Title: 2D to 4D Correspondence: Towers of Kinks versus Towers of Monopoles

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Abstract: <span>Two-dimensional
models provide for a very attractive playground being a theory imitating some
of the main features of QCD. Those include the asymptotic freedom, mass gap,
confinement, chiral symmetry breaking and others. Furthermore, there is a correspondence between the spectra of
four-dimensional SQCD and N=(2,2) CP(N-1) sigma model which was discovered more
than a decade ago. This correspondence was explained later when it was found
that SQCD supports non-Abelian strings with confined monopoles. The kinks of
the two-dimensional theory are the monopoles attached to the strings. Thus,
analysis of two-dimensional sigma models gives a deeper insight into the four-dimensional SQCD, in particular,
into its strong dynamics.<br>
<br>
We study the BPS spectrum of the N=(2,2) CP(N-1) model with the Z_N-symmetric
twisted mass terms. We focus on analysis of the "extra" towers of states
found previously and compare them to the states that can be identified in the
quasiclassical domain. Exact analysis of the strong-coupling states shows that
not all of them survive when passing to the weak-coupling domain. Some of the states decay on
the curves of the marginal stability (CMS). Thus, most of the strong coupling
states do not exist at weak coupling and cannot be classified quasiclassically.
This result lifts to four dimensions. In terms of the four-dimensional theory,
the "extra" states are the strong coupling dyons, while the
quasiclassical bound states are the bound states of dyons and quarks.</span>

# Two-dimensional — four-dimensional duality: Towers of kinks $\leftrightarrow$ towers of monopoles in $\mathcal{N} = 2$ theories

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April 5, 2013

Two-dimensional models have interesting similarities to 4-d gauge theories

$\mathrm{CP}^{N-1}$ theory has been shown to have chiral symmetry breaking, mass gap, asymptotic freedom, *etc* and all these properties are much easier to show in two dimensions

In general, two-dimensional theories are simpler and two-dimensional methods are more powerful

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There is also a two-dimensional — four-dimensional duality of their BPS spectra

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| $\mathcal{N}=2$ $N_c = N_f$ SYM in four dimensions at the root of the first baryonic Higgs branch | $\longleftrightarrow$ | $\mathcal{N}=(2,2)$ $\mathrm{CP}^{N-1}$ theory in two dimensions |
| $\downarrow$ | | $\downarrow$ |
| supports non-Abelian vortex strings | | kinks interpolate between worldsheet vacua |
| monopoles | $\longleftrightarrow$ | kinks |
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At strong coupling — we consider $\mathrm{CP}^{N-1}$ theory
Strong coupling spectrum is accessible via the mirror theory

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At strong coupling — we consider $CP^{N-1}$ theory
Strong coupling spectrum is accessible via the mirror theory

## $\mathrm{CP}^{N-1}$ theory with *twisted* masses:

$$r\left(\left|\mathcal{D}_\mu n^l\right|^2 + |\sigma - m^l|^2\left|n^l\right|^2 + i\,D\left(\left|n^l\right|^2 - 1\right) + \ldots\right)$$
$$+ \frac{1}{4e^2}F_{\mu\nu}^2 + \frac{1}{e^2}\left|\partial_\mu\sigma\right|^2 + \frac{1}{2e^2}D^2 + \ldots,$$

in the $e^2 \to \infty$ limit

The theory has $N$ vacua — both classically and exactly

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$\mathrm{CP}^{N-1}$ model with $\mathcal{Z}_N$ twisted masses

$$m_l = m_0 \cdot e^{2\pi i l/N}$$

in this case $\mathcal{Z}_N \subset U_R(1)$ remains unbroken

# Exact superpotential

The theory possesses an "exact" superpotential of Veneziano-Yankielowicz type

$$\mathcal{W}_{\rm eff} \;=\; -\,i\,\tau\hat{\sigma} \;+\; \frac{1}{2\pi}\sum_{l}\left(\hat{\sigma}-m_l\right)\left(\ln\frac{\hat{\sigma}-m_l}{\mu}-1\right)$$

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Vacuum values:

$$\mathcal{W}_{\text{eff}} \;=\; -\,\frac{1}{2\pi}\left(N\,\sigma_p \,+\, \sum_l m_l\,\ln\,(\sigma_p \,-\, m_l)\right)$$

where

$$\sigma_p \;=\; \sigma_0 \cdot e^{2\pi i p/N}$$

$$\sigma_0 \;=\; \sqrt[N]{1 \,+\, m_0^N}$$

vacuum equation

$$\prod_l (\,\sigma \,-\, m_l\,) \;=\; 1\,.$$

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# Mirror dual of $\mathrm{CP}^{N-1}$

The mirror dual is the affine Toda theory

$$\mathcal{W}_{\text{mirror}}^{\mathrm{CP}^{N-1}} = -\frac{1}{2\pi}\left(x_1 + x_2 + \ldots + x_n + \sum m_l \ln x_l\right),$$

$$x_1\, x_2 \ldots x_n = 1$$

Only the superpotential is known in that theory

The vacuum values coincide with $\mathcal{W}_{\text{eff}}(\sigma_p)$ upon identification

$$x_l^{(p)} = \sigma_p - m_l$$

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Based on work of K.Hori and C.Vafa, $N$ mirror kinks can be found at strong coupling

$$|\, m_k \,| \;\; \leq \;\; 1$$

Limiting to the sector interpolating between $0^{\text{th}}$ and $1^{\text{st}}$ vacua

$$\mathcal{Z} \;=\; \mathcal{W}(\sigma_1) \,-\, \mathcal{W}(\sigma_0) \,+\, i\, m_k\,, \qquad\qquad k = 0, ..., N-1,$$

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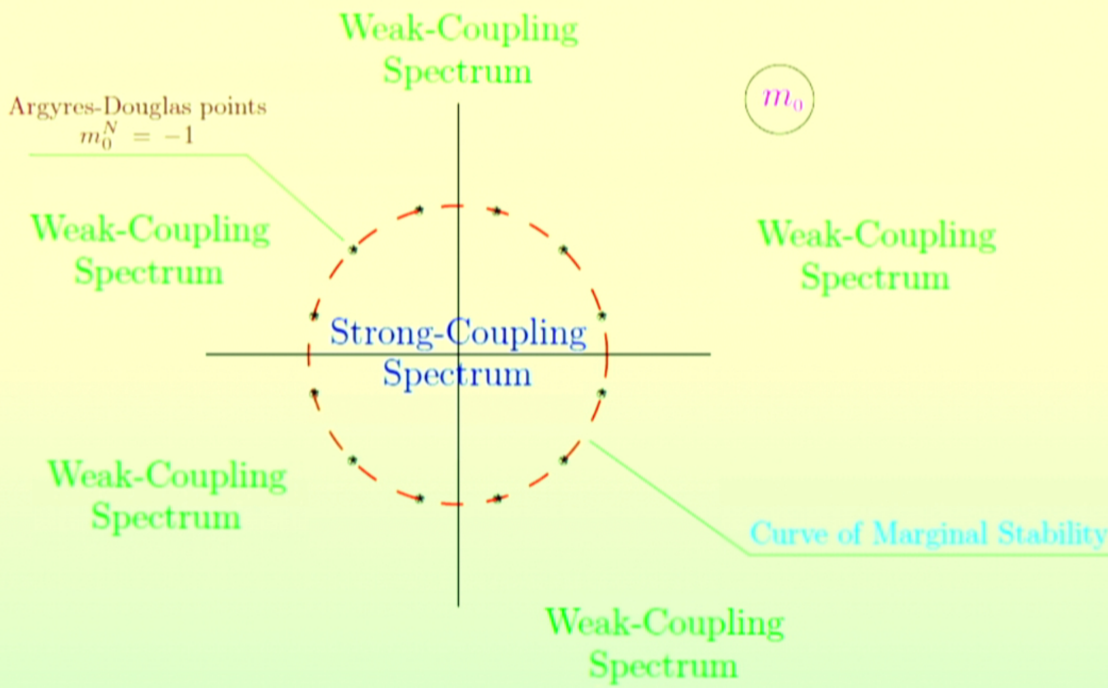
Weak-Coupling Spectrum
$m_0$
Argyres-Douglas points
$m_0^N = -1$
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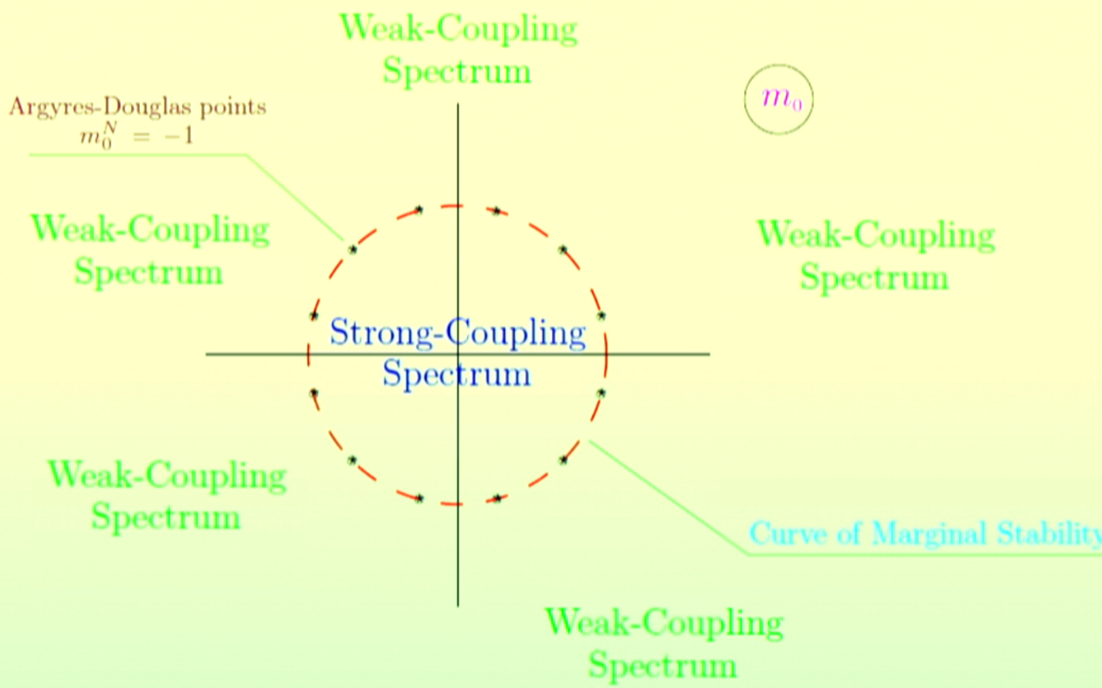
Curve of Marginal Stability
Weak-Coupling Spectrum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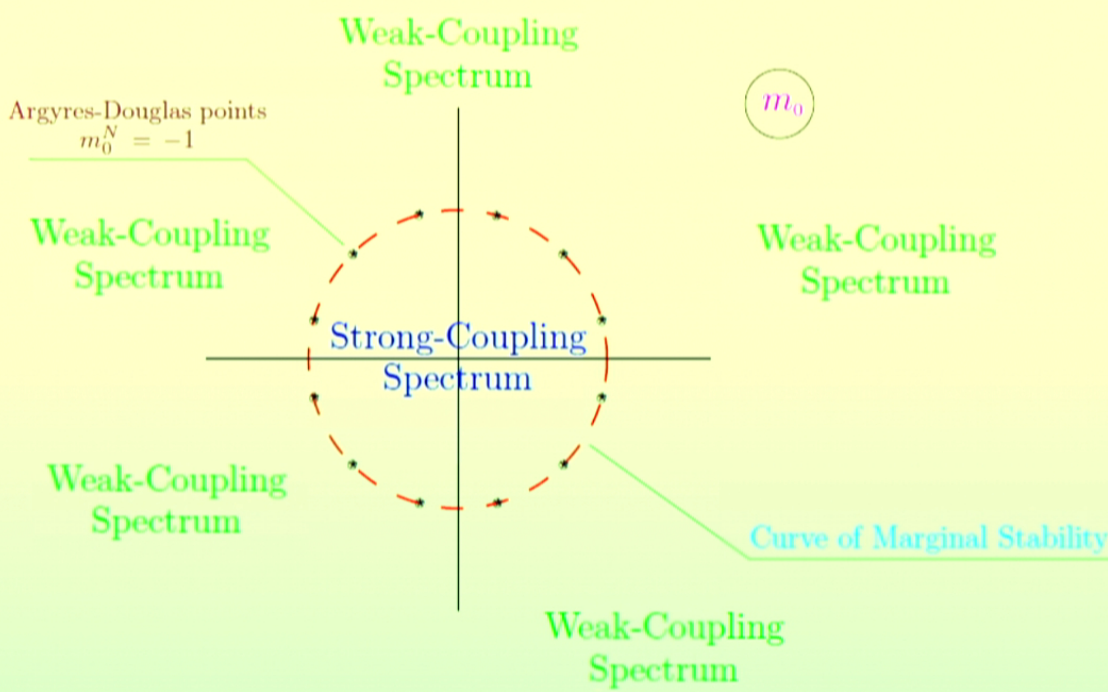
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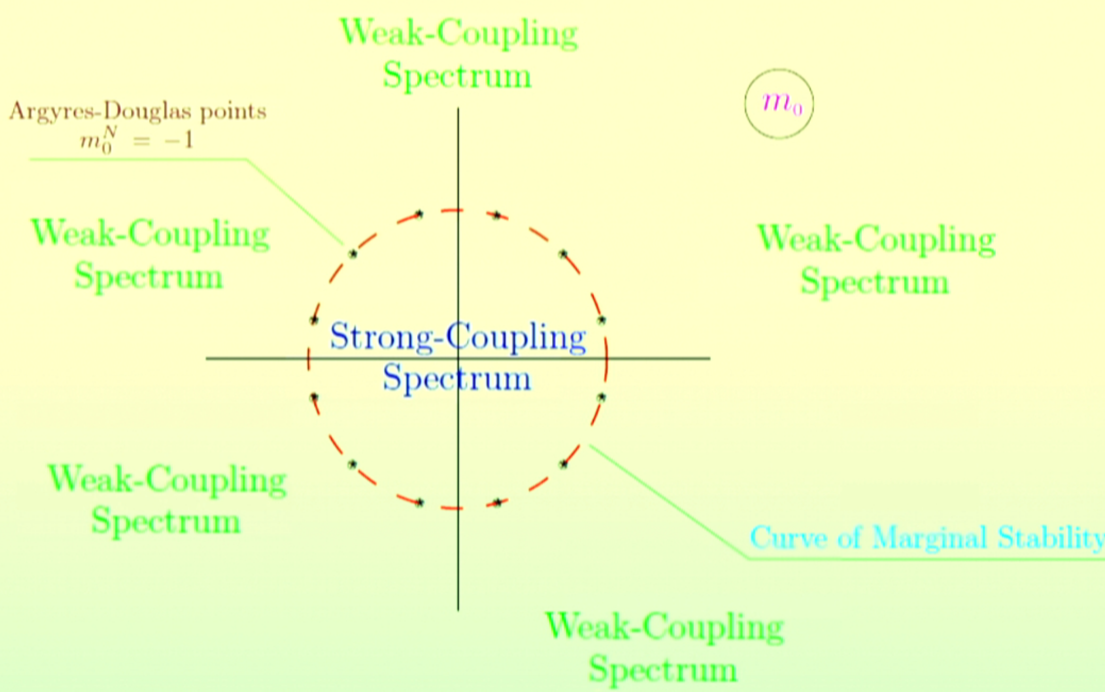
The spectrum

$$\mathcal{Z} \;=\; \mathcal{W}(\sigma_1) \;-\; \mathcal{W}(\sigma_0) \;+\; i\, m_k\,, \qquad\qquad k \,=\, 0\,, ..., N-1,$$

describes $N$ dyonic kinks (of either $\mathrm{CP}^{N-1}$ or the mirror theory) in the fundamental of $\mathrm{SU}(N)$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Previously known picture

Weak coupling:

$$\mathcal{Q}_{ik}, \quad \mathcal{M}_{ik}, \quad \mathcal{D}^{n}_{ik}$$

$\mathcal{D}^{n}_{ik} + Q$ — bound states

$$\mathcal{Q}_{ik} = i\,(m_i - m_k)$$

$\mathcal{M}_{ik}$ — purely topological kink interpolating from $(k) \to (i)$

$\mathcal{D}^{n}_{ik} = \mathcal{M}_{ik} + i\,n\,(m_i - m_k)$ — tower of dyonic kinks upon quasiclassical quantization

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# $CP^2$ theory

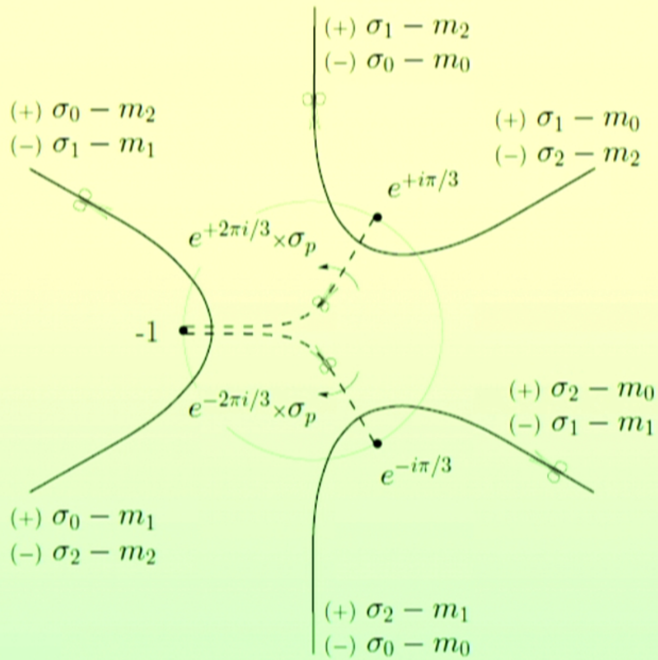
We focus on $CP^2$ model — the first non-trivial theory.

We find the curves of marginal stability (*c.m.s.*) — analogues of wall crossing — for this model, where the spectrum can change due to decays

*c.m.s.* are a supersymmetric version of a "phase transition"

there are practically no phase transitions in supersymmetric theories except for, perhaps, when supersymmetry is broken, or for theories with $N_c \to \infty$

# Moduli space of $\mathrm{CP}^2$ — plane of $m_0$




there are three $\mathcal{Z}_3$–equivalent sectors

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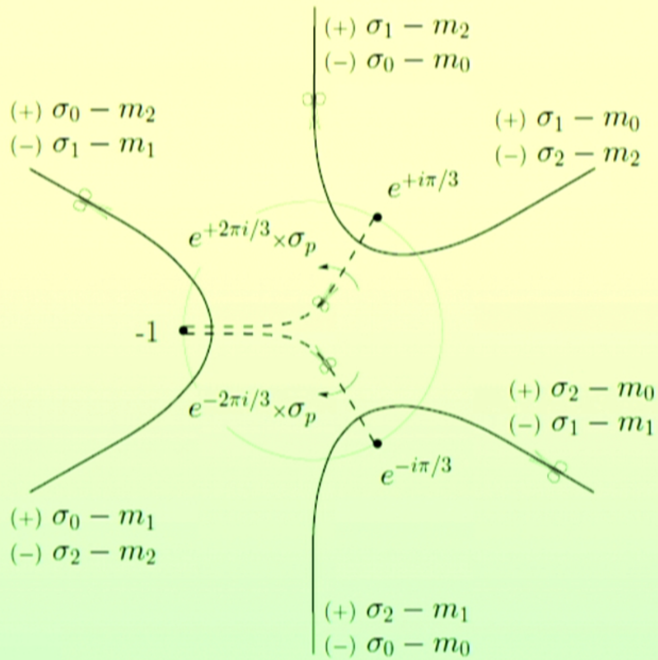
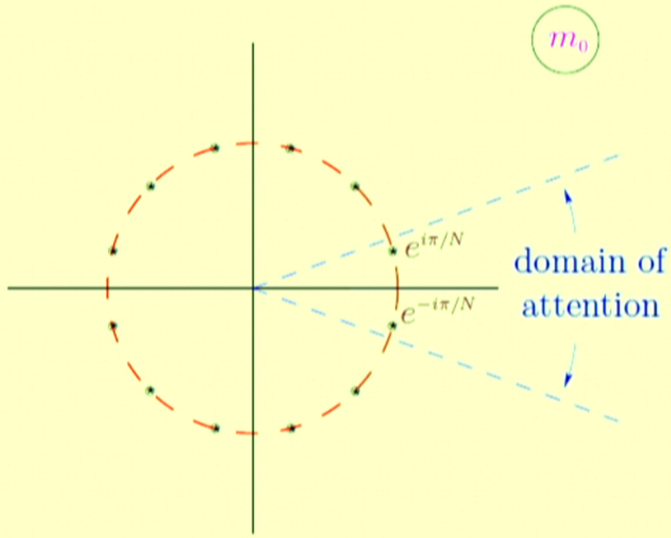



there are three $\mathcal{Z}_3$–equivalent sectors

So we choose only one topological sector:

kinks: $(0) \longrightarrow (1)$

and one sector in $m_0$-plane:




the other two sectors are completely equivalent

other kinks have the same masses, just central charges shifted by a phase

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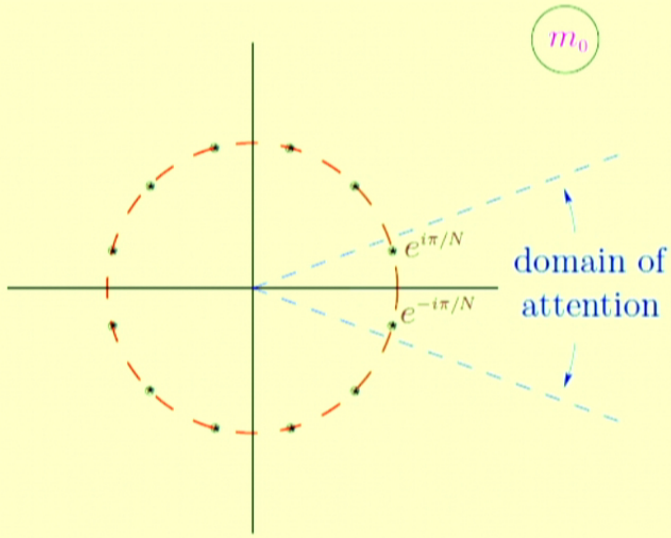


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decays $\Longleftarrow$ $\mathcal{Q}_{ik}$

$\mathcal{M}_{10}$

$\mathcal{D}_{10}^{(n)}$

$\mathcal{D}_{10}^{(n)}\,\mathcal{Q}_{02}$

# Weak coupling spectrum

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| decay all but one | $\Longleftarrow$ | $\mathcal{D}_{10}^{(n)}$ |
| all decay | $\Longleftarrow$ | $\mathcal{D}_{10}^{(n)}\,\mathcal{Q}_{02}$ |

# Strong coupling spectrum

$$\mathcal{W}_1 - \mathcal{W}_0 + i\,m_0$$

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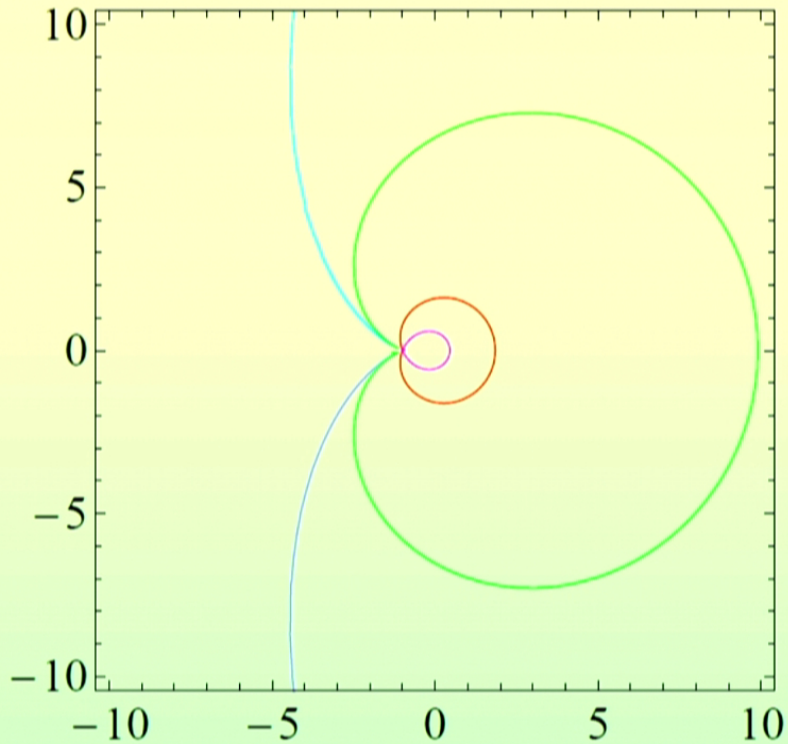
# Strong coupling spectrum

$\mathcal{M}_{10} \quad \Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\, m_0$ — next lightest

$\mathcal{D}_{10} \quad \Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\, m_1$ — the heavier

decays $\Longleftarrow \quad \mathcal{W}_1 - \mathcal{W}_0 + i\, m_2$ — the lightest

## c.m.s.



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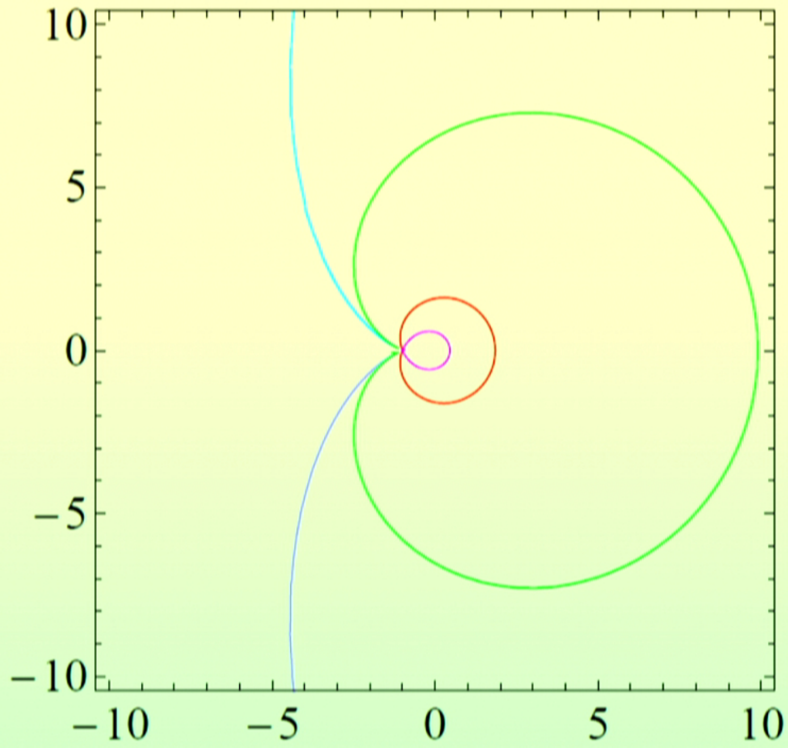

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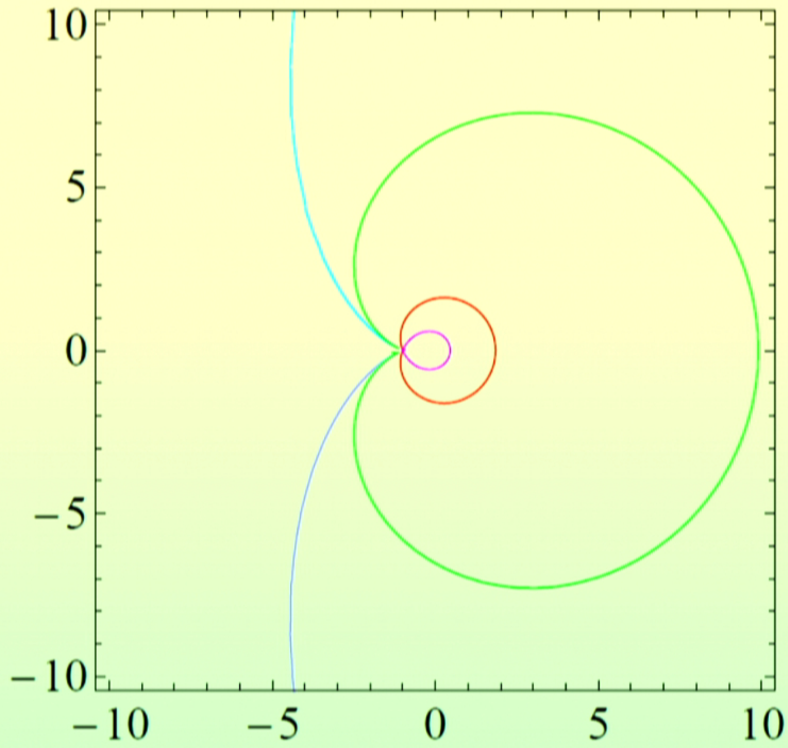

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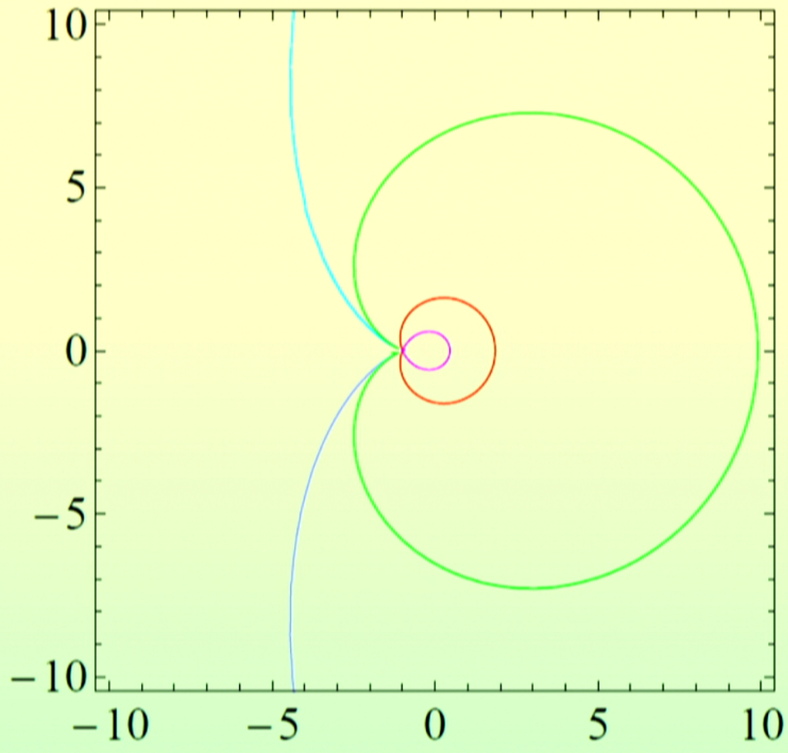

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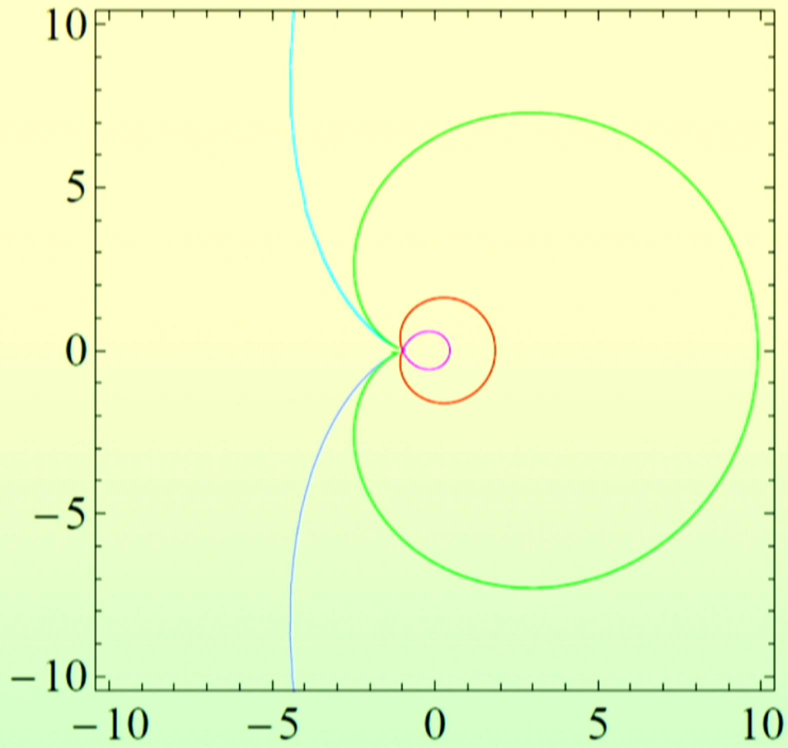

# Decays

Simple decays:

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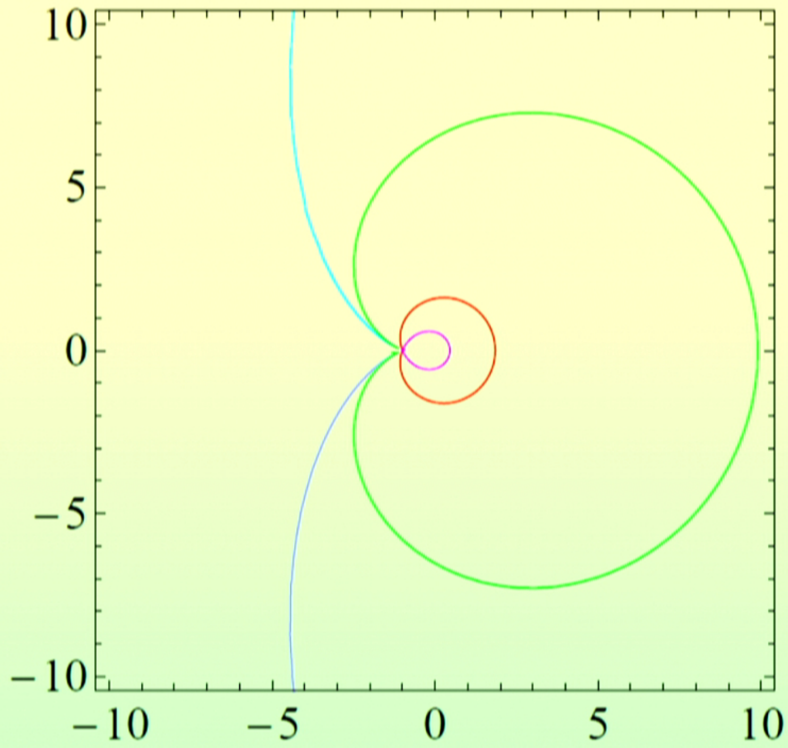

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Two states become part of the "monopole" tower

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All extra states decay

(Indeed for even $N$ there are *no bound states*)

We can infer that most likely they decay for *all $N$*

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