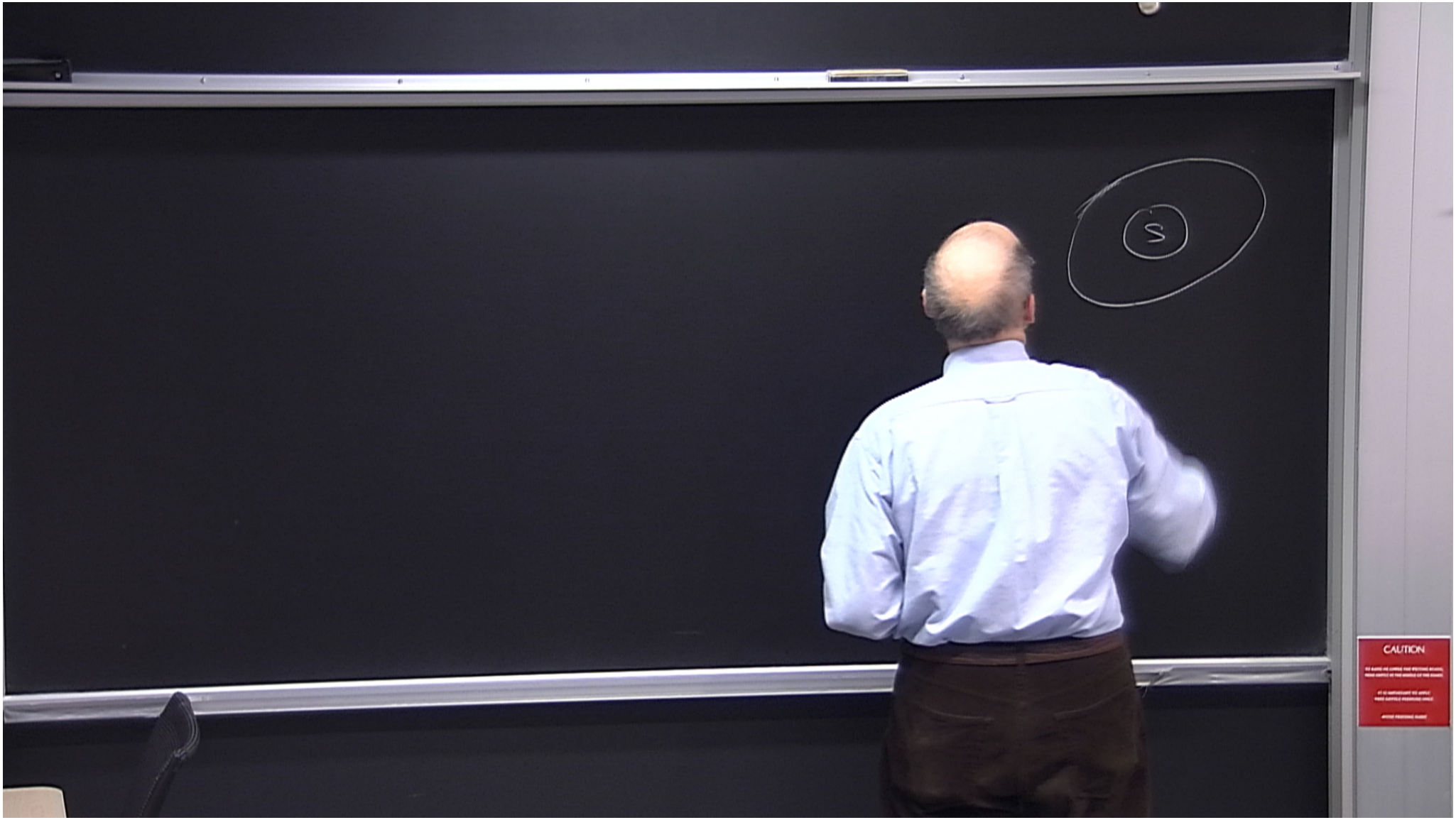
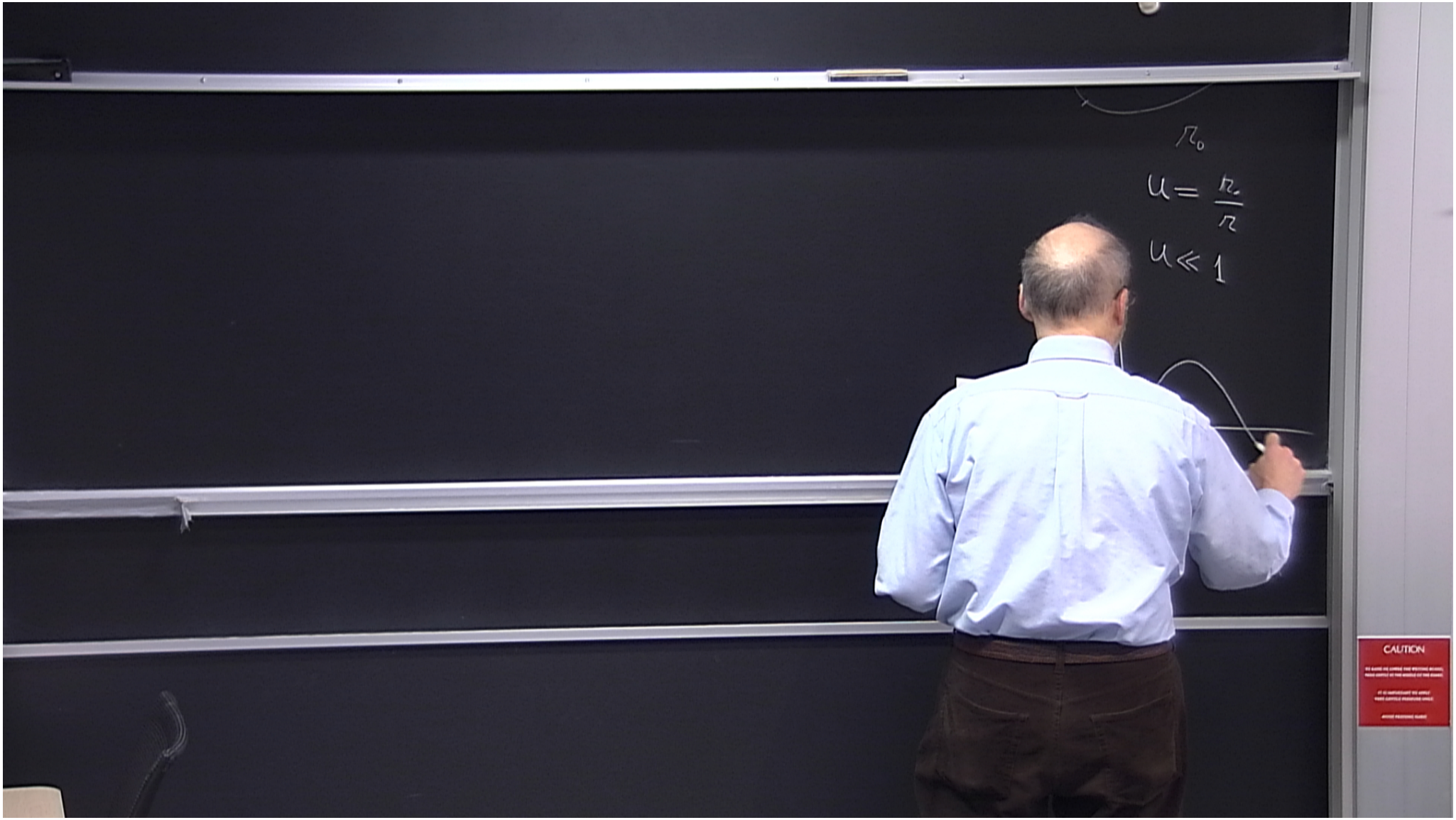
Title: Advanced General Relativity - Lecture 21

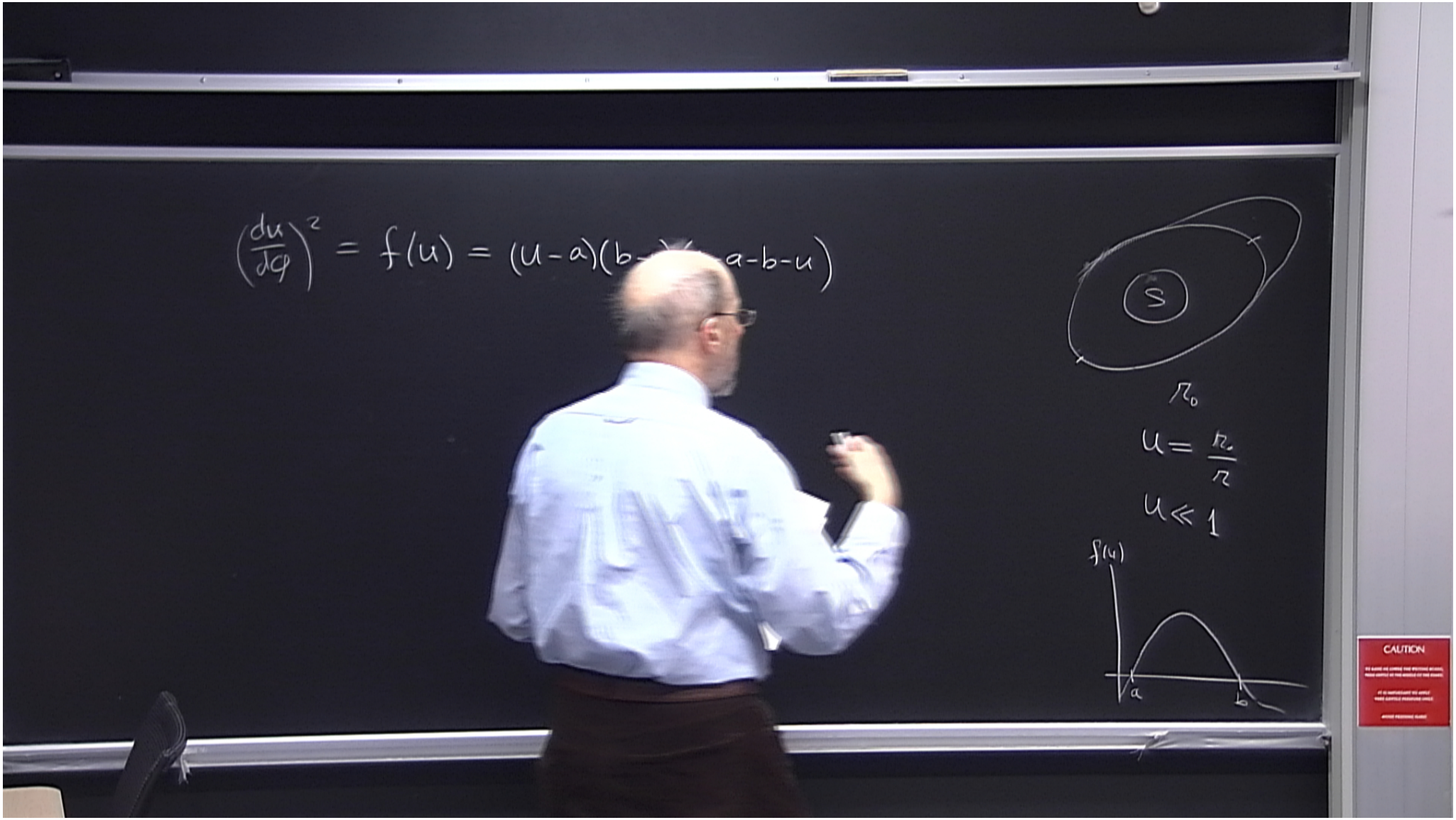
Date: Apr 02, 2013 05:00 PM

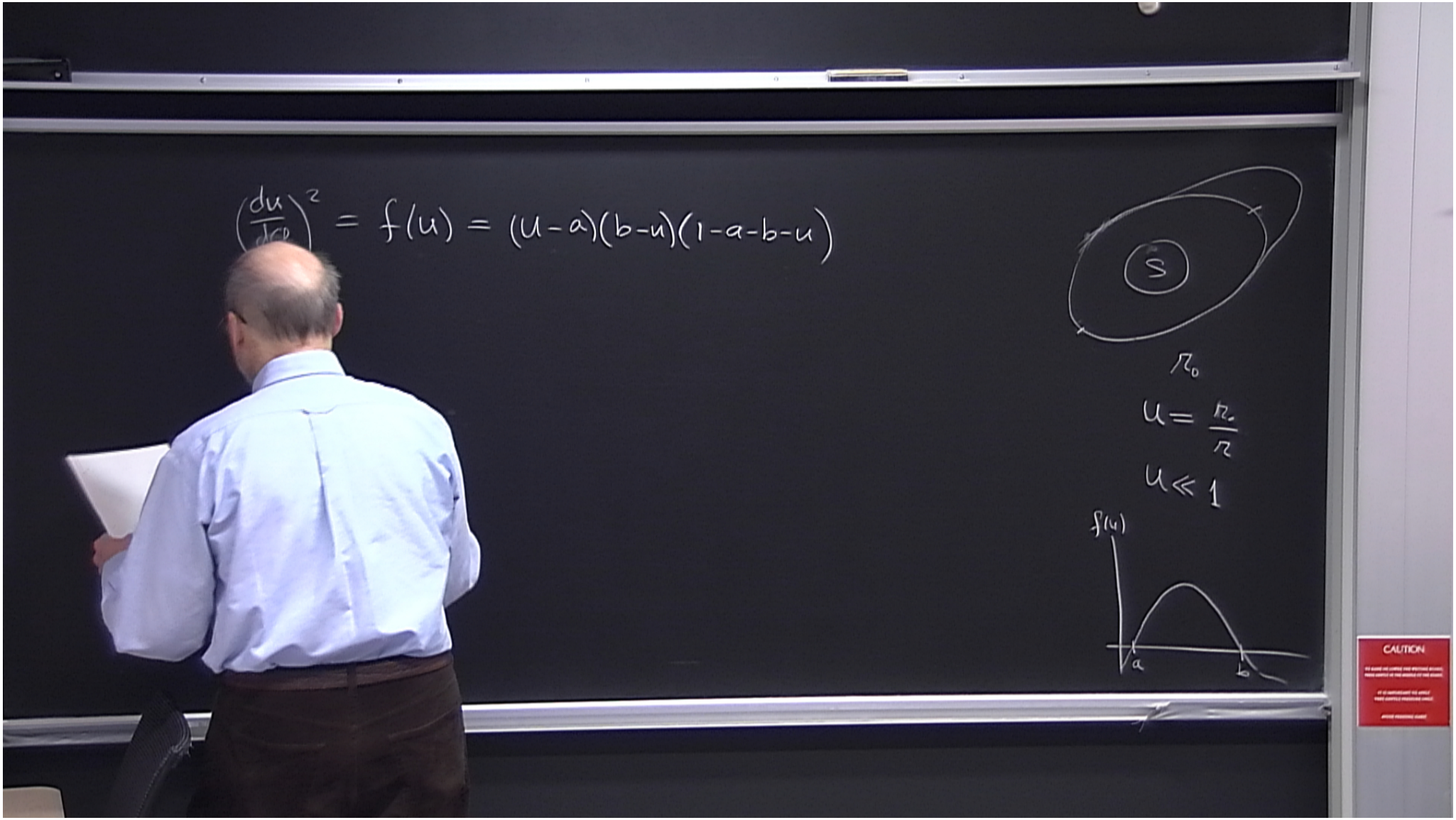
URL: http://pirsa.org/13040001

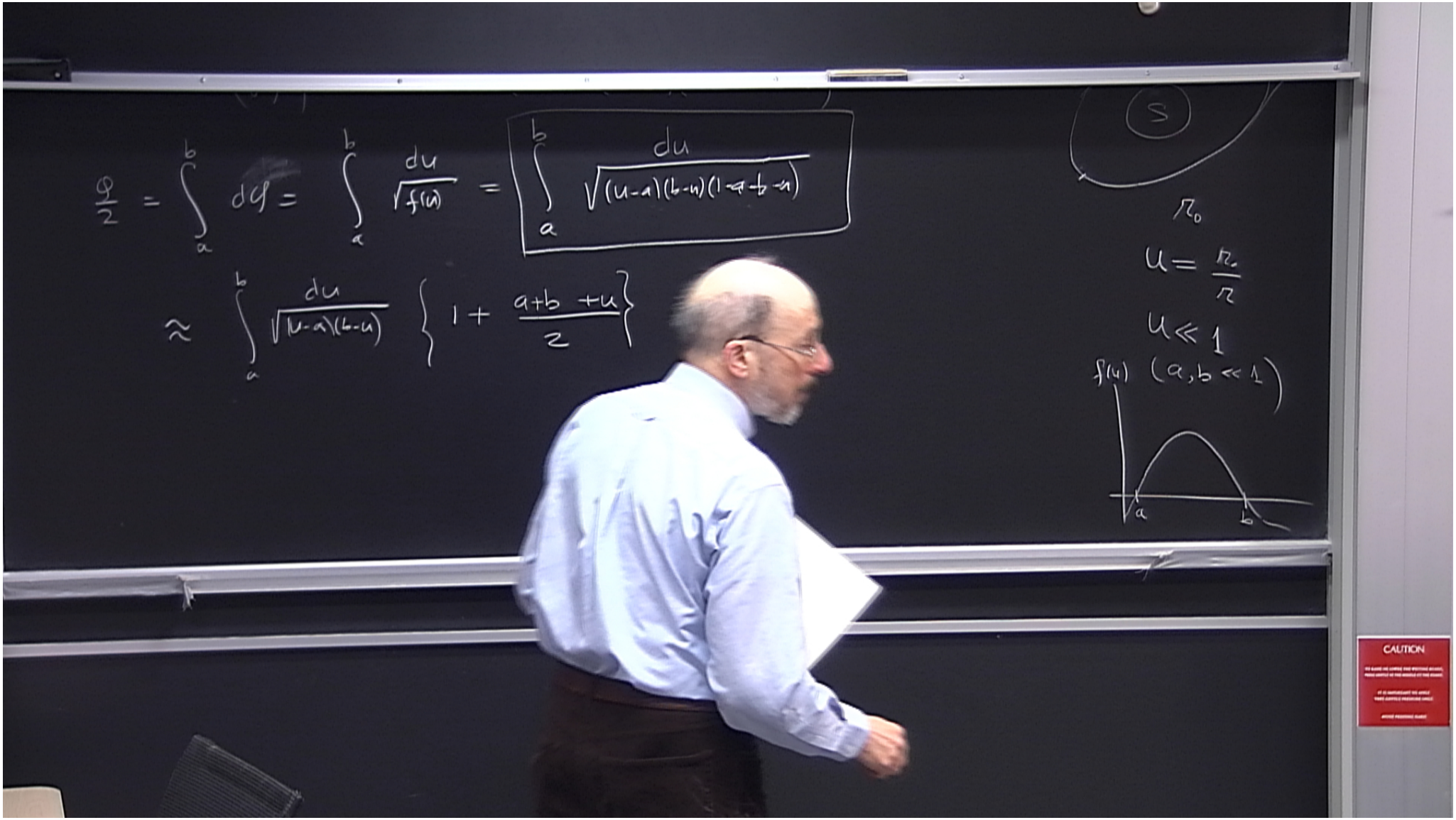
Abstract:

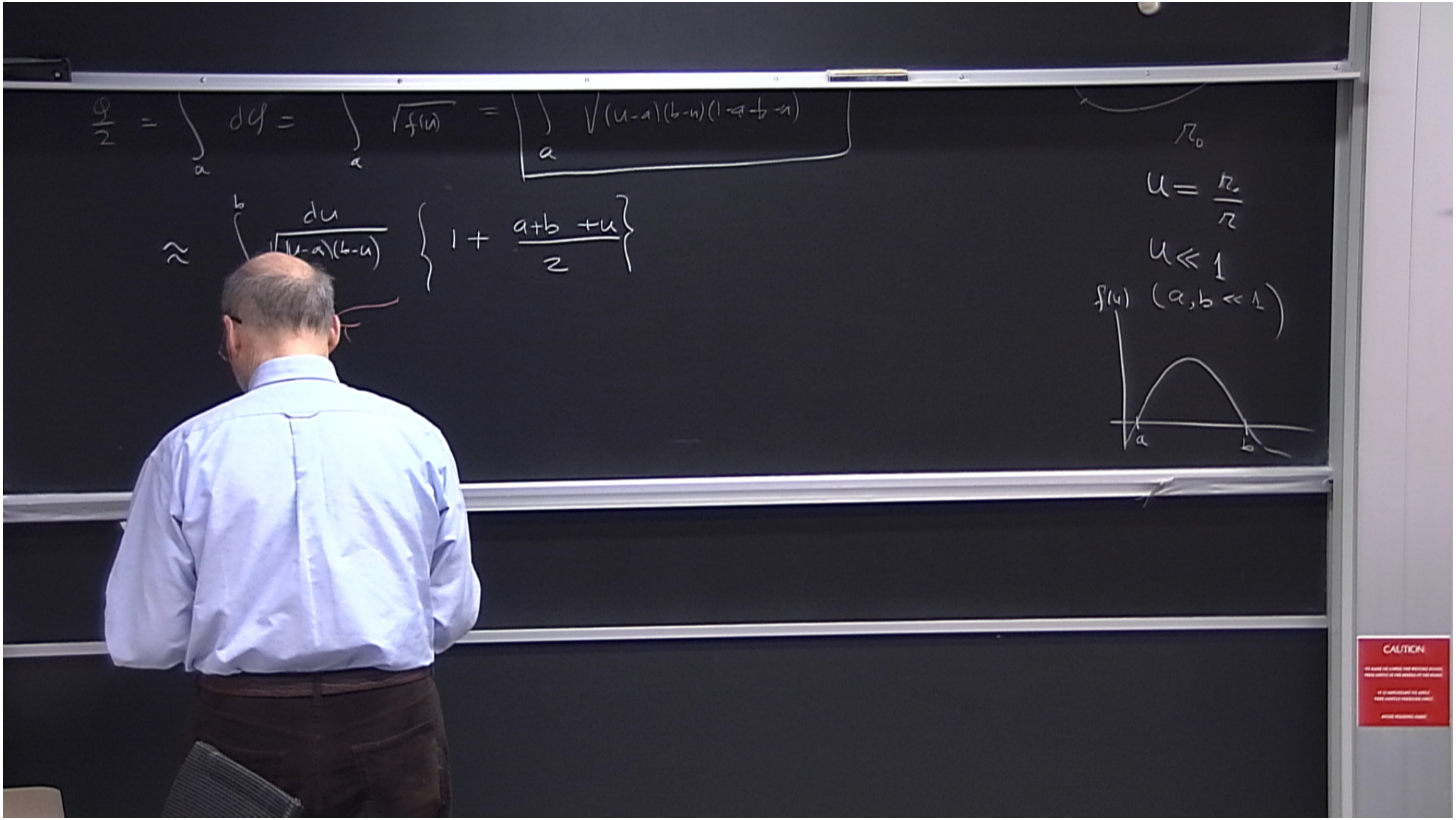

S
CAUTION


$r_0$
$u = \frac{r_0}{r}$
$u \ll 1$
CAUTION


$\left(\frac{du}{d\varphi}\right)^2 = f(u) = (u-a)(b-$ $a-b-u)$
S
$\pi_0$
$u = \frac{\pi_0}{\pi}$
$u \ll 1$
f(u)
a
b
CAUTION


$\left(\frac{du}{d\varphi}\right)^2 = f(u) = (u-a)(b-u)(1-a-b-u)$
S
$\pi_0$
$u = \frac{\pi_0}{\pi}$
$u \ll 1$
f(u)
a
b
CAUTION


$\frac{\varphi}{2} = \int_a^b d\varphi = \int_a^b \frac{du}{\sqrt{f(u)}} = \int_a^b \frac{du}{\sqrt{(u-a)(b-u)(1-a-b-u)}}$
$\approx \int_a^b \frac{du}{\sqrt{(u-a)(b-u)}} \left\{ 1 + \frac{a+b+u}{2} \right\}$
S
$\pi_0$
$u = \frac{\pi_0}{\pi}$
$u \ll 1$
$f(u)$ $(a, b \ll 1)$
a
b
CAUTION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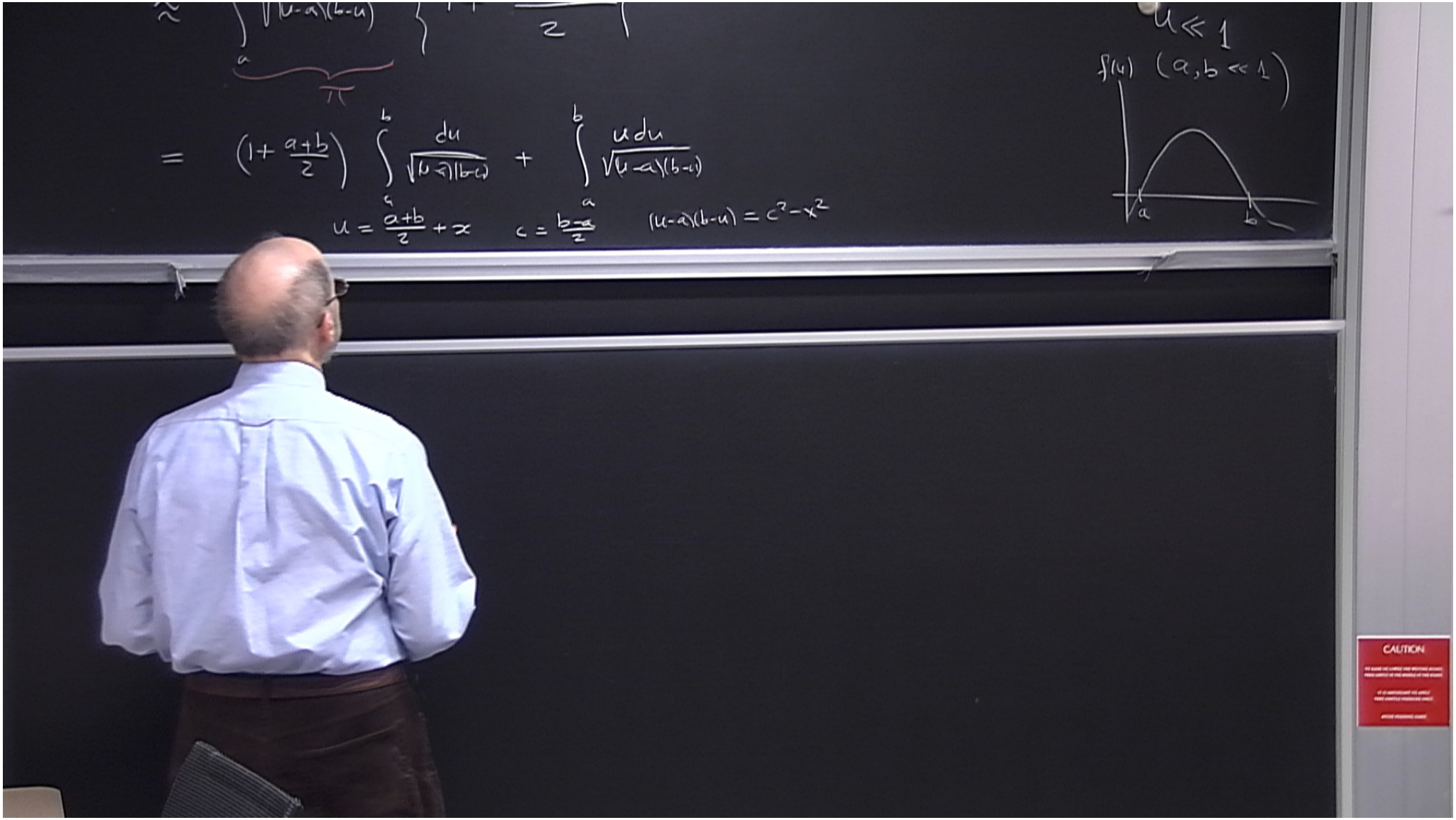
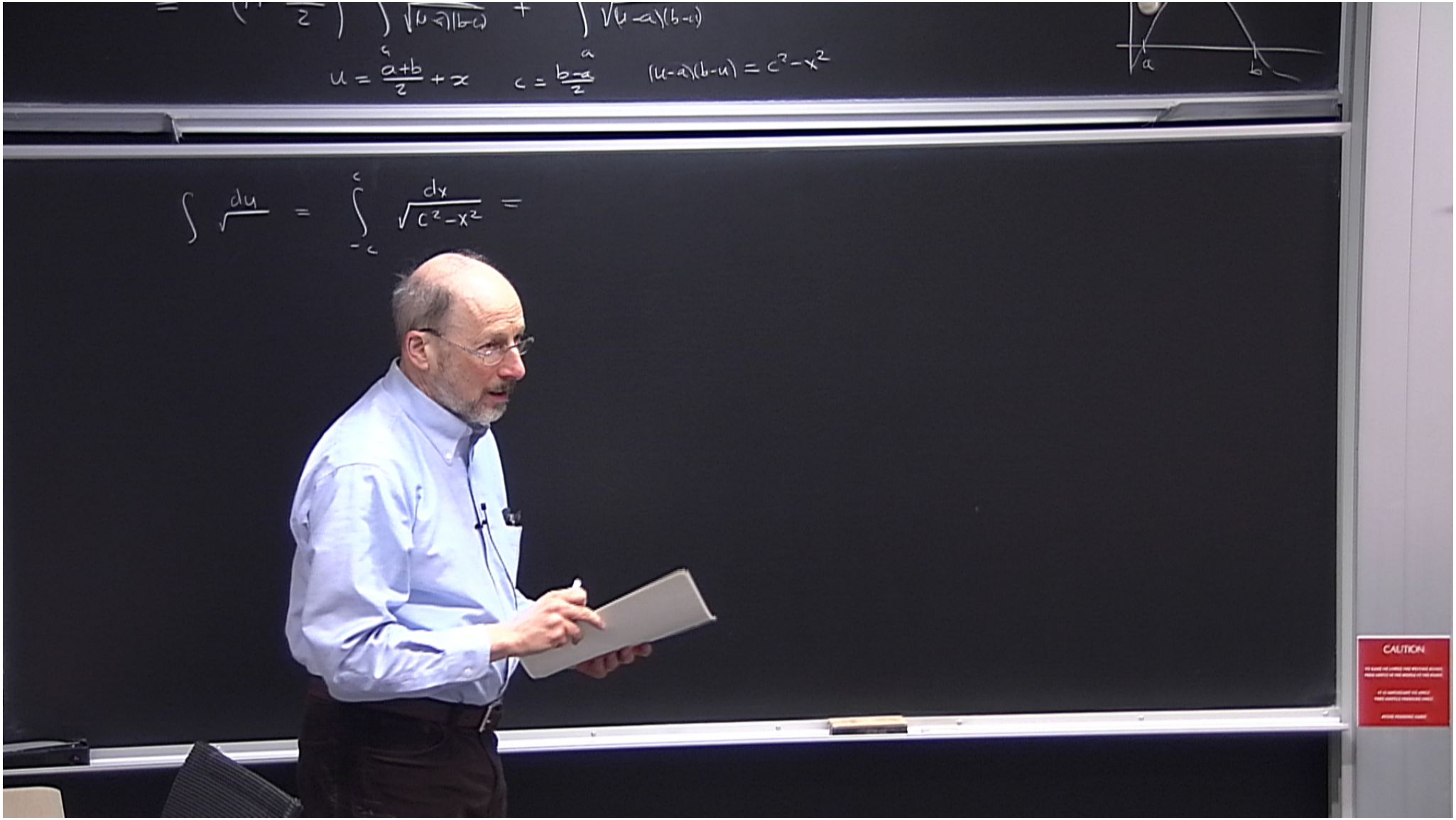
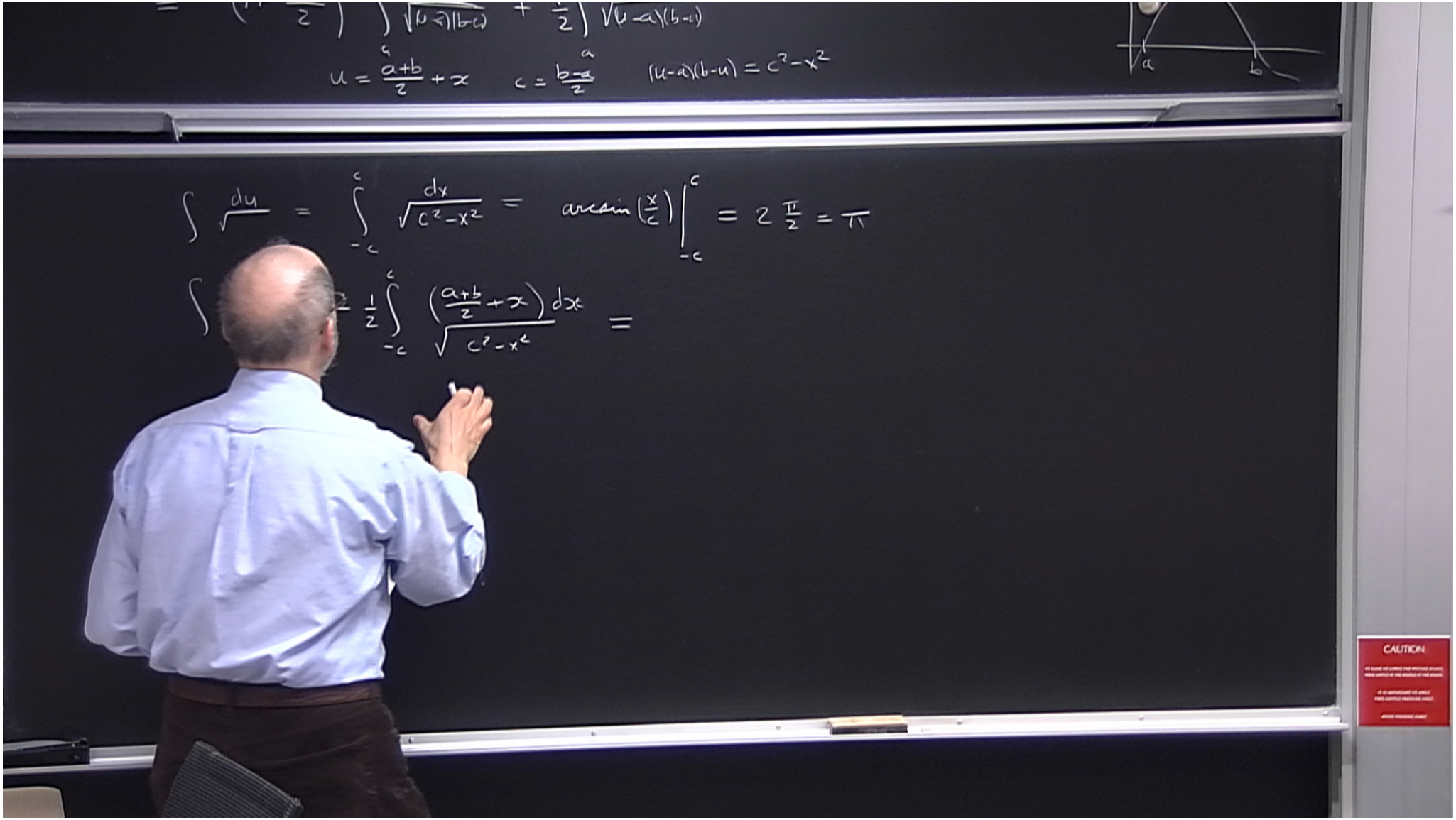
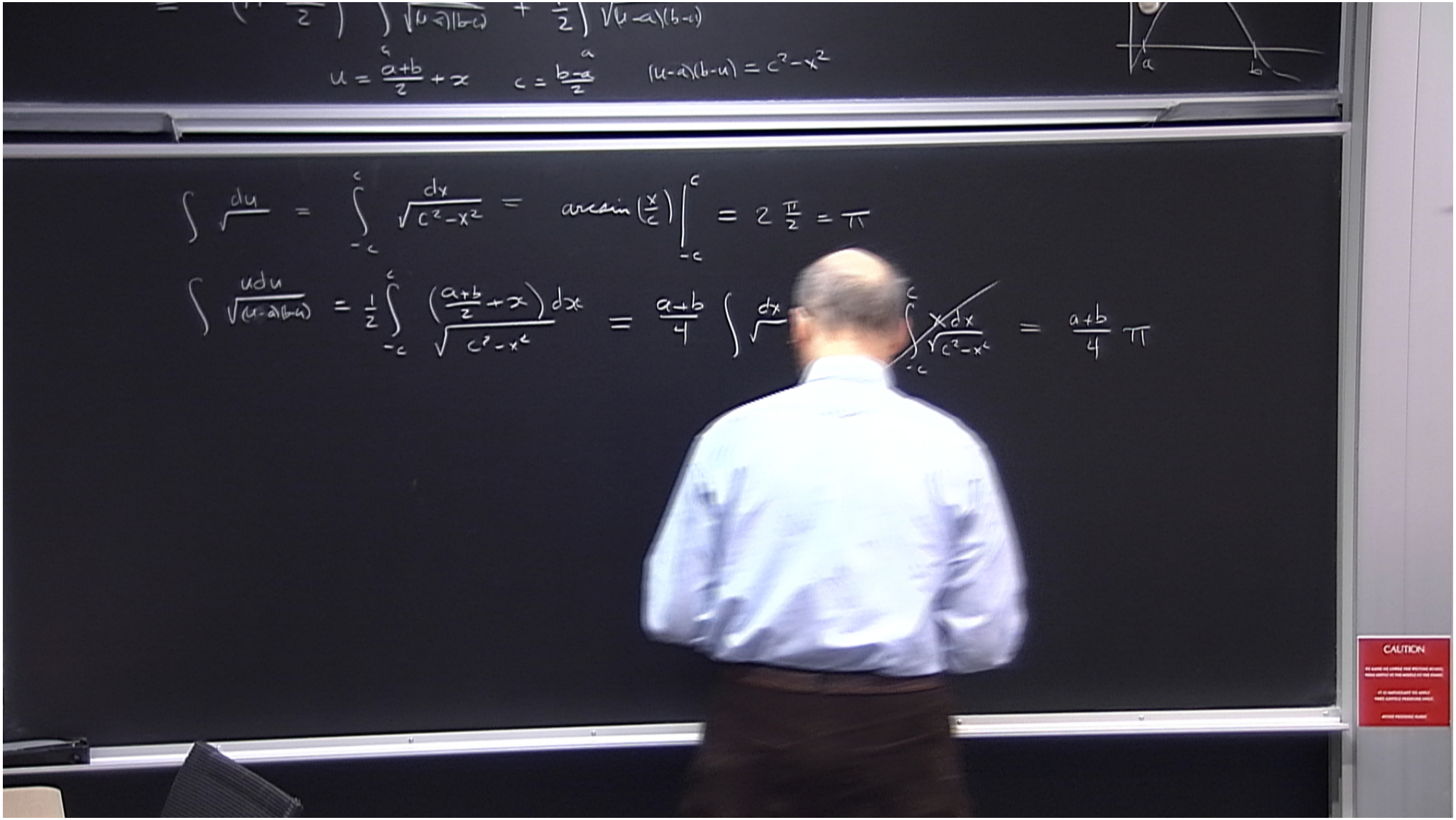
$\approx \int_a^b \frac{du}{\sqrt{(u-a)(b-u)}} \left\{ 1 + \frac{a+b+u}{2} \right\}$
$\pi_0$
$u = \frac{\pi_0}{\pi}$
$u \ll 1$
$f(u)$ $(a, b \ll 1)$
a
b
CAUTION


$u \ll 1$
$f(u)$ $(a,b \ll 1)$
$= \left(1+\frac{a+b}{2}\right) \int_a^b \frac{du}{\sqrt{(u-a)(b-u)}} + \int_a^b \frac{u\,du}{\sqrt{(u-a)(b-u)}}$
$u = \frac{a+b}{2} + x \quad c = \frac{b-a}{2} \quad (u-a)(b-u) = c^2 - x^2$
CAUTION


$u = \frac{a+b}{2} + x$
$c = \frac{b-a}{2}$
$(u-a)(b-u) = c^2 - x^2$
$\int \frac{du}{\sqrt{\quad}} = \int_{-c}^{c} \frac{dx}{\sqrt{c^2-x^2}} =$
a
b
CAUTION


$u = \frac{a+b}{2} + x \qquad c = \frac{b-a}{2} \qquad (u-a)(b-u) = c^2 - x^2$
$\int \frac{du}{\sqrt{}} = \int_{-c}^{c} \frac{dx}{\sqrt{c^2-x^2}} = \arcsin\left(\frac{x}{c}\right)\Big|_{-c}^{c} = 2\frac{\pi}{2} = \pi$
$\frac{1}{2}\int_{-c}^{c} \frac{\left(\frac{a+b}{2}+x\right)dx}{\sqrt{c^2-x^2}} =$
CAUTION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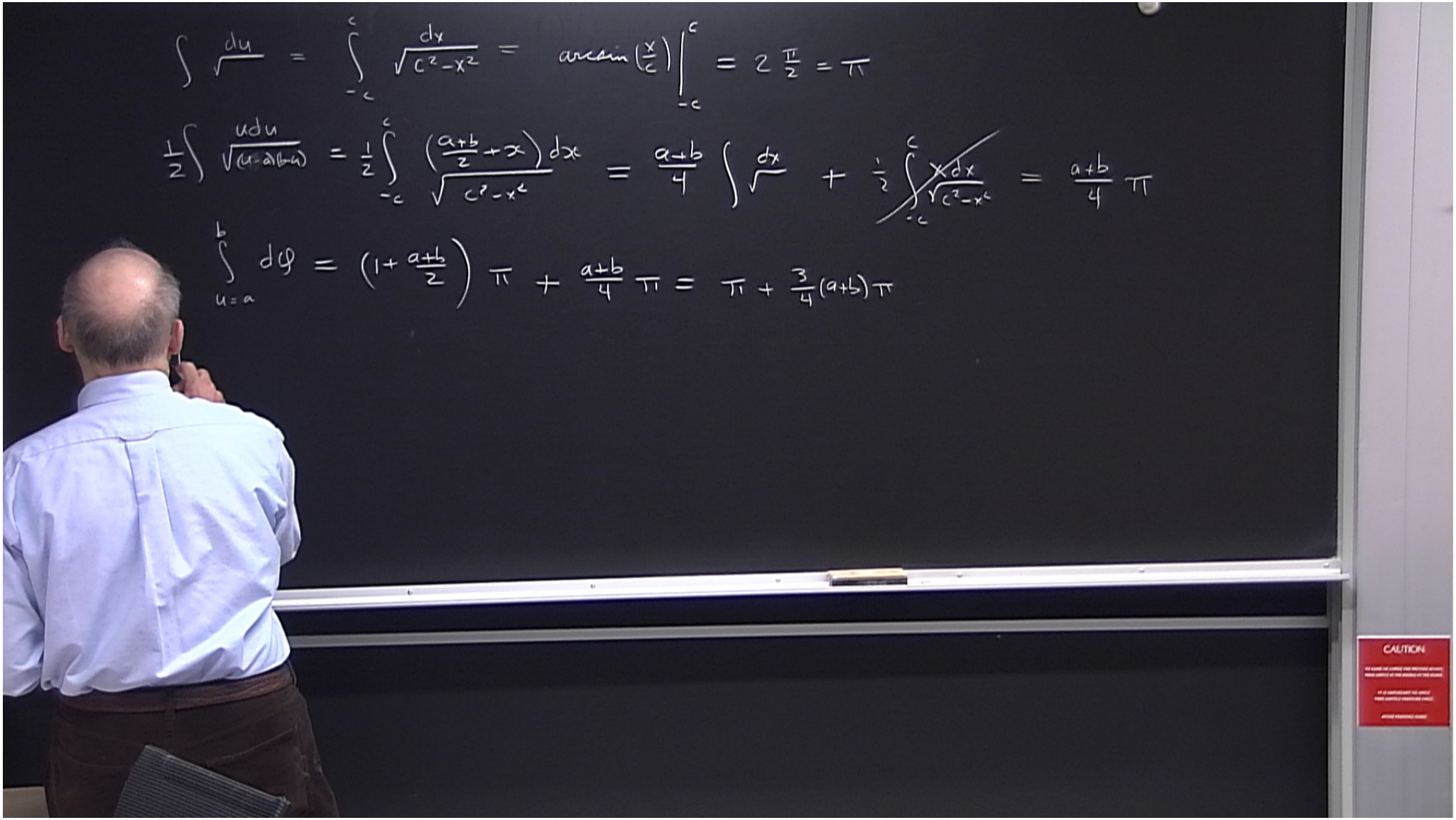
$\int \frac{u\,du}{\sqrt{(u-a)(b-u)}} = \frac{1}{2}\int_{-c}^{c} \frac{\left(\frac{a+b}{2}+x\right)dx}{\sqrt{c^2-x^2}} = \frac{a+b}{4}\int \frac{dx}{\sqrt{\quad}} + \int_{-c}^{c} \frac{x\,dx}{\sqrt{c^2-x^2}} = \frac{a+b}{4}\pi$
CAUTION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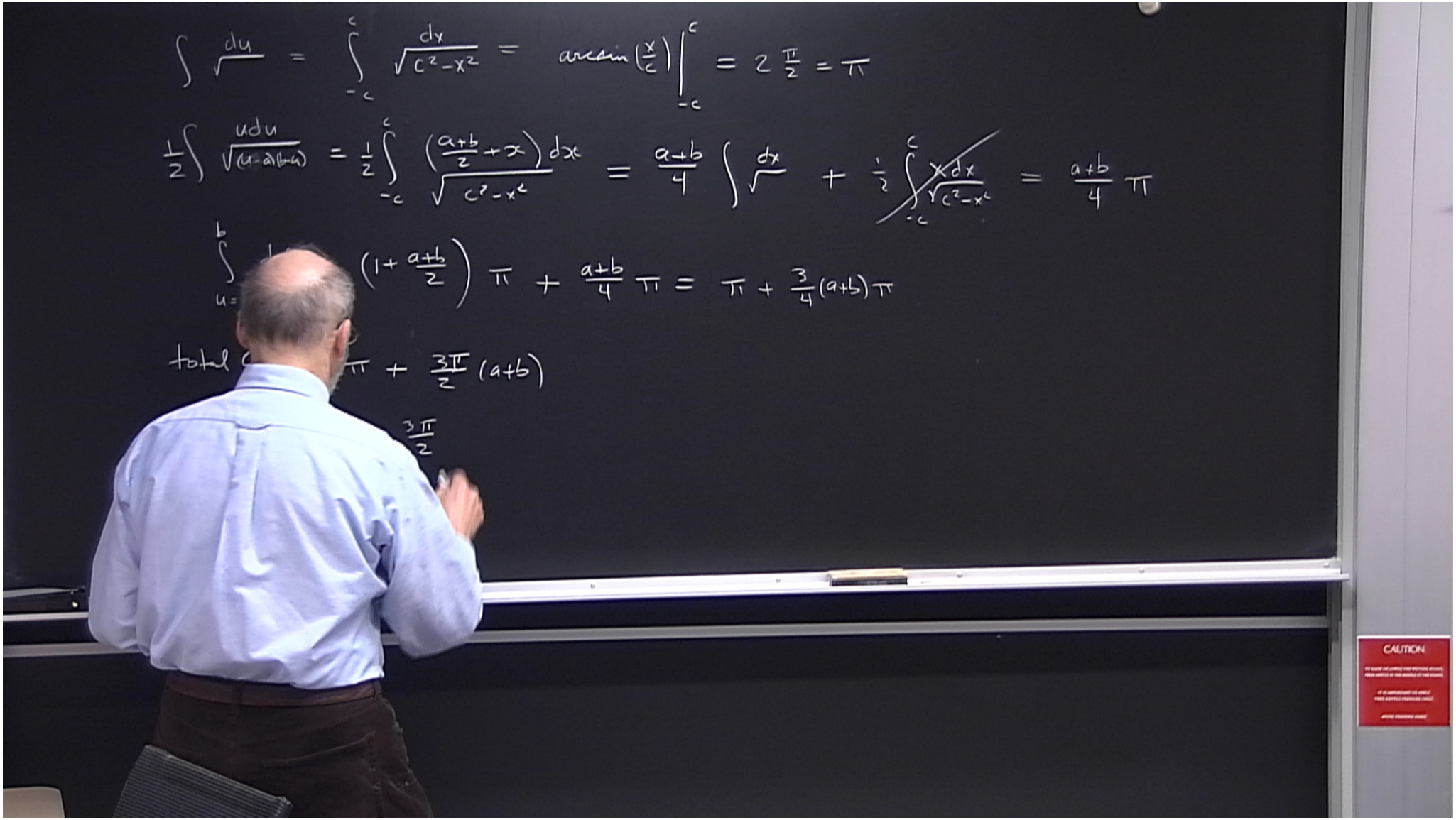
$\int_{u=a}^{b} d\varphi = \left(1+\frac{a+b}{2}\right)\pi + \frac{a+b}{4}\pi = \pi + \frac{3}{4}(a+b)\pi$
CAUTION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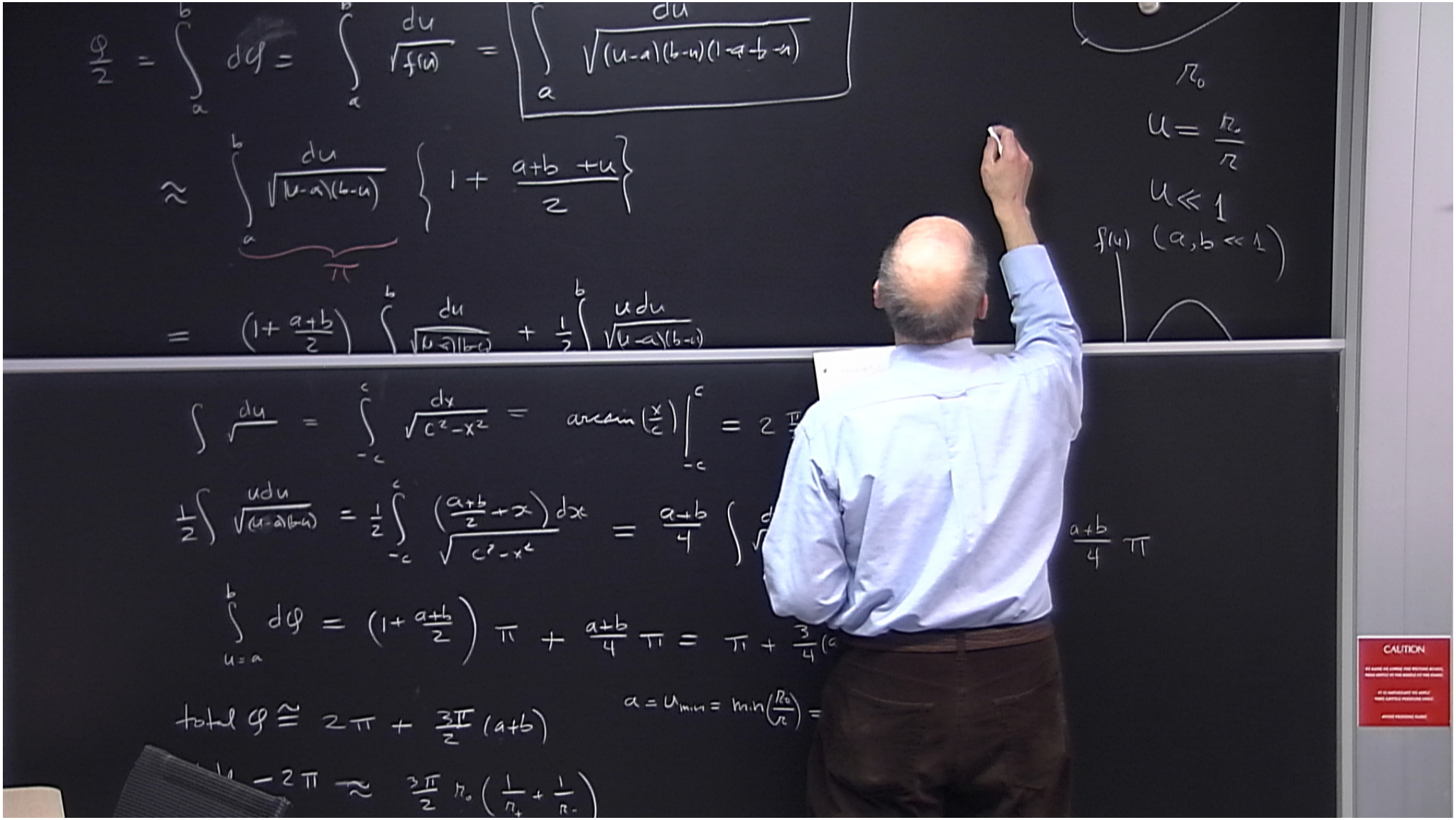
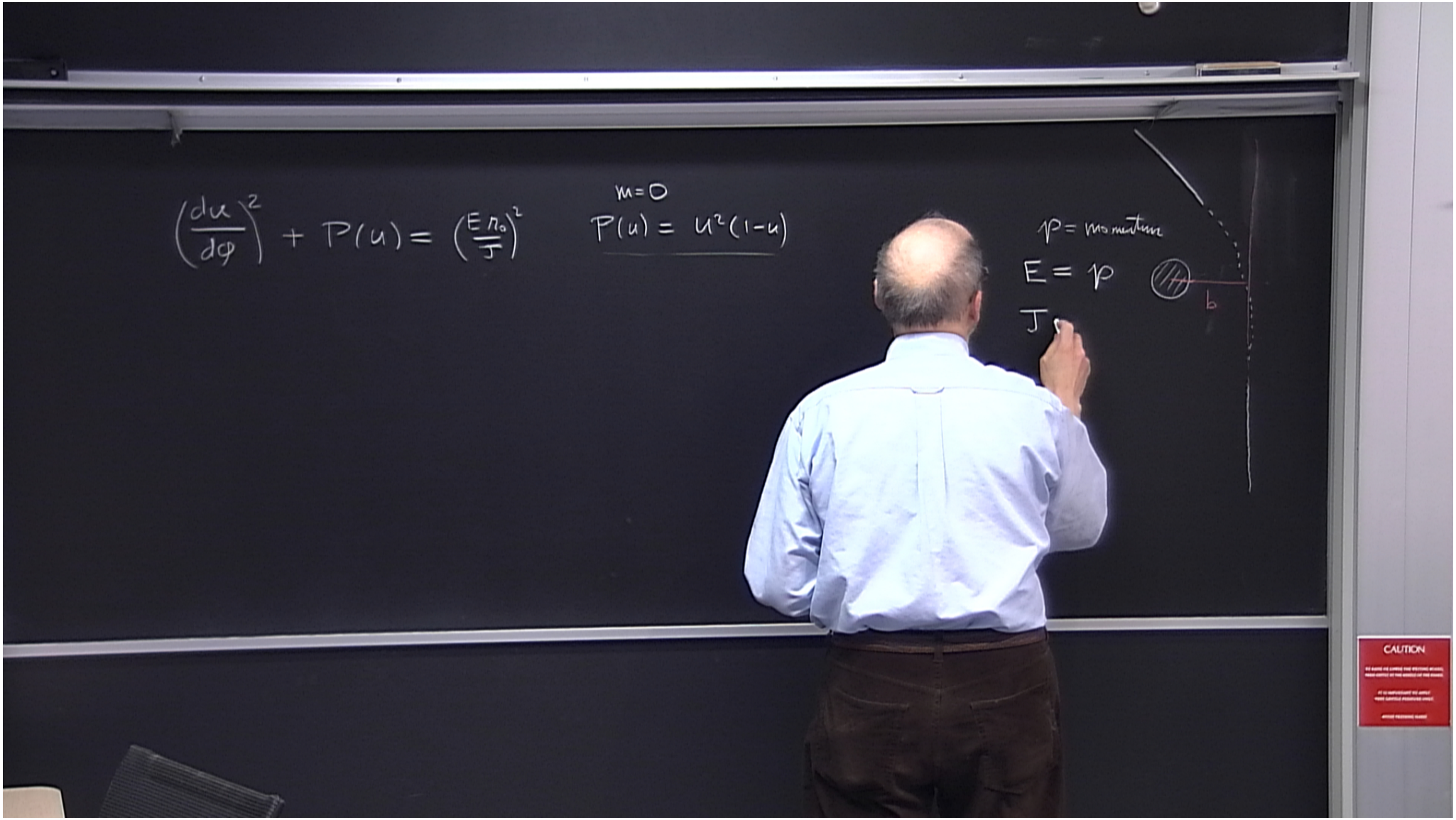
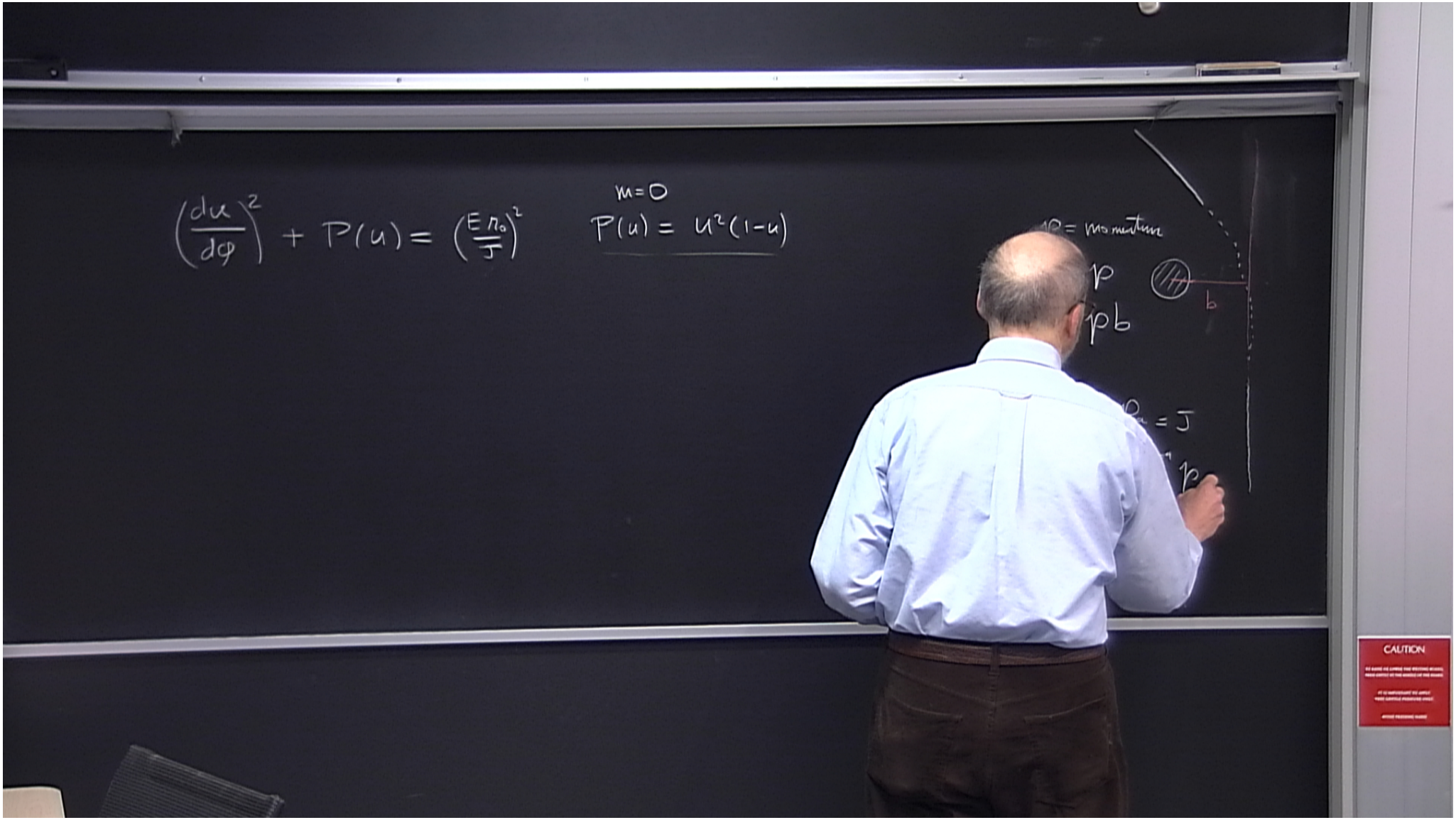
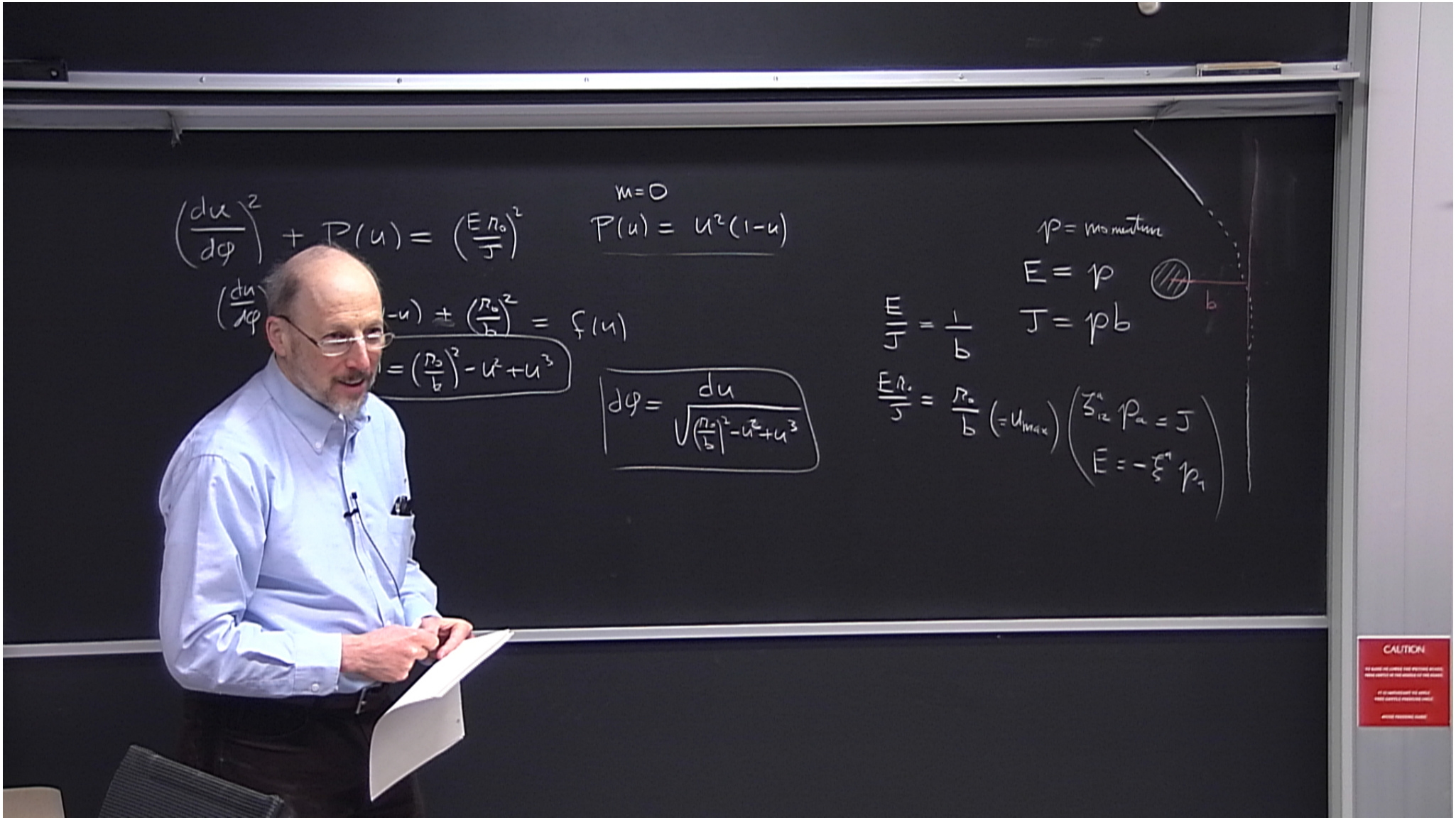
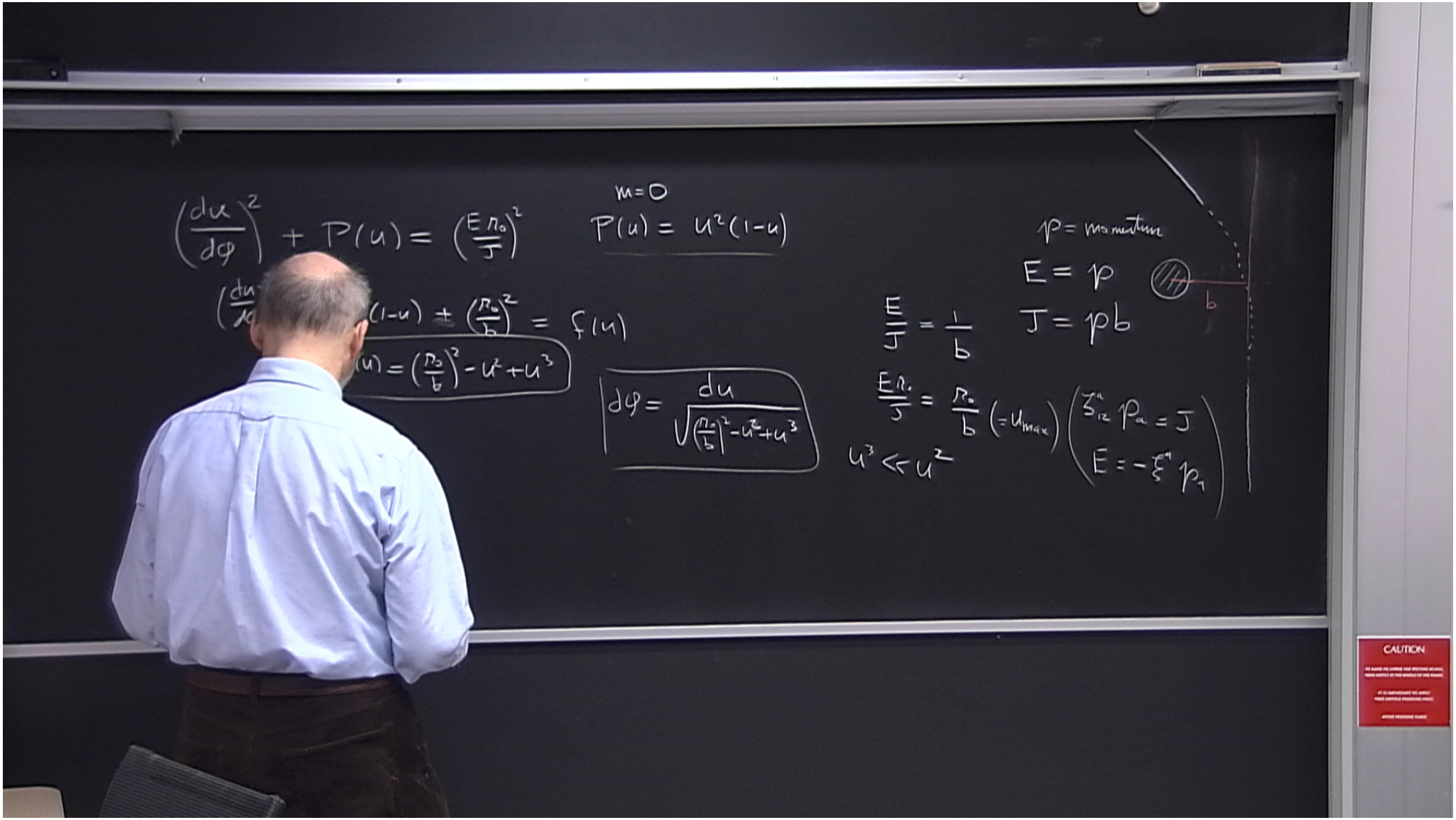
$\left(1+\frac{a+b}{2}\right)\pi + \frac{a+b}{4}\pi = \pi + \frac{3}{4}(a+b)\pi$
$\pi + \frac{3\pi}{2}(a+b)$
$\frac{3\pi}{2}$
CAUTION

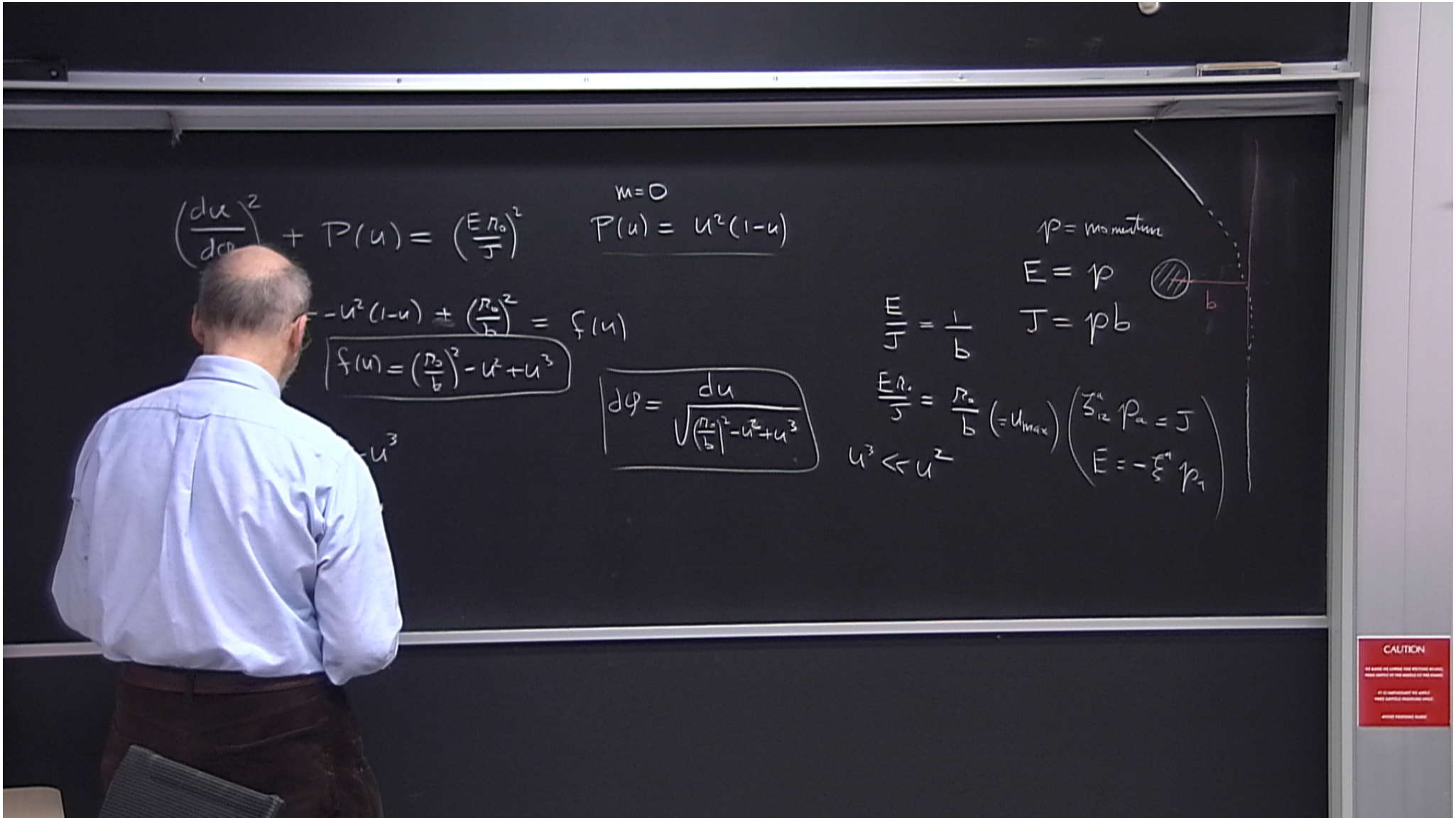

$\frac{\varphi}{2} = \int_a^b d\varphi = \int_a^b \frac{du}{\sqrt{f(u)}} = \int_a^c \frac{du}{\sqrt{(u-a)(b-u)(1-a-b-u)}}$
$\approx \int_a^b \frac{du}{\sqrt{(u-a)(b-u)}} \left\{ 1 + \frac{a+b+u}{2} \right\}$
$\pi$
$= \left(1 + \frac{a+b}{2}\right) \int_a^b \frac{du}{\sqrt{(u-a)(b-u)}} + \frac{1}{2} \int_a^b \frac{u\,du}{\sqrt{(u-a)(b-u)}}$
$\int \frac{du}{\sqrt{\quad}} = \int_{-c}^{c} \frac{dx}{\sqrt{c^2-x^2}} = \arcsin\left(\frac{x}{c}\right)\Big|_{-c}^{c} = 2\,\frac{\pi}{2}$
$\frac{1}{2} \int \frac{u\,du}{\sqrt{(u-a)(b-u)}} = \frac{1}{2} \int_{-c}^{c} \frac{\left(\frac{a+b}{2} + x\right) dx}{\sqrt{c^2-x^2}} = \frac{a+b}{4} \int \frac{d}{\sqrt{\quad}}$
$\frac{a+b}{4}\,\pi$
$\int_{u=a}^{b} d\varphi = \left(1 + \frac{a+b}{2}\right)\pi + \frac{a+b}{4}\pi = \pi + \frac{3}{4}(a$
$a = u_{min} = \min\left(\frac{R_0}{r}\right) =$
total $\varphi \cong 2\pi + \frac{3\pi}{2}(a+b)$
$- 2\pi \approx \frac{3\pi}{2} R_0 \left(\frac{1}{R_+} + \frac{1}{R_-}\right)$
$R_0$
$u = \frac{R_0}{r}$
$u \ll 1$
$f(u)$ $(a, b \ll 1)$
CAUTION

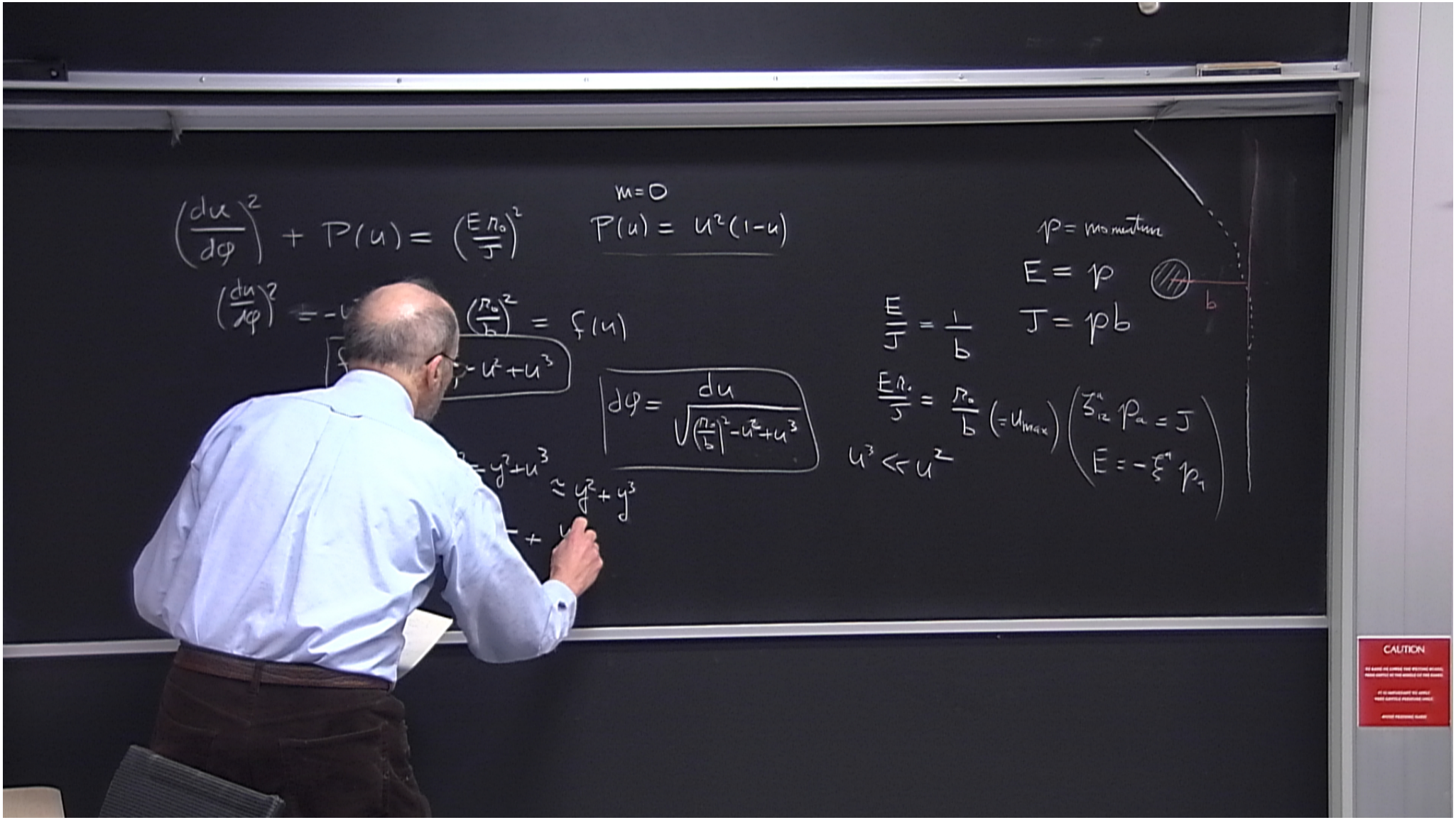

$\left(\frac{du}{d\varphi}\right)^2 + P(u) = \left(\frac{E r_0}{J}\right)^2$
m=0
$P(u) = u^2(1-u)$
p = momentum
E = p
b
J
CAUTION

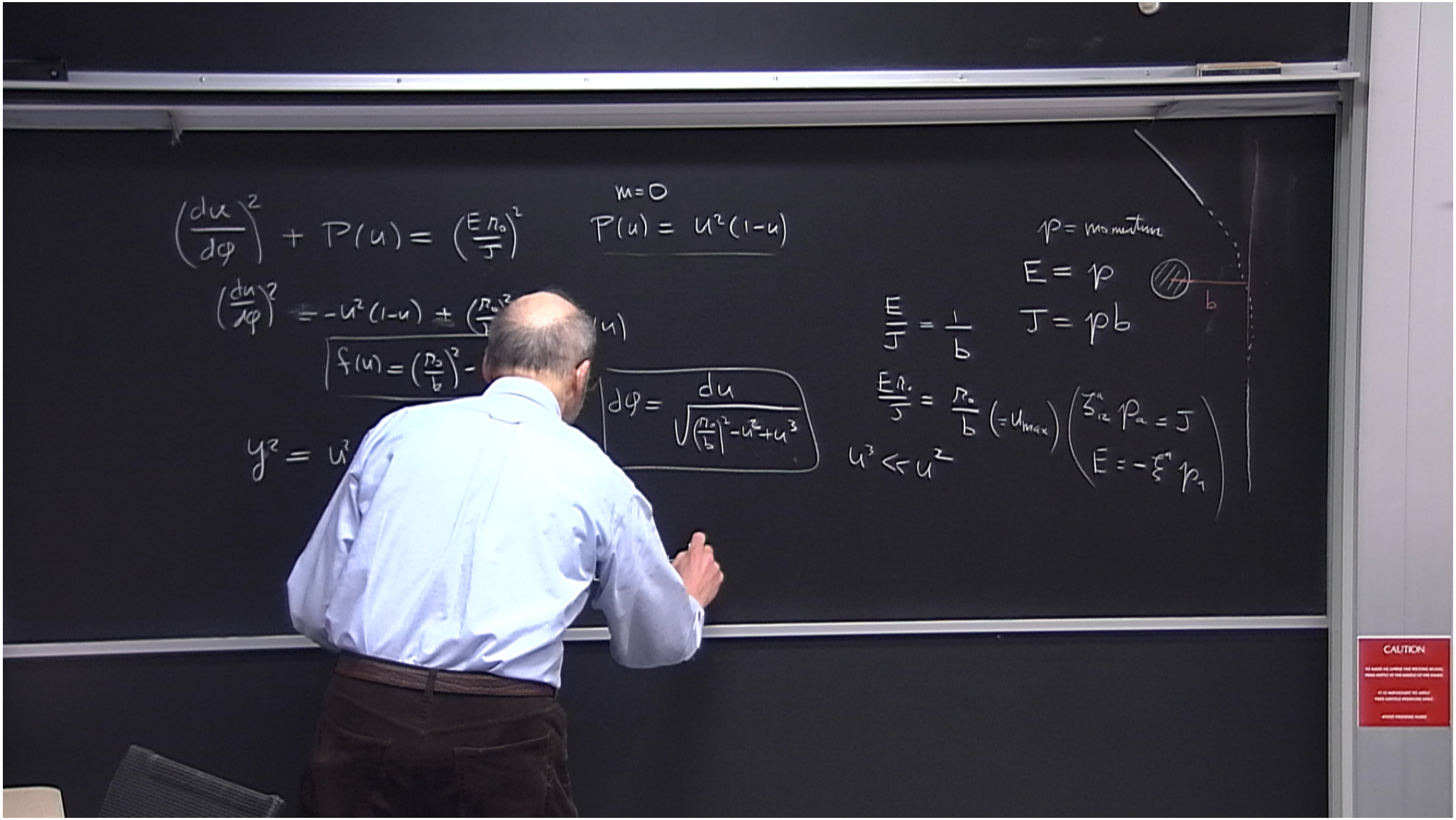

$\left(\frac{du}{d\varphi}\right)^2 + P(u) = \left(\frac{E n_0}{J}\right)^2$
$m=0$
$P(u) = u^2(1-u)$
momentum
$p$
$pb$
$b$
$= J$
CAUTION


$\left(\frac{du}{d\varphi}\right)^2 + P(u) = \left(\frac{E r_0}{J}\right)^2$
$m=0$
$P(u) = u^2(1-u)$
$p$ = momentum
$E = p$
$J = pb$
$\frac{E}{J} = \frac{1}{b}$
$\frac{E r_0}{J} = \frac{r_0}{b} (= u_{max})$
$d\varphi = \frac{du}{\sqrt{\left(\frac{r_0}{b}\right)^2 - u^2 + u^3}}$
CAUTION


$\left(\frac{du}{d\varphi}\right)^2 + P(u) = \left(\frac{E r_0}{J}\right)^2$
$m=0$
$P(u) = u^2(1-u)$
$p$ = momentum
$E = p$
$J = pb$
$\frac{E}{J} = \frac{1}{b}$
$\frac{E r_0}{J} = \frac{r_0}{b} (= u_{max})$
$u^3 \ll u^2$
$d\varphi = \frac{du}{\sqrt{\left(\frac{r_0}{b}\right)^2 - u^2 + u^3}}$
b
CAUTION


$\left(\frac{du}{d\varphi}\right)^2 + P(u) = \left(\frac{E r_0}{J}\right)^2$
$m=0$
$P(u) = u^2(1-u)$
$p$ = momentum
$E = p$
$J = pb$
$\frac{E}{J} = \frac{1}{b}$
$-u^2(1-u) + \left(\frac{r_0}{b}\right)^2 = f(u)$
$f(u) = \left(\frac{r_0}{b}\right)^2 - u^2 + u^3$
$d\varphi = \frac{du}{\sqrt{\left(\frac{r_0}{b}\right)^2 - u^2 + u^3}}$
$\frac{E r_0}{J} = \frac{r_0}{b} \left(= u_{max}\right)$
$u^3 \ll u^2$
$\xi^a_{12} p_a = J$
$E = -\xi^a p_a$
b
CAUTION

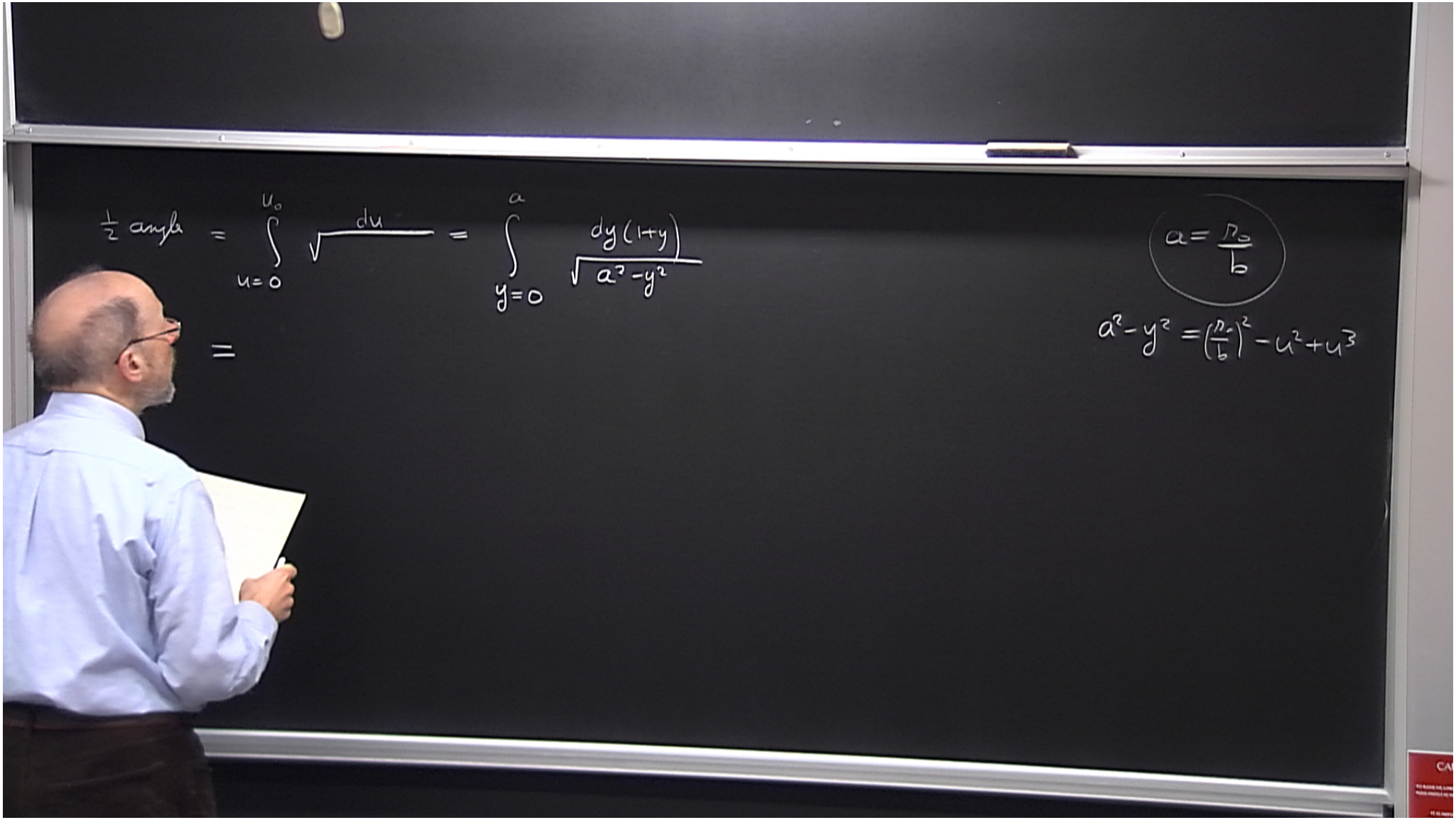

m=0
P(u) = u²(1-u)
p = momentum
E = p
J = pb
E/J = 1/b
u³ ≪ u²
CAUTION

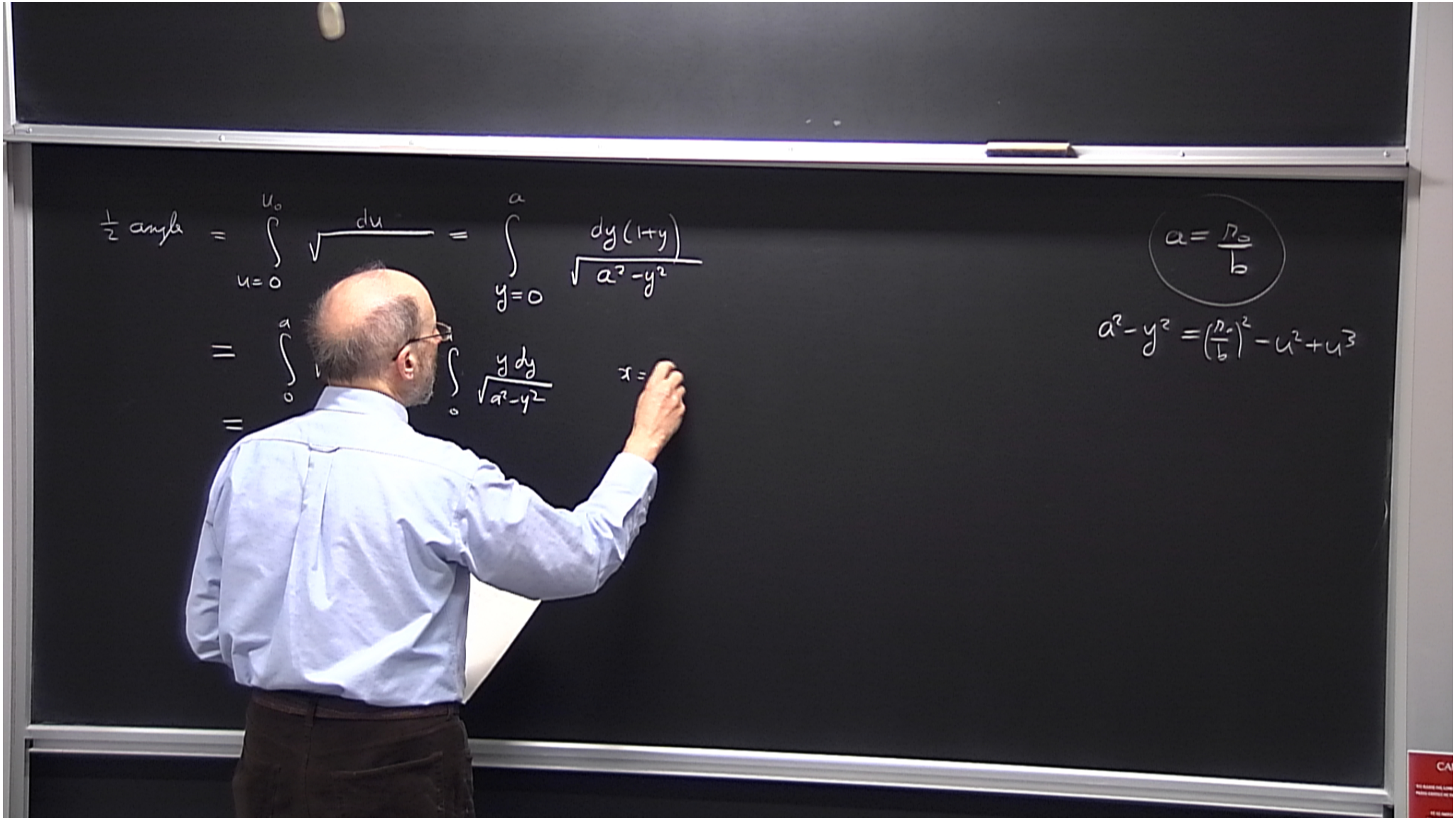

$\left(\frac{du}{d\varphi}\right)^2 + P(u) = \left(\frac{E\,n_0}{J}\right)^2$
$m=0$
$P(u) = u^2(1-u)$
$p$ = momentum
$E = p$
$J = pb$
$\frac{E}{J} = \frac{1}{b}$
$\left(\frac{du}{d\varphi}\right)^2 = -u^2(1-u) + \left(\frac{n_0}{J}\right)^2$
$f(u) = \left(\frac{n_0}{b}\right)^2 - u^2 + u^3$
$d\varphi = \frac{du}{\sqrt{\left(\frac{n_0}{b}\right)^2 - u^2 + u^3}}$
$\frac{E\,n_0}{J} = \frac{n_0}{b} \; (= u_{max})$
$\xi^a_{12}\, p_a = J$
$E = -\xi^a p_a$
$y^2 = u^2$
$u^3 \ll u^2$
b
CAUTION

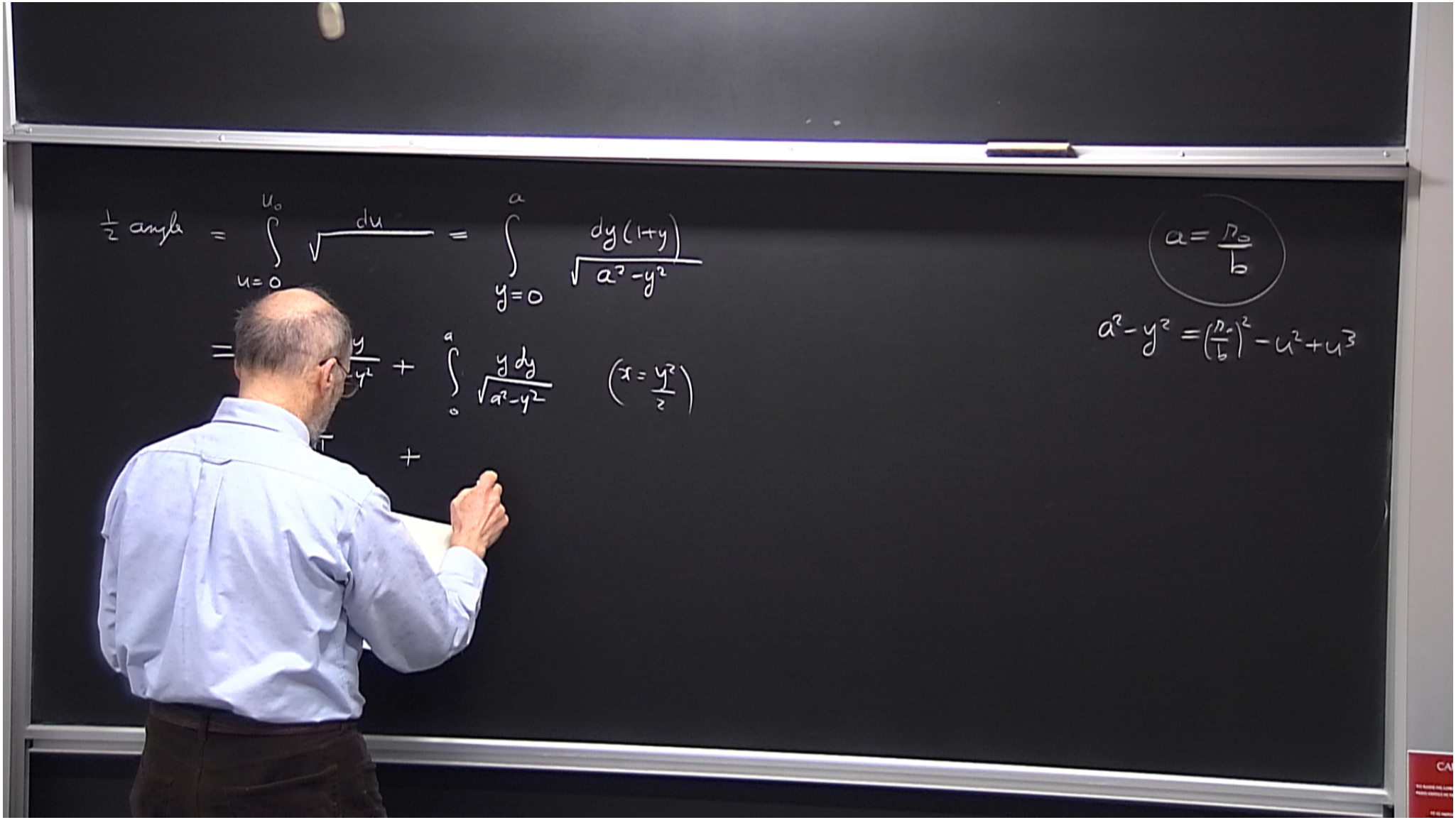
$$\tfrac{1}{2}\,\text{angle} = \int_{u=0}^{u_0} \frac{du}{\sqrt{\quad}} = \int_{y=0}^{a} \frac{dy\,(1+y)}{\sqrt{a^2-y^2}}$$

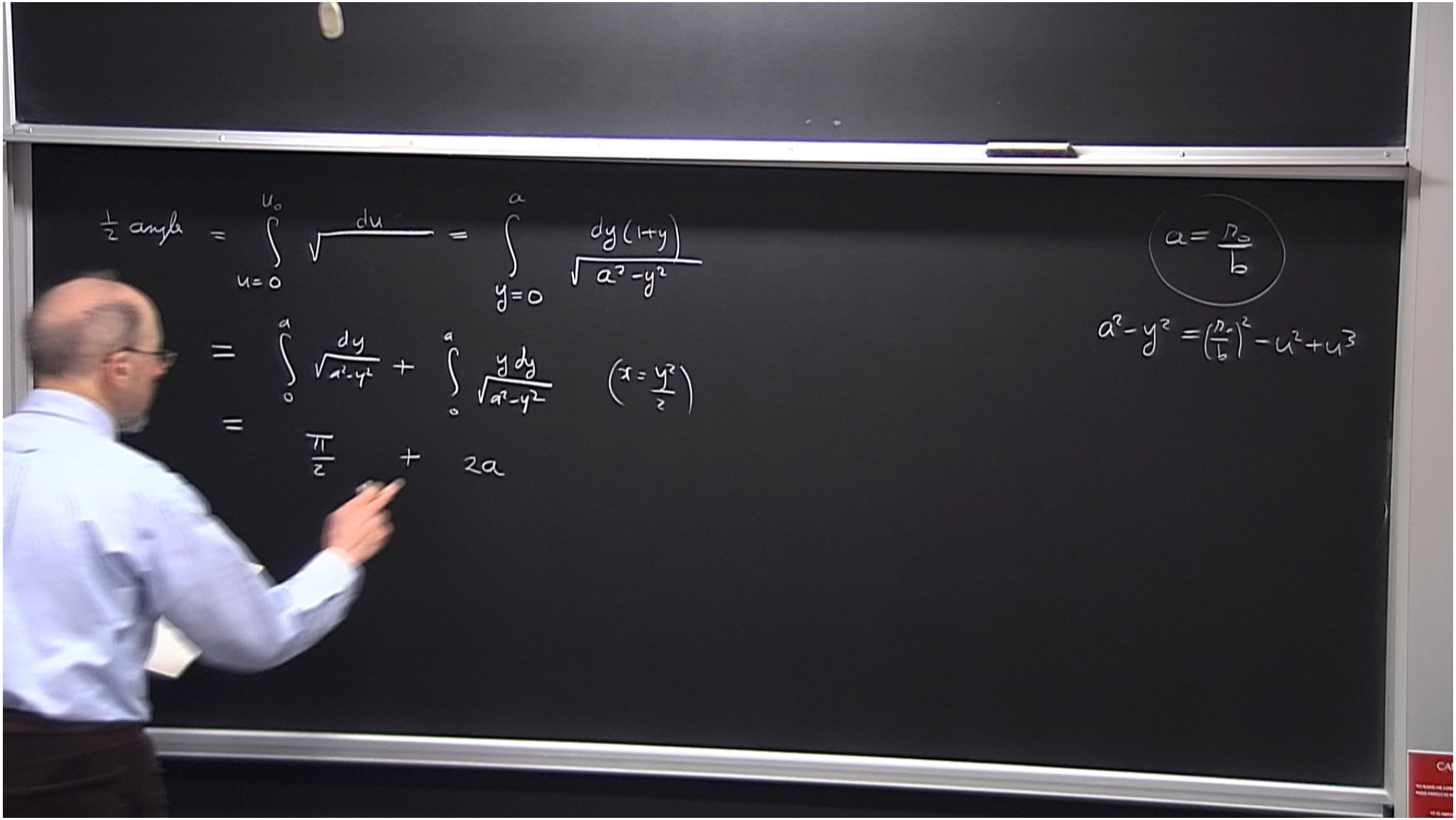
$$a = \frac{R_0}{b}$$

$$a^2 - y^2 =$$


$\frac{1}{2}$ angle $= \int_{u=0}^{u_0} \frac{du}{\sqrt{\quad}} = \int_{y=0}^{a} \frac{dy(1+y)}{\sqrt{a^2-y^2}}$
$=$
$a = \frac{r_0}{b}$
$a^2 - y^2 = \left(\frac{r_0}{b}\right)^2 - u^2 + u^3$


$\frac{1}{2}$ angle $= \int_{u=0}^{u_0} \frac{du}{\sqrt{\quad}} = \int_{y=0}^{a} \frac{dy(1+y)}{\sqrt{a^2-y^2}}$
$= \int_0^a \int_0^{\quad} \frac{y\,dy}{\sqrt{a^2-y^2}}$
$=$
$x =$
$a = \frac{r_0}{b}$
$a^2 - y^2 = \left(\frac{r_0}{b}\right)^2 - u^2 + u^3$


$\frac{1}{2}$ angle $= \int_{u=0}^{u_0} \frac{du}{\sqrt{\quad}} = \int_{y=0}^{a} \frac{dy(1+y)}{\sqrt{a^2-y^2}}$
$a = \frac{n_0}{b}$
$a^2 - y^2 = \left(\frac{n_0}{b}\right)^2 - u^2 + u^3$
$= \quad + \int_0^a \frac{y\,dy}{\sqrt{a^2-y^2}} \quad \left(x = \frac{y^2}{2}\right)$
$+$


$\frac{1}{2}$ angle $= \int_{u=0}^{u_0} \frac{du}{\sqrt{\quad}} = \int_{y=0}^{a} \frac{dy(1+y)}{\sqrt{a^2-y^2}}$
$a = \frac{r_0}{b}$
$a^2 - y^2 = \left(\frac{r_0}{b}\right)^2 - u^2 + u^3$
$= \int_0^a \frac{dy}{\sqrt{a^2-y^2}} + \int_0^a \frac{y\,dy}{\sqrt{a^2-y^2}} \quad \left(x = \frac{y^2}{2}\right)$
$= \frac{\pi}{2} + 2a$

$$\tfrac{1}{2}\text{ angle} = \int_{u=0}^{u_0} \frac{du}{\sqrt{\quad}} = \int_{y=0}^{a} \frac{dy\,(1+y)}{\sqrt{a^2-y^2}}$$

$$= \int_0^a \frac{dy}{\sqrt{a^2-y^2}} + \int_0^a \frac{y\,dy}{\sqrt{a^2-y^2}} \qquad \left(x = \frac{y^2}{2}\right)$$

$$= \frac{\pi}{2} + a$$

$$\text{angle} = \pi + 2a = \pi + \frac{2r_0}{b} \quad \text{deflection}$$

$$a = \frac{r_0}{b}$$

$$a^2 - y^2 = \left(\frac{r_0}{b}\right)^2 - u^2 + u^3$$

$\frac{1}{2}$ angle $= \int_{u=0}^{u_0} \frac{du}{\sqrt{\quad}} = \int_{y=0}^{a} \frac{dy(1+y)}{\sqrt{a^2-y^2}}$

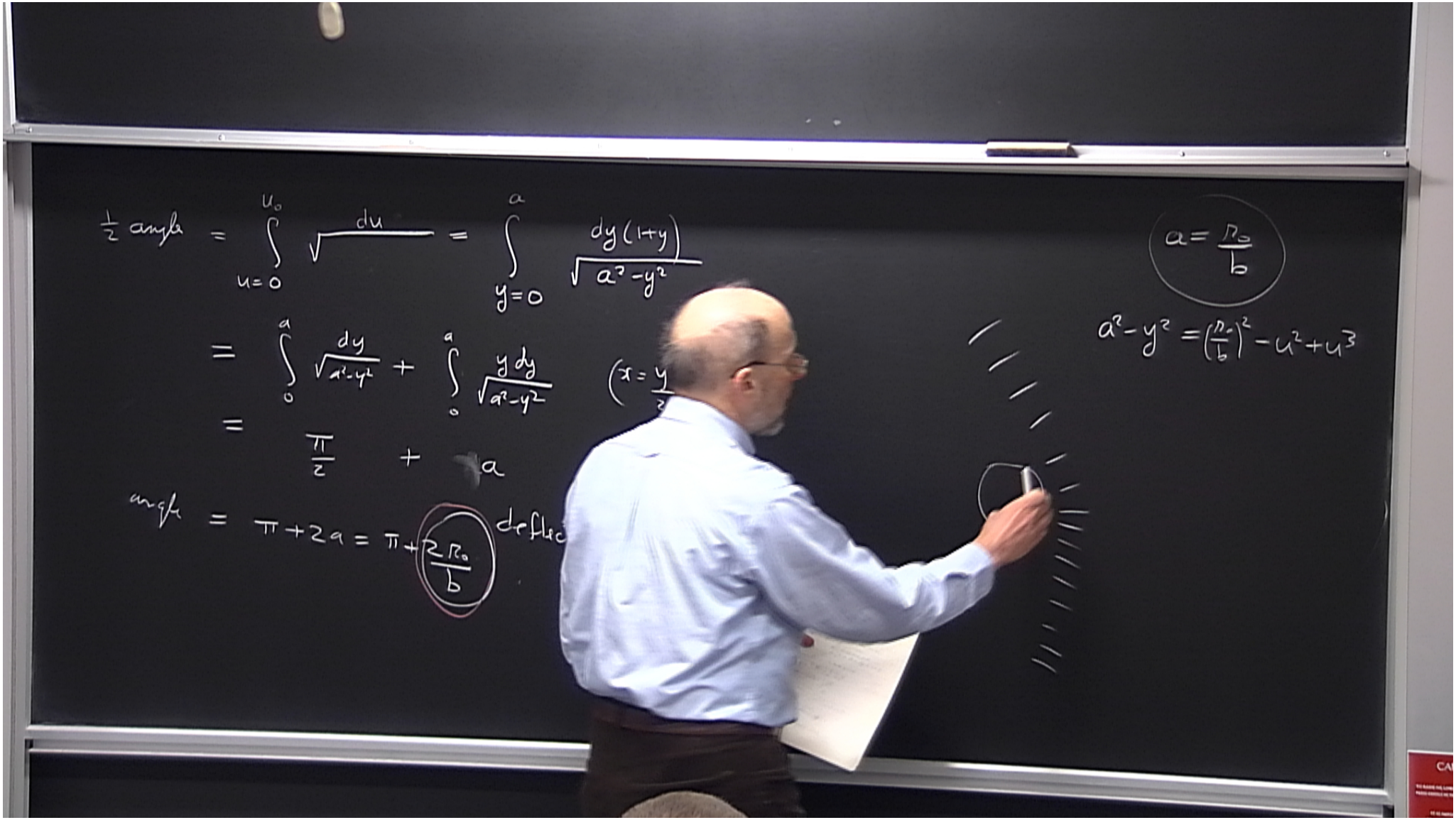
$a = \frac{r_0}{b}$

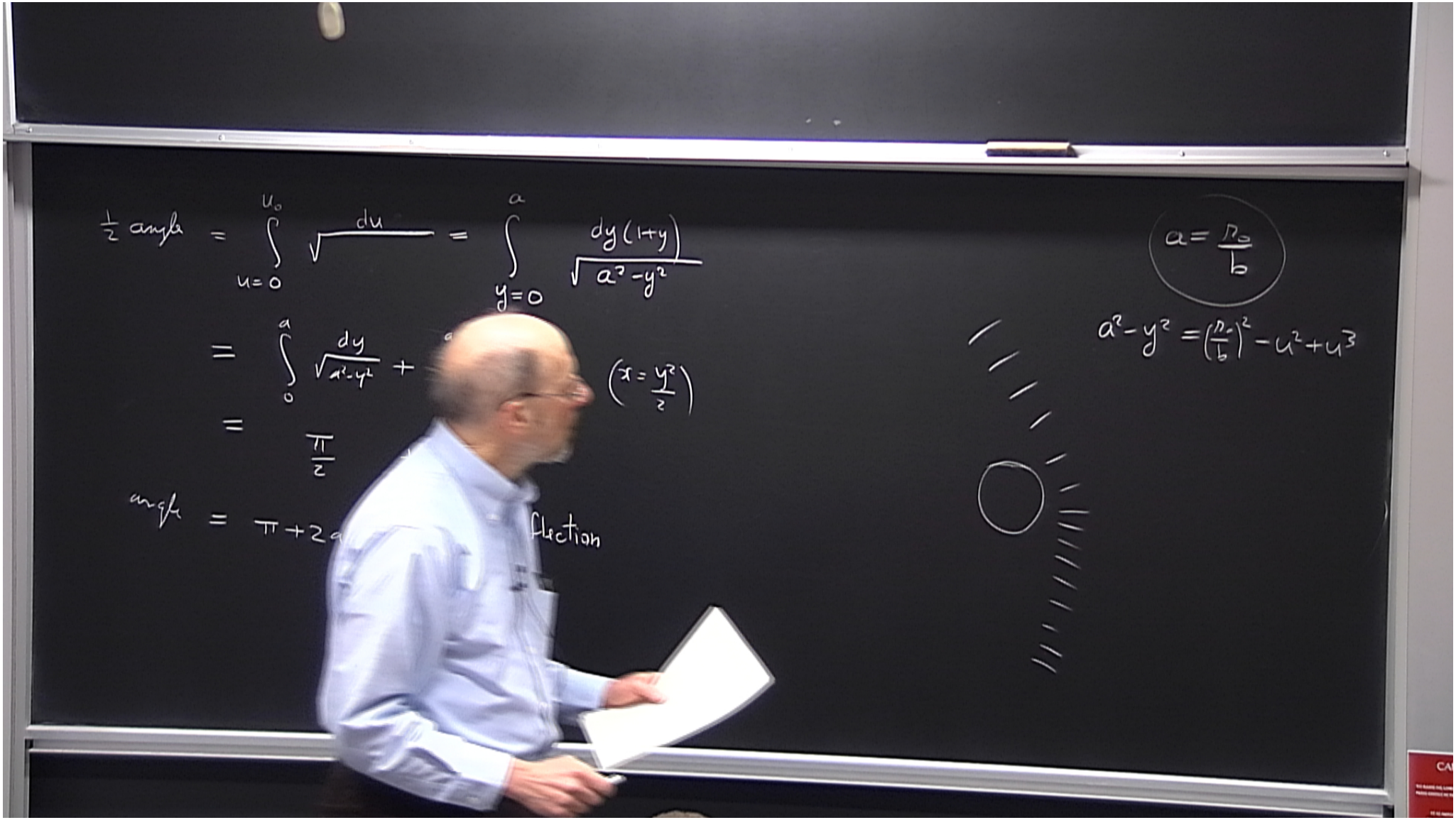
$a^2 - y^2 = \left(\frac{r_0}{b}\right)^2 - u^2 + u^3$

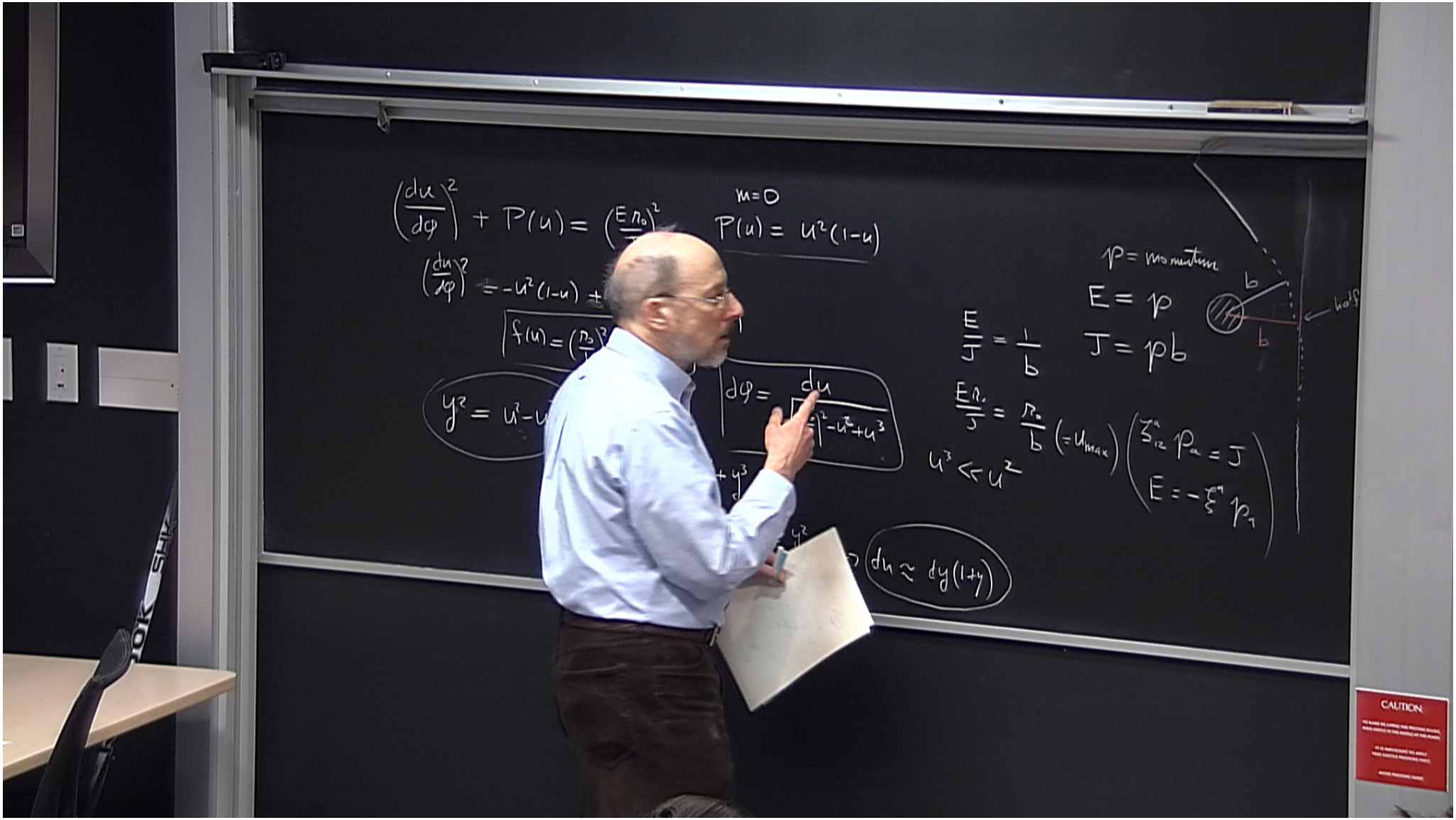
$= \int_0^a \frac{dy}{\sqrt{a^2-y^2}} + \int_0^a \frac{y\,dy}{\sqrt{a^2-y^2}} \quad \left(x = \frac{y^2}{2}\right)$

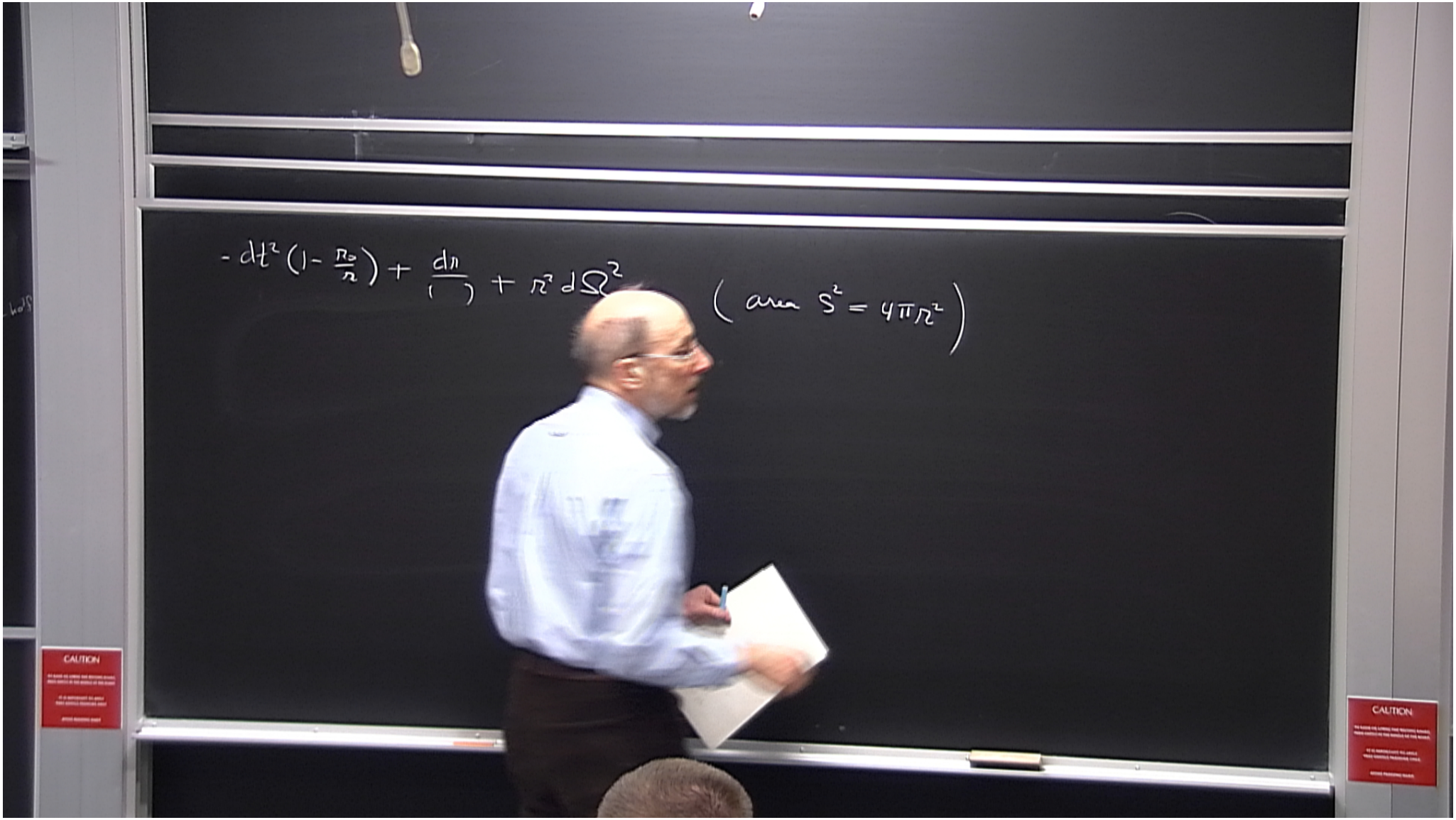
$= \frac{\pi}{2} + a$

angle $= \pi + 2a = \pi + \frac{2r_0}{b}$ deflection

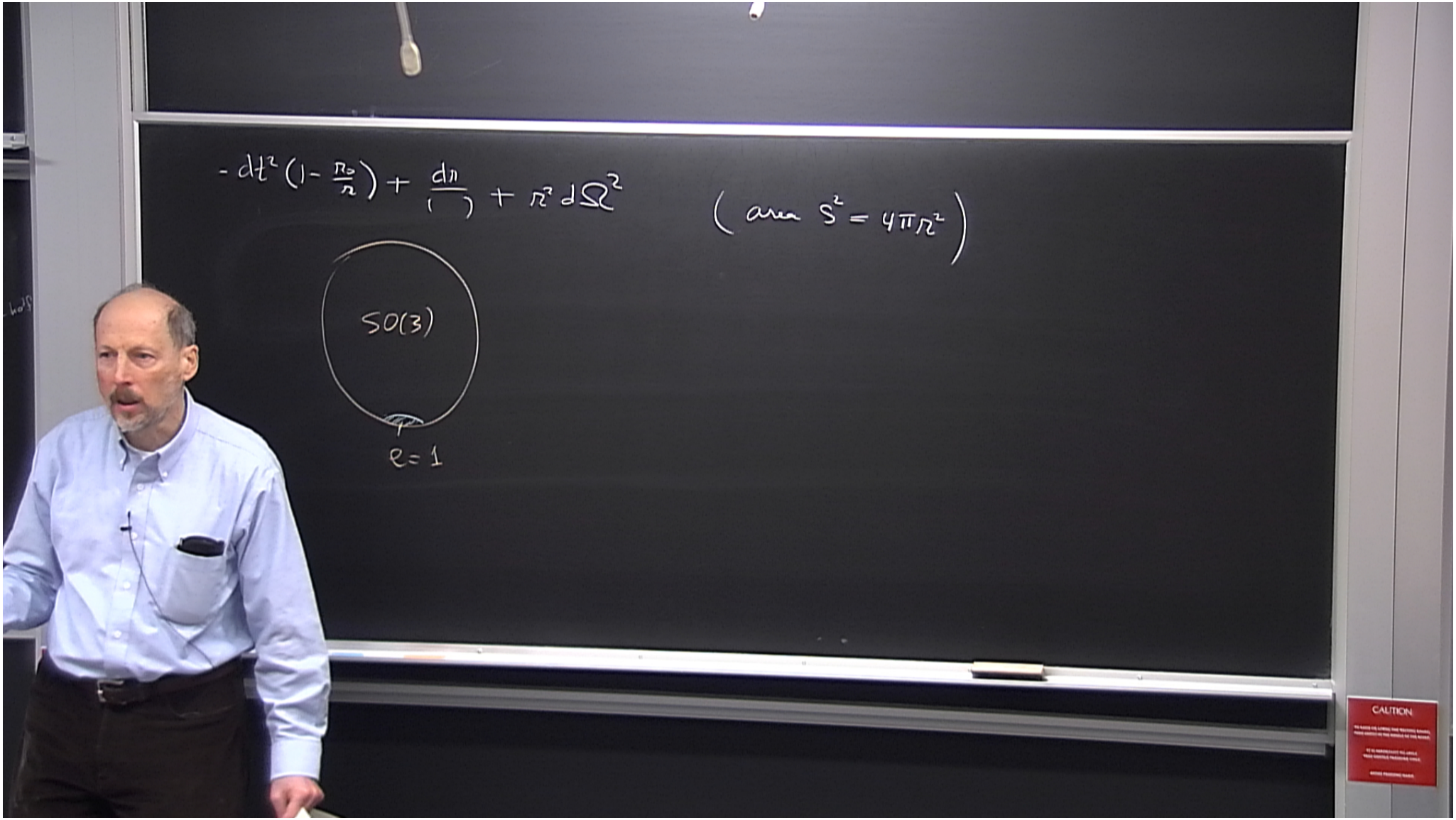

½ angle = ∫ du/√ = ∫ dy(1+y)/√(a²−y²)
a = r₀/b
a²−y² = (r₀/b)² − u² + u³
= ∫ dy/√(a²−y²) + ∫ y dy/√(a²−y²)
= π/2 + a
angle = π + 2a = π + 2r₀/b

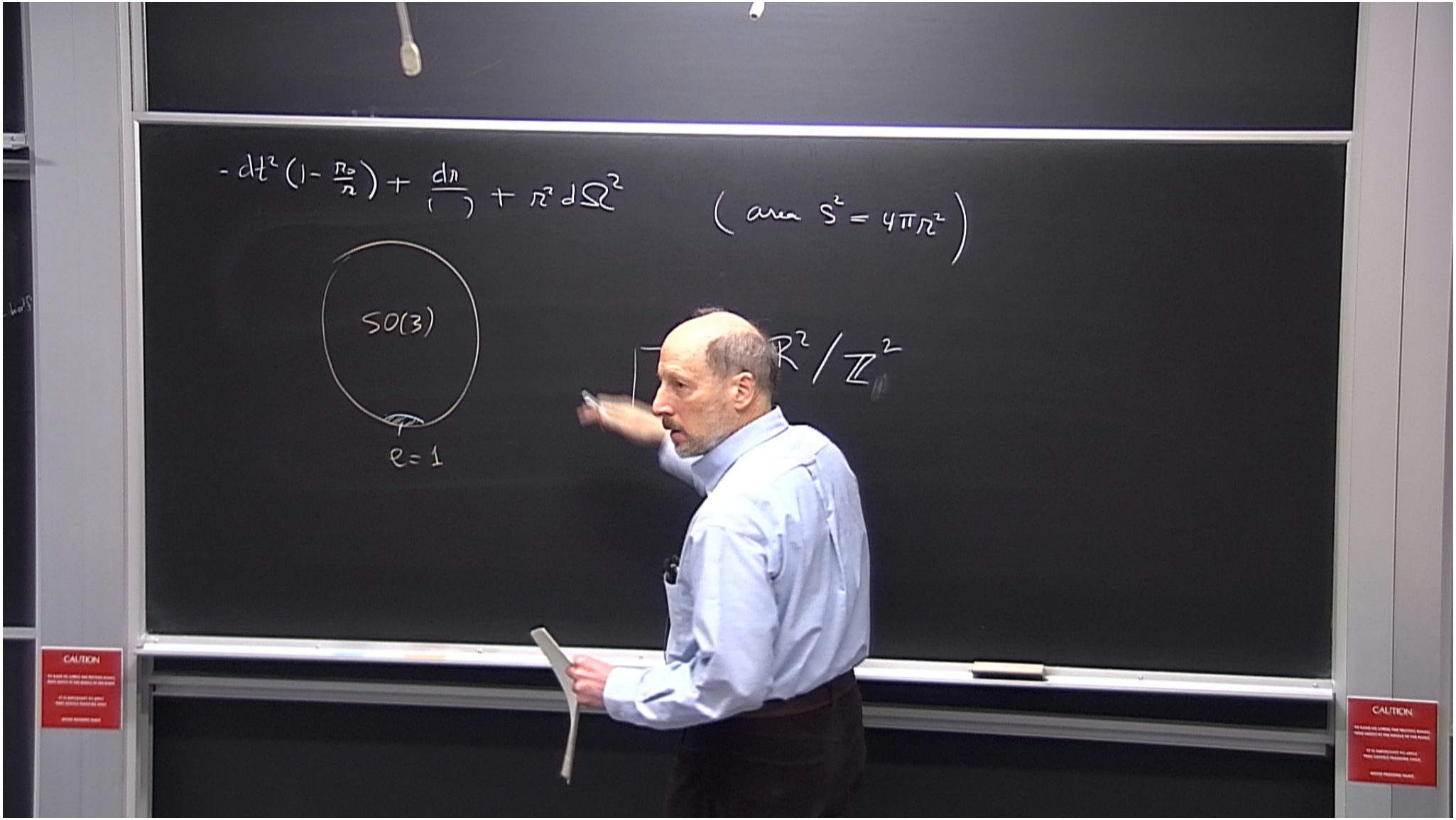

$\frac{1}{2}$ angle $= \int_{u=0}^{u_0} \frac{du}{\sqrt{\quad}} = \int_{y=0}^{a} \frac{dy(1+y)}{\sqrt{a^2-y^2}}$
$a = \frac{r_0}{b}$
$a^2 - y^2 = \left(\frac{r_0}{b}\right)^2 - u^2 + u^3$
$= \int_0^a \frac{dy}{\sqrt{a^2-y^2}} +$
$\left(x = \frac{y^2}{2}\right)$
$= \frac{\pi}{2}$
angle $= \pi + 2$
flection

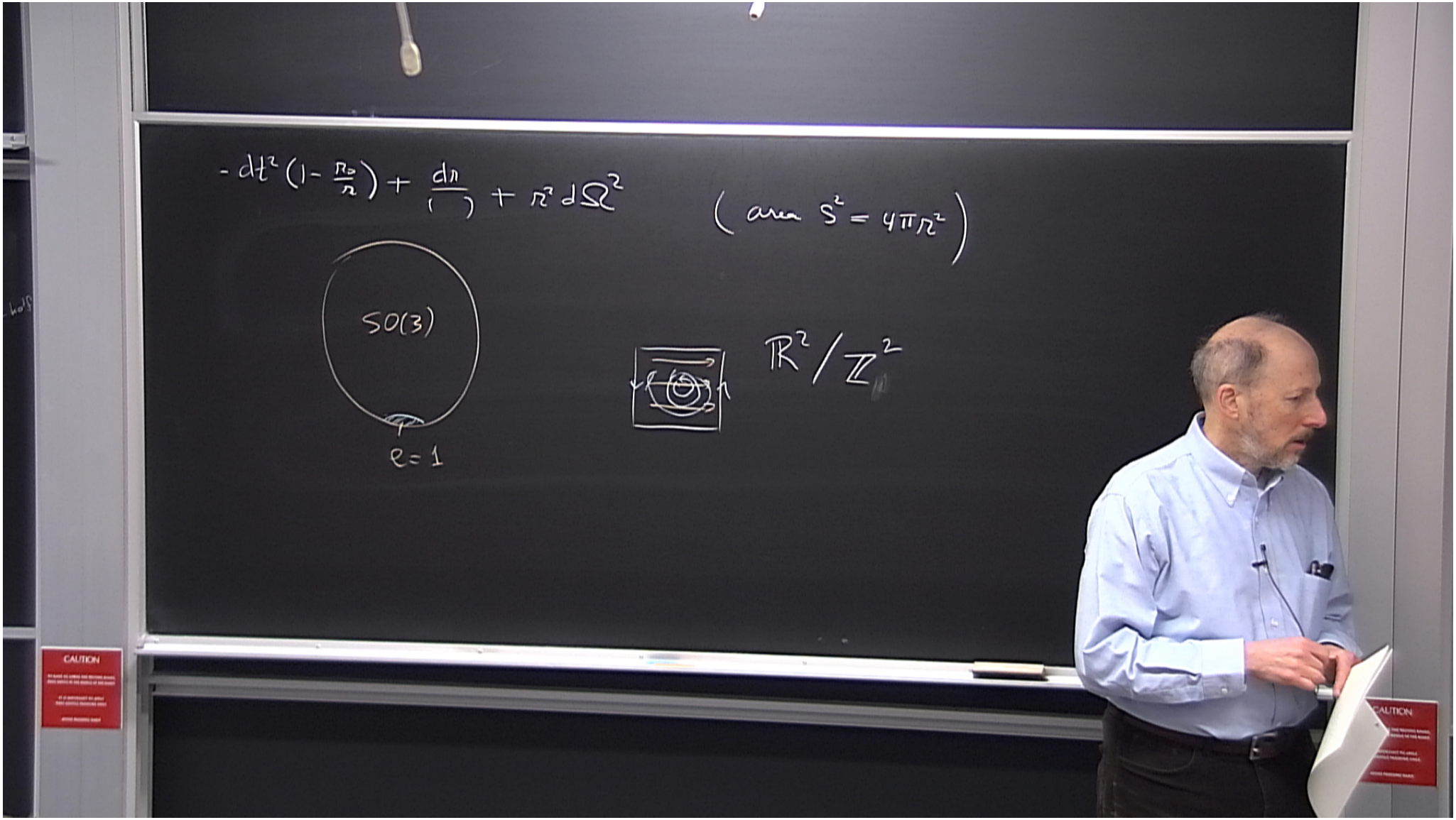

$\left(\frac{du}{d\varphi}\right)^2 + P(u) = \left(\frac{E r_0}{J}\right)^2$
$m=0$
$P(u) = u^2(1-u)$
$\left(\frac{du}{d\varphi}\right)^2 = -u^2(1-u) +$
$f(u) = \left(\frac{r_0}{J}\right)^2$
$y^2 = u^2 - u^3$
$d\varphi = \frac{du}{\sqrt{\quad -u^2+u^3}}$
$+y^3$
$u^3 \ll u^2$
$du \approx dy(1+y)$
$p$ = momentum
$E = p$
$J = pb$
$\frac{E}{J} = \frac{1}{b}$
$\frac{E r_0}{J} = \frac{r_0}{b} (= u_{max})$
$\xi^a_{12} p_a = J$
$E = -\xi^a p_a$
b
b
half
CAUTION

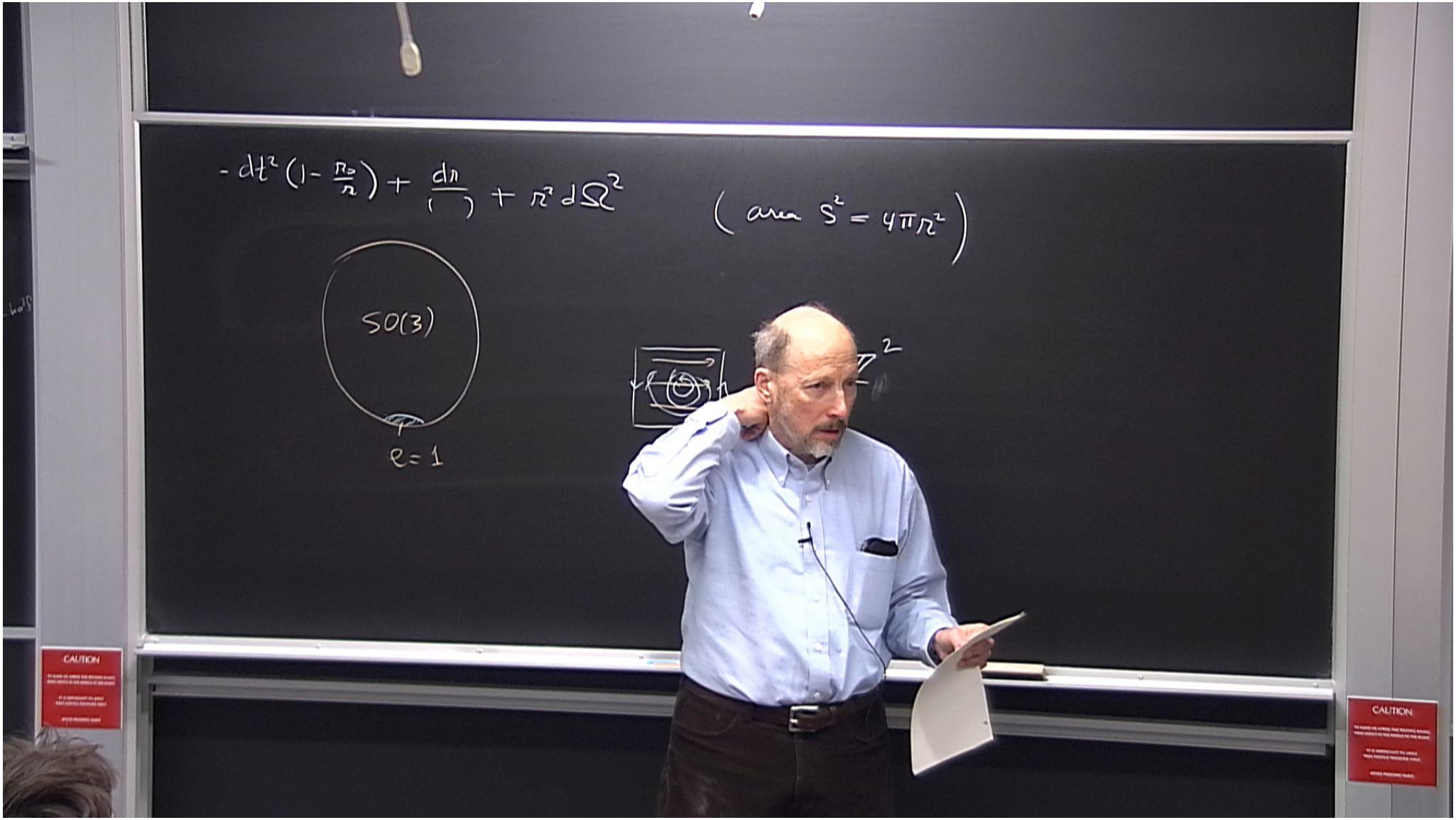

$-dt^2\left(1-\frac{r_0}{r}\right)+\frac{dr}{(\ )}+r^2 d\Omega^2$
$\left(\text{area } S^2 = 4\pi r^2\right)$
CAUTION
CAUTION

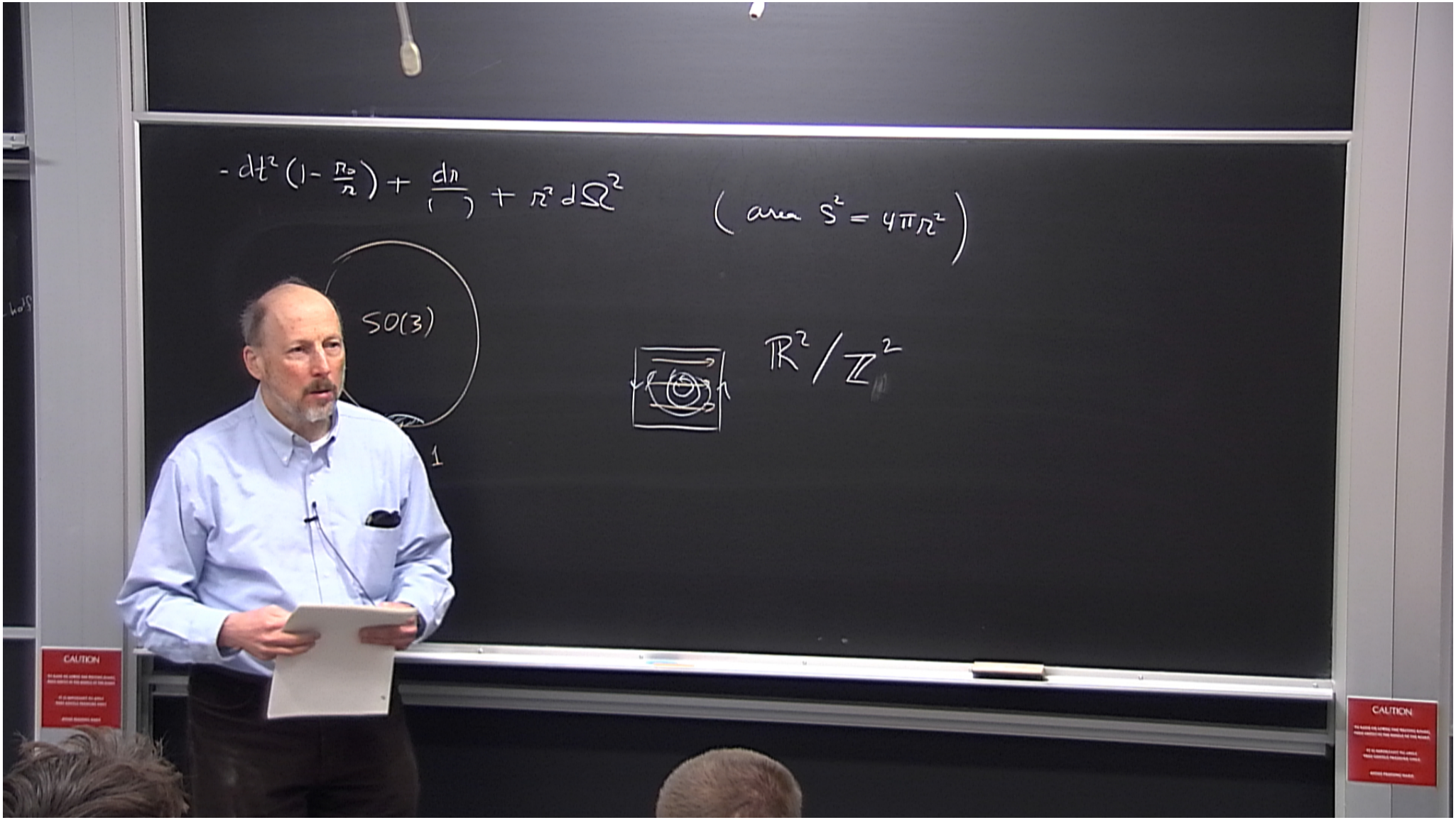
$$-dt^2\left(1-\frac{r_0}{r}\right)+\frac{dr}{(\quad)}+r^2 d\Omega^2 \qquad \left(\text{area } S^2 = 4\pi r^2\right)$$

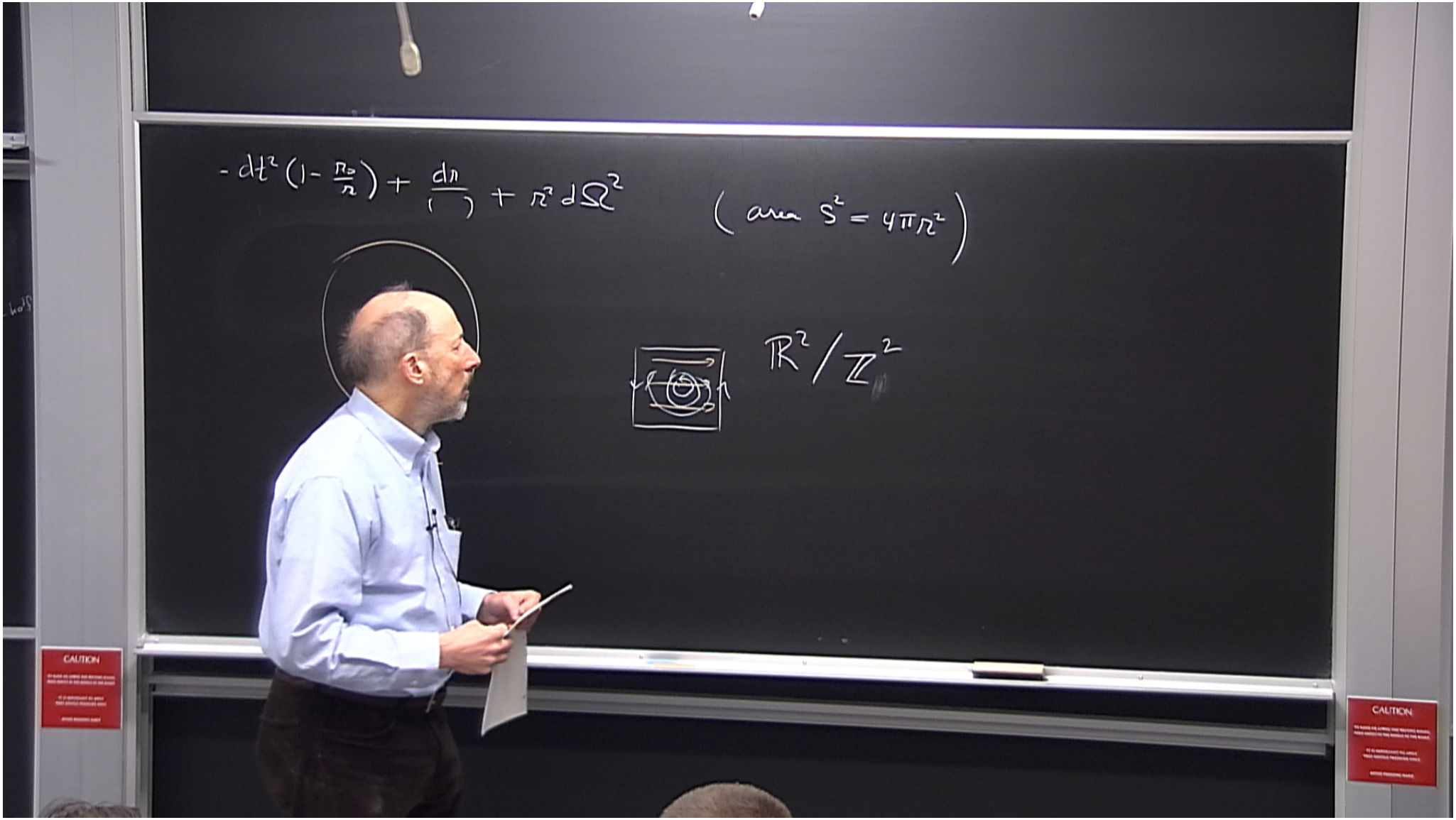

$-dt^2\left(1-\frac{r_0}{r}\right)+\frac{dr}{(\ )}+r^2 d\Omega^2$
$\left(\text{area } S^2 = 4\pi r^2\right)$
SO(3)
e = 1
CAUTION


$-dt^2\left(1-\frac{r_0}{r}\right)+\frac{dr}{(\ )}+r^2 d\Omega^2$
$\left(\text{area } S^2 = 4\pi r^2\right)$
SO(3)
$R^2/\mathbb{Z}^2$
e = 1
CAUTION
CAUTION


$-dt^2\left(1-\frac{r_0}{r}\right)+\frac{dr}{(\ )}+r^2 d\Omega^2$
$\left(\text{area } S^2 = 4\pi r^2\right)$
SO(3)
$e=1$
$\mathbb{R}^2/\mathbb{Z}^2$
CAUTION


$-dt^2\left(1-\frac{r_0}{r}\right)+\frac{dr}{(\ )}+r^2 d\Omega^2$
$\left(\text{area } S^2 = 4\pi r^2\right)$
SO(3)
e = 1
CAUTION
CAUTION


$-dt^2\left(1-\frac{r_0}{r}\right)+\frac{dr}{(\ )}+r^2 d\Omega^2$
$\left(\text{area } S^2 = 4\pi r^2\right)$
SO(3)
1
$\mathbb{R}^2/\mathbb{Z}^2$
CAUTION
CAUTION


$-dt^2\left(1-\frac{r_0}{r}\right)+\frac{dr}{(\ )}+r^2 d\Omega^2$
$\left(\text{area } S^2 = 4\pi r^2\right)$
$\mathbb{R}^2/\mathbb{Z}^2$
CAUTION
CAUTION

$\frac{1}{2}$ angle $= \int_{u=0}^{u_0} \frac{du}{\sqrt{\quad}} = \int_{y=0}^{a} \frac{dy(1+y)}{\sqrt{a^2-y^2}}$

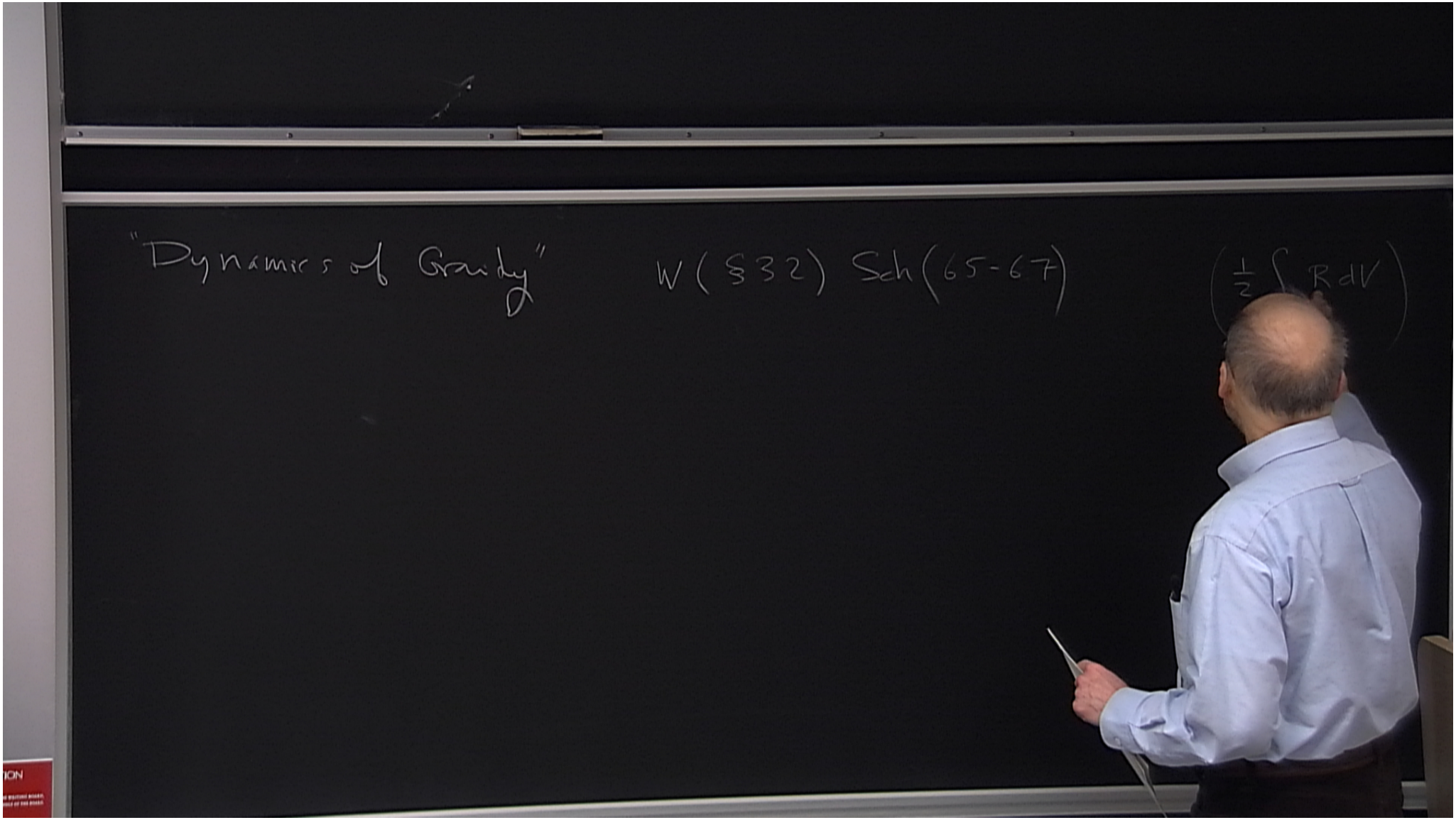
$= \int_0^a \frac{dy}{\sqrt{a^2-y^2}} + \int_0^a \frac{y\,dy}{\sqrt{a^2-y^2}} \quad \left(x = \frac{y^2}{2}\right)$

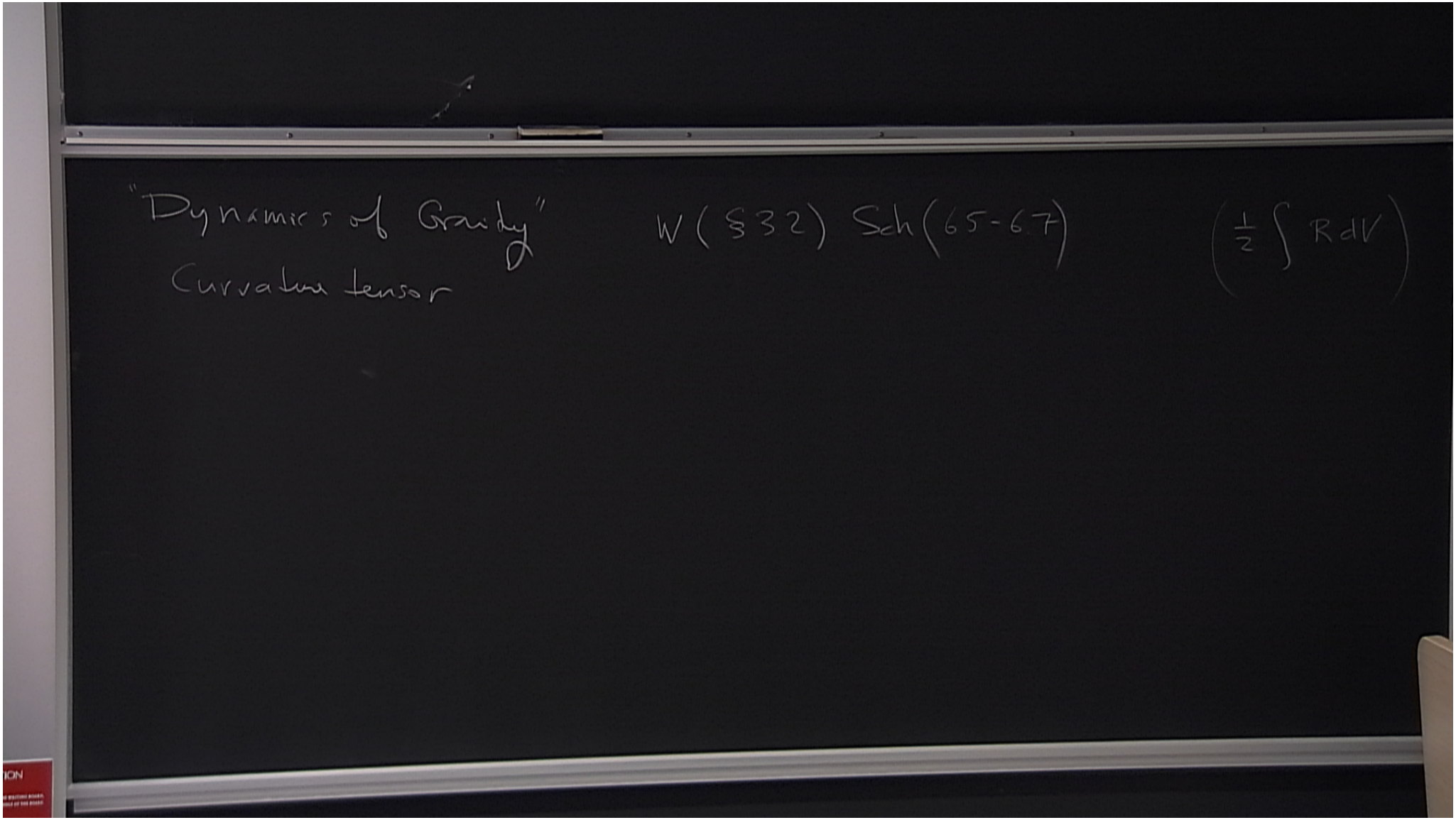
$= \frac{\pi}{2} + a$

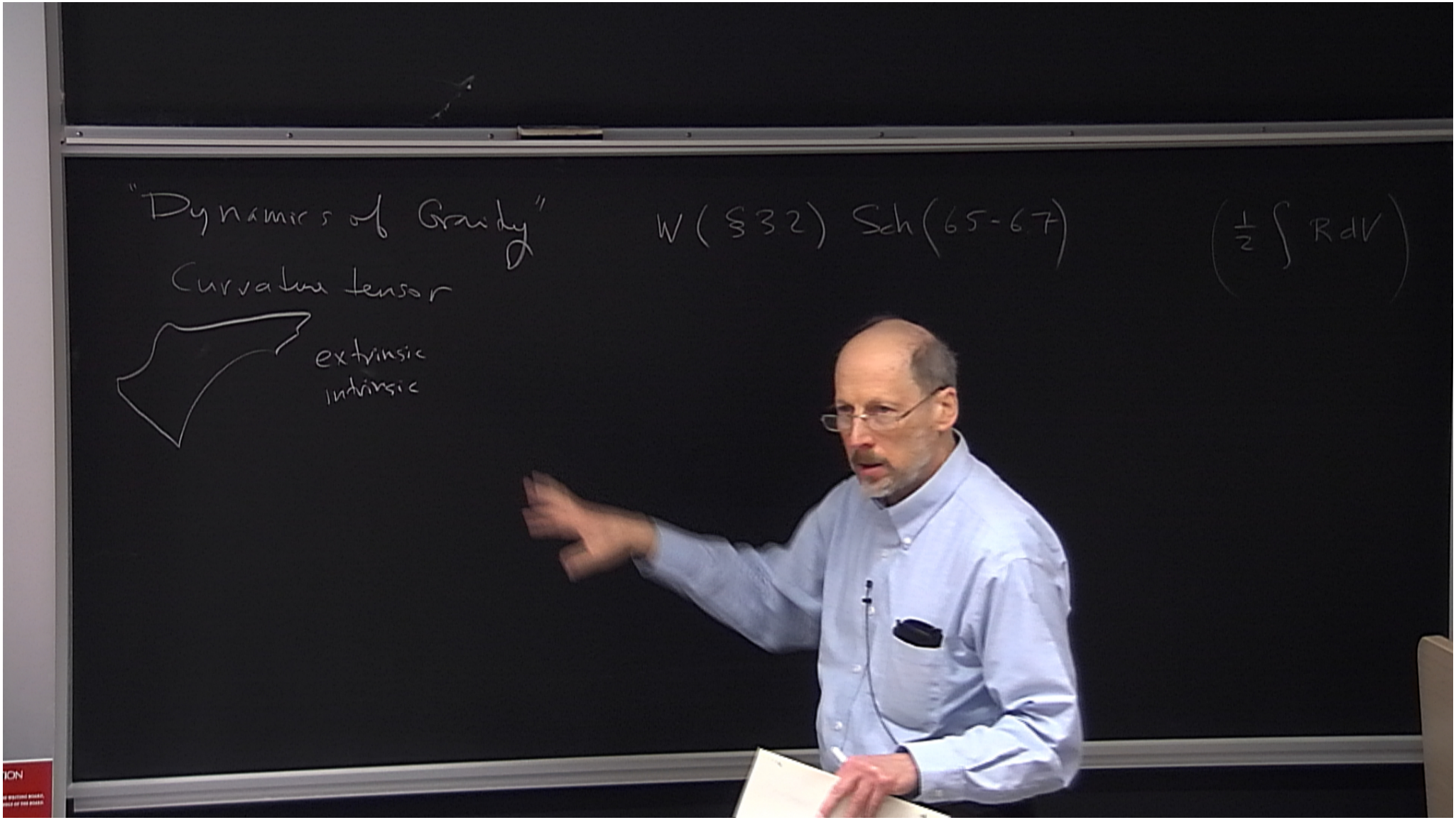
angle $= \pi + 2a = \pi + \frac{2R_0}{b}$ deflection

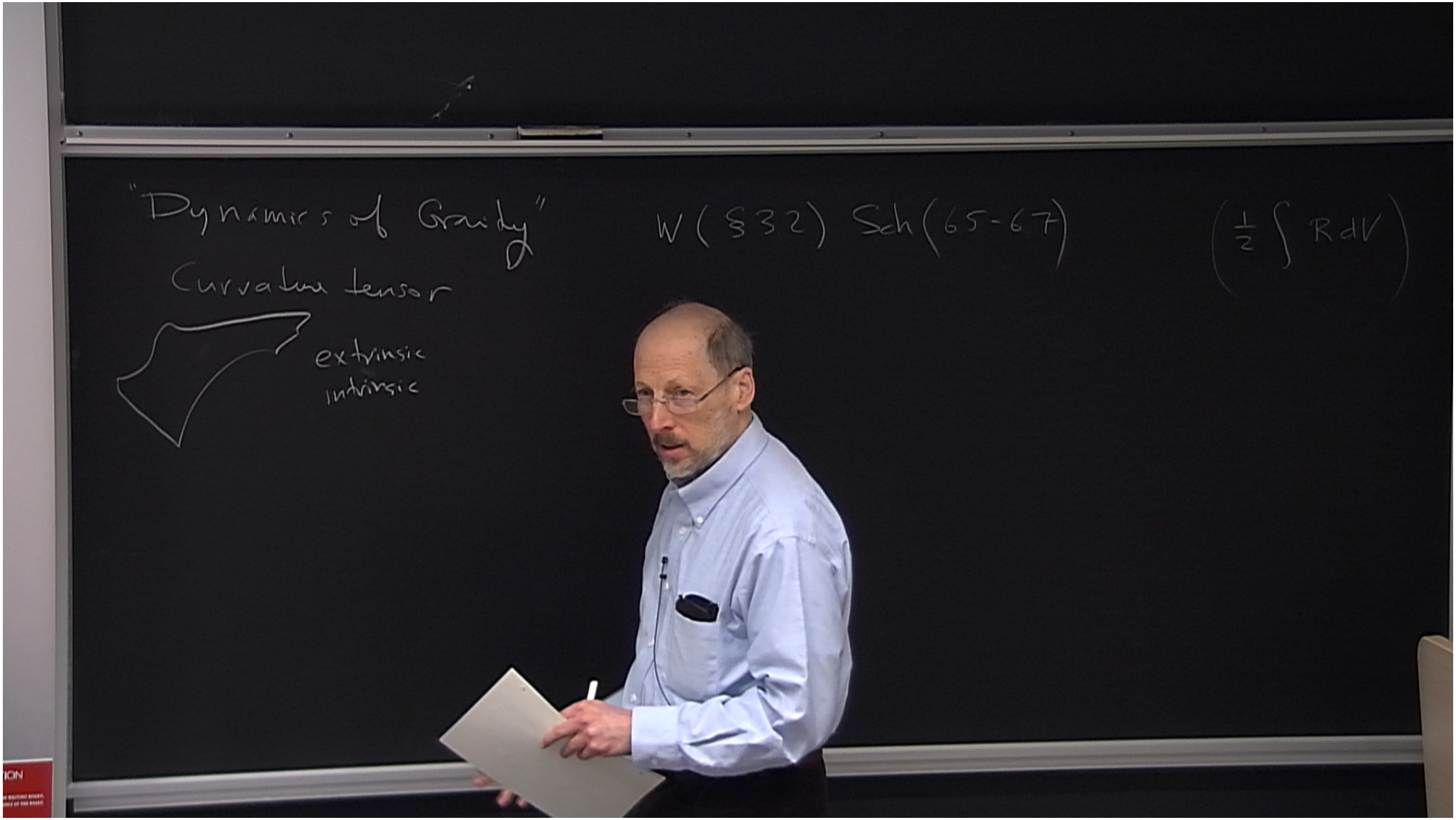
$a = \frac{R_0}{b}$

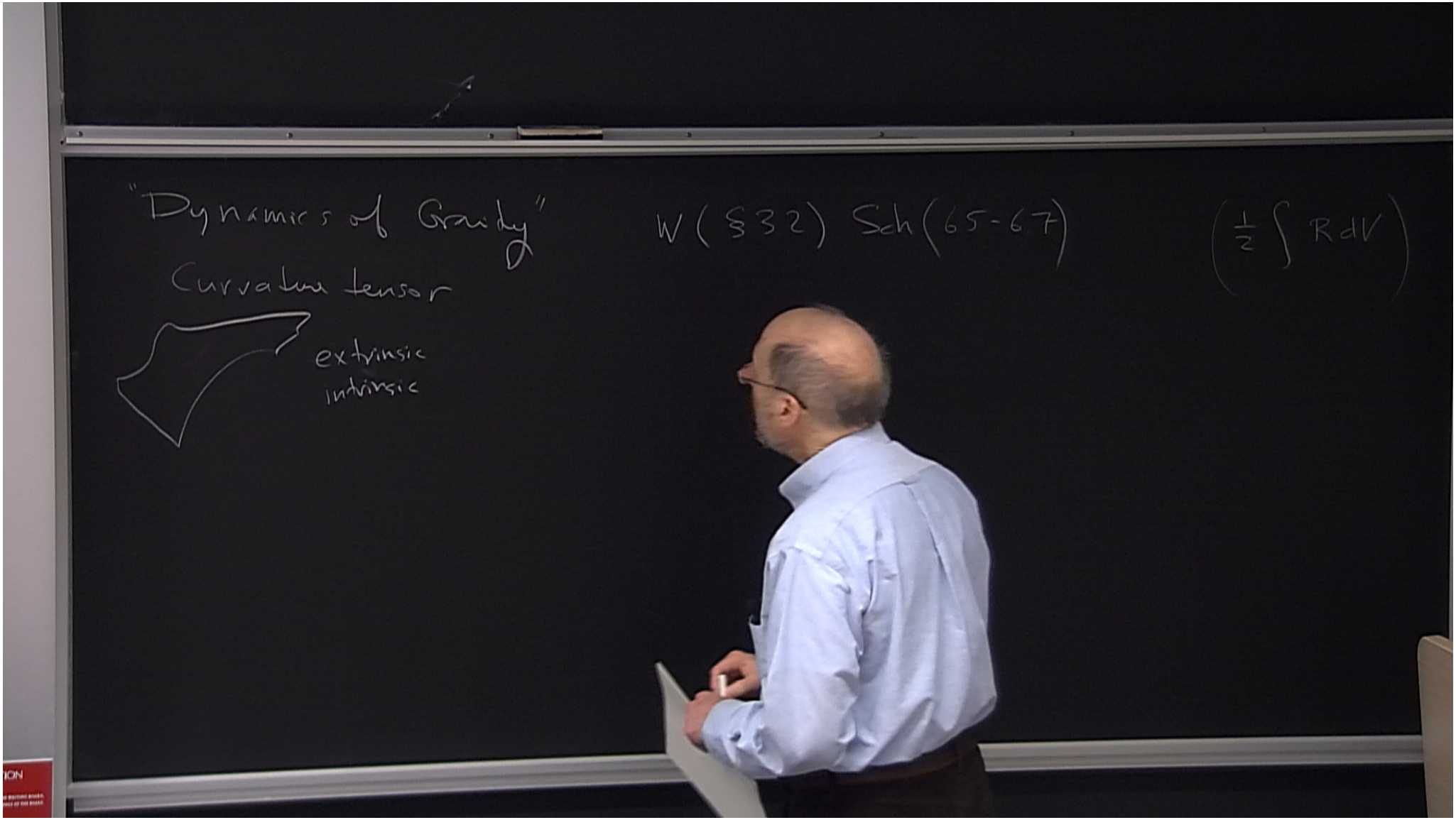
$a^2 - y^2 = \left(\frac{R_0}{b}\right)^2 - u^2 + u^3$

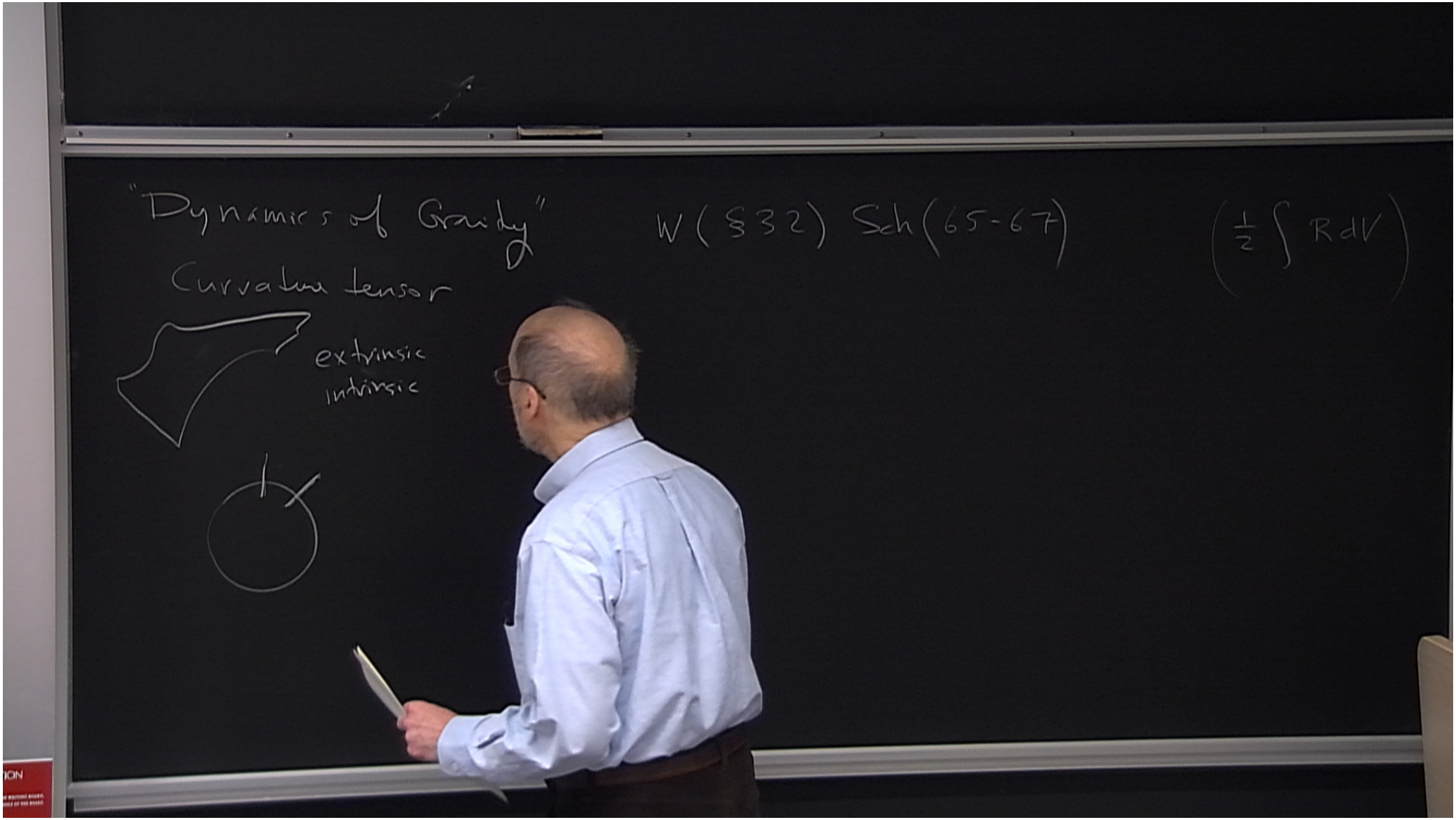

"Dynamics of Gravity"
W (§32) Sch (65-67)
$\frac{1}{2}\int R\,dV$

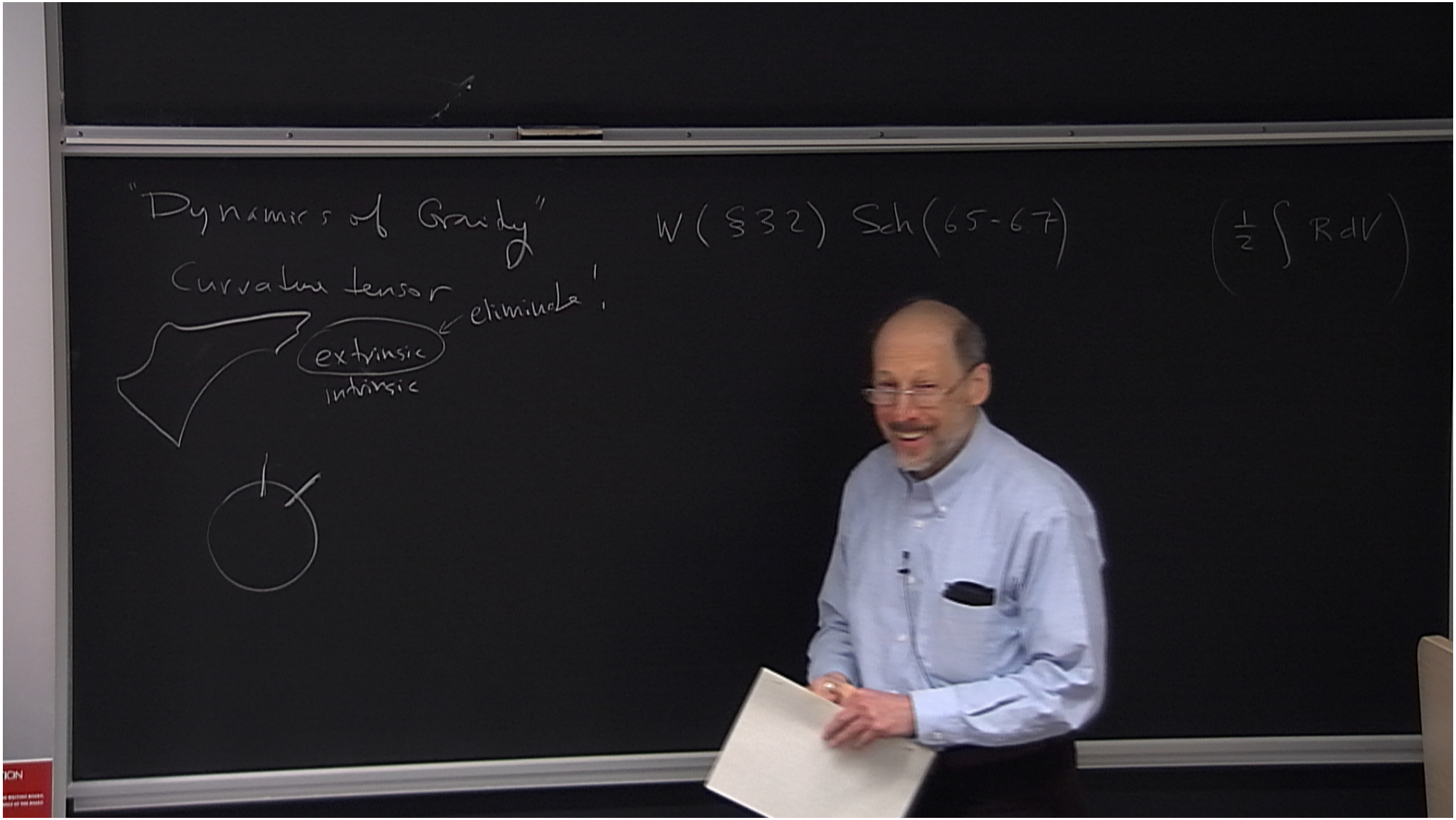

"Dynamics of Gravity"
W (§32) Sch (65-67)
$\left(\frac{1}{2}\int R\,dV\right)$
Curvature tensor

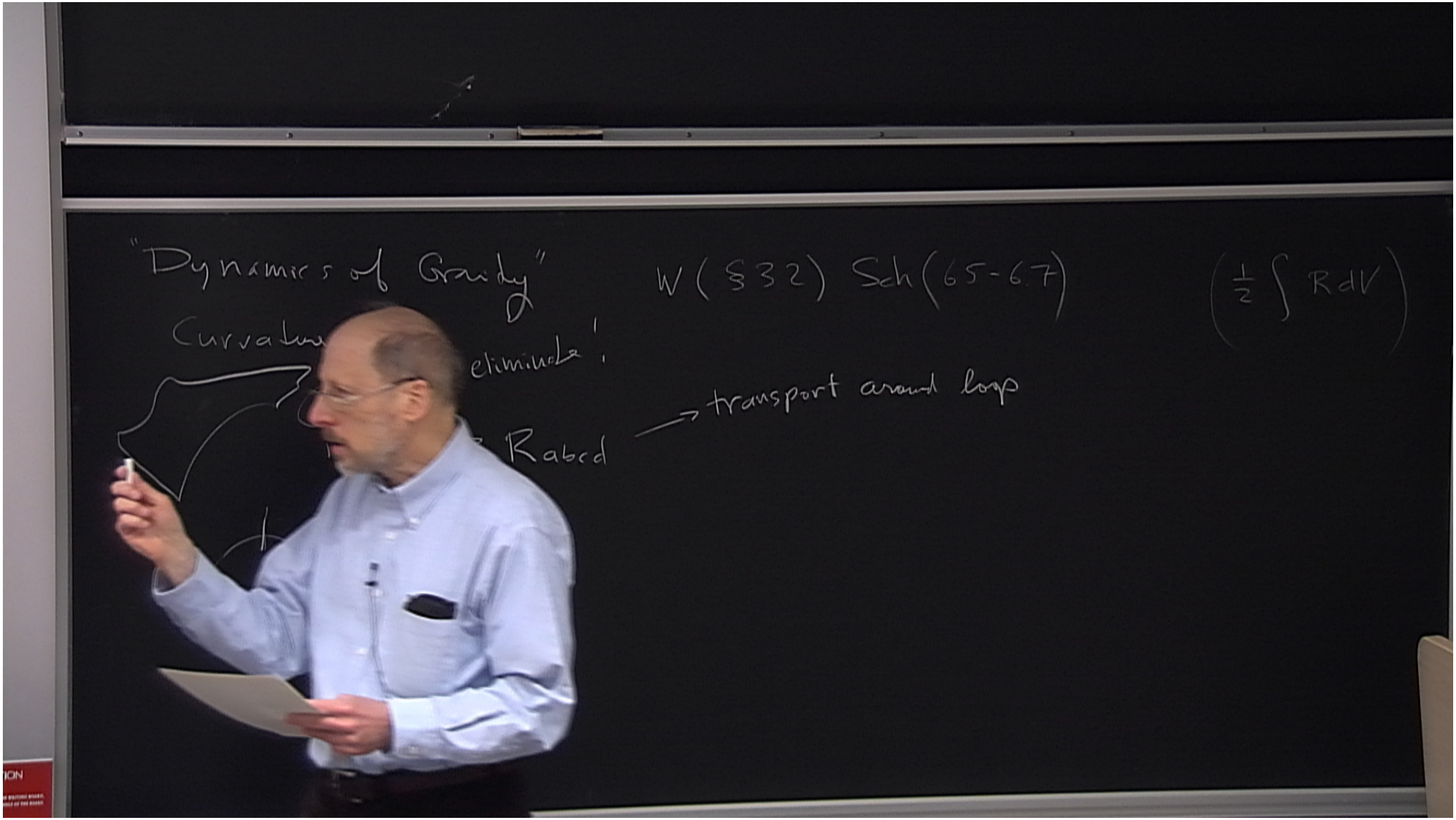

"Dynamics of Gravity"
W(§32) Sch(65-67)
$\frac{1}{2}\int R\,dV$
Curvature tensor
extrinsic
intrinsic

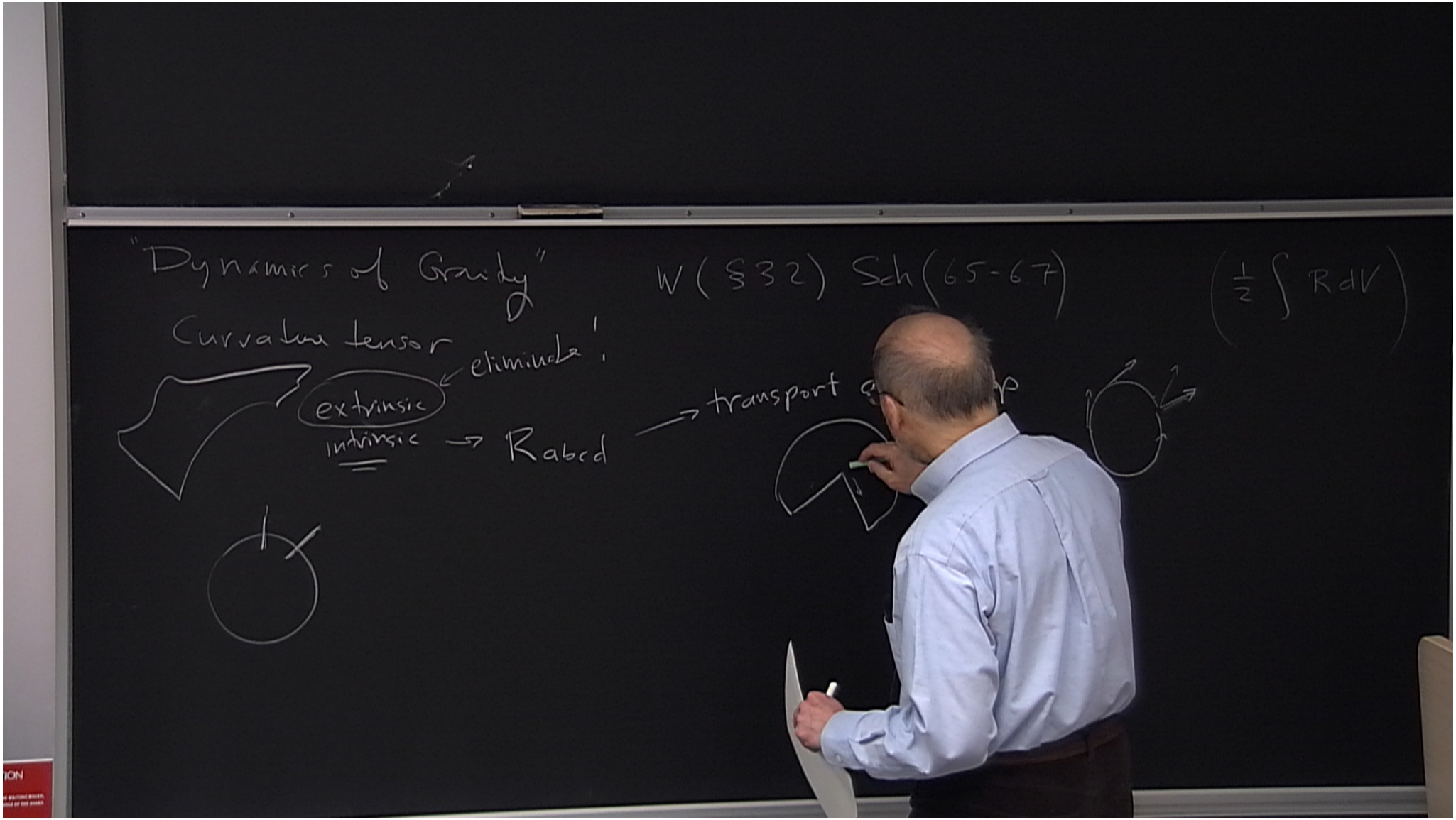

"Dynamics of Gravity"
W(§32) Sch(65-67)
$\left(\frac{1}{2}\int R\,dV\right)$
Curvature tensor
extrinsic
intrinsic

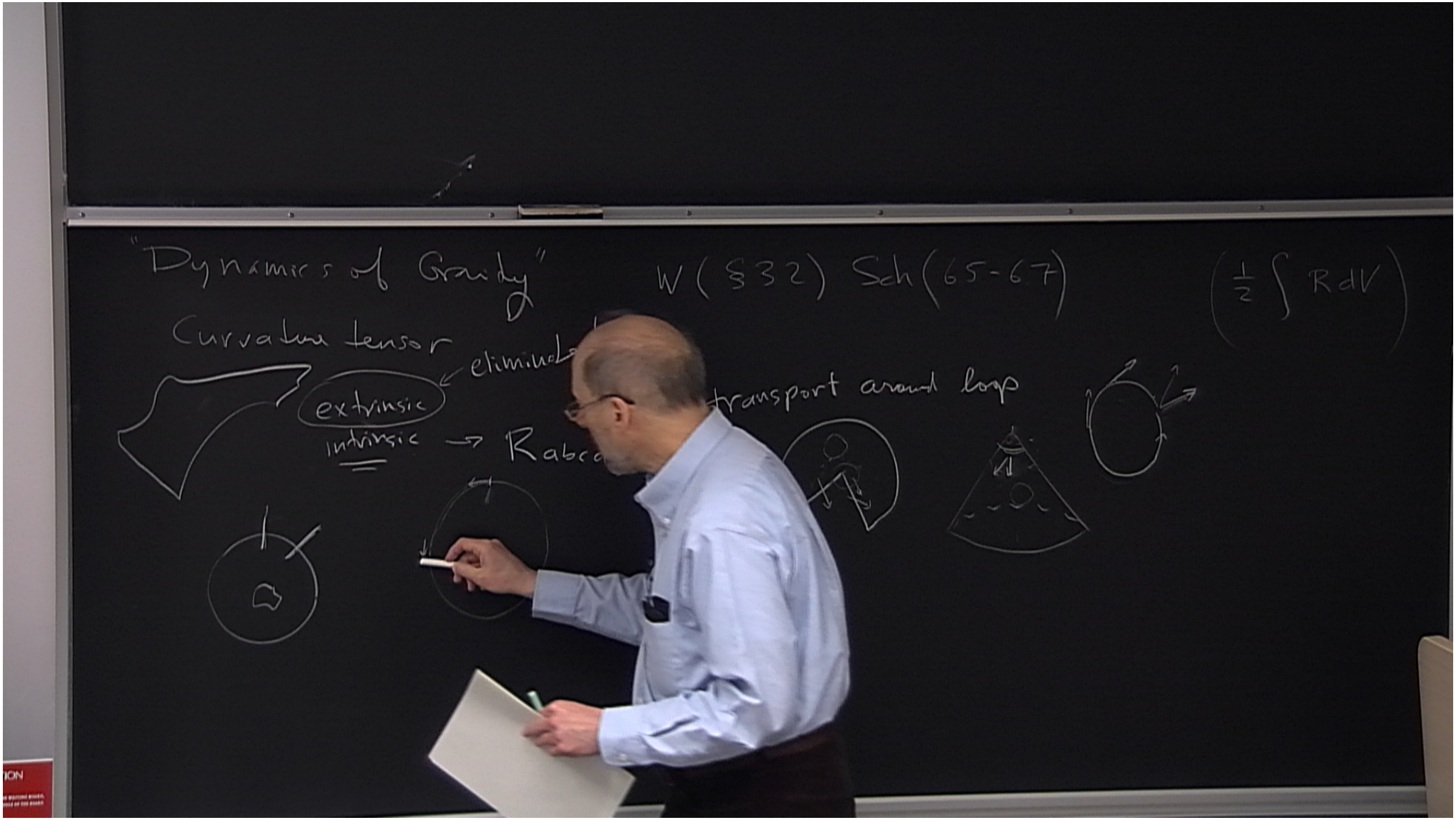

"Dynamics of Gravity"
W(§32) Sch(65-67)
$\left(\frac{1}{2}\int R\,dV\right)$
Curvature tensor
extrinsic
intrinsic

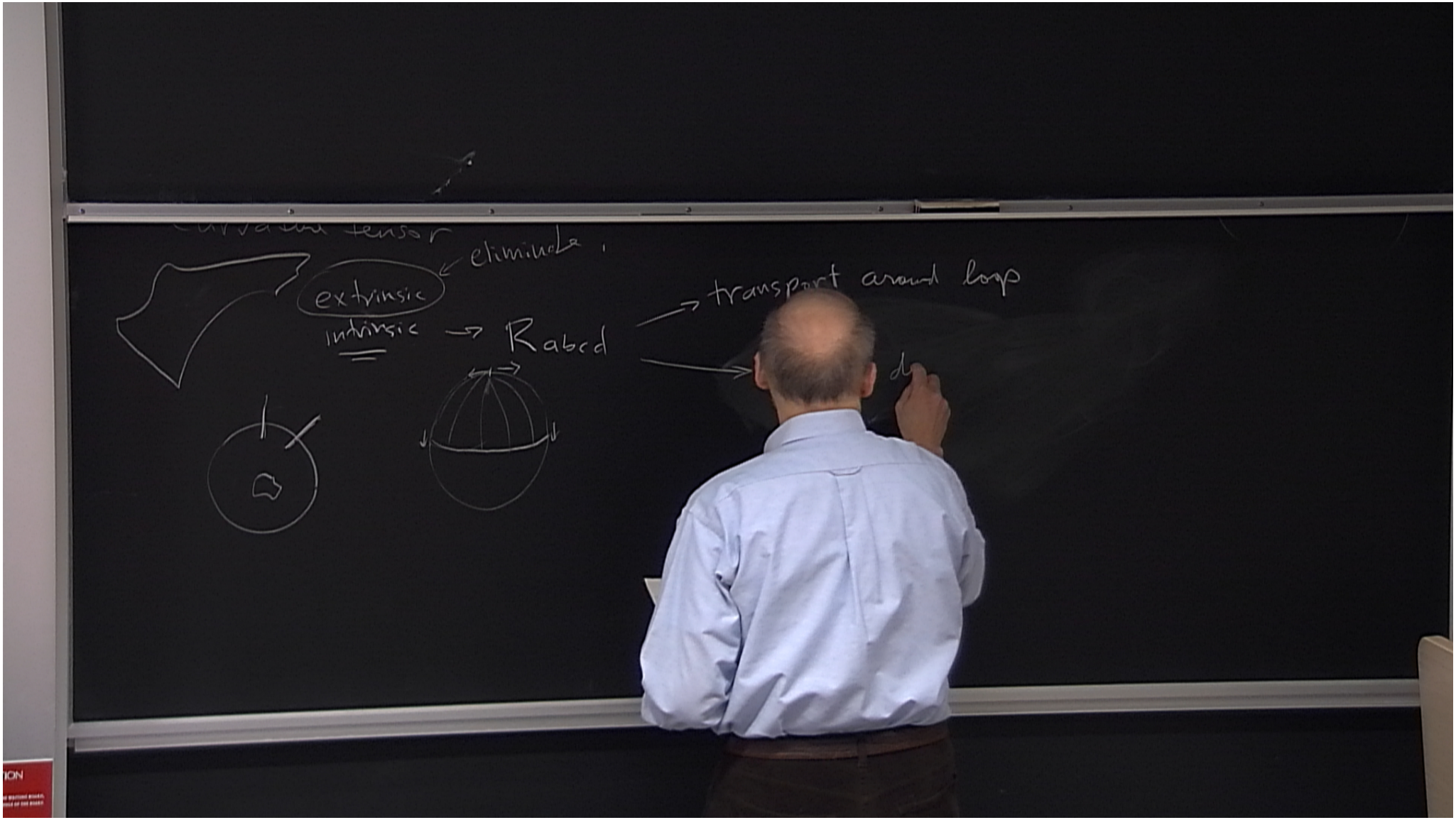

"Dynamics of Gravity"
W (§32) Sch (65-67)
$\left(\frac{1}{2}\int R\,dV\right)$
Curvature tensor
extrinsic
intrinsic

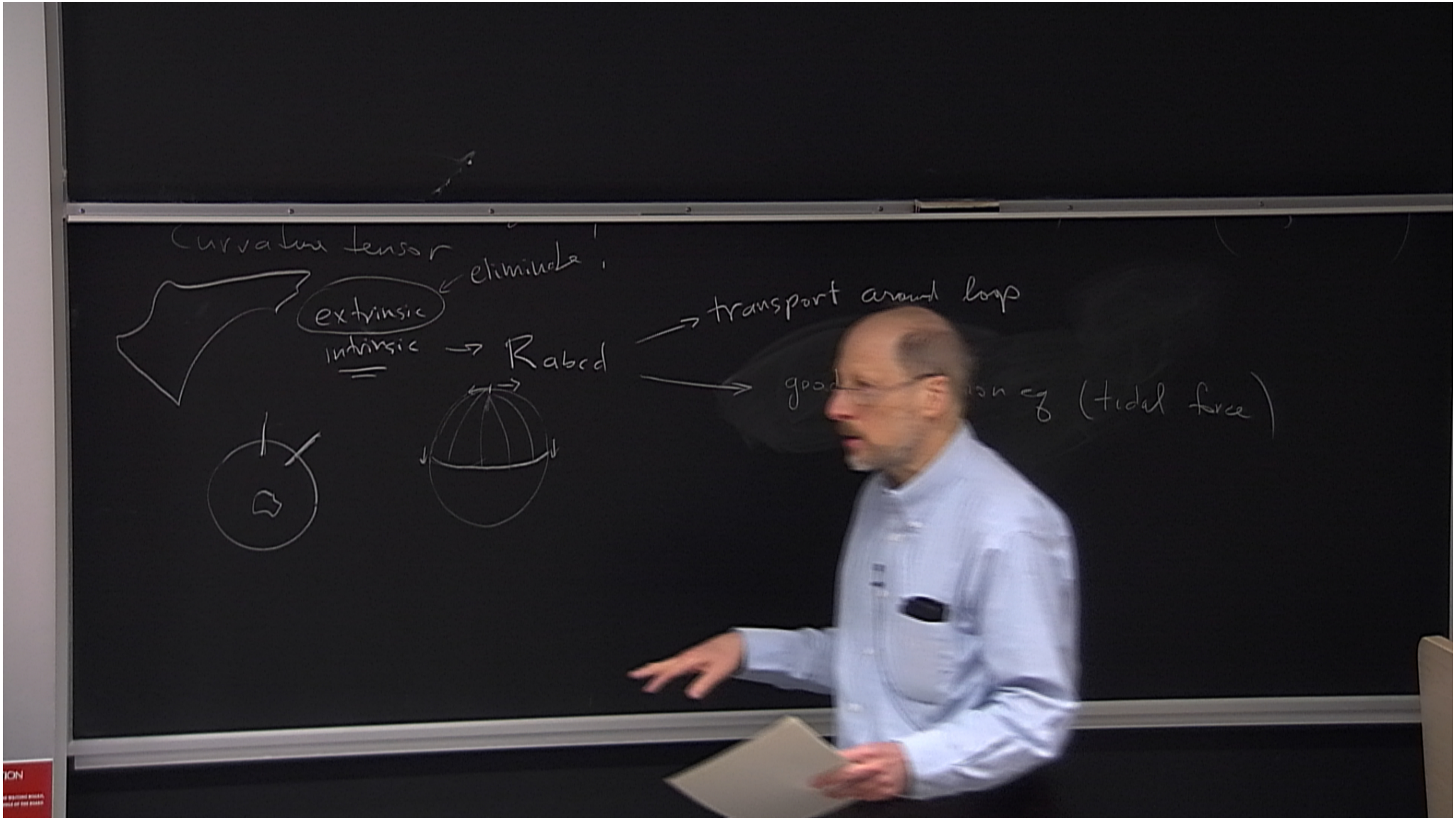

"Dynamics of Gravity"
W (§32) Sch (65-67)
$\frac{1}{2}\int R\,dV$
Curvature tensor
eliminate!
extrinsic
intrinsic

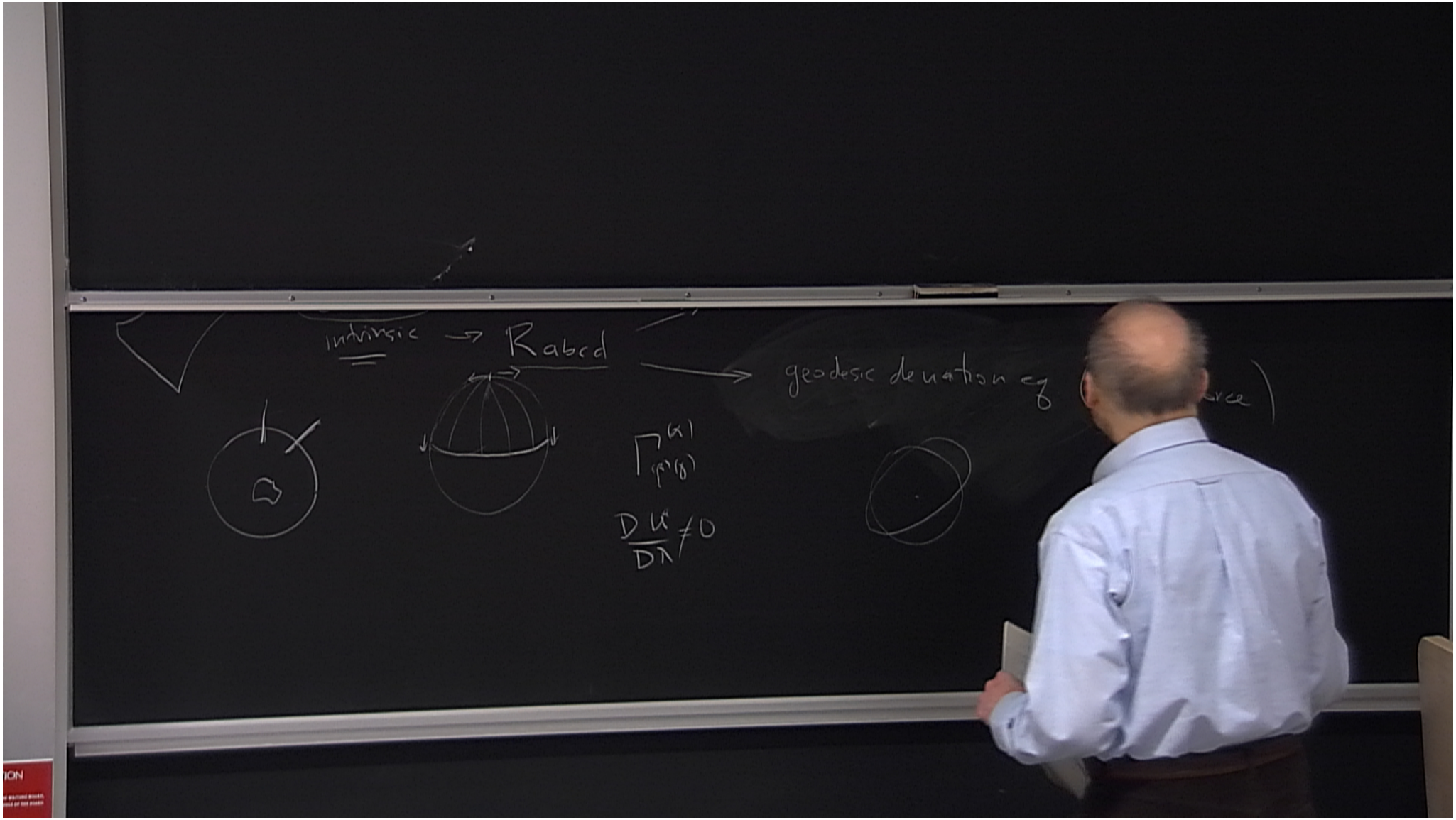

"Dynamics of Gravity"
W (§32) Sch (65-67)
$\left(\frac{1}{2}\int R\,dV\right)$
eliminate!
$R_{abcd}$
→ transport around loop

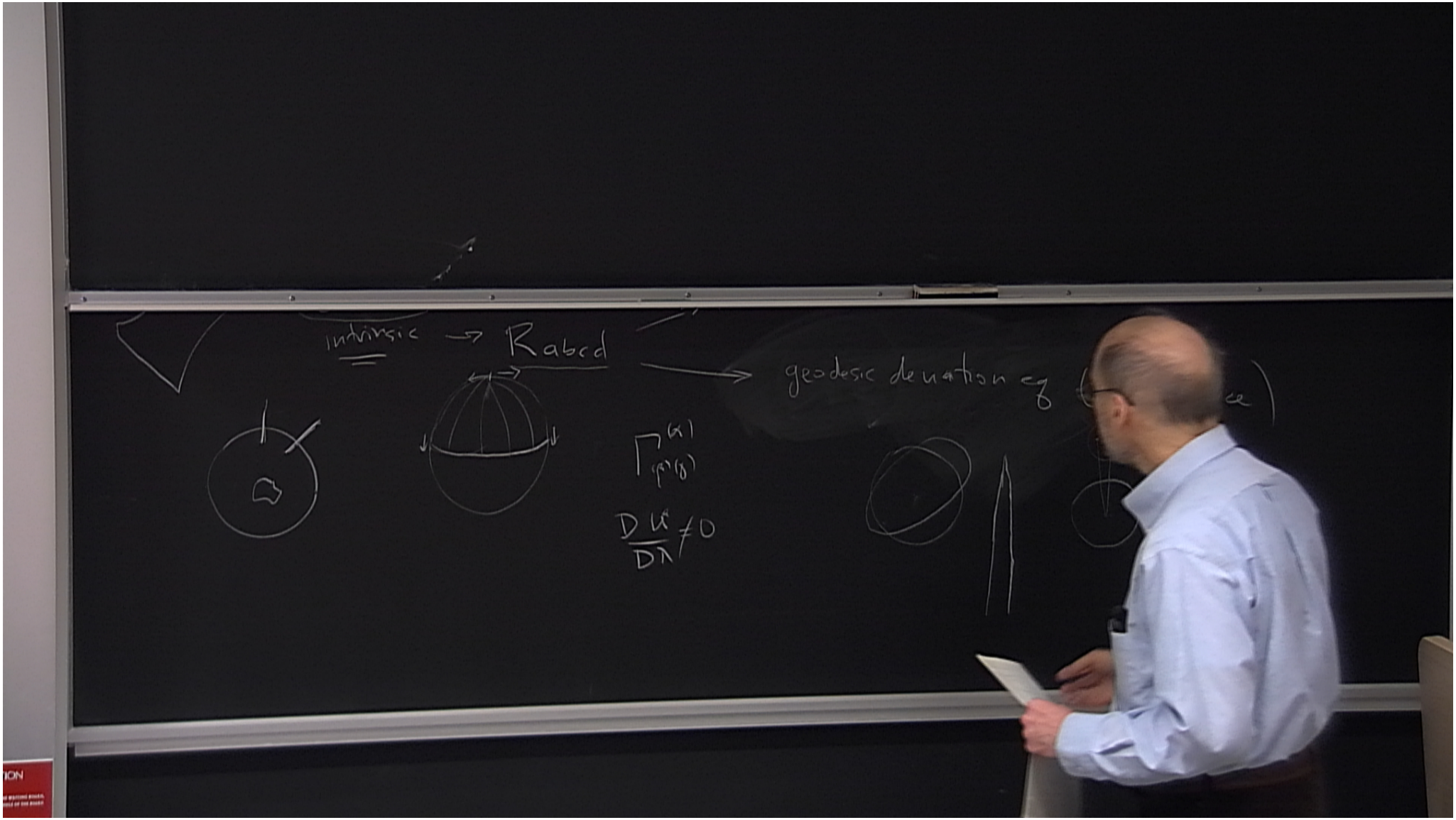

"Dynamics of Gravity"
W (§32) Sch (65-67)
$\frac{1}{2}\int R\,dV$
Curvature tensor
eliminate!
extrinsic
intrinsic → Rabcd
transport

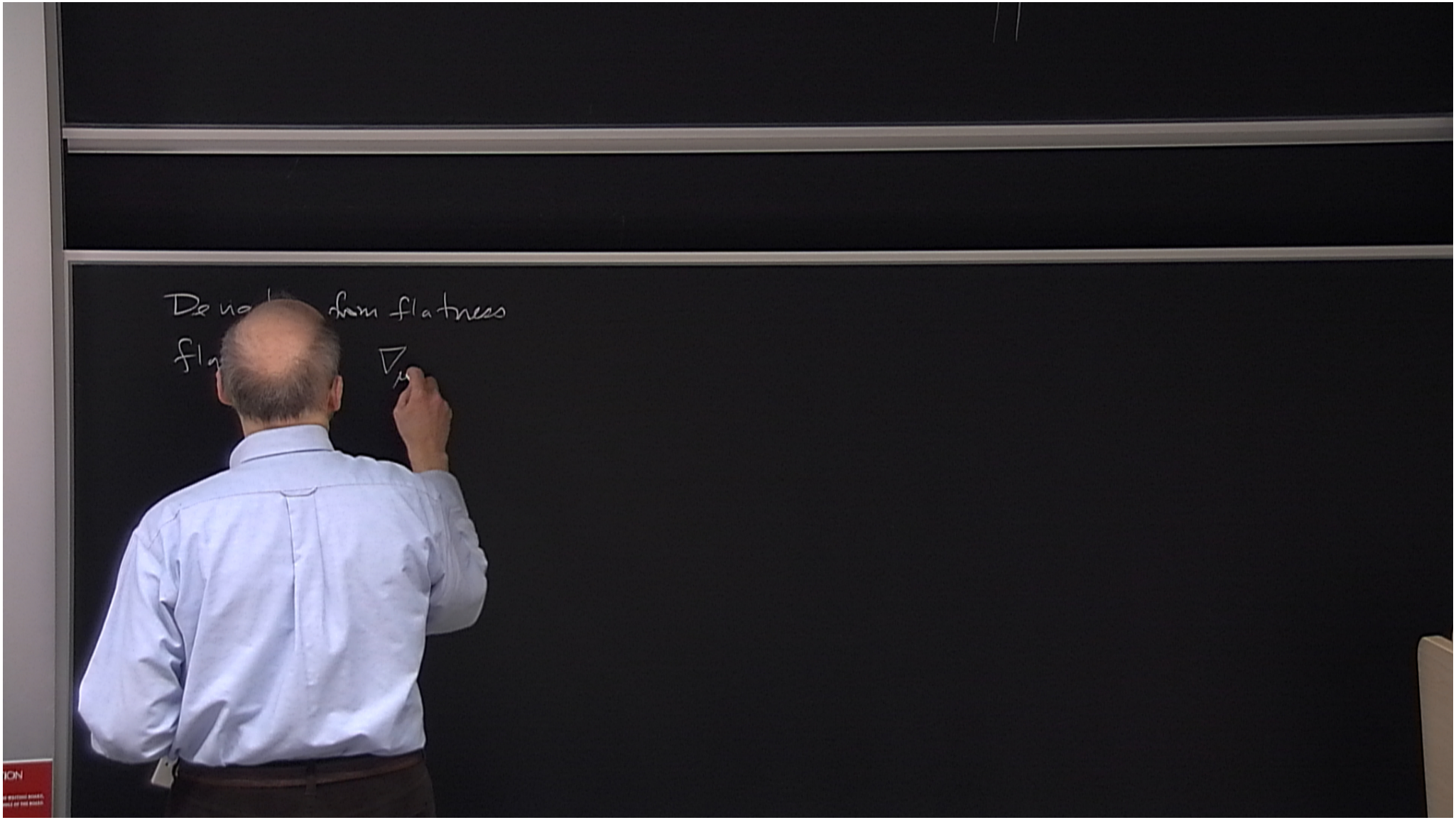

"Dynamics of Gravity"
W(§32) Sch(65-67)
$\frac{1}{2}\int R\,dV$
Curvature tensor
extrinsic
intrinsic
transport around loop

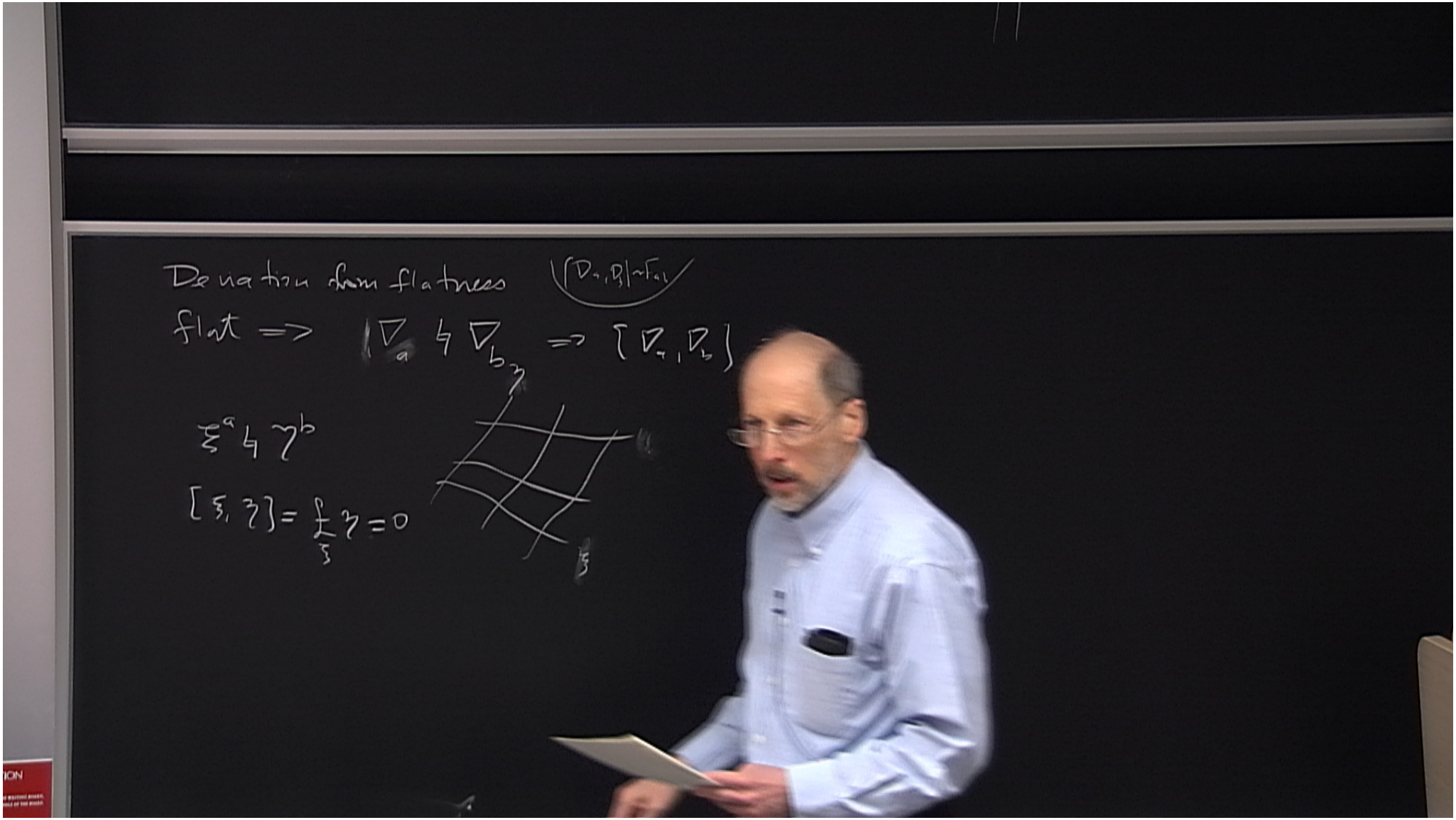

curvature tensor
eliminate
extrinsic
intrinsic
Rabcd
transport around loop

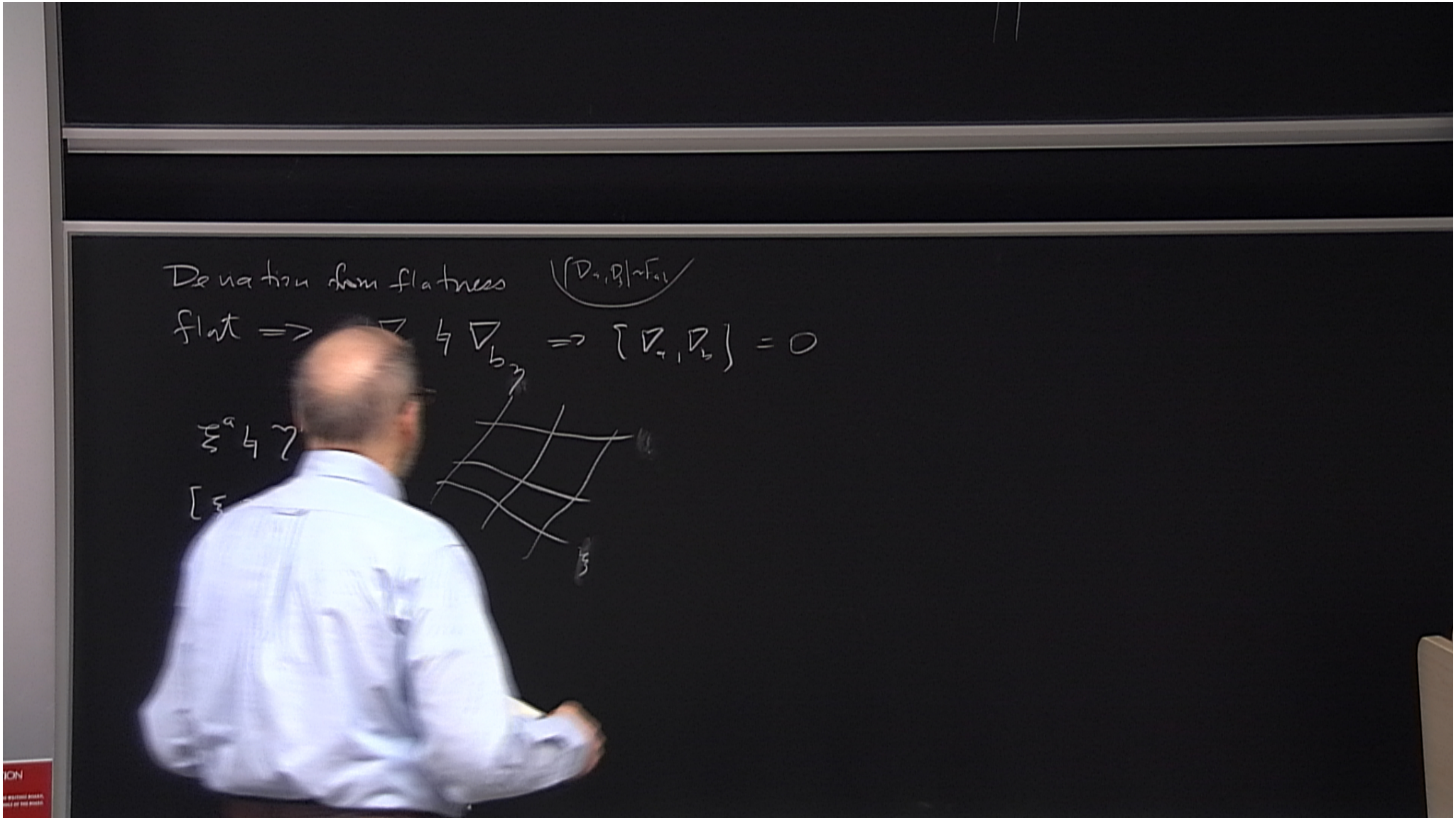

Curvature tensor
eliminate
extrinsic
intrinsic
Rabcd
transport around loop
(tidal force)

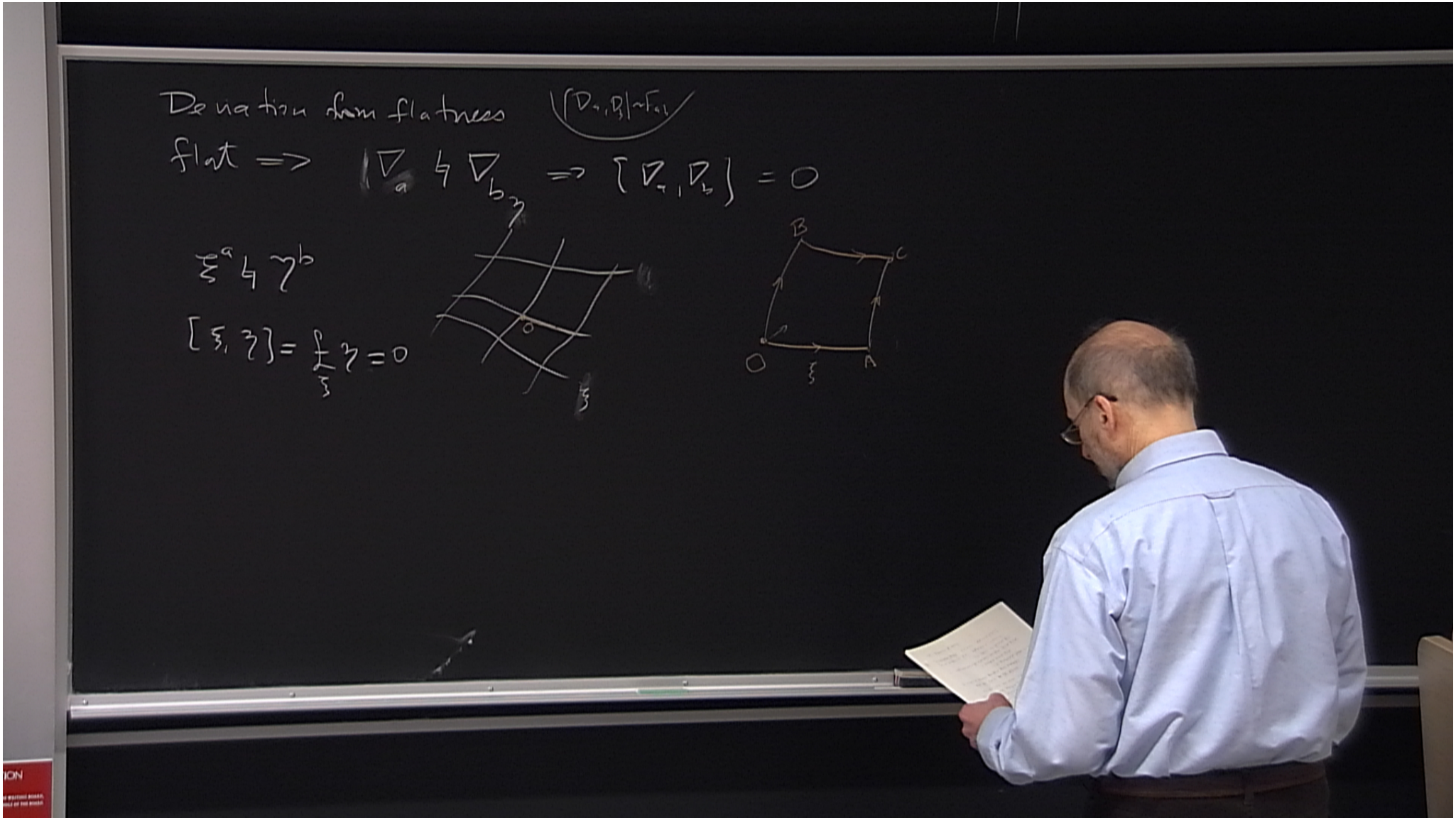

intrinsic → Rabcd
geodesic deviation eq
$\Gamma^{(x)}_{(p)(q)}$
$\frac{D u^a}{D\lambda} \neq 0$

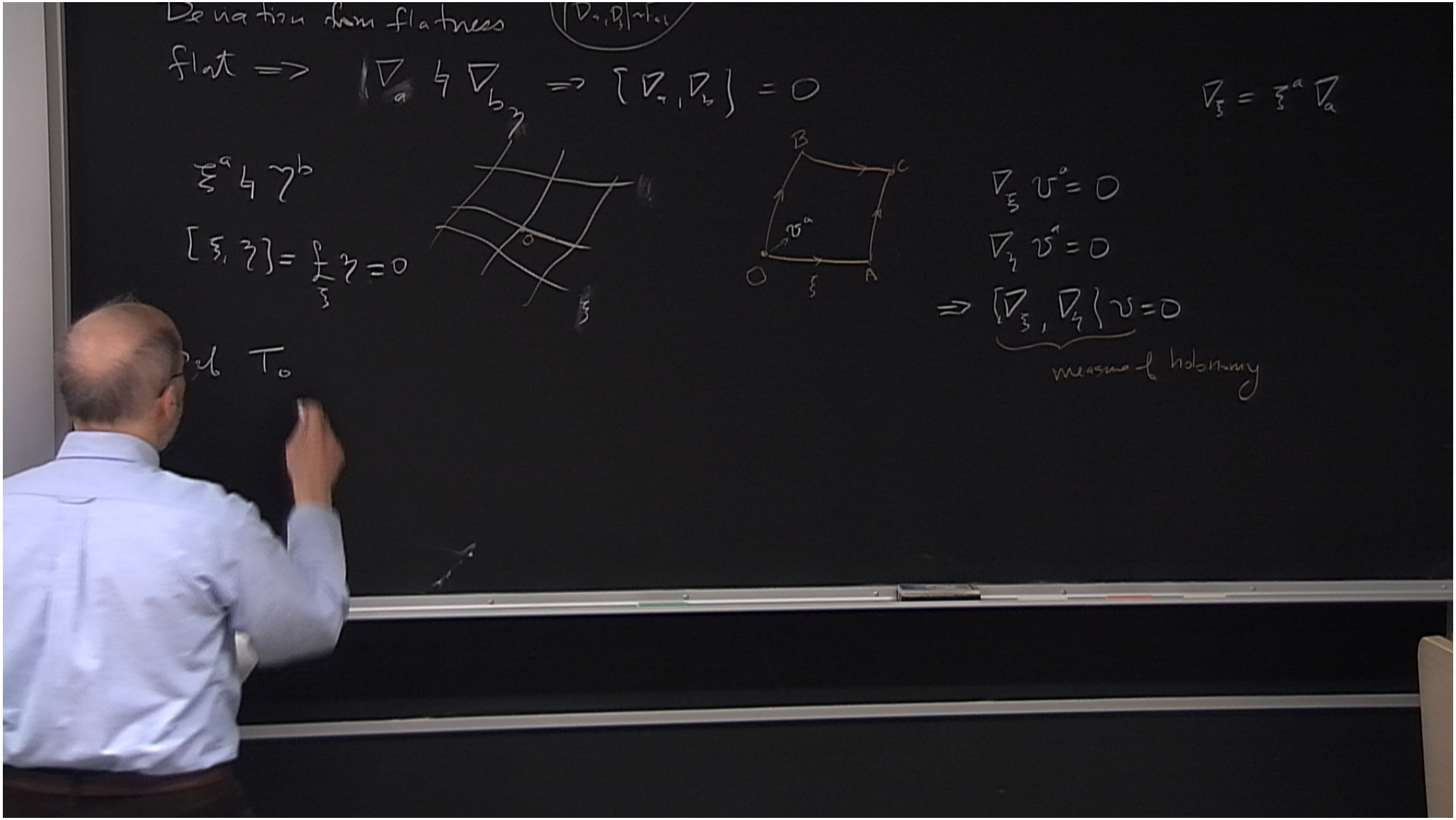

intrinsic → Rabcd
geodesic deviation eq
$\Gamma^{(\alpha)}_{(\beta\gamma)}$
$\frac{D u^a}{D\lambda} \neq 0$

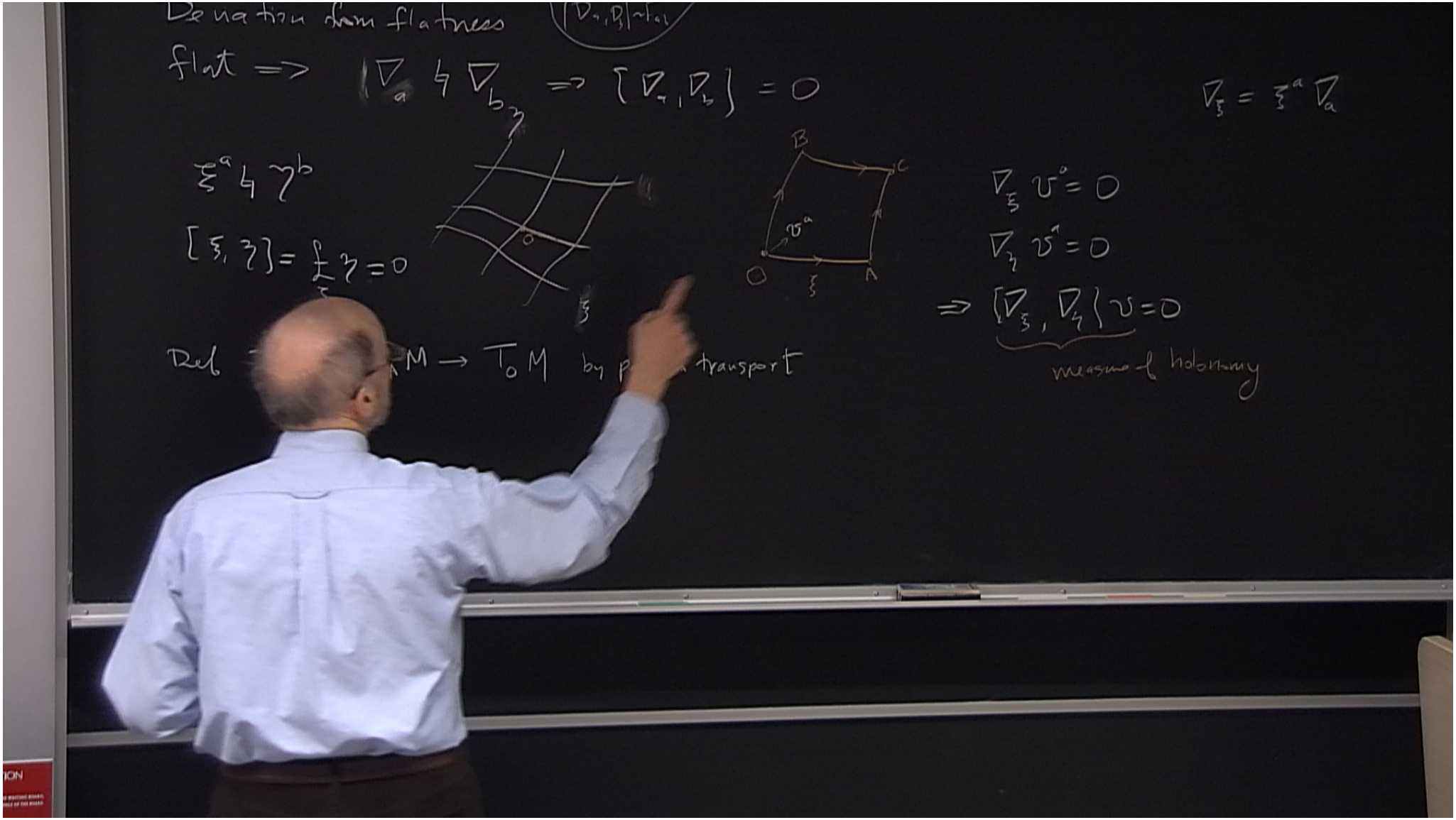

flatness

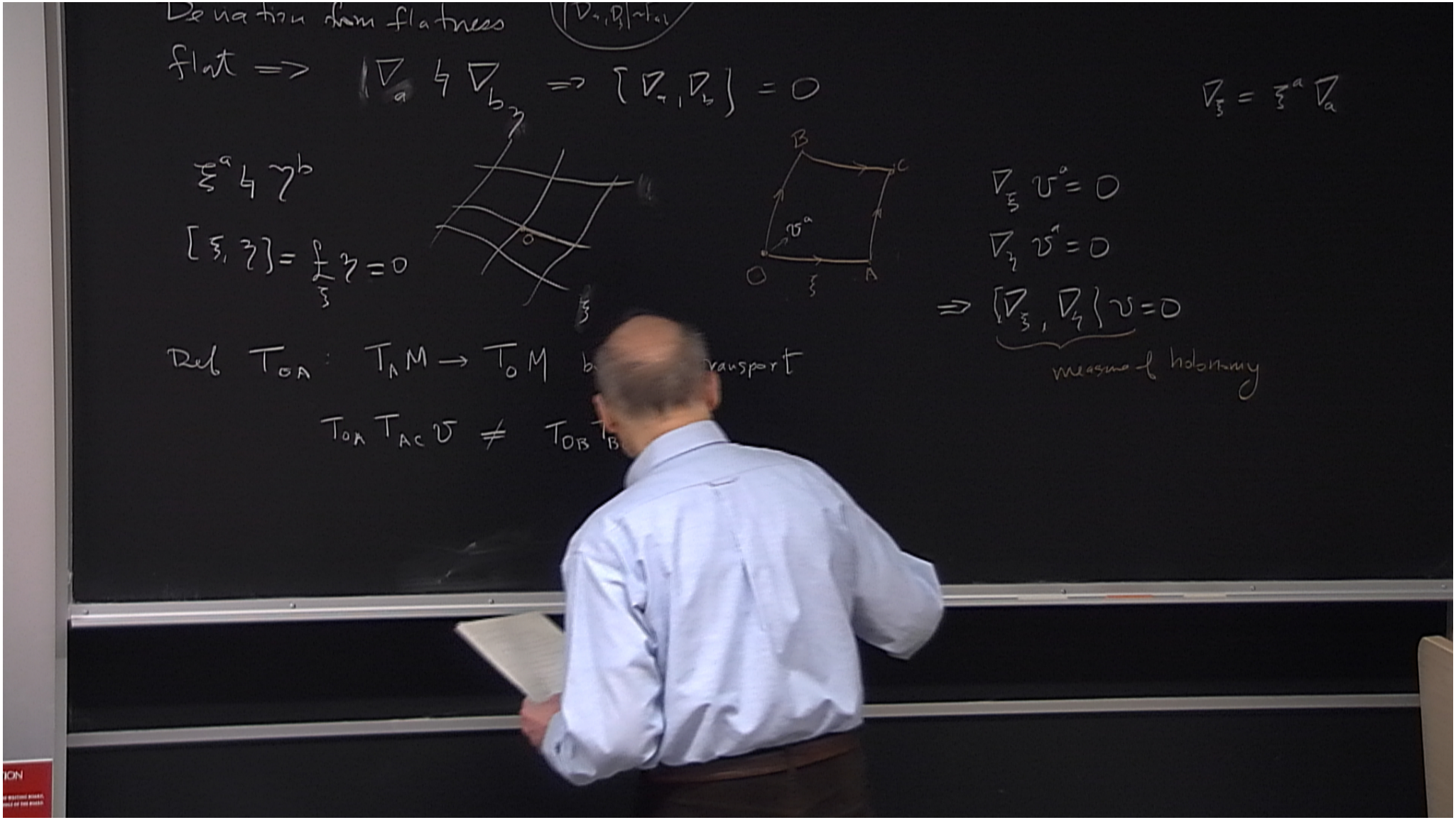

Deviation from flatness
$[D_a, D_b]\sim F_{ab}$
flat $\Rightarrow$ $\nabla_a \nleftrightarrow \nabla_b$ $\Rightarrow$ $[\nabla_a, \nabla_b]$
$\xi^a \nleftrightarrow \eta^b$
$[\xi, \eta] = \pounds_\xi \eta = 0$

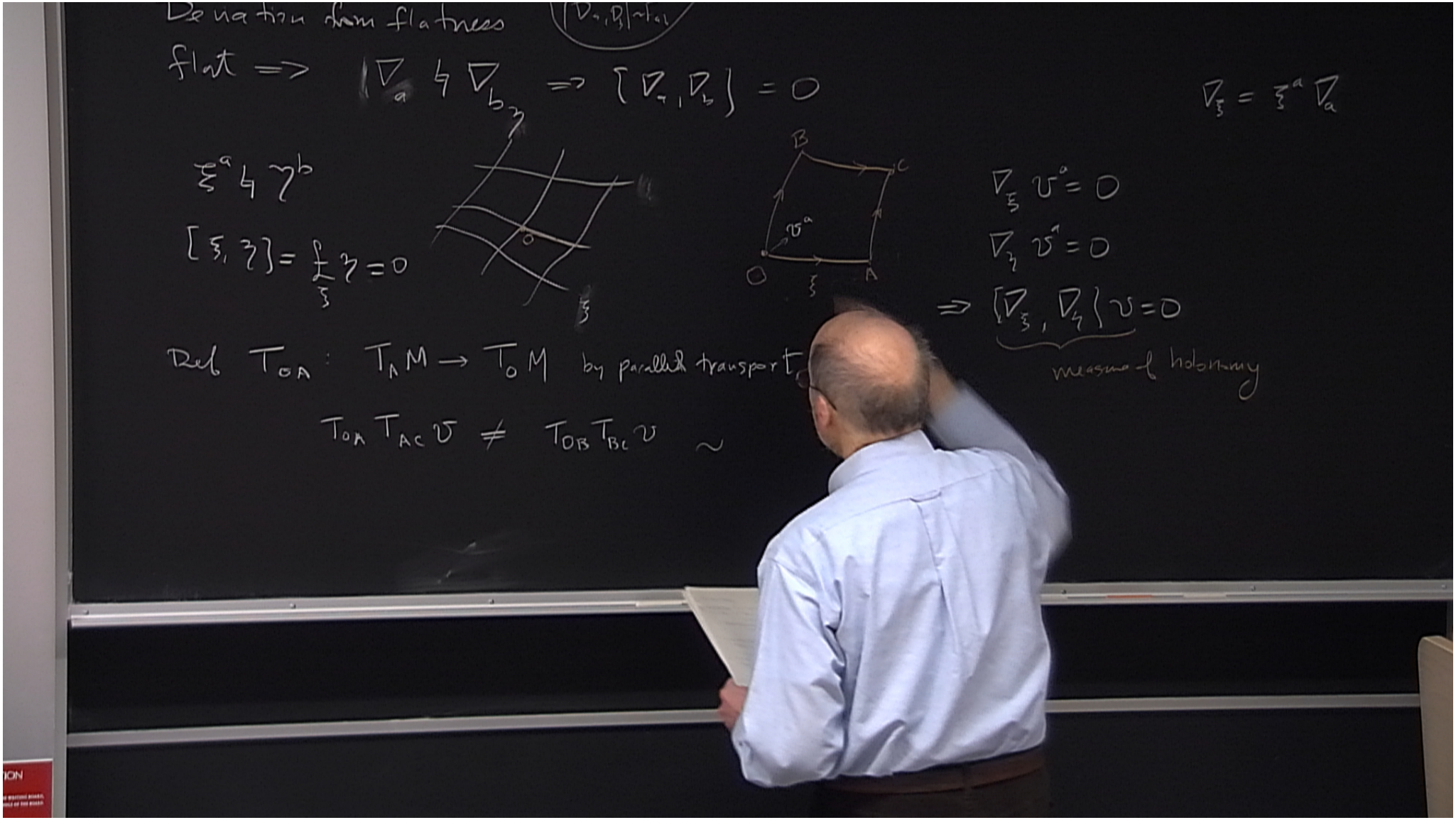

Deviation from flatness
$[\nabla_a, \nabla_b] \sim F_{ab}$
flat =>
$\Rightarrow [\nabla_a, \nabla_b] = 0$

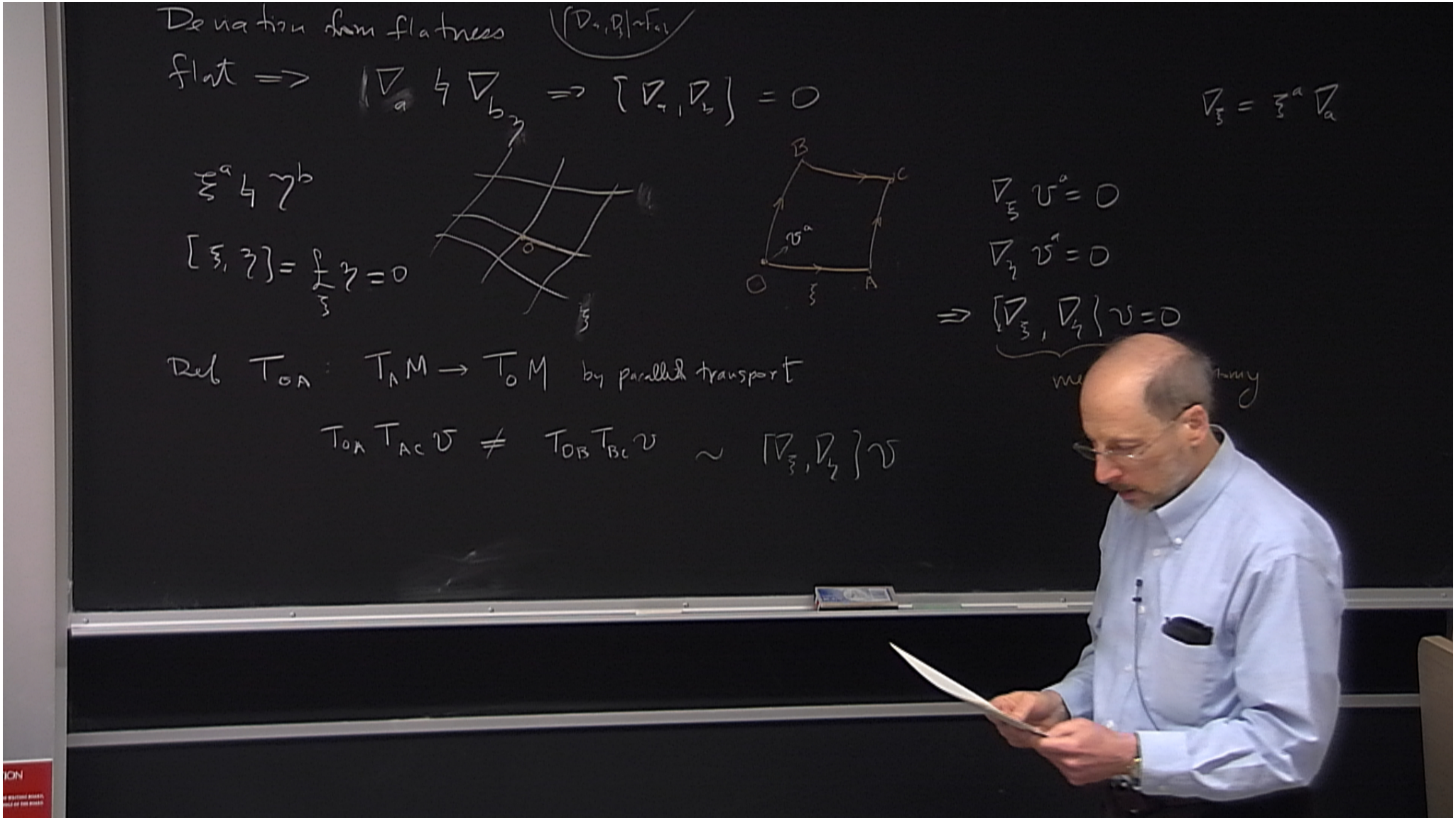

Deviation from flatness
flat =>
=>
B
C
O
A

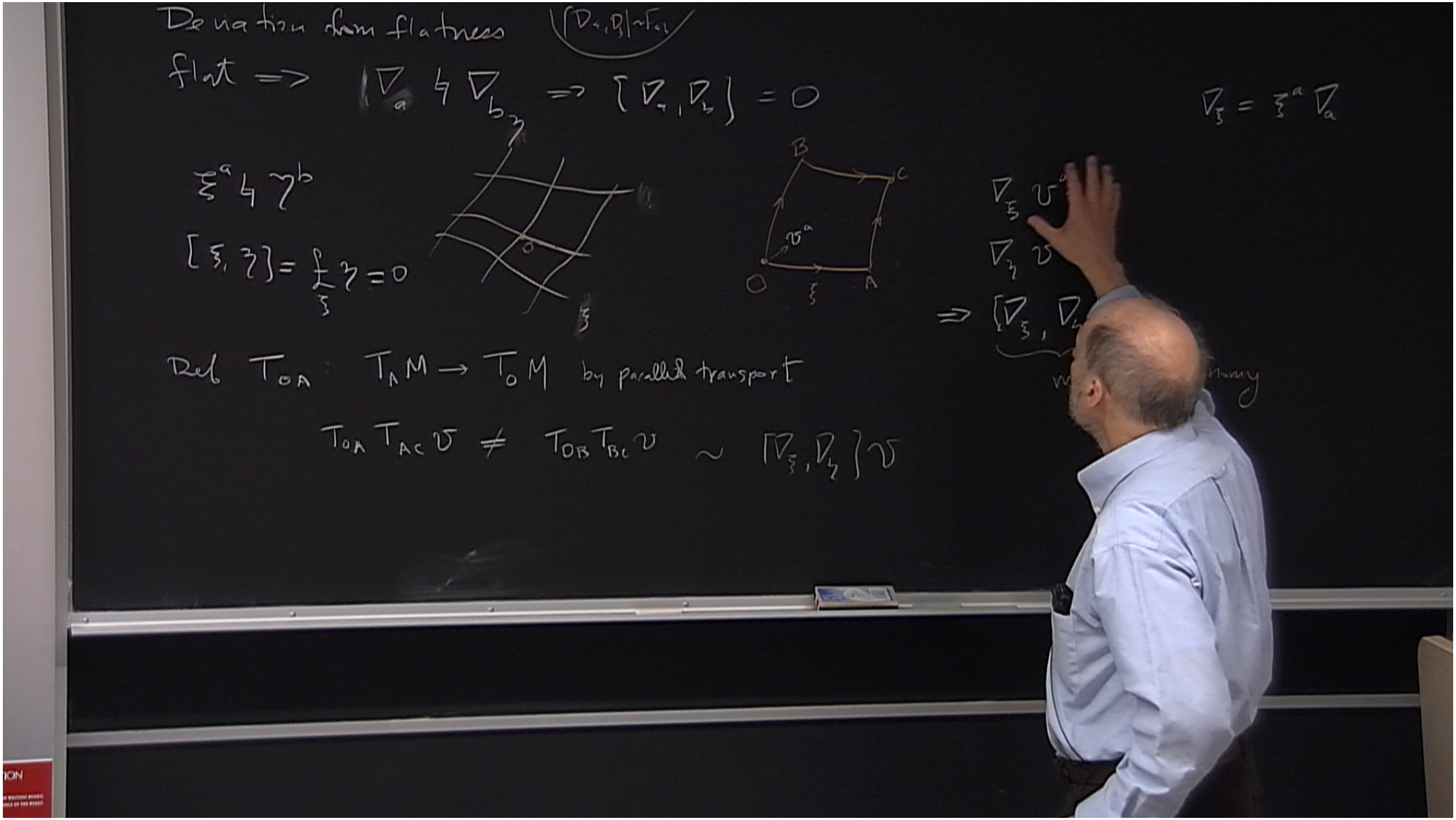

Deviation from flatness
flat =>
$\nabla_a \nabla_b = \nabla_b \nabla_a \Rightarrow [\nabla_a, \nabla_b] = 0$
$\nabla_\xi = \xi^a \nabla_a$
$\xi^a \nabla_a \gamma^b$
$[\xi, \gamma] = \mathcal{L}_\xi \gamma = 0$
$\nabla_\xi v^a = 0$
$\nabla_\gamma v^a = 0$
$\Rightarrow [\nabla_\xi, \nabla_\gamma] v = 0$
measure of holonomy
B
C
O
A
$v^a$
$\xi$
$\gamma$


Deviation from flatness
flat ⇒ ∇a ↯ ∇b ⇒ [∇a, ∇b] = 0
∇ξ = ξa ∇a
ξa ↯ ηb
[ξ, η] = £ η = 0
B
C
O
A
ξ
∇ξ v^a = 0
∇η v^a = 0
⇒ [∇ξ, ∇η] v = 0
measure of holonomy
Def
M → T0 M by p
transport


Deviation from flatness
flat ⇒ ∇a ↯ ∇b ⇒ [∇a, ∇b] = 0
∇ξ = ξa ∇a
ξa ↯ ηb
[ξ, η] = £ξ η = 0
B
C
O
A
ξ
∇ξ va = 0
∇η va = 0
⇒ [∇ξ, ∇η] v = 0
measure of holonomy
Def TOA : TAM → TOM
TOA TAC v ≠ TOB


Deviation from flatness
flat ⇒ ∇a ↯ ∇b ⇒ [∇a, ∇b] = 0
∇ξ = ξa ∇a
ξa ↯ ηb
[ξ, η] = £ξ η = 0
B
C
O
A
ξ
va
∇ξ va = 0
∇η va = 0
⇒ [∇ξ, ∇η] v = 0
measure of holonomy
Def TOA : TAM → TOM by parallel transport
TOA TAC v ≠ TOB TBC v ~


Deviation from flatness
$[\nabla_a, \nabla_b] = F_{ab}$
flat $\Rightarrow$ $\nabla_a \nmid \nabla_b \Rightarrow [\nabla_a, \nabla_b] = 0$
$\nabla_\xi = \xi^a \nabla_a$
$\xi^a \nmid \gamma^b$
$[\xi, \gamma] = \mathcal{L}_\xi \gamma = 0$
$\nabla_\xi v^a = 0$
$\nabla_\gamma v^a = 0$
$\Rightarrow [\nabla_\xi, \nabla_\gamma] v = 0$
Def $T_{0A}$ : $T_A M \to T_0 M$ by parallel transport
$T_{0A} T_{AC} v \neq T_{0B} T_{Bc} v \sim [\nabla_\xi, \nabla_\gamma] v$


Deviation from flatness
flat ⇒ ∇a ↯ ∇b ⇒ [∇a, ∇b] = 0
∇ξ = ξ^a ∇a
ξ^a ↯ η^b
[ξ, η] = £ξ η = 0
Def T_OA : T_A M → T_O M by parallel transport
T_OA T_AC v ≠ T_OB T_BC v ~ [∇ξ, ∇η] v