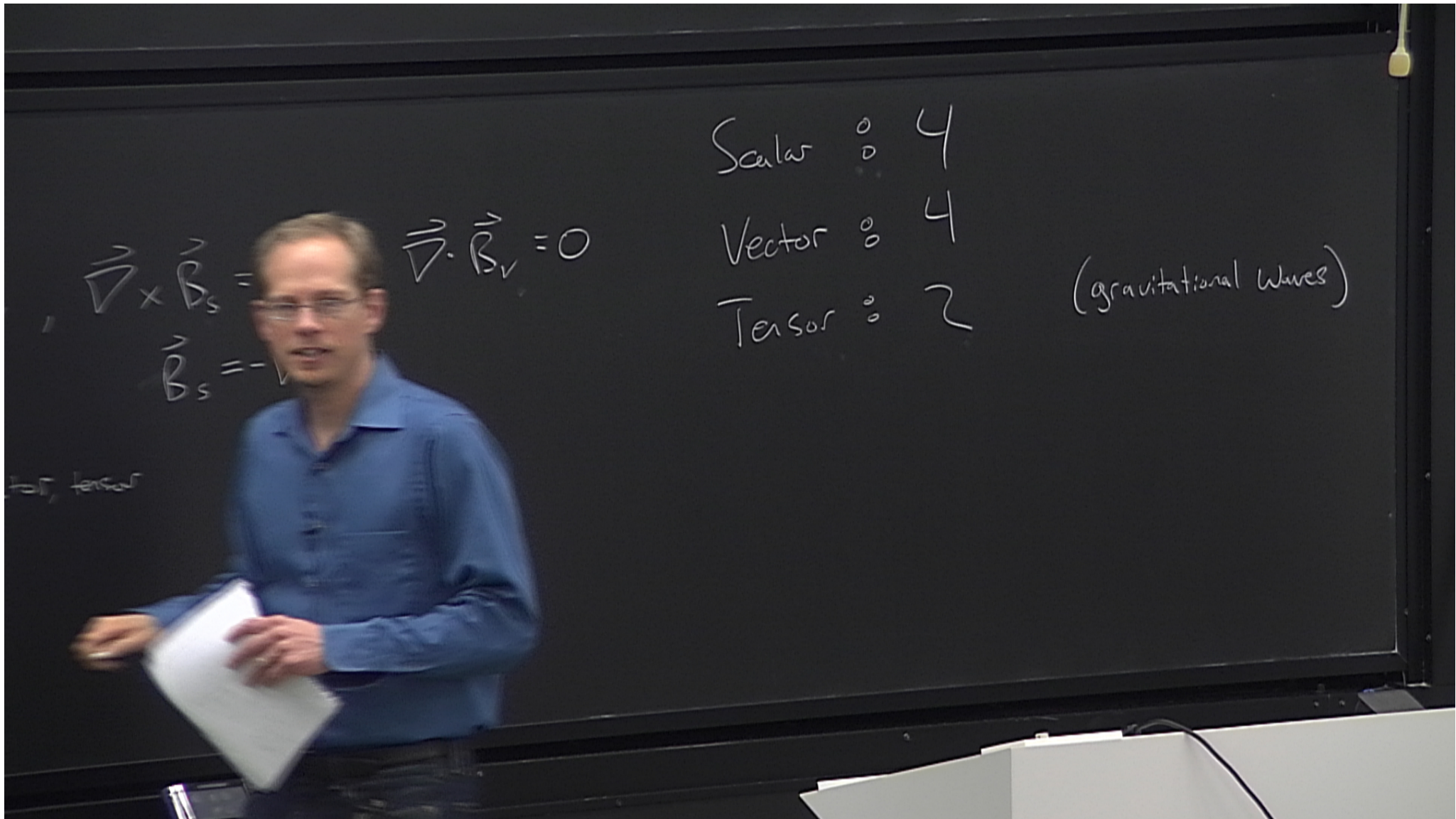
Scalars: $A'(x') = A(x)$

Vectors: $B_{x'} = R^i_{x'} B_i$

Tensors: $E_{\kappa x'} = R^i_{\kappa} R^j_{x'} \tilde{E}_{ij} \rightarrow \text{scalar, vector, tensor}$

$$\vec{B} = \vec{B}_s + \vec{B}_v, \quad \vec{\nabla} \times \vec{B}_s = 0, \quad \vec{\nabla} \cdot \vec{B}_v = 0$$

$$\vec{B}_s = -\vec{\nabla} B$$



$$\vec{\nabla} \times \vec{B}_s = 0, \quad \vec{\nabla} \cdot \vec{B}_v = 0$$

$$\vec{B}_s = -\vec{\nabla} B$$

scalar, tensor

Scalar : 4

Vector : 4

Tensor : 2

(gravitational waves)

$$\vec{\nabla} \times \vec{B}_s = 0, \quad \vec{\nabla} \cdot \vec{B}_v = 0$$

$$\vec{B}_s = -\vec{\nabla} B$$

scalar, tensor

Scalar	0	4	(Source Fluctuations)
Vector	0	4	(
Tensor	0	2	(gravitational na)

$$\vec{\nabla} \times \vec{B}_s = 0, \quad \vec{\nabla} \cdot \vec{B}_v = 0$$

$$\vec{B}_s = -\vec{\nabla} B$$

total tensor

Scalar	0	4	(Source Fluctuations)
Vector	0	4	(decay)
Tensor	0	2	(gravitational waves)
		10	

Scalar parts of A, B, C, E

$$\partial S^2 = -(1+2A)\partial t^2 - 2a(t)\partial_i B \partial X^i \partial t + a^2(t) \left[(1+2C)g_{ij} - 2\partial_i \partial_j E \right] \partial X^i \partial X^j$$

Gauge Freedom

Scalar parts ϕ, A, B, C, E

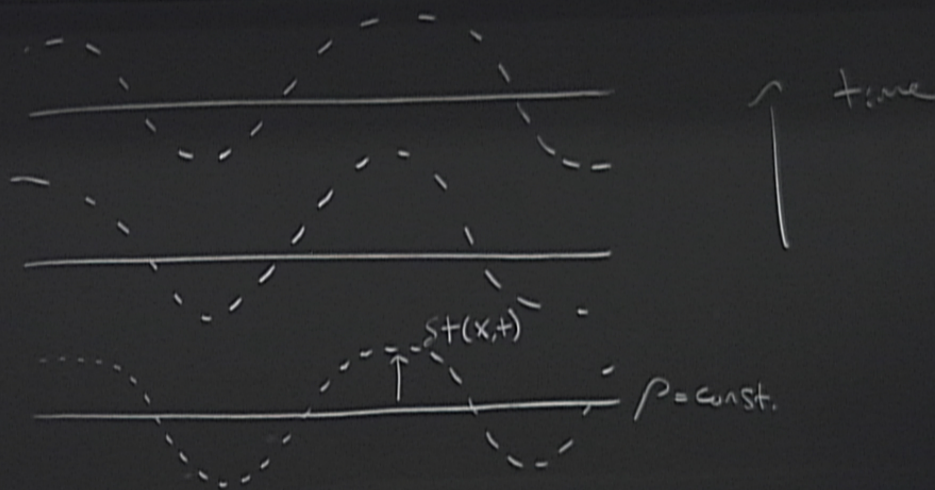
$$\partial S^2 = -(1+2A)\partial t^2 - 2a(t)\partial_t B \partial x^i \partial t + a^2(t) \left[(1+2C)g_{ij} - 2\partial_i \partial_j E \right] \partial x^i \partial x^j$$

Gauge Freedom

Scalar parts of A, B, C, E

$$\partial S^2 = -(1+2A)\partial t^2 - 2a(t)\partial_t B \partial X^i \partial t + a^2(t) \left[(1+2C)g_{ij} - 2\partial_i \partial_j E \right] \partial X^i \partial X^j$$

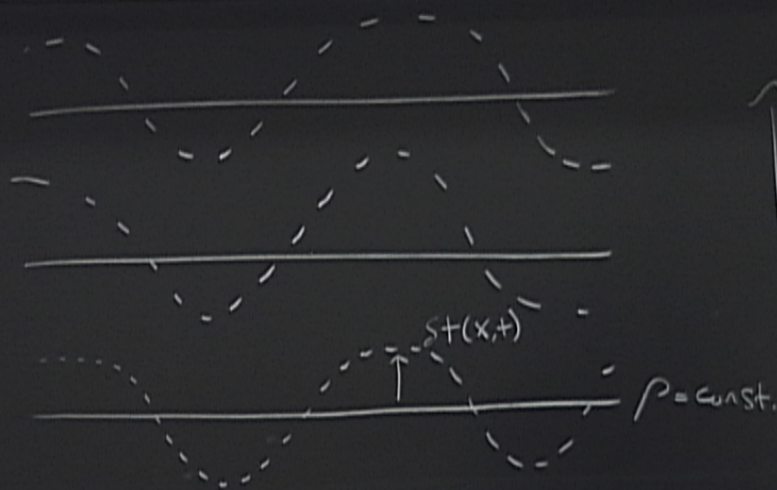
Gauge Freedom



$$\tilde{t} = t + S(\vec{x})$$

$$P(t) = P(\tilde{t})$$

$$= \tilde{P}(\tilde{t})$$



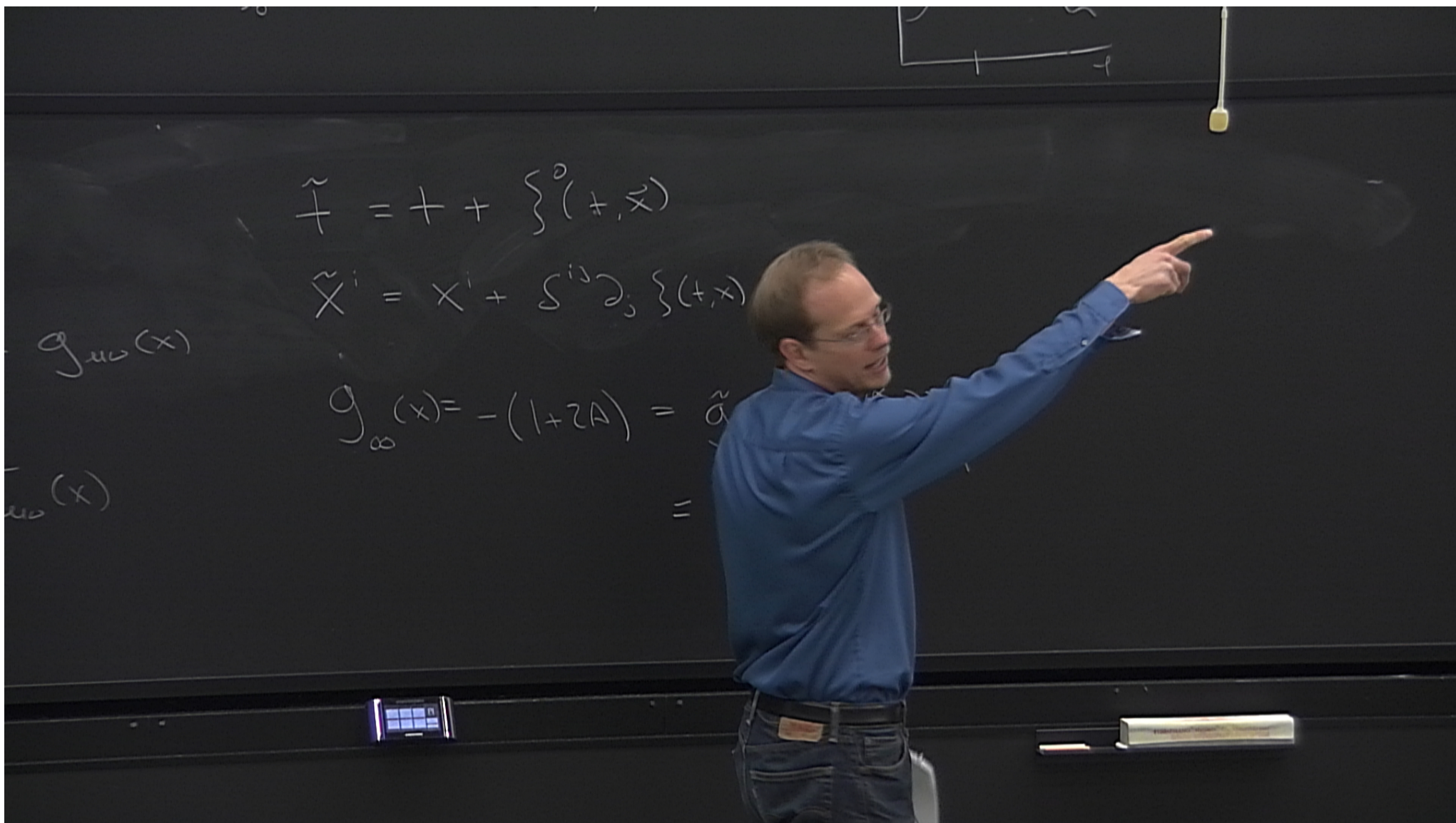
$$\begin{aligned}
 \tilde{t} &= t + s(t, \vec{x}) \\
 \tilde{p}(t) &= p(\tilde{t} - s(t)) \\
 &= \tilde{p}(\tilde{t}) - \underbrace{\frac{\partial p}{\partial t}}_{\delta p} s(t) \\
 &= p(t) + \delta p
 \end{aligned}$$



$$L = a(t) \sqrt{e_M} \Delta\theta$$

General coordinate trans.

$$\tilde{g}_{\alpha\beta}(\tilde{x}) \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial \tilde{x}^\beta}{\partial x^\nu} = g_{\mu\nu}(x)$$



$$\tilde{t} = t + \xi(t, \vec{x})$$

$$\tilde{x}^i = x^i + \delta^{ij} \partial_j \xi(t, \vec{x})$$

$$g_{\mu\nu}(x)$$

$$g_{\alpha\beta}(x) = -(1+2A) = \tilde{g}_{\alpha\beta}(\tilde{x}) \frac{\partial x^\alpha}{\partial \tilde{x}^0} \frac{\partial x^\beta}{\partial \tilde{x}^0}$$

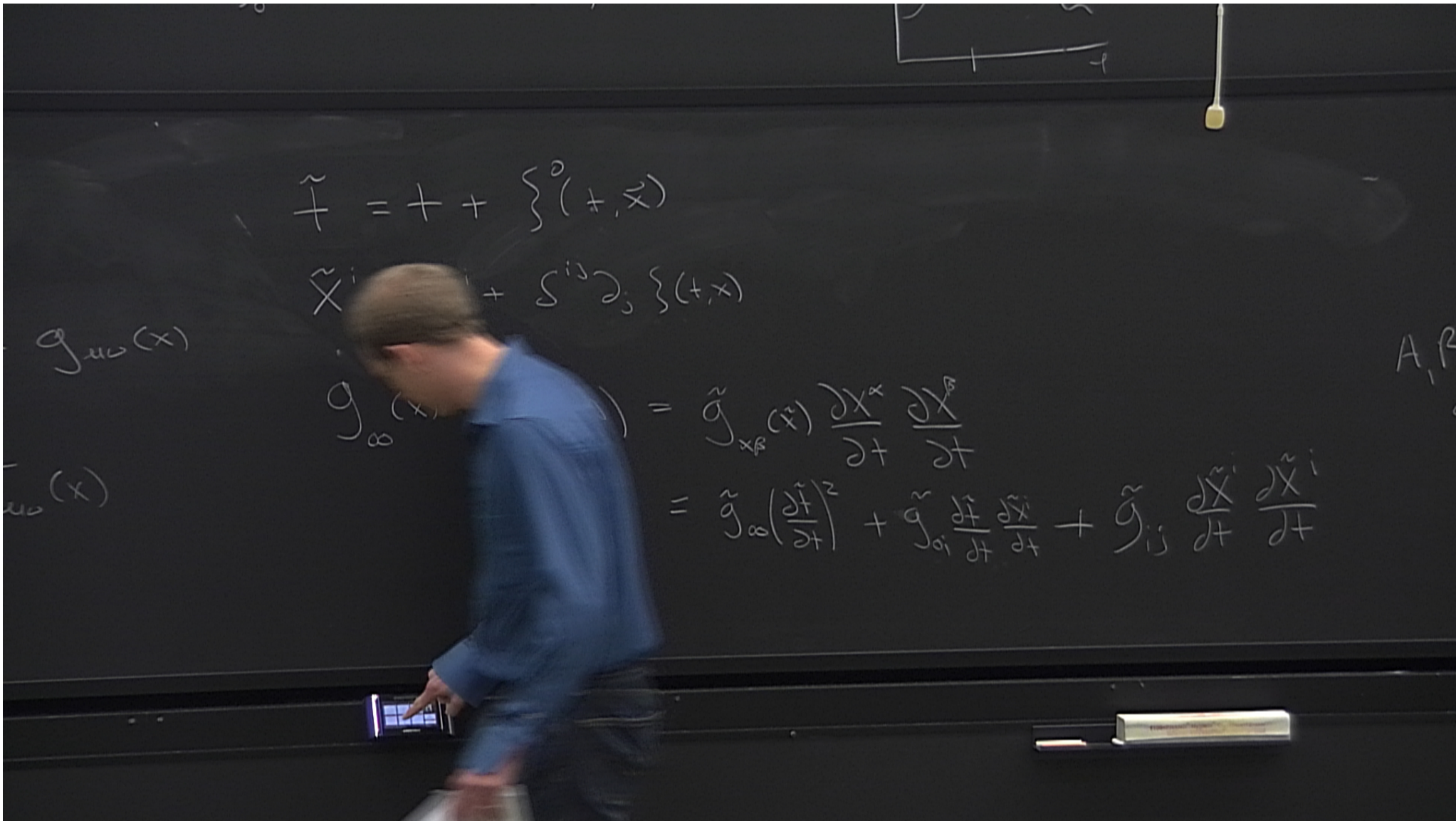
$$g_{\mu\nu}(x)$$

$$= \tilde{g}_{00} \left(\frac{\partial \tilde{t}}{\partial t} \right)^2 + \tilde{g}_{0i} \frac{\partial \tilde{t}}{\partial t} \frac{\partial \tilde{x}^i}{\partial t} + \tilde{g}_{ij} \frac{\partial \tilde{x}^i}{\partial t} \frac{\partial \tilde{x}^j}{\partial t}$$

$$\tilde{t} = t + \xi(t, \vec{x})$$

$$\tilde{x}^i = x^i + \delta^{ij} \partial_j \xi(t, \vec{x})$$

$$g_{\infty}(x) = -(1+2A) = \tilde{g}_{\tilde{x}^{\mu}}(\tilde{x}) \frac{\partial x^{\mu}}{\partial \tilde{x}^{\mu}} = \tilde{g}_{\infty} \left(\frac{\partial \tilde{t}}{\partial t} \right)^2 + \tilde{g}_{0i} \frac{\partial \tilde{t}}{\partial t} \frac{\partial \tilde{x}^i}{\partial t} +$$



$$\tilde{g}_{0i} = 2a\partial_i \tilde{B}$$

$$\frac{d\tilde{t}}{dt} = 1 + \frac{d\tilde{\xi}^0}{dt}$$

$$\frac{d\tilde{x}^i}{dt} = \delta^{ij} \frac{d\tilde{\xi}^j}{dt + dx^j}$$

$$\tilde{X}^i = X^i + \delta^{ij} \partial_j \xi(t, x)$$

$$g_{\infty}(x) = -(1+2A) = \tilde{g}_{x^\beta}(x) \frac{\partial X^\alpha}{\partial t} \frac{\partial X^\beta}{\partial t}$$

$$= \tilde{g}_{\infty} \left(\frac{\partial \tilde{t}}{\partial t} \right)^2 + \tilde{g}_{\alpha_1} \frac{\partial \tilde{t}}{\partial t} \frac{\partial \tilde{X}^{\alpha_1}}{\partial t} + \tilde{g}_{ij} \frac{\partial \tilde{X}^i}{\partial t} \frac{\partial \tilde{X}^j}{\partial t}$$

$\mathcal{O}(1) \mathcal{O}(2) \mathcal{O}(1)$
 $\mathcal{O}(2)$

$$A, B, C, E \ll 1$$

$$\xi^0, \partial, \xi \ll 1$$

$$\tilde{x}^i = x^i + \delta^{ij} \partial_j \xi(t, x)$$

$$g_{\alpha\beta}(x) = -(1+2A) = \tilde{g}_{\alpha\beta}(\tilde{x}) \frac{\partial x^\alpha}{\partial t} \frac{\partial x^\beta}{\partial t}$$

$$= \tilde{g}_{00} \left(\frac{\partial t}{\partial t} \right)^2 + \tilde{g}_{0i} \frac{\partial t}{\partial t} \frac{\partial \tilde{x}^i}{\partial t} + \tilde{g}_{ij} \frac{\partial \tilde{x}^i}{\partial t} \frac{\partial \tilde{x}^j}{\partial t}$$

$\begin{matrix} \mathcal{O}(1) & \mathcal{O}(2) & \mathcal{O}(1) \\ \mathcal{O}(2) \end{matrix}$

$$A, B, C, E \ll 1$$

$$\xi^0, \xi^1, \xi^2 \ll 1$$

$$\tilde{g}_{0i} = 2a\partial_i \tilde{B}$$

$$\frac{d\tilde{t}}{dt} = 1 + \frac{\partial \tilde{\xi}^0}{\partial t}$$

$$\frac{d\tilde{x}^i}{dt} = \delta^{ij} \frac{\partial \tilde{\xi}^j}{\partial t}$$

$$\Rightarrow -(1+2A) = -(1+2\tilde{A}) \left(1 + 2\frac{\partial \tilde{\xi}^0}{\partial t}\right)$$

$$1+2A = 1+2\tilde{A} + 2\frac{\partial \tilde{\xi}^0}{\partial t}$$

$$\tilde{A} = A$$

$$\tilde{g}_{0i} = 2a\partial_i \tilde{\Phi}$$

$$\frac{d\tilde{t}}{dt} = 1 + 2\frac{\partial \tilde{\Phi}}{\partial t}$$

$$\frac{d\tilde{x}^i}{dt} = \dot{x}^i \frac{\partial \tilde{x}^i}{\partial x^i}$$

$$\Rightarrow -(1+2A) = -(1+2\tilde{A})\left(1+2\frac{\partial \tilde{\Phi}}{\partial t}\right)$$

$$X + \dots + X + 2\tilde{A} + 2\frac{\partial \tilde{\Phi}}{\partial t}$$

A -

$$\tilde{g}_{0i} = 2a\partial_i \tilde{B}$$

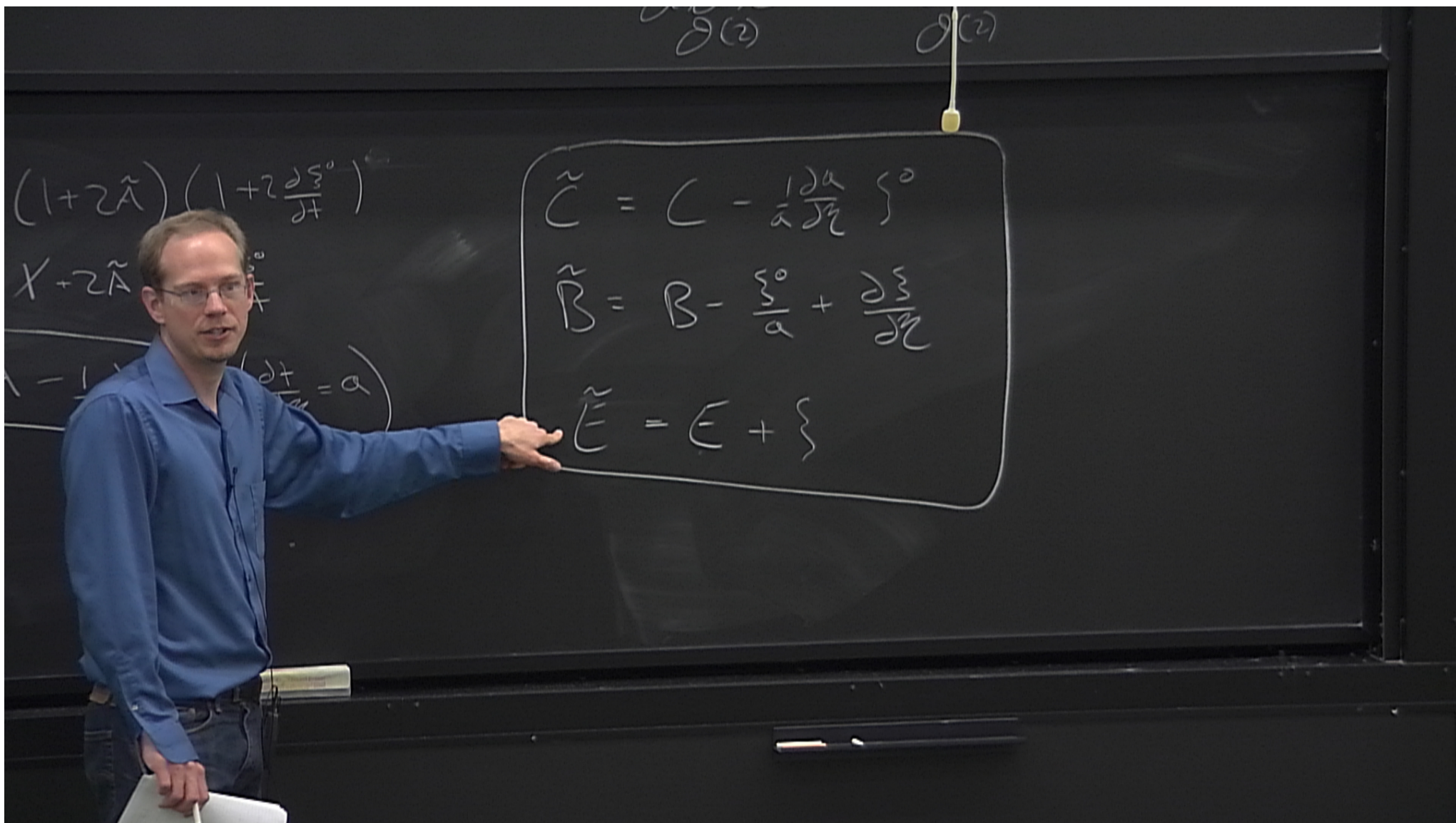
$$\frac{d\tilde{t}}{dt} = 1 + \frac{\partial \tilde{\xi}^0}{\partial t}$$

$$\frac{d\tilde{x}^i}{dt} = \delta^{ij} \frac{\partial \tilde{\xi}^j}{\partial t}$$

$$\Rightarrow -(1+2A) = -(1+2\tilde{A}) \left(1 + 2 \frac{\partial \tilde{\xi}^0}{\partial t}\right)$$

$$1+2A = 1+2\tilde{A} + 2 \frac{\partial \tilde{\xi}^0}{\partial t}$$

$$2A - 2\tilde{A} = 2 \frac{\partial \tilde{\xi}^0}{\partial t}$$



Perturbed Universe

$$\partial s^2 = a^2(\eta) \left[-(1+\Psi) \partial t^2 + (1+2\Phi) \partial x^2 \right]$$

$\Psi, \Phi \ll 1$

Conformal Newtonian Gauge

Perturbed Universe

$$ds^2 = a^2(\eta) \left[dt^2 + (1 + 2\Phi) d\vec{x}^2 \right]$$

Conformal Gauge

$$A = \Psi$$

Synchronous

$$A = B = 0$$

$$\partial S^2 = \alpha^2(r) \left[-dt^2 + ((1+2C) - 2\partial_j E) dx^i dx^j \right]$$

Synchronous

$$A = B = 0$$

$$\partial S^2 = \alpha^2(x) \left[-\partial \eta^2 + ((1+2c) - 2\partial_j E) \partial x^j \partial x^i \right]$$

Not unique!

$$\xi^0 = \xi^0(x) \quad \text{and} \quad \xi = \xi^0(x) \left(\frac{\partial \eta}{\partial} + \xi^{(1)}(x) \right)$$

$$\downarrow$$
$$A = \tilde{A}$$

Synchronous

$$A = B = 0$$

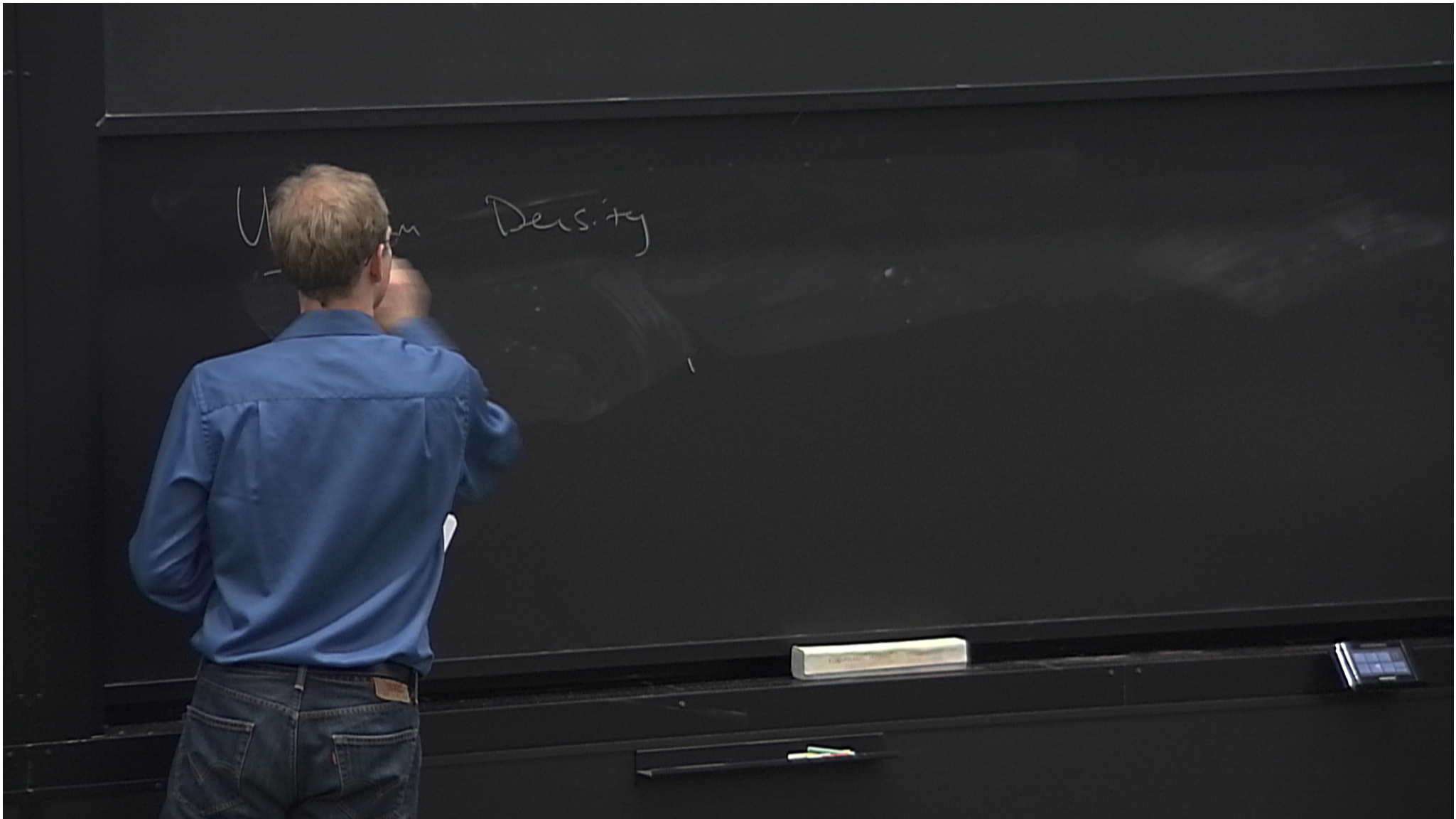
$$\partial S^2 = \alpha^2(x) \left[-\partial^2 \eta^2 + ((1+2c) - 2\partial_j E) \partial x^j \partial x^i \right]$$

Not unique!

$$\xi^0 = \xi^0(x) \quad \text{and} \quad \xi = \xi^0(x) \left(\frac{\partial \eta}{\partial x} + \xi^{(1)}(x) \right)$$

$$\downarrow$$
$$A = \tilde{A}$$

$$\downarrow$$
$$B = \tilde{B}$$



Uniform Density

$$E=0, \quad C=\zeta$$

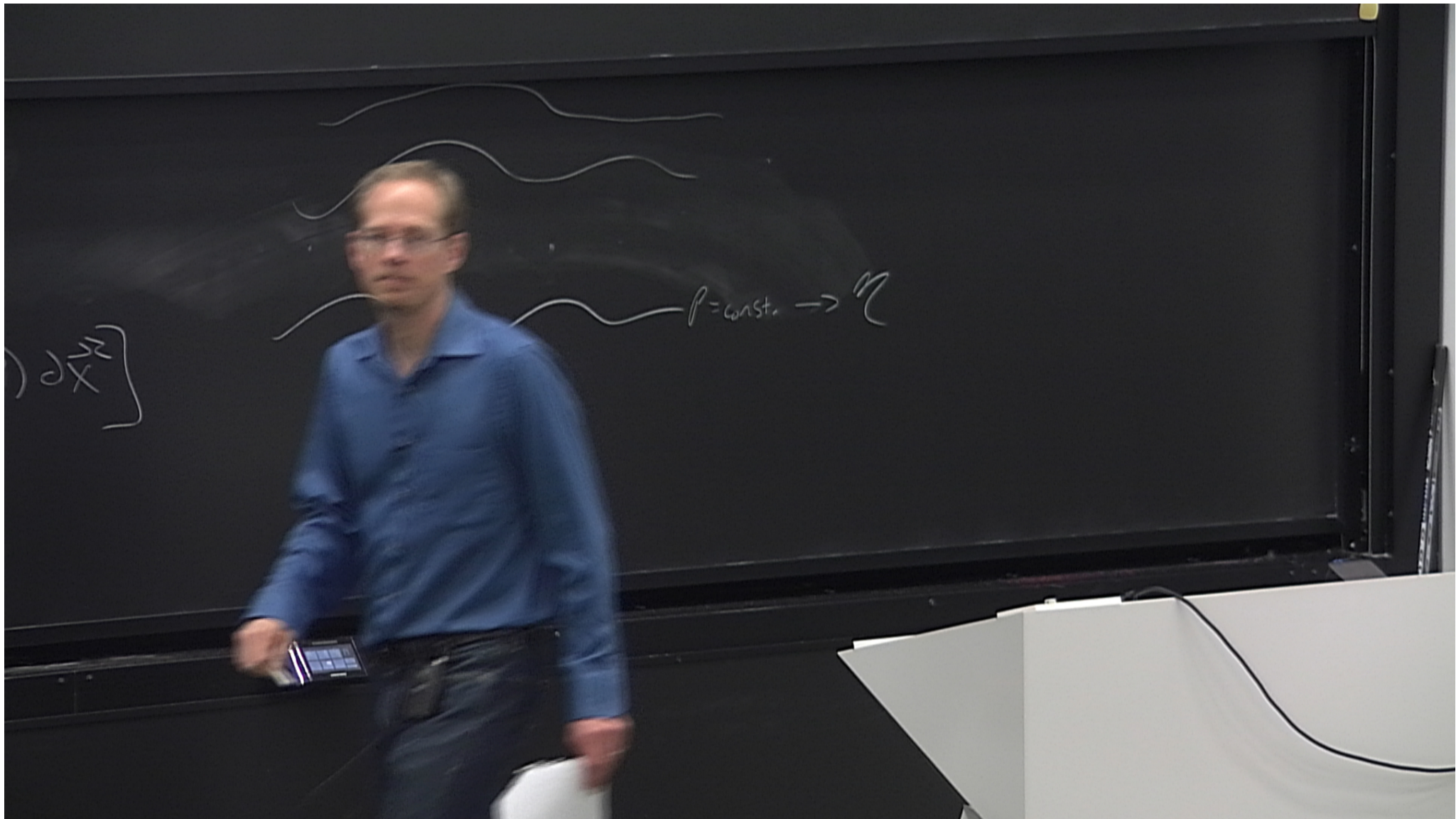
$$ds^2 = a^2 \left[-(1+2A)d\eta^2 + 2\partial_i B \partial X^i d\eta + (1+2\phi) d\vec{X}^2 \right]$$

Uniform Density

$$E=0, \quad C=\zeta$$

$$ds^2 = a^2 \left[-(1+2A)d\eta^2 + 2\partial_i B \partial X^i d\eta + (1+2\phi) d\vec{X}^2 \right]$$

$$\delta P = 0$$



$$\Phi_A = A + \frac{1}{a} \frac{\partial}{\partial \eta} \left[a \left(\frac{\partial E}{\partial \eta} - B \right) \right]$$

$$\Phi_H = -C + \frac{\partial a}{\partial \eta} \left(B - \frac{\partial E}{\partial \eta} \right)$$

Synchronous

$$A =$$

$$\partial S^2 =$$

Not w

Gauge Invariant

$$\Phi_A = A + \frac{1}{a} \frac{\partial}{\partial \eta} \left[a \left(\frac{\partial E}{\partial \eta} - B \right) \right]$$

$$= -C + \frac{\partial a}{\partial \eta} \left(B - \frac{\partial E}{\partial \eta} \right)$$

Synchron

$$A =$$

$$\partial S^2 =$$

Not w

Gauge Invariant

$$\Phi_A = A + \frac{1}{a} \frac{\partial}{\partial \eta} \left[a \left(\frac{\partial E}{\partial \eta} - B \right) \right]$$

$$\Phi_H = -C + \frac{\partial a}{\partial \eta} \left(B - \frac{\partial E}{\partial \eta} \right)$$

Synchronous

$$A =$$

$$\partial S^2 =$$

Not w