Title: 12/13 PSI - Explorations in Condensed Matter Lecture 10

Date: Apr 19, 2013 09:00 AM

URL: http://pirsa.org/13040054

Abstract:

4-CORNER SOUTHWEST ONTARIO CONDENSED MATTER SYMPOSIUM 2013

Conference Date:
Thursday, April 25, 2013 (All day)

- Lukasz Cincio, “tensor networks: an overview”, 4pm

EMERGENCE AND ENTANGLEMENT II

Conference Date:
Monday, May 6, 2013 (All day) to Friday, May 10, 2013 (All day)

- Subir Sachdev (entanglement) [*colloquium]
- Tadashi Takayanagi (entanglement)
- Roger Melko (entanglement)
- Tarun Grover (entanglement)
- Hong Liu (entanglement)

- Frank Verstraete (MPS)
- Glen Evenbly (MERA)
- Steve White (MPS in 2D)
- Leon Balents (MPS in 2D)
- Philippe Corboz (PEPS)
- Norbert Schuch (PEPS)

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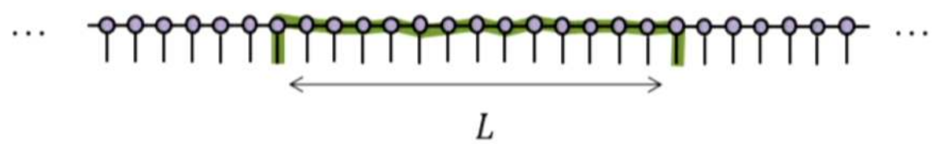
CORRELATIONS and DISTANCE

matrix product state (MPS)

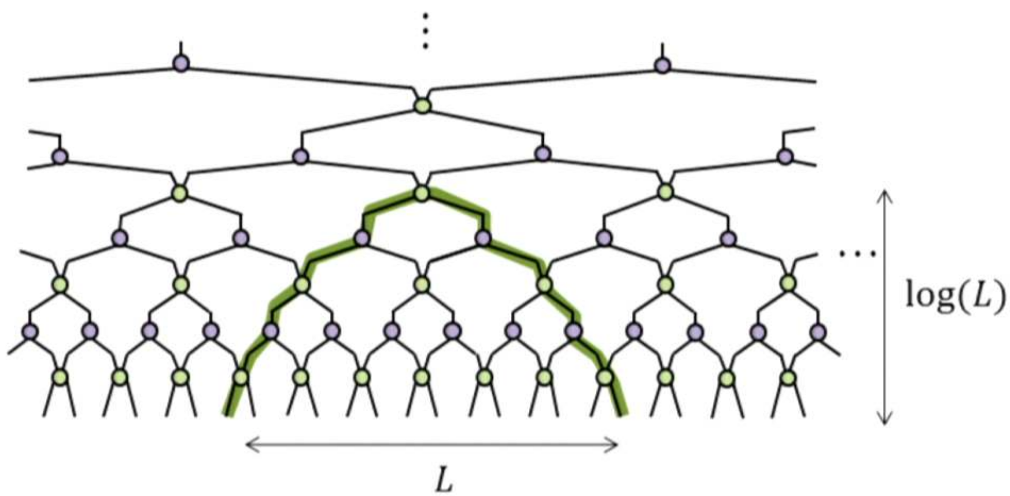


exponential correlators

$C(L) \approx e^{-L/\xi}$

$[C(L) \approx \lambda^{L}]$

multi-scale entanglement renormalization ansatz (MERA)



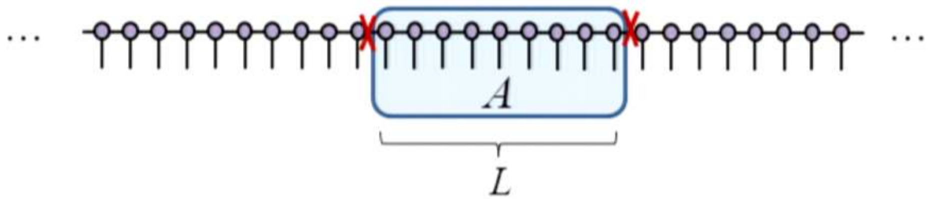
polynomial correlators

$C(L) \approx L^{-p}$

$[C(L) \approx \lambda^{\log(L)}]$

ENTANGLEMENT ENTROPY and CONNECTIVITY

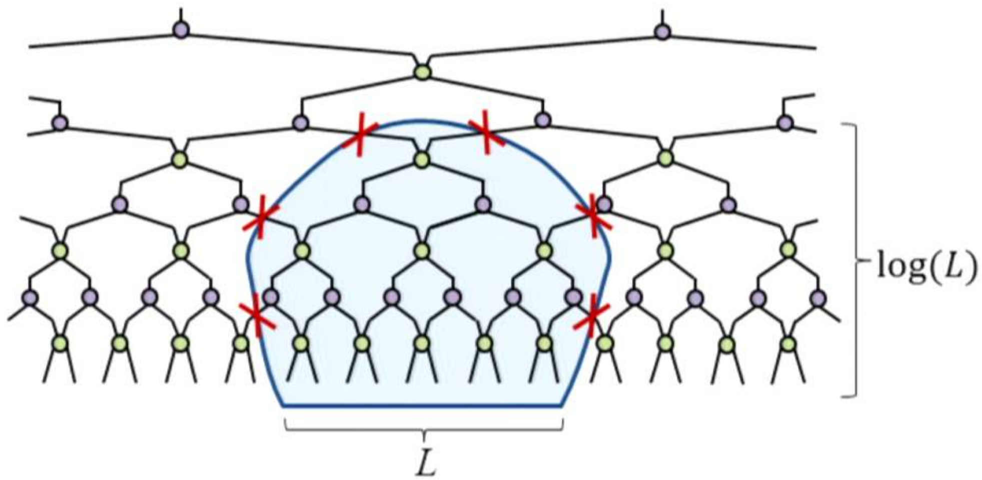
matrix product state (MPS)



constant entropy

$S_L \approx const$

multi-scale entanglement renormalization ansatz (MERA)

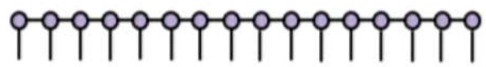


logarithmic entropy

$S_L \approx \log(L)$

D=1 spatial dimensions

matrix product state
(MPS)

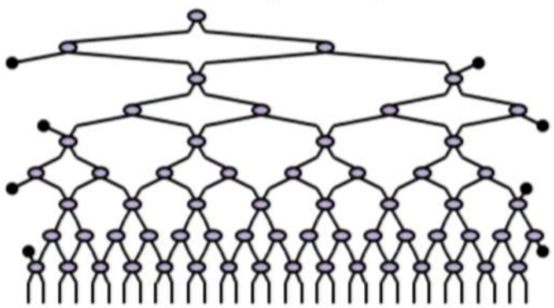


$$C(L) \approx e^{-L/\xi}$$

$$S_L \approx const \ \ (= L^{D-1})$$

(gapped systems)

multi-scale entanglement renormalization ansatz
(MERA)



$$C(L) \approx L^{-p}$$

$$S_L \approx \log L \quad (= L^{D-1} \log L)$$

(critical systems)

D=1 spatial dimensions

matrix product state

(MPS)

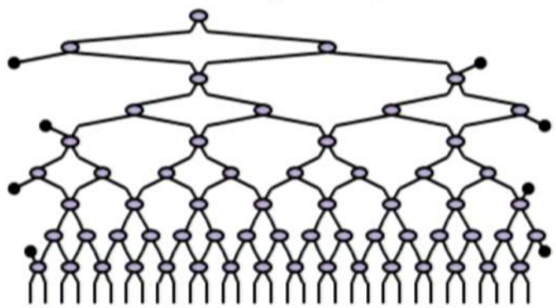
$C(L) \approx e^{-L/\xi}$

$S_L \approx const \ \ (= L^{D-1})$

(gapped systems)

multi-scale entanglement renormalization ansatz

(MERA)



$C(L) \approx L^{-p}$

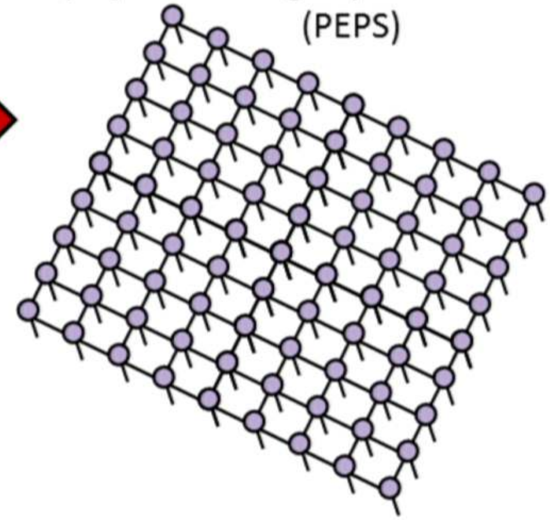
$S_L \approx \log L \quad (= L^{D-1} \log L)$

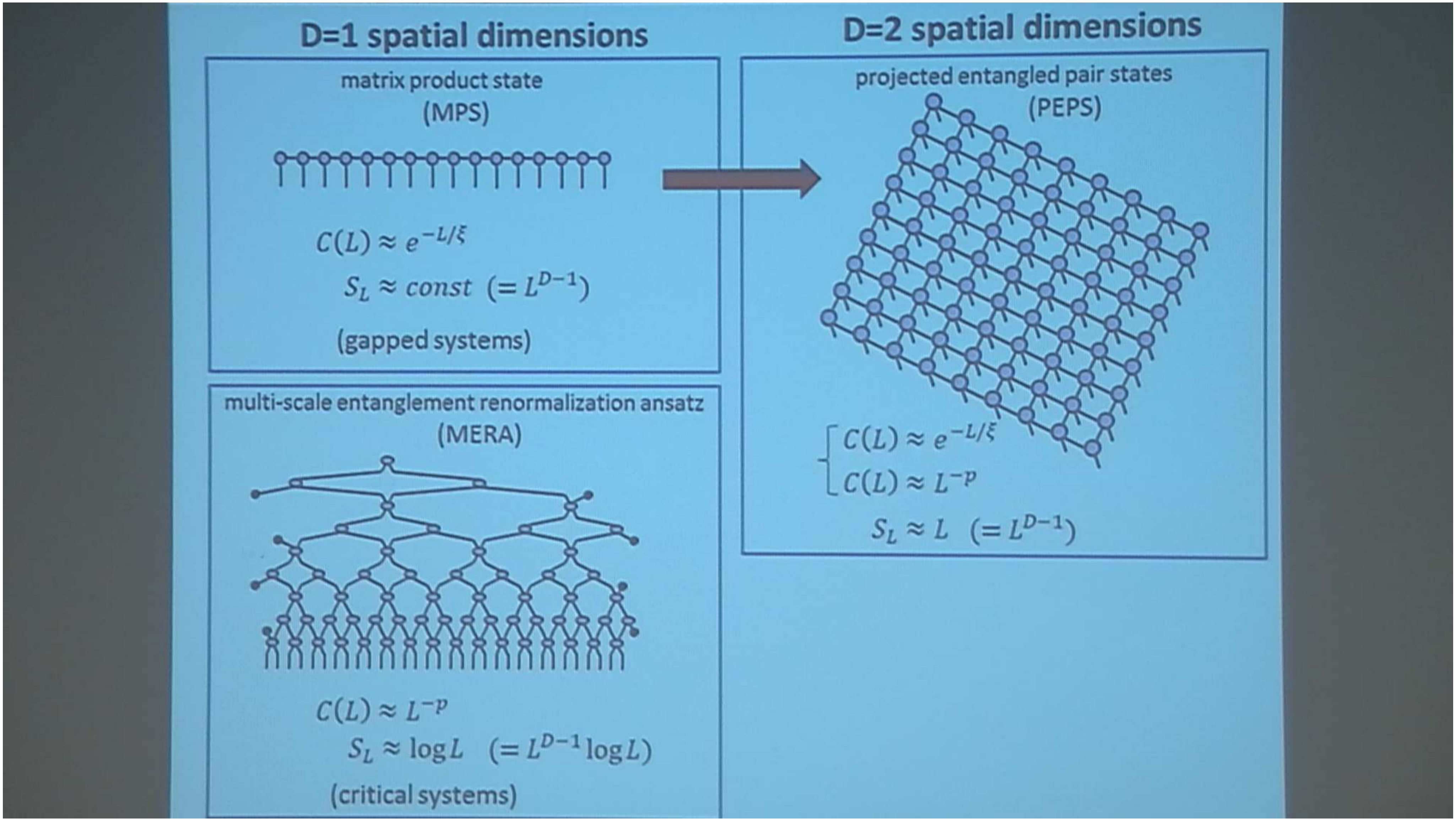
(critical systems)

D=2 spatial dimensions

projected entangled pair states

(PEPS)




D=1 spatial dimensions
matrix product state
(MPS)
$C(L) \approx e^{-L/\xi}$
$S_L \approx const \ (= L^{D-1})$
(gapped systems)
multi-scale entanglement renormalization ansatz
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$C(L) \approx L^{-p}$
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(critical systems)
D=2 spatial dimensions
projected entangled pair states
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$C(L) \approx e^{-L/\xi}$
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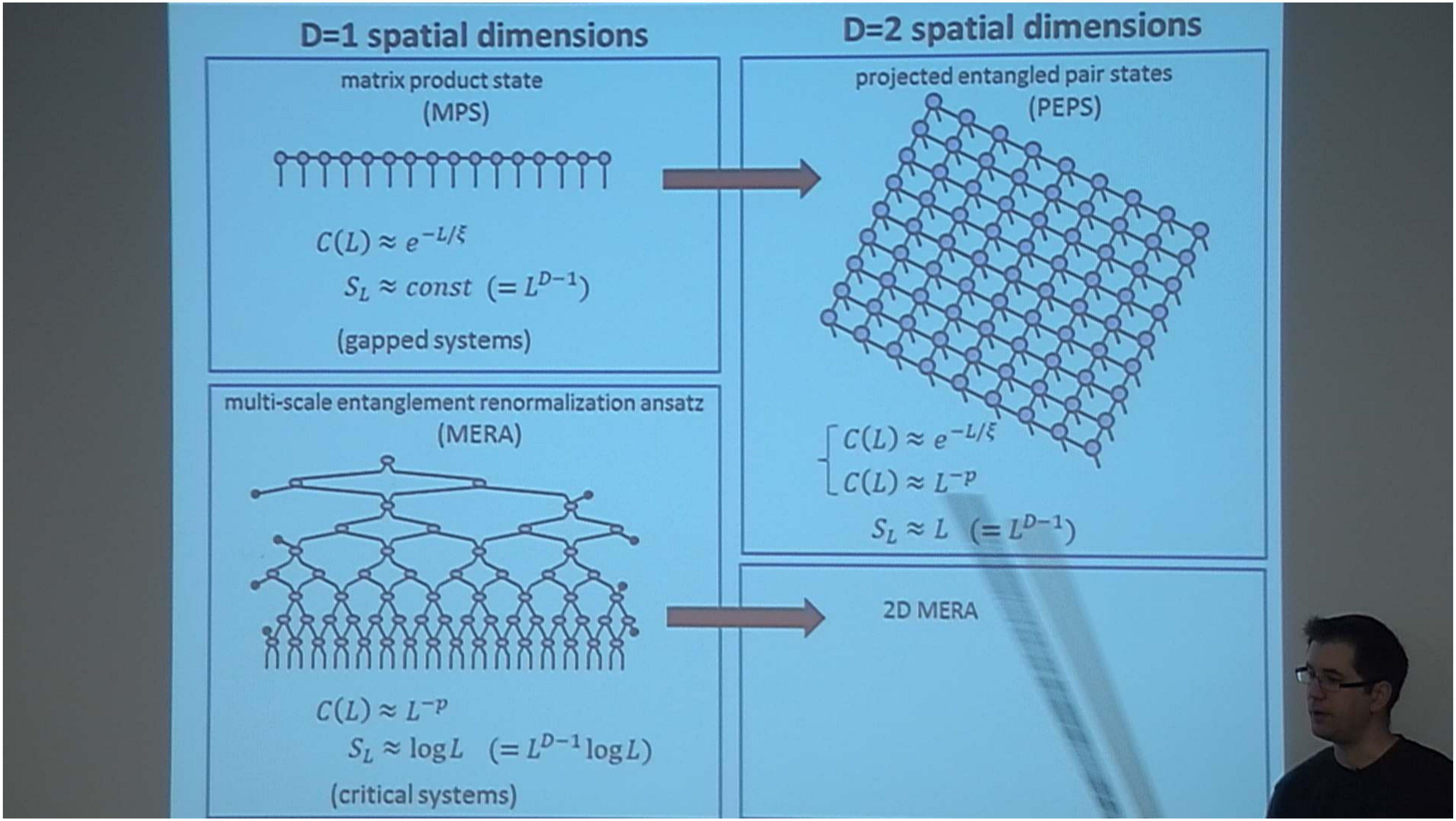
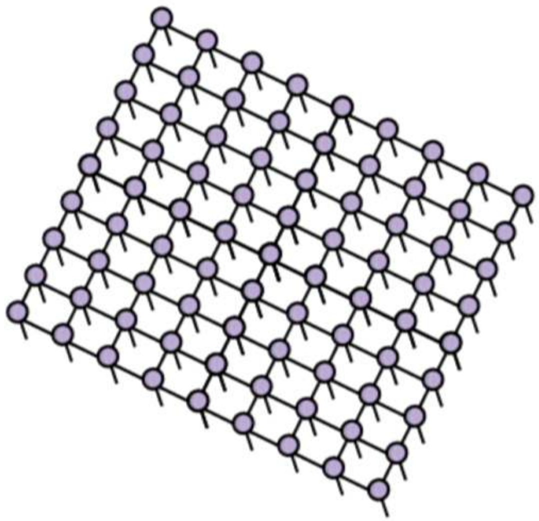

D=1 spatial dimensions
D=2 spatial dimensions
matrix product state
(MPS)
$C(L) \approx e^{-L/\xi}$
$S_L \approx const \ (= L^{D-1})$
(gapped systems)
projected entangled pair states
(PEPS)
$C(L) \approx e^{-L/\xi}$
$C(L) \approx L^{-p}$
$S_L \approx L \ (= L^{D-1})$
multi-scale entanglement renormalization ansatz
(MERA)
$C(L) \approx L^{-p}$
$S_L \approx \log L \ (= L^{D-1} \log L)$
(critical systems)
2D MERA

9-Projected entangled pair states (PEPS)



exponential or polynomial correlations

$$C(L) \approx e^{-L/\xi}$$

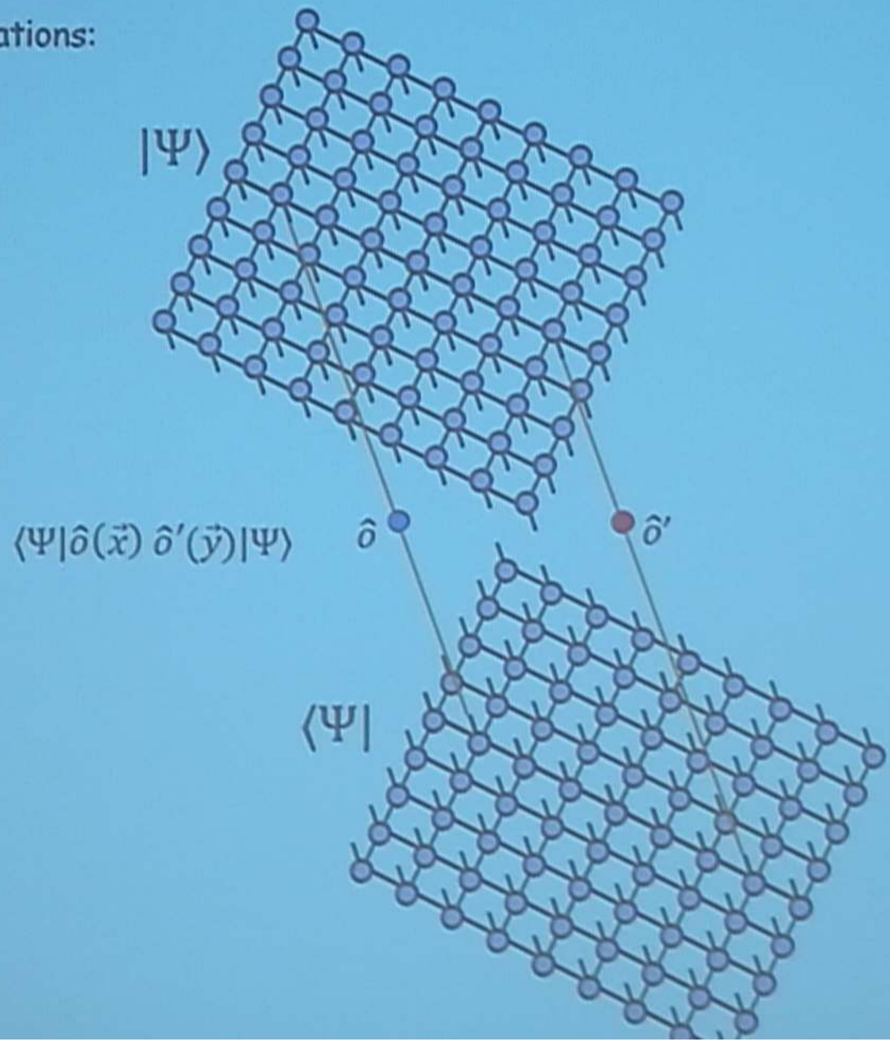
$$C(L) \approx L^{-p}$$

boundary law for entanglement entropy

$$S_L \approx L \quad (= L^{D-1})$$

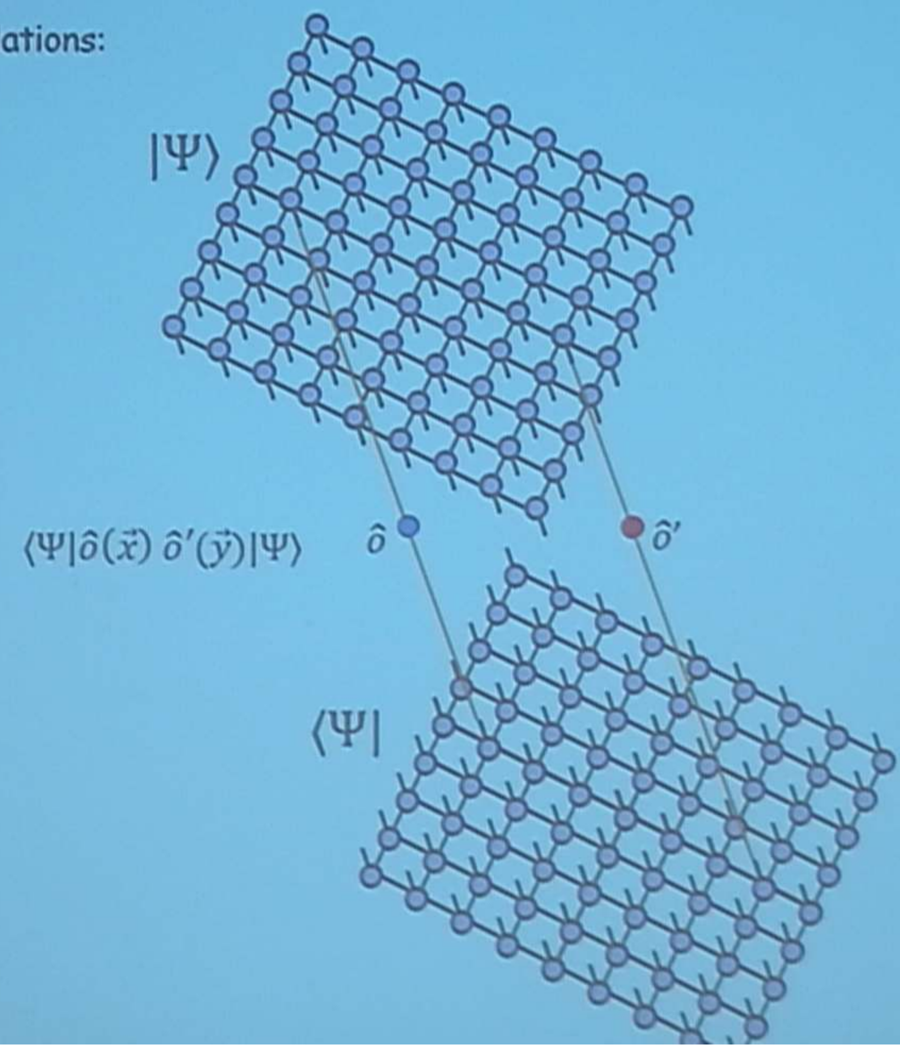
Projected entangled pair states (PEPS)

Correlations:



Projected entangled pair states (PEPS)

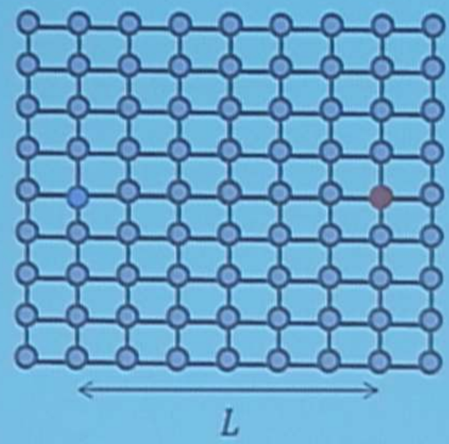
Correlations:



Projected entangled pair states (PEPS)

Correlations:

$\langle\Psi|\hat{o}(\vec{x})\;\hat{o}'(\vec{y})|\Psi\rangle$



Projected entangled pair states (PEPS)

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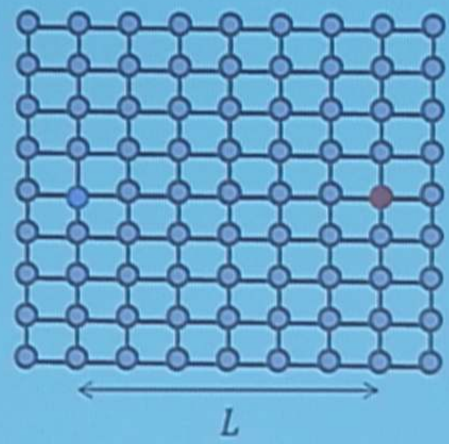
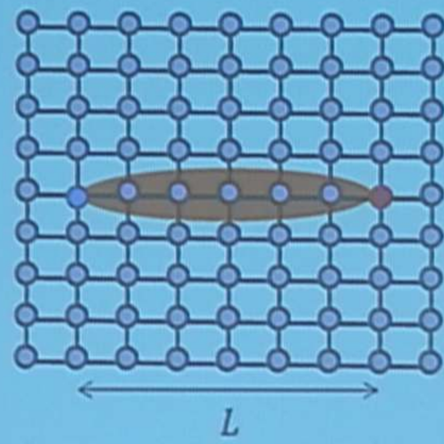


Projected entangled pair states (PEPS)

Correlations:

$\langle\Psi|\hat{o}(\vec{x})\;\hat{o}'(\vec{y})|\Psi\rangle$



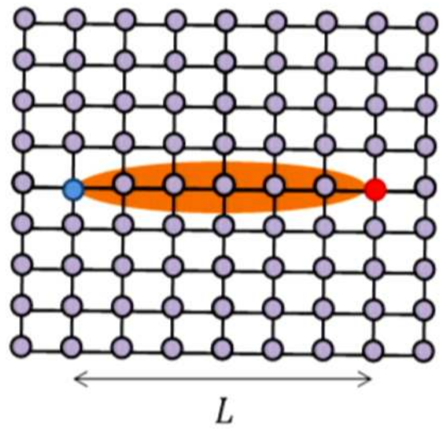
exponential correlations

$C(L) \approx e^{-L/\xi}$

Projected entangled pair states (PEPS)

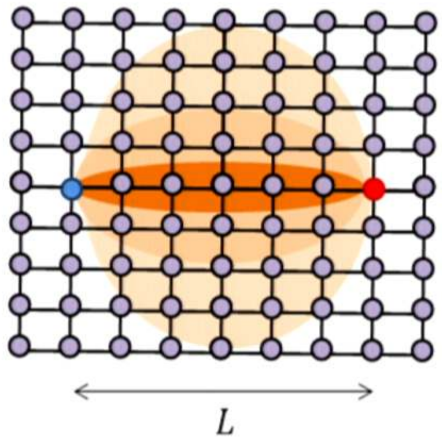
Correlations:

$\langle\Psi|\hat{o}(\vec{x})\ \hat{o}'(\vec{y})|\Psi\rangle$



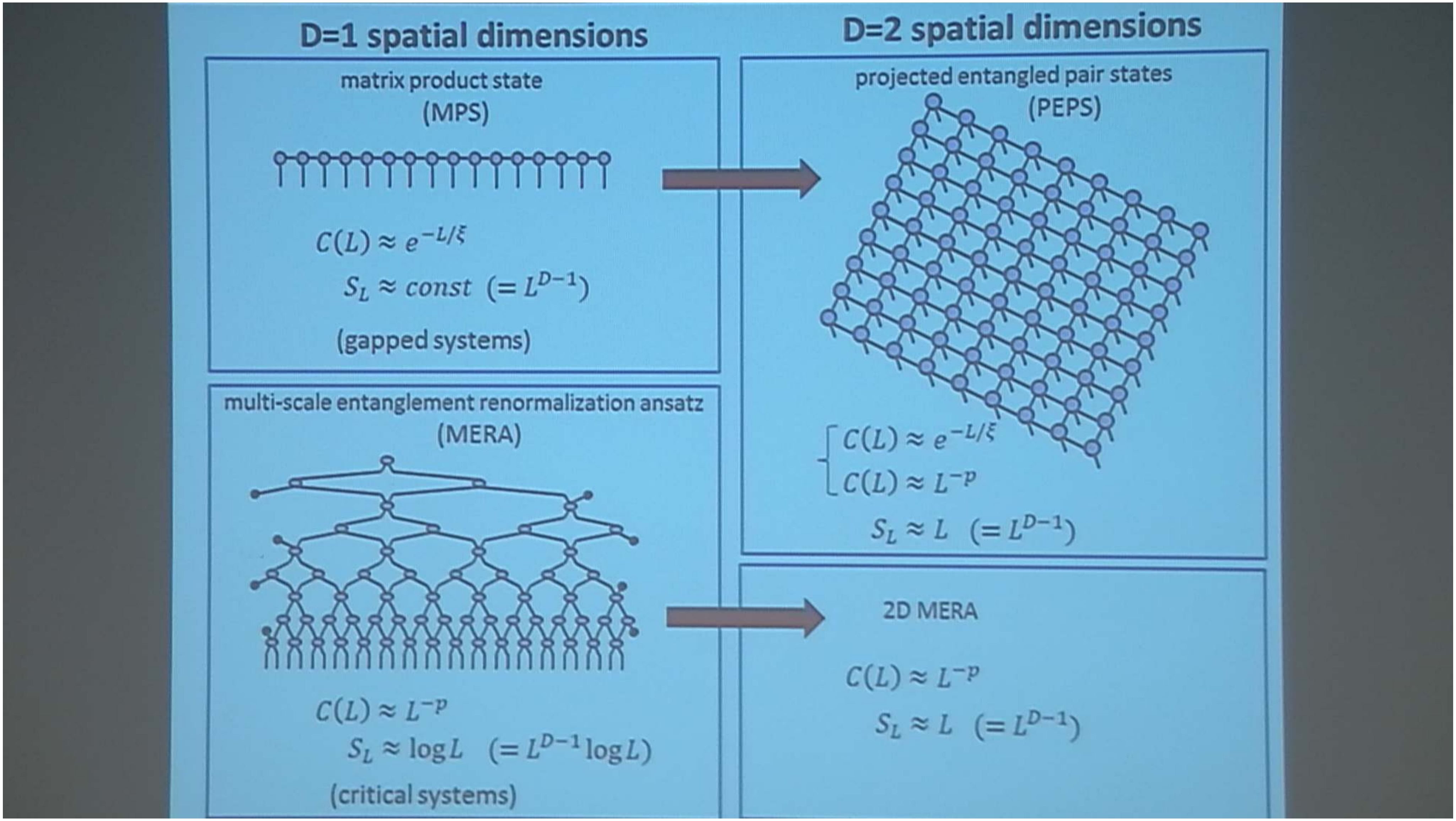
exponential correlations

$$C(L) \approx e^{-L/\xi}$$



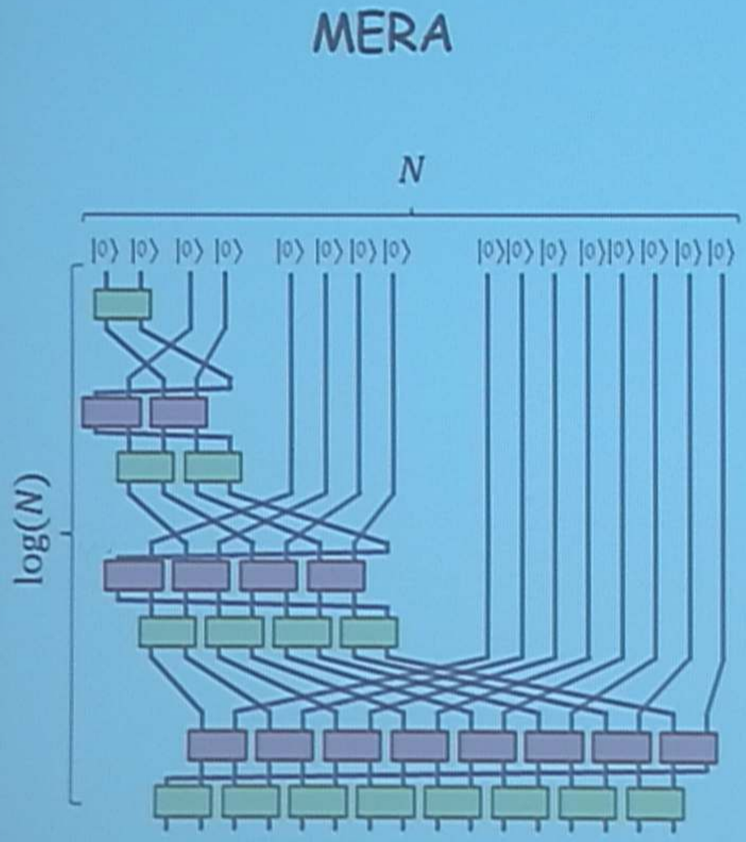
polynomial correlations

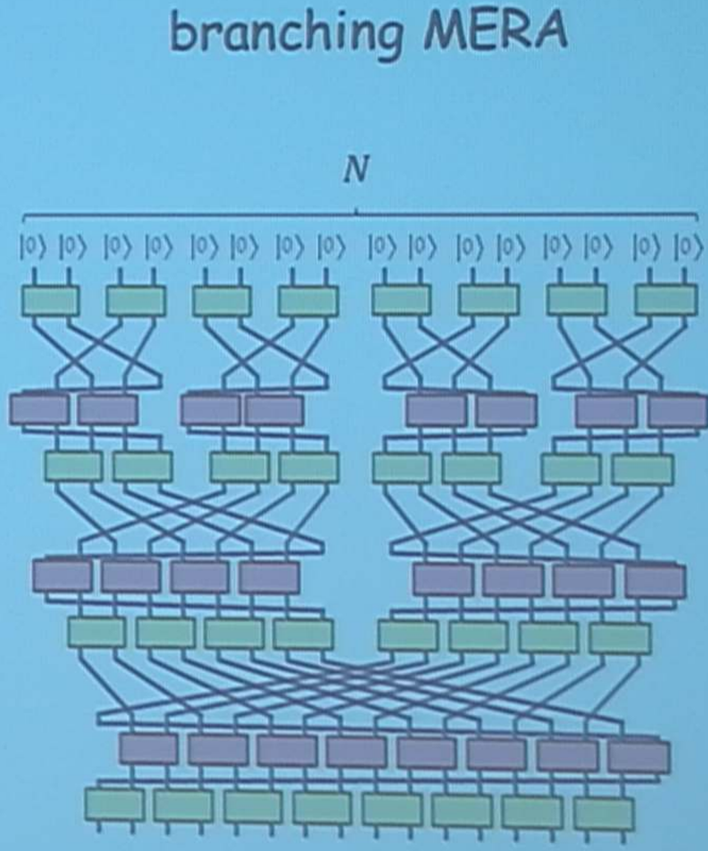
$$C(L) \approx L^{-p}$$



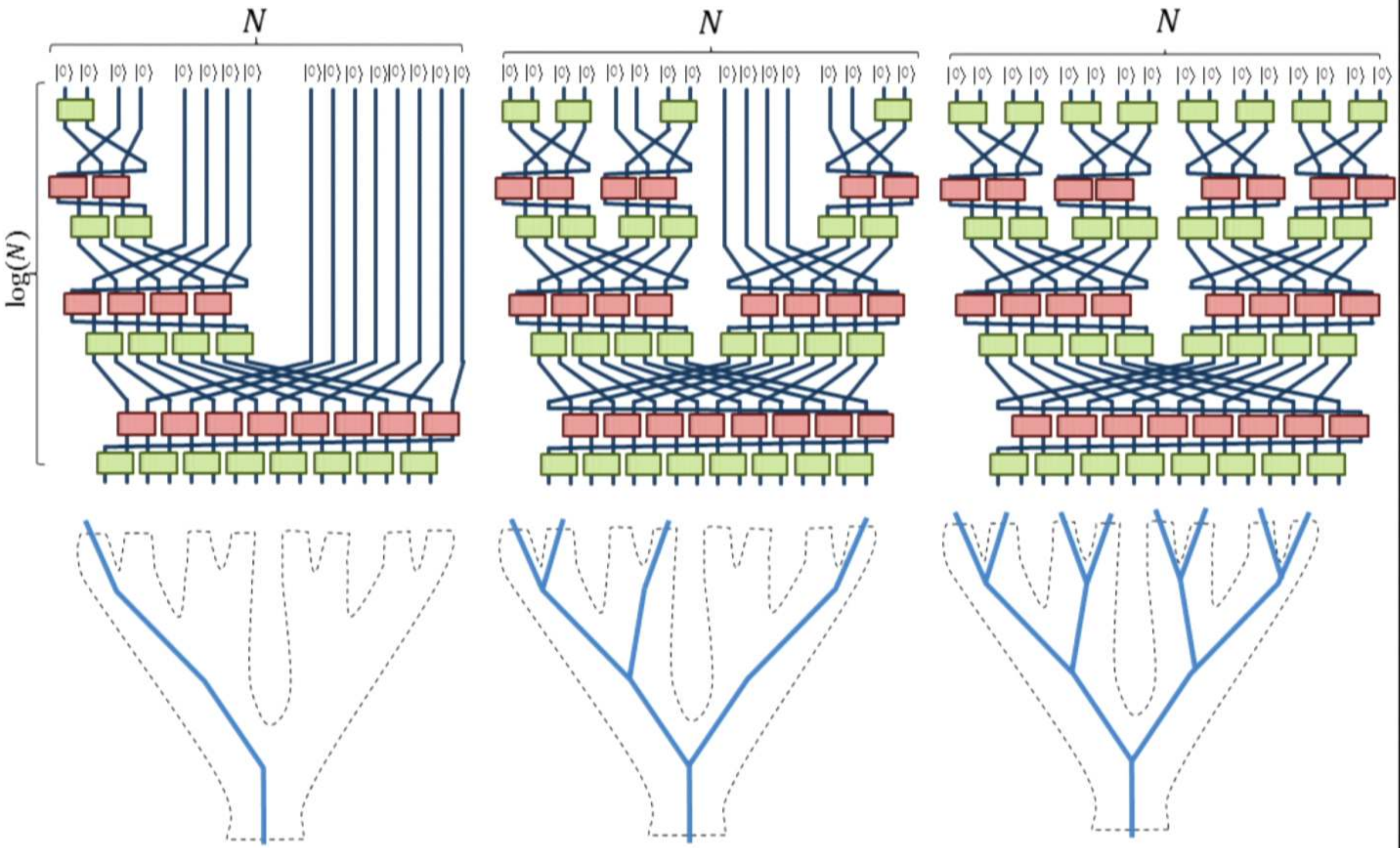
D=1 spatial dimensions
matrix product state
(MPS)
$C(L) \approx e^{-L/\xi}$
$S_L \approx const \ (= L^{D-1})$
(gapped systems)
multi-scale entanglement renormalization ansatz
(MERA)
$C(L) \approx L^{-p}$
$S_L \approx \log L \quad (= L^{D-1} \log L)$
(critical systems)
D=2 spatial dimensions
projected entangled pair states
(PEPS)
$C(L) \approx e^{-L/\xi}$
$C(L) \approx L^{-p}$
$S_L \approx L \ (= L^{D-1})$
2D MERA
$C(L) \approx L^{-p}$
$S_L \approx L \ (= L^{D-1})$

10-Branching MERA

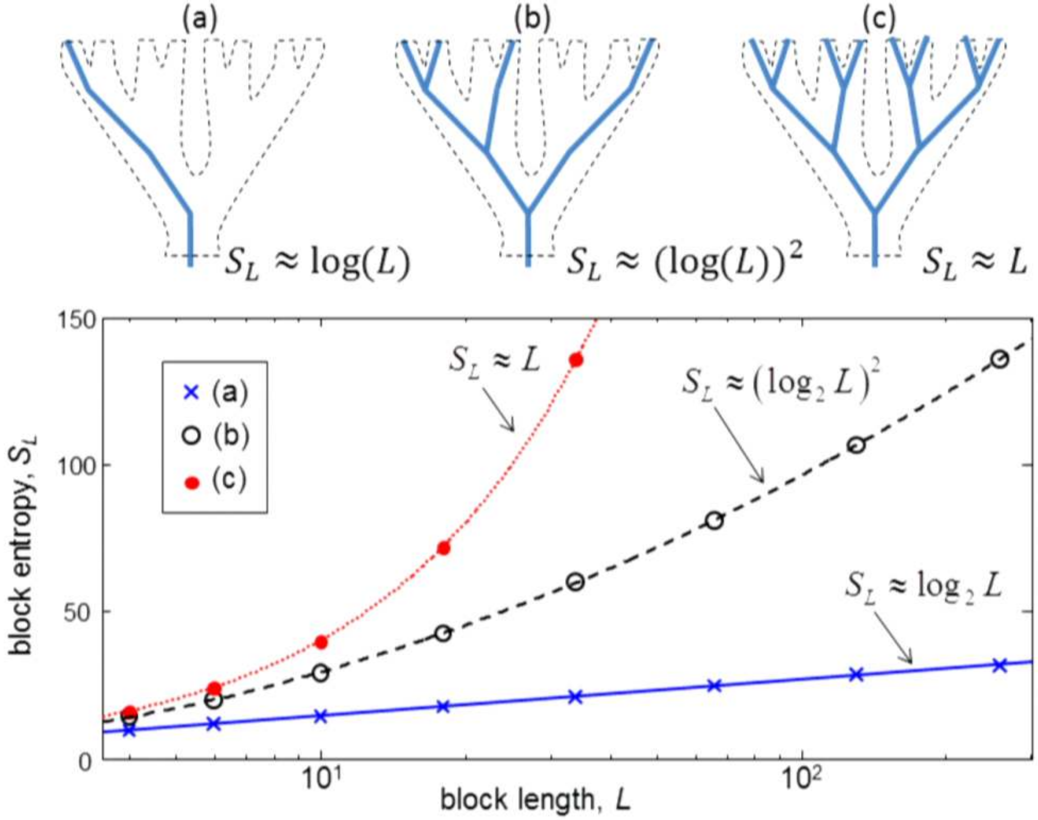


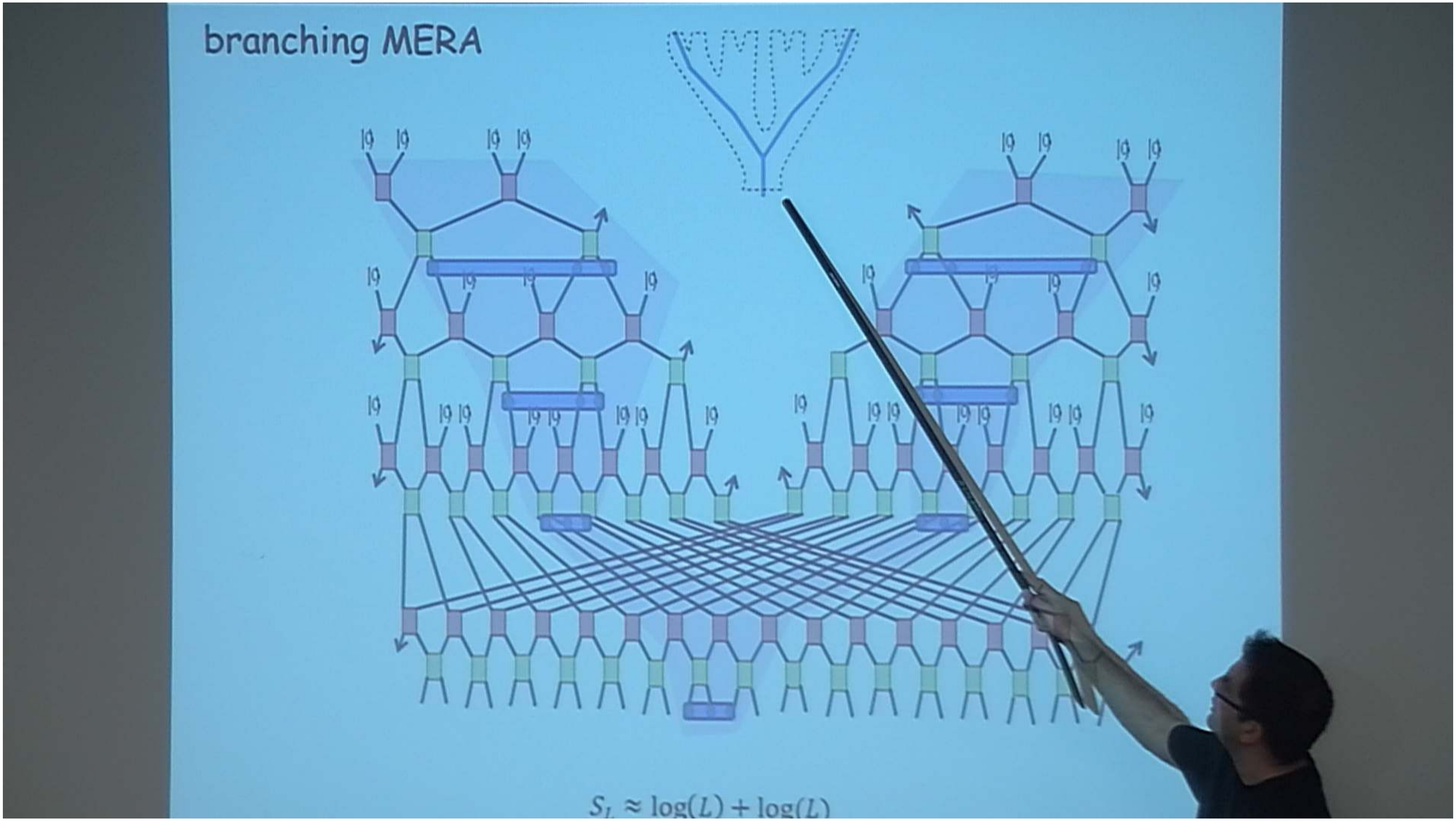


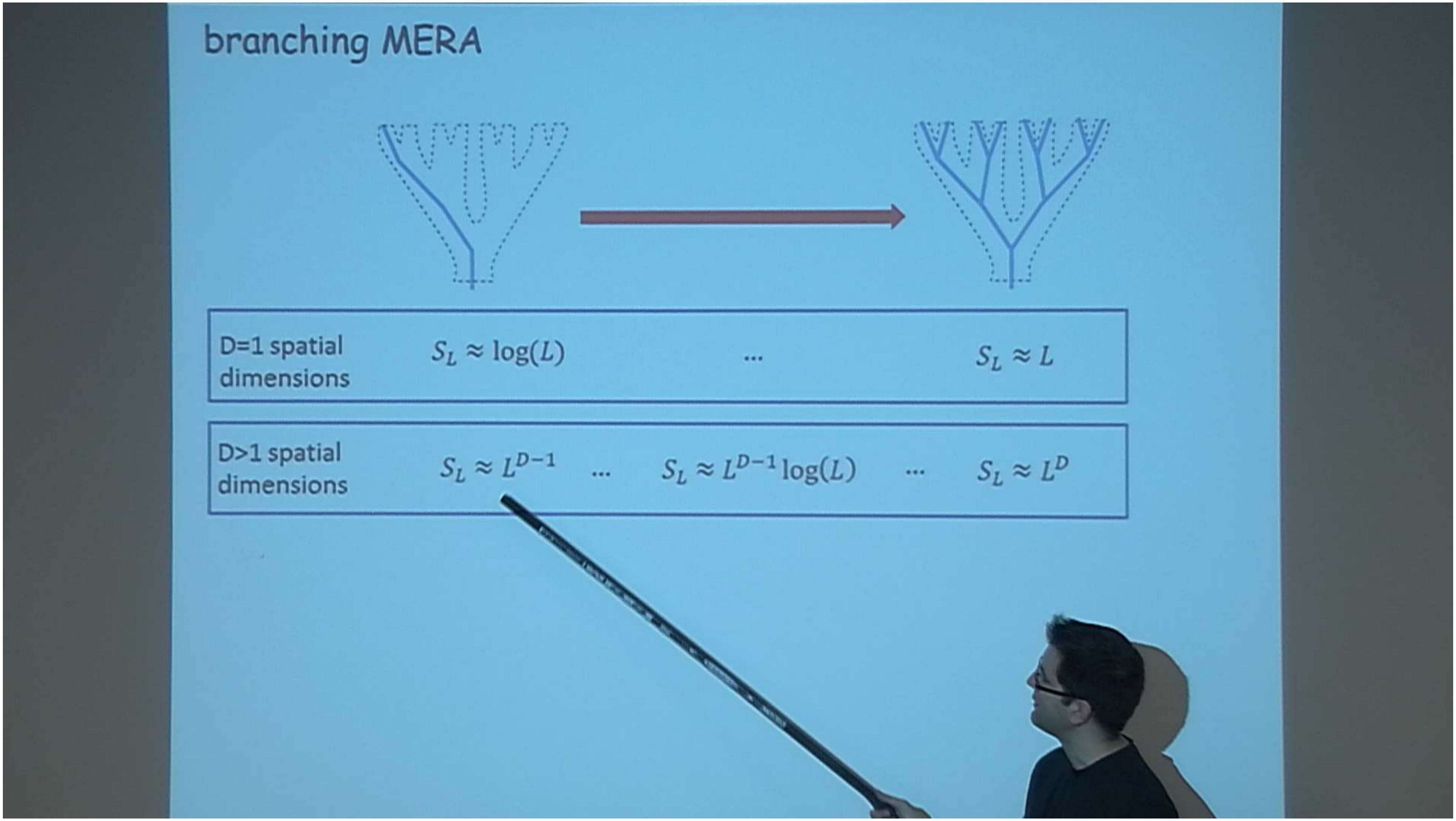
MERA | branching MERA



branching MERA

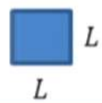
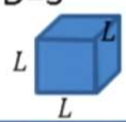



branching MERA
$S_L \approx \log(L) + \log(L)$


branching MERA
D=1 spatial dimensions
$S_L \approx \log(L)$
...
$S_L \approx L$
D>1 spatial dimensions
$S_L \approx L^{D-1}$
...
$S_L \approx L^{D-1}\log(L)$
...
$S_L \approx L^D$

11-Summary/outlook

Scaling of entanglement entropy (first week)

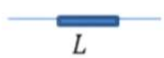
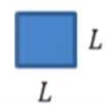
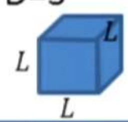
Dimension	gapped $\Delta > 0$	gapless $\Delta = 0$ **no** (D-1)- dimensional Fermi surface	gapless $\Delta = 0$ (D-1)- dimensional Fermi surface
D=1 	$S_L \approx const$	N/A	$S_L \approx \log(L)$
D=2 	$S_L \approx L$	$S_L \approx L$	$S_L \approx L \log(L)$
D=3 	$S_L \approx L^2$	$S_L \approx L^2$	$S_L \approx L^2 \log(L)$

$S_L \approx L^{D-1}$
boundary law

$S_L \approx L^{D-1}\log(L)$
boundary law with logarithmic correction

11-Summary/outlook

Tensor networks (second week)

Dimension	gapped $\Delta > 0$	gapless $\Delta = 0$	
		no (D-1)- dimensional Fermi surface	(D-1)- dimensional Fermi surface
D=1 	$S_L \approx const$ MPS	N/A	$S_L \approx \log(L)$
D=2 	$S_L \approx L$	$S_L \approx L$	$S_L \approx L\log(L)$
D=3 	$S_L \approx L^2$	$S_L \approx L^2$	$S_L \approx L^2\log(L)$

$S_L \approx L^{D-1}$

boundary law

$S_L \approx L^{D-1}\log(L)$

boundary law with logarithmic correction

11-Summary/outlook

Tensor networks (second week)

Dimension	gapped $\Delta > 0$	gapless $\Delta = 0$ — **no** (D-1)- dimensional Fermi surface	gapless $\Delta = 0$ — (D-1)- dimensional Fermi surface
D=1 (L)	$S_L \approx const$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2 (L, L)	$S_L \approx L$	$S_L \approx L$	$S_L \approx L\log(L)$
D=3 (L, L, L)	$S_L \approx L^2$	$S_L \approx L^2$	$S_L \approx L^2\log(L)$

Gapped and gapless without a Fermi surface: $S_L \approx L^{D-1}$

boundary law

Gapless with a Fermi surface: $S_L \approx L^{D-1}\log(L)$

boundary law with logarithmic correction

11-Summary/outlook

Tensor networks (second week)

Dimension	gapped $\Delta > 0$	gapless $\Delta = 0$ — **no** (D-1)- dimensional Fermi surface	gapless $\Delta = 0$ — (D-1)- dimensional Fermi surface
D=1	$S_L \approx const$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2	$S_L \approx L$ PEPS	$S_L \approx L$ PEPS/MERA	$S_L \approx L \log(L)$
D=3	$S_L \approx L^2$ PEPS	$S_L \approx L^2$ PEPS/MERA	$S_L \approx L^2 \log(L)$

Gapped and gapless without Fermi surface: $S_L \approx L^{D-1}$ boundary law

With Fermi surface: $S_L \approx L^{D-1} \log(L)$ boundary law with logarithmic correction

11-Summary/outlook

Tensor networks (second week)

Dimension	gapped $\Delta > 0$	gapless $\Delta = 0$ — **no** (D-1)- dimensional Fermi surface	gapless $\Delta = 0$ — (D-1)- dimensional Fermi surface
D=1 (L)	$S_L \approx const$ MPS	N/A	$S_L \approx \log(L)$ MERA
D=2 (L, L)	$S_L \approx L$ PEPS	$S_L \approx L$ PEPS/MERA	$S_L \approx L\log(L)$ branching MERA
D=3 (L, L, L)	$S_L \approx L^2$ PEPS	$S_L \approx L^2$ PEPS/MERA	$S_L \approx L^2\log(L)$ branching MERA

$S_L \approx L^{D-1}$

boundary law

$S_L \approx L^{D-1}\log(L)$

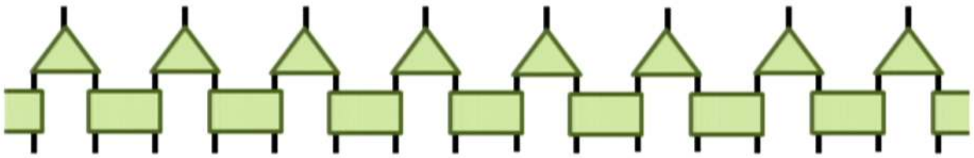
boundary law with logarithmic correction

11-Summary/outlook

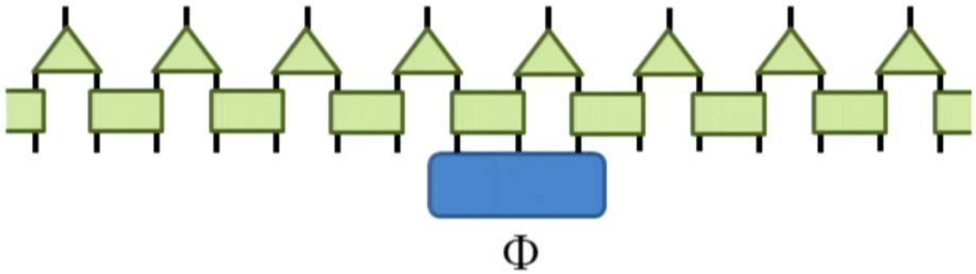
Current applications of tensor networks:

- Basis for non-perturbative numerical approaches to
  a. frustrated antiferromagnets (2D)
  b. interacting fermions (2D)
  c. topologically ordered phases (2D)
  d. quantum criticality/phase transitions
- Classification of symmetry protected gapped phases
  a. complete classification in 1D (MPS!)
  b. partial classification in 2D, 3D, etc
- Non-perturbative lattice realization of
  a. (real space) renormalization group
  b. conformal field theories
  c. quantum field theories
  d. the holographic principle

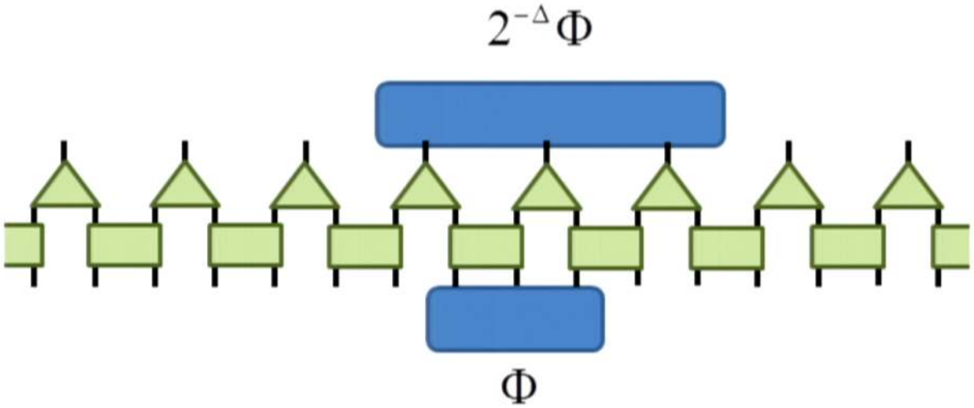
Scaling operators/scaling dimensions



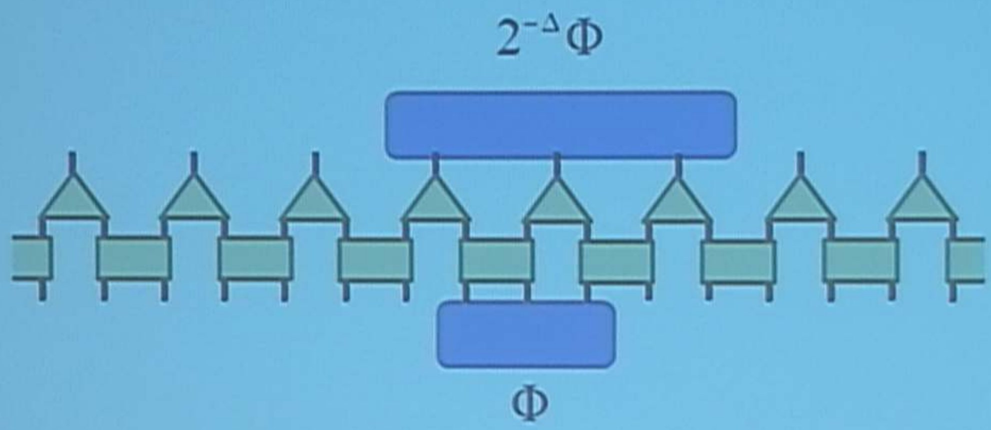
Scaling operators/scaling dimensions



Scaling operators/scaling dimensions

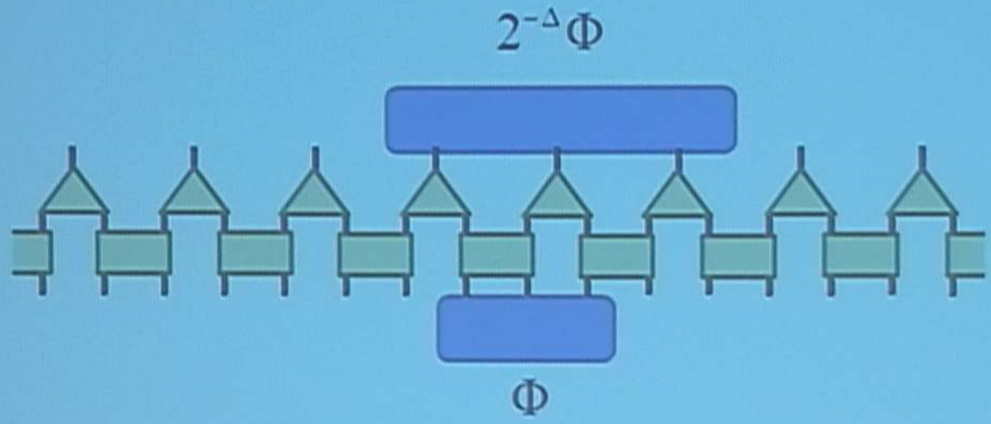


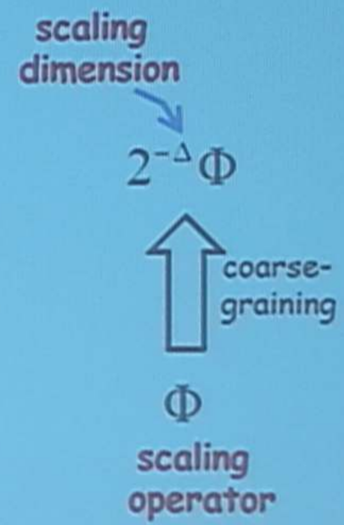
Scaling operators/scaling dimensions



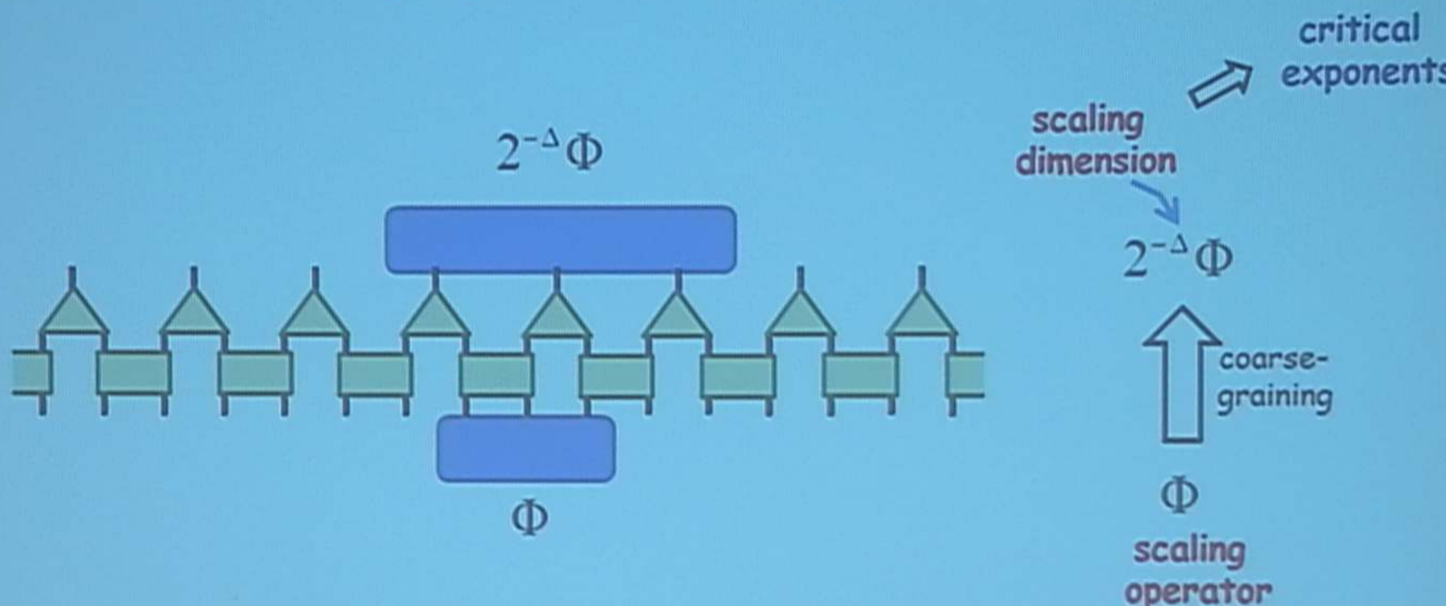
Φ
scaling operator

Scaling operators/scaling dimensions





Scaling operators/scaling dimensions

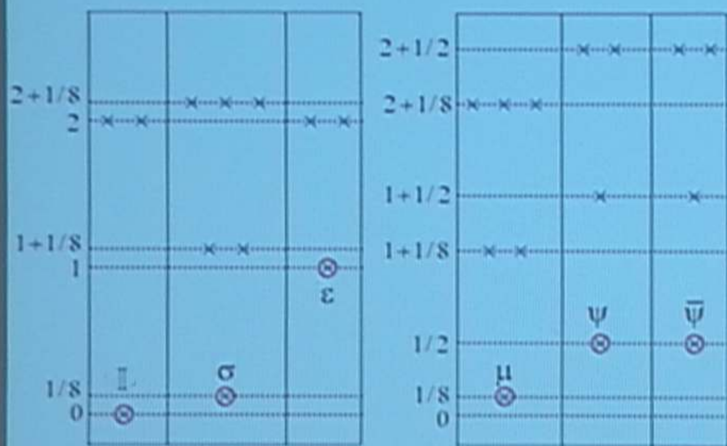


Example: quantum Ising model

scaling operator Φ	scaling dimension Δ
identity $\mathbb{I}$	0
spin σ	$\frac{1}{8}$
energy ε	1

Example: operator content of quantum Ising model

scaling operators/dimensions:



		scaling dimension (exact)	scaling dimension (MERA)	error
identity	$\mathbb{I}$	0	0	----
spin	σ	0.125	0.124997	0.003%
energy	ε	1	0.99993	0.007%
disorder	μ	0.125	0.1250002	0.0002%
fermions	ψ	0.5	0.5	$<10^{-8}$ %
	$\overline{\psi}$	0.5	0.5	$<10^{-8}$ %

OPE for local & non-local primary fields

$$C_{\varepsilon\sigma\sigma} = 1/2 \qquad C_{\varepsilon\psi\bar{\psi}} = i$$

$$C_{\varepsilon\mu\mu} = -1/2 \qquad C_{\varepsilon\bar{\psi}\psi} = -i$$

$$C_{\psi\mu\sigma} = e^{-i\pi/4}/\sqrt{2}$$

$$(\pm 6\times 10^{-4})$$

$$C_{\bar{\psi}\mu\sigma} = e^{i\pi/4}/\sqrt{2}$$

OPE for local & non-local primary fields

$C_{\varepsilon\sigma\sigma} = 1/2$

$C_{\varepsilon\mu\mu} = -1/2$

$C_{\psi\mu\sigma} = e^{-i\pi/4}/\sqrt{2}$

$C_{\bar\psi\mu\sigma} = e^{i\pi/4}/\sqrt{2}$

$C_{\varepsilon\psi\bar\psi} = i$

$C_{\varepsilon\bar\psi\psi} = -i$

$(\pm 6\times 10^{-4})$

$\Rightarrow$

fusion rules

$$\varepsilon \times \varepsilon = I$$
$$\sigma \times \sigma = I + \varepsilon$$
$$\sigma \times \varepsilon = \sigma$$
$$\mu \times \mu = I + \varepsilon$$
$$\mu \times \varepsilon = \mu$$
$$\psi \times \psi = I$$
$$\bar\psi \times \bar\psi = I$$
$$\psi \times \bar\psi = \varepsilon$$
$$\psi \times \varepsilon = \bar\psi$$
$$\bar\psi \times \varepsilon = \psi$$
$$\cdots$$

OPE for local & non-local primary fields

$C_{\varepsilon\sigma\sigma} = 1/2$ $\qquad C_{\varepsilon\psi\bar{\psi}} = i$

$C_{\varepsilon\mu\mu} = -1/2$ $\qquad C_{\varepsilon\bar{\psi}\psi} = -i$

$C_{\psi\mu\sigma} = e^{-i\pi/4}/\sqrt{2}$

$(\pm 6\times 10^{-4})$

$C_{\bar{\psi}\mu\sigma} = e^{i\pi/4}/\sqrt{2}$

$\Rightarrow$

fusion rules

$$\varepsilon \times \varepsilon = \mathrm{I}$$
$$\sigma \times \sigma = \mathrm{I} + \varepsilon$$
$$\sigma \times \varepsilon = \sigma$$
$$\mu \times \mu = \mathrm{I} + \varepsilon$$
$$\mu \times \varepsilon = \mu$$
$$\psi \times \psi = \mathrm{I}$$
$$\bar{\psi} \times \bar{\psi} = \mathrm{I}$$
$$\psi \times \bar{\psi} = \varepsilon$$
$$\psi \times \varepsilon = \bar{\psi}$$
$$\bar{\psi} \times \varepsilon = \psi$$
$$\cdots$$

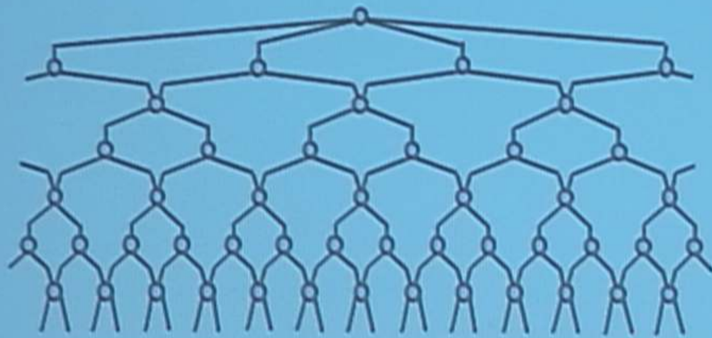
$\{\mathrm{I}, \varepsilon, \sigma, \mu, \psi, \bar{\psi}\}$

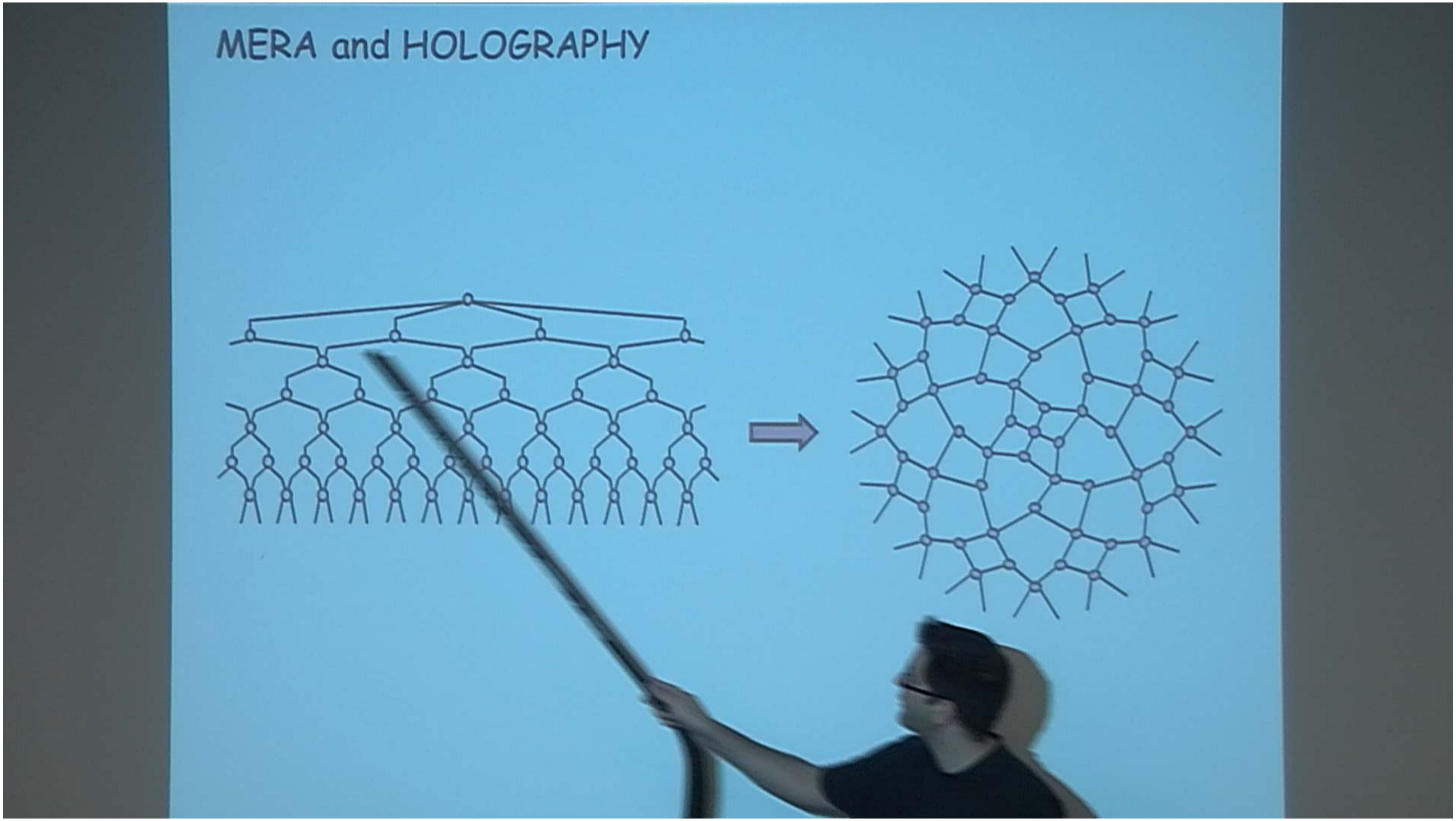
local and semi-local subalgebras

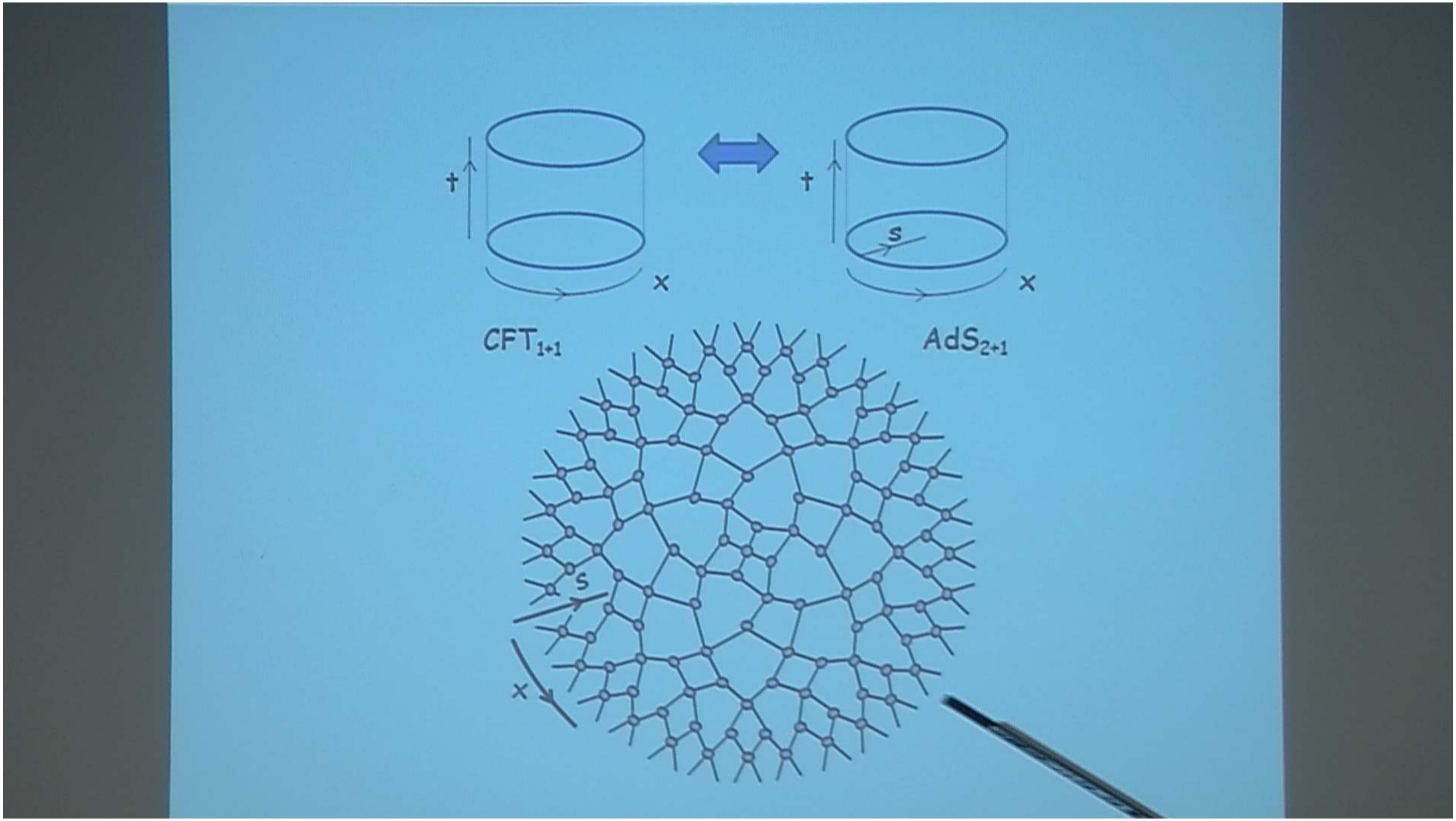
$\{\mathrm{I}, \varepsilon\}$ $\qquad \{\mathrm{I}, \varepsilon, \sigma\}$

$\{\mathrm{I}, \varepsilon, \mu\}$ $\qquad \{\mathrm{I}, \varepsilon, \psi, \bar{\psi}\}$

MERA and HOLOGRAPHY




MERA and HOLOGRAPHY


t
x
CFT1+1
t
s
x
AdS2+1
s
x