Title: Effective field theory approach to quasi-single field inflation

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Abstract: We apply the effective field theory approach to quasi-single field inflation, which contains an additional scalar field with Hubble scale mass other than inflaton.
Based on the time-dependent spatial diffeomorphism, which is not broken by the time-dependent background evolution, the most generic action of quasi-single field inflation is constructed up to third order fluctuations. Using the obtained action,
the effects of the additional massive scalar field on the primordial curvature perturbations are discussed. In
particular, we calculate the power spectrum and discuss the momentum-dependence of three point functions in the squeezed limit for general settings of quasi-single field inflation. Our
framework can be also applied to inflation models with heavy particles. We make a qualitative discussion on the effects of heavy particles during inflation and that of sharp turning trajectory in our framework.

Effective field theory approach to quasi-single field inflation

MASAHIDE YAMAGUCHI

(Tokyo Institute of Technology)

03/12/13 @Perimeter Institute

arXiv:1211.1624, Toshifumi Noumi, MY, and Daisuke Yokoyama

$c = \hbar = M_{\rm pl}^2 = 1/(8\pi G) = 1$

Contents

Inflation

Inflation can naturally solve the problems of the standard big bang cosmology.

- **The horizon problem**
- **The flatness problem**
- **The origin of density fluctuations**
- **The monopole problem**
- …

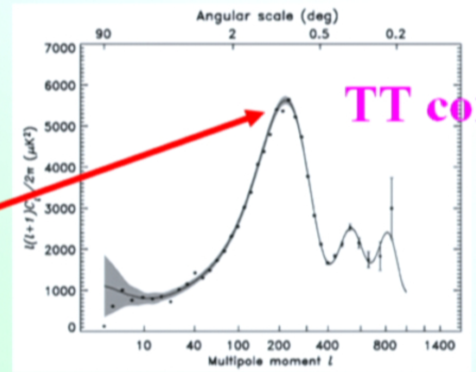
General prediction of inflation

- Spatially flat universe
- Almost scale invariant, adiabatic, and Gaussian primordial density fluctuations

General predictions of inflation

- **Spatially flat universe**

$$\Omega_{\text{total}} \simeq 1.0$$

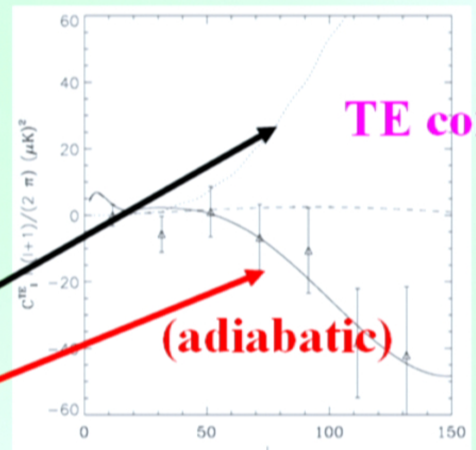


TT correlation

- **Almost scale invariant, adiabatic, and Gaussian primordial density fluctuations**

Causal seed models

Inflation models



TE correlation

(adiabatic)

WMAP1

There are still rooms for $|f_{NL}| = O(10)$ and $O(1)$% isocurvature perturbations.

Classification of inflation

- **Single field inflation :**

Scalar field ϕ ←⇒ curvature perturbation ζ
($m_\phi \ll H$) (adiabatic mode)

- **Multiple field inflation :**

Scalar fields ϕ_i ←⇒ curvature perturbation ζ
($m_{\phi_i} \ll H$) (adiabatic mode)
\+ isocurvature perturbations S_{ij}

A number of effectively massless ($m \ll H$) fields is important.

Is this kind of classification sufficient ?

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Natural Hubble mass in supergravity

In supergravity,

$V \neq 0$ (inflation) $\Longrightarrow$ **SUSY breaking**

(mediated by gravity) $\Big\downarrow$

soft breaking masses : $m^2 \sim V\,G \sim H^2$

In supergravity, a situation naturally happens, in which there is only a light field and the masses of other fields are comparable to the Hubble parameter.

(Note also that a non-minimal coupling $R\phi^2$ leads to $m \sim H$)

This model is called quasi-single field inflation

(Chen & Wang 2009)

Effects of isocurvaton σ with m ~ H

Since m ~ H, the isocurvaton σ can contribute to the curvature perturbation around the horizon exit.

Adiabatic mode | **Isocurvaton**

$$\pi = \frac{\delta\phi}{\dot{\phi}}$$

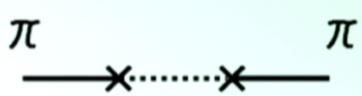
mixing

$\dot{\pi}\sigma$

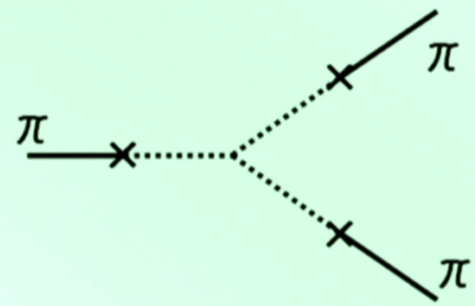
σ

(σ^3 coupling)

- **modify powerspectrum :**



- **generate large bispectrum :**



We would like to investigate this kind of feature in more detail.

Original model of quasi-single field inflation

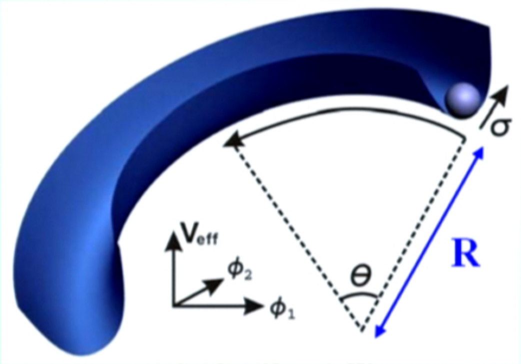
(Chen & Wang 2009)

$$S_{\text{matter}} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}(R+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\text{sr}}(\theta) - V(\sigma)\right].$$

$$V''(\sigma_0) \gtrsim \mathcal{O}(H^2).$$

Background (homogeneous) EOMs :

$$\begin{cases} 3M_{\text{Pl}}H^2 = \frac{1}{2}R^2\dot\theta_0^2 + V + V_{\text{sr}}, \\ R^2\ddot\theta_0 + 3R^2H\dot\theta_0 + V_{\text{sr}}'(\theta) = 0. \\ \sigma_0 = \text{const.} \equiv 0, \quad V'(\sigma_0 = 0) = R\dot\theta_0^2. \end{cases}$$



(Credit: Chen & Wang)

$\sigma_0 = 0$ is the minimum of the effective potential :

$$V_{\text{eff}} = -\frac{\dot\theta_0^2}{2}(R+\sigma)^2 + V(\sigma).$$

Centrifugal force caused by the turning angular velocity

Due to the centrifugal force, $\sigma_0 = 0$ is not a minimum of the original potential V.

Two different approaches to inflation

- Consider a model with a particular type of potential and kinetic term, which is well motivated by particle physics.

- Consider a quite generic action permitted by the symmetry preserved during inflation, that is, a time-dependent spatial diffeomorphism invariance. Inflation spontaneously breaks time diffeomorphism inv. (Effective field theory of inflation)

(Cheung et al. 2008)

Two approaches are complementary.

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Effective field theory of inflation

(Single-field case)

Basic idea of effective field theory inflation

(Cheung et al. 2008)

Inflation must end to be followed by hot big bang Universe.

⇒ $\dot{\phi}(t) \neq 0.$

⇒ **spontaneously breaks time diffeomorphism inv.**

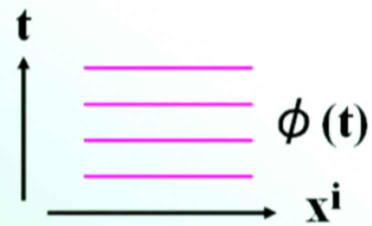
⇔ **Time-dependent spatial diffeo is unbroken.**

$$\delta x^i = \epsilon^i(t, x^i)$$

⇒ **In the low energy effective theory, any term respecting the unbroken symmetry is allowed.**

We can investigate the properties of perturbations generated during inflation without resort to a particular Lagrangian.

Unitary(Comoving) gauge (single field case)



Unitary(Comoving) gauge :

Time slice (t = const. hypersurface) coincides with ϕ = const. hypersurface.

$$\phi(t,\boldsymbol{x}) = \phi_0(t) + \underbrace{\cancel{\delta\phi(t,\boldsymbol{x})}}_{=0}.$$

The scalar field perturbation is eaten by the metric.

The graviton (metric) has three degree of freedom:

- **Curvature perturbation** ζ
- **Tensor perturbations** γ_{ij}

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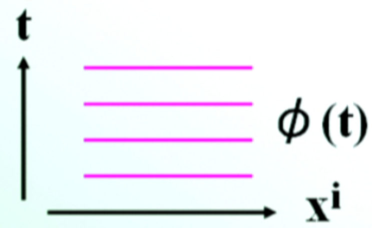

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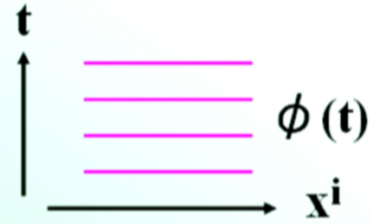

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Expanding around FLRW background

Fluctuations around FLRW background:

$$\begin{cases} \delta g^{00} = g^{00} + 1, \\ \delta K_{\mu\nu} = K_{\mu\nu} - H h_{\mu\nu}, \\ \delta R_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - 2(H^2 + k/a^2) h_{\mu[\rho]} h_{\sigma]\nu} + (\dot{H} + H^2)(h_{\mu\rho}\delta^0_\nu \delta^0_\sigma + (3\,\text{perms})). \end{cases}$$

- **0th and 1st order in fluctuations:**

kinetic(g^{00} dot{ϕ}^{2}) & potential energy of the background scalar field

$$\Rightarrow \quad S^{(0)+(1)} = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\rm pl}^2 R - c(t) g^{00} - \Lambda(t) \right].$$

(Terms such as $\partial^0 g^{00}, K^\mu_\mu, R^{00}$ can be absorbed into the above terms by integration by parts)

Variation w.r.t. g^{00} & g^{ij}

$$\Rightarrow \quad \begin{cases} 3M_{\rm pl}^2 H^2 = c(t) + \Lambda(t), \\ \dot{H} M_{\rm pl}^2 = -c(t). \end{cases}$$

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Action in unitary gauge (single field case)

Any quantities respecting the time-dependent diffeo. inv.

- **4-dim scalar**
- **generic function of t, f(t)**
- $\partial_\mu t = \delta^0_\mu$ **in unitary gauge, which allows any tensor with 0 upper index (g^{00}, R^{00}, …)**
- **Extrinsic curvature :** $K_{\mu\nu} = h^\sigma_\mu \nabla_\sigma n_\nu .$

(All covariant derivatives of n_μ can be written using $K_{\mu\nu}$ and derivatives of g^{00})

$$n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu}\partial_\mu t \partial_\nu t}}$$ **: unit vector orthogonal to t =const.**

$$h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$$ **: projection tensor to t =const.**

Note that ${}^{(3)}R_{\alpha\beta\gamma\delta} = h^\mu_\alpha h^\nu_\beta h^\rho_\gamma h^\sigma_\delta R_{\mu\nu\rho\sigma} - K_{\alpha\gamma}K_{\beta\delta} + K_{\beta\gamma}K_{\alpha\delta}.$ **${}^{(3)}R$ is redundant.**

$$\Rightarrow \quad S = \int d^4x \sqrt{-g} F(g^{\mu\nu}, g_{\mu\nu}, K_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\mu, \delta^0_\mu, t)$$

(all indices are contracted)

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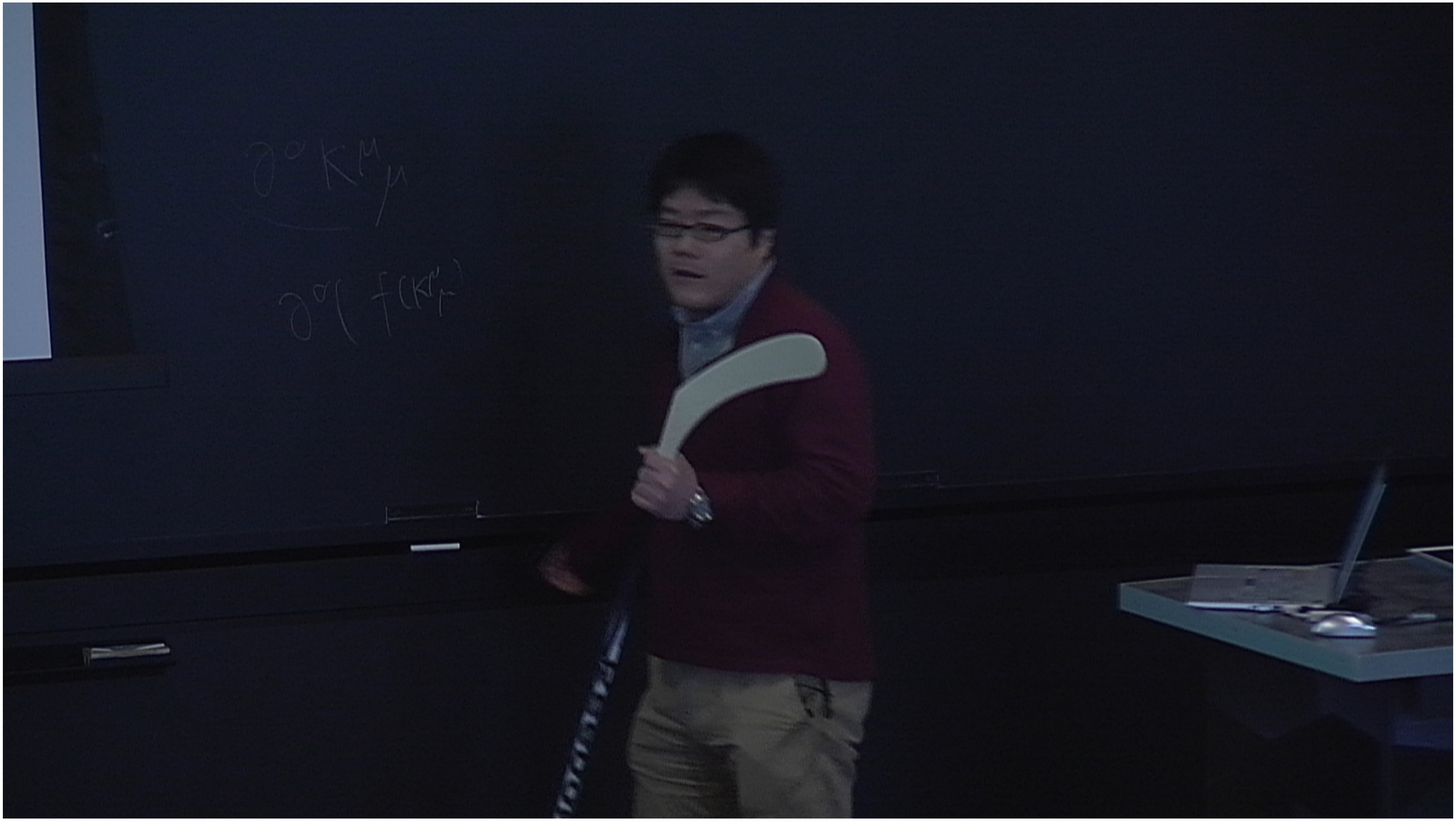

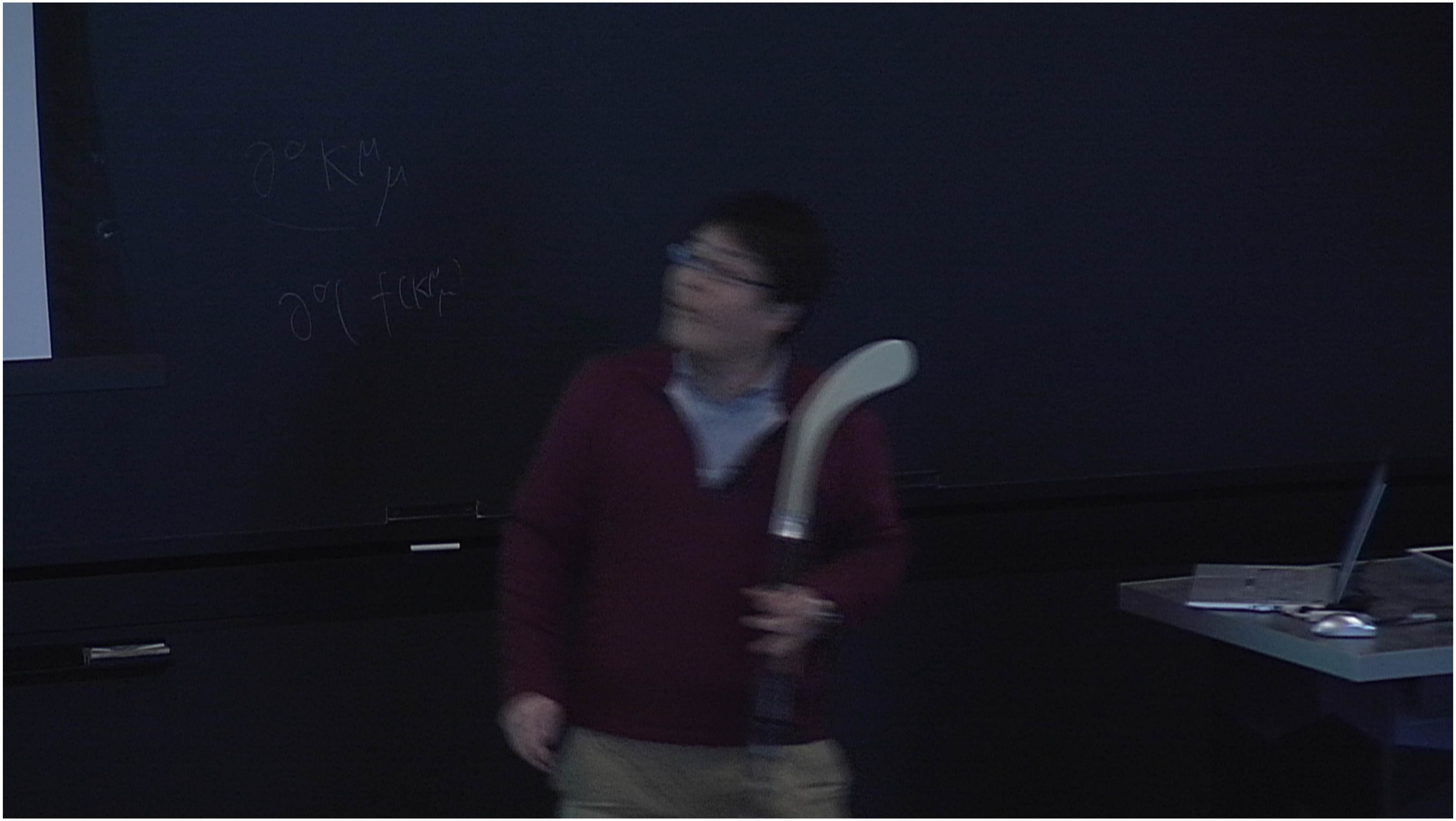

Expanding around FLRW background II

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R + M_{\rm pl}^2\dot{H}(t)g^{00} - M_{\rm pl}^2(3H^2(t) + \dot{H}(t))\right.$$
$$\left. + F^{(2)+(3)+\cdots}\left(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\rho\sigma}; \delta^0_\mu, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, t\right)\right].$$

$$F^{(2)+(3)+\cdots} = \frac{1}{2}M_2^4(t)\left(\delta g^{00}\right)^2 + \frac{1}{3!}M_3^4(t)\left(\delta g^{00}\right)^3$$
$$+\frac{1}{2}\lambda_1^3(t)\delta g^{00}\delta K^\mu_\mu + \frac{1}{2}\lambda_2^2(t)\delta K^\mu_\mu K^\nu_\nu + \frac{1}{2}\lambda_2^3(t)\delta K^\mu_\nu K^\nu_\mu + \cdots.$$

This is the most general action in unitary gauge ($\delta\phi(t, \boldsymbol{x}) = 0$), which satisfies the time-dependent spatial diffeo. invariance.

Note, however, that time diffeo. :
$t \longrightarrow \tilde{t} = t + \xi^0(x),\ \boldsymbol{x} \longrightarrow \tilde{\boldsymbol{x}}$ **is broken.**

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Stuckelberg trick

In order to (apparently) recover broken time diffeo. :

$$t \longrightarrow \tilde{t} = t + \xi^0(x), \;\; \boldsymbol{x} \longrightarrow \tilde{\boldsymbol{x}},$$

we introduce the Stuckelberg field π , which corresponds to the Goldstone boson and transforms as

$$\pi(x) \longrightarrow \tilde{\pi}(\tilde{x}(x)) = \pi(x) - \xi^0(x).$$

$\Rightarrow$ t + π (x) is invariant under this time diffeo.

We have only to make the following replacements :

$$\begin{cases} f(t) \longrightarrow f(t+\pi), \\ \delta^0_\mu \longrightarrow \partial_\mu(t+\pi) = \delta^0_\mu(1+\dot{\pi}) + \delta^i_\mu \partial_i \pi, \\ g^{00} \longrightarrow g^{\mu\nu}\dfrac{\partial(t+\pi)}{\partial x^\mu}\dfrac{\partial(t+\pi)}{\partial x^\nu} = (1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi})g^{0i}\partial_i\pi + g^{ij}\partial_i\pi\partial_j\pi, \\ g^{o\nu} \longrightarrow g^{\mu\nu}\dfrac{\partial(t+\pi)}{\partial x^\mu} = (1+\dot{\pi})g^{0\nu} + g^{i\nu}\partial_i\pi, \\ \ldots \end{cases}$$

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$$\begin{cases} f(t) \longrightarrow f(t+\pi), \\ \delta^0_\mu \longrightarrow \partial_\mu(t+\pi) = \delta^0_\mu(1+\dot{\pi}) + \delta^i_\mu \partial_i \pi, \\ g^{00} \longrightarrow g^{\mu\nu} \dfrac{\partial(t+\pi)}{\partial x^\mu} \dfrac{\partial(t+\pi)}{\partial x^\nu} = (1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi}) g^{0i} \partial_i \pi + g^{ij} \partial_i \pi \partial_j \pi, \\ g^{o\nu} \longrightarrow g^{\mu\nu} \dfrac{\partial(t+\pi)}{\partial x^\mu} = (1+\dot{\pi}) g^{0\nu} + g^{i\nu} \partial_i \pi, \\ \dots \end{cases}$$

Expanding around FLRW background II

$$S=\int d^4x\sqrt{-g}\Big[\frac{1}{2}M_{\rm pl}^2R+M_{\rm pl}^2\dot H(t)g^{00}-M_{\rm pl}^2(3H^2(t)+\dot H(t))$$
$$+F^{(2)+(3)+\cdots}\left(\delta g^{00},\delta K_{\mu\nu},\delta R_{\mu\nu\rho\sigma};\delta^0_\mu,g_{\mu\nu},g^{\mu\nu},\nabla_\mu,t\right)\Big].$$

$$F^{(2)+(3)+\cdots}=\frac{1}{2}M_2^4(t)\left(\delta g^{00}\right)^2+\frac{1}{3!}M_3^4(t)\left(\delta g^{00}\right)^3$$
$$+\frac{1}{2}\lambda_1^3(t)\delta g^{00}\delta K^\mu_\mu+\frac{1}{2}\lambda_2^2(t)\delta K^\mu_\mu K^\nu_\nu+\frac{1}{2}\lambda_2^3(t)\delta K^\mu_\nu K^\nu_\mu+\cdots.$$

This is the most general action in unitary gauge ($\delta\phi(t,\boldsymbol{x})=0$), which satisfies the time-dependent spatial diffeo. invariance.

Note, however, that time diffeo. :
$t\longrightarrow\tilde t=t+\xi^0(x),\ \ \boldsymbol{x}\longrightarrow\tilde{\boldsymbol{x}}$ **is broken.**

Mixing terms

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2}M_{\rm pl}^2R-M_{\rm pl}^2\left[3H^2(t+\pi)+\dot{H}(t+\pi)\right]\right.$$
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($\pi = 0$ recovers the action in unitary gauge)

Mixing terms between π & g have fewer derivatives :

e.g. $M_{\rm pl}^2\dot{H}\cdot(1+\dot{\pi})^2g^{00}\ni M_{\rm pl}^2\dot{H}\left(-\dot{\pi}^2+2\dot{\pi}\delta g^{00}\right)$

Canonical normalization
$(\pi_c\sim M_{\rm pl}(-\dot{H})^{1/2}\pi,\ \delta g_c^{00}\sim M_{\rm pl}\delta^{00})$ $\Rightarrow$ $-\dot{\pi}_c^2+2\epsilon^{1/2}H\dot{\pi}_c\delta g_c^{00}$

The Goldstone boson π decouples from graviton g at the energy scale E >> E_{mix} = $\varepsilon^{1/2}$ H.

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We have only to consider self interaction terms for π in DL.

$$\begin{aligned} S &= \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2R - M_{\rm pl}^2\dot{H}\left(\dot\pi^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + \frac{1}{2}M_2^4\left(\dot\pi^2 + 2\dot\pi - \frac{(\partial_i\pi)^2}{a^2}\right)^2 - \frac{1}{6}M_3^4\left(\dot\pi^2 + 2\dot\pi - \frac{(\partial_i\pi)^2}{a^2}\right)^3 + \cdots\right] \\ &= \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2R - M_{\rm pl}^2\dot{H}\left(\dot\pi^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot\pi^2 + \dot\pi^3 - \dot\pi\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot\pi^3 + \cdots\right] \\ &= \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2R - M_{\rm pl}^2\dot{H}c_\pi^{-2}\left(\dot\pi^2 - c_\pi^2\frac{(\partial_i\pi)^2}{a^2}\right) - M_{\rm pl}^2\dot{H}(c_\pi^{-2}-1)\left(\dot\pi^3 - \dot\pi\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot\pi^3 + \cdots\right] \end{aligned}$$

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- **It is manifest which term leads to a non-trivial sound velocity.**
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Expanding around FLRW background II

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R + M_{\rm pl}^2 \dot{H}(t) g^{00} - M_{\rm pl}^2(3H^2(t)+\dot{H}(t)) + F^{(2)+(3)+\cdots}\left(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\rho\sigma}; \delta^0_\mu, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, t\right)\right].$$

$$F^{(2)+(3)+\cdots} = \frac{1}{2}M_2^4(t)\left(\delta g^{00}\right)^2 + \frac{1}{3!}M_3^4(t)\left(\delta g^{00}\right)^3 + \frac{1}{2}\lambda_1^3(t)\delta g^{00}\delta K^\mu_\mu + \frac{1}{2}\lambda_2^2(t)\delta K^\mu_\mu K^\nu_\nu + \frac{1}{2}\lambda_2^3(t)\delta K^\mu_\nu K^\nu_\mu + \cdots.$$

This is the most general action in unitary gauge ($\delta\phi(t,\boldsymbol{x}) = 0$), which satisfies the time-dependent spatial diffeo. invariance.

Note, however, that time diffeo. :
$t \longrightarrow \tilde{t} = t + \xi^0(x),\ \boldsymbol{x} \longrightarrow \tilde{\boldsymbol{x}}$ **is broken.**

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Relation between π and ζ

Flat gauge :

$$dl^2 = a^2(t)\delta_{ij}dx^i dx^j,$$

Unitary gauge (π =0) :

$$dl^2 = a^2(t)(1+2\zeta)\delta_{ij}dx^i dx^j.$$

$$a^2(t) = a^2(\tilde{t}-\pi) = a^2(\tilde{t})(1-2H\pi)$$

gauge transformation

$$\xi^0(x) = \pi(x)$$

$$\begin{cases} t \longrightarrow \tilde{t} = t + \xi^0(x), \\ \pi(x) \longrightarrow \tilde{\pi}(\tilde{x}(x)) = \pi(x) - \xi^0(x). \end{cases}$$

$$\Rightarrow \quad \zeta = -H\pi$$

We can easily evaluate observable quantities like powerspectrum.

Strategy of effective field theory inflation

Action in unitary gauge,
which respects time dependent spatial diffeo.

↓ Stuckelberg trick

Action for Goldstone boson π

↓ Decoupling limit

Simplified action without mixing with g

↓ Relation between π & ζ
($\zeta \sim -H\pi$)

Easy calculations for powerspectrum & bispectrum for ζ

Effective field theory approach to quasi-single field inflation

Action in unitary gauge (multi field case)

(See also Senatore & Zaldarriaga 2010)

D.O.F = three physical modes of graviton
(adiabatic mode and tensor modes)
+ additional scalar field σ
(isocurvature mode)

- **Action in unitary gauge :**

$$S = \int d^4x \sqrt{-g} F(g^{\mu\nu}, g_{\mu\nu}, K_{\mu\nu}, R_{\mu\nu\rho\sigma}, \sigma, \nabla_\mu, \delta^0_\mu, t)$$

- **Expanding around FLRW background :**

$$\begin{aligned} S = \int d^4x \sqrt{-g} \Big[& \frac{1}{2} M_{\rm pl}^2 R + M_{\rm pl}^2 \dot{H}(t) g^{00} - M_{\rm pl}^2 (3H^2(t) + \dot{H}(t)) \\ & + F^{(2)+(3)+\cdots} \left(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\rho\sigma}, \sigma; \delta^0_\mu, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, t \right) \Big] \\ = S_{\rm grav} + S_\sigma + S_{\rm mix}. & \end{aligned}$$

We can construct a completely general action for multiple field case.

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We can construct a completely general action for multiple field case.

Action up to the second order

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R + M_{\rm pl}^2\dot H(t)g^{00} - M_{\rm pl}^2(3H^2(t)+\dot H(t)) + F^{(2)+(3)+\cdots}\left(\delta g^{00},\delta K_{\mu\nu},\delta R_{\mu\nu\rho\sigma},\sigma;\delta^0_\mu,g_{\mu\nu},g^{\mu\nu},\nabla_\mu,t\right)\right]$$

$$= S_{\rm grav} + S_\sigma + S_{\rm mix}.$$

$$S^{(2)}_{\rm grav} = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R + M_{\rm pl}^2\dot H(t)g^{00} - M_{\rm pl}^2(3H^2(t)+\dot H(t)) + \frac{1}{2}M_2^4(t)\left(\delta g^{00}\right)^2\right].$$

Stuckelberg trick
&
Decoupling limit

$$S^{(2)}_{\rm grav} = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R - M_{\rm pl}^2\dot H c_\pi^{-2}\left(\dot\pi^2 - c_\pi^2\frac{(\partial_i\pi)^2}{a^2}\right) - M_{\rm pl}^2\dot H(c_\pi^{-2}-1)\left(\dot\pi^3 - \dot\pi\frac{(\partial_i\pi)^2}{a^2}\right) + \cdots\right]$$

The origins of a non-trivial sound velocity and large non-G are clear.

Action up to the second order III

$$S^{(2)}_{\rm mix} = \int d^4x\sqrt{-g}\left[\beta_1(t)\delta g^{00}\sigma + \beta_2(t)\delta g^{00}\partial^0\sigma + \beta_3(t)\delta K^\mu_\mu\sigma\right]$$
$$= \int d^4x\sqrt{-g}\left[\beta_1(t)\delta g^{00}\sigma + \beta_2(t)\delta g^{00}\partial^0\sigma + \beta_3(t)\partial^0\sigma - \left(\dot\beta_3(t) + 3H(t)\beta_3(t)\right)\sigma\right].$$

(Typically, δ K terms contain more derivatives and hence can be neglected. But, this term can survive.)

Stuckelberg trick & Decoupling limit

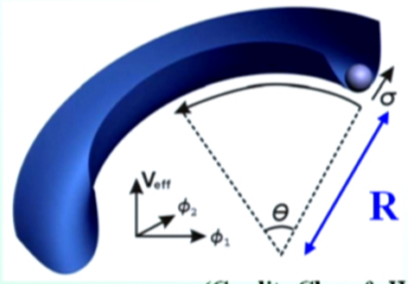
$$S^{(2)}_{\rm mix} = \int d^4x\, a^3\left[(-2\beta_1 + \dot\beta_3)\dot\pi\sigma + (2\beta_2 - \beta_3)\dot\pi\dot\sigma + \beta_3\frac{\partial_i\pi\partial_i\sigma}{a^2} - \beta_1\left(\dot\pi^2 - \frac{(\partial_i\pi)^2}{a^2}\right)\sigma + 3\beta_2\dot\pi^2\dot\sigma - 2\beta_2\dot\pi\frac{\partial_i\pi\partial_i\sigma}{a^2} - \beta_2\frac{(\partial_i\pi)^2}{a^2}\dot\sigma + \cdots\right]$$

We have mixing terms between π & σ coming from β s, which can cause large non-Gaussianities for π (ζ).

Correspondence to the original model of quasi-single field inflation

$$S_{\rm matter} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}(R+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)\Big].$$

$$V''(\sigma_0 = 0) \gtrsim \mathcal{O}(H^2).$$



(Credit: Chen & Wang)

Background solutions : $\theta_0(t),\ \sigma_0(t) = 0.$

In unitary gauge : $\delta\theta = \theta - \theta_0 = 0.$

$$\begin{aligned} S_{\rm matter} &= \int d^4x\sqrt{-g}\Big[-\frac{1}{2}(R+\sigma)^2 g^{00}\dot\theta_0^2 - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\,\partial_\nu\sigma - V_{\rm sr}(\theta_0) - V(\sigma)\Big] \\ &= \int d^4x\sqrt{-g}\Big[-\frac{1}{2}R^2\dot\theta_0^2\, g^{00} - (V_{\rm sr}(\theta_0) + V(0)) - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\,\partial_\nu\sigma - \frac{1}{2}\big(V''(0) - \dot\theta_0^2\big)\sigma^2 \\ &\qquad - R\,\dot\theta_0^2\,\delta g^{00}\sigma - \frac{V'''(0)}{3!}\sigma^3 - \frac{1}{2}\dot\theta_0^2\,\delta g^{00}\sigma^2 + \mathcal{O}(\sigma^4)\Big], \end{aligned}$$

(By use of the background EOMs, this can be rewritten in terms of H.)

$$\begin{aligned} S_{\rm matter} &= \int d^4x\sqrt{-g}\Big[M_{\rm Pl}^2\dot H g^{00} - M_{\rm Pl}^2(3H^2+\dot H) - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\,\partial_\nu\sigma - \frac{1}{2}\Big(V''(0) + \frac{2M_{\rm Pl}^2\dot H}{R^2}\Big)\sigma^2 \\ &\qquad + \frac{2M_{\rm Pl}^2\dot H}{R}\,\delta g^{00}\sigma - \frac{V'''(0)}{3!}\sigma^3 + \frac{M_{\rm Pl}^2\dot H}{R^2}\,\delta g^{00}\sigma^2 + \mathcal{O}(\delta\sigma^4)\Big]. \end{aligned}$$

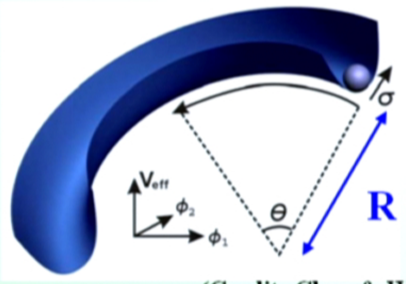
$$\alpha_1 = 1,\ \alpha_3 = V''(0) + \frac{2M_{\rm Pl}^2\dot H}{R^2},\ \beta_1 = \frac{2M_{\rm Pl}^2\dot H}{R},\ \gamma_1 = -\frac{1}{3!}V'''(0),\ \bar\gamma_1 = \frac{M_{\rm Pl}^2\dot H}{R^2},\ (\text{others}) = 0.$$

Our framework can easily accommodate the original model.

Correspondence to the original model of quasi-single field inflation

$$S_{\rm matter} = \int d^4x\sqrt{-g}\,[-\frac{1}{2}(R+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{\rm sr}(\theta) - V(\sigma)]\,.$$

$$V''(\sigma_0 = 0) \gtrsim \mathcal{O}(H^2).$$



(Credit: Chen & Wang)

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Effects of heavy particles

Simple example :

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2\dot{H}g^{00} - M_{\rm Pl}^2(3H^2+\dot{H}) - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\,\partial_\nu\sigma - \frac{m^2}{2}\sigma^2 + \beta\,\delta g^{00}\sigma\right].$$

m : mass of σ

β : mixing between adiabatic mode and σ

For m >> H, the kinetic term of σ is irrelevant.

$$S \sim \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2\dot{H}g^{00} - M_{\rm Pl}^2(3H^2+\dot{H}) - \frac{m^2}{2}(\sigma - \frac{\beta}{m^2}\delta g^{00})^2 + \frac{\beta^2}{2m^2}(\delta g^{00})^2\right].$$

σ can quickly responds to the variation of the adiabatic mode δ g^{00}.

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Integration out of σ

Modify sound velocity, $c_\pi^2 \equiv \dfrac{-M_{\rm pl}^2\dot{H}}{-M_{\rm pl}^2\dot{H} + 2\beta^2/m^2}$ ⟵ **Coefficient of (δ g^{00})^2**

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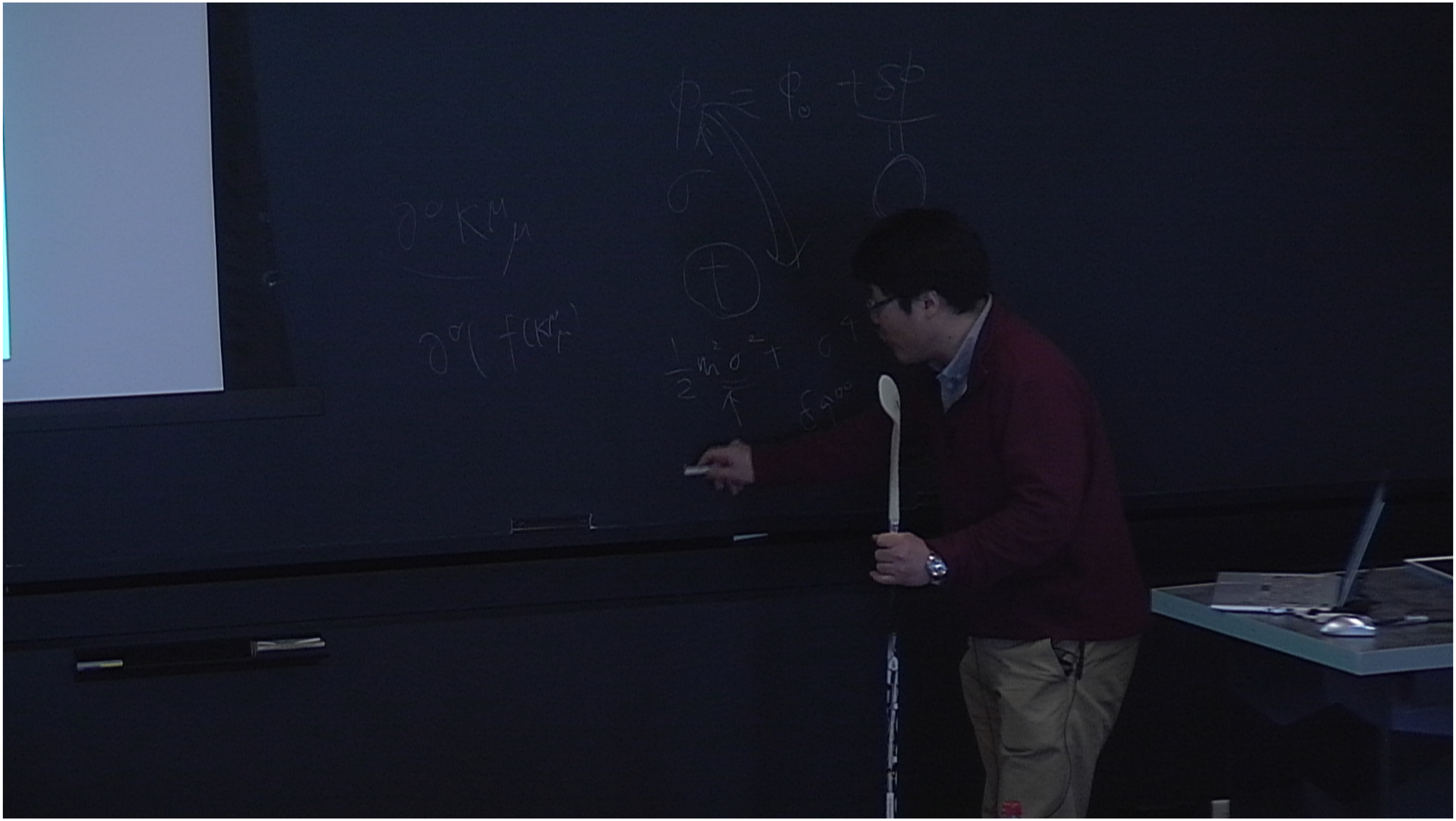


Powerspectrum

Almost constant turn case :

Assumption: time dependence of $\epsilon, \tilde{\epsilon}, \alpha_\sigma, c_\sigma, m_\sigma$ is negligible.

$$\begin{cases} u_k = \dfrac{1}{2M_{\rm Pl}\tilde{\epsilon}^{1/2}(c_\pi k)^{3/2}}(1 + ic_\pi k\tau)e^{-ic_\pi k\tau} = \dfrac{1}{2M_{\rm Pl}\tilde{\epsilon}^{1/2}(c_\pi k)^{3/2}}(1 - ix)e^{ix}\,, \\ v_k = -ie^{\frac{i}{2}\pi\nu + \frac{i}{4}\pi}\dfrac{\sqrt{\pi}H}{2\alpha_\sigma}(-\tau)^{3/2}H_\nu^{(1)}(-c_\sigma k\tau) = -ie^{\frac{i}{2}\pi\nu + \frac{i}{4}\pi}\dfrac{\sqrt{\pi}H}{2\alpha_\sigma(c_\pi k)^{3/2}}x^{3/2}H_\nu^{(1)}(r_s x) \end{cases}$$

(τ = - 1/(aH) : conformal time,   x = - c π k τ ,   rs = c σ /c π)

$$\nu = \sqrt{\frac{9}{4} - \frac{m_\sigma^2}{H^2}} \ \text{ for } m_\sigma < \frac{3}{2}H\,, \qquad \nu = i\sqrt{\frac{m_\sigma^2}{H^2} - \frac{9}{4}} \ \text{ for } m_\sigma > \frac{3}{2}H\,.$$

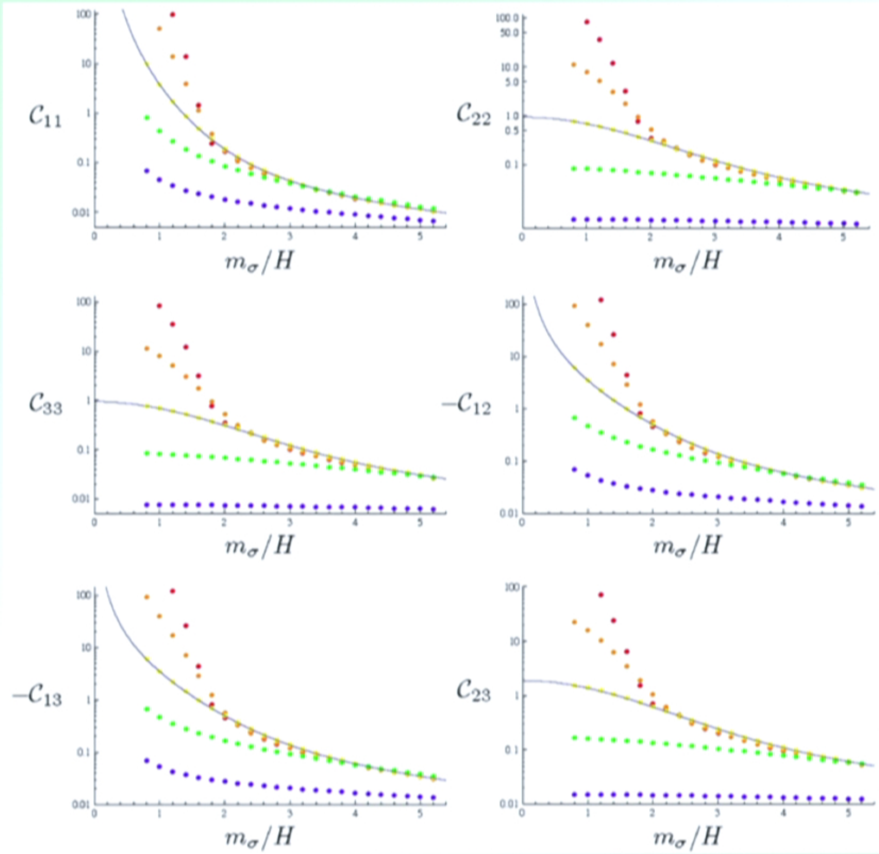
$$\langle \zeta_{\mathbf{k}}(t)\zeta_{\mathbf{k}'}(t)\rangle = H^2\langle \pi_{\mathbf{k}}(t)\pi_{\mathbf{k}'}(t)\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k} + \mathbf{k}')\frac{2\pi^2}{k^3}\mathcal{P}_\zeta(k)$$

$$\mathcal{P}_\zeta(k) = \frac{H^2}{8\pi^2 M_{\rm Pl}^2\epsilon\, c_\pi}(1 + C)\,.$$

$$C = \frac{c_\pi^2}{\alpha_\sigma^2 M_{\rm Pl}^2(-\dot{H})}\left(\frac{\tilde{\beta}_1^2}{H^2}\mathcal{C}_{11} + \tilde{\beta}_2^2\mathcal{C}_{22} + \tilde{\beta}_3^2\mathcal{C}_{33} + \frac{\tilde{\beta}_1}{H}\tilde{\beta}_2\mathcal{C}_{12} + \frac{\tilde{\beta}_1}{H}\tilde{\beta}_3\mathcal{C}_{13} + \tilde{\beta}_2\tilde{\beta}_3\mathcal{C}_{23}\right).$$

Cij is an integration of H ν .

Cij for fixed rs = c σ / c π



Note that all C's vanish in the heavy mass limit.

The dots are numerical results for rs=0.1 (red), 0.3 (orange), 1 (yellow), 3 (green), and 10 (blue).
The curves are analytic results for rs=1, which well coincide with the numerical results.

Summary

- We have constructed the most generic action in the unitary gauge for multiple field inflation.
- Taking decoupling limit is justified for adequate slow-roll parameters and makes it clear.
- In the constant turning case, we have calculated the power spectrum of scalar perturbations numerically for general values of $r_s = c_\sigma / c_\pi$ and analytically for $r_s = 1$.
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