Title: Evidence for a new particle on the worldsheet of the QCD flux tube

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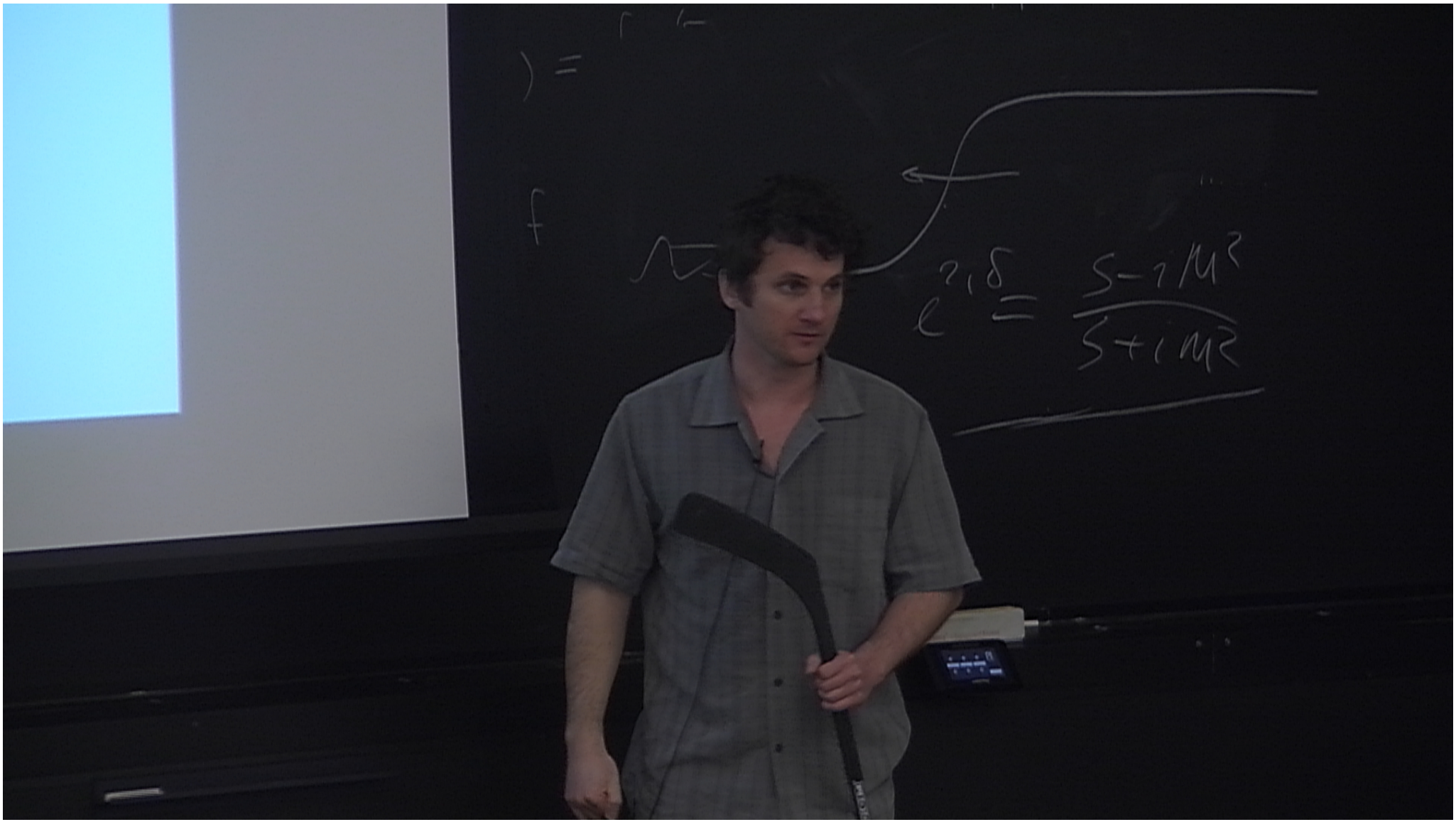
Abstract: We propose a new
approach for the calculation of the spectrum of excitations of QCD flux tubes. It relies on the fact that the worldsheet theory is integrable at low energies. With this approach, energy levels can be calculated for much shorter flux tubes than was previously possible, allowing for a quantitative comparison with existing lattice data. The improved theoretical control makes it manifest that existing lattice data provides strong evidence for a new pseudoscalar particle localized on the QCD fluxtube - the worldsheet axion.



Evidence for a new particle on the worldsheet of the QCD fluxtube

SD, Raphael Flauger, Victor Gorbenko, 1203.1054, 1205.6805, 1301.2325
+more to appear

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Two faces of the story:

- ✷ QCD (4D, pure glue) today
- ✷ Gravity (2D, integrable)

Athenodorou,Bringoltz,Teper '1007.4720

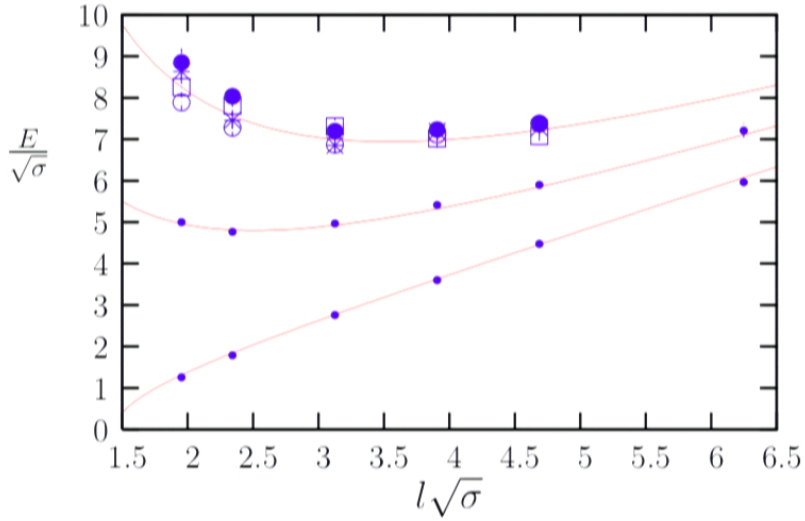
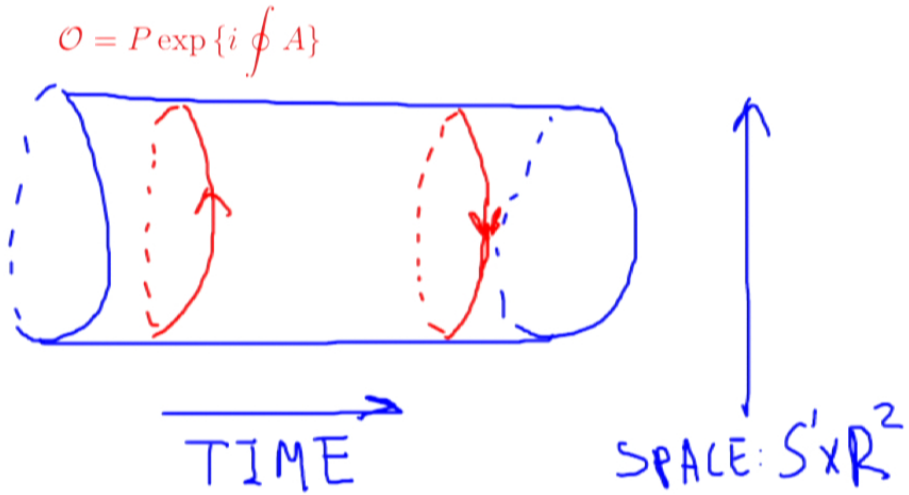


Figure 4: Lightest flux tube energies for longitudinal momenta $q = 0$, •, $q = 1$, •, and $q = 2$ in SU(3) at $\beta = 6.0625$. The four $q = 2$ states are $J^{P_t} = 0^+(\star)$, $1^{\pm}(\circ)$, $2^+(\square)$, $2^-(\bullet)$. Lines are Nambu-Goto predictions.

What is being measured?



$$\int \mathcal{D}A e^{-S_{YM}} \mathcal{O}(0)\mathcal{O}^\dagger(t) \to e^{-E_{\mathcal{O}}t} + \ldots$$

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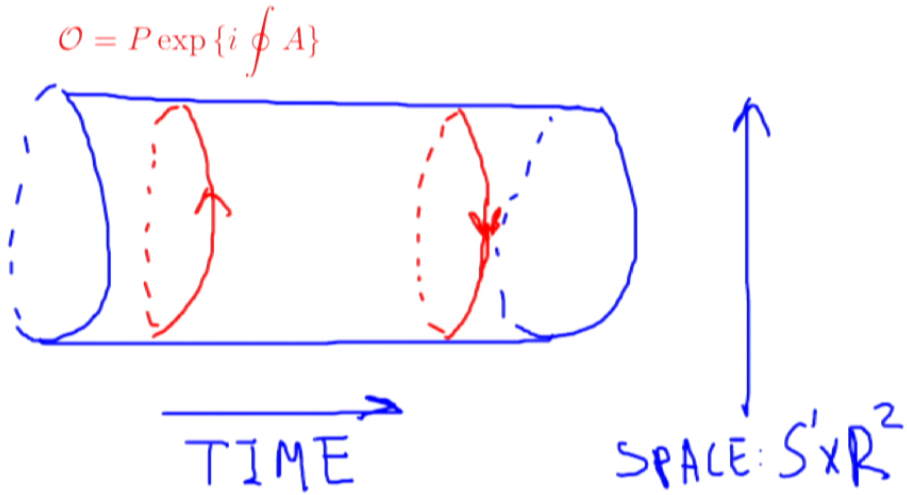
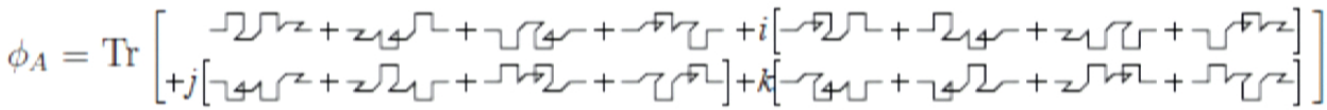
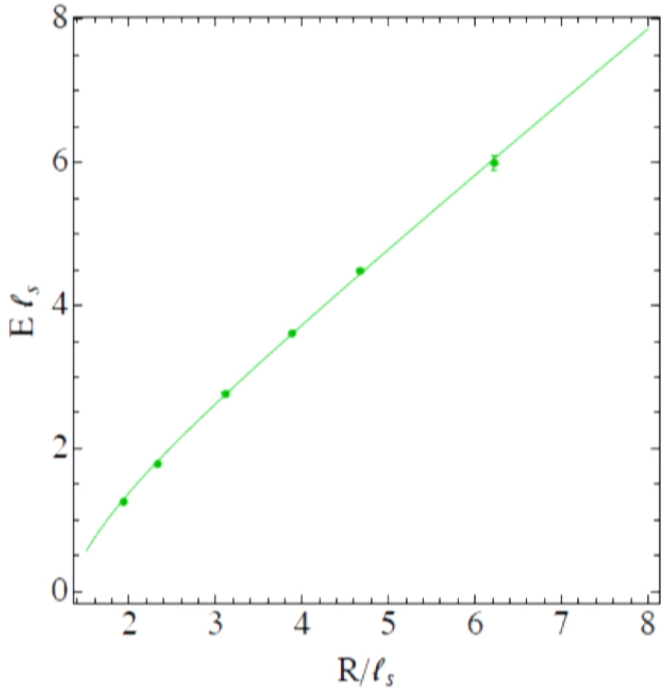


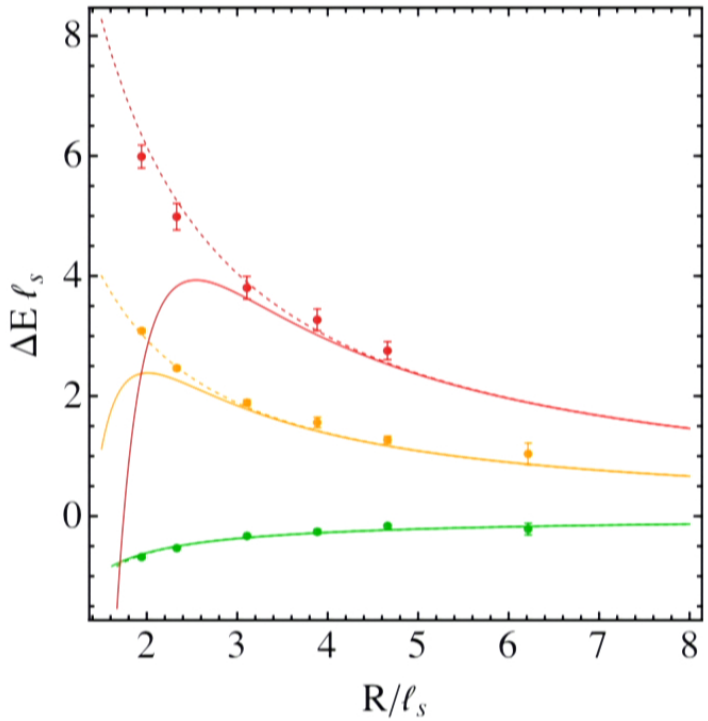


(35)

Puzzle #1: Remarkable agreement with a theory



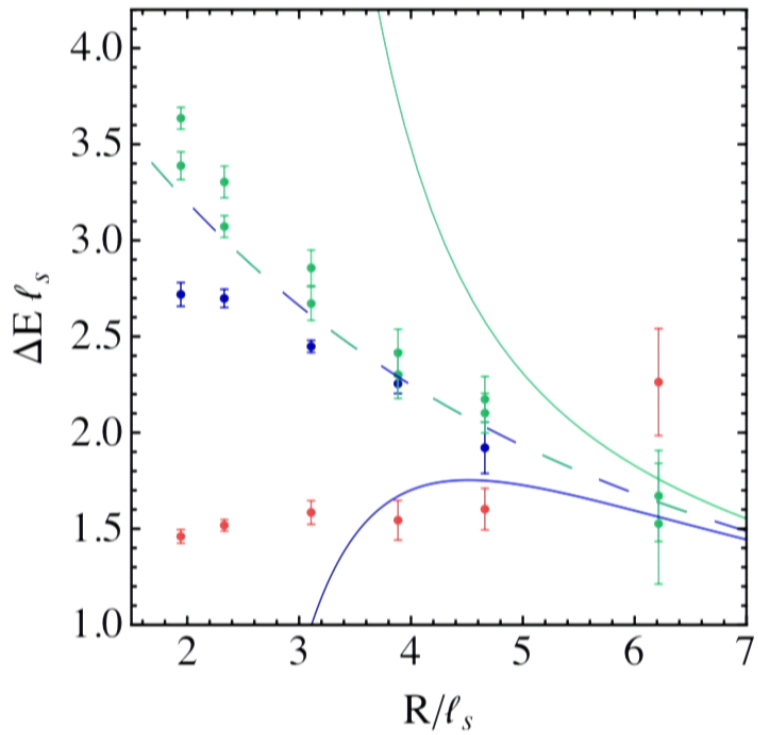
Puzzle #2: The theory is known to be wrong



Dashed --- light cone quantized bosonic string

Solid --- standard ℓ_s/R effective field theory expansion

Puzzle #3: More is going on for some levels



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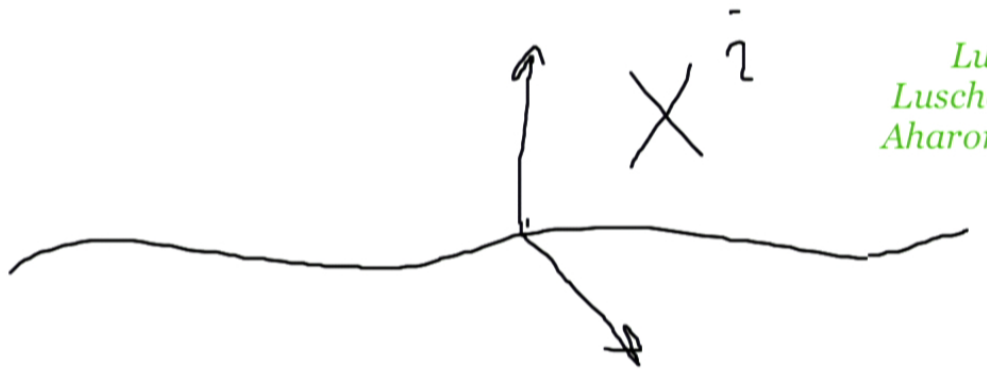
(Long) String as seen by an Effective Field Theorist



Luscher '81
Luscher, Weisz '04
Aharony et al '07-11

Theory of Goldstone Bosons

$$ISO(1, D-1) \to ISO(1,1) \times SO(D-2)$$

$$\delta_\epsilon^{\alpha i} X^j = -\epsilon(\delta^{ij}\sigma^\alpha + X^i \partial^\alpha X^j)$$

CCWZ construction

$$X^\mu = (\sigma^\alpha, X^i(\sigma)) \quad h_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu$$

$$S_{string} = -\int d^2\sigma \sqrt{-\det h_{\alpha\beta}} \left(\ell_s^{-2} + \frac{1}{\alpha_0} \left(K^i_{\alpha\beta}\right)^2 + \dots \right)$$

Nambu-Goto rigidity

Perturbatively:

$$S_{string} = -\ell_s^{-2} \int d^2\sigma \frac{1}{2}(\partial_\alpha X^i)^2 + c_2(\partial_\alpha X^i)^4 + c_3(\partial_\alpha X^i \partial_\beta X^j)^2 + \dots$$

$$c_2 = -\frac{1}{8} \quad c_3 = \frac{1}{4}$$

Interacting, in fact non-renormalizable, healthy effective field theory with cutoff ℓ_s

Why D=26 is special?

Theory is renormalizable (in some sense)

Nambu-Goto Spectrum

$$E_{LC}(N,\tilde{N}) = \sqrt{\frac{4\pi^2(N-\tilde{N})^2}{R^2} + \frac{R^2}{\ell_s^4} + \frac{4\pi}{\ell_s^2}\left(N+\tilde{N}-\frac{D-2}{12}\right)}$$

Comes from quantization in the light cone gauge

Goddard, Goldstone, Rebbi, Thorn'73 +winding

Crucial property: no splittings between different SO(D-2) multiplets

~~Nambu-Goto~~ Spectrum

"Light Cone" or GGRT

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Consistent with target space Lorentz symmetry only at D=26. What it has to do with D=4 spectrum?

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General SO(D-2) invariant amplitude:

$$\mathcal{M}_{ij,kl} = A\delta_{ij}\delta_{kl} + B\delta_{ik}\delta_{jl} + C\delta_{il}\delta_{jk}$$

annihilation

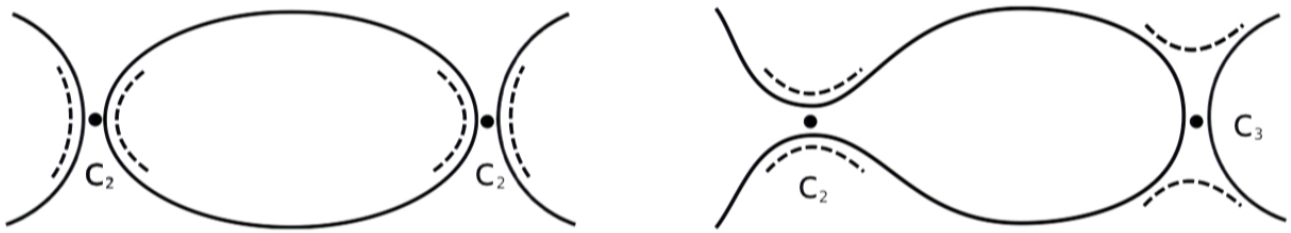
$$A(s,t,u) = A(s,u,t) = B(t,s,u) = C(u,t,s)$$

Tree level:

$$\mathcal{M}_{ij,kl} = -\frac{\ell_s^2}{2}(\delta^{ik}\delta^{jl}su + \delta^{il}\delta^{jk}st)$$

No annihilations for Nambu-Goto!

One-loop:



Finite part:

~18 diagrams

$$\mathcal{M}_{ij,kl} = -\ell_s^4 \frac{D-26}{192\pi}\left(s^3\delta_{ij}\delta_{kl} + t^3\delta_{ik}\delta_{jl} + u^3\delta_{il}\delta_{jk}\right)$$

$$+$$

$$-\frac{\ell_s^4}{16\pi}\left((s^2u\log\frac{t}{s} + su^2\log\frac{t}{u})\delta_{ik}\delta_{jl} + (s^2t\log\frac{u}{s} + st^2\log\frac{u}{t})\delta_{il}\delta_{jk}\right)$$

gives rise to annihilations!

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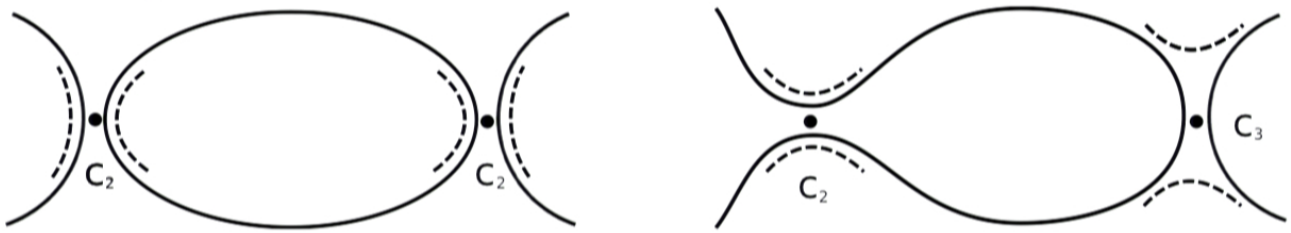

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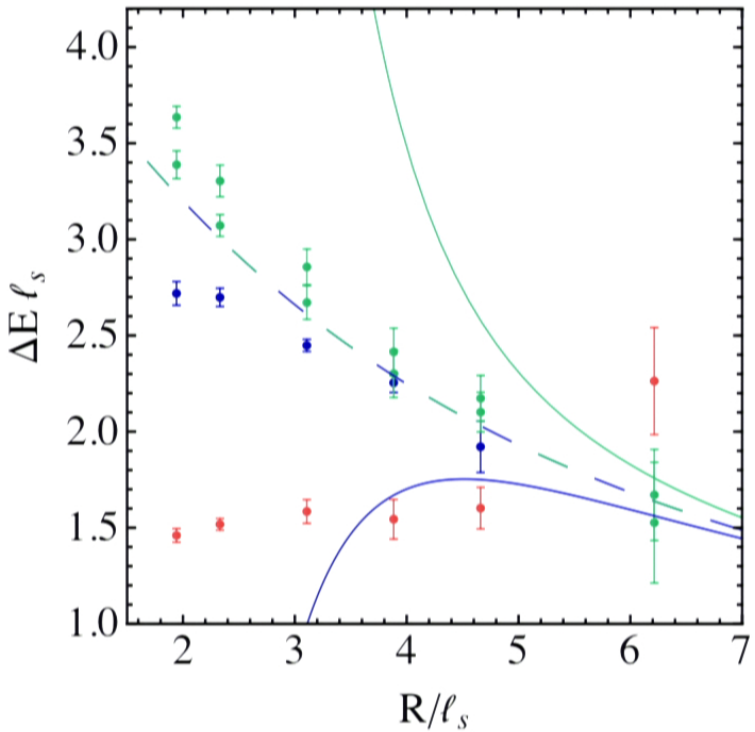


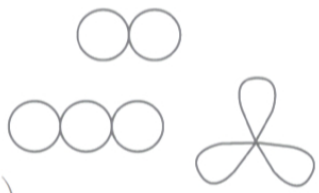
R^{-5} splittings in SO(D-2) multiplets

$$\mathcal{L}_{QCD\ string} = \mathcal{L}_{light\ cone} - \frac{D-26}{192\pi}\partial_\alpha\partial_\beta X^i\partial^\alpha\partial^\beta X^i\partial_\gamma X^j\partial^\gamma X^j + \ldots$$

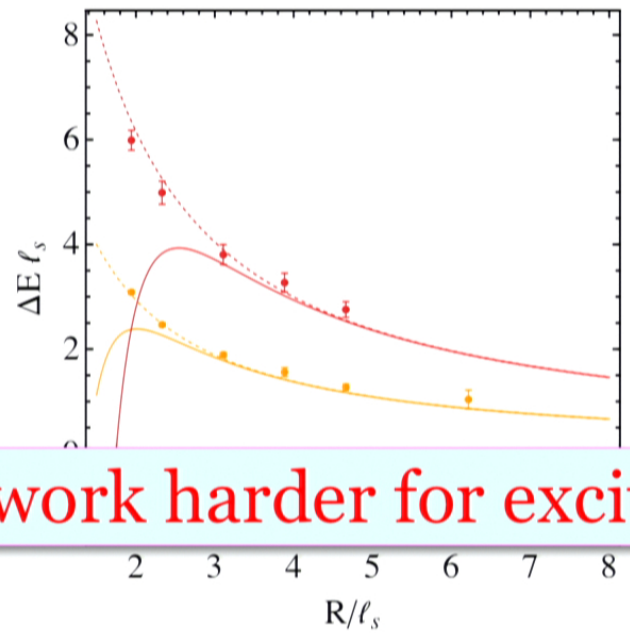
Explains the ground state data

$$E_0(R) = \frac{R}{\ell_s^2}\left(1 - \frac{(D-2)\pi}{6}\left(\frac{\ell_s^2}{R}\right)^2 - \frac{(D-2)^2\pi^2}{72}\left(\frac{\ell_s^2}{R}\right)^4 - \frac{(D-2)^3\pi^3}{432}\left(\frac{\ell_s^2}{R}\right)^6 + \text{non-universal higher order terms}\right)$$

free theory of (D-2) bosons (with c.c.)



no (D-26)



Need to work harder for excited states!

GGRT spectrum:

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for excited states:

$$E = \ell_s^{-1}\mathcal{E}(p_i\ell_s, \ell_s/R)$$

Let's try to disentagle these two expansions

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Finite volume spectrum in two steps:

1) Find infinite volume S-matrix

2) Extract finite volume spectrum from the S-matrix

1) is a standard perturbative expansion in $p\ell_s$

2) perturbatively in massive theories (Luscher)

exactly in integrable 2d theories through TBA

Relativistic string is neither massive nor integrable...

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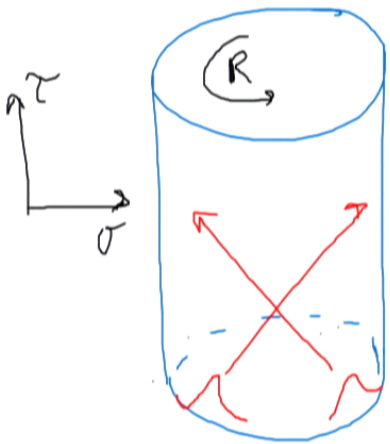
Luscher's Prescription

Luscher '86

Consider a two-particle state on a cylinder at $R \to \infty$:



$$|\psi(t)\rangle = e^{-iE_2 t}|\psi(0)\rangle = e^{-i(2E_1 t - 2\delta \frac{t}{\Delta t})}|\psi(0)\rangle$$

$$\Delta t = \frac{R}{2}$$

$$2\delta(s) = -\lim_{R\to\infty} \frac{R}{2}(E_2 - 2E_1)$$

$$s = \frac{16\pi^2 N^2}{R^2}$$

We find

$$e^{2i\delta_{GGRT}(s)} = e^{is\ell_s^2/4}$$

- Polynomially bounded on the physical sheet
- Agrees with tree-level and one-loop results
- One can reconstruct the entire string spectrum using thermodynamic Bethe Ansatz
- No poles anywhere. A cut all the way to infinity with an infinite number of broad resonances

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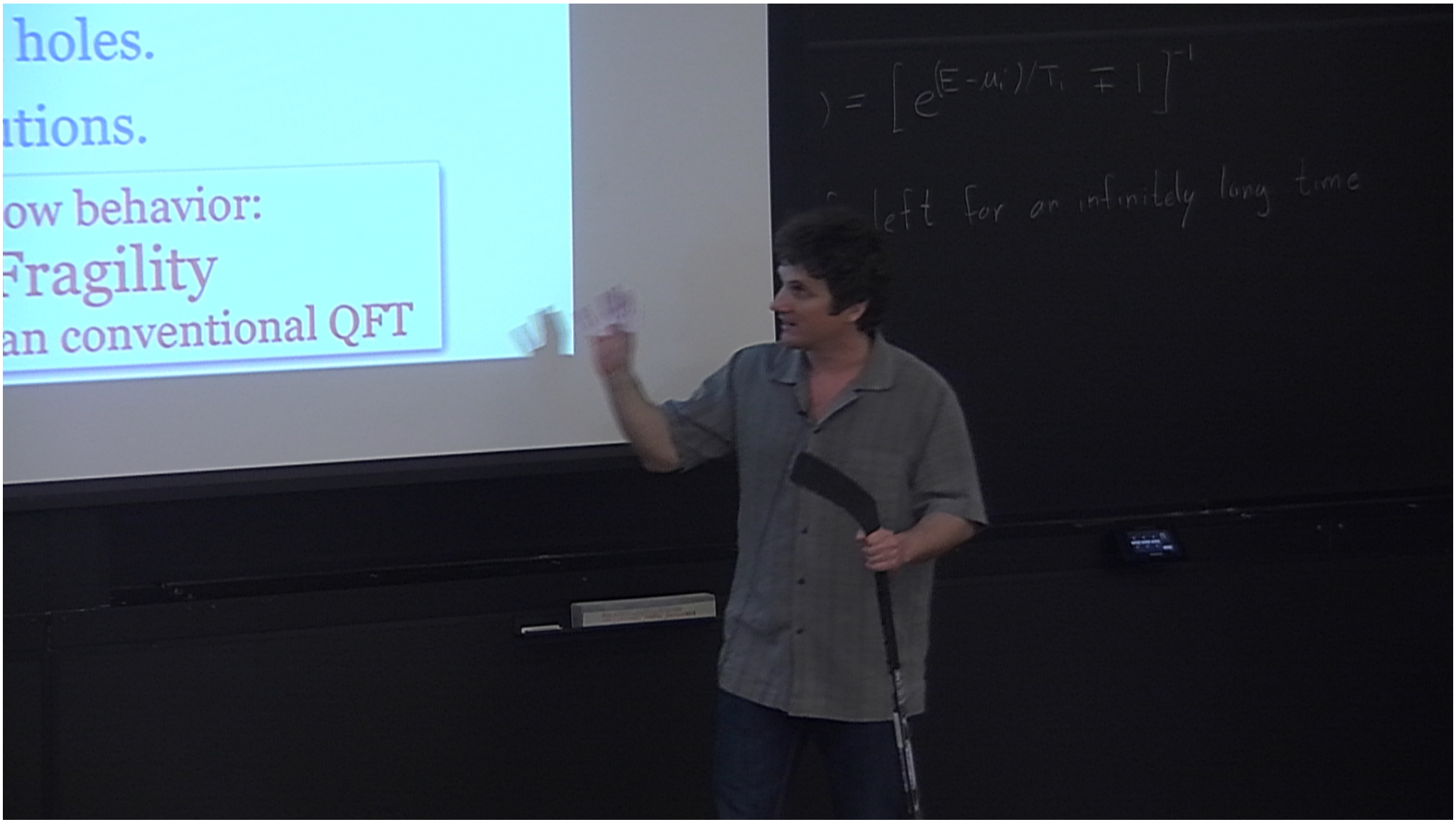
Some properties of the theory

- ✳No UV fixed point.
- ✳No UV central charge.
- ✳No local off-shell observables.
- ✳Maximal achievable (Hagedorn) temperature.
- ✳Minimal length.
- ✳Integrable cousins of black holes.
- ✳Big Bang cosmological solutions.

A new type of RG flow behavior:

Asymptotic Fragility

Theory of gravity rather than conventional QFT


holes.
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ow behavior:
Fragility
an conventional QFT
left for an infinitely long time


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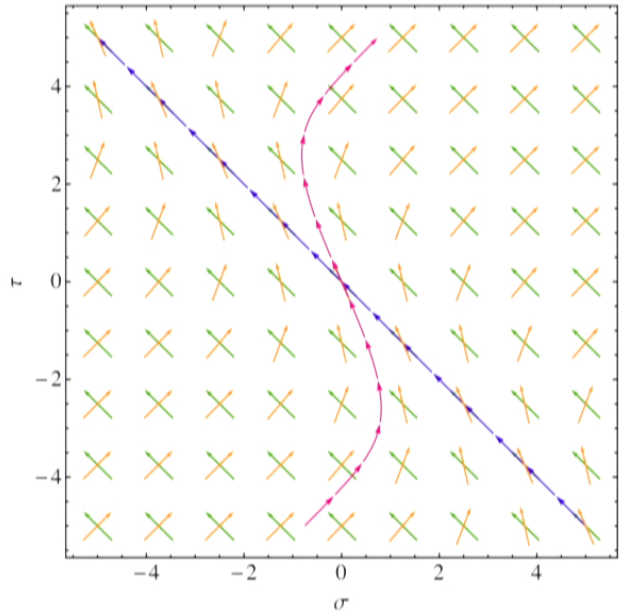
Classical Origin of the Time Delay

$X^i_{cl}(\tau + \sigma)$ is a solution



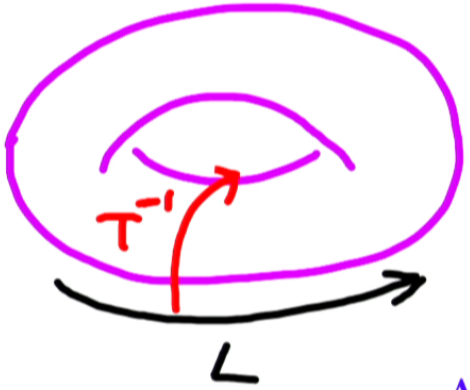
$$\Delta t = \int_{-\infty}^{\infty} dz X'^2_{cl}$$

exactly reproduces the quantum answer

Free string spectrum circa 2012

Thermodynamic Bethe Ansatz

Zamolodchikov '91



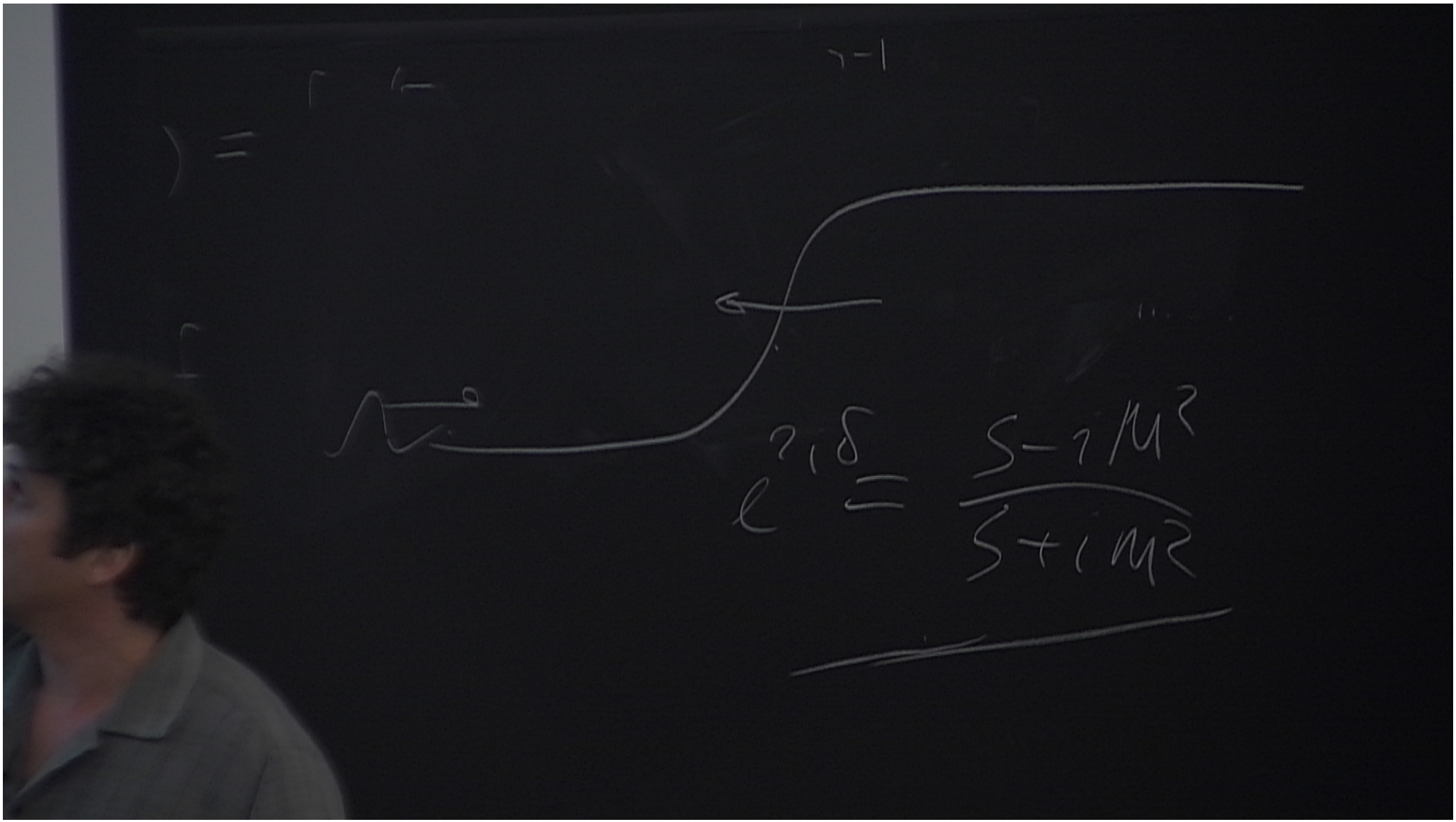
in thermodynamic (large L) limit

$$Z(T,L) = e^{-LE_0(1/T)} = e^{-Lf(T)/T}$$

$$+$$

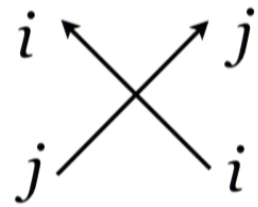
Asymptotic Bethe Ansatz

$$p^{(i)}_{k\,R}L + \sum_{i=1}^{D-2}\int_0^\infty 2\delta(p^{(i)}_{k\,R},p)\rho^i_{1L}(p)dp = 2\pi n^{(i)}_{k\,R}$$


$e^{2i\delta} = \frac{s - iM^2}{s + iM^2}$

Asymptotic Bethe Ansatz

$$\Psi(x_1, x_2) = \langle 0|X^i(x_1)X^j(x_2)|p_L^{(i)}, p_R^{(j)}|\rangle$$



$x_1 > x_2 \qquad \Psi(x_1, x_2) = e^{-ip_L x_1} e^{ip_R x_2}$

$x_1 > x_2 \qquad \Psi(x_1, x_2) = e^{-ip_L x_1} e^{ip_R x_2} e^{2i\delta(p_L, p_R)}$

periodicity: $\quad e^{-ip_{L,R}} = e^{2i\delta(p_L, p_R)}$

$$p_{L,R} + 2\delta(p_L, p_R) = 2\pi n_{L,R}$$

NB: particles are getting softer!

after taking the continuum limit
minimization of the free energy results in

$$\epsilon_L^i(p) = p\left[1 + \frac{\ell_s^2 T}{2\pi}\sum_{j=1}^{D-2}\int_0^\infty dp' \ln\left(1 - e^{-\epsilon_R^j(p')/T}\right)\right]$$

where

$$f = \frac{T}{2\pi}\sum_{j=1}^{D-2}\int_0^\infty dp' \ln\left(1 - e^{-\epsilon_L^j(p')/T}\right) + (L \to R)$$

reproduces the correct ground state energy

$$f(T) = \frac{1}{\ell_s^2}(\sqrt{1 - T^2/T_H^2} - 1)$$

$$T_H = \frac{1}{\ell_s}\sqrt{\frac{3}{\pi(D-2)}}$$

Excited States TBA

Dorey, Tateo '96

general idea: excited states can be obtained by analytic continuation of the ground state

$$\hat{p}^{(i)}_{k\,L} R + \sum_{j,m} 2\delta(\hat{p}^{(i)}_{k\,L}, \hat{p}^{(j)}_{m\,R}) N^{(j)}_{m\,R} - i \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \frac{d\,2\delta(i\hat{p}^{(i)}_{k\,L}, p')}{dp'} \ln\left(1 - e^{-R\epsilon^j_R(p')}\right) = 2\pi n^{(i)}_{k\,L}$$

$$\epsilon^i_L(p) = p + \frac{i}{R} \sum_{j,k} 2\delta(p, -i\hat{p}^{(j)}_{k\,R}) N^{(j)}_{k\,R} + \frac{1}{2\pi R} \sum_{j=1}^{D-2} \int_0^\infty dp' \frac{d\,2\delta(p,p')}{dp'} \ln\left(1 - e^{-R\epsilon^j_R(p')}\right)$$

$$E(R) = R + \sum_{j,k} p^{(j)}_{k\,L} + \sum_{j=1}^{D-2} \int_0^\infty \frac{dp'}{2\pi} \ln\left(1 - e^{-R\epsilon^j_L(p')}\right)$$

+right-movers

Exactly reproduces all of the light cone spectrum

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Asymptotic Bethe Ansatz

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finite size corrections

Asymptotic Bethe Ansatz

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+right-movers

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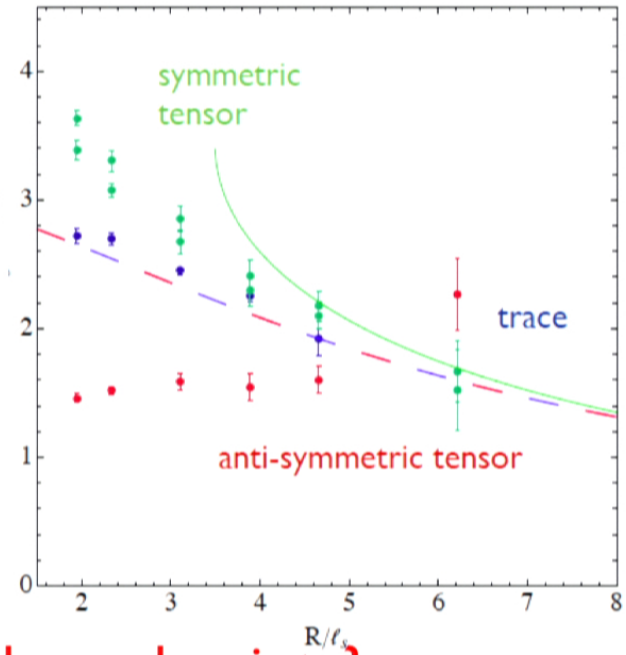
Colliding left- and right-movers

ABA phase shift from 1-loop $\mathcal{O}(p^4)$ computation with GGRT TBA for windings

$$p\ell_s < 1.8$$



What are the red points?

A new massive state appearing as a resonance in the antisymmetric channel!

see also arXiv:1007.4720
Athenodorou, Bringoltz, Teper

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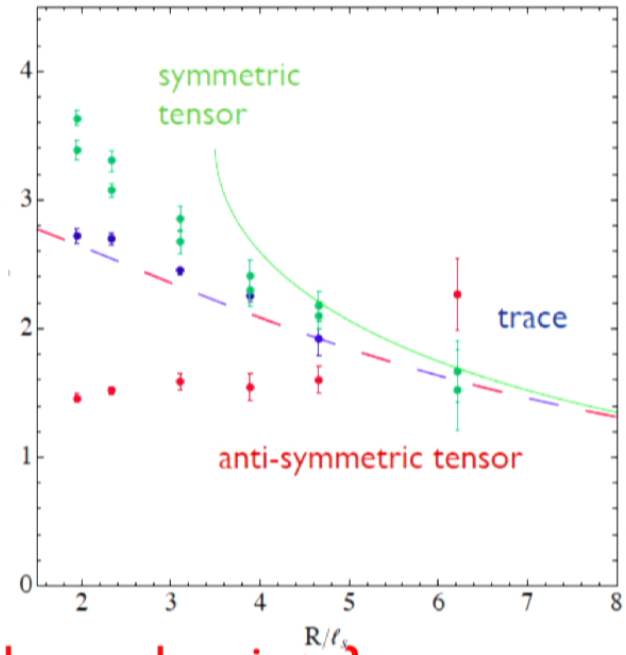
ABA phase shift from 1-loop $\mathcal{O}(p^4)$ computation with GGRT TBA for windings

$$p\ell_s < 1.8$$



What are the red points?

A new massive state appearing as a resonance in the antisymmetric channel!

see also arXiv:1007.4720
Athenodorou, Bringoltz, Teper

How do we include this massive state?

Contributes to scattering of Goldstone's and changes the phase shifts. In particular, it appears as a resonance in the antisymmetric channel. We can calculate contributions from

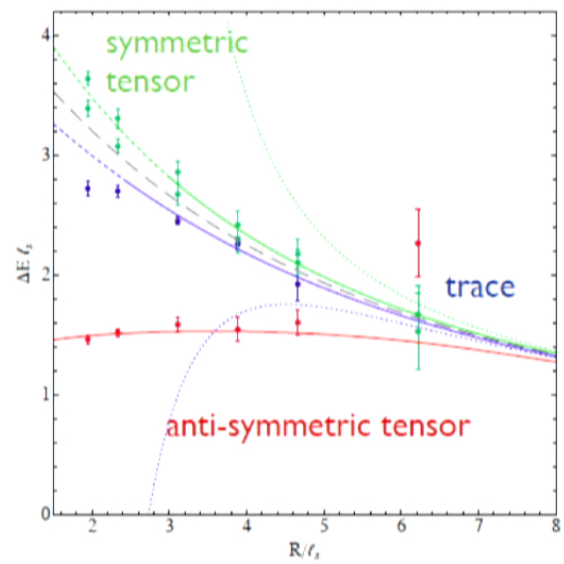
$$S = \int d^2\sigma \frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi - \frac{1}{2}m^2\phi^2 + \frac{\alpha}{8\pi}\phi\epsilon^{\alpha\beta}\epsilon_{ij}K^i_{\alpha\gamma}K^{j\,\gamma}_{\beta}$$

Including the resonant s-channel contribution

$$\delta(s) = \arctan\left(\frac{m\Gamma(s/m)^3}{m^2 - s}\right)$$

$$m \sim 1.85\ell_s^{-1} \qquad \Gamma \sim 0.4\ell_s^{-1}$$

as well as perturbative non-resonant contributions in crossed channels



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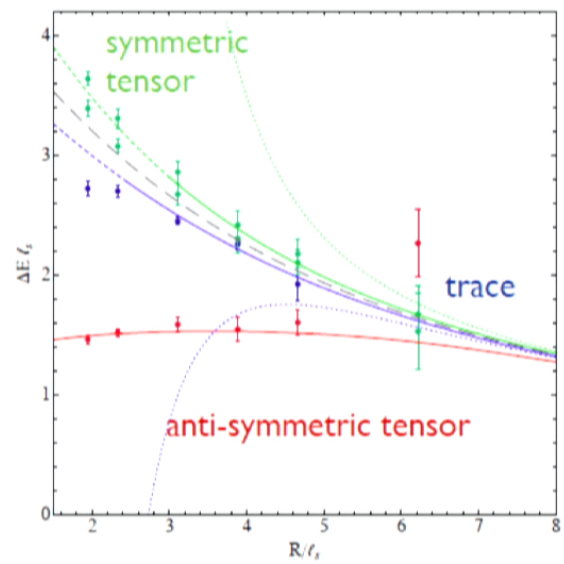
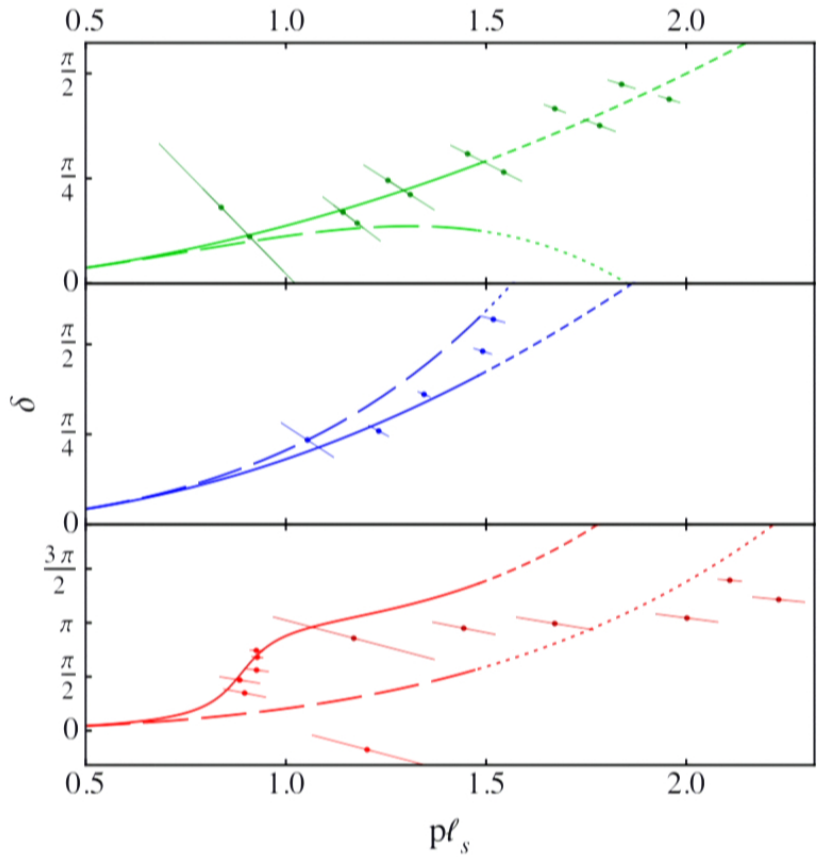


Reverting the logic: S-matrix from finite volume spectrum



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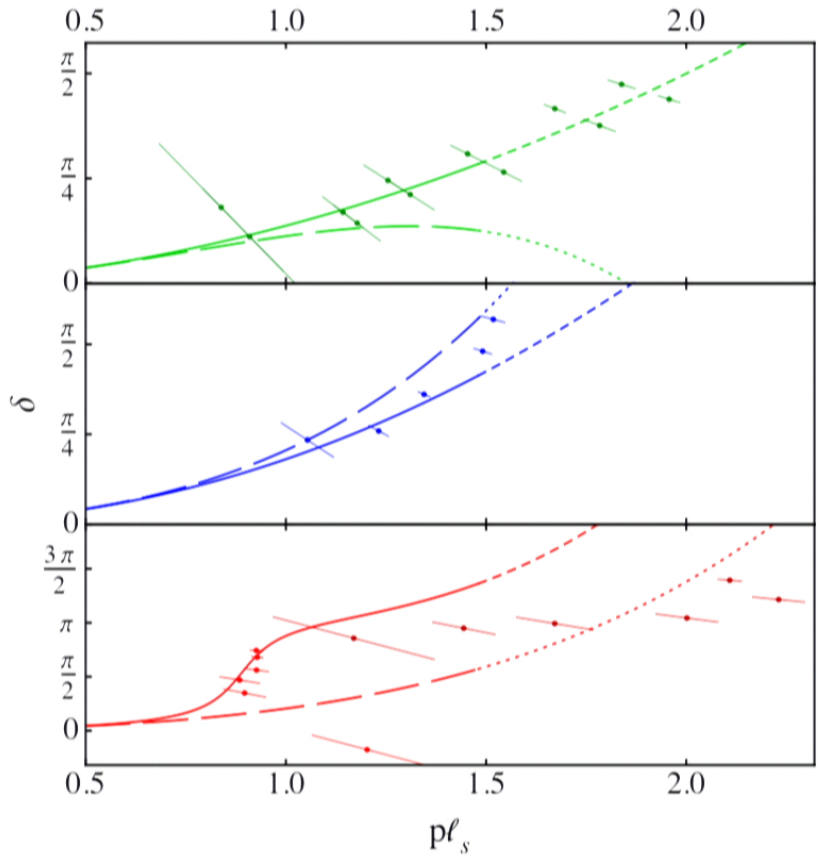
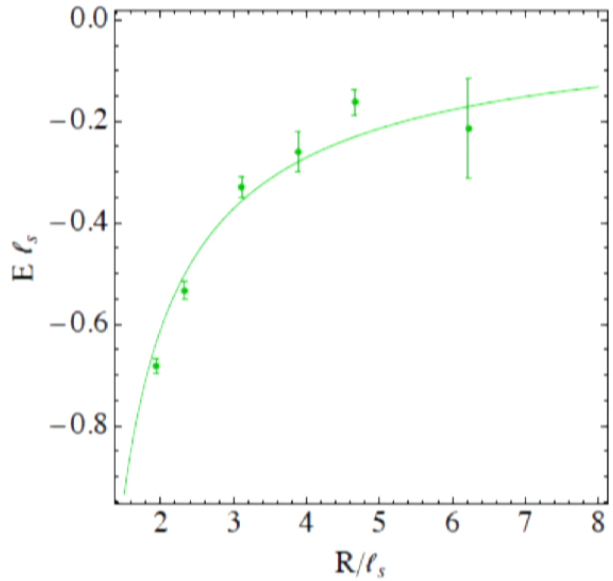
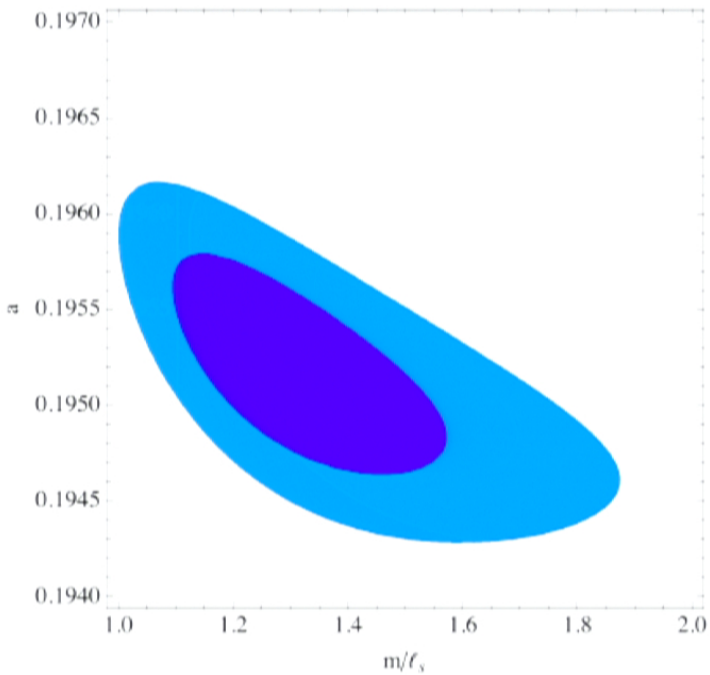


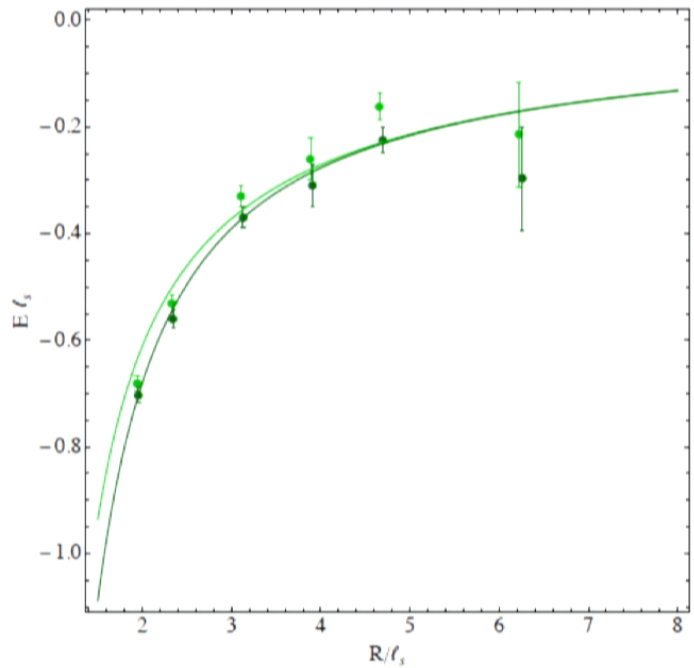
Is this the lightest massive state, or there is a hidden valley?



A massive particle contributes into the Casimir energy

$$\Delta E(R) = -\frac{m}{\pi}\sum_n K_1(mnR)$$

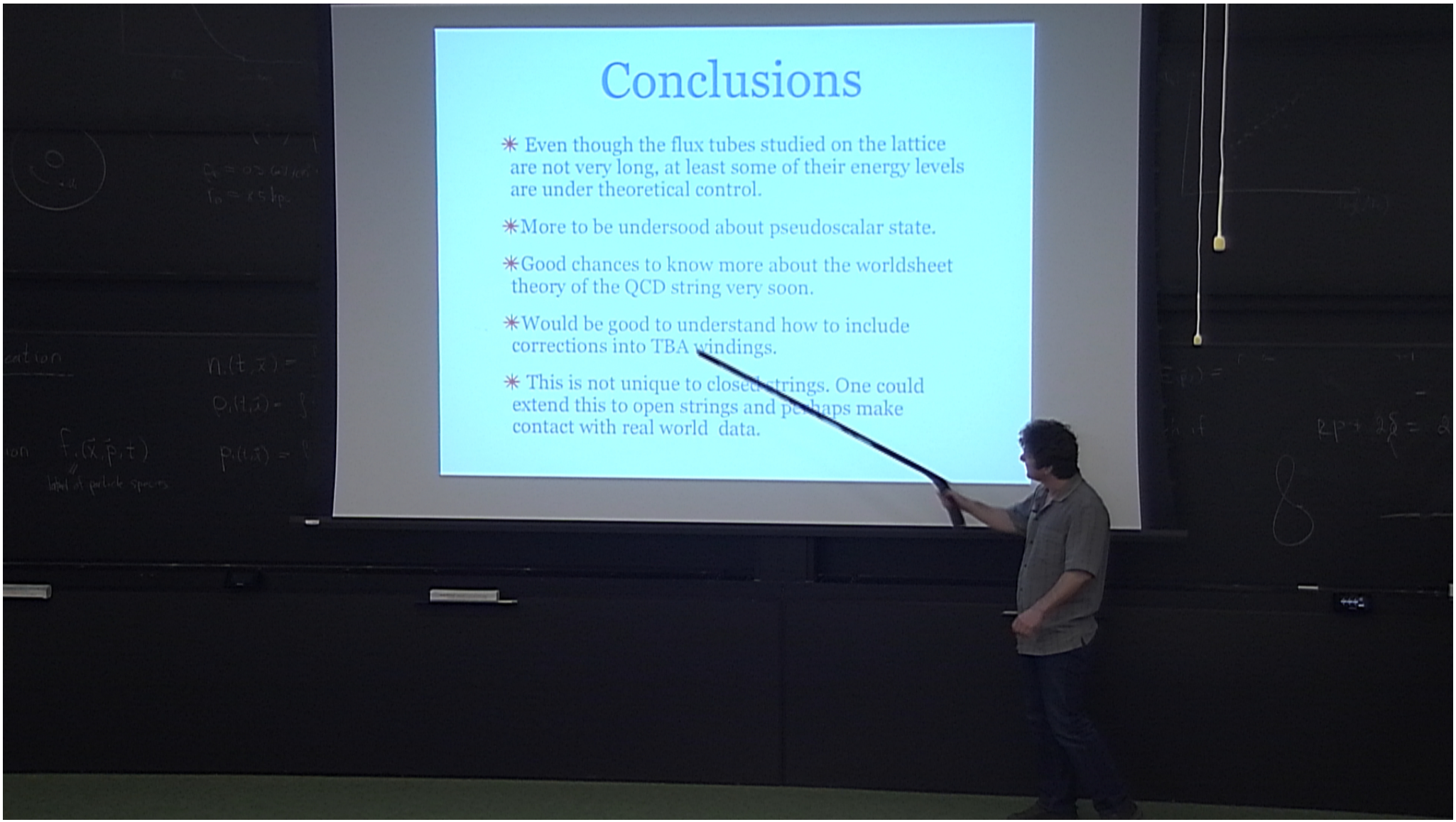




$\Delta\chi^2 \approx 21$ for one new parameter. Remains to be seen whether this is due to "new physics" or systematics

Conclusions

- Even though the flux tubes studied on the lattice are not very long, at least some of their energy levels are under theoretical control.
- More to be undersood about pseudoscalar state.
- Good chances to know more about the worldsheet theory of the QCD string very soon.
- Would be good to understand how to include corrections into TBA windings.
- This is not unique to closed strings. One could extend this to open strings and perhaps make contact with real world data.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