Title: Renormalization of Entanglement Entropy

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Abstract: The Standard Model Higgs boson may be mixed with another scalar that does not couple singly to gauge bosons or fermions. The electroweak quantum numbers of such an additional scalar can be determined by measuring the quartic Higgs-Higgs-vector-vector couplings, which contributeâ€”along with the coveted triple Higgs couplingâ€”to double Higgs production in e+eâˆ’ collisions. We show that simultaneous sensitivity to the quartic Higgs-Higgs-vector-vector coupling and the triple Higgs coupling can be obtained using measurements of the double Higgs production cross section at two different e+eâˆ’ center-of-mass energies. Kinematic distributions of the two Higgs bosons in the final state could provide additional discriminating power.

Renormalization of Entanglement Entropy

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arXiv:1302.1878 (with J. Cooperman)

Introduction

$$E = mc^2$$

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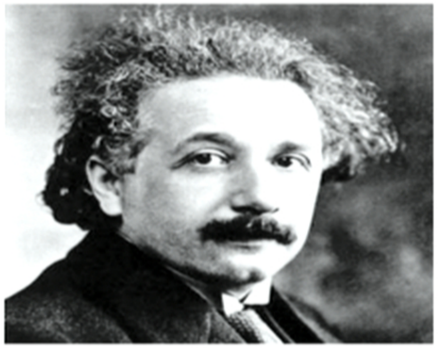

$$S_{\mathrm{BH}} = \frac{1}{4}\frac{c^2}{G_N \hbar} A$$

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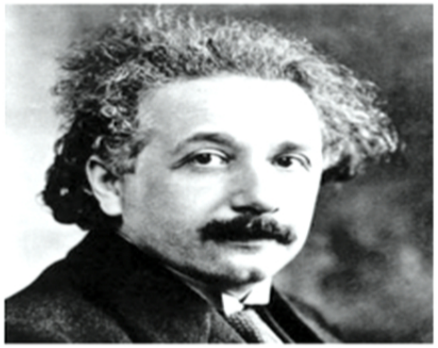

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???

Black Hole Thermodynamics

$$|\Psi\rangle = \sum_{a,b} c_{ab} |a\rangle |b\rangle$$

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$$|\Psi\rangle = \sum_{a,b} c_{ab}|a\rangle|b\rangle$$

$$\rho_A = \sum_b |a'\rangle c^*_{a'b} c_{ab} \langle a|$$

$=$ reduced density matrix describing A

$A =$ QFT outside BH horizon

$\Rightarrow \quad \rho_A = e^{-H/T_{\rm BH}}$

The Big Idea

Sorkin 1983
Bombelli, Kaul, Lee, Sorkin 1986
Srednicki 1993
Frolov, Novikov 1993

Entanglement entropy

$$S_{\mathrm{ent}} = -\,\mathrm{Tr}(\rho_A \ln \rho_A) = -\,\mathrm{Tr}(\rho_B \ln \rho_B)$$

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Entanglement entropy

$$S_{\text{ent}} = -\operatorname{Tr}(\rho_A \ln \rho_A) = -\operatorname{Tr}(\rho_B \ln \rho_B)$$

$$\stackrel{?}{=} S_{\text{BH}}$$

$$S_{\text{ent}} = c\,\Lambda_{\text{UV}}^2 A$$

holography?

The Big Idea (cont'd)

Susskind, Uglum 1994
Jacobson 1994

S_{BH} is also UV divergent!

$$S_{\mathrm{BH}} = \frac{1}{4} M_{\mathrm{P}}^2 A \qquad M_{\mathrm{P}}^2 = M_{\mathrm{P}0}^2 + c' \Lambda_{\mathrm{UV}}^2 + \cdots$$

Do UV divergences match?

$$S_{\mathrm{ent}} = c\, \Lambda_{\mathrm{UV}}^2 A$$

$$c \stackrel{?}{=} \frac{1}{4} c'$$

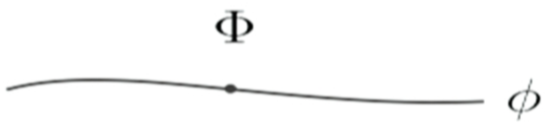
Can be checked in flat spacetime...

Geometrical Entropy

Callan, Wilczek 1994

S_{ent} depends on entangling surface and state

$|\Psi\rangle =$ path integral



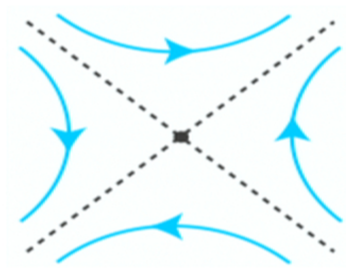
$$\langle\phi|\Psi\rangle = \int_{\Phi(0)=\phi} d[\Phi]\, e^{iS[\Phi]}$$

Callan-Wilczek Formula

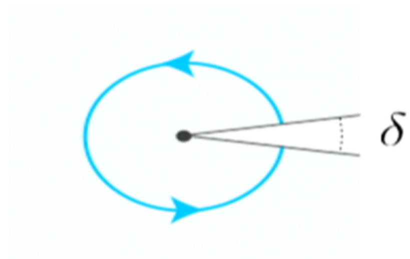
$$S_{\text{ent}} = -\lim_{\delta\to 0}\left(2\pi\frac{\partial}{\partial\delta}+1\right)W_\delta \qquad e^{iW_\delta} = \int d[\Phi]\, e^{iS_\delta[\Phi]}$$

$\delta =$ deficit angle at entangling surface

Makes sense for spacetimes with boost/rotation symmetry about entangling surface



Minkowski



Euclidean

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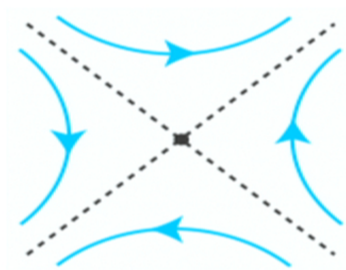

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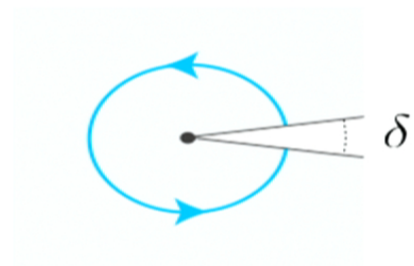

Euclidean

This Talk

- Derivation of CW formula without “replica trick”
- Counterterms for EE!
- UV divergences localized on entangling surface don’t contribute to EE

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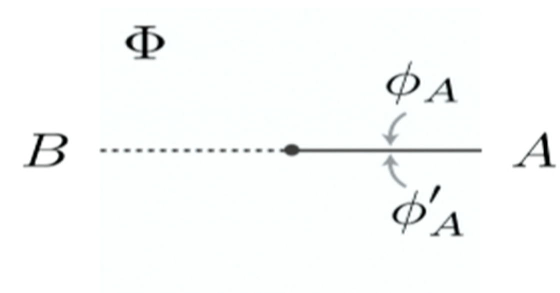
- Derivation of CW formula without "replica trick"
- Counterterms for EE!
- UV divergences localized on entangling surface don't contribute to EE
- Subleading divergent terms in EE depend on state
- Check that EE = BH entropy (Wald)

Speculations at end!

Proof of CW

$$\langle \phi'_A | \rho_A | \phi_A \rangle = \int d[\phi_B] \int d[\Phi] e^{-S[\Phi]}$$

$$\Phi(0+) = (\phi_A, \phi_B)$$
$$\Phi(0-) = (\phi'_A, \phi_B)$$



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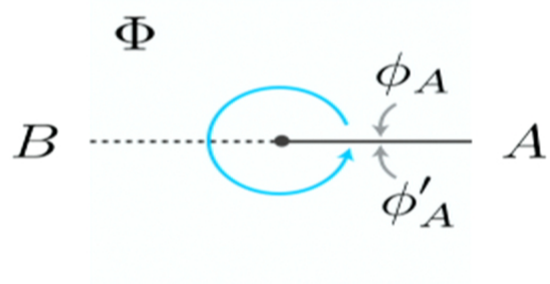
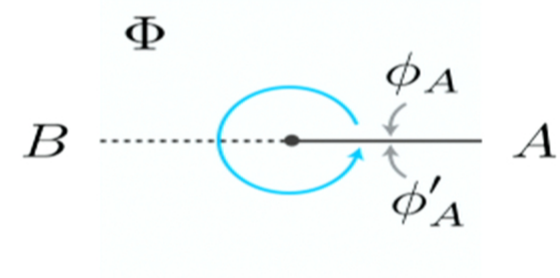


Rotation invariance ⇒ $\rho_A = e^{-2\pi K}$

$K =$ rotation generator

Proof of CW

$$\langle \phi'_A | \rho_A | \phi_A \rangle = \int d[\phi_B] \int_{\substack{\Phi(0+) = (\phi_A, \phi_B) \\ \Phi(0-) = (\phi'_A, \phi_B)}} d[\Phi] e^{-S[\Phi]}$$



Rotation invariance $\Rightarrow$ $\rho_A = e^{-2\pi K}$

$K =$ rotation generator

$$S_{\text{ent}} = \lim_{n \to 1} \left(\frac{\partial}{\partial n} + 1 \right) \ln \operatorname{Tr} \rho^n$$

$\operatorname{Tr} \rho^n = e^{-2\pi n K}$

$=$ path integral on space with deficit angle

$\delta = 2\pi(1-n)$

UV Divergences

EE is UV divergent due to entanglement of UV modes across boundary

$$S_A = S_B$$

$\Rightarrow$ UV divergent terms localized on boundary

UV Divergences

EE is UV divergent due to entanglement of UV modes across boundary

$$S_A = S_B$$

$\Rightarrow$ UV divergent terms localized on boundary

$$S_{\rm ent} = \int d^2y \sqrt{\gamma}\left[\Lambda_{\rm UV}^2 + \ln\Lambda_{\rm UV}\, R(\gamma) + \cdots\right]$$

To keep geometrical interpretation,
need generally covariant regulator

Counterterms

W_δ includes counterterms $\Rightarrow$ so does EE

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Changing cutoff $\Leftrightarrow$ adding counterterm

No manifest state counting interpretation,
but $S_{\rm ent} = S_{\rm BH}$ in all cases where we can compare

Speculation

alert!

$S_{\rm ent}$ is well-defined in quantum gravity
$\Rightarrow$ counterterms parameterize entanglement of modes above cutoff

No Counterterms?

Kabat 1995; Fursaev, Solodukhin, Jacobson, Wall,...

EE is UV divergent?

$$S_{\rm ent} = c\,\Lambda_{\rm UV}^2 A$$

Connection to $S_{\rm BH}$ requires generally covariant regulator

(Pauli-Villars, dim reg, heat kernel,...)

Must also regularize conical singularity:

$$R(\tilde{g}) \underset{\ell \to 0}{\longrightarrow} R(g) + \delta\,\delta^2(x)$$

No state counting interpretation

$c < 0$ in some theories

Curvature Couplings

$\mathcal{L}_{\rm int} = \frac{1}{2}\xi R(g)\phi^2$

$S_{\rm ent} = c\,\Lambda_{\rm UV}^2 \qquad M_{\rm P}^2 = c'\,\Lambda_{\rm UV}^2$

Generally covariant regulator

$\Rightarrow c = \frac{1}{4}c'$ for all ξ

$\Rightarrow S_{\rm ent} = \frac{1}{4}M_{\rm P}^2$ including counterterms

No counterterms for $S_{\rm ent}$?

$c\,\Lambda_{\rm UV}^2 \neq S_{\rm ent}$

CW ⇒ SU?

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$$W_\delta = \int d^4x\sqrt{g}\left[\Lambda^4+\Lambda^2 R(g)+\ln\Lambda R^2(g)+\cdots\right]$$
$$+\int d^2y\sqrt{\gamma}\left[\Lambda^2+\ln\Lambda R(\gamma)+\cdots\right]$$

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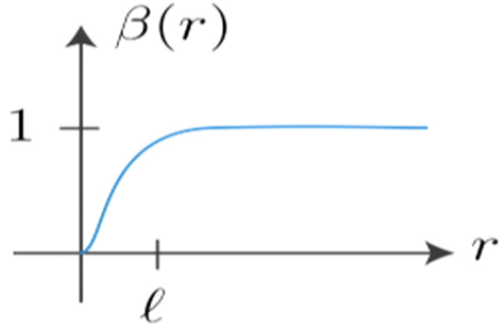
Proof

Smooth conical singularity:

$$d\tilde{s}^2 = dr^2 + \rho^2(r,y)\left[1 - \epsilon\beta(r)\right]^2 d\theta^2 + \gamma_{ij}(r,y)dy^i dy^j$$



$$\epsilon = \frac{\delta}{2\pi}$$

Smooth metric ⇒ only bulk UV divergences

Show $\ell \to 0$ limit exists for fixed Λ at $O(\epsilon)$

Proof (cont'd)

$$\beta(r) = \Theta_+(r) = \lim_{\ell \to 0} \Theta(r - \ell)$$

Discontinuous metric? 

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Discontinuous metric?

$\epsilon\, \Theta_+(r)$ is a well defined distribution at $O(\epsilon)$

$$W = \int d^D x \sqrt{g} \sum_n \mathcal{O}_n(g)$$

$O(\epsilon)$ term in $\mathcal{O}_n = \beta'(r) I[\mathcal{O}_n]$

Proof (cont'd)

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Calculations

Reproduce standard results

$$d\tilde{s}^2 = dr^2 + \rho^2(r,y)\left[1 - \epsilon\beta(r)\right]^2 d\theta^2 + \gamma_{ij}(r,y)dy^i dy^j$$

$$R(\tilde{g}) = R(g) + \epsilon\left[\frac{4\rho'\beta'}{\rho} + \beta'\gamma^{ij}\gamma'_{ij} + 2\beta''\right] + O(\epsilon^2)$$

At $r = 0$:

$$\rho = r + O(r^3) \qquad \partial_r^m \rho| = 0, \quad m = 2,4,6,\ldots$$

$$\partial_r \gamma_{ij}| = 0, \quad n = 1,3,5,\ldots$$

$$\Rightarrow I_1[R] = 2$$

$$\Rightarrow S_{\text{ent}} = \frac{1}{4} M_{\text{P}}^2 A$$

Calculations (con't)

Subleading terms:

$$I_1[f(R_{\mu\nu\rho\sigma})] = \frac{\partial f}{\partial R_{\mu\nu\rho\sigma}} \left(P_{\mu\rho}P_{\nu\sigma} - P_{\mu\sigma}P_{\nu\rho}\right)$$

$$P_{rr} = 1 \qquad P_{\theta\theta} = \rho^2$$

Leading subleading terms: (Fursaev, Solodukhin 1995)

$$\Delta W = \int d^D x \sqrt{g}\, \frac{1}{M^{D-4}} \left[c_1 R^2(g) + c_2 R^2_{\mu\nu}(g) + c_3 R^2_{\mu\nu\rho\sigma}(g)\right]$$

Calculations (cont'd)

New results: terms involving covariant derivatives

$$\Delta W = \int d^D x \sqrt{g}\ (\nabla_\mu R_{\nu\rho\sigma\tau})^2$$

State Dependence

Subleading terms in EE depend on extrinsic geometry of entangling surface

$$ds^2 = \underbrace{dr^2 + \rho^2 d\theta^2}_{\text{extrinsic}} + \underbrace{\gamma_{ij} dy^i dy^j}_{\text{intrinsic}}$$

Black Hole Entropy

Black Hole Entropy

Higher curvature terms in action
$\Leftrightarrow$ corrections to classical black hole dynamics

$\Rightarrow$ Generalized entropy formula for black holes with bifurcate Killing horizon (Wald 1993)

We find agreement for all cases we have computed

$$W = \int d^D x \sqrt{\gamma} \left[f(R_{\mu\nu\rho\sigma}) + (\nabla_\mu R_{\nu\rho\sigma\tau})^2 \right]$$

Speculation

Renormalization of EE suggests that EE is a well-defined observable in quantum gravity

UV cutoff = matching scale

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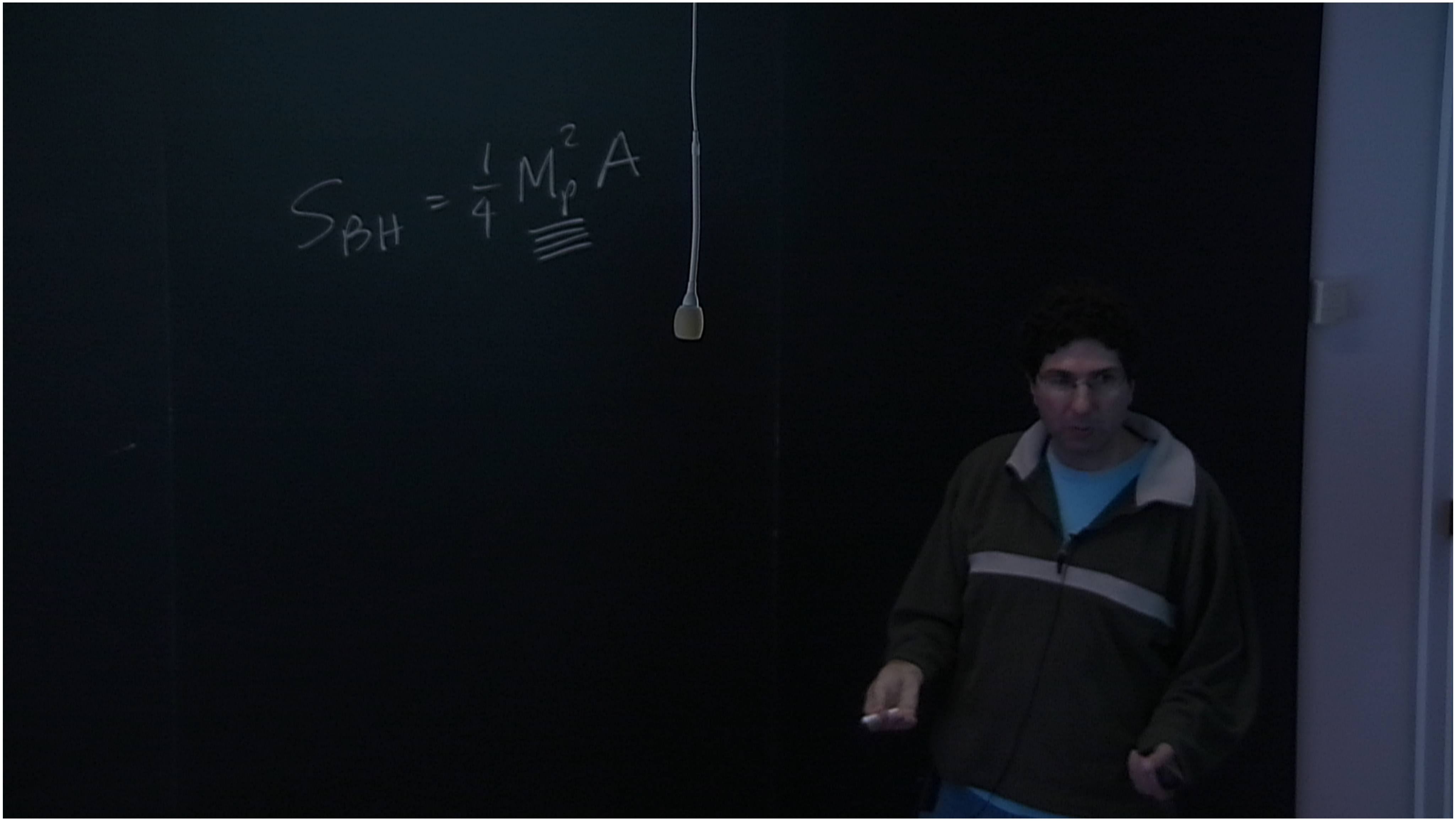
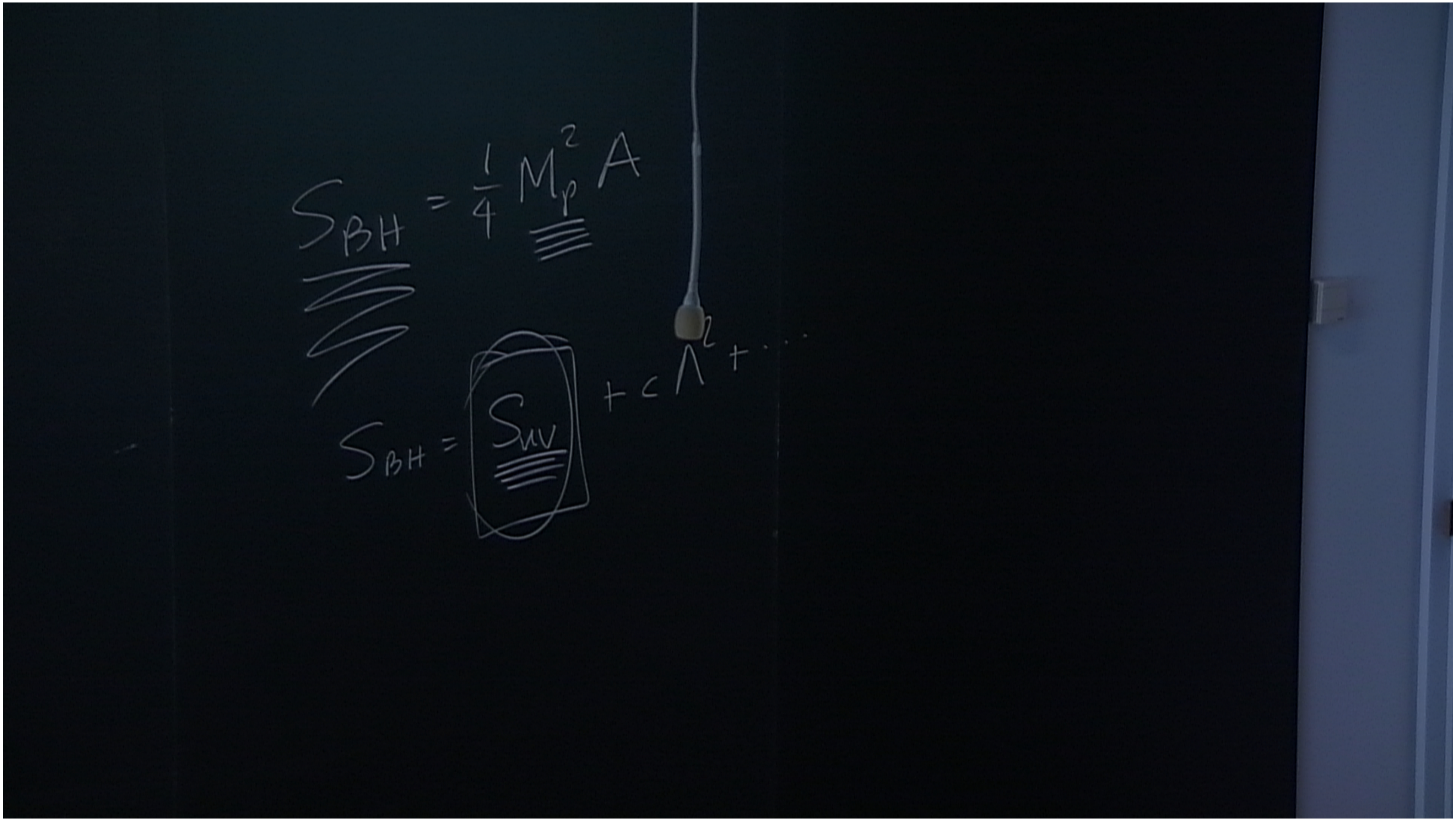
$\Rightarrow S_{\mathrm{ent}} \simeq \frac{1}{4} M_{\mathrm{P}}^{D-2} A_{D-2}$ for any boundary

String theory state counting gives UV contribution to EE?


$S_{BH} = \frac{1}{4} M_p^2 A$


S_BH = 1/4 M_p^2 A
S_BH = S_UV + c Λ^2 + . . .

Conclusions