Title: Neorealism and the Internal Language of Topoi

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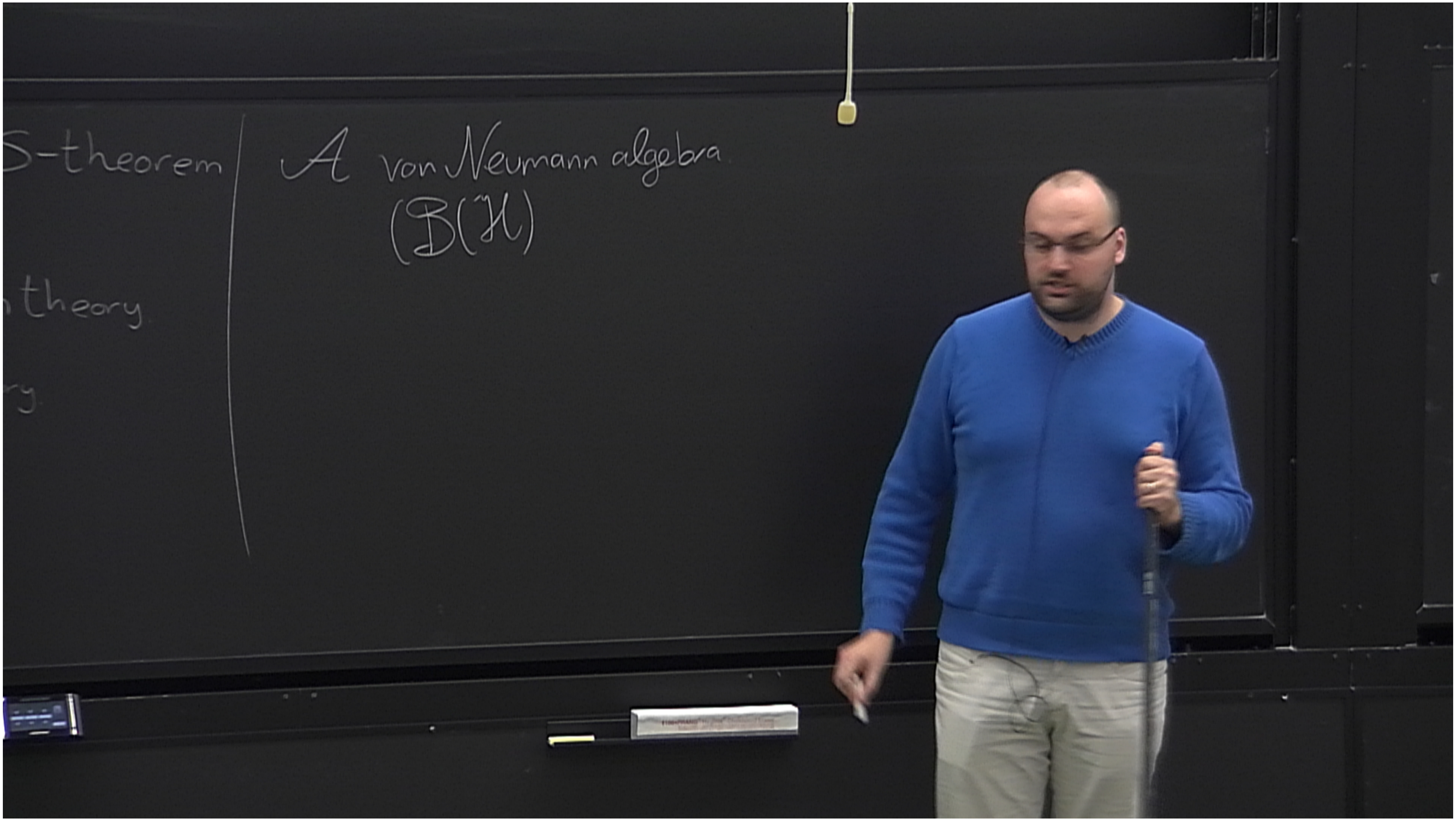
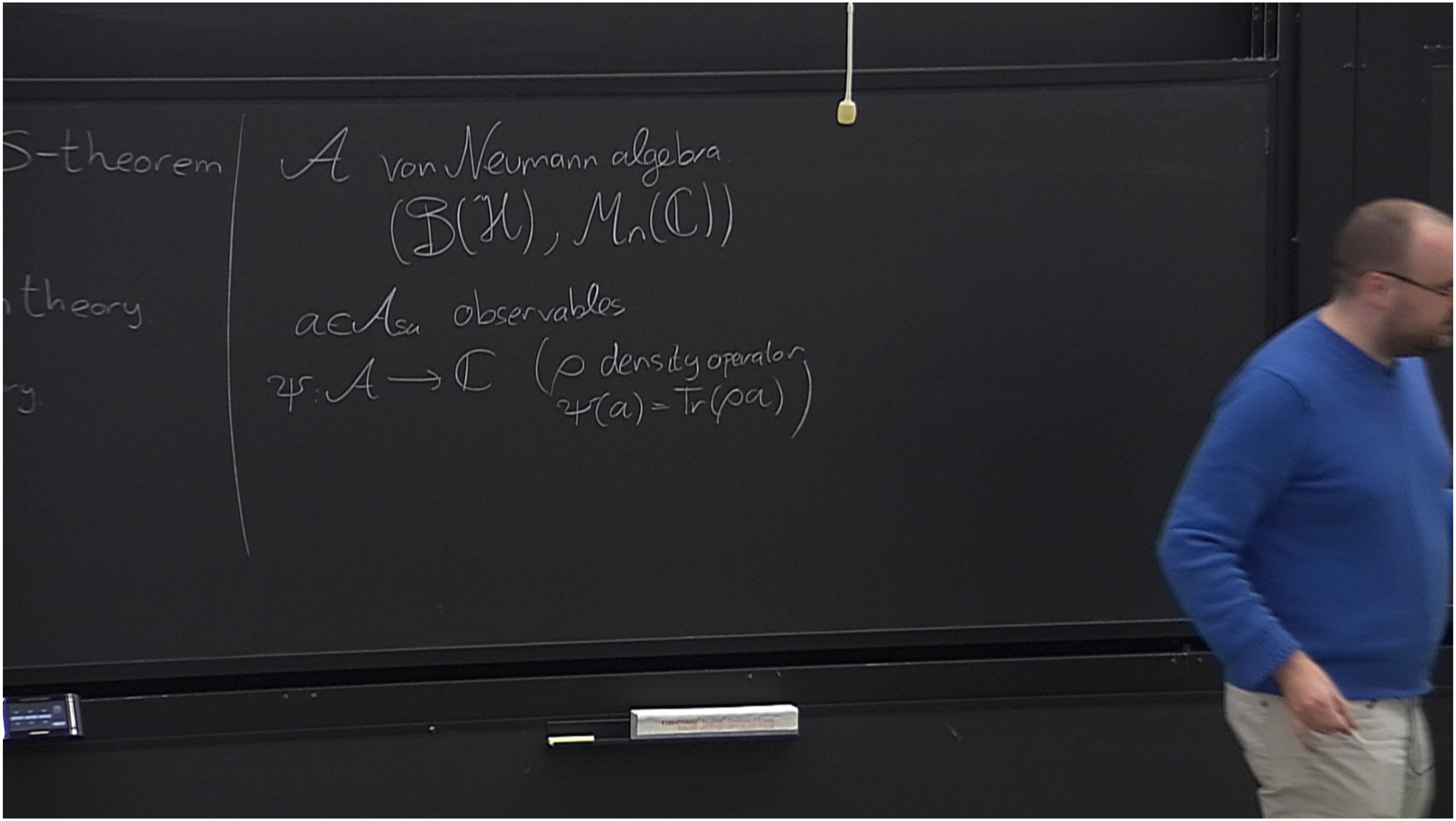
Abstract: Crudely formulated, the idea of neorealism, in the way that Chris Isham and Andreas Doering use it, means that each theory of physics, in its mathematical formulation should share certain structural properties of classical physics. These properties are chosen to allow some degree of realism in the interpretation (for example, physical variables always have values). Apart from restricting the form of physical theories, neorealism does increase freedom in the shape of physical theories in another way. Theories of physics may be interpreted in other topoi than the category of sets and functions.

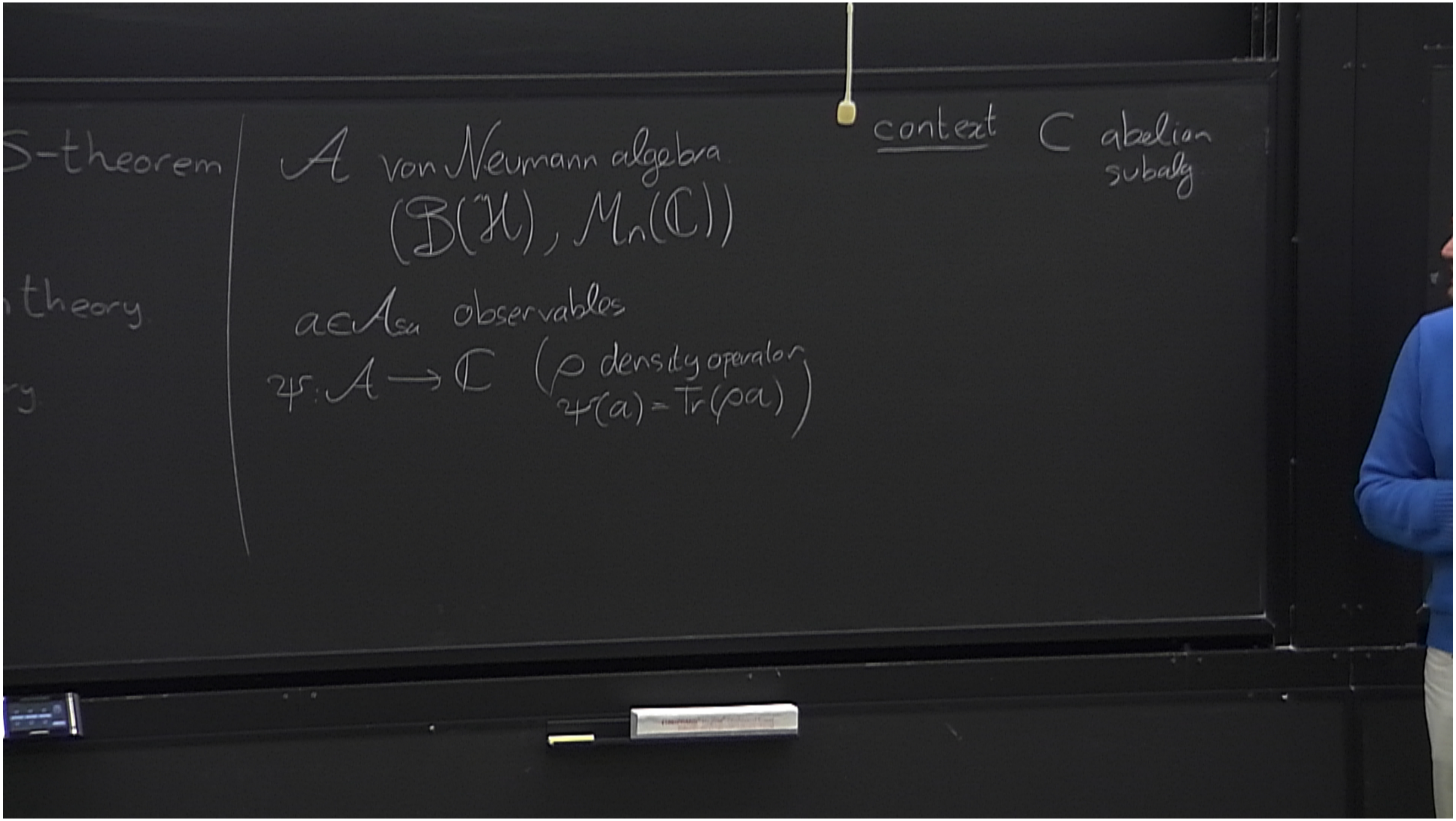
In my talk I will concentrate on two topos models for quantum theory. The contravariant model of Butterfield, Isham and Doering on the one hand, and the covariant model of Heunen, Landsman and Spitters on the other. I will argue that when we think of the topoi as generalized categories of sets (i.e. when we use the internal perspective of the topoi at hand), these two models are closely related, and both resemble classical physics.

I will assume no background knowledge in topos theory.

1. Butterfield & Isham - A topos perspective on the KS-theorem
2. Döring & Isham — What is a thing?
3. Heunen, Landsman & Spitters — A topos for algebraic quantum theory
4. W. A comparison of two topos-theoretic approaches to quantum theory
5. W. Neorealism and the internal language of topoi


S-theorem
A von Neumann algebra
(B(H)
theory
ry


S-theorem
$\mathcal{A}$ von Neumann algebra
$(\mathcal{B}(\mathcal{H}), M_n(\mathbb{C}))$
theory
$a \in \mathcal{A}_{sa}$ observables
$\psi : \mathcal{A} \longrightarrow \mathbb{C}$ (ρ density operator
$\psi(a) = \mathrm{Tr}(\rho a)$)
ry.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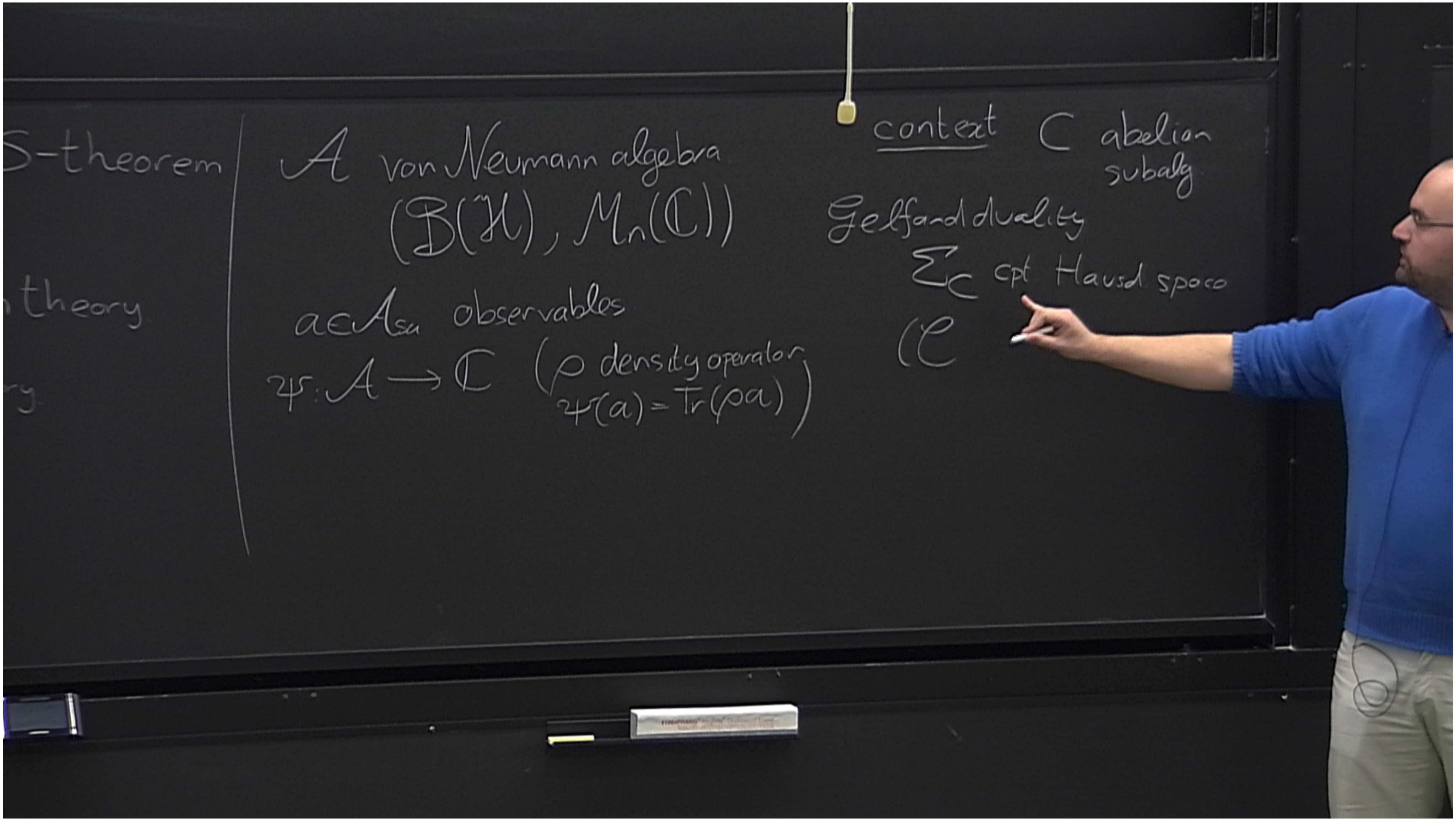
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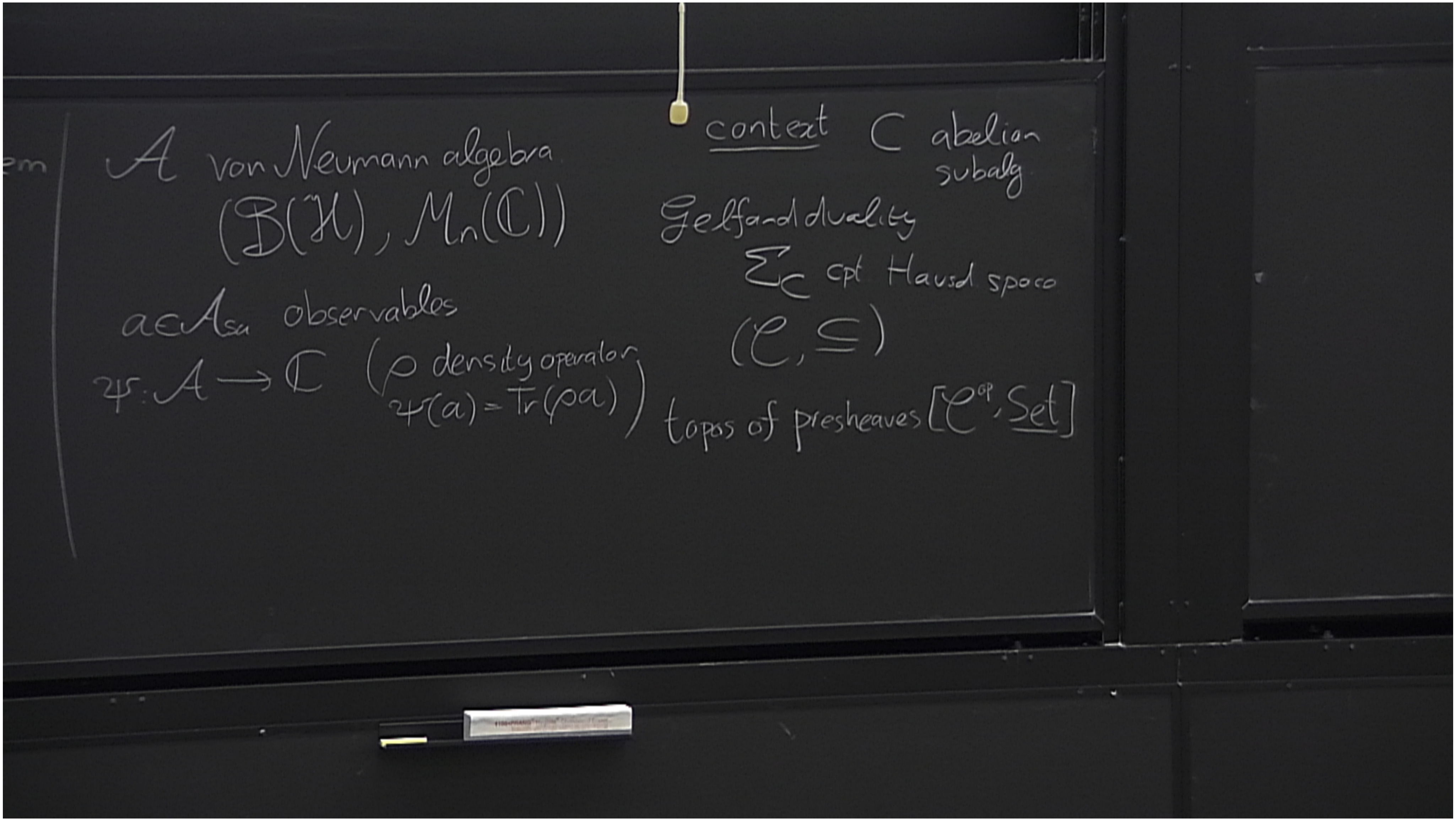
Gelfand duality
Σ_C cpt. Hausd. space
$(\mathcal{C}$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context C abelian subalg
Gelfand duality
Σ_C cpt Hausd space
$(\mathcal{C}, \subseteq)$
topos of presheaves $[\mathcal{C}^{op}, \underline{Set}]$

$\forall C \in \mathcal{C}$ set $\underline{\Sigma}(C) = \Sigma_C$

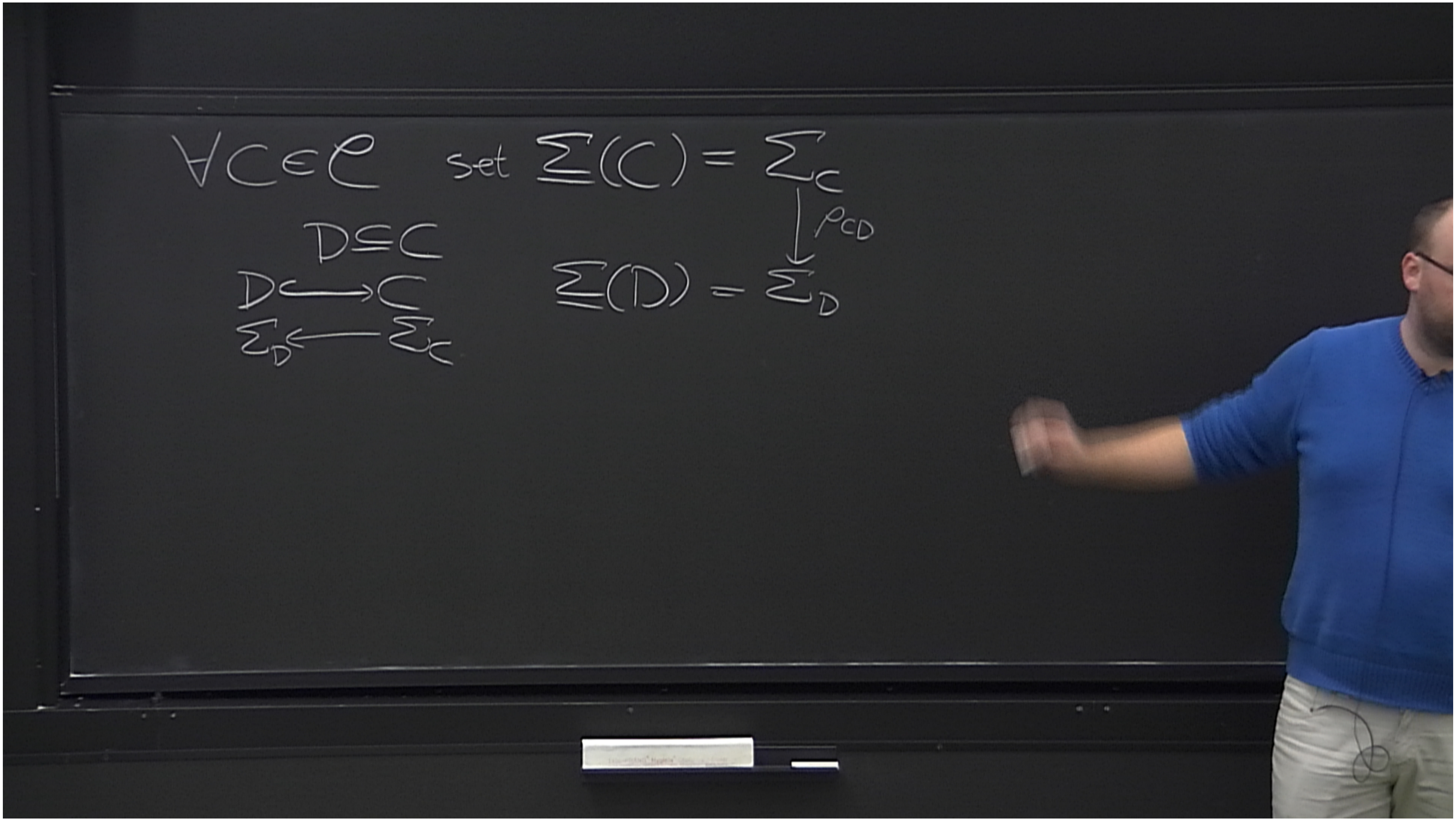
$$\begin{array}{c} \Sigma_C \\ \Big\downarrow \rho_{CD} \\ \Sigma_D \end{array}$$

$D \subseteq C$

$D \hookrightarrow C$

$\Sigma_D \longleftarrow \Sigma_C$

$\underline{\Sigma}(D) = \Sigma_D$



$\forall C \in \mathcal{C}$ set $\underline{\Sigma}(C) = \Sigma_C$
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$\underline{\Sigma}(D) = \Sigma_D$
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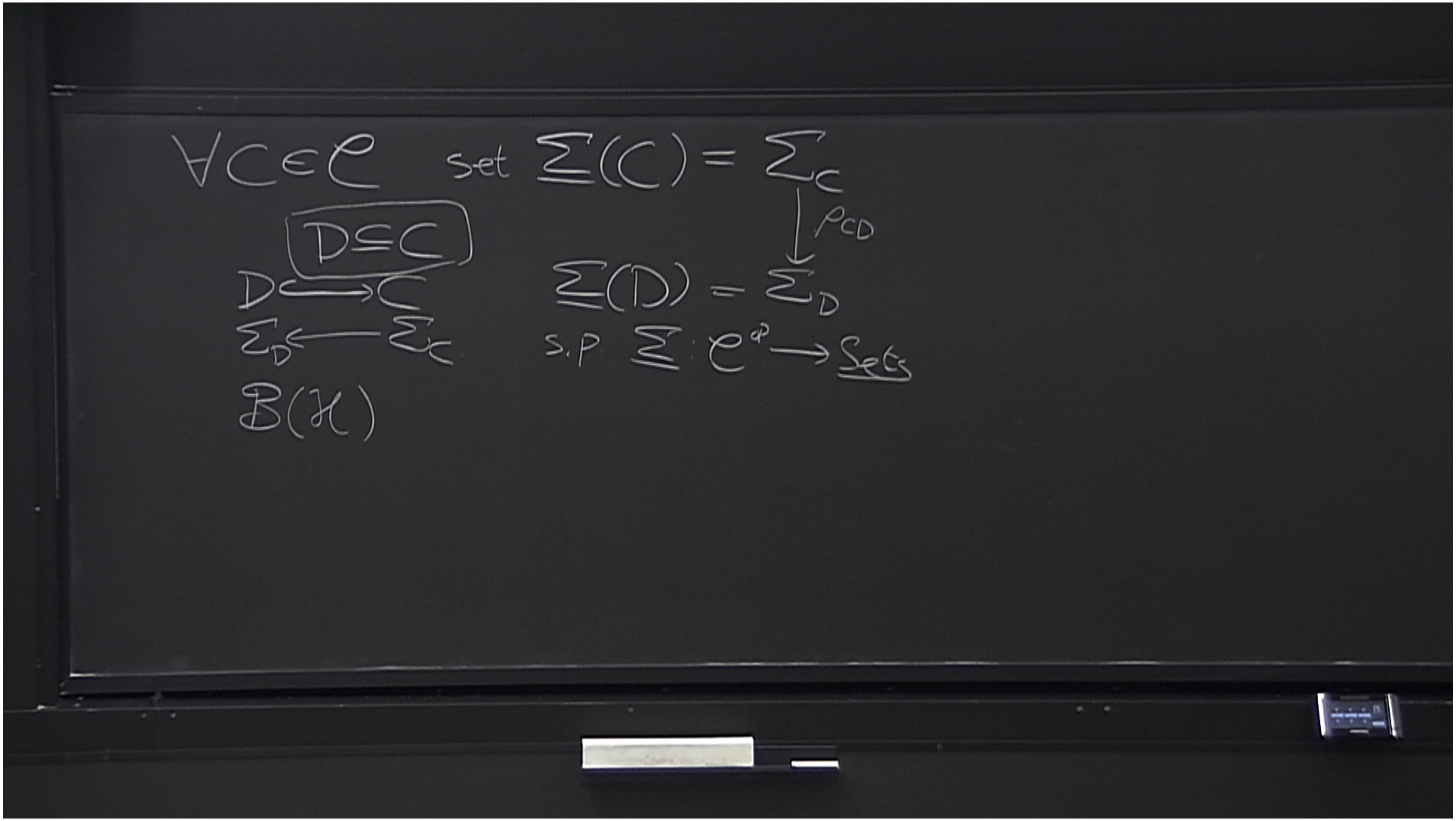
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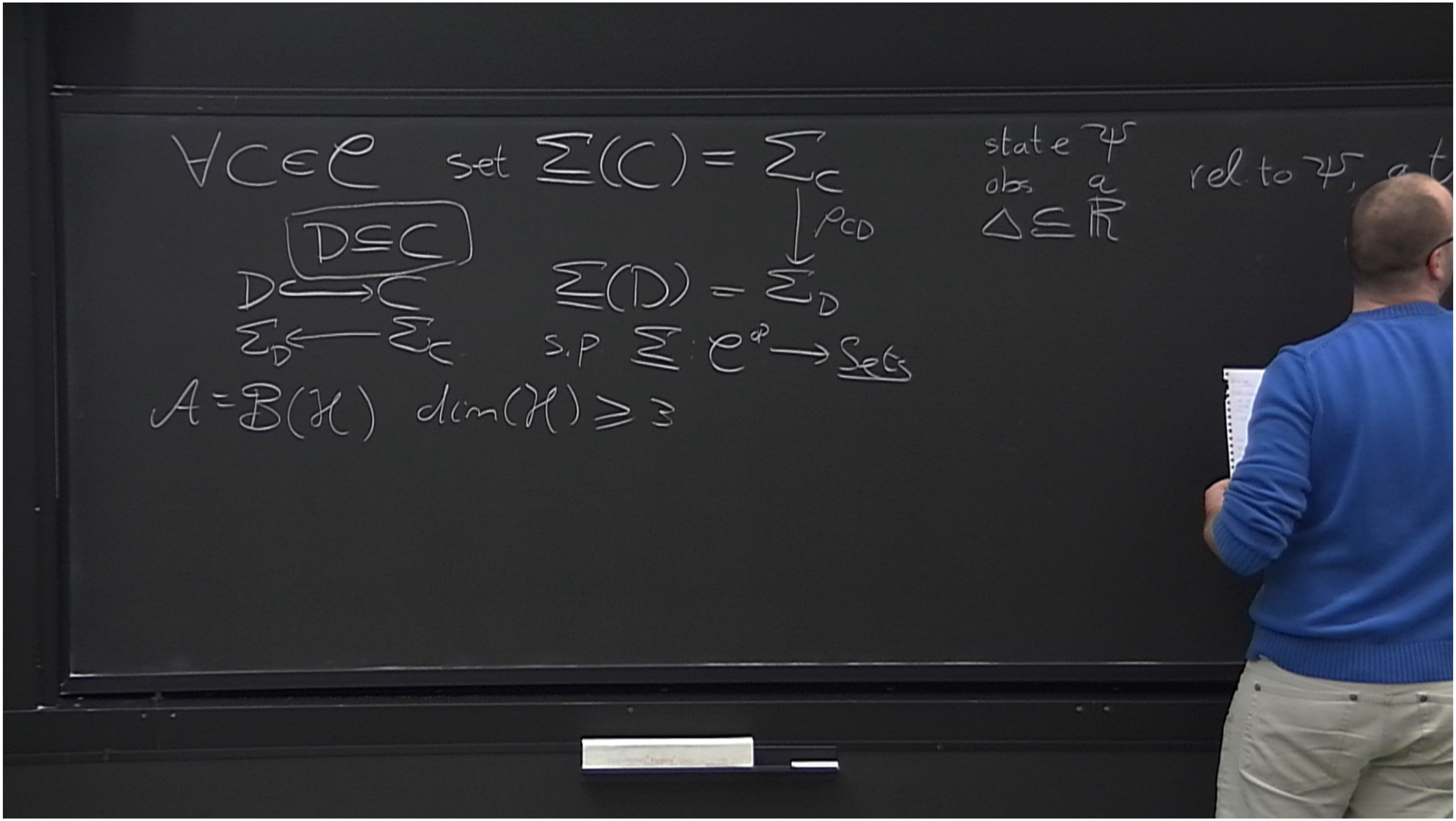
s.p $\underline{\Sigma} : \mathcal{C}^{op} \longrightarrow \underline{Sets}$
$\mathcal{B}(\mathcal{H})$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$D \hookrightarrow C$
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s.p. $\underline{\Sigma} : \mathcal{C}^{op} \to \underline{Sets}$
$\mathcal{A} = \mathcal{B}(\mathcal{H})$ $\dim(\mathcal{H}) \geq 3$
state ψ
obs a
$\Delta \subseteq \mathbb{R}$
rel. to ψ,

et $\Sigma(C) = \Sigma_C$

$$\begin{array}{c} \Sigma_C \\ \downarrow \rho_{CD} \\ \Sigma(D) = \Sigma_D \end{array}$$

s.p. $\Sigma : \mathcal{C}^{op} \longrightarrow \underline{Sets}$

$n(\mathcal{H}) \geq 3$

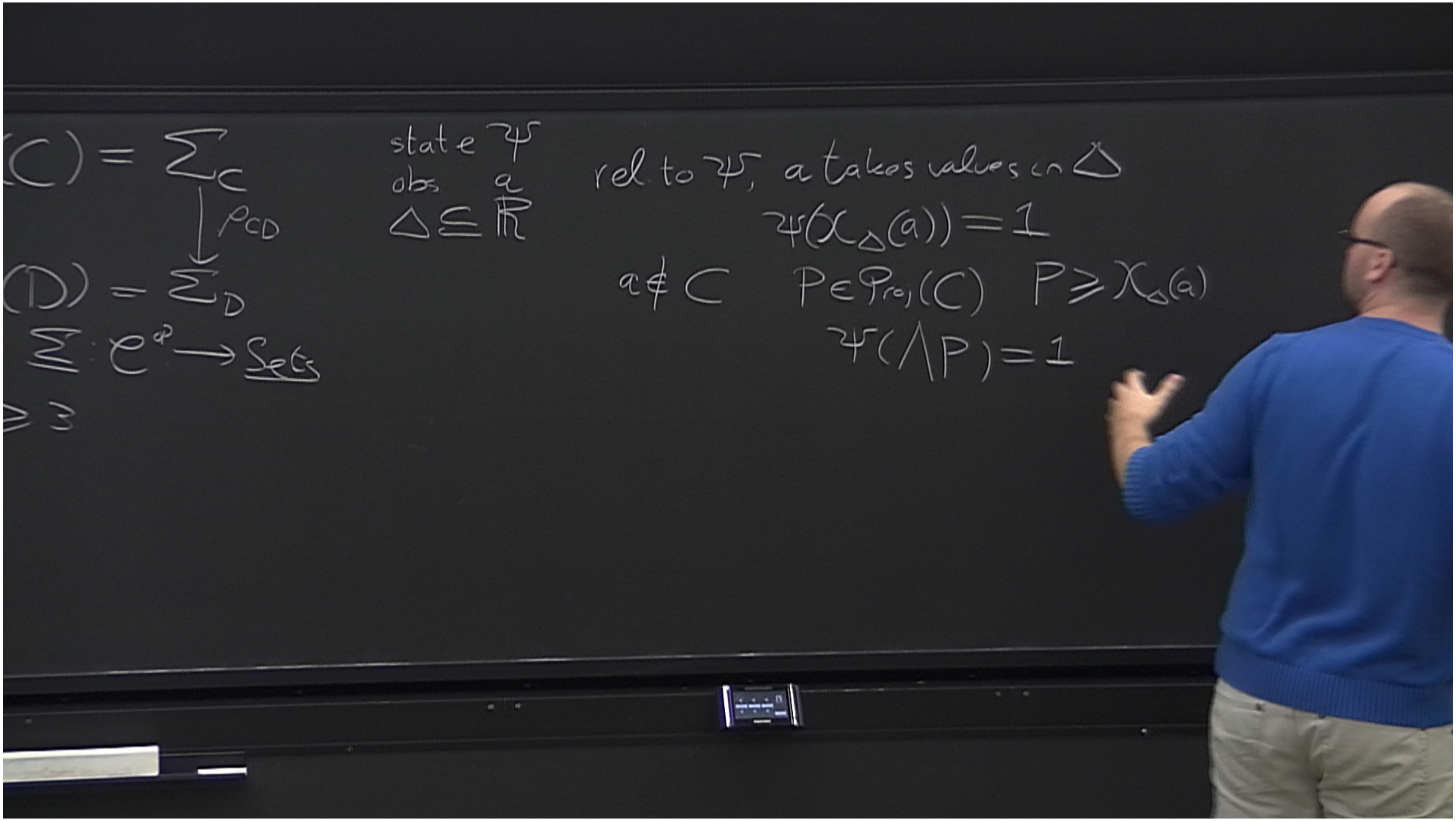
state ψ

obs a

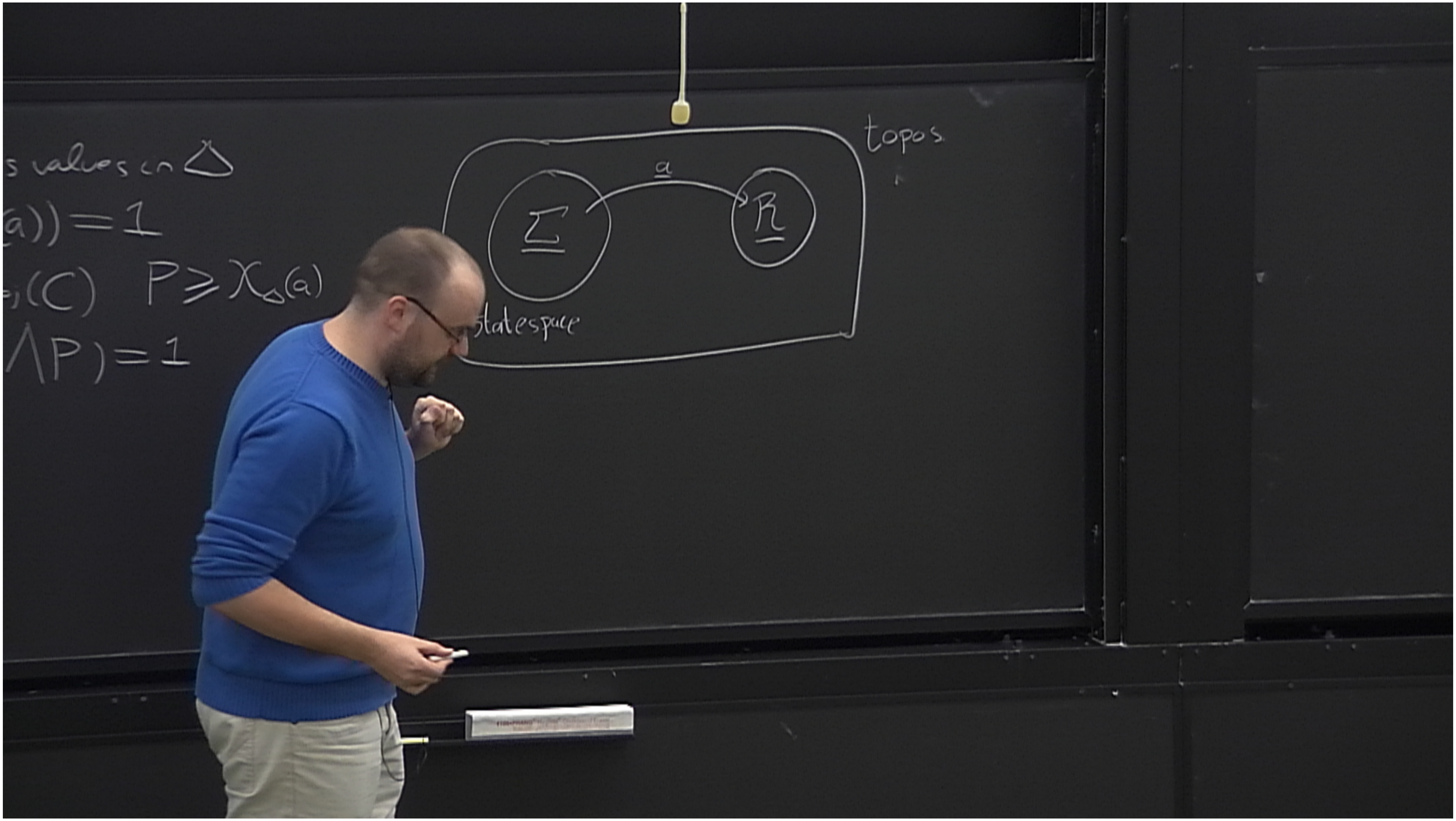
$\Delta \subseteq \mathbb{R}$

rel. to ψ, a takes values in Δ

$$\psi(\chi_\Delta(a)) = 1$$



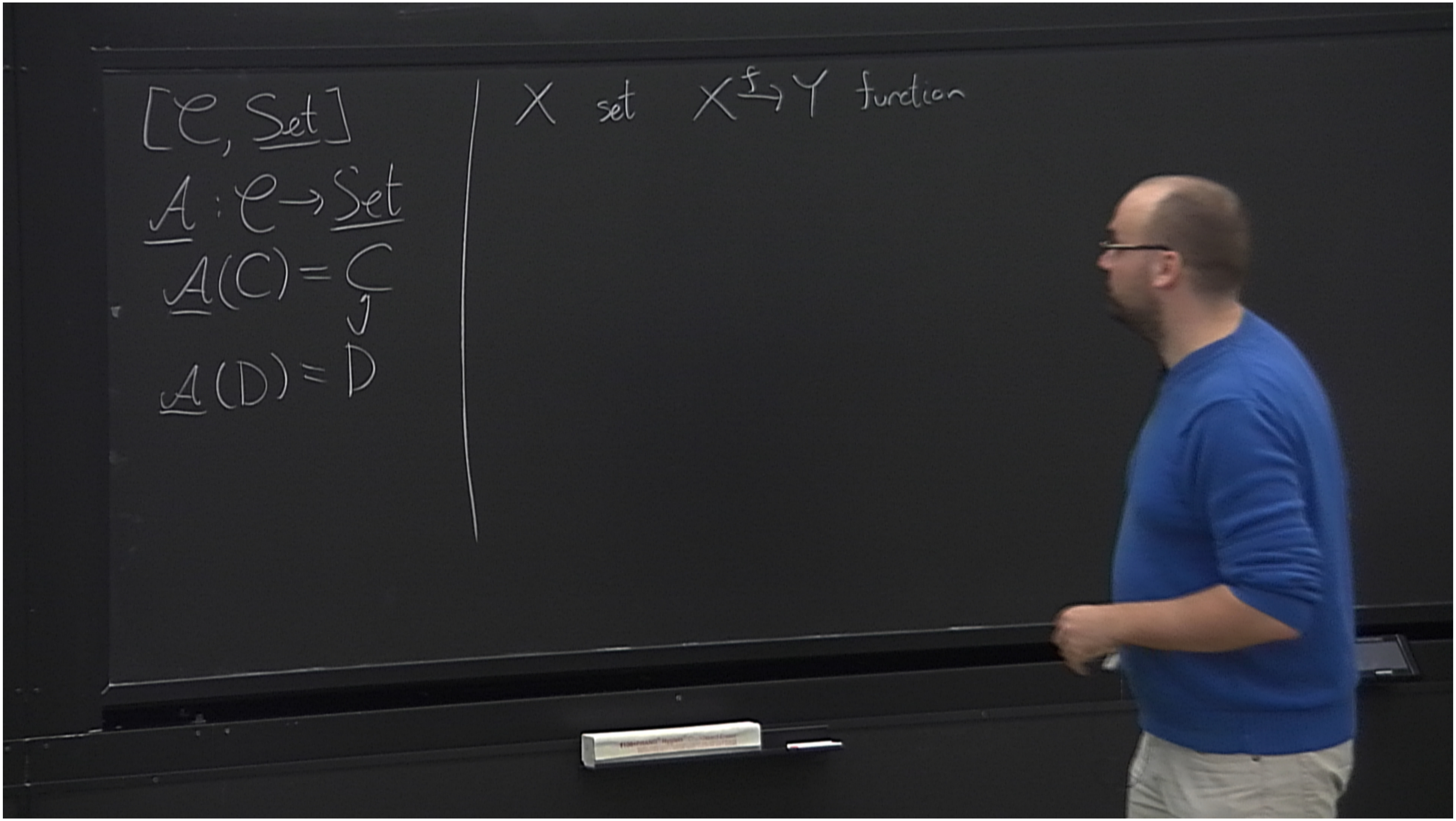
$(C) = \Sigma_C$
ρ_{CD}
$(D) = \Sigma_D$
$\Sigma : \mathcal{C}^{op} \longrightarrow \underline{Sets}$
≥ 3
state ψ
obs a
$\Delta \subseteq \mathbb{R}$
rel. to ψ, a takes values in Δ
$\psi(\chi_\Delta(a)) = 1$
$a \notin C$ $P \in \mathcal{P}_{roj}(C)$ $P \geq \chi_\Delta(a)$
$\psi(\bigwedge P) = 1$


topos
statespace

$$[\mathcal{C}, \underline{Set}]$$

$$\underline{A} : \mathcal{C} \to \underline{Set}$$

$$\begin{array}{l}
\underline{A}(C) = C \\
\qquad\qquad\ \ \downarrow \\
\underline{A}(D) = D
\end{array}$$


$[\mathcal{C}, \underline{Set}]$
$\underline{A} : \mathcal{C} \to \underline{Set}$
$\underline{A}(C) = C$
$\underline{A}(D) = D$
X set
$X \xrightarrow{f} Y$ function

$[\mathcal{C}, \underline{Set}]$

$\underline{A} : \mathcal{C} \to \underline{Set}$

$$\begin{array}{l} \underline{A}(C) = C \\ \qquad\quad\;\; \downarrow \\ \underline{A}(D) = D \end{array}$$

X set $\quad X \xrightarrow{f} Y$ function

$x \in X \quad 1 \xrightarrow{x} X$

$[\mathcal{C}, \underline{Set}]$

$\underline{A} : \mathcal{C} \to \underline{Set}$

$$\begin{array}{l} \underline{A}(C) = C \\ \qquad\qquad\ \ \downarrow \\ \underline{A}(D) = D \end{array}$$

X set $\quad X \xrightarrow{f} Y$ function

$x \in X \quad Y \xrightarrow{x} X$

$OX \subseteq PX, \ \emptyset \in OX \wedge X \in OX \wedge \ldots$

$[\mathcal{C}, \underline{Set}]$

$\underline{A} : \mathcal{C} \to \underline{Set}$

$\underline{A}(C) = C$

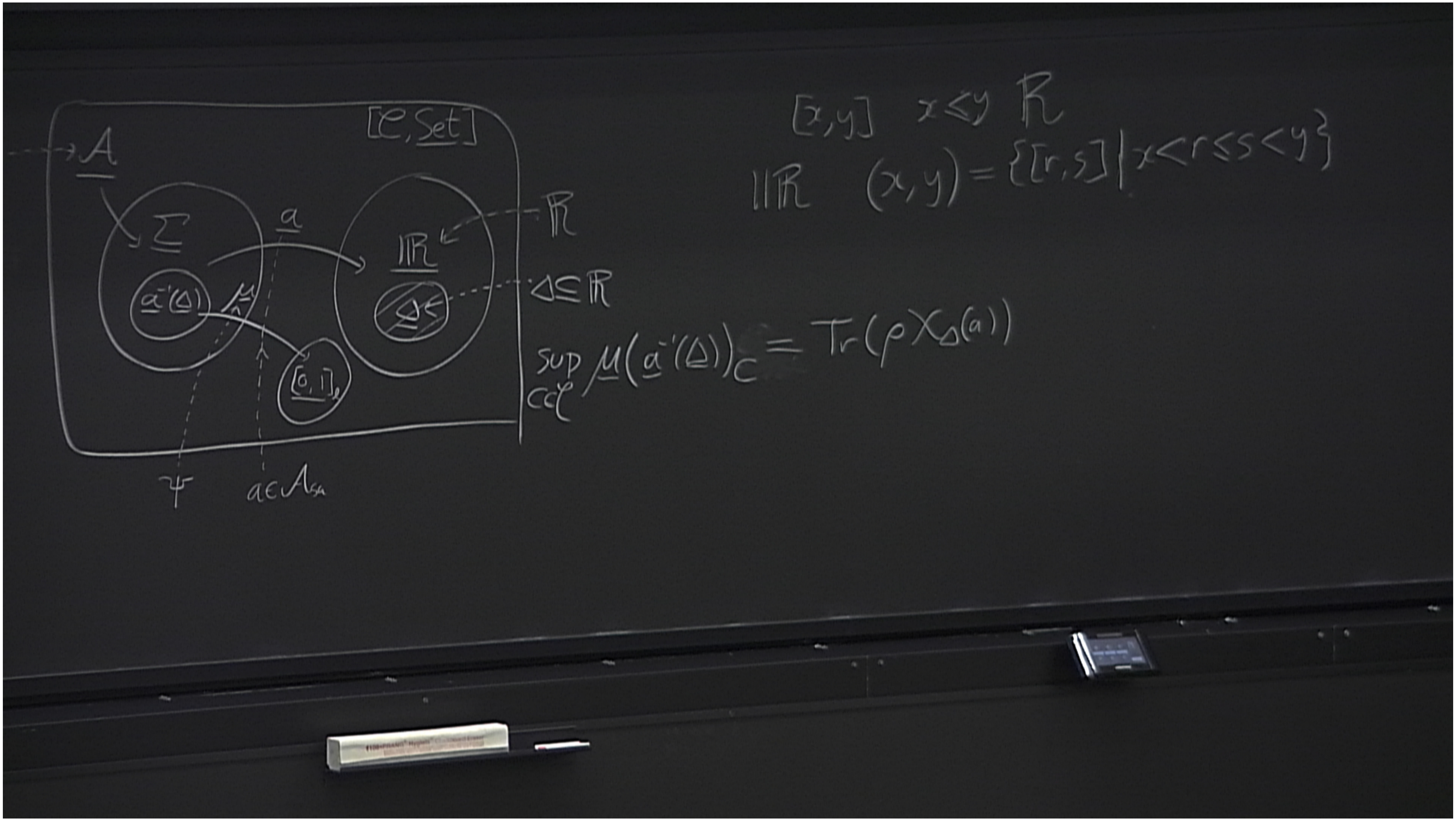
$\downarrow$

$\underline{A}(D) = D$

X set $X \xrightarrow{f} Y$ function

$x \in X$ $Y \xrightarrow{x} X$

$\mathcal{O}X \subseteq \mathcal{P}X$, $\emptyset \in \mathcal{O}X \wedge X \in \mathcal{O}X \wedge \ldots$


[x,y] x≤y R
IR (x,y) = {[r,s] | x<r≤s<y}
[C, Set]
sup μ(a⁻¹(Δ))_C = Tr(ρ χ_Δ(a))
C∈C
Δ⊆R
ψ
a∈A_sa