Title: The correlator of two vector and one axial current in QCD

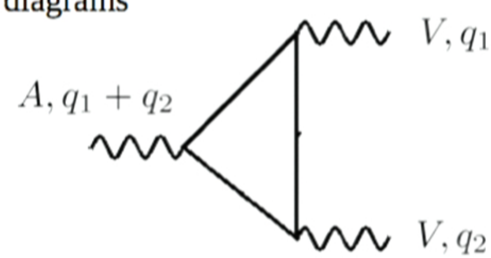
Date: Mar 05, 2013 01:00 PM

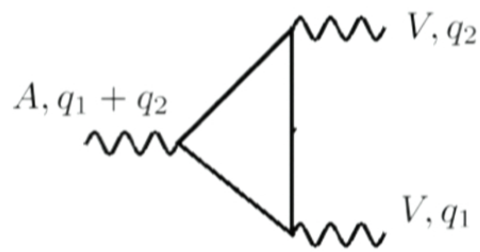
URL: http://pirsa.org/13030099

Abstract: It is known that the correlator of one axial and two vector currents, that receives leading contributions through one-loop fermion triangle diagrams, is not modified by QCD radiative corrections at two loops. It was suggested that this non-renormalization of the VVA correlator persists in higher orders in perturbative QCD as well. To check this assertion, I compute the three-loop QCD corrections to the VAA-correlator using the technique of asymptotic expansions. I find that these corrections do not vanish and that they are proportional to the QCD beta-function. I will also review some properties of the VVA correlator that were discovered in recent years.

Introduction

- The correlator of two vector currents and one axial current $\langle 0|T V_\mu(x_1) V_\nu(x_2) A_\rho(0)|0\rangle$ is a remarkable object with rich history
- It was instrumental for understanding many important phenomena in quantum field theory including
  - anomalous behavior of the axial current — Adler, Bardeen, Jackiw
  - the non-renormalization of the anomaly — Adler, Bardeen,
  - pion decay to two photons and the emergence of color
  - anomaly matching condition
  - solution of the U(1) problem — t'Hooft
- In perturbation theory, the VVA correlator first appears through famous triangle diagrams





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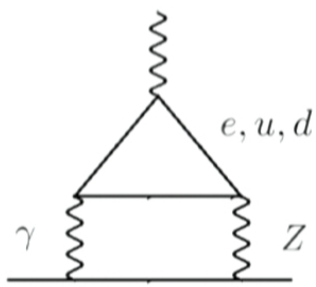
- In recent years, there was a renewed interest in the VVA correlator. The driving force was the need to understand certain types of electroweak contributions to the muon anomalous magnetic moment
- The vector part of the Z coupling to fermions does not contribute due to Furry theorem and we are left with a perfect example of the VVA correlator in the limit when one of the currents (magnetic field) is soft.
- Explicit computations showed that the result is sensitive to the infra-red region of the loop momenta – which can be probed by introducing constituent quark masses
- Although electroweak corrections to muon g-2 are not large, their non-perturbative sensitivity motivated new studies of both perturbative and non-perturbative contributions to VVA correlator



$$a_\mu^{\rm EW}([e,u,d]) \approx -\frac{\sqrt{2}G_\mu m_\mu^2}{16\pi^2}\frac{\alpha}{\pi}\left[\ln\frac{m_u^8}{m_\mu^6 m_d^2}+\frac{17}{2}\right].$$

Kuraev, Kuhto, Silagadze, Czarnecki, Marciano, Vainshtein, de Rafael, Knecht, Peris

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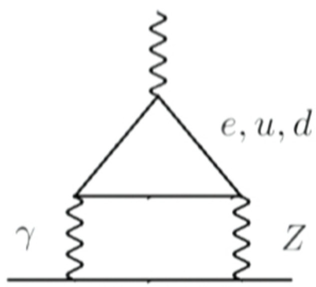

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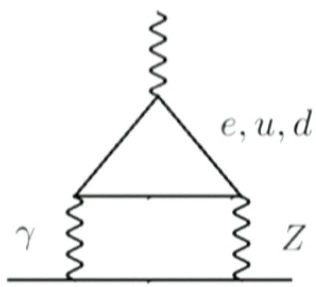

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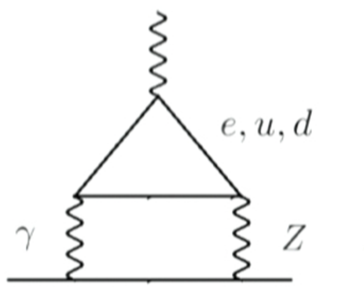

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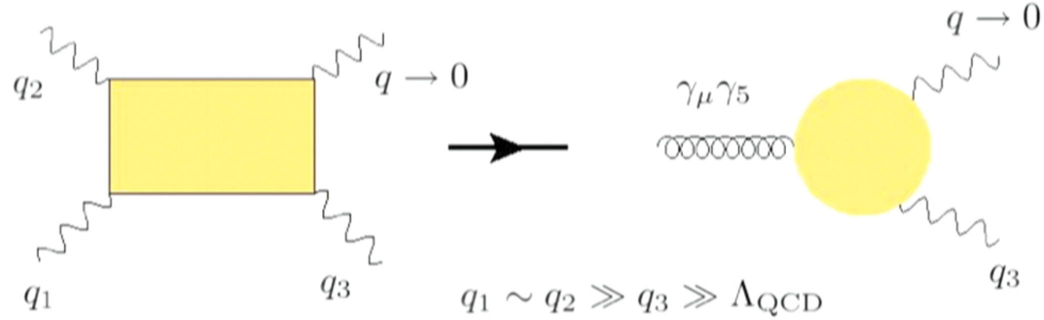

$$i \int \mathrm{d}^4x \mathrm{d}^4y e^{-iq_1x - iq_2y} T\{j_{\mu_1}(x), j_{\mu_2}(y)\} = \int \mathrm{d}^4z e^{-i(q_1+q_2)z} \frac{2i}{q^2} \epsilon_{\mu_1\mu_2\delta\rho} q_3^\delta j_5^\rho(z) + \ldots$$

In that limit, the four-point function maps on a the three-point function of an axial current and two vector currents where one of the vector currents is soft

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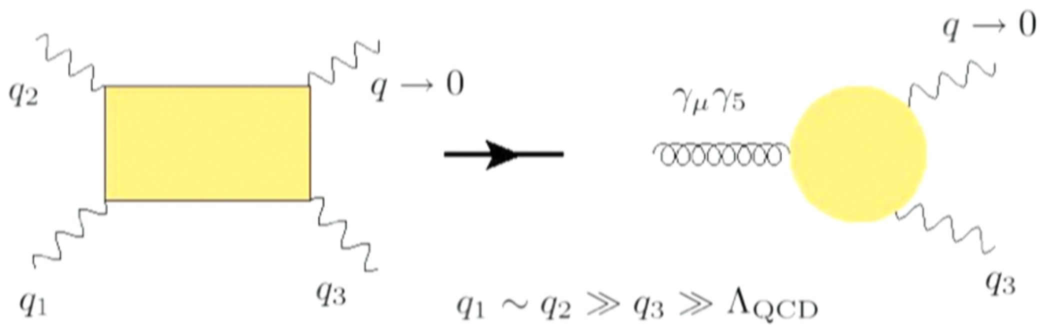

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- Many aspects of hadronic contributions to g-2 require simultaneous consideration of long-distance and short-discance effects that need to be combined in a consistent way. This is a difficult problem in QCD
- Holographic AdS/QCD models provide a way to do that. This was one of the motivations to study the VVA correlator, as needed for g-2, within holographic models and these studies led to an interesting result
- It was argued by Son and Yamamoto, that a robust relation exists between two form-factors that describe the VVA correlator in the limit when one of the vector currents becomes soft, and the difference of two-point correlators of vector and axial currents
- This relation (valid for arbitrary q^2) is supposed to be a sharp prediction of a holographic framework based on 5-d Yang-Mills-Chern-Simons Lagrangian, with chiral symmetry broken by boundary conditions, valid for arbitrary values of

$$w_L(q^2) - 2w_T(q^2) = -\frac{2N_c}{f_\pi^2}\left(\Pi_{VV}(q^2) - \Pi_{AA}(q^2)\right)$$

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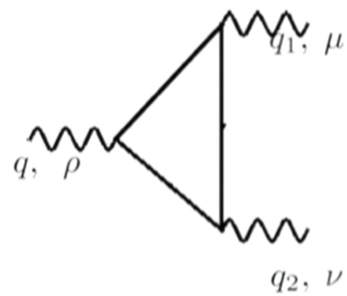
VVA correlator in perturbation theory

- The divergence of the axial current is determined by the anomaly equation which completely fixes the longitudinal form factor of the VVA correlator

$$\partial_\mu A^\mu = \frac{N_c}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\rho}F^{\alpha\rho}$$



$$q^\rho W_{\mu\nu\rho} = -w_L(q_1^2, q_2^2, q^2)q^2\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta = -\frac{iN_c}{2\pi^2}\epsilon_{\alpha\beta\mu\nu}q_1^\alpha q_2^\beta$$

$$w_L = \frac{iN_c}{2\pi^2 q^2}$$

As shown by Adler and Bardeen, the anomaly equation does not receive any corrections in perturbation theory; this implies that the above expression for the longitudinal form factor is exact in perturbative QCD

Since there is no Adler-Bardeen theorem for any form-factor other than longitudinal, we expect that other form-factors receive perturbative corrections. Is this really so?

VVA correlator in perturbation theory

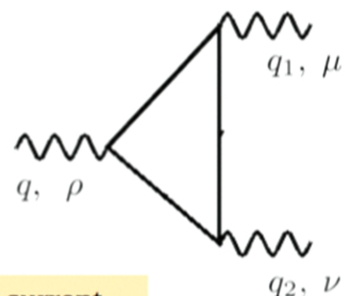
- We can write a general parametrization of the VVA correlator using the vector current conservation and the Bose symmetry. We find four form-factors in a general case.

$$W_{\mu\nu\rho}(q_1,q_2) = w_L(q_1^2,q_2^2,q^2)t^{(1)}_{\mu\nu\rho} + w_T^{(+)}(q_1^2,q_2^2,q^2)t^{(+)}_{\mu\nu\rho}$$
$$+w_T^{(-)}(q_1^2,q_2^2,q^2)t^{(-)}_{\mu\nu\rho} + \tilde{w}_T^{(-)}(q_1^2,q_2^2,q^2)\tilde{t}^{(-)}_{\mu\nu\rho}$$

$$t^{(1)}_{\mu\nu\rho} = -q_\rho \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \qquad t^{(-)}_{\mu\nu\rho} = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \kappa_\rho - \frac{\kappa \cdot q}{q^2}\epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta q_\rho$$
$$t^{(+)}_{\mu\nu\rho} = q_{1\nu}\epsilon_{\mu\rho\alpha\beta} q_1^\alpha q_2^\beta - q_{2\mu}\epsilon_{\nu\rho\alpha\beta} q_1^\alpha q_2^\beta - q_1\cdot q_2 \epsilon_{\mu\nu\rho\alpha}\kappa^\alpha - 2\frac{q_1\cdot q_2}{q^2}\epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta q_\rho$$
$$\tilde{t}^{(-)}_{\mu\nu\rho} = q_{1\nu}\epsilon_{\mu\rho\alpha\beta} q_1^\alpha q_2^\beta + q_{2\mu}\epsilon_{\nu\rho\alpha\beta} q_1^\alpha q_2^\beta - q_1\cdot q_2 \epsilon_{\mu\nu\rho\alpha} q^\alpha.$$

$$q = q_1 + q_2, \quad \kappa = q_1 - q_2.$$

$$q^\rho t^{(1)}_{\mu\nu\rho} = -q^2 \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta. \qquad q^\rho t^{\pm}_{\mu\nu\rho} = 0, \quad q^\rho \tilde{t}^{(-)}_{\mu\nu\rho} = 0$$



Only the first form factor gives non-vanishing divergence of the axial current

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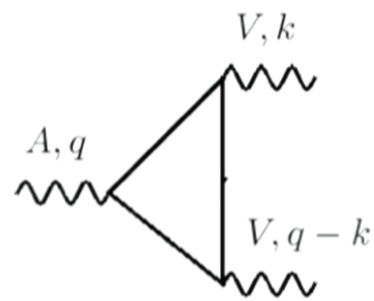
- For the muon anomalous magnetic moment applications, the VVA correlation is needed in the limit when one of the vector currents is soft $k \ll q$ where $q_2 = k, \ \ q_1 = q - k$
- In this limit, to first order in the soft momentum, the correlator is described by only two form-factors, one longitudinal and one transversal; the new transversal form-factor is a sum of the two form-factors introduced previously ($f_{\mu\nu} = \epsilon_\mu k_\nu - \epsilon_\nu k_\mu$)

$$W_{\mu\nu\rho}\epsilon^\nu \approx w_L(q^2) q_\rho q^\sigma \tilde{f}_{\sigma\mu} + w_T(q^2)\left(-q^2 \tilde{f}_{\mu\nu} + q_\mu q^\sigma \tilde{f}_{\sigma\rho} - q_\nu q^\sigma \tilde{f}_{\sigma\mu}\right) + \mathcal{O}(k^2)$$

- At one-loop, we find

$$w_L(q^2) = 2w_T(q^2) = \frac{iN_c}{2\pi^2 q^2}$$

Bell, Jackiw, Adler, Rosenberg



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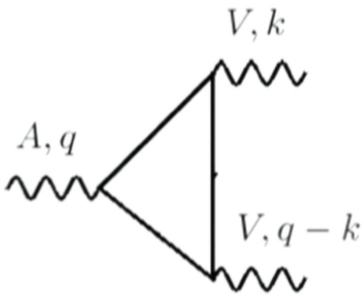

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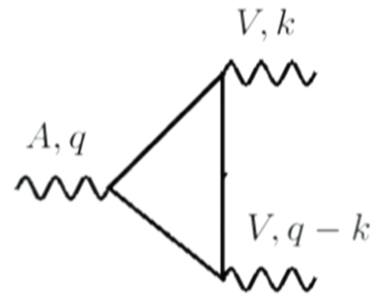

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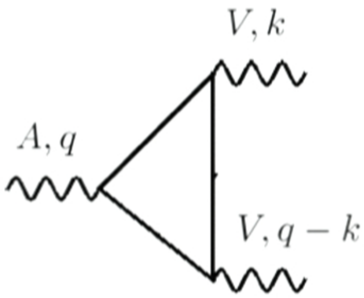

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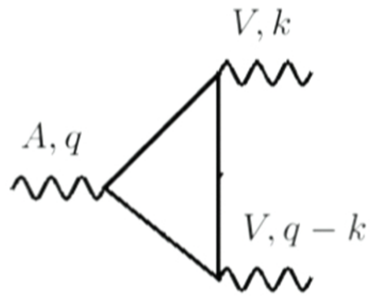

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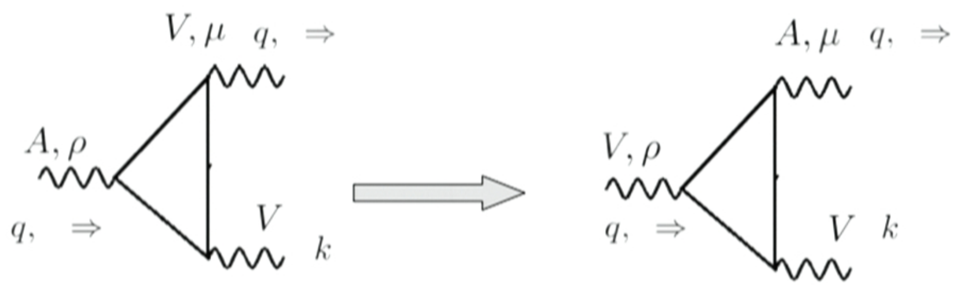
- To prove the relation $w_L(q^2) = 2w_T(q^2)$ we can move γ_5 from the axial to the vector current; this requires anti-commuting γ_5 with even number of Dirac matrices at any order in perturbation theory
- Note also, that if axial and vector currents are permuted, we need to change the direction of the large momentum flow, q → - q. We therefore get an identity



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$$T_{\mu\rho} = W_{\mu\nu\rho}\epsilon^\nu \qquad T_{\mu\rho}(q,k) = T_{\rho\mu}(-q,k)$$

Vainshtein

VVA correlator in perturbation theory

- This symmetry looks peculiar when we confront it with explicit parametrization of the correlator

$$T_{\mu\rho} = (w_L - w_T) q_\rho q^\sigma \tilde{f}_{\sigma\mu} + w_T q_\mu q^\sigma \tilde{f}_{\sigma\rho} - w_T q^2 f_{\mu\rho}$$

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- The last term does not satisfy the required symmetry but this is just a subtraction term whose presence enforces conservation of the vector current. The subtraction term can not be expected to satisfy the symmetry (cf. Pauli-Villars).
- The first two terms can be reconciled with the V-A symmetry by requiring that

$$w_L - w_T = w_T \quad \Leftrightarrow \quad w_L = 2w_T$$

- Since the longitudinal form factor is protected from high-order QCD effects by the non-renormalization of the axial anomaly, the above equation implies that the transversal form factor is also protected
- We conclude that the result for the VVA correlator in the soft-V limit is exact in perturbative QCD

$$T_{\mu\rho} = \frac{iN_c}{4\pi^2 q^2} \left[q_\rho q^\sigma \tilde{f}_{\sigma\mu} + q_\mu q^\sigma \tilde{f}_{\sigma\rho} - q^2 f_{\mu\rho} \right]$$

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$$w_L - w_T = w_T \quad \Leftrightarrow \quad w_L = 2w_T$$

- Since the longitudinal form factor is protected from high-order QCD effects by the non-renormalization of the axial anomaly, the above equation implies that the transversal form factor is also protected
- We conclude that the result for the VVA correlator in the soft-V limit is exact in perturbative QCD

$$T_{\mu\rho} = \frac{iN_c}{4\pi^2 q^2} \left[q_\rho q^\sigma \tilde{f}_{\sigma\mu} + q_\mu q^\sigma \tilde{f}_{\sigma\rho} - q^2 f_{\mu\rho} \right]$$

Vainshtein

VVA correlator in perturbation theory

- This symmetry looks peculiar when we confront it with explicit parametrization of the correlator

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Vainshtein

VVA in perturbation theory

- There are three relations between the form-factor that follow from the V-A symmetry

Knecht, Peris, Perrottet, de Rafael

$$\left[w_T^{(+)} + w_T^{(-)}\right](q_1^2, q_2^2, q^2) - \left[w_T^{(+)} + w_T^{(-)}\right](q^2, q_2^2, q_1^2) = 0$$

$$\left[\tilde{w}_T^{(-)} + w_T^{(-)}\right](q_1^2, q_2^2, q^2) + \left[\tilde{w}_T^{(-)} + w_T^{(-)}\right](q^2, q_2^2, q_1^2) = 0$$

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$$= -\left[2\frac{q_2 \cdot q_1}{q_1^2} w_T^{(+)}(q^2, q_2^2, q_1^2) - 2\frac{q_1 \cdot q_2}{q_1^2} \tilde{w}_T^{(-)}(q^2, q_2^2, q_1^2)\right]$$

In contrast to the soft-V case, the above relations between form-factors do not fully fix the VVA correlator in perturbative QCD

The soft-V limit for the form-factors can be derived from these equations $\quad w_L(q^2) = 2w_T(q^2) = 2\left(w_T^{(+)}(q^2, 0, q^2) + \tilde{w}_T^{(-)}(q^2, 0, q^2)\right)$

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VVA in perturbation theory

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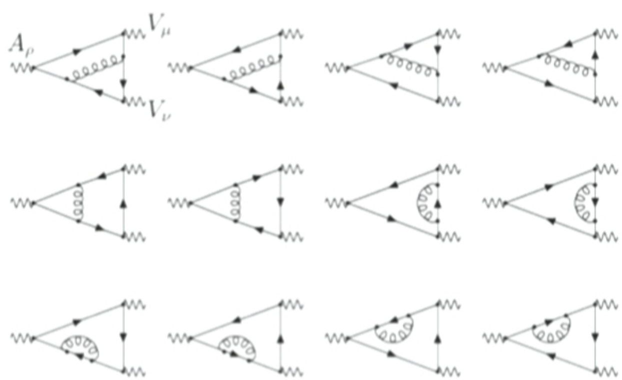
VVA in perturbation theory

- It is possible to test these general results by performing explicit two-loop computation of the VVA correlator. Such a calculation was undertaken by F.Jegerlehner and O.Tarasov in 2005.

- The calculation is highly non-trivial; there are 12 two-loop diagrams; they depend on three kinematic invariants $q_1^2,\ q_2^2,\ q_3^2$ and already the one-loop result for the form-factors (below) suggests that calculation of the two-loop form-factors will lead to very very complex results



$$q_3^2 \Delta^2 w_{2,T}^{(+)}(q_1^2, q_2^2, q_3^2) =$$

$$8[6xy + (x+y)\Delta]\Phi(x,y) + 8\Delta$$

$$-4\,[6x + \Delta]\,(x - y - 1)L_x$$

$$+4\,[6y + \Delta]\,(x - y + 1)L_y$$

$$L_x = \ln x, \quad L_y = \ln y, \quad x = \frac{q_1^2}{q_3^2}, \quad y = \frac{q_2^2}{q_3^2}.$$

$$\Phi(x,y) = \frac{1}{\lambda}\left\{2(\mathrm{Li}_2(-\rho x) + \mathrm{Li}_2(-\rho y)) + \ln\frac{y}{x}\ln\frac{1+\rho x}{1+\rho y} + \ln(\rho x)\ln(\rho y) + \frac{\pi^2}{3}\right\}.$$

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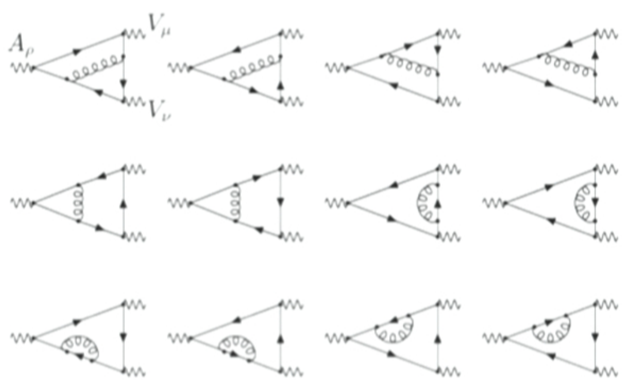

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VVA in perturbation theory

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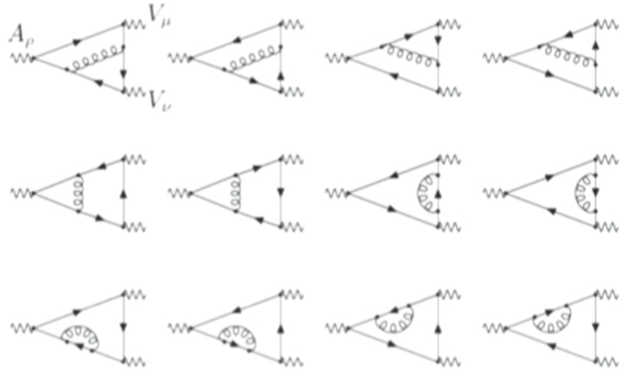
- The finite correction must be removed by the renormalization of the axial current, to make the result consistent with the non-renormalization of the axial anomaly

$$W_{\mu\nu\rho}(q_1, q_2) = \left(1 + \frac{\alpha_s}{\pi} C\right) W^{(0)}_{\mu\nu\rho} + \mathcal{O}(\alpha_s^2) \Rightarrow W^{(0)}_{\mu\nu\rho}(q_1, q_2) + \mathcal{O}(\alpha_s^2)$$

Jegerlehner, Tarasov



The two-loop correction to the full VVA correlator vanishes.

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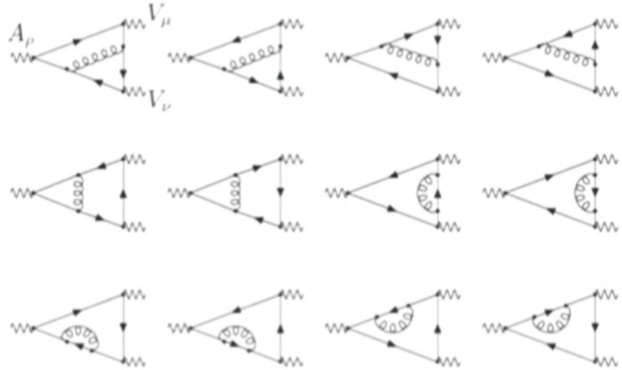

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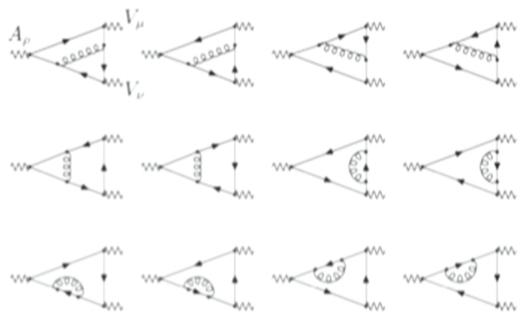
VVA in perturbation theory

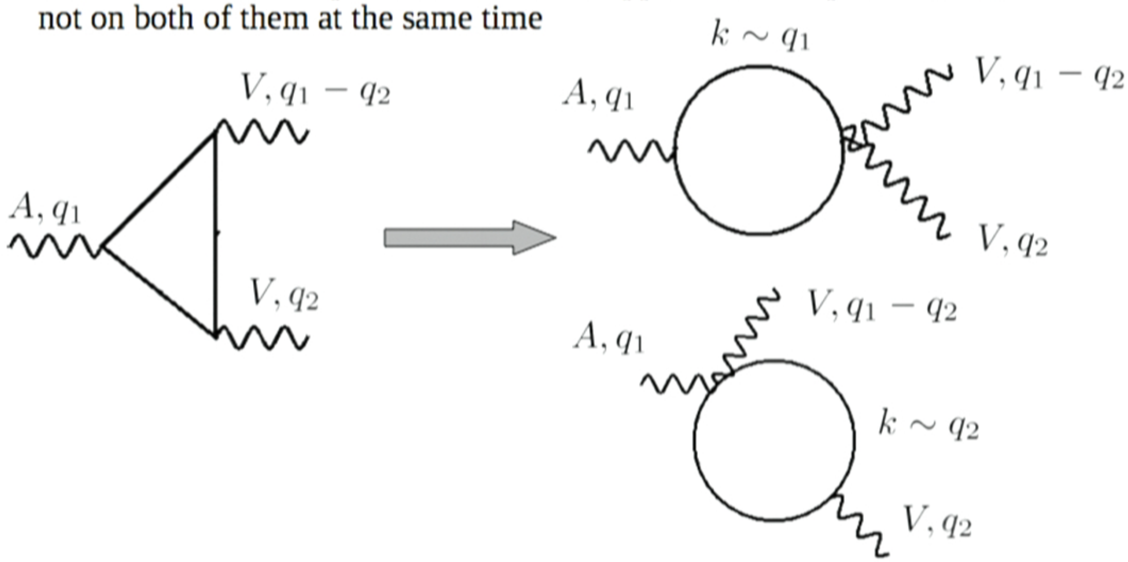
- In fact, one can understand the non-renormalization of the VVA correlator by employing an old (1971) observation by Schreier who pointed out that – under the assumption of conformal invariance – functional form of various two-point and three-point functions is significantly restricted
- In particular the VVA correlator is fixed to be given by the one-loop expression up to a overall constant, that can be determined by the requirement that the longitudinal form-factor equals to its one-loop expression
- While massless QCD is not conformally or scale-invariant, violation of these symmetries is ultimately related to the running of the coupling constant. But such are not present in both one- and two-loop contributions to the VVA correlator, making them amenable to Schreier's argument



We therefore conclude that the absence of the two-loop QCD corrections to VVA correlator is the consequence of the delayed violation of conformal symmetry in this observable and it is not expected to be an all-order QCD result for the VVA correlator

VVA correlator and the two-point functions

- A useful approximate way to compute the VVA correlator at three-loops is provided by the soft-V limit. This means that the large momentum $q_1 \gg q_2$ flows from an axial current to one of the vector currents, while the second vector current carries away a much smaller momentum
- It is intuitively clear that an expansion in the small momentum of the second vector current maps a three-point function on a collection of two-point functions and their derivatives. The two-point functions that appear either depend on q_1 or on q_2 but not on both of them at the same time



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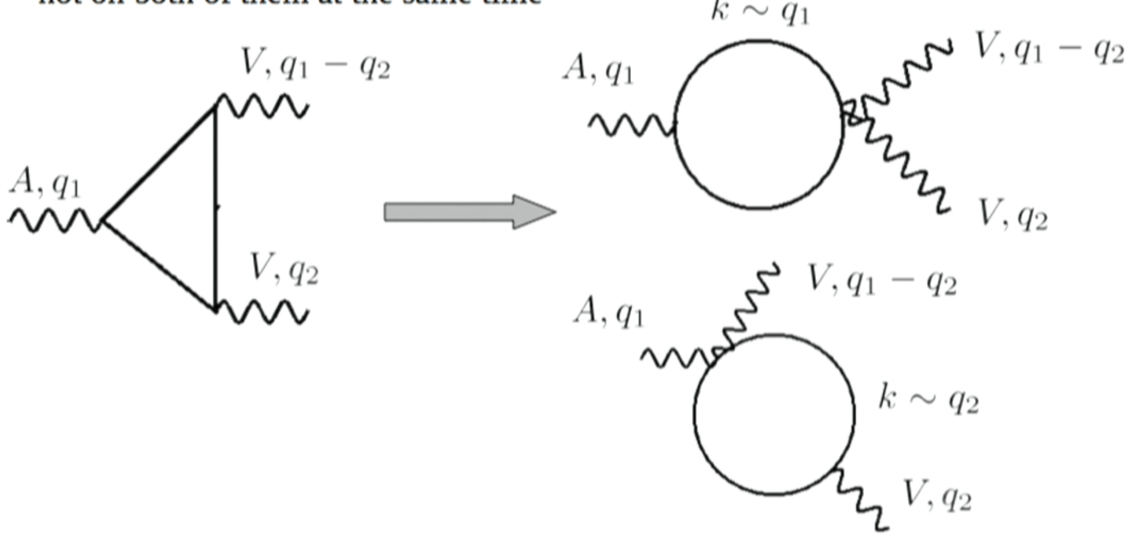
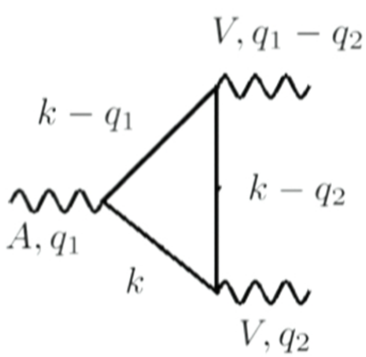
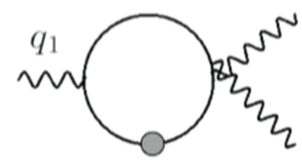


VVA correlator and the three-point functions

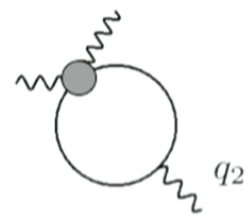
- At any fixed loop order, the expansion can be constructed to any order in q_2/q_1 . The recipe is simple:
  - classify each virtual line in a diagram as carrying momentum either of order q_1 or q_2 and impose momentum conservation in each vertex
  - Taylor expand in all small momenta (both external and loop) where possible
  - regularize the resulting integrals dimensionally; ignore possible overlap regions and integrate over all possible values of the loop momenta
- For example, at one-loop there are two contributing regions and a single-type of integrals that need to be evaluated



$$k \sim q_1; \quad \frac{1}{(k-q_2)^2} \sim \frac{1}{k^2} + \frac{2q_2 \cdot k}{k^4} + \ldots$$



$$k \sim q_2; \quad \frac{1}{(k-q_1)^2} \sim \frac{1}{q_1^2} + \frac{2k \cdot q_1}{q_1^4} + ..$$



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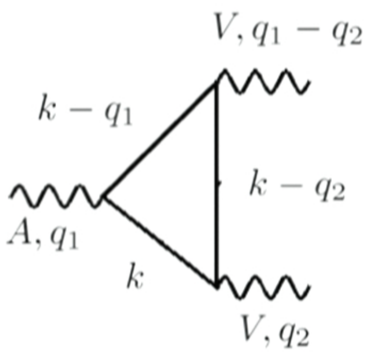

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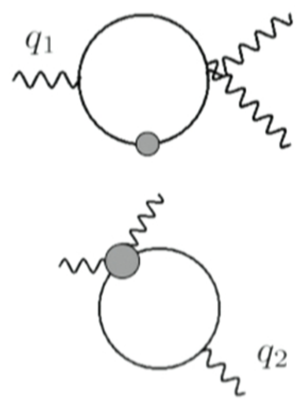


Axial current in dimensional regularization

- The important point of the above construction is the need to define Feynman integrals in dimensional regularization; this makes calculations technically feasible.
- However, it creates a problem for the axial current since it can not be continued dimensionally in a straightforward way (the t'Hooft-Veltman prescription is, of course, a possibility but it is impractical beyond one-loop)
- The useful way of defining the axial current was suggested by Larin and we follow it here. The renormalization constants that restore vector - axial Ward identities have been computed to two-loops and we can use them in our computation

$$A_\rho = \frac{i}{6}\epsilon_{\rho\alpha'\beta'\lambda'}\bar{\psi}\Gamma^{\alpha'\beta'\lambda'}\psi, \quad \Gamma^{\alpha\beta\lambda} = \frac{1}{6}\left[\gamma^\alpha\gamma^\beta\gamma^\lambda - \gamma^\alpha\gamma^\lambda\gamma^\beta + ...\right).$$

$$A_\rho = Z_A A_\rho^{\rm bare}$$

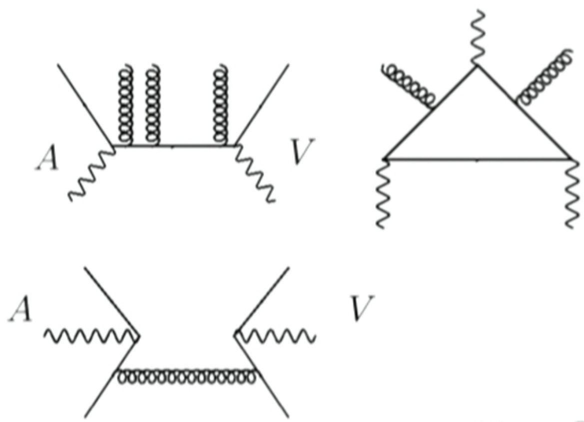
$$Z_A = \left(1 + \frac{2a_s^2 C_F}{\epsilon}\beta_0\right) \times \left(1 - 4C_F a_s + C_F a_s^2\left(22C_F - \frac{107}{9}C_A + \frac{2}{9}n_f\right)\right)$$

$$T_{\alpha\beta\lambda;\mu\nu} = -i\epsilon_{\alpha\beta\lambda\rho}W_{\mu\nu\rho}(q_1, q_2) \qquad \epsilon_{\alpha\beta\lambda\rho}\epsilon_{\rho\alpha'\beta'\lambda'} = g_{\alpha\alpha'}g_{\beta\beta'}g_{\lambda\lambda'} - g_{\alpha\alpha'}g_{\beta\lambda'}g_{\lambda'\beta'} + ...$$

VVA correlator beyond perturbation theory

- What happens to the VVA correlator beyond perturbation theory? This question was studied both in the chiral limit and beyond
- Note that the equation $w_L = 2w_T$ can only hold perturbatively. Longitudinal form-factor is protected from non-perturbative corrections in the chiral limit (t'Hooft anomaly matching condition) but the transversal form-factor must receive them

Knecht, Peris, Perrottet, de Rafael



Vanishing contributions to the OPE

The non-vanishing contribution requires breaking continuous fermion line since otherwise $T_{\mu\rho}(q) = T_{\rho\mu}(-q)$ holds

$$\mathcal{O}^{(6)}_{\alpha\beta} = \bar{q} T^a \gamma_\alpha \gamma_5 q \otimes \bar{q} T^a \gamma_\beta q - (\alpha \leftrightarrow \beta)$$

$$\Delta T_{\mu\rho} = -\frac{16\pi\alpha_s Q_q}{q^6}\left(-q^2 g_{\mu\alpha} g_{\rho\beta} + q_\mu q_\alpha g_{\rho\beta} - q_\rho q_\alpha g_{\mu\beta}\right) \times \langle 0|\mathcal{O}^{(6)}_{\alpha\beta}|\gamma\rangle$$

$$w_T[u,d] = \frac{1}{m_{a_1}^2 - m_\rho^2}\left(\frac{m_{a_1}^2 - m_\pi^2}{q^2 - m_\rho^2} - \frac{m_\rho^2 - m_\pi^2}{q^2 - m_{a_1}^2}\right)$$

Czarnecki, Marciano, Vainshtein