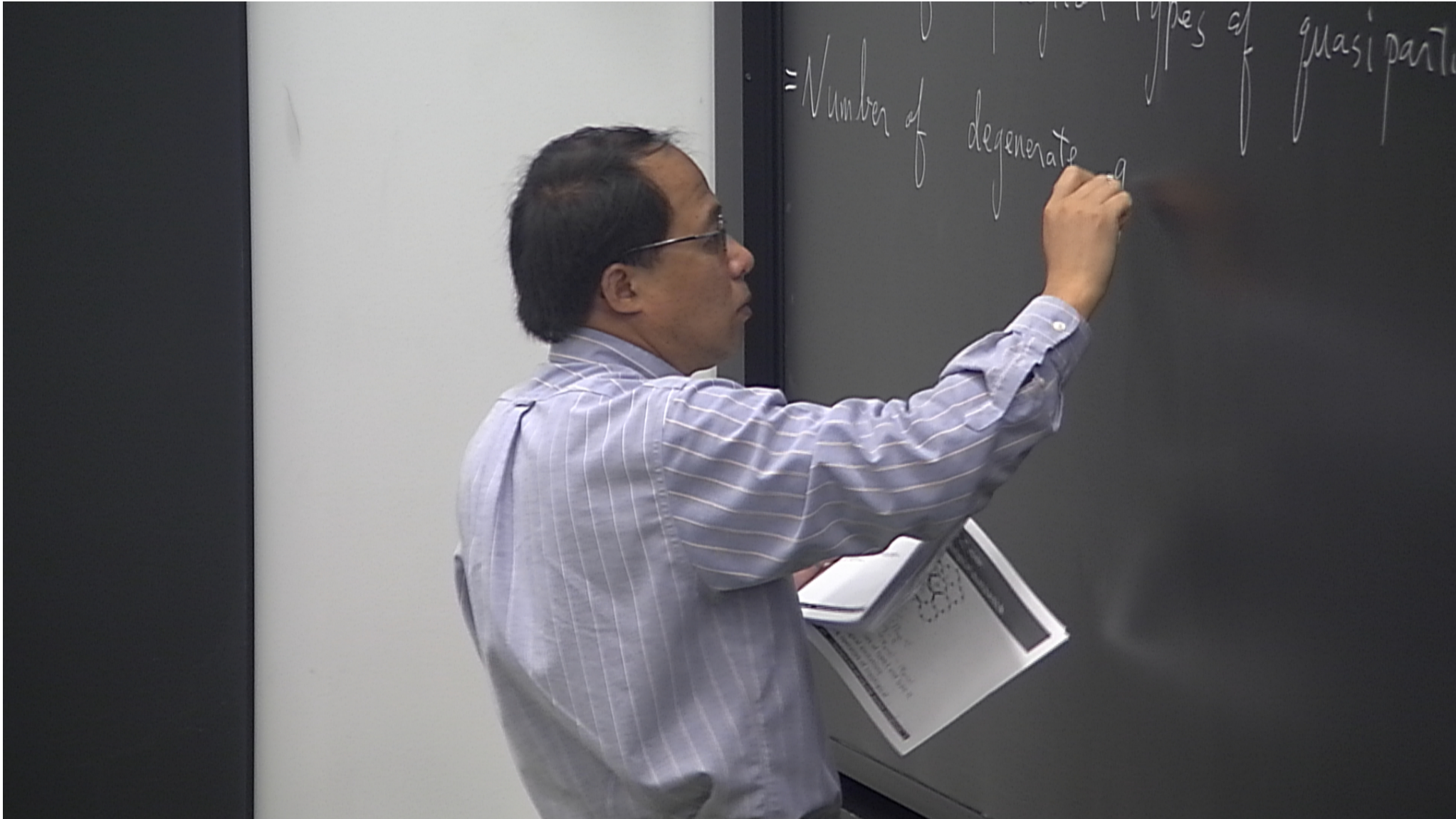


Title: 12/13 PSI - Explorations in Condensed Matter Lecture 15

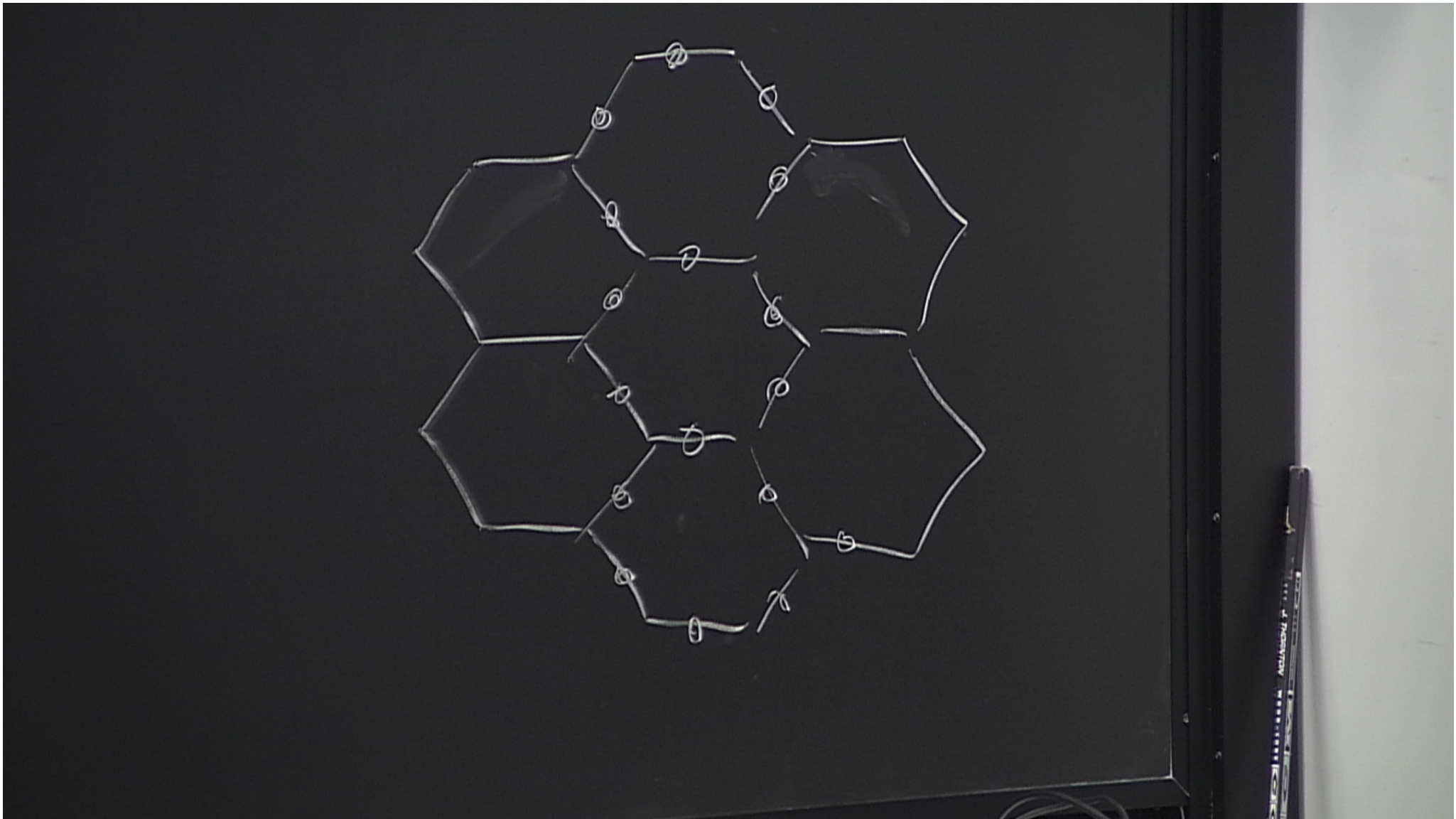
Date: Apr 26, 2013 09:00 AM

URL: <http://pirsa.org/13040061>

Abstract:



Number of topological types of quasiparticles $\longleftrightarrow$ # of string types
= Number of degenerate ground states of torus

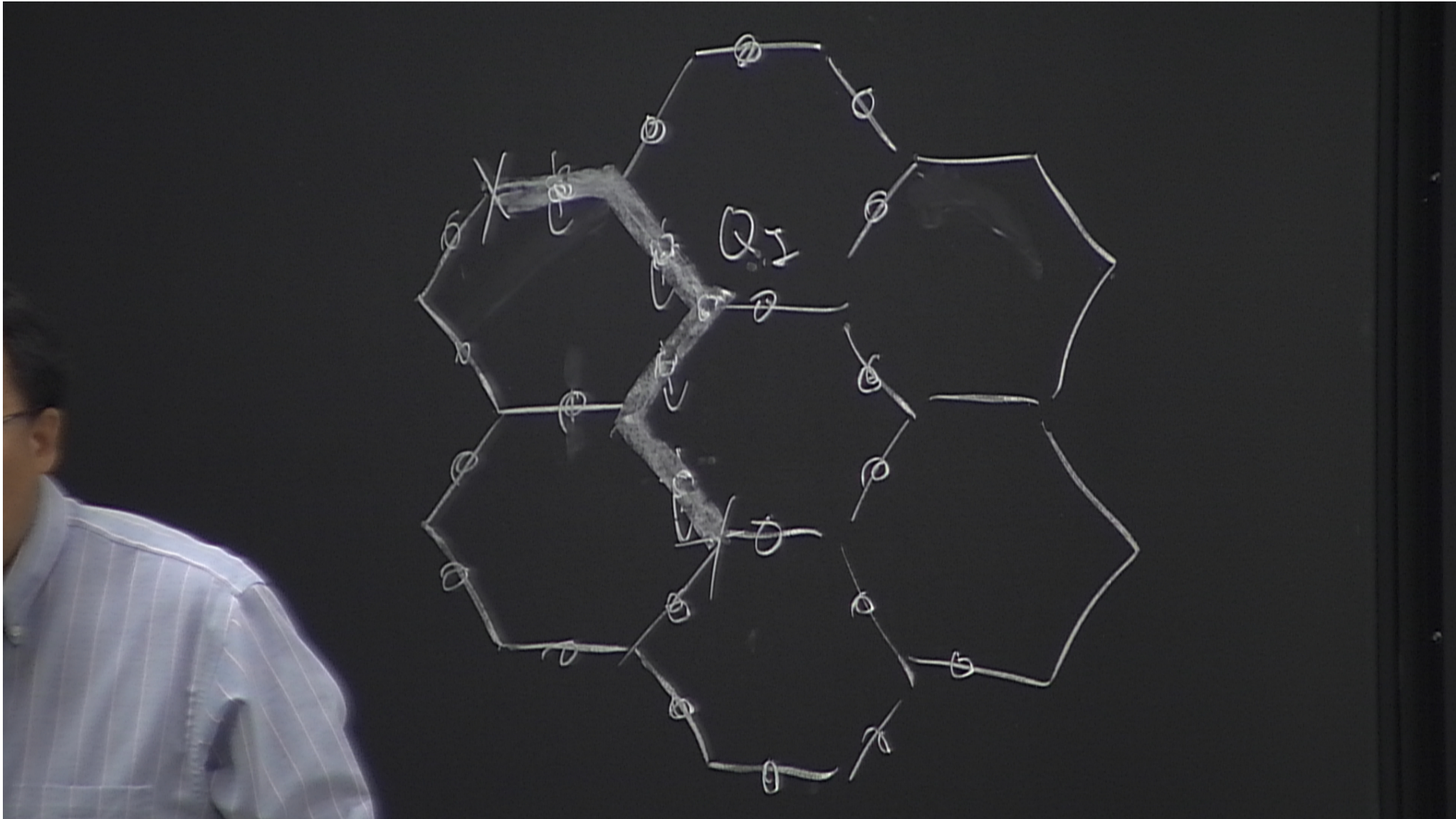


$$H = -U \sum_I \prod_{\substack{i \in \\ \text{legs of } I}} \sigma_i^z - g \sum_P \prod_{\substack{i \in \\ \text{edges of } P}} \sigma_i^x$$

$$H = -U \sum_I \prod_{i \in \text{legs of } I} \sigma_i^z - g \sum_P \prod_{i \in \text{edges of } P} \sigma_i^x$$

string operator

$$W_I = \prod_{i \in \text{string}} \sigma_i^x$$



$$H = - \bigcup_I \sum_I \prod_{\substack{i \in \\ \text{legs of } I}} \sigma_i^z - g \sum_P \prod_{\substack{i \in \\ \text{edges of } P}} \sigma_i^x$$

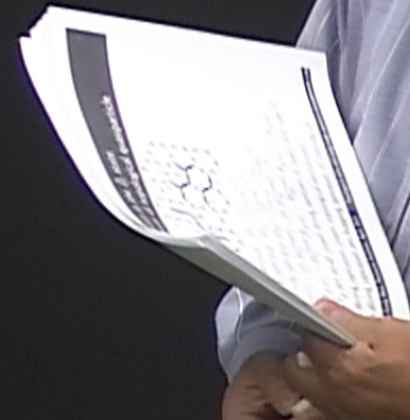
$\underbrace{\hspace{10em}}_{Q_I}$

string operator

$$W_I = \prod_{i \in \text{string}} \sigma_i^x$$

$$\underline{[W_3^{\text{closed}}, H] = 0}$$

$$\left[W_3^{\text{closed}}, H \right] = 0$$



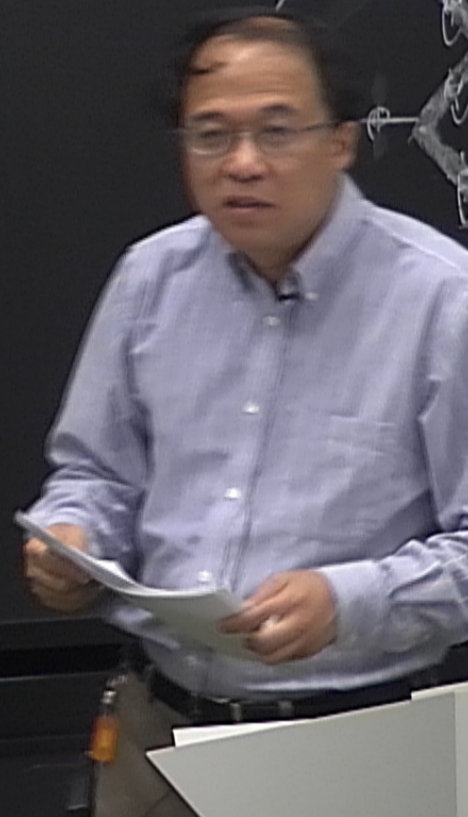
$$\sum_P \prod_{i \in \text{edges of } P} \sigma_i^x$$

$$W_1 W_2 = W_3$$

$$\textcircled{1} \textcircled{2} \rightarrow \textcircled{3}$$

$$\prod_{i \in \text{string}} \sigma_i^x$$

$$\underline{[W_3^{\text{closed}}, H] = 0}$$



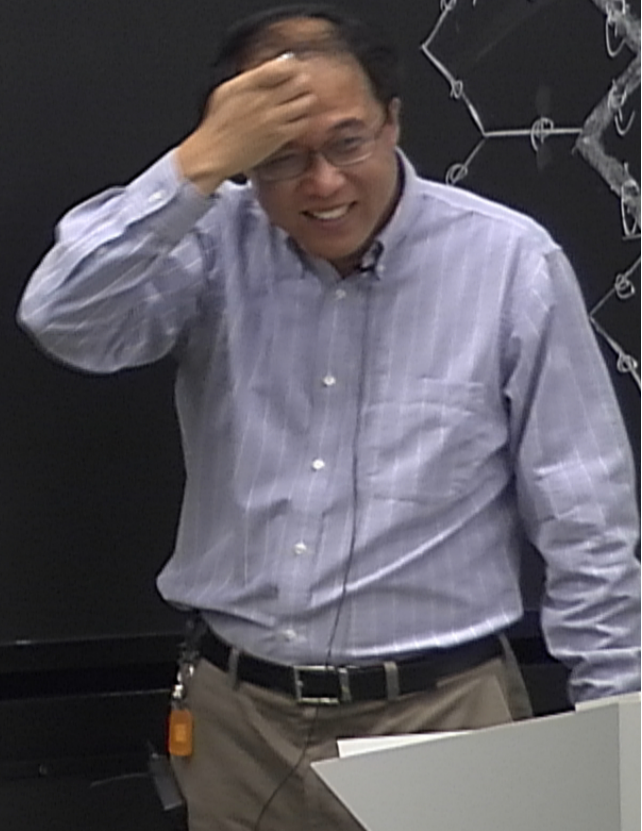
$$\sum_P \prod_{i \in \text{edges of } P} \sigma_i^x$$

$$W_1 + W_2 = W_3$$

$$\textcircled{1} \textcircled{2} \rightarrow \textcircled{3}$$

$$\prod_{i \in \text{string}} \sigma_i^x$$

$$\underline{[W_3^{\text{closed}}, H] = 0}$$

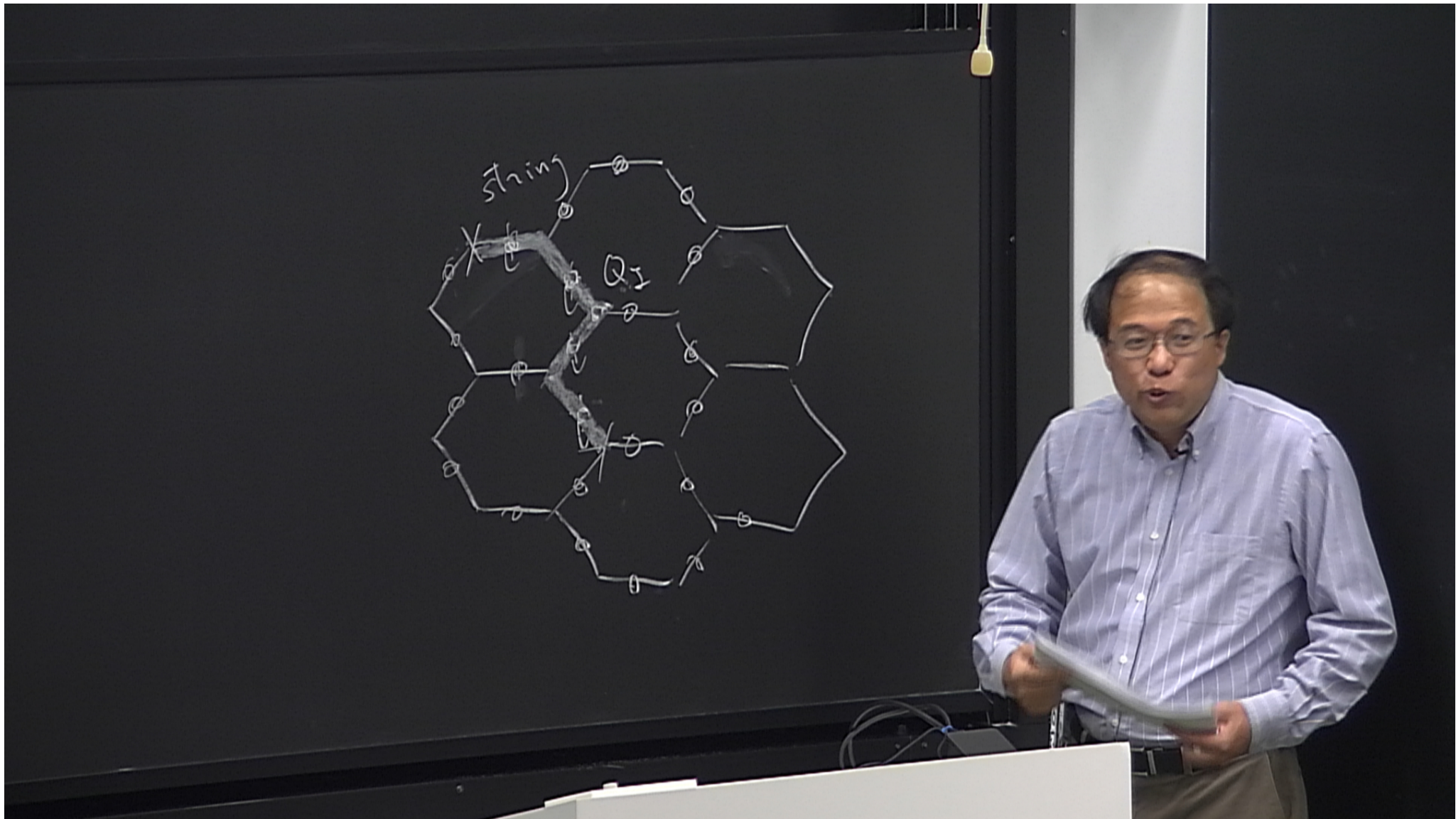


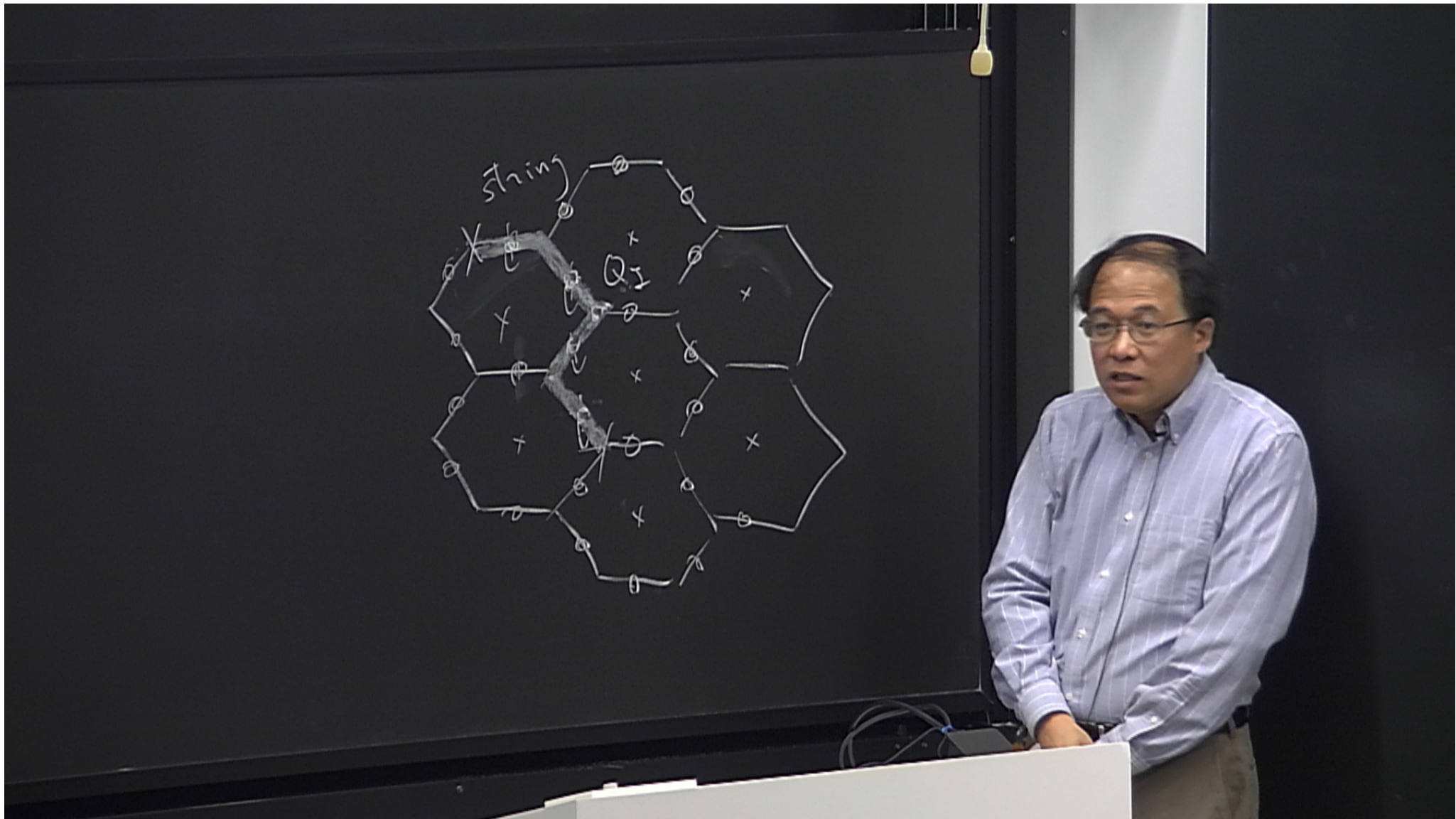
$i \in$
 legs of I
 Q_I

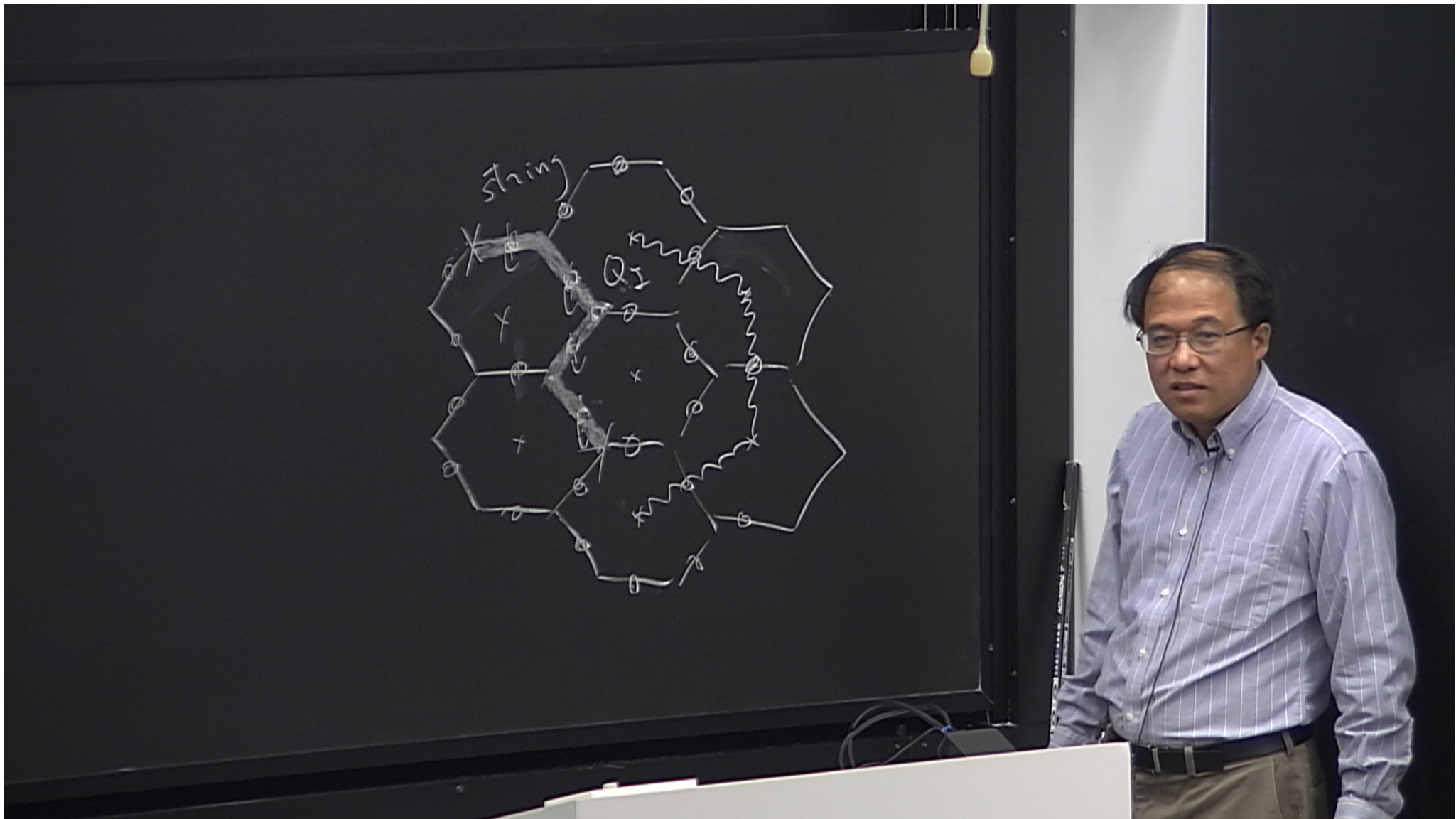
edges of P

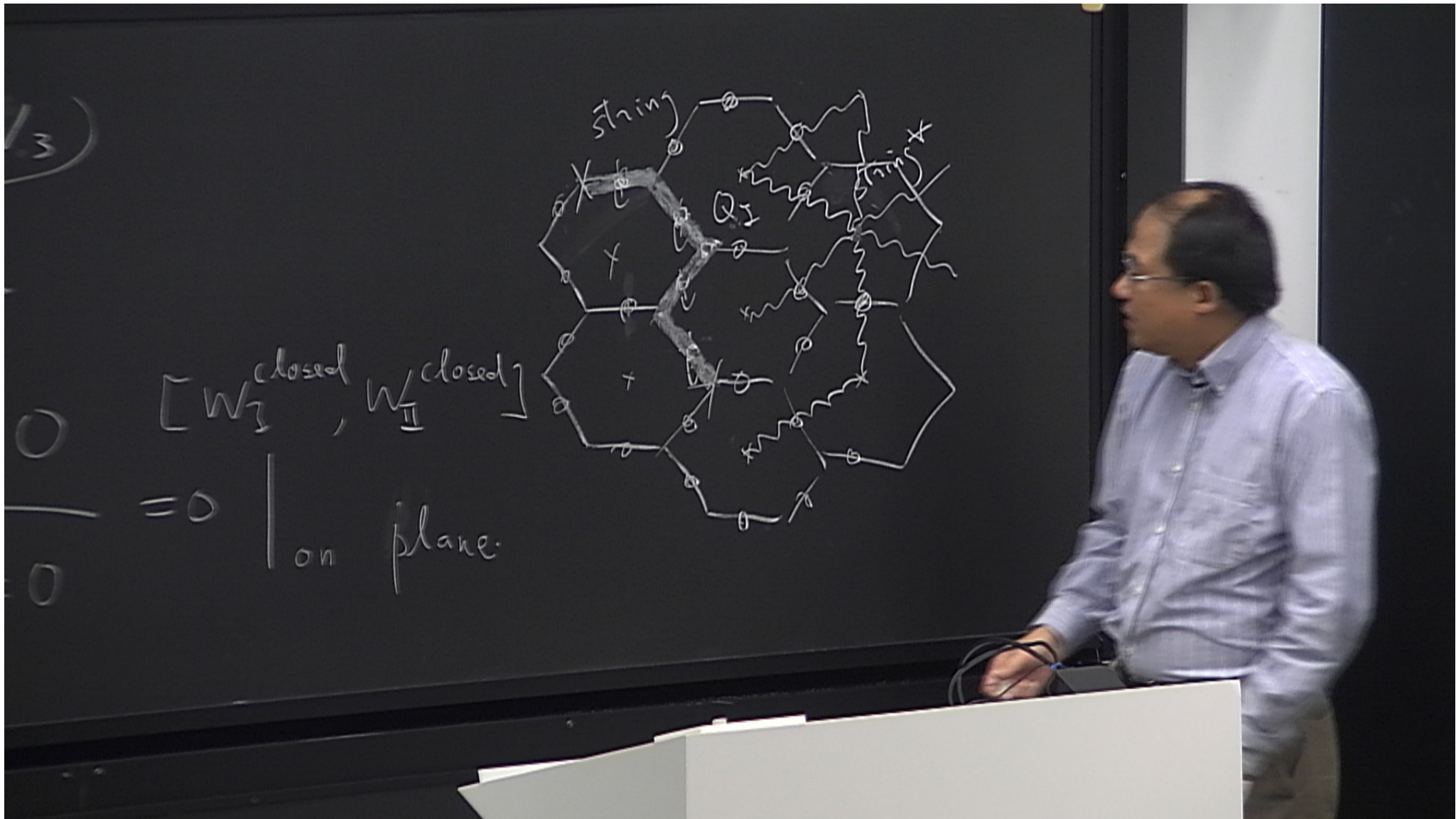
type-I string operator: $W_I = \prod_{i \in \text{string}} \sigma_i^x$

type-II string operator: $W_{II} = \prod_{i \in \text{string}^*} \sigma_i^z$









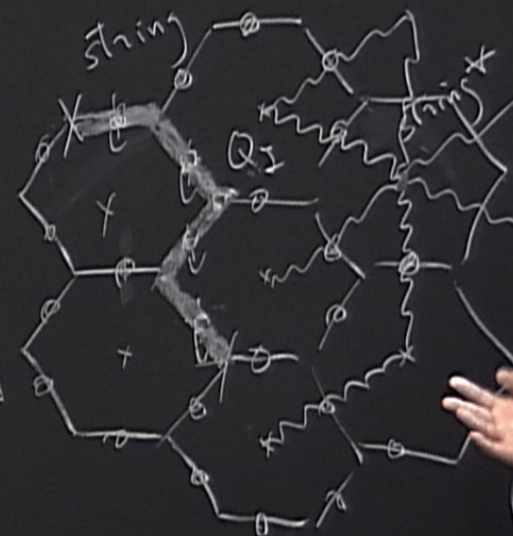
$$W_1 + W_2 = W_3$$

$$\textcircled{1} \textcircled{2} \rightarrow \textcircled{3}$$

$$[W_3^{\text{closed}}, H] = 0$$

$$[W_4^{\text{closed}}, H] = 0$$

$$[W_3^{\text{closed}}, W_4^{\text{closed}}] = 0 \quad \text{on plane.}$$

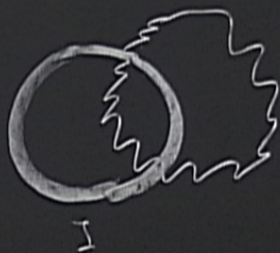


$$W_1 + W_2 = W_3$$

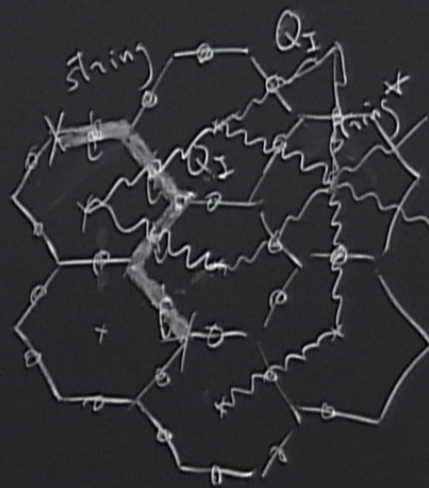
$$\textcircled{1} \textcircled{2} \rightarrow \textcircled{3}$$

$$[W_3^{\text{closed}}, H] = 0$$

$$[W_{II}^{\text{closed}}, H] = 0$$



$$[W_3^{\text{closed}}, W_{II}^{\text{closed}}] = 0 \quad \text{on plane.}$$



$$H = -U \sum_I \prod_{\substack{i \in \\ \text{legs of } I}} \sigma_i^z - g \sum_P \prod_{\substack{i \in \\ \text{edges of } P}} \sigma_i^x$$

$\underbrace{\quad}_{Q_I}$
 $\underbrace{\quad}_{F_P}$

$$W_I W_{II} = W_{III}$$

$$\textcircled{1} \textcircled{2} \rightarrow \textcircled{3}$$



pe-I string operator: $W_I = \prod_{i \in \text{string}} \sigma_i^x$

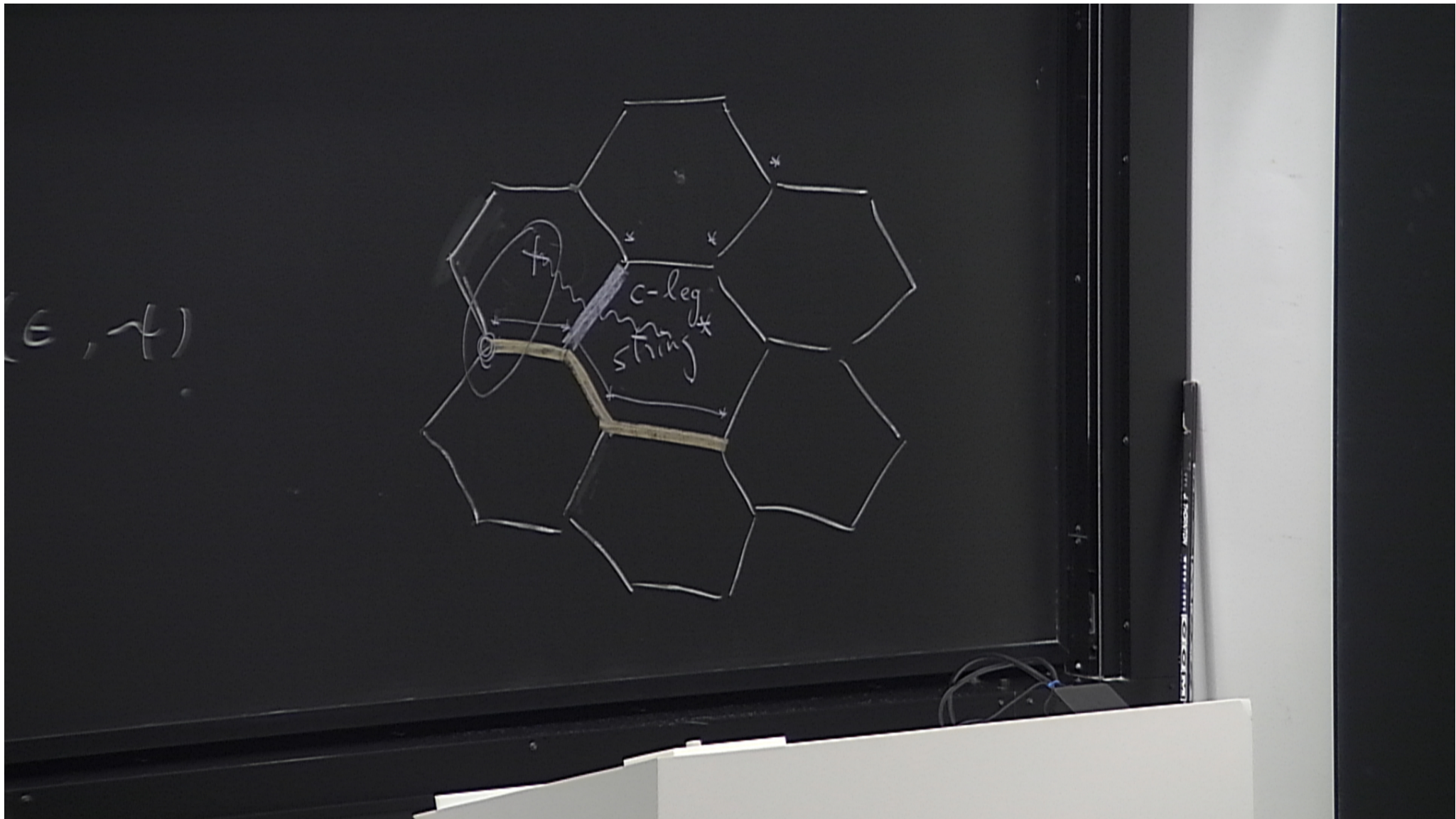
pe-II string operator: $W_{II} = \prod_{i \in \text{string}^*} \sigma_i^z$

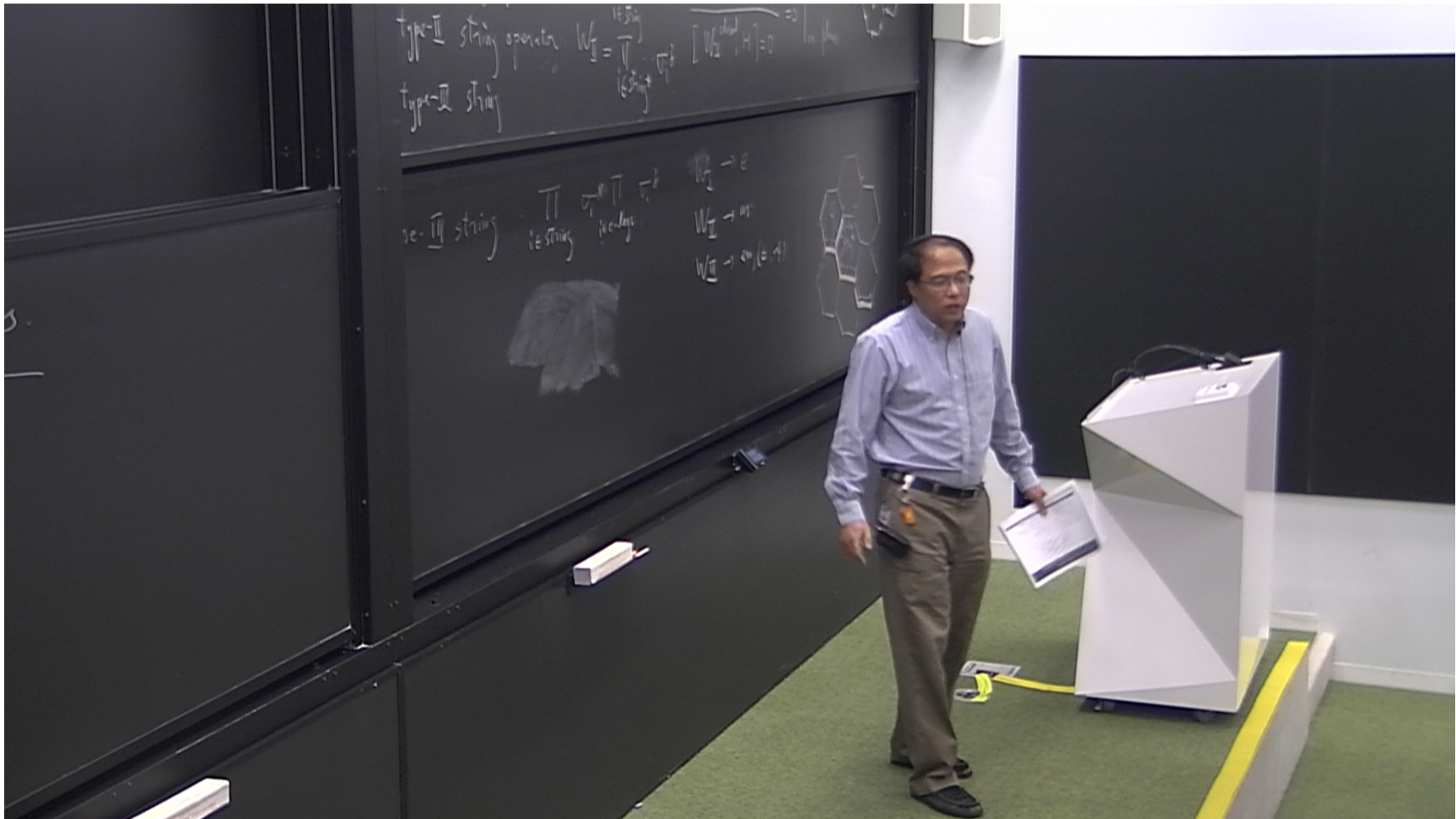
pe-III string

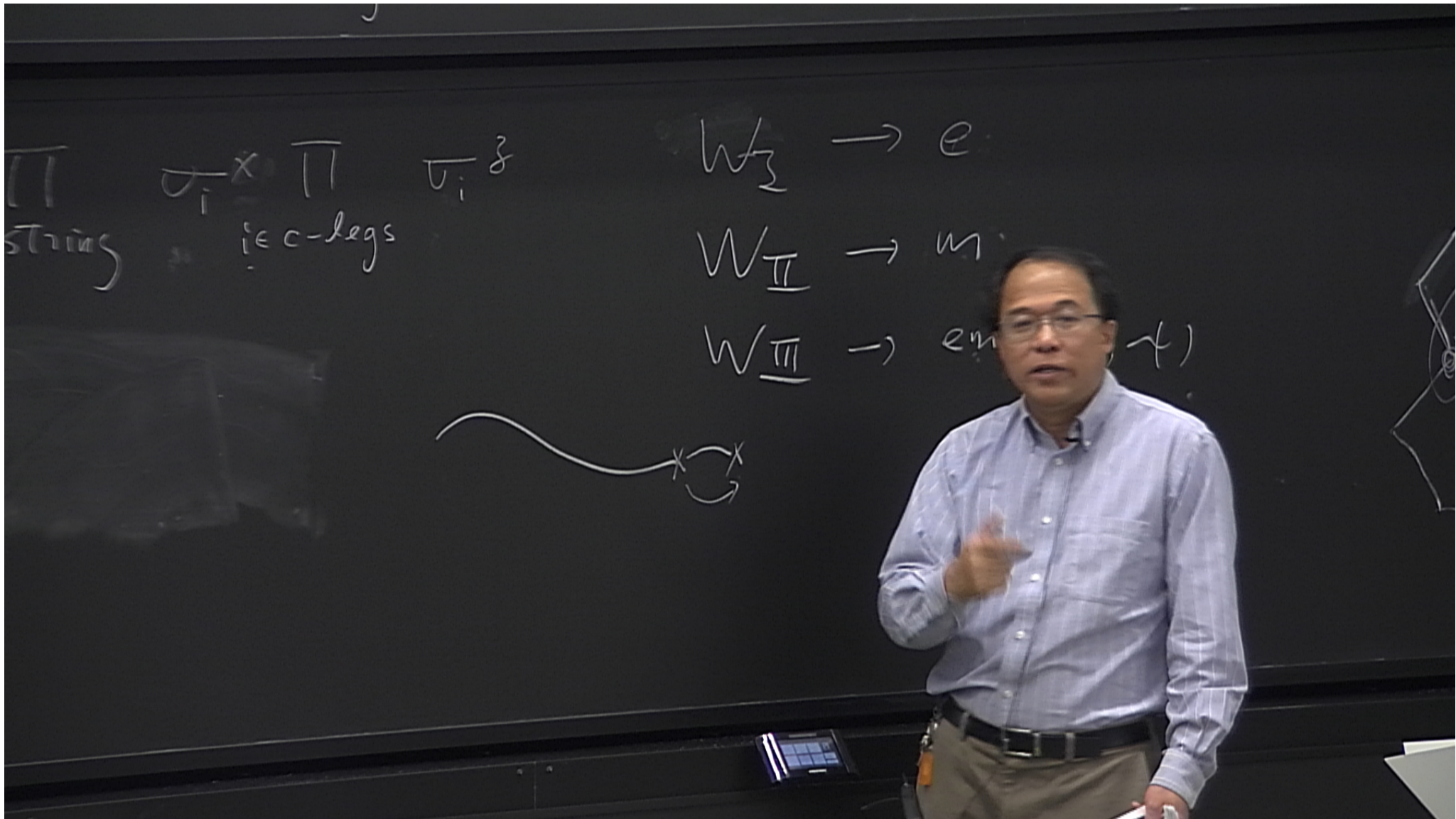
$$[W_I^{\text{closed}}, H] = 0$$

$$[W_{II}^{\text{closed}}, H] = 0$$

= 0 | on

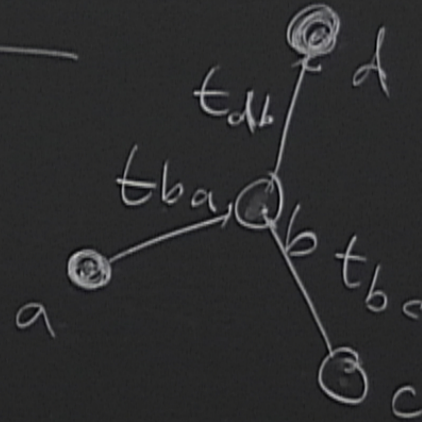




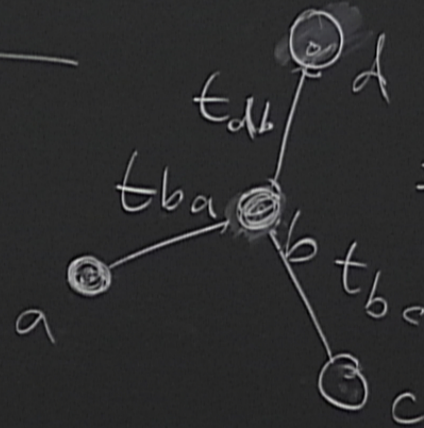




$\longleftrightarrow$ # of string types

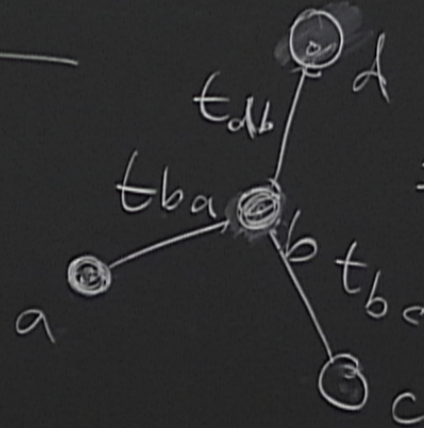


$\longleftrightarrow$ # of string types



$$t_{bc} t_{db} t_{ba} = \pm t_{ba} t_{db} t_{bc}$$

$\longleftrightarrow$ # of string types



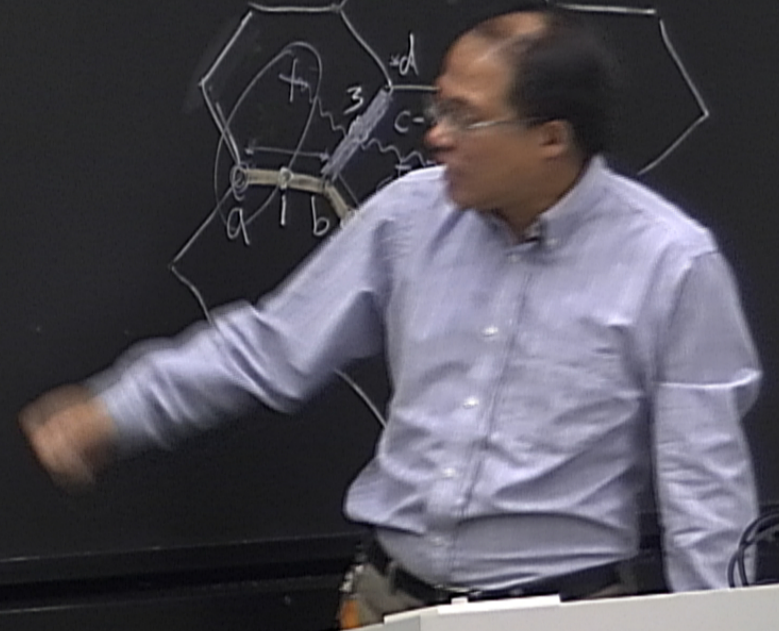
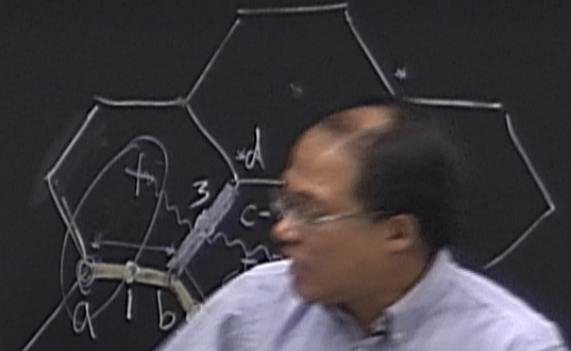
$$t_{bc} t_{db} t_{ba} = t_{ba} t_{db} t_{bc}$$

Π $\bar{\nu}_i$
c-legs

$$W_I \rightarrow e \rightarrow \text{boson}$$

$$W_{II} \rightarrow m \rightarrow \text{boson}$$

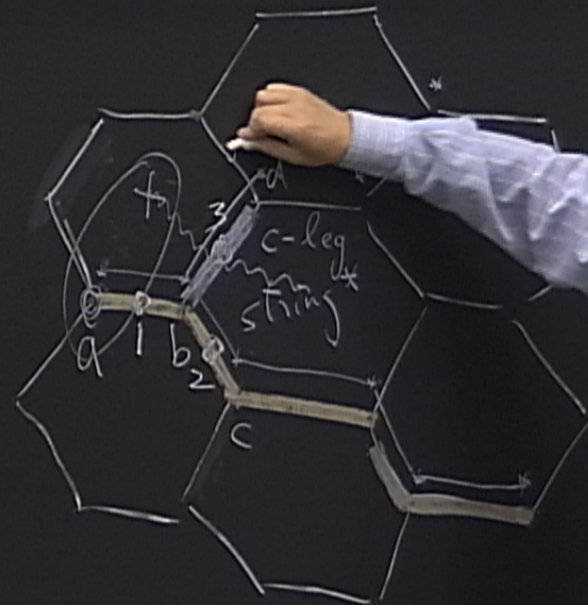
$$W_{III} \rightarrow em (e, \gamma)$$



$$W_I \rightarrow e + \text{boson}$$

$$W_{II} \rightarrow m \rightarrow \text{boson}$$

$$W_{III} \rightarrow em \ (e, \gamma) \rightarrow \text{fermion}$$



type-III string

$\prod_{i \in \text{string}}$

$\sigma_i^x \prod_{i \in c\text{-legs}}$

σ_i^z

W_3

W_{II}

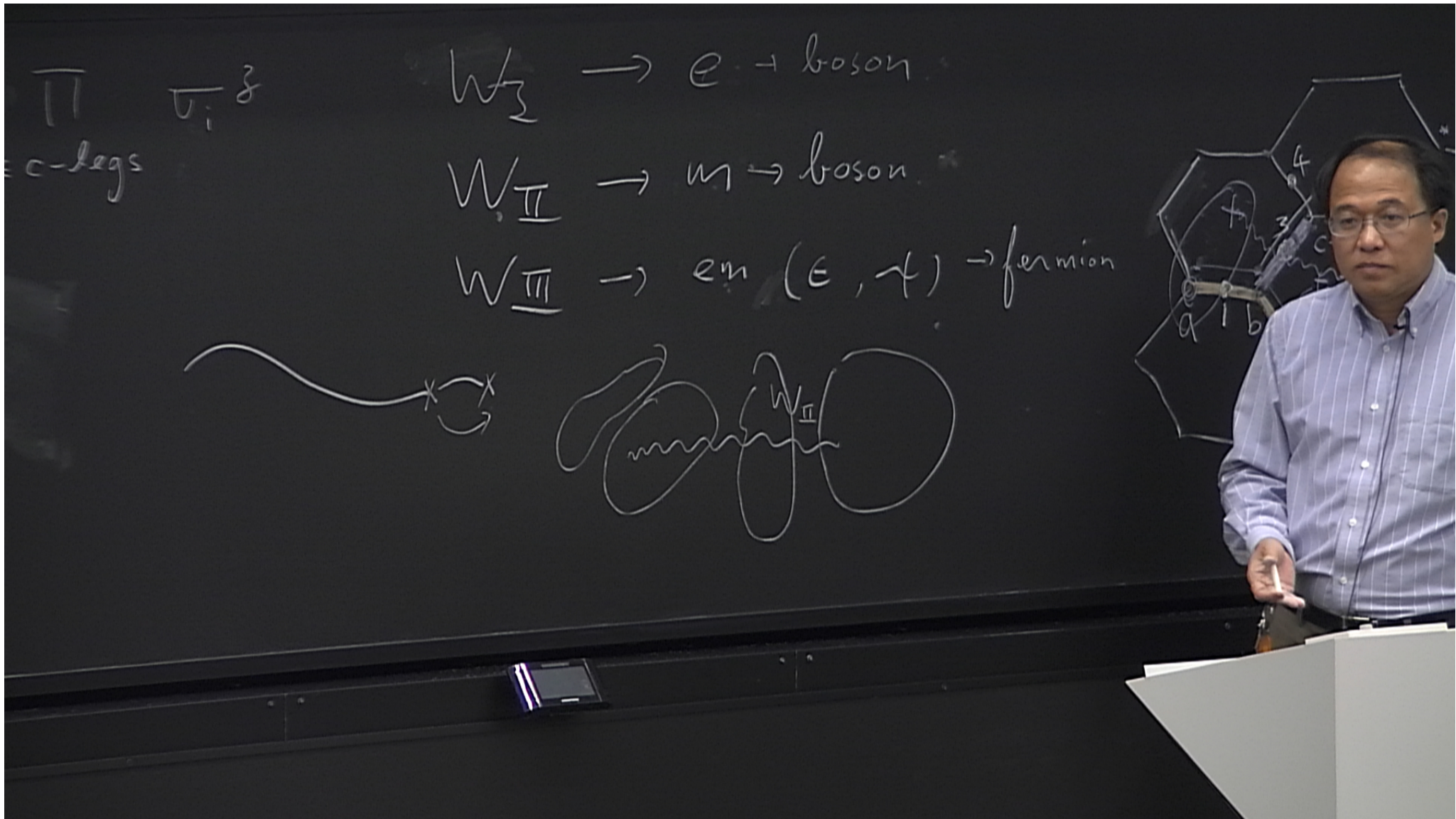
W_{III}

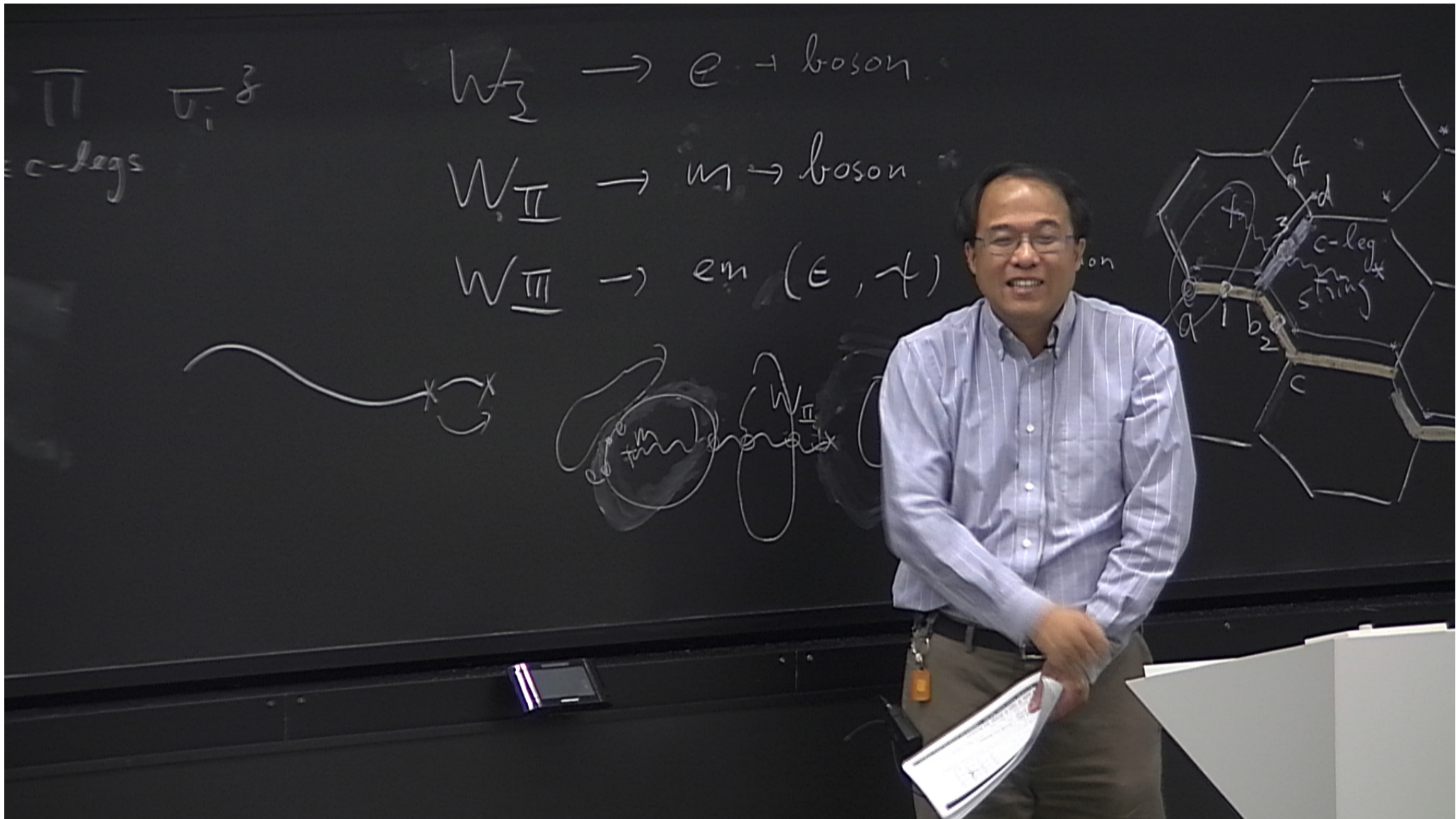
type-II $t_{ba} = \sigma_1^x$

$t_{db} = \sigma_3^x$

t_{bc}

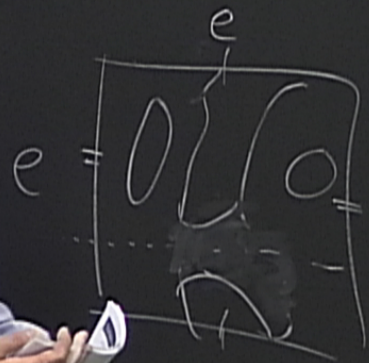






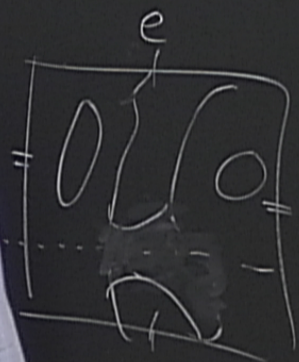


Number of topological types of quasiparticle
 = Number of degenerate ground states of torus



4 sectors (ee) (eo) (oe) (oo)

Number of topological types of quasiparticles
 = Number of degenerate ground states of torus



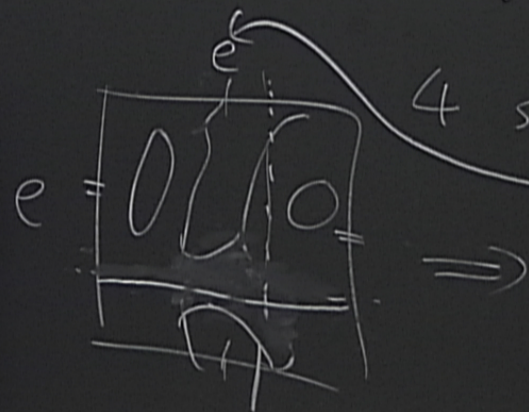
4 sectors

(ee) (eo) (oe) (oo)

$\Rightarrow$

$(ee) + (eo)$	$(ee) - (eo)$
$(oe) + (oo)$	$(oe) - (oo)$

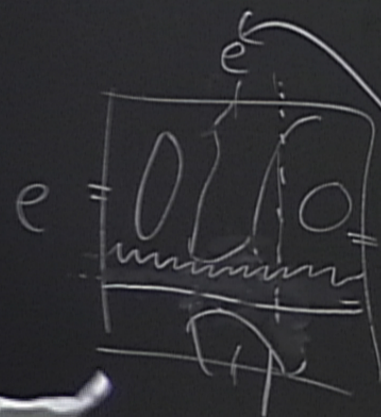
Number of topological types of quasiparticle
 = Number of degenerate ground states of torus



4 sectors (ee) (eo) (oe) (oo)

$(ee) + (eo)$	$(ee) - (eo)$
$(oe) + (oo)$	$(oe) - (oo)$

Number of topological types of quasiparticles $\longleftrightarrow$
 Number of degenerate ground states of torus



4 sectors (ee) (eo) (oe) (oo)

$$\begin{array}{ccc}
 (ee) + (eo) & \xrightarrow{\text{type I}} & (ee) - (eo) \\
 (oe) + (oo) & \xrightarrow{\text{type II}} & (oe) - (oo)
 \end{array}$$



of topological types of quasiparticles
of degenerate ground states of torus

$\longleftrightarrow$ # of s

e^{-L}



$\equiv e^{-L}$

4 sectors (ee) (eo) (oe) (oo)

$\Rightarrow$

$$\begin{array}{ccc}
 (ee) + (eo) & & (ee) - (eo) \\
 \downarrow \text{type I} & \curvearrowright & \uparrow \text{type II} \\
 (oe) + (oo) & & (oe) - (oo)
 \end{array}$$