Title: A Self-consistent Model of the Black Hole Evaporation

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Abstract: We construct a
self-consistent model which describes a black hole from formation to evaporation including the back reaction from the Hawking radiation. In the case where a null shell collapses, at the beginning the evaporation occurs, but it stops eventually, and a horizon and singularity appear. On the other hand, in the generic collapse process of a continuously distributed null matter, the black hole evaporates completely without forming a macroscopically large horizon nor singularity. We also find a stationary solution in the heat bath, which can be regarded as a normal thermodynamic object. (hep-th: 1302.4733)

A Self-consistent Model of the Black Hole Evaporation

Kyoto University

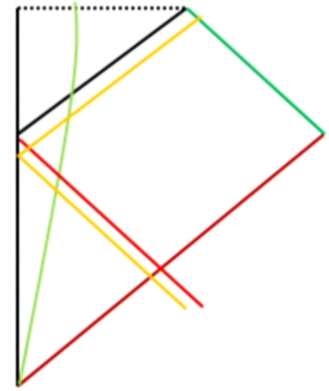
Yuki Yokokura

(with H. Kawai and Y. Matsuo)

[hep-th:1302.4733]

@ Perimeter Institute 2013 3/7

Introduction 1 : Is horizon really formed?



QM+GR⇒Hawking radiation

⇒What is the origin of the entropy $S_{BH} = \frac{A}{4}$?
How does the information come back?

⇒Usually:

- Neglect effect from the Hawking radiation in a collapse process
- Assume formation of horizon
- Use a static black hole geomerty

⇒Question:
when we take quantum effect into account in the collapse process, is the horizon really formed?

⇒We tried to solve the semi-classical Einstein equation by including the back reaction of the collapsing matter and radiation:

$$G_{\mu\nu} = 8\pi G \langle T_{\mu\nu} \rangle$$

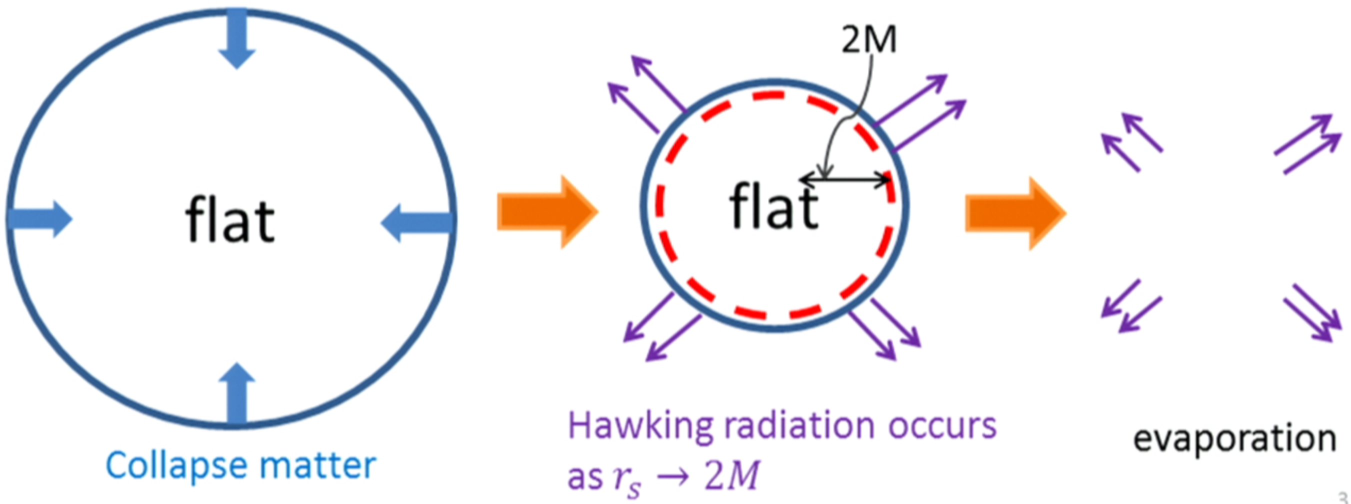
Collapsing matter and Hawking radiation

Introduction2:
A naive viewpoint from an outside observer

- In classical collapse process, from viewpoint of an outside observer, it takes infinite time to form a horizon and singularity.

⇒Introduce quantum effects

⇒If BH evaporates in finite time from viewpoint of the outside observer, then the horizon and singularity will not occur.



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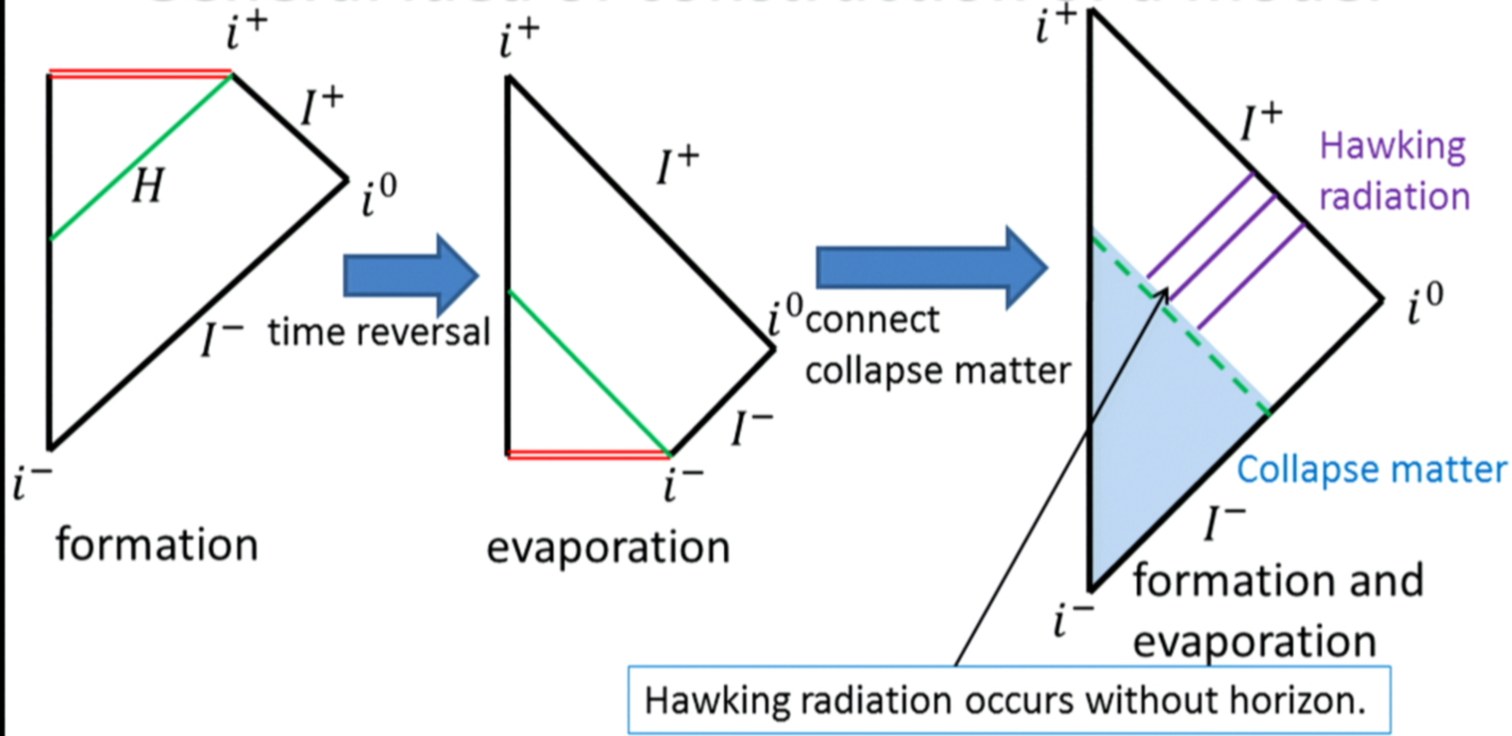
Introduction3:
General idea of construction of a model



A simple model=flat + null shell+ outgoing Vaidya

⇒Question:

Is this a solution of the semi-classical Einstein equation?

Intoduction4: Our results

- Construct a self-consistent model which describes a black hole from formation to evaporation including the back reaction from the Hawking radiation.
- In the case where a null shell collapses, at the beginning the evaporation occurs, but it stops eventually, and a horizon and singularity appear.
- In the generic collapse process of a continuously distributed null matter, the black hole evaporates almost completely without forming a macroscopically large horizon nor singularity.
- Find a stationary solution in the heat bath

Note

We mean by "black hole" not one that has an event horizon defined globally as in the rigorous sense, but one that is formed in a semi-classical collapse process.

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Talk Plan

- 1 Introduction
- 2 Construction of a model and a flux formula
- 3 A single null collapse
- 4 Generic collapse and stationary solution
- 5 Summery and Discussions

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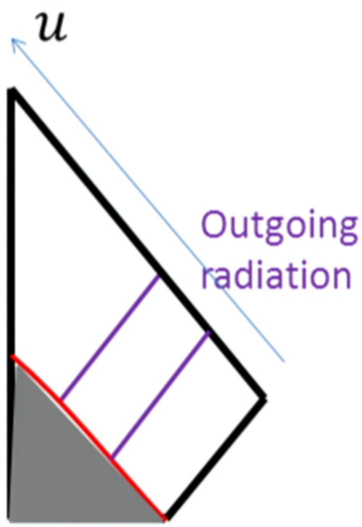
2-1: Outgoing Vaidya metric

$$ds^2 = -\left(1 - \frac{a(u)}{r}\right) du^2 - 2dudr + r^2 d\Omega^2$$

[Vaidya 1951]

∼This describes outgoing radiation without the gray-body factor.

(The Bondi mass $m(u) = \frac{a(u)}{2G}$)



- spherically symmetric
- Traceless : $G^{\mu}{}_{\mu} = 0$
- the only non-zero component : $G_{uu} = -\frac{\dot{a}(u)}{r^2}$

<u>The physical meanings</u>

- Assume massless particle
- Neglect the Weyl anomaly
- Neglect partial waves with $l \gg 1$
- Neglect the gray-body factor

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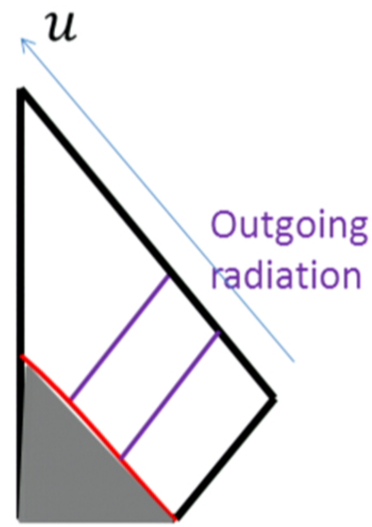

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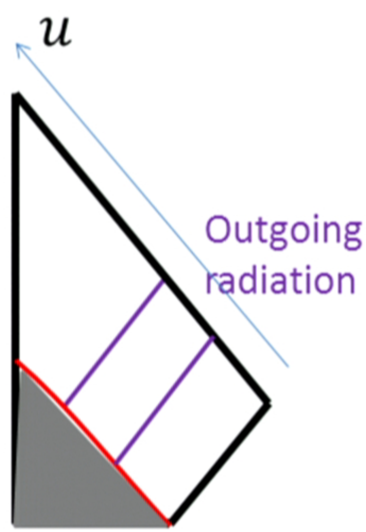

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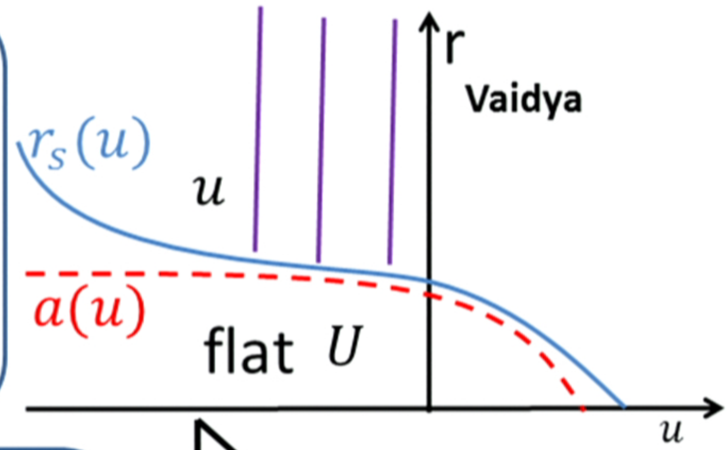
2-2:Connecting the two metrics on the shell

Flat metric:

$$ds^2 = -dU^2 - 2dUdr + r^2 d\Omega^2$$

junction condition on the shell r_s:

$$\frac{r_s(u)-a(u)}{r_s(u)}du = -2dr_s = dU$$



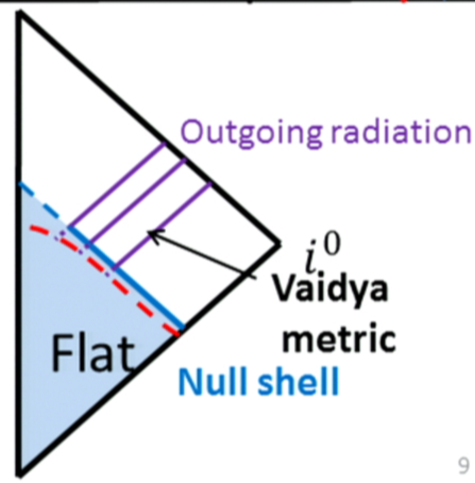
Surface energy-momentum on the shell

[Barrabes and Israel 1991]

surface energy density: $\mu = \frac{a}{8\pi G r_s{}^2}$

surface pressure: $p = \frac{-r_s \dot{a}}{4\pi G (r_s - a)^2} > 0$

(The work done by the shell as it contracts is transformed to the Hawking radiation.)



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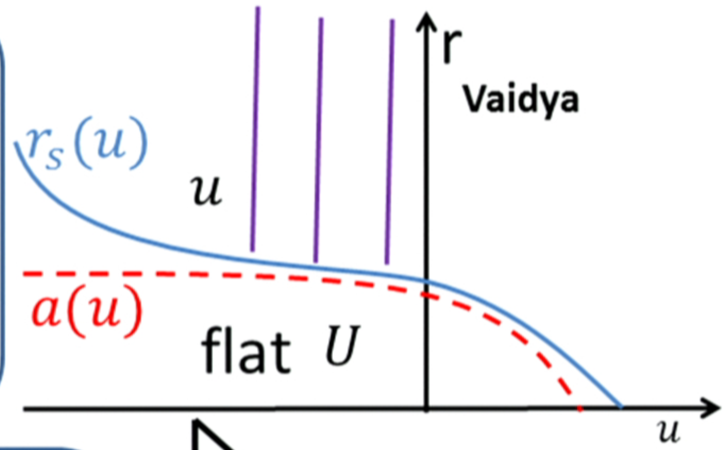

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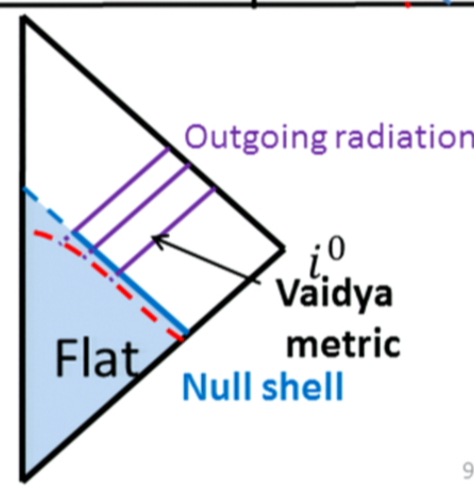

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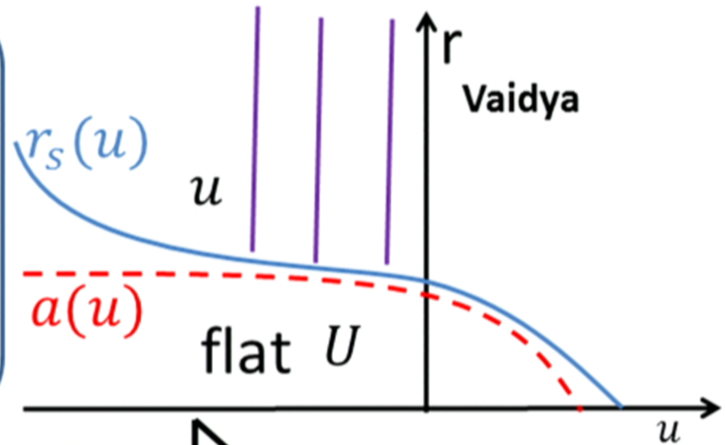

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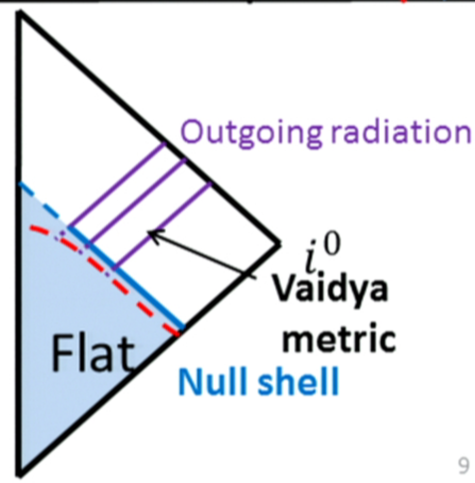

2-3: General behavior of the shell

The equation of motion for $r_s(u)$ is given by

$$\frac{dr_s}{du} = -\frac{r_s(u) - a(u)}{2r_s(u)}$$

where $a(u)$ is a given function.

$\Rightarrow r_s$ approaches a in time $\sim a$ if

$$a \ll \frac{a}{|\dot{a}|} (\Leftrightarrow |\dot{a}| \ll 1)$$

Cf. In the case of the Hawking radiation $\frac{a}{|\dot{a}|} \sim a^3$

$\Rightarrow$ The general solution is given by

$$\boldsymbol{r_s(u) \approx a(u) - 2a(u)\dot{a}(u) + Ca(u)e^{-\frac{u}{2a(u)}}}$$

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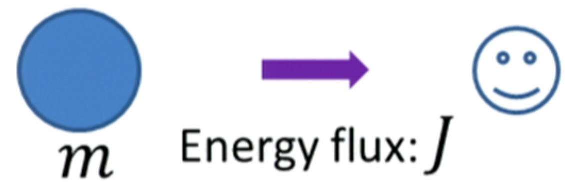
2-4: Flux formula

- We here derive a flux formula



$$\frac{dm}{du} = -J(u)$$

<u>Derivation</u>

- $J(u) \equiv 4\pi r^2 < 0|:T_{uu}(u):|0 >$ at $r \gg a$
- Minkowski vacuum:$a_\omega|0> = 0, \omega > 0$ (in Heisenberg picture)
- Point-splitting regularization:
- Eikonal approximation
- Only use the s-wave

$$J(u) = \frac{\hbar}{8\pi}\left[\frac{\ddot{U}(u)^2}{\dot{U}(u)^2} - \frac{2\dddot{U}(u)}{3\dot{U}\ (u)}\right] \equiv \frac{\hbar}{8\pi}\{u, U\}$$

Appendix2: derivation of the Flux formula

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Derivation

- $J(u) \equiv 4\pi r^2 < 0|{:}T_{uu}(u){:}|0 >$ at $r \gg a$
- Heisenberg picture:$a_\omega|0> = 0, \omega > 0$
- Point-splitting regularization:
  $< 0|{:}T_{uu}(u){:}|0 > = \lim_{u' \to u}[< 0|{:}\partial_u\phi(u)\partial_u\phi(u'){:}|0 > - < 0|{:}\partial_u\phi_0(u)\partial_u\phi_0(u'){:}|0 >]$
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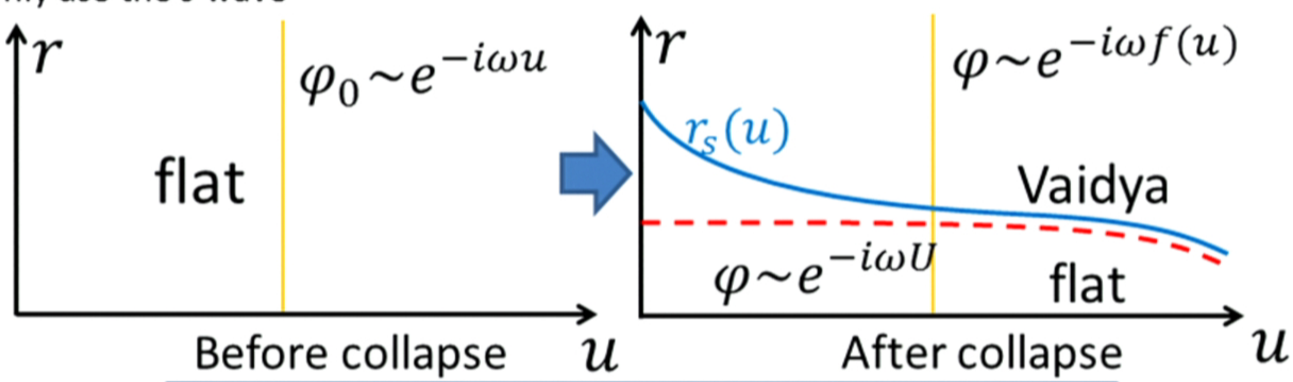

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2-6: Test of the equations

- Let's test the equations in the case $\dot{a} = 0$.

$$\Rightarrow r_s(u) = a_0 + C a_0 e^{-\frac{u}{2a_0}}$$

$$\Rightarrow J(u) = \frac{\hbar}{8\pi}\left[\frac{\ddot{r}_s(u)^2}{\dot{r}_s(u)^2} - \frac{2\dddot{r}_s(u)}{3\dot{r}_s\,(u)}\right] = \frac{\hbar}{96\pi {a_0}^2} = \frac{\pi}{6\hbar}{T_H}^2$$

$\Rightarrow$1D black body radiation with $T_H = \dfrac{\hbar}{4\pi a_0}$

Note: The Planck distribution can be also derived in the same setup.
$\Rightarrow$Then the horizon structure is not used. All that is necessary is that the affine parameters are related exponentially.[Barcelo et al 2011]

$$U(u) = -2r_s(u) \sim a(u_*) e^{-\frac{u-u_*}{2a(u_*)}}$$

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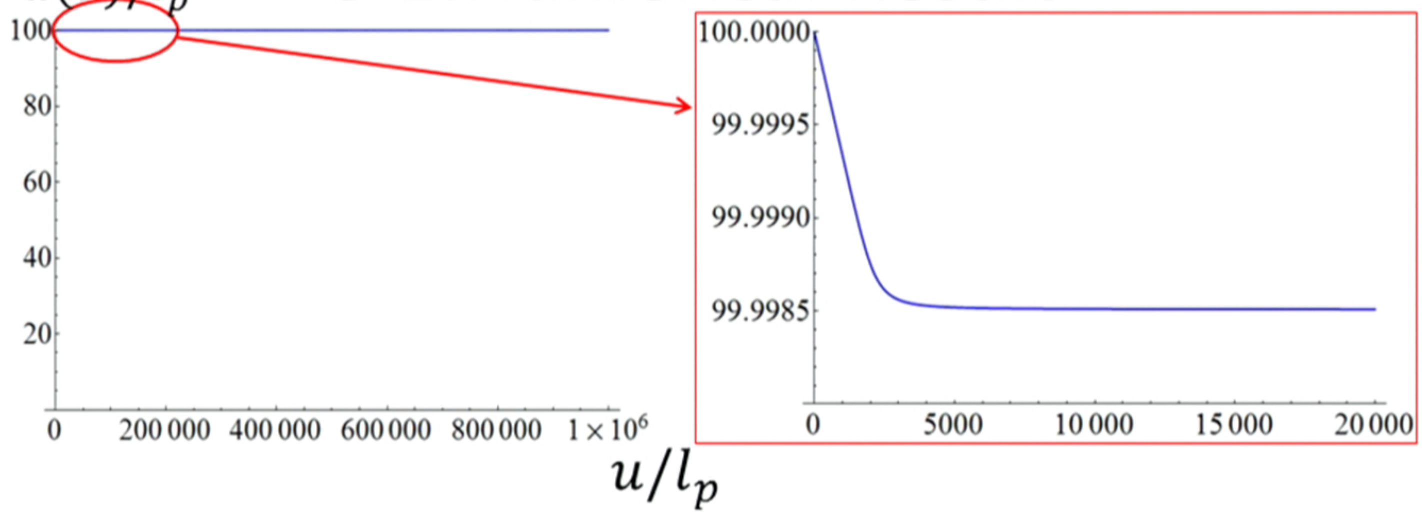
3-1:Numerical Result



⇒ At the beginning, the radiation occurs, but it stops eventually.

⇒It cannot evaporate even though the back reaction is taken into account.

⇒The horizon and singularity appear (at $u = \infty$).

Initial conditions:

$$r_s(0) = na(0), \qquad \dot{r}_s(0) = -\frac{n-1}{2n}, \qquad \ddot{r}_s(0) = 0,$$

$$a(0) \gg l_p, \qquad n > 1, \qquad n \approx 1$$

3-2:Analytical result
～Why does the evaporation stop?～

From the general behavior of the shell, for large u,

$$r_s(u) \approx a(u) - 2a(u)\dot{a}(u) + Ca(u)e^{-\frac{u}{2a(u)}}$$

This term will damp as time passes.

$\Rightarrow$ $r_s(u) \approx a(u)$

$\Rightarrow$In this case, we can solve the equations analytically:

$$u = \frac{e^{-\frac{D^2}{2}}}{6\pi B}\int_D^{\xi} d\xi' \, e^{\frac{1}{4}\xi'^2}$$

$$a(u) = a(0) - B\int_D^{\xi} d\xi' \, e^{\frac{1}{4}\xi'^2}$$

where B, D are integration constants, and B is small and positive.

$\Rightarrow$$a(u)$ will not necessarily vanish as $u \to \infty$.

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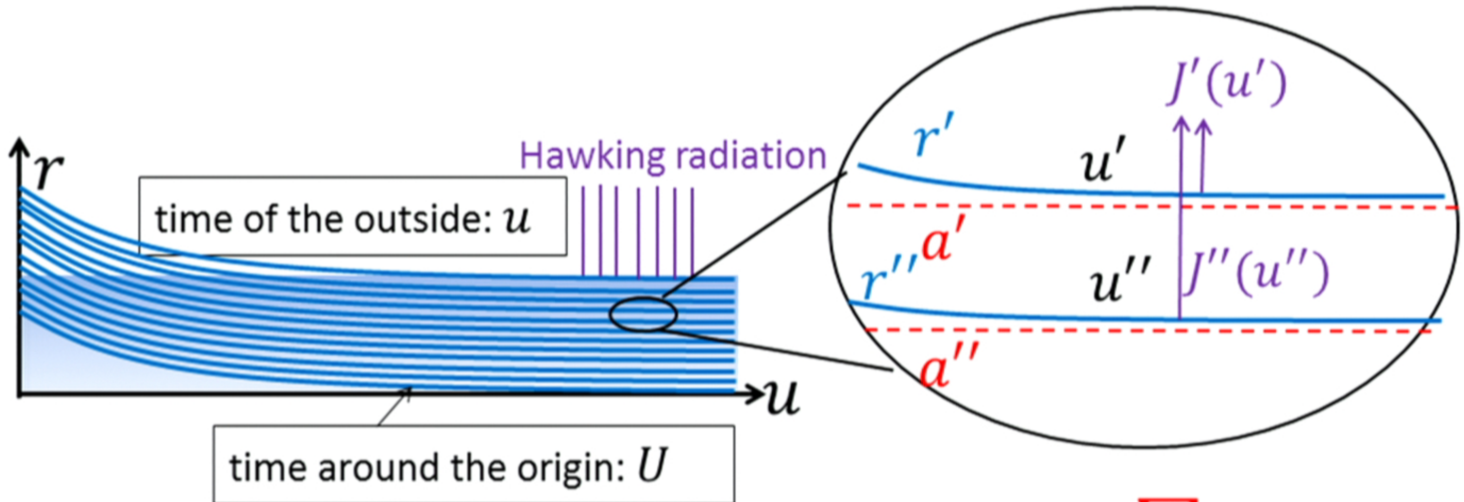
4-1: the general collapse process of a continuously distributed null matter

- Suppose that a continuously distributed null matter collapses. Each shell approaches the Schwarzschild radius.



$$a' = \sum_{under\ the\ sell} \delta a$$

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4-2: Equations for each layer

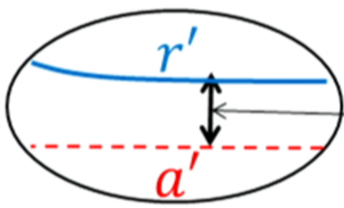
For each shell, the following equations hold in its time $\boldsymbol{u'}$, and we consider the case where time has passed sufficiently $u' \gg 1$.

- $ds^2 = -\left(1 - \frac{a'(u')}{r}\right) du'^2 - 2du'dr + r^2 d\Omega^2$
- $\frac{dr'}{du'} = -\frac{r'(u') - a'(u')}{2r'(u')} \Rightarrow r'(u') \approx a' - 2a'\frac{da'}{du'} + C\, a' e^{-\frac{u'}{2a'}}$
- $\frac{da'}{du'} = -2GJ'(u') = -\frac{N {l_p}^2}{4\pi}\{u', U\}$

N is the degrees of freedom of the fields. (Cf. In the standard model, $N \sim 100$.)

Assumption: $\frac{da'}{du'} = -f(a')$

Cf. Hawking $\frac{da}{du} \sim -\frac{1}{a^2}$



$$\rho' \equiv r' - a' \approx -2a'\frac{da'}{du'} = 2a' f(a')$$

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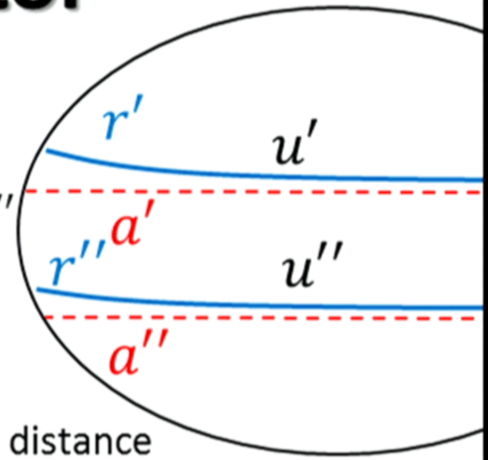
4-3: Redshift factor



- We estimate the redshift factor here.

Look at the shell r' from the both sides

$$\frac{r'-a'}{r'}du' = dr' = \frac{r'-a''}{r'}du''$$

If $a' - a'' = da$ is small,

$$\frac{du''}{du'} = \frac{r'-a'}{r'-a''} = 1 - \frac{da}{\rho'}$$

By integrating it, we obtain the redshift factor for finite distance

$$\frac{du''}{du'} = exp\left(-\int_{a''}^{a'} \frac{d\bar{a}}{\rho(\bar{a})}\right)$$

- We introduce

$$\xi' \equiv \frac{d}{du'}\log\left(\frac{dU}{du'}\right) = \frac{d}{du'}\left(-\int_{0}^{a'(u')} \frac{d\bar{a}}{\rho(\bar{a})}\right) = \frac{1}{2a'}$$

$$\rho' = -2a'\frac{da'}{du'}$$

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4-4:Hawking Radiation from each shell

If the flux formula is rewritten in ξ',

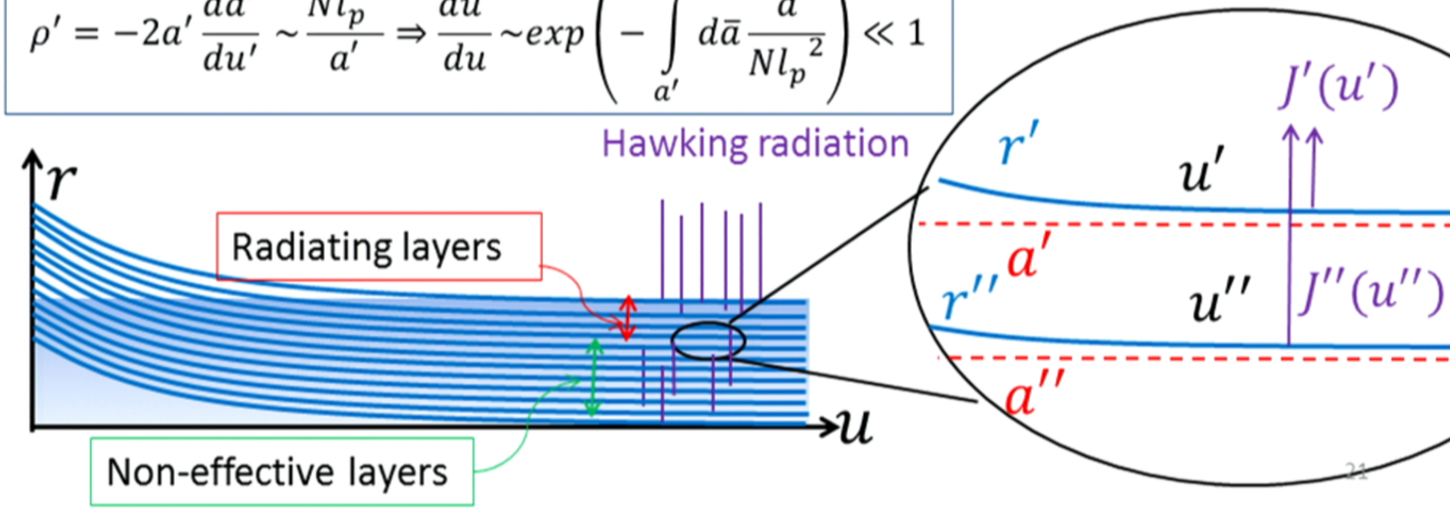
$$J' = -\frac{N\hbar}{24\pi}\left(\xi'^2 - \frac{d\xi'}{du'}\right) \approx \frac{N}{6\hbar}T'_H{}^2$$

$$\xi' = \frac{1}{2a'}$$

⇒The Hawking radiation occurs from each shell.

⇒But the radiation is emitted substantially only from the outermost region because of the large redshift.

$$\rho' = -2a'\frac{da'}{du'} \sim \frac{N l_p{}^2}{a'} \Rightarrow \frac{du'}{du} \sim exp\left(-\int_{a'}^{a} d\bar{a}\,\frac{\bar{a}}{N l_p{}^2}\right) \ll 1$$



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4-5:The Stationary Metric

- Suppose that the black hole is put in the heat bath, and it becomes stationary. Then the inside metric is given by

$$ds^2 = -\frac{N{l_p}^2 r^2}{24\pi a^4} e^{-\frac{24\pi}{N{l_p}^2}(a^2 - r^2)} dt^2 + \frac{24\pi r^2}{N{l_p}^2} dr^2 + r^2 d\Omega^2$$

- This is smoothly connected to the Schwarzschild metric at $r = a + \frac{N{l_p}^2}{24\pi a}$.
- This does not have a horizon.
- This does not have a semi-classical limit ($\hbar \to 0$) because we has made it in the self-consistent manner. (cf. Ising model + mean field approximation)
- The time around the origin T is almost frozen from viewpoint of the outside

$$dT = \frac{N{l_p}^2}{a^2} e^{-\frac{12\pi}{N{l_p}^2} a^2} dt$$

- This does not have a large curvature compared with ${l_p}^{-2}$ if $N > 100$:

$$at\ \ r \sim \sqrt{N} l_p \quad R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \sim \frac{10000}{N^2 {l_p}^4} < \frac{1}{{l_p}^4}$$

- This also takes into account the back reaction from the Weyl anomaly.

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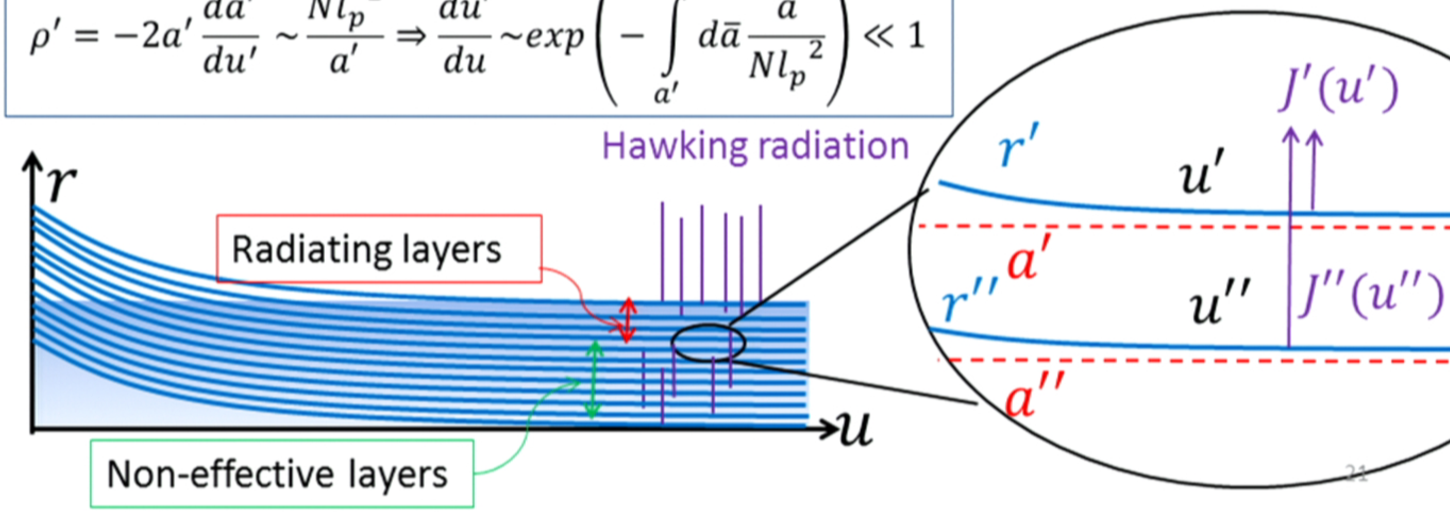

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m Energy flux: J

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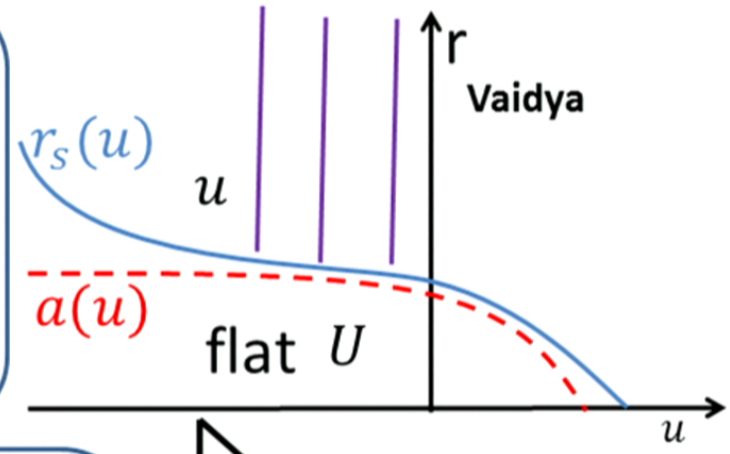

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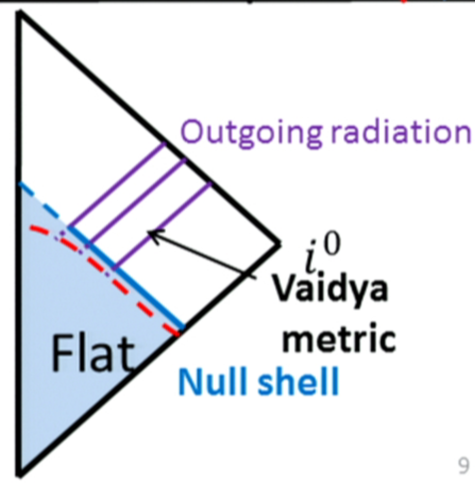

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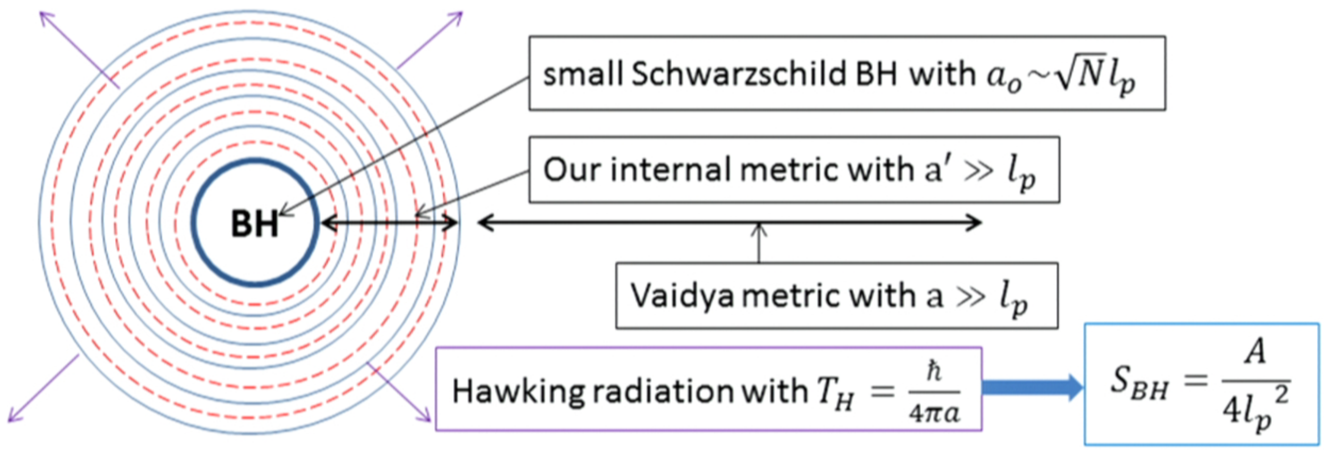
4-6: How does the black hole evaporate?

- The time evolution of total mass is determined by

$$\frac{da}{du} = -\frac{N{l_p}^2}{48\pi a^2}$$

Actually it evaporates gradually from the outermost shell as if one peels off an onion.

- However, the innermost shell with $a_0 \sim \sqrt{N} l_p$ cannot evaporate forever because of the same reason in the a single shell case. It has the horizon and singularity, which are not macroscopically large.



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5-2:Discussion
What is the origin of the entropy?
How does the information come back?

- Our stationary solution has neither horizon nor singularity, so the information inside the hole must come back after evaporation. However, we don't understand the mechanism clearly yet.
- For example, suppose that we throw a newspaper into the stationary black hole described by our metric. It will behave like another null shell going to the hole as it approaches the surface. Clearly its energy will be transformed into the Hawking radiation by our mechanism.
- However, the radiation itself comes from the quantum field on the past infinity, or the vacuum. How will the information of the newspaper come back?
- A clue to this problem is that we have taken the expectation value of the energy-momentum tensor in our self-consistent equations, which might correspond to the coarse-graining procedure in the ordinary statistical mechanics.
- The microscopic origin of the black hole also comes from the same point.