Title: Dissecting holography with higher spins

Date: Mar 25, 2013 02:00 PM

URL: http://www.pirsa.org/13030098

Abstract: The holographic correspondence is a powerful duality between a quantum theory of gravity and a quantum gauge theory in one lower space-time dimension. Higher spin gravity theories, i.e. gravity theories that also contain (gauge) fields of spins greater than 2, play a special role in holography. I will explain consistent interacting higher spin gravity theories in anti-de Sitter space, their duality with gauged conformal vector models, and their connection to string theory.

Perimeter Institute
March 25, 2013

Dissecting holography with higher spins

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What is quantum gravity?

To answer this question, we need to address (as in any quantum theory):

1. What is the Hilbert space of states?

2. What are the observables? (S-matrix?)

3. How/what can we calculate? (perturbation theory, and beyond perturbation theory.)

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1. What is the Hilbert space of states?

The answer to this question depends on the asymptotic boundary condition of the space-time.

Two basic examples:

Flat space-time: scattering states (in/out states) of gravitons, etc.

Anti-de Sitter space-time: normalizable states, typically of discrete spectrum.

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3. What/how can we calculate?

In flat space-time

String perturbation theory: a UV complete, but perturbative, framework for computing S-matrix elements.

Scattering at very high energies: black hole creation and decay... - semi-classical calculation of Hawking radiation, incomplete framework and not yielding a unitary S-matrix.

In AdS space-time

Via holography, all correlation functions could be computed from the dual conformal field theory.

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Example I.

11-dimensional M-theory on the spacetime

$$ds^2 = f(r)dy^2 - 2dydt + \sum_{i=1}^{9} dx_i dx_i,$$

where $r^2 = \sum x_i^2$, $f(r) = N/r^7$. y is periodically identified: $y \sim y + 2\pi$.

Dual theory: (Banks-Fischler-Shenker-Susskind)

16-supercharge SU(N) gauged matrix quantum mechanics.

$$H = \mathrm{Tr}\left(\frac{1}{2}P_i P_i - \frac{1}{4}[X_i, X_j]^2 - \frac{1}{2}\Theta^T \Gamma^i [X_i, \Theta]\right)$$

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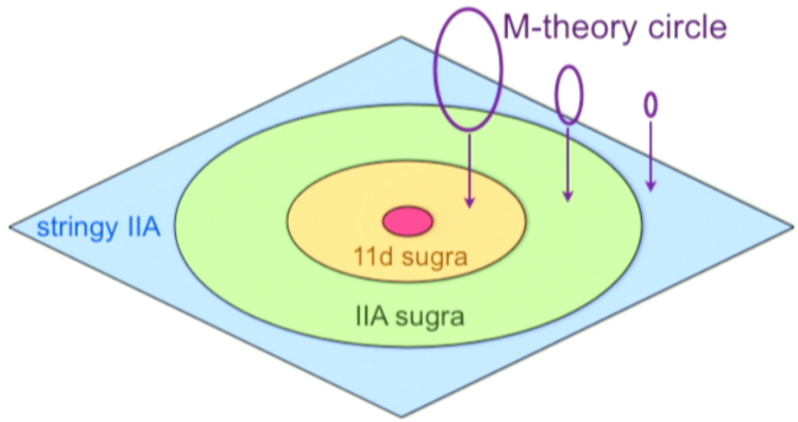

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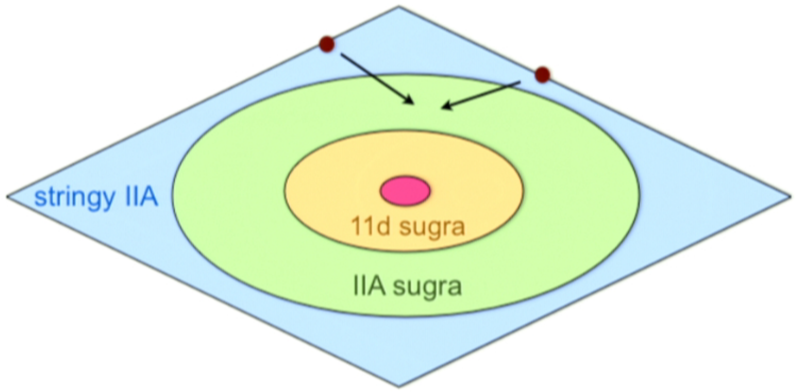

Scattering of D0-branes in IIA (gravitons in M-theory)

$\updownarrow$

Scattering of eigenvalues in matrix quantum mechanics

The duality with BFSS matrix quantum mechanics in principle allows for the extraction of flat spacetime S-matrix of D0-brane scattering in IIA string theory and graviton scattering in M-theory (at all energies). To do so one must work in the non-perturbative regime of the SU(N) matrix quantum mechanics at large N, however. This is hard, but there is limited progress.

How hard is it? Even in the case of SU(2) gauge group, the matrix quantum mechanics involves a 2^{24} component wave function in 27 coordinates.

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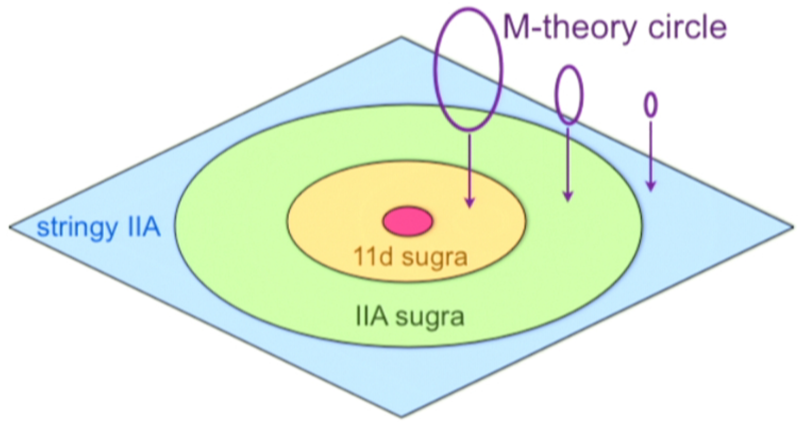

type IIA string theory

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Example II.

Type IIB string theory on the spacetime $AdS_5 x S^5$

$$ds^2 = R^2\left(\frac{-dt^2 + dx_i dx_i + dz^2}{z^2} + d\Omega_5^2 \right).$$

Dual theory: (Maldacena)

16-supercharge SU(N) Yang-Mills theory ($\mathcal{N}$=4 SYM).

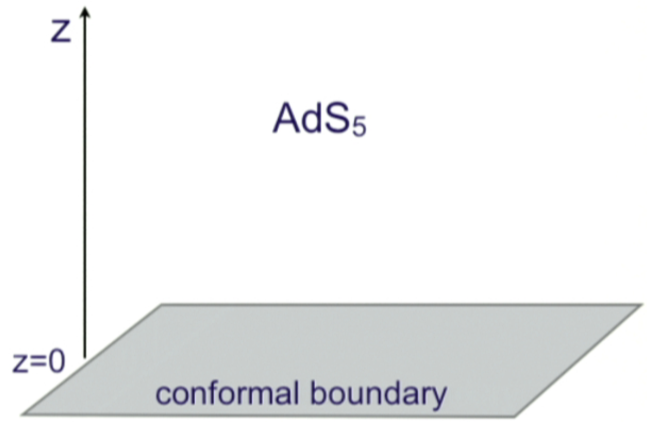
Type IIB on $AdS_5 x S^5$

two parameters: string coupling g_s, and flux N through the S^5.

$$g_s = g_{YM}^2$$

$\mathcal{N}$=4 SYM with gauge group SU(N)

a conformal field theory - exactly marginal coupling g_{YM}.



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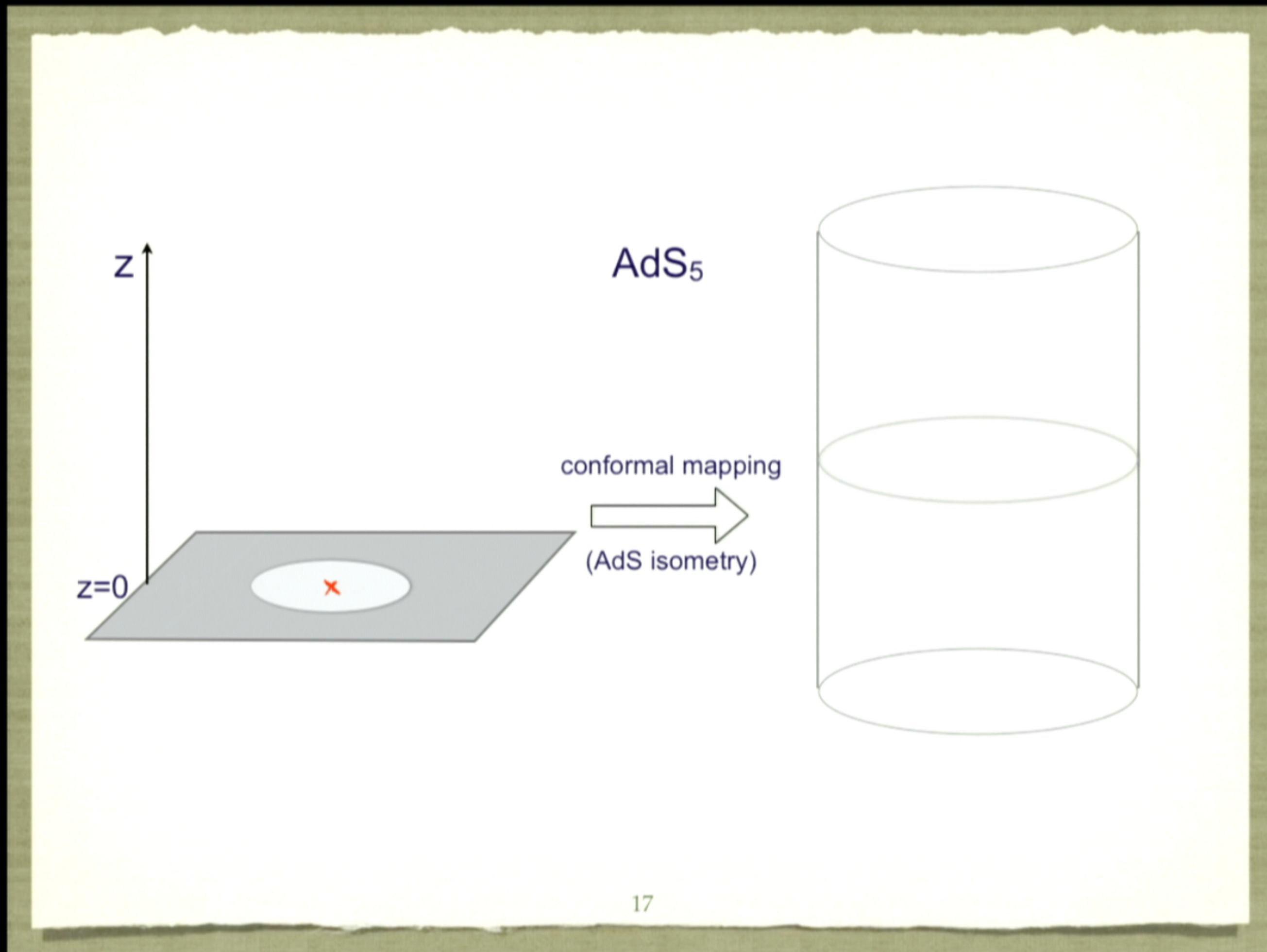
AdS$_5$
z
z=0
conformal mapping
(AdS isometry)
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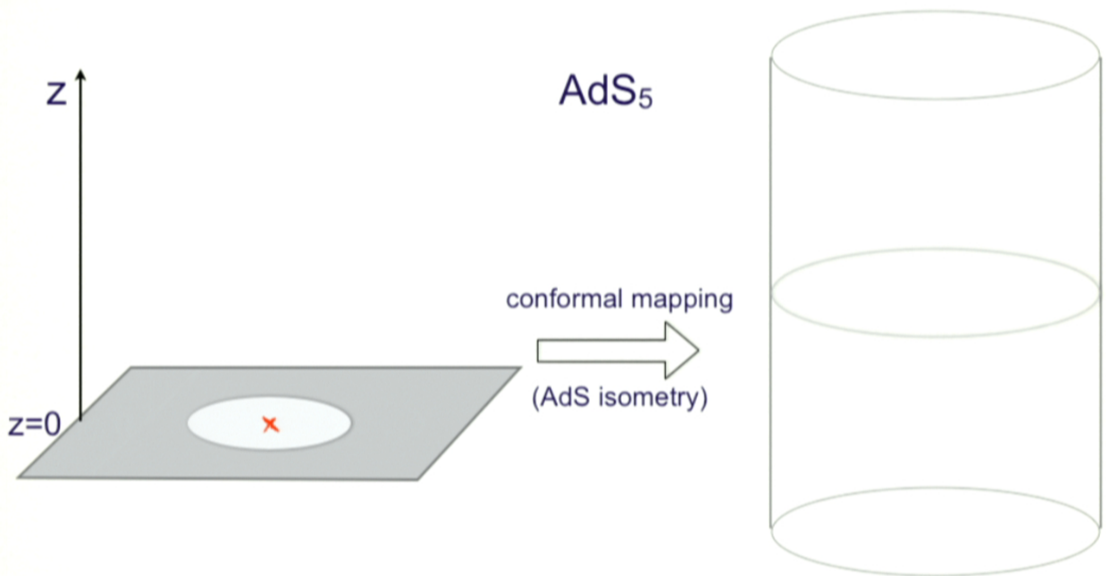

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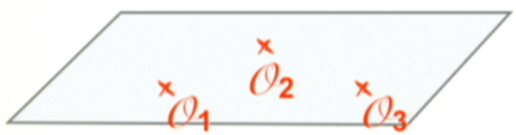
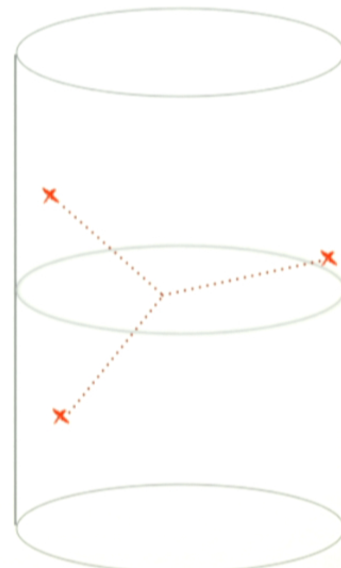
Observables

Correlation functions of gauge invariant operators in SU(N) $\mathcal{N}$=4 SYM ⟺ Boundary correlators of string fields in AdS_5



⟺



Why is this duality useful:

Gravity ⇒ gauge theory: the strong ‘t Hooft coupling regime of $\mathcal{N}$=4 SYM at large N is mapped to semi-classical supergravity in AdS5xS5.

Gauge theory ⇒ gravity: in principle a non-perturbatively defined framework for quantum gravity, provided sufficient computational power on the gauge theory side.

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Example III.

Vasiliev's higher spin gauge theory in AdS_4

$$d\mathbb{A} + \mathbb{A} * \mathbb{A} = f_*(\mathbb{B} * K)dz^2 + \overline{f}_*(\mathbb{B} * \overline{K})d\overline{z}^2.$$

Dual theory: (Klebanov-Polyakov, Sezgin-Sundell, Giombi-Minwalla-Prakash-Trivedi-Wadia-XY)

3-dimensional Chern-Simons vector model

$$S = \frac{k}{4\pi}\int \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right) + \int \overline{\psi}\gamma^{\mu} D_{\mu}\psi$$

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The spectrum: an infinite tower of gauge fields in AdS_4 of spin $s=0,1,2,3,...,\infty$. These are mapped to the very simple spectrum of single-trace operators in the conformal vector model.

The correlation functions: computable on both gravity and field theory sides in overlapping regime.

In the bulk: full classical theory known (“halfway” between supergravity and full fledged string field theory in AdS)

On the boundary: exactly solvable at large N (not relying on supersymmetry or integrability)

A good testbed for holography.

Higher spin gauge theories

We know from AdS/CFT that higher spin gauge theory in AdS must exist, because there are dual large N free CFTs with conserved higher spin currents.

e.g. d=4, $\mathcal{N}$=4 SYM in the zero 't Hooft coupling limit

⇒ tensionless limit of type IIB string theory in

$AdS_5 x S^5$ must reduce to a higher spin gauge theory (coupled to infinite towers of massive fields.) [Ferrara-Fronsdal '98, Haggi-Mani-Sundborg '00, Konstein-Vasiliev-Zaikin '00, Witten '01 talk, Beisert-Bianchi-Morales-Samtleben '04]

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We know from AdS/CFT that higher spin <u>gauge</u> theory in AdS must exist, because there are dual large N free CFTs with <u>conserved</u> higher spin currents.

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Long before AdS/CFT...

Fradkin-Vasiliev '87: cubic vertex for higher spin gauge fields in AdS_4.

Vasiliev '90-92: constructed full nonlinear equation of motion for higher spin gauge fields in AdS_4.

Vasiliev's system are a set of classical nonlinear gauge invariant equations describing an infinite tower of interacting higher spin fields in AdS_4.

The bosonic theory involves all nonnegative integer spins s=0,1,2,3,.... There is a consistent truncation ("minimal bosonic theory") to only the even spins but no further.

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Vasiliev's frame-like formalism

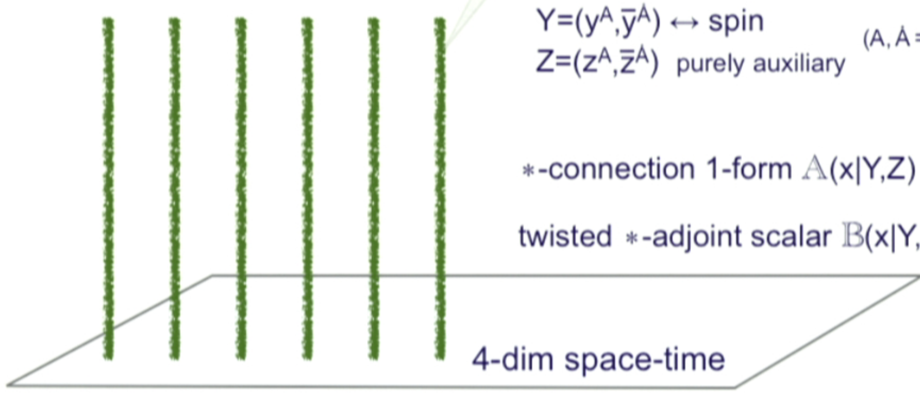
twistors Y, Z

noncommutative $*$ product

$Y=(y^A,\bar{y}^{\dot{A}}) \leftrightarrow$ spin
$Z=(z^A,\bar{z}^{\dot{A}})$ purely auxiliary
$(A, \dot{A} = 1,2)$

$*$-connection 1-form $\mathbb{A}(x|Y,Z)$

twisted $*$-adjoint scalar $\mathbb{B}(x|Y,Z)$



Vasiliev's frame-like formalism

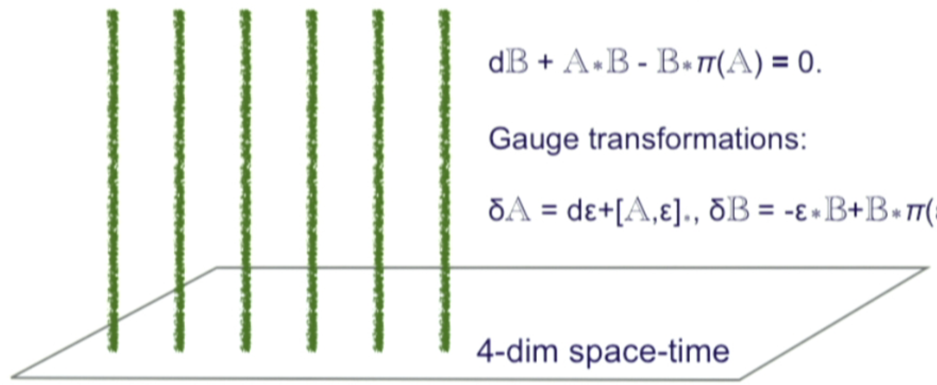
Equations of motion:

$d\mathbb{A} + \mathbb{A}*\mathbb{A} = f_*(\mathbb{B}*K)\, dz^2 + \overline{f}_*(\mathbb{B}*\overline{K})\, d\overline{z}^2.$

$d\mathbb{B} + \mathbb{A}*\mathbb{B} - \mathbb{B}*\pi(\mathbb{A}) = 0.$

Gauge transformations:

$\delta\mathbb{A} = d\varepsilon+[\mathbb{A},\varepsilon]_*,\ \delta\mathbb{B} = -\varepsilon*\mathbb{B}+\mathbb{B}*\pi(\varepsilon).$



The function f(X) can be put to the form $f(X) = X \exp(i\theta(X))$ by field redefinition. $\theta(X)$ is a real, even function that controls interactions of Vasiliev theory.

If impose parity symmetry, then only two inequivalent theories:

A-type: $f(X)=X$.

B-type: $f(X)=iX$.

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The formalism is background independent.

There is an (AdS_4) vacuum solution that preserves so(3,2) symmetry:

$$\mathbb{A}=W_0(x|Y), \quad \mathbb{B}=0.$$

Here W_0 is constructed out of the spin connection and vierbein of AdS_4, and obeys $dW_0+W_0 * W_0=0$.

Next, linearizing Vasiliev's equation around the vacuum solution, one finds an infinite tower of higher spin gauge fields propagating in AdS_4. The details are lengthy and I won't explain here.

Roughly speaking, $\mathbb{B}$ contains higher spin Weyl tensors, whereas $\mathbb{A}$ contains the metric-like higher spin symmetric traceless tensor fields. Either $\mathbb{B}$ or $\mathbb{A}$ contains all physical degrees of freedom.

The linearized Vasiliev equation can be shown to be equivalent to Fronsdal's equations for free HS gauge fields in AdS_4, of the stand-looking form

$$(\Box - m^2)\varphi_{\mu_1\mu_2\cdots\mu_s} + \cdots = 0.$$

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Proceeding to nonlinear orders, one can compute the boundary correlation functions of Vasiliev theory in AdS4.

Tree-level 3-point function computed in [Giombi-Yin, '09, '10, '12].

$\Leftarrow$ how to compute a boundary 4-point function in Vasiliev theory

Some exact solutions to the full nonlinear equations are known, e.g. black-hole like solutions of [Didenko-Vasiliev], and purely scalar solution of [Sezgin-Sundell].

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The simplest example

The free $O(N)$ vector model is the theory of N free massless scalar fields ϕ_i ($i=1,\dots,N$) in 3d, restrict to the $O(N)$ singlet sector. This CFT has conserved currents of the form $J_{\mu_1\dots\mu_s} = \phi_i \partial_{(\mu_1}\dots\partial_{\mu_s)}\phi_i + \dots$ for each even integer s.

The critical $O(N)$ model can be described as the IR (Wilson-Fisher) fixed point of a double trace deformation of the free CFT. Restriction to $O(N)$ singlet sector is important for large N factorization. Higher spin symmetries are broken by $1/N$ effects.

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Conjecture (Klebanov-Polyakov, Sezgin-Sundell '02):
Vasiliev's minimal bosonic theory in AdS_4 is holographically dual to the free or critical O(N) vector model.

	A-type	B-type
Δ=1	free O(N) boson	critical O(N) fermion (Gross-Neveu)
Δ=2	critical O(N) boson (Wilson-Fisher)	free O(N) fermion

AdS boundary condition on bulk scalar field

Matching of 3-point functions at tree level in the bulk theory and leading order in the 1/N expansion of boundary CFT verified in [Giombi-XY '09, '10]

Maldacena and Zhiboedov ('11) showed that in CFT_3, exact higher spin symmetry implies free correlators.

Chern-Simons vector model

[Giombi-Minwalla-Prakash-Trivedi-Wadia-XY, Aharony-Gur-Ari-Yacobi, '11, Chang-Minwalla-Sharma-XY '12, Aharony-Giombi-Gur-Ari-Maldacena-Yacobi, '12]

Take U(N) or SU(N) Chern-Simons theory, and couple it to massless matter fields.

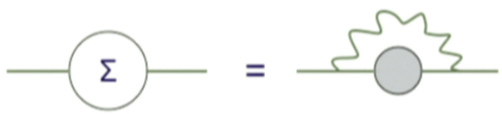
CS-boson VM: the matter fields are a set of (complex) scalars ϕ in the fundamental representation of the gauge group.

CS-fermion VM: the matter fields are fermions ψ.

Supersymmetric CS-VM: the matter fields involves both bosons and fermions. SUSY may be preserved by turning on appropriate double trace and triple trace marginal couplings.

Exact solution at infinite *N*

Schwinger-Dyson equations (Chern-Simons in lightcone gauge)



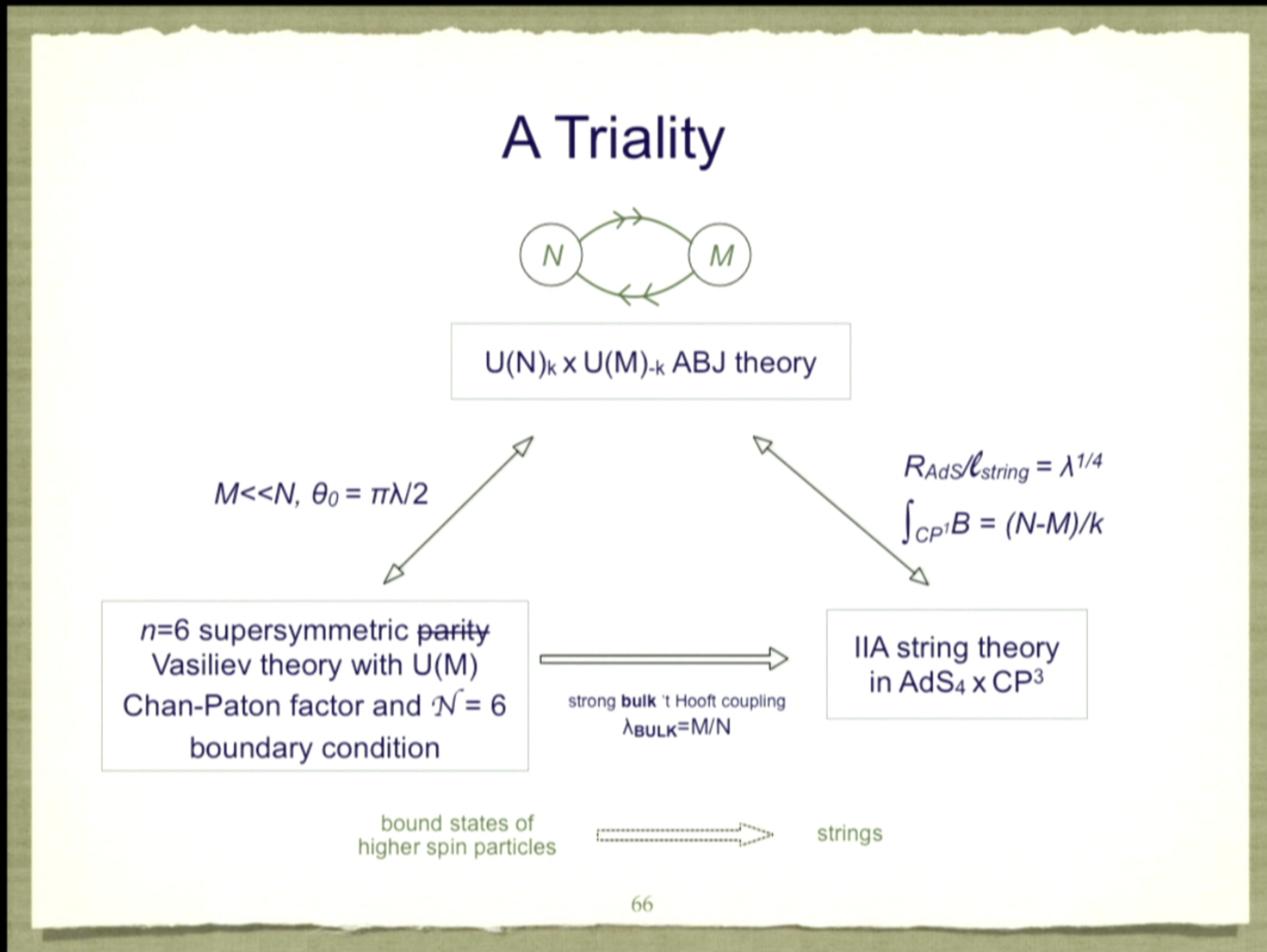
⇒ solve exact planar matter self energy

[Giombi-Minwalla-Prakash-Trivedi-Wadia-XY, '11]

⇒ solve exact planar 4-point vertex

Can compute exact three-point function of single-trace operators at infinite N, finite λ.

[Aharony-Gur-Ari-Yacobi, Gui-Ari-Yacobi, '12]


A Triality
N
M
U(N)k x U(M)-k ABJ theory
M<<N, θ0 = πλ/2
RAdS/ℓstring = λ1/4
∫CP1B = (N-M)/k
n=6 supersymmetric parity Vasiliev theory with U(M) Chan-Paton factor and 𝒩 = 6 boundary condition
strong bulk 't Hooft coupling
λBULK=M/N
IIA string theory in AdS4 x CP3
bound states of higher spin particles
strings
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Higher spin gauge theories are non-local. Are they *really* close to semi-classical gravity?

The AdS/CFT correspondence really has two parts: the emergence of holography at large N, and the emergence of bulk locality at strong 't Hooft coupling.

The higher spin/vector model duality has essentially demystified the "holography" part.

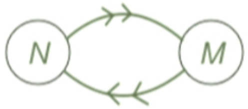
The true mystery lies in bulk locality. From the perspective of ABJ triality, locality must arise from strongly coupled nonabelian Vasiliev theory in AdS_4. This is the hard part.

Thank you!

These no-go theorems could be evaded with non-minimal coupling and non-local interactions of massless higher spin fields.

Gauge invariant <u>cubic</u> vertex of massless higher spin fields in flat space does exist. For spins $s_1 \geq s_2 \geq s_3$, n-derivative interaction, $s_1+s_2-s_3 \leq n \leq s_1+s_2+s_3$. [Metsaev '05, Manvelyan-Mkrtchyan-Rühl '10, Sagnotti-Taronna '10]

A Triality



U(N)$_k$ x U(M)$_{-k}$ ABJ theory

$M << N$, $\theta_0 = \pi\lambda/2$

$R_{AdS}/\ell_{string} = \lambda^{1/4}$

$\int_{CP^1} B = (N-M)/k$

n=6 supersymmetric ~~parity~~ Vasiliev theory with U(M) Chan-Paton factor and $\mathcal{N} = 6$ boundary condition

strong **bulk** 't Hooft coupling $\lambda_{\mathbf{BULK}}$=M/N

IIA string theory in $AdS_4 \times CP^3$

bound states of higher spin particles ⇒ strings

66