

Title: Large N Chern-Simons-matter theories and three dimensional bosonization

Date: Mar 12, 2013 02:30 PM

URL: <http://pirsa.org/13030106>

Abstract: I will discuss the conformal theories of N complex scalars or fermions in $2+1$ dimensions, coupled to a $U(N)$ Chern-Simons (CS) theory at level k . In the large N limit these theories have a high-spin symmetry, and, as I will review, they are dual to Vasiliev's high-spin gravity theories on four dimensional anti-de Sitter space. Maldacena and Zhiboedov showed that the high-spin symmetry determines the 2-point and 3-point functions of these theories at large N , up to two parameters. The duality to Vasiliev's gravity suggests that there is some mapping between the CS+scalar and CS+fermion theories (when one adds a quartic coupling to the scalar theories, to flow to their "critical fixed point"). We compute explicitly the large N limit of some 2-point and 3-point functions in these theories. We show that the results match with the general results of Maldacena and Zhiboedov, and they are consistent with an equivalence between the critical theory of N scalars coupled to a $U(N)$ CS theory at level k , and the theory of k fermions coupled to a $U(k)$ CS theory at level $(N-1/2)$. We conjecture that this large N equivalence may be an exact duality between the scalar and fermion theories also at finite N , thus providing a bosonization of the fermionic theory. Similar results hold for real scalars (fermions) when $U(N)$ is replaced by $O(N)$.

Large N Chern-Simons-matter theories and 3d bosonization

- 1) Introduction + Motivations
- 2) Review of free/critical vector models $\leftrightarrow$ high spin gravity
- 3) CS-matter theories at large N: spectrum and fixed points
- 4) Computation of 2-point and 3-point functions
- 5) Bosonization in three dimensions

$$\mathcal{L}_b = \frac{ik}{4\pi} \text{Tr}(A \wedge A + \frac{2}{3} A^3) + |\mathcal{D}_\mu \phi|^2$$

$$\left(+ \frac{\lambda_4}{2N} (|\phi|^2)^2 + \frac{\lambda_6}{6N^2} (|\phi|^2)^3 \right)$$

$$\mathcal{L}_f = \frac{ik}{4\pi} \text{Tr}(A \wedge A + \frac{2}{3} A^3) + \bar{\Psi} \gamma^\mu \mathcal{D}_\mu \Psi$$

$$G = U(N)$$

$$(\mathcal{D}_\mu \phi_i)^2 \quad i=1, \dots, N$$

$U(N)$ -singlet

$$J_0 = \phi^\dagger \phi \quad \Delta=1$$

$$J_S = \phi^\dagger \partial_{\mu_1} \dots \partial_{\mu_S} \phi \quad \Delta=S+1$$

3d CFT $\leftrightarrow$ quantum gravity
on AdS_4

$N \rightarrow \infty \leftrightarrow$ weakly coupled
 $g \sim \frac{1}{N}$

$\Delta_{S \gg 2} \not\rightarrow 1 ? \leftarrow L \gg l_s$ weakly curved

Main references:

OA, Gur-Ari, Yacoby	1110.4382
Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin	1110.4386
Maldacena, Zhiboetov	$\begin{cases} 1112.1016 \\ 1204.3882 \end{cases}$
OA, Gur-Ari, Yacoby	1207.4593
Chang, Minwalla, Sharma, Yin	1207.4485
Gur-Ari, Yacoby	1211.1866
OA, Giombi, Gur-Ari, Maldacena, Yacoby	1211.4843
Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama	1301.6169

3d CFT $\leftrightarrow$ quantum gravity
on AdS_4

$N \rightarrow \infty \leftrightarrow$ weakly coupled
 $g \sim \frac{1}{N}$

$\Delta_{S \geq 2} \gg 1 ? \leftarrow \frac{L \gg l_s}{AdS_4}$ weakly curved

massless spin S
 $S = 1, 2, 3, \dots$

scalar $m^2 = -\frac{2}{L_{AdS}^2}$

Main references:

OA, Gur-Ari, Yacoby 1110.4382
Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin 1110.4386
Maldacena, Zhiboetov $\begin{cases} 1112.1016 \\ 1204.3882 \end{cases}$
OA, Gur-Ari, Yacoby 1207.4593
Chang, Minwalla, Sharma, Yin 1207.4485
Gur-Ari, Yacoby 1211.1866
OA, Giombi, Gur-Ari, Maldacena, Yacoby 1211.4047
Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama 1305.1866

Large N Chern-Simons-matter theories and 3d bosonization

- 1) Introduction + Motivations
- 2) Review of free/critical vector models $\leftrightarrow$ high spin gravity
- 3) CS-matter theories at large N: spectrum and fixed points
- 4) Computation of 2-point and 3-point functions
- 5) Bosonization in three dimensions

$$\mathcal{L}_b = \frac{ik}{4\pi} \text{Tr}(A \wedge dA + \frac{2}{3} A^3) + |\mathcal{D}_\mu \phi|^2$$

$$\left(+ \frac{\lambda_4}{2N} (|\phi|^2)^2 + \frac{\lambda_6}{6N^2} (|\phi|^2)^3 \right)$$

$$\mathcal{L}_f = \frac{ik}{4\pi} \text{Tr}(A \wedge dA + \frac{2}{3} A^3) + \bar{\Psi} \gamma^\mu \mathcal{D}_\mu \Psi$$

$$G = U(N)$$

$$(\partial_\mu \phi_i)^2 \quad i=1, \dots, N$$

$U(N)$ -singlet

$$J_0 = \phi^\dagger \phi \quad \Delta=1, \text{ parity-even}$$

$$J_S = \phi^\dagger \partial_{\mu_1} \dots \partial_{\mu_S} \phi \quad \Delta=S+1$$

3d CFT $\leftrightarrow$ quantum gravity
on AdS_4

$N \rightarrow \infty \leftrightarrow$ weakly coupled
 $g \sim \frac{1}{N}$

$\Delta_{S \gg 2} \gg 1 ? \leftarrow \frac{L \gg l_s}{AdS_4}$ weakly curved

massless spin S
 $S=1, 2, 3, \dots$

scalar $m^2 = -\frac{2}{L_{AdS}^2}$
parity-even

Main references:

OA, Gur-Ari, Yacoby	1110.4382
Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin	1110.4386
Maldacena, Zhiboedov	$\begin{cases} 1112.1016 \\ 1204.3882 \end{cases}$
OA, Gur-Ari, Yacoby	1207.4593
Chang, Minwalla, Sharma, Yin	1207.4485
Gur-Ari, Yacoby	1211.1866
OA, Giombi, Gur-Ari, Maldacena, Yacoby	1211.4843
Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama	1301.6169

motivations

U(1) vector model $\longleftrightarrow$ Vasiliev type A

1) Condensed matter

2) SUSY generalizations

non-SUSY

Motivations

U(1) vector model $\longleftrightarrow$ Vasiliev type A

$\Delta_\phi = 1, \Delta_S = S+1$

Condensed matter

SUSY generalizations

non-SUSY

motivations

$U(1)$ vector model $\longleftrightarrow$ Vasiliev type A

$\Delta_\phi = 1, \Delta_S = S+1$

Condensed matter

SUSY generalization

non-SUSY

U(1) vector model $\longleftrightarrow$ Vasiliev type A

$$\Delta_0 = 1, \Delta_S = S + 1$$

$$\downarrow R + (\mathcal{J}_0)^2$$

$$\mathcal{L} = (\partial_\mu \phi)^2 + (|\phi|^2)^2$$

critical U(1) model

$\longleftrightarrow$ Vasiliev type A

$$\Delta_0 = 2 + O(\frac{1}{N}), \Delta_S = S + 1 + O(\frac{1}{N})$$

Large N Chern-Simons-matter theories and 3d bosonization

- 1) Introduction + Motivations
- 2) Review of free/critical vector models $\leftrightarrow$ high spin gravity
- 3) CS-matter theories at large N: Spectrum and fixed points
- 4) Computation of 2-point and 3-point functions
- 5) Bosonization in three dimensions

$$\mathcal{L}_b = \frac{ik}{4\pi} \text{Tr}(A \wedge dA + \frac{2}{3} A^3) + |D_\mu \phi|^2 + \frac{\lambda_4}{2N} (|\phi|^2)^2 + \frac{\lambda_6}{6N^2} (|\phi|^2)^3$$

$$\mathcal{L}_f = \frac{ik}{4\pi} \text{Tr}(A \wedge dA + \frac{2}{3} A^3) + \bar{\Psi} \gamma^\mu D_\mu \Psi$$

$$G = U(N)$$

$$\mathcal{L} = \bar{\Psi}_i \gamma^\mu \partial_\mu \Psi_i \quad i=1, \dots, N$$

$U(N)$ -singlet

$$J_0 = \phi^\dagger \phi \quad \Delta=1, \text{ parity-even}$$

$$J_s = \phi^\dagger \partial_{\mu_1} \dots \partial_{\mu_s} \phi \quad \Delta=s+1$$

Large N Chern-Simons-matter theories and 3d bosonization

- 1) Introduction + Motivations
- 2) Review of free/critical vector models $\leftrightarrow$ high spin gravity
- 3) CS-matter theories at large N: Spectrum and fixed points
- 4) Computation of 2-point and 3-point functions
- 5) Bosonization in three dimensions

$$\mathcal{L}_b = \frac{ik}{4\pi} \text{Tr}(A \wedge dA + \frac{2}{3} A^3) + |D_\mu \phi|^2 + \frac{\lambda_4}{2N} (|\phi|^2)^2 + \frac{\lambda_6}{6N^2} (|\phi|^2)^3$$

$$\mathcal{L}_f = \frac{ik}{4\pi} \text{Tr}(A \wedge dA + \frac{2}{3} A^3) + \bar{\Psi} \gamma^\mu D_\mu \Psi$$

$$G = U(N)$$

$$\mathcal{L} = \bar{\Psi}_i \gamma^\mu \partial_\mu \Psi_i \quad i=1, \dots, N$$

$U(N)$ -singlet

$$J_0 = \bar{\Psi} \Psi \quad \Delta = 2, \text{ parity-odd}$$

$$J_5 = \bar{\Psi} \gamma^5 \Psi \quad \Delta = 5+1$$

3d CFT $\leftrightarrow$ quantum gravity
on AdS_4

$N \rightarrow \infty \leftrightarrow$ weakly coupled
 $g \sim \frac{1}{N}$

$\Delta_{S \geq 2} \not\rightarrow 1 ? \leftarrow \frac{L \gg l_s}{AdS_4}$ weakly curved

massless spin S
 $S = 1, 2, 3, \dots$

scalar $m^2 = -\frac{2}{L_{AdS}^2}$
parity-odd

Main references:

OA, Gur-Ari, Yacoby	1110.4382
Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin	1110.4386
Maldacena, Zhiboedov	$\begin{cases} 1112.1016 \\ 1204.3882 \end{cases}$
OA, Gur-Ari, Yacoby	1207.4593
Chang, Minwalla, Sharma, Yin	1207.4485
Gur-Ari, Yacoby	1211.1866
OA, Giombi, Gur-Ari, Maldacena, Yacoby	1211.4843
Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama	1301.6169

U(1) vector model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 1, \Delta_S = S+1, g \sim \frac{1}{\sqrt{N}}$

$$\downarrow R + (J_0)^2$$

$\mathcal{L} = (\partial_\mu \phi)^2 + (|\phi|^2)^2 \longleftrightarrow$ Vasiliev type A
 critical U(1) model $\Delta_0 = 2 + O(\frac{1}{N}), \Delta_S = S+1 + O(\frac{1}{N})$

N free fermions $\longleftrightarrow$ Vasiliev type A

U(1) vector model

Vasiliev type A

$$\Delta_0 = 1, \Delta_S = S+1, g \sim \frac{1}{\sqrt{N}}$$

$$\downarrow R + (J_0)^2$$

$$\mathcal{L} = (\partial_\mu \phi)^2 + (|\phi|^2)^2$$

critical U(1) model

Vasiliev type A

$$\Delta_0 = 2 + o\left(\frac{1}{N}\right), \Delta_S = S+1 + o\left(\frac{1}{N}\right)$$

Gross-Neveu model

Vasiliev type B

$$\Delta_0 = 1 + o\left(\frac{1}{N}\right)$$

$$\Delta_S = S+1 + o\left(\frac{1}{N}\right)$$

N free fermions

Vasiliev B

$$\Delta_0 = 2$$

$U(1)$ vector model

Vasiliev type A

$$\Delta_0 = 1, \Delta_S = S+1, g \sim \frac{1}{\sqrt{N}}$$

$$\downarrow R + (\bar{J}_0)^2$$

$$\mathcal{L} = (\partial_\mu \phi)^2 + (|\phi|^2)^2$$

critical $U(1)$ model

Vasiliev type A

$$\Delta_0 = 2 + o(\frac{1}{N}), \Delta_S = S+1 + o(\frac{1}{N})$$

Gross-Neveu
model
 $\bar{\Psi}\Psi$

Vasiliev type B

$$\Delta_0 = 1 + o(\frac{1}{N})$$

$$\Delta_S = S+1 + o(\frac{1}{N})$$

$$\downarrow$$

$$\mathcal{L} + \sigma \bar{\Psi}\Psi$$

N free fermions $\Rightarrow$

Vasiliev
type B

$$\Delta_0 = 2, \Delta_S = S+1$$

free
U(N) vector model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 1, \Delta_S = S+1, g \sim \frac{1}{\sqrt{N}}$

$$\downarrow R + (J_0)^2$$

$\mathcal{L} = (\partial_\mu \phi)^2 + (|\phi|^2)^2$
 critical U(N) model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 2 + O(\frac{1}{N}), \Delta_S = S+1 + O(\frac{1}{N})$

Gross-Neveu model $\longleftrightarrow$ Vasiliev type B
 $\Delta_0 = 1 + O(\frac{1}{N})$
 $\Delta_S = S+1 + O(\frac{1}{N})$
 $\bar{\Psi}\Psi$

$$\downarrow \int \bar{\Psi}\Psi$$

N free fermions $\Rightarrow$ 3
 $\Delta_0 = 1$

free $\nearrow$
 $U(N)$ vector model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 1, \Delta_S = S+1, g \sim \frac{1}{\sqrt{N}}$

$$\downarrow R + (J_0)^2$$

$\mathcal{L} = (\partial_\mu \phi)^2 + (|\phi|^2)^2$
critical $U(N)$ model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 2 + O(\frac{1}{N}), \Delta_S = S+1 + O(\frac{1}{N})$

$$\lambda = \frac{N}{R}$$

$\sigma ?$
Gross-Neveu model $\longleftrightarrow$ Vasiliev type B
 $\bar{\Psi}\Psi$
 $\Delta_0 = 1 + O(\frac{1}{N}), \Delta_S = S+1 + O(\frac{1}{N})$

$$\downarrow \int (1 + \sigma \bar{\Psi}\Psi)$$

N free fermions $\Rightarrow$ Vasiliev type B
 $\Delta_0 = 2, \Delta_S = S+1$

free $\nearrow$
 $U(N)$ vector model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 1, \Delta_S = S+1, g \sim \frac{1}{\sqrt{N}}$

$\downarrow R + (J_0)^2$

$\mathcal{L} = (\partial_\mu \phi)^2 + (|\phi|^2)^2 \longleftrightarrow$ Vasiliev type A
 critical $U(N)$ model $\Delta_0 = 2 + o(\frac{1}{N}), \Delta_S = S+1 + o(\frac{1}{N})$

$$\lambda = \frac{N}{R}$$

$\sigma ? \nearrow$
 Gross-Neveu model $\longleftrightarrow$ Vasiliev type B
 $\bar{\Psi}\Psi$ $\Delta_0 = 1 + o(\frac{1}{N}), \Delta_S = S+1 + o(\frac{1}{N})$

$\downarrow \int \bar{\Psi}\Psi$

N free fermions $\Rightarrow$ Vasiliev type B
 $\Delta_0 = 2, \Delta_S = S+1$

free $\nearrow$
 $U(1)$ vector model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 1, \Delta_S = S+1, g \sim \frac{1}{\sqrt{N}}$

$\downarrow R + (\mathcal{J}_0)^2$

$\mathcal{L} = (\partial_\mu \phi)^2 + (|\phi|^2)^2$
 critical $U(1)$ model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 2 + o(\frac{1}{N}), \Delta_S = S+1 + o(\frac{1}{N})$

$\lambda = \frac{N}{R}$

$\theta = 0$
 $\nearrow$

Gross-Neveu
 model
 $\bar{\Psi}\Psi$

$\nearrow 1 + \sigma \bar{\Psi}\Psi$

N free fermions
 $\Delta_0 = \alpha$

$\theta = \frac{\pi}{2}$
 $\nearrow$

Vasiliev type B
 $\Delta_0 = 1 + o(\frac{1}{N})$
 $\Delta_S = S+1 + o(\frac{1}{N})$

Vasiliev type B
 $\Delta_0 = 1$

free $\nearrow$
 $U(1)$ vector model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 1, \Delta_S = S+1, g \sim \frac{1}{\sqrt{N}}$

$$\downarrow R + (\mathcal{J}_0)^2$$

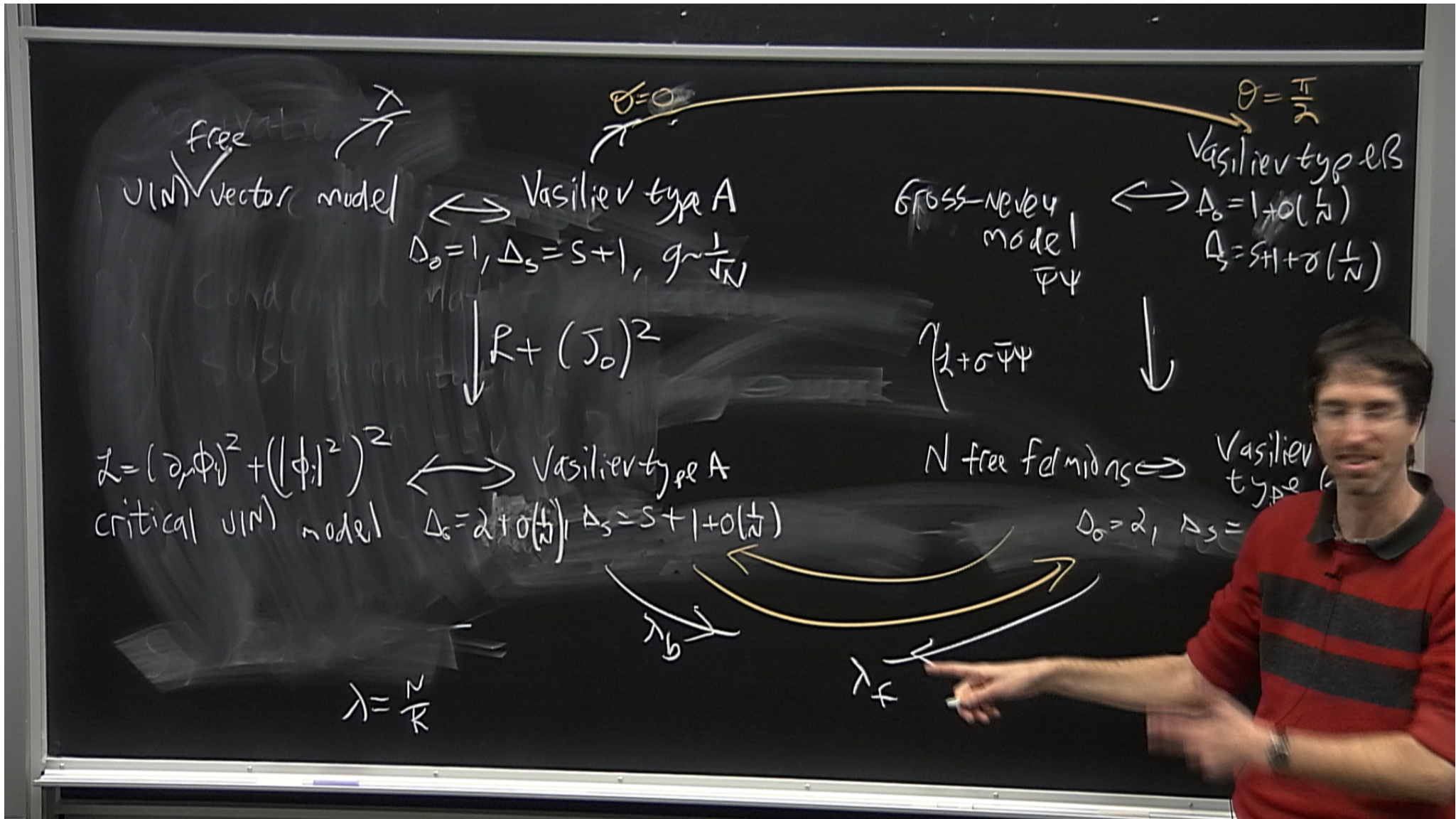
$\mathcal{L} = (\partial_\mu \phi)^2 + (|\phi|^2)^2$
 critical $U(1)$ model $\longleftrightarrow$ Vasiliev type A
 $\Delta_0 = 2 + o(\frac{1}{N}), \Delta_S = S+1 + o(\frac{1}{N})$

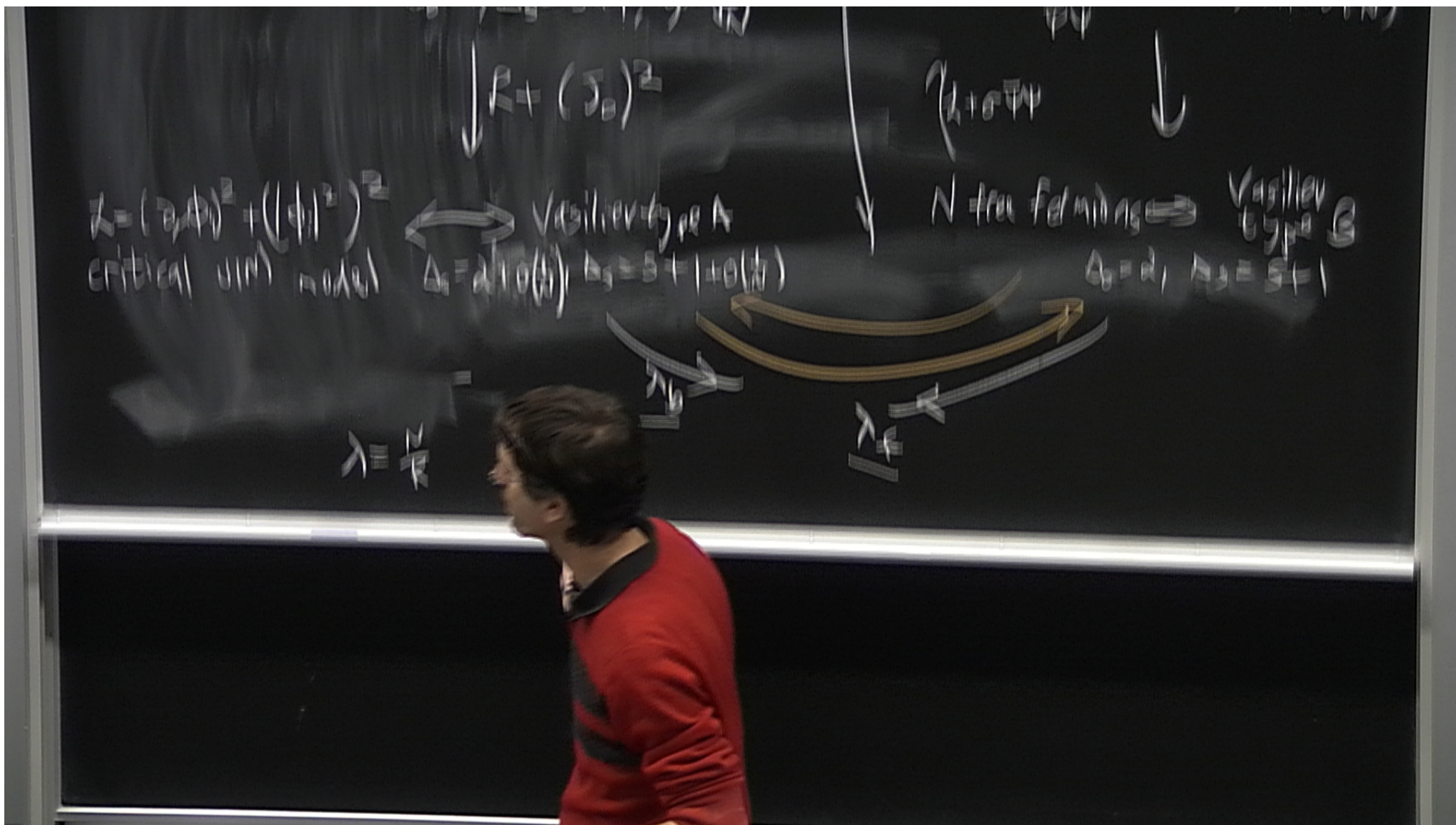
$$\lambda = \frac{N}{R}$$

$\theta = 0$ $\nearrow$ $\theta = \frac{\pi}{2}$
 Gross-Neveu model $\longleftrightarrow$ Vasiliev type B
 $\Delta_0 = 1 + o(\frac{1}{N}), \Delta_S = S+1 + o(\frac{1}{N})$

$$\downarrow \int (1 + \sigma \bar{\Psi} \Psi)$$

N free fermions $\longleftrightarrow$ Vasiliev type B
 $\Delta_0 = 2, \Delta_S = S+1$





3) CFTs ?

Fermionic: $N, K \checkmark$

Bosonic: $\beta_{\lambda_b}(\lambda, \lambda_b) = 0 \quad ?$

$\lambda = 0: \beta_{\lambda_b} = 0$

$\lambda \neq 0: \beta_{\lambda_b} = 0$



3) CFT_S ?

Fermionic: $N, K \checkmark$

Bosonic: $\beta_{\lambda_b}(\lambda, \lambda_b) = 0 \quad ?$

$\lambda = 0: \beta_{\lambda_b} = 0$

$\lambda \neq 0: \beta_{\lambda_b} = 0$

$N \rightarrow \infty \quad \lambda_b$ is main



3) CFT_S ?

Fermionic: $N, K \checkmark$

Bosonic: $\beta_{\lambda_6}(\lambda, \lambda_6) = 0 \quad ?$

$$\lambda = 0: \beta_{\lambda_6} = O\left(\frac{1}{N}\right)$$

$$\lambda \neq 0: \beta_{\lambda_6} = O\left(\frac{1}{N}\right)$$

$N \rightarrow \infty$ λ_6 is marginal. λ, λ_6 CFT $\checkmark$



3) CFTs?

Fermionic: $N, K \checkmark$

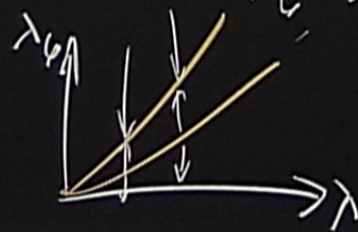
Bosonic: $\beta_{\lambda_6}(\lambda, \lambda_6) = 0 \quad ? \quad \text{CFT} \quad \checkmark$

$$\lambda = 0: \quad \beta_{\lambda_6} = O\left(\frac{1}{N}\right)$$

$$\lambda \neq 0: \quad \beta_{\lambda_6} = O\left(\frac{1}{N}\right)$$



$N \rightarrow \infty$ λ_6 is marginal.



$$\lambda \propto \lambda^2$$

λ, λ_6 CFT $\checkmark$

3) CFTs ?

Fermionic: $N, K \checkmark$

Bosonic: $\beta_{\lambda_6}(\lambda, \lambda_6) = 0 \quad ?$

CFT $\times$

$$\lambda = 0: \quad \beta_{\lambda_6} = O\left(\frac{1}{N}\right)$$

$$\lambda \neq 0: \quad \beta_{\lambda_6} = O\left(\frac{1}{N}\right)$$

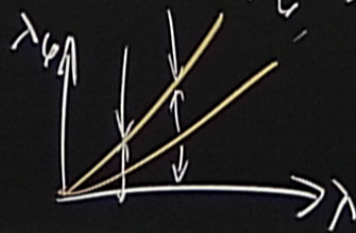


$$V = (|\phi|^2)^3 (4\pi^2 - \lambda_6)$$

$N \rightarrow \infty$ λ_6 is marginal.

λ, λ_6 CFT $\checkmark$

$$\lambda \propto \lambda^2$$



3)

$$\sum_{m_1, \dots, m_{s-1}} J_{m_1, \dots, m_{s-1}} = \overbrace{\text{No single-trace operator}} + \sum_{s', s''} J_{s'} \times J_{s''} + \text{triple-trace}$$

3)

$$\text{Tr} J_{m_1 \dots m_{s-1}} = \overbrace{\text{No single-trace operator}} + \frac{1}{N} \sum_{s', s''} J_{s'} \times J_{s''} + \text{triple-trace}$$

3)

$$\sum_{m_1, \dots, m_{s-1}} J_{m_1, \dots, m_{s-1}} = \overbrace{\text{No single-trace operator}} + \frac{1}{N} \sum_{s', s''} J_{s'} \times J_{s''} + \text{triple-trace}$$

$$\Delta = s+1 + O\left(\frac{1}{N}\right)$$

Bosonic

N_b, λ_b

Fermionic

N_f, λ_f

3)

$$\partial^M J_{m_1 m_2 \dots m_{s-1}} = \overbrace{\text{No single-trace operator}} + \frac{1}{N} \sum_{s', s''} J_{s'} \times J_{s''} + \text{triple-trace}$$

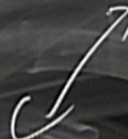
$$\Delta = s+1 + O\left(\frac{1}{N}\right)$$

Bosonic

N_b, λ_b

Fermionic

N_f, λ_f



g

$$4) \quad \gamma(J) = \frac{1}{N} J \times J + \dots \rightarrow ?$$

compute all 2-point and 3-pt functions up to 2 parameters

$$4) \quad \gamma(J) = \frac{1}{N} J \times J + \dots \rightarrow ?$$

compute all 2-point and 3-pt functions up to 2 parameters

$$\langle J_s J_s \rangle = \bar{N} \langle J_s J_s \rangle_{\text{free boson}}$$

$$4) \quad \gamma(J^b) = \frac{1}{N} J \cdot J + \dots \rightarrow ?$$

compute all 2-point and 3-pt functions up to 2 parameters

$$\langle J_s J_s \rangle = \frac{N}{2} \langle J_s J_s \rangle_{\text{free boson}}, \quad s=1,2,3, \dots$$

$$\partial J_4 = \frac{1}{N} \partial J_2 \cdot \partial J_0 \varepsilon, \quad \underline{\lambda}$$

$$4) \quad \gamma(J^b) = \frac{1}{N} J \cdot J + \dots \rightarrow ?$$

compute all 2-point and 3-pt functions up to 2 parameters

$$\langle T_s T_s \rangle = \tilde{N} \langle T_s T_s \rangle_{\text{free boson}}, \quad s=1,2,3, \dots$$

$$\partial \cdot T_4 = \frac{1}{N} \partial T_2 \cdot \partial T_0 \varepsilon, \quad \tilde{\lambda}$$

$$\langle T_{s_1} T_{s_2} T_{s_3} \rangle = \tilde{N} \left[\frac{1}{1+\tilde{\lambda}^2} \langle T_{s_1} T_{s_2} T_{s_3} \rangle_{\text{free boson}} + \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} \langle T_{s_1} T_{s_2} T_{s_3} \rangle_{\text{free fermion}} + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} \langle T_{s_1} T_{s_2} T_{s_3} \rangle_{\text{odd}} \right]$$

$$4) \quad \gamma(J^b) = \frac{1}{N} J \cdot J + \dots \rightarrow ?$$

compute all 2-point and 3-pt functions up to 2 parameters

$$\langle T_s T_s \rangle = \tilde{N} \langle T_s T_s \rangle_{\text{free boson}}, \quad s=1,2,3, \dots$$

$$\partial \cdot T_4 = \frac{1}{N} \partial T_2 \cdot \partial T_0 \varepsilon, \quad \tilde{\lambda}$$

$$\langle T_{s_1} T_{s_2} T_{s_3} \rangle = \tilde{N} \left[\frac{1}{1+\tilde{\lambda}^2} \langle T_{s_1} T_{s_2} T_{s_3} \rangle_{\text{free boson}} + \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} \langle T_{s_1} T_{s_2} T_{s_3} \rangle_{\text{free fermion}} + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} \langle T_{s_1} T_{s_2} T_{s_3} \rangle_{\text{odd}} \right] \quad s_1, s_2, s_3 = 1, 2, 3, \dots$$

3d CFT $\longleftrightarrow$ quantum gravity
on AdS_4

4 different ways to compute correlator:

- bosonic N_b, λ_b
- fermionic N_f, λ_f
- Vasiliev gravity ϕ
- MZ $N, \tilde{\lambda}$

Main references:

OA, Gur-Ari, Yacoby	1110.4382
Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin	1110.4386
Maldacena, Zhiboetov	$\begin{cases} 1112.1016 \\ 1204.3882 \end{cases}$
OA, Gur-Ari, Yacoby	1207.4593
Chang, Minwalla, Sharma, Yin	1207.4485
Gur-Ari, Yacoby	1211.1866
OA, Giombi, Gur-Ari, Maldacena, Yacoby	1211.4843
Jain, Minwalla, Sharma, Trivedi, Wadia, Yokoyama	1301.6169

$$4) \quad A_- = 0 \quad X^\pm = (X^1 \pm iX^2)$$

$$\langle \bar{\Psi} \Psi \rangle: \text{---} \textcircled{1PI} \text{---} = \text{---} + \text{---} \text{ (loop diagram) }$$

$$\langle \bar{\Psi} \Psi(p) \rangle = \frac{1}{i\not{p} + \lambda\sqrt{p_1^2 + p_2^2} - i\lambda^2 \not{p}_+ \gamma^+}$$

$$\langle \phi^\dagger \phi(p) \rangle = \frac{1}{p^2}$$

$$4) \quad A_- = 0 \quad X^\pm = (X^1 \pm iX^2)$$

$$\langle \bar{\Psi} \Psi \rangle: \text{---} \textcircled{1PI} \text{---} = \text{---} + \text{---} \textcircled{\text{cloud}} \text{---}$$

$$\langle \bar{\Psi} \Psi(p) \rangle = \frac{1}{i \not{p} + \lambda \sqrt{p_1^2 + p_2^2} - i \lambda^2 \not{p}_+ \gamma^+}$$

$$\langle \phi^\dagger \phi(p) \rangle = \frac{1}{p^2}$$

$$\langle J, \bar{\Psi} \Psi \rangle: \text{---} \textcircled{\text{V}} \text{---} = \text{---} \textcircled{\text{V}} \text{---} + \text{---} \textcircled{\text{V}} \text{---}$$

$$4) \quad A_- = 0 \quad X^\pm = (X^1 \pm iX^2)$$

$$\langle \bar{\Psi} \Psi \rangle: \text{---} \textcircled{1PI} \text{---} = \text{---} + \text{---} \textcircled{\text{cloud}} \text{---}$$

$$\langle \bar{\Psi} \Psi(p) \rangle = \frac{1}{i \not{p} + \lambda \sqrt{p_1^2 + p_2^2} - i \lambda^2 \not{p}_+ \gamma^+}$$

$$\langle \phi^\dagger \phi(p) \rangle = \frac{1}{p^2}$$

$$\langle \pi(p) \bar{\Psi} \Psi \rangle: \text{---} \textcircled{\text{V}} \text{---} = \text{---} \textcircled{\text{V}} \text{---} + \text{---} \textcircled{\text{V}} \text{---}$$

$$\langle \pi_1 \pi_1 \rangle = \text{---} \textcircled{\text{loop}} \text{---}$$

$$4) \quad A_- = 0 \quad X^\pm = (X^1 \pm iX^2)$$

$$\langle \bar{\Psi} \Psi \rangle: \text{---} \textcircled{1PI} \text{---} = \text{---} + \text{---} \textcircled{\text{cloud}} \text{---}$$

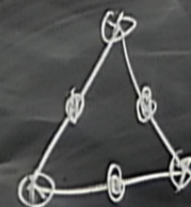
$$\langle \bar{\Psi} \Psi(p) \rangle = \frac{1}{i \not{p} + \lambda \sqrt{p_1^2 + p_2^2} - i \lambda^2 \not{p}_+ \gamma^+}$$

$$\langle \phi^\dagger \phi(p) \rangle = \frac{1}{p^2}$$

$$\langle T(\bar{\Psi} \Psi) \rangle: \text{---} \textcircled{\times} \text{---} = \text{---} \textcircled{\times} \text{---} + \text{---} \textcircled{\text{cloud}} \text{---}$$

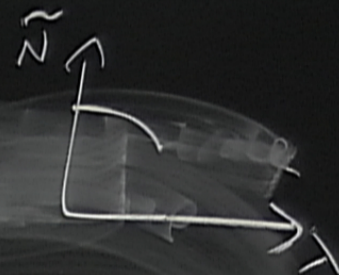
$$\langle T_1 T_1 \rangle = \text{---} \textcircled{\times} \text{---}$$

$$\langle T_1 T_1 T_1 \rangle =$$



4)

$$\langle T T \rangle = 2N \underbrace{\frac{\sin(\pi\lambda)}{\pi\lambda}}_{\tilde{N}} \langle T T \rangle_{\text{free red boson}}$$



$$\langle J_1 \bar{\psi} \psi \rangle = \text{diagram} = \text{diagram} + \text{diagram}$$

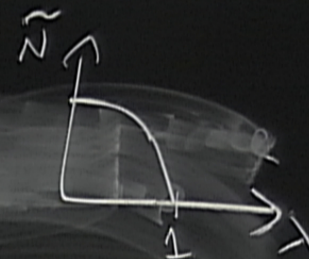
$$\langle J_1 J_1 \rangle = \text{diagram}$$

$$\langle J_1 J_1 J_1 \rangle =$$



4)

$$\langle T T \rangle = 2N \underbrace{\frac{\sin(\pi\lambda)}{\pi\lambda}}_{\tilde{N}} \langle T T \rangle_{\text{free real boson}}$$



$$k =$$

$$\langle J(q) \bar{\psi} \psi \rangle$$



$$\langle J_1 J_1 \rangle =$$

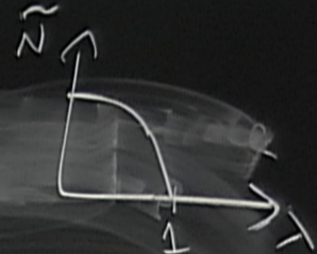


$$\langle J_1 J_1 J_1 \rangle =$$



4)

$$\langle T T \rangle = 2N \underbrace{\frac{\sin(\pi\lambda)}{\pi\lambda}}_{\tilde{N}} \langle T T \rangle_{\text{free real boson}}$$



$$|k| = |k_{\text{YM}}| + N \rightarrow |k| \geq N \rightarrow \lambda \leq 1$$

$$\langle J(q) \bar{\psi} \psi \rangle$$



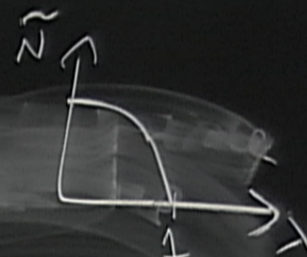
$$\langle J_1 J_1 \rangle =$$

$$\langle J_1 J_1 J_1 \rangle =$$



4)

$$\langle T T \rangle = 2 N_b \underbrace{\frac{\sin(\pi \lambda_b)}{\pi \lambda_b}}_{\tilde{N}} \langle T T \rangle_{\text{free real boson}}$$



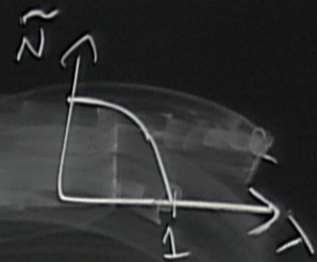
$$|K| = |K_{YM}| + N \longrightarrow |K| \geq N \longrightarrow \lambda \leq 1$$

$$\tilde{\lambda} = \tan\left(\frac{\pi \lambda_b}{2}\right)$$

$$\langle J_1 J_1 \rangle = \text{bubble diagram}$$

4)

$$\langle T T \rangle = 2N_b \underbrace{\frac{\sin(\pi\lambda_b)}{\pi\lambda_b}}_{\tilde{N}} \langle T T \rangle_{\text{free real boson}}$$



$$|K| = |K_{\text{eff}}| + N \rightarrow |K| \geq N \rightarrow \lambda \leq 1$$

$$\tilde{\lambda} = \tan\left(\frac{\pi\lambda_b}{2}\right)$$

Fermionic:

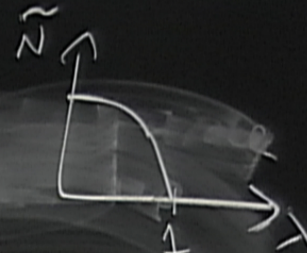
$$\tilde{N} = \frac{2N\sin(\pi\lambda_f)}{\pi\lambda_f}$$

$$\tilde{\lambda} = -\cot\left(\frac{\pi\lambda_f}{2}\right)$$

$$U(N_b)_{K_b} \leftrightarrow U(K_b - N_b)_{-K_b}$$

4)

$$\langle T T \rangle = 2N_b \underbrace{\frac{\sin(\pi\lambda_b)}{\pi\lambda_b}}_{\tilde{N}} \langle T T \rangle_{\text{free real boson}}$$



$$|K| = |K_{YM}| + N \rightarrow |K| \geq N \rightarrow \lambda \leq 1$$

$$\tilde{\lambda} = \tan\left(\frac{\pi\lambda_b}{2}\right)$$

Fermionic:

$$\tilde{N} = \frac{2N\sin(\pi\lambda_f)}{\pi\lambda_f}$$

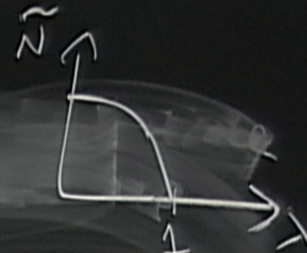
$$\tilde{\lambda} = -\cot\left(\frac{\pi\lambda_f}{2}\right)$$

$$U(N_b)_{K_b} \leftrightarrow U(K_b - N_b)_{-K_b}$$

$$U(N)_K + \text{critical bosons} \leftrightarrow U(K)_N + \text{free fermions}$$

4)

$$\langle T T \rangle = 2N_b \underbrace{\frac{\sin(\pi\lambda_b)}{\pi\lambda_b}}_{\tilde{N}} \langle T T \rangle_{\text{free real boson}}$$



$$|K| = |K_{\text{YM}}| + N \rightarrow |K| \geq N \rightarrow \lambda \leq 1$$

$$\tilde{\lambda} = \tan\left(\frac{\pi\lambda_b}{2}\right)$$

Fermionic:

$$\tilde{N} = \frac{2N\sin(\pi\lambda_f)}{\pi\lambda_f}$$

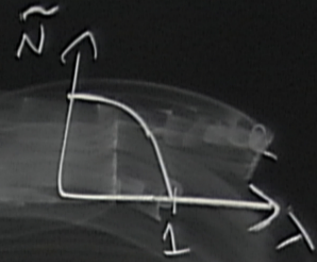
$$\tilde{\lambda} = -\cot\left(\frac{\pi\lambda_f}{2}\right)$$

$$U(N_b)_{K_b} \leftrightarrow U(K_b - N_b)_{-K_b}$$

$$U(N)_K + \text{critical bosons} \leftrightarrow U(K)_N + \text{free fermions}$$

4)

$$\langle T T \rangle = 2 N_b \frac{\sin(\pi \lambda_b)}{\pi \lambda_b} \langle T T \rangle_{\text{free real boson}}$$



$$|K| = |K_{\text{YM}}| + N \rightarrow |K| \geq N \rightarrow \lambda \leq 1$$

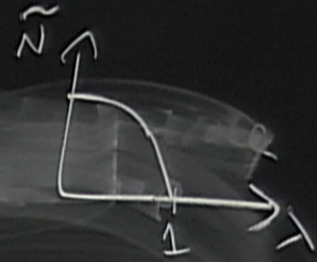
- Evidence:
- 1) Spectrum, 2-point, 3-point
 - 2) Thermal partition functions on $R^2 \times S^1, S^2 \times S^1$
 - 3) Gravity dual

$$U(N_b)_{K_b} \leftrightarrow U(K_b - N_b)_{-K_b}$$

$$U(N)_K + \text{critical bosons} \leftrightarrow \frac{U(K)_N}{+ \text{free fermions}}$$

4)

$$\langle T T \rangle = 2 N \underbrace{\frac{\sin(\pi \lambda)}{\pi \lambda}}_{\tilde{N}} \langle T T \rangle_{\text{free real boson}}$$



$$|K| = |K_{\text{YM}}| + N \rightarrow |K| \geq N \rightarrow \lambda \leq 1$$

Evidence: 1) Spectrum, 2-point, 3-point

2) Thermal partition functions
on $R^2 \times S^1, S^2 \times S^1$

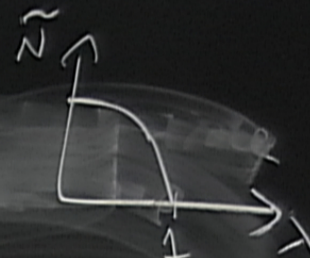
3) Gravity dual

$$U(N_b)_{K_b} \leftrightarrow U(K_b - N_b)_{-K_b}$$

$$U(N)_K + \text{critical bosons} \leftrightarrow \frac{U(K)_N}{+ \text{free fermions}}$$

4)

$$\langle T T \rangle = 2 N_b \frac{\sin(\pi \lambda_b)}{\pi \lambda_b} \langle T T \rangle_{\text{free real boson}}$$



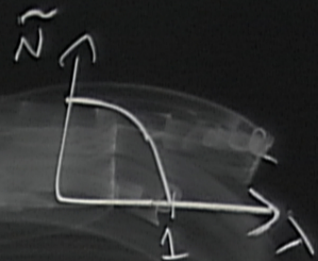
$$|K| = |K_M| + N \rightarrow |K| \geq N \rightarrow \lambda \leq 1$$

$$U(N_b)_{K_b} \leftrightarrow U(K_b - N_b)_{-K_b}$$

$$U(N)_K + \text{critical bosons} \leftrightarrow \frac{U(K)_{N+1/2}}{+ \text{free fermions}}$$

4)

$$\langle T T \rangle = 2 N_b \frac{\sin(\pi \lambda_b)}{\pi \lambda_b} \langle T T \rangle_{\text{free real boson}}$$



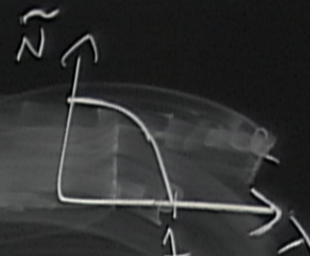
$$|K| = |K_M| + N \rightarrow |K| \geq N \rightarrow \lambda \leq 1$$

$$U(N_b)_{K_b} \leftrightarrow U(K_b)$$

$$\frac{U(N)_K + \text{critical bosons}}{2 + \sigma|\phi|^2 + m\sigma} \leftrightarrow \frac{U(K)}{+ \text{free fermions}}$$

4)

$$\langle T T \rangle = 2 N_b \underbrace{\frac{\sin(\pi \lambda_b)}{\pi \lambda_b}}_{\sim 2} \langle T T \rangle_{\text{free real boson}}$$



$$|K| = |K_M| + N \rightarrow |K| \geq N \rightarrow \lambda \leq 1$$

$$U(N_b)_{K_b} \leftrightarrow U(K_b - N_b)_{-K_b}$$

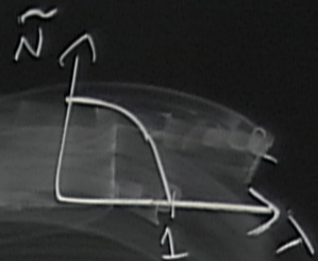
$$\frac{U(N)_K + \text{critical bosons}}{\mathcal{L} + \sigma |\phi|^2 + m \sigma} \leftrightarrow \frac{U(K)_{N+1/2}}{+ \text{free fermions}}$$

$\swarrow \text{MFT } \mu > 0$
 $U(K)_{-N}$

$\searrow \text{MFT } \mu < 0$
 $U(K)_{-N+1}$

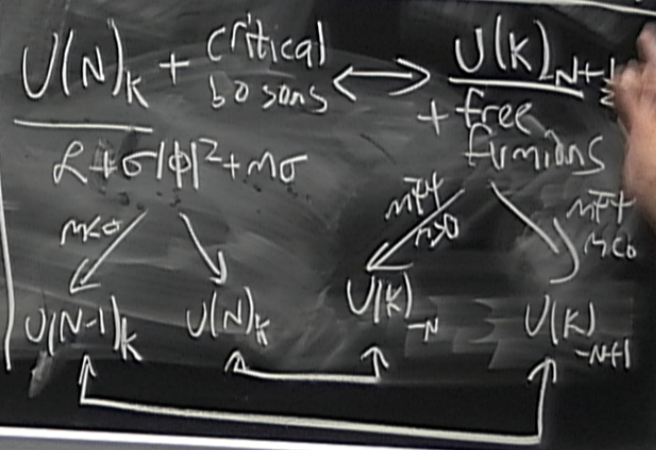
4)

$$\langle T T \rangle = 2 N_b \frac{\sin(\pi \lambda_b)}{\pi \lambda_b} \langle T T \rangle_{\text{free real boson}}$$



$$|K| = |K_M| + N \rightarrow |K| \geq N \rightarrow \lambda \leq 1$$

$$U(N_b)_{K_b} \leftrightarrow U(K_b - N_b)_{-K_b}$$



CAUTION
DO NOT TOUCH THE BOARD WHEN
IT IS BEING USED BY THE BOARD