Title: Canonical gravity using unconstrained null initial data

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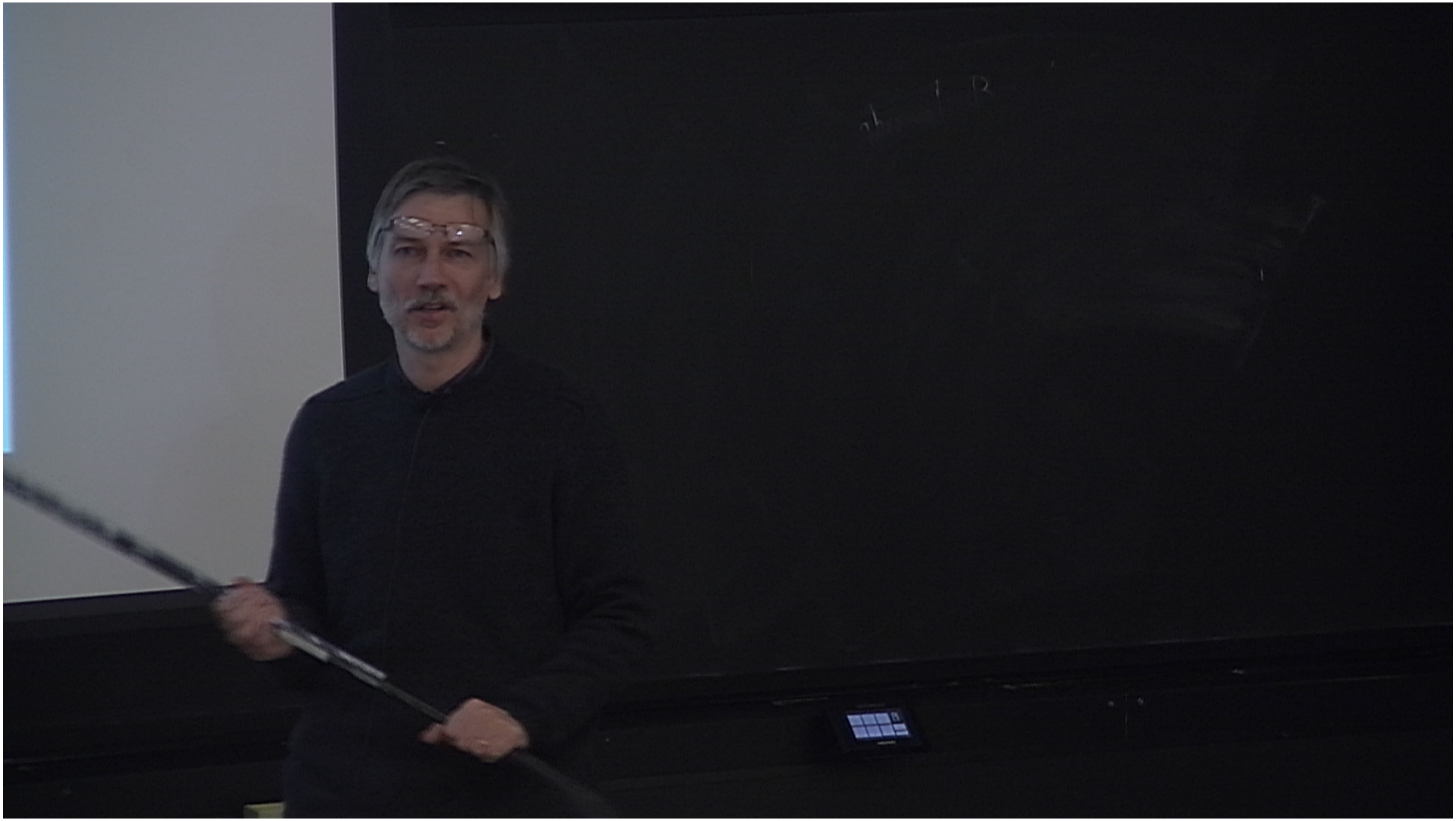
Abstract:



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ASPECTS OF CANONICAL GRAVITY USING NULL INITIAL DATA

Michael Reisenberger

Universidad de la República, Uruguay

PI, Waterloo, 28/3/2013

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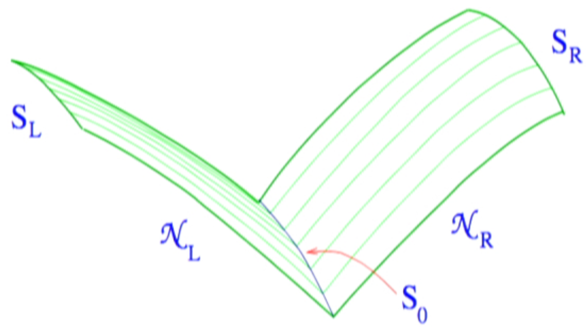
[1] arXiv: gr-qc/0703134, arXiv: 0712.2541, PRL **101**, 211101 (2008), arXiv: 1211.3880,

Plan of talk

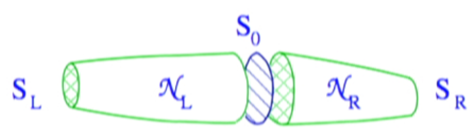
- Double null sheets as initial data hypersurfaces
- Advantages of null canonical gravity
- - the holographic principle
- The Poisson brackets
- How can one understand the holographic entropy bound?
- Klein-Gordon field in terms of null initial data in curved spacetime
- Inconclusion and a conjecture on holography.

DOUBLE NULL SHEETS AS INITIAL DATA HYPERSURFACES

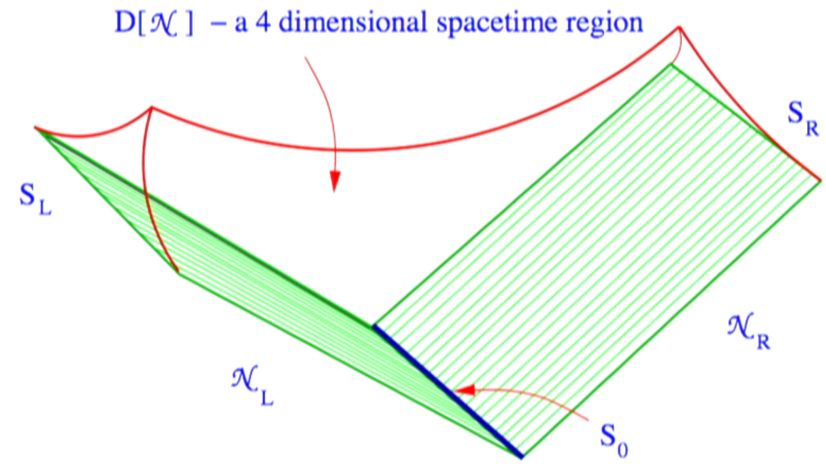
- A double null sheet is a pair of intersecting null hypersurfaces (or "lightfronts") - like an open book in spacetime.



- $\mathcal{N}_R, \mathcal{N}_L$ are 3-surfaces swept out by null geodesics emerging normally from the two sides of 2-disk S_0.

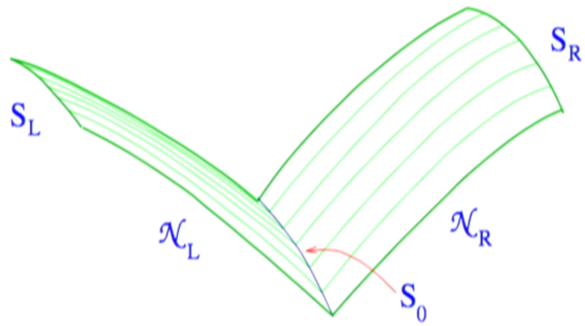


- initial data on $\mathcal{N} = \mathcal{N}_L \cup \mathcal{N}_R$ specifies solution in domain of dependence $D[\mathcal{N}]$



Advantages of null canonical gravity

- No constraints -can identify free, complete data ($\sim$ 1962 Sachs, Bondi, van der Burg, Metzner, Penrose, Dautcourt)
- Lorentzian
- Observables - main free initial data has direct interpretation in terms of test lightrays $\rightarrow$ allow formulation of observables
- There is a natural, preferred, class of time evolutions



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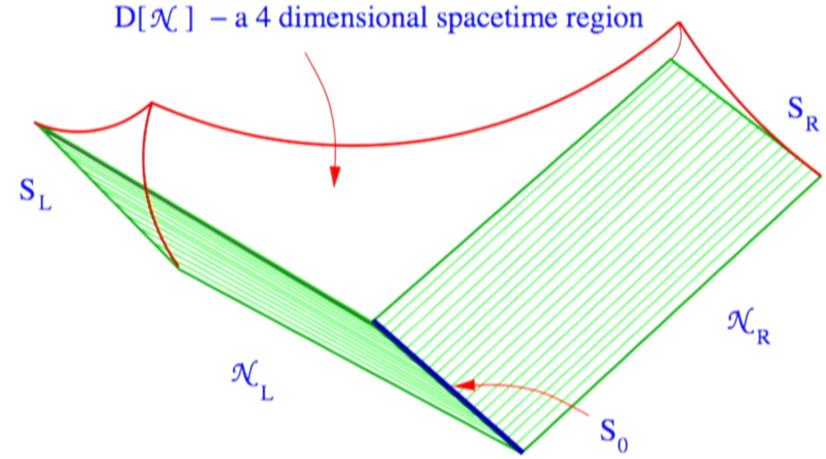

Double null sheets as initial data hypersurfaces

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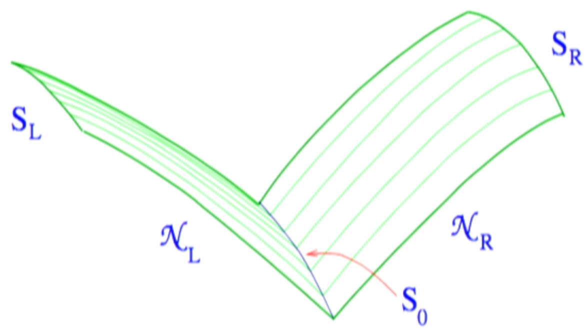

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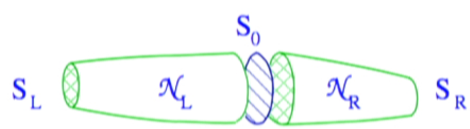

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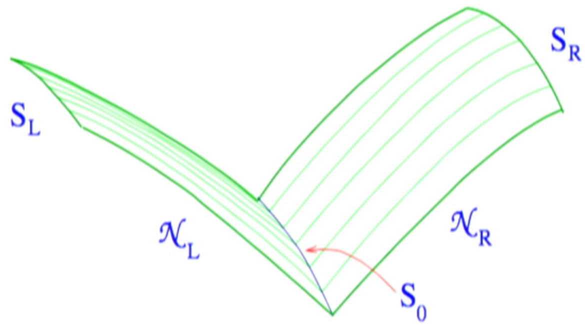

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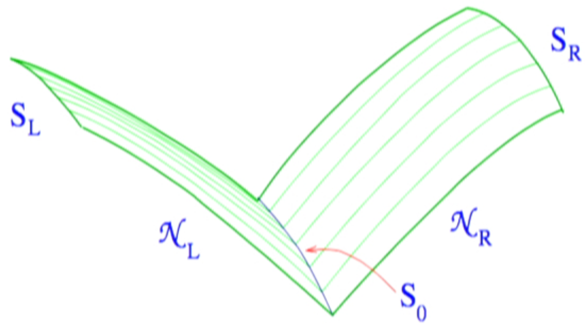


- Holography Beckenstein - 't Hooft - Susskind - Bousso bound: If generators of a branch ($\mathcal{N}_R$ say) are non-expanding at S_0 then they argue

$$\text{Entropy on } \mathcal{N}_R \leq \frac{\text{Area}[S_0]}{4\text{A}_{Planck}}$$

with saturation possible.

  - Normally the highest entropy thermodynamic macrostate of a system has essentially *all* microstates. This suggests

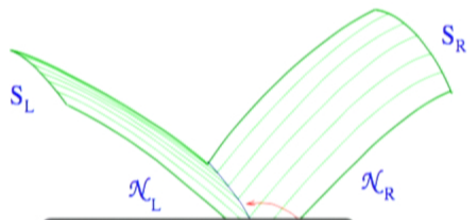
$$dimH_{\mathcal{N}_R} = e^{\frac{A[S_0]}{4A_{Planck}}}$$

or

$$dimH_{\mathcal{N}} = e^{\frac{A[S_0]}{2A_{Planck}}}$$

with $H_{\mathcal{N}}$ the Hilbert space of gravity and matter in $D[\mathcal{N}]$.

  - Canonical GR on $\mathcal{N}$ seems ideal framework to check this.



THE POISSON BRACKETS FOR FREE DATA ON $\mathcal{N}$ FOR CLASSICAL VACUUM GR

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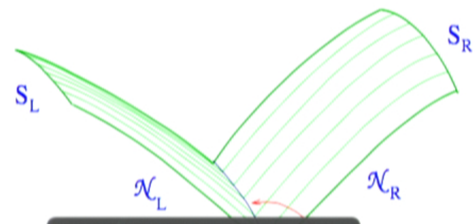

The Poisson brackets for free data on $\mathcal{N}$ for classical vacuum GR

Brackets not shown vanish.

$$\begin{aligned}\{\mu(\mathbf{1}),\bar{\mu}(\mathbf{2})\} &= 4\pi G\,\frac{1}{\rho_0}\,\delta^2(\theta_2-\theta_1)\,H(\mathbf{1},\mathbf{2})\Big[\frac{1-\mu\bar{\mu}}{\nu_A}\Big]_{\mathbf{1}} \\ &\quad\times\Big[\frac{1-\mu\bar{\mu}}{\nu_A}\Big]_{\mathbf{2}}\, e^{\int_1^2(\bar{\mu}d\mu-\mu d\bar{\mu})/(1-\mu\bar{\mu})}\end{aligned}$$

for $\mathbf{1},\mathbf{2}$ in the same branch, $\mathcal{N}_A$.

$$\begin{aligned}\{\rho_0(\theta_1),\lambda(\theta_2)\} &= 8\pi G\delta^2(\theta_2-\theta_1) \\ \{\rho_0(\theta),\tau[f]\} &= -8\pi G \pounds_f\rho_0(\theta) \\ \{\lambda(\theta),\tau[f]\} &= -8\pi G\Big[\pounds_f\lambda+\frac{\pounds_f\mu}{(1-\mu\bar{\mu})^2}(\partial_{\nu_R}\bar{\mu}-\partial_{\nu_L}\bar{\mu})\Big]_\theta\end{aligned}$$

$$\begin{aligned}\{\tau[f_1],\tau[f_2]\} &= -16\pi G\Big[\tau[[f_1,f_2]]-\frac{1}{2}\int_{S_0}\pounds_{[f_1,f_2]}\epsilon \\ &\quad+\int_{S_0}\Big[\frac{\pounds_{f_1}\mu}{(1-\mu\bar{\mu})^2}\{\epsilon\pounds_{f_2}\bar{\mu}-\frac{1}{2}\pounds_{f_2}\epsilon(\partial_{\nu_R}\bar{\mu}+\partial_{\nu_L}\bar{\mu})\}-(1\leftrightarrow 2)\Big]\Big].\end{aligned}$$

For $\mathbf{1}$ in $\mathcal{N}_R - S_0$

$$\{\mu(\mathbf{1}),\lambda(\theta_2)\} = 4\pi G\frac{1}{\rho_0}\delta^2(\theta_2-\theta_1)[\nu_R\partial_{\nu_R}\mu]_1$$

$$\{\mu(\mathbf{1}),\tau[f]\} = -16\pi G\Big[\pounds_f\mu-\frac{1}{4}\frac{\pounds_f\rho_0}{\rho_0}\nu_R\partial_{\nu_R}\mu\Big]_1.$$

For $\mathbf{1}$ in S_0

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For $\mathbf{1}$ in $\mathcal{N}_L - S_0$

$$\begin{aligned}\{\mu(\mathbf{1}),\lambda(\theta_2)\} &= 4\pi G\frac{1}{\rho_0}\delta^2(\theta_2-\theta_1)[v_L\partial_{v_L}\mu]_{\mathbf{1}} \\ \{\mu(\mathbf{1}),\tau[f]\} &= -4\pi G\Big[\frac{\pounds_f\rho_0}{\rho_0}v_L\partial_{v_L}\mu\Big]_{\mathbf{1}}.\end{aligned}$$

For $\mathbf{1}\in\mathcal{N}_R$ (including $\mathbf{1}\in S_0$)

$$\begin{aligned}\{\bar\mu(\mathbf{1}),\lambda(\theta_2)\} &= 4\pi G\frac{1}{\rho_0}\delta^2(\theta_2-\theta_1)\Big[(v_R\partial_{v_R}\bar\mu)_{\mathbf{1}} \\ &\quad+\Big(\frac{1}{v_R}\Big)_{\mathbf{1}}e^{-2\int_{\mathbf{1}_0}^{\mathbf{1}}(\mu d\bar\mu)/(1-\mu\bar\mu)}(\partial_{v_L}\bar\mu)_{\mathbf{1}_0}\Big] \\ \{\bar\mu(\mathbf{1}),\tau[f]\} &= -8\pi G\Big[\Big(2\pounds_f\bar\mu-\frac{1}{2}\frac{\pounds_f\rho_0}{\rho_0}v_R\partial_{v_R}\bar\mu\Big)_{\mathbf{1}} \\ &\quad-\Big(\pounds_f\bar\mu-\frac{1}{2}\frac{\pounds_f\rho_0}{\rho_0}\partial_{v_L}\bar\mu\Big)_{\mathbf{1}_0}\Big(\frac{1}{v_R}\Big)_{\mathbf{1}}e^{-2\int_{\mathbf{1}_0}^{\mathbf{1}}(\mu d\bar\mu)/(1-\mu\bar\mu)}\Big]\end{aligned}$$

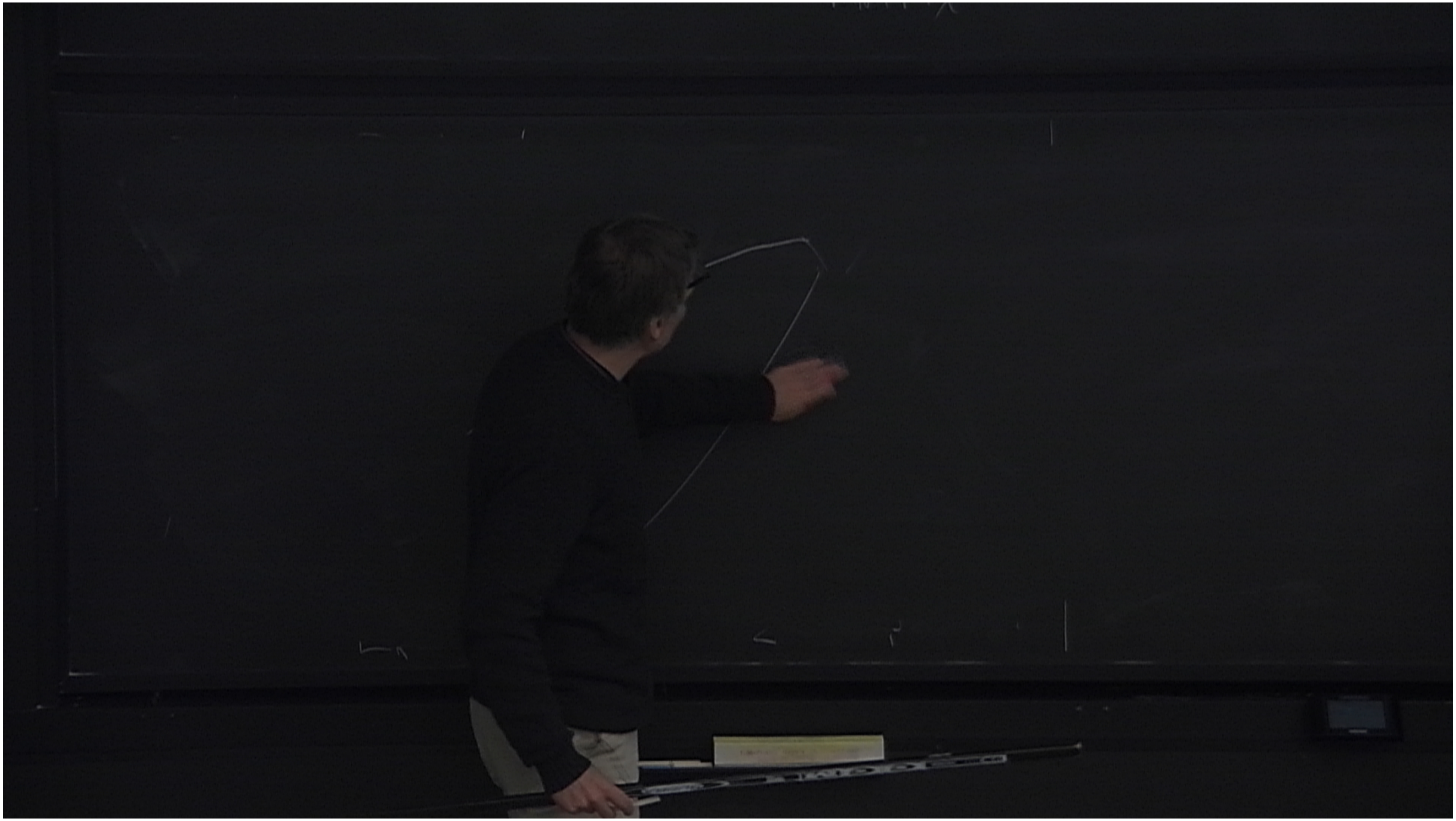
where $\mathbf{1}_0\in S_0$ is the origin of the generator through $\mathbf{1}$.
For $\mathbf{1}\in\mathcal{N}_L$

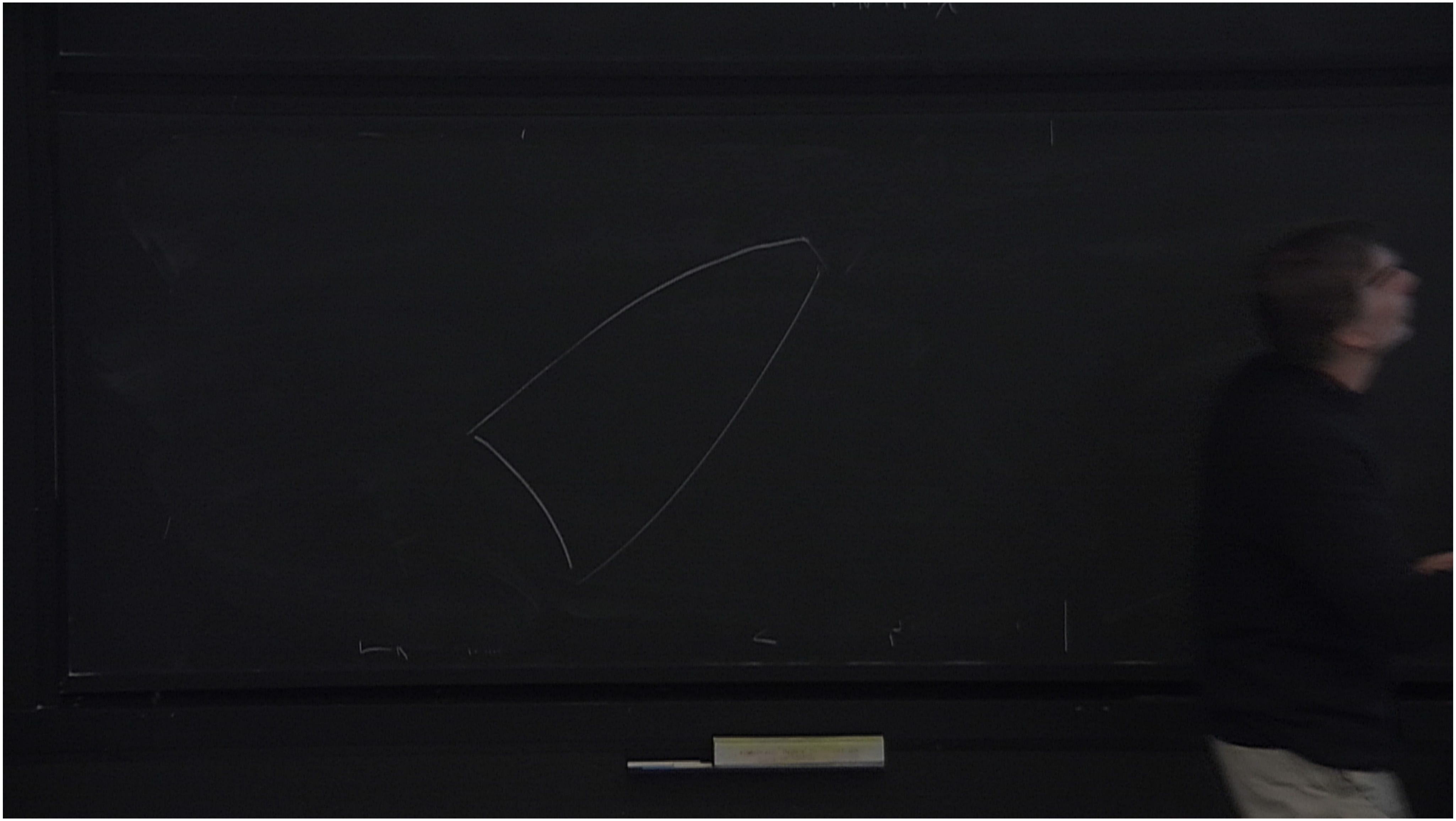
$$\begin{aligned}\{\bar\mu(\mathbf{1}),\lambda(\theta_2)\} &= 4\pi G\frac{1}{\rho_0}\delta^2(\theta_2-\theta_1)\Big[(v_L\partial_{v_L}\bar\mu)_{\mathbf{1}} \\ &\quad+\Big(\frac{1}{v_L}\Big)_{\mathbf{1}}e^{-2\int_{\mathbf{1}_0}^{\mathbf{1}}(\mu d\bar\mu)/(1-\mu\bar\mu)}(\partial_{v_R}\bar\mu)_{\mathbf{1}_0}\Big] \\ \{\bar\mu(\mathbf{1}),\tau[f]\} &= -8\pi G\Big[\Big(\frac{1}{2}\frac{\pounds_f\rho_0}{\rho_0}v_L\partial_{v_L}\bar\mu\Big)_{\mathbf{1}} \\ &\quad+\Big(\pounds_f\bar\mu-\frac{1}{2}\frac{\pounds_f\rho_0}{\rho_0}\partial_{v_R}\bar\mu\Big)_{\mathbf{1}_0}\Big(\frac{1}{v_L}\Big)_{\mathbf{1}}e^{-2\int_{\mathbf{1}_0}^{\mathbf{1}}(\mu d\bar\mu)/(1-\mu\bar\mu)}\Big].\end{aligned}$$

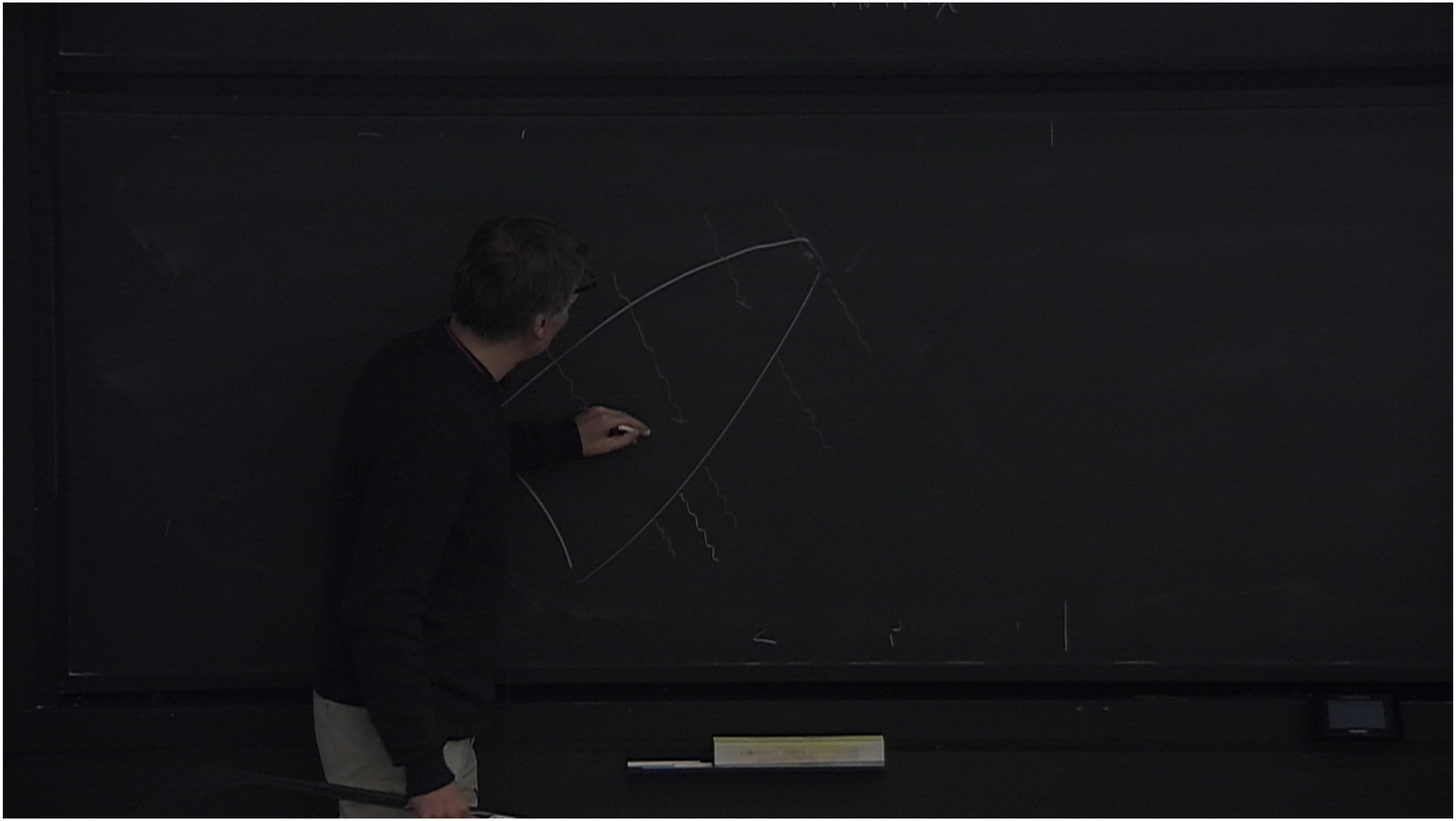
How can one understand the holographic entropy bound?

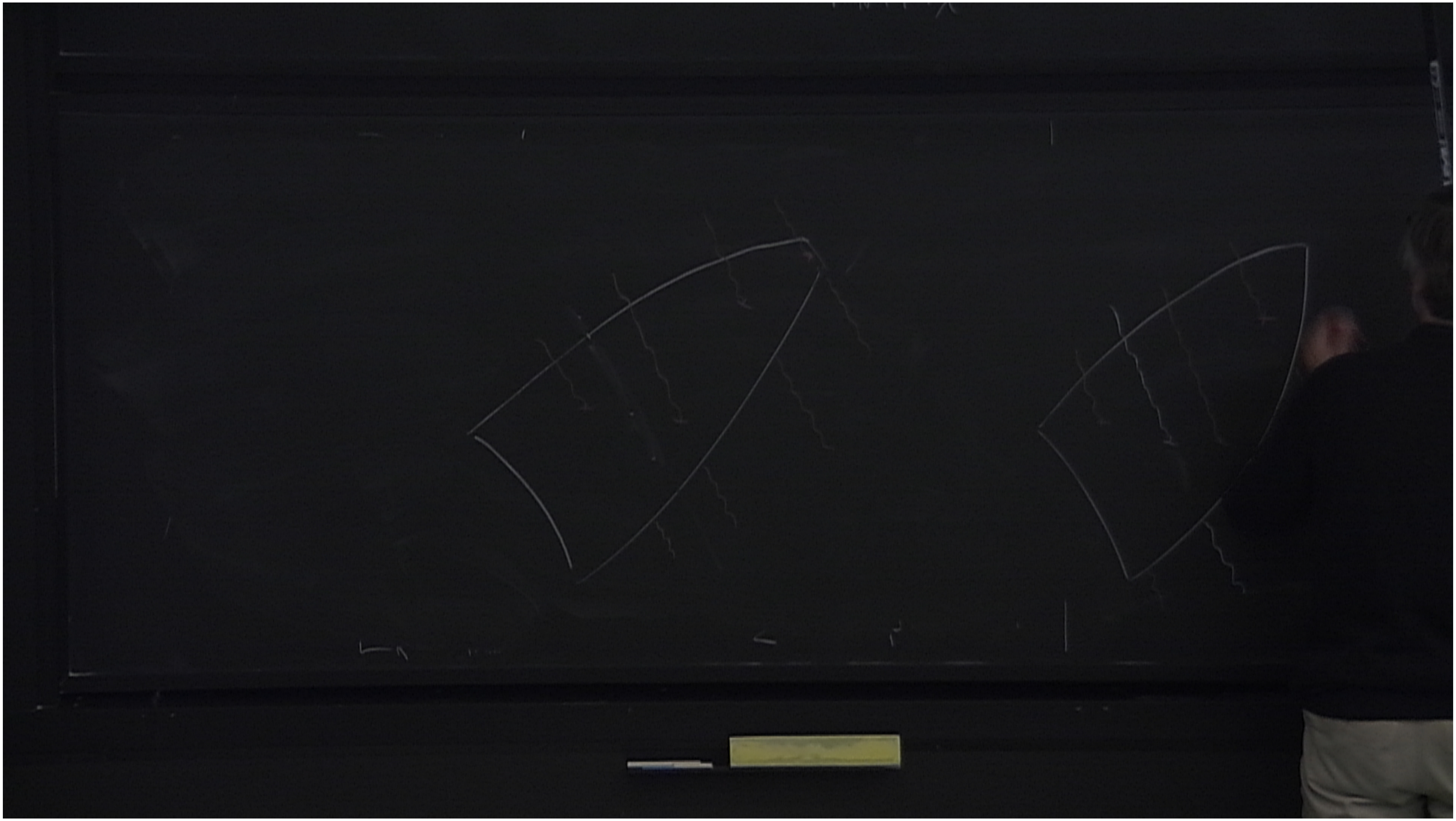
Let's try to understand why the Hilbert space of a scalar field interacting semiclassically with gravity should satisfy a holographic bound on its dimensionality.

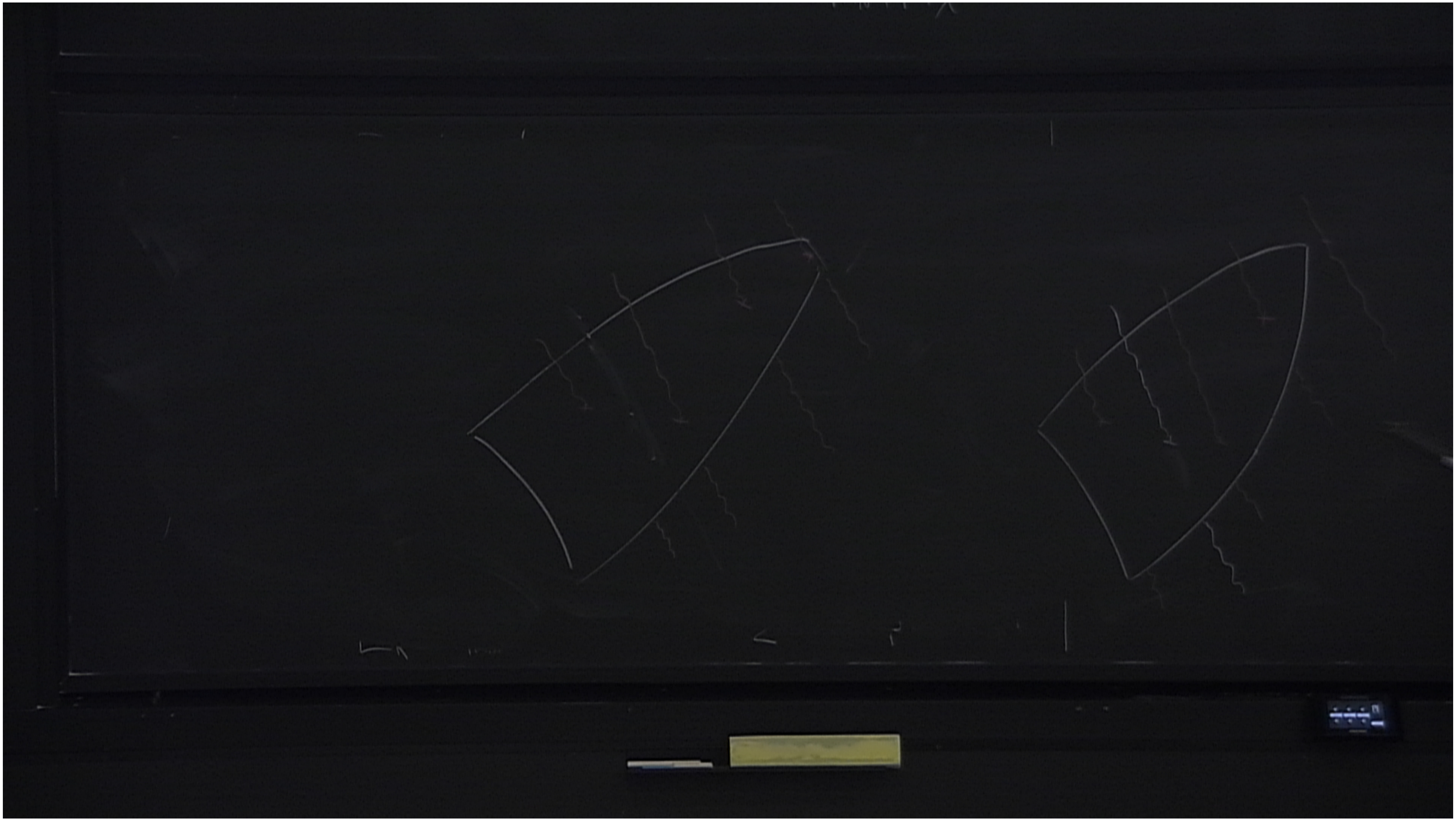
- A simple picture: Suppose n quanta of the scalar field cross a branch $\mathcal{N}_R$ of $\mathcal{N}$.
- Suppose we try to stuff one more quantum through $\mathcal{N}_R$. The generators converge more strongly and the quantum that formerly at the tip of $\mathcal{N}_R$ falls off. The number of quanta on $\mathcal{N}_R$ remains n.
- Suppose we glue together two identical double null sheets $\mathcal{N}$s so they form a single double null sheet $\mathcal{N}'$ with twice the cross sectional area A_{S_0}. If the Hilbert space $\mathcal{H}_\mathcal{N}$ for data on $\mathcal{N}$ has dimension N then it seems reasonable to suppose that the Hilbert space of $\mathcal{N}'$ should have dimension N^2, since the points in the two $\mathcal{N}$s are spacelike to each other. Thus the log of the dimensionality of $\mathcal{H}_\mathcal{N}$ should be extensive in A_{S_0}.

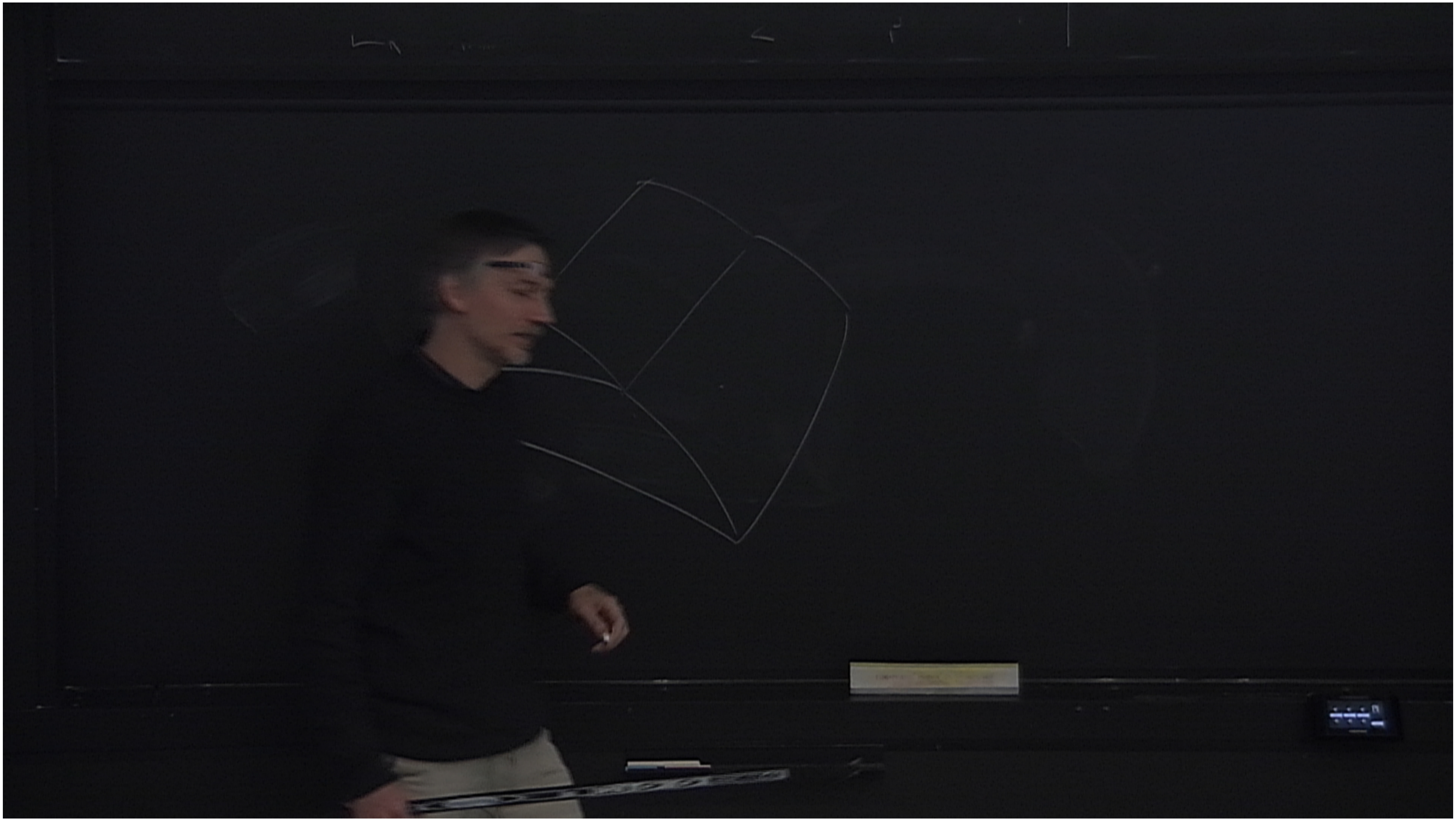


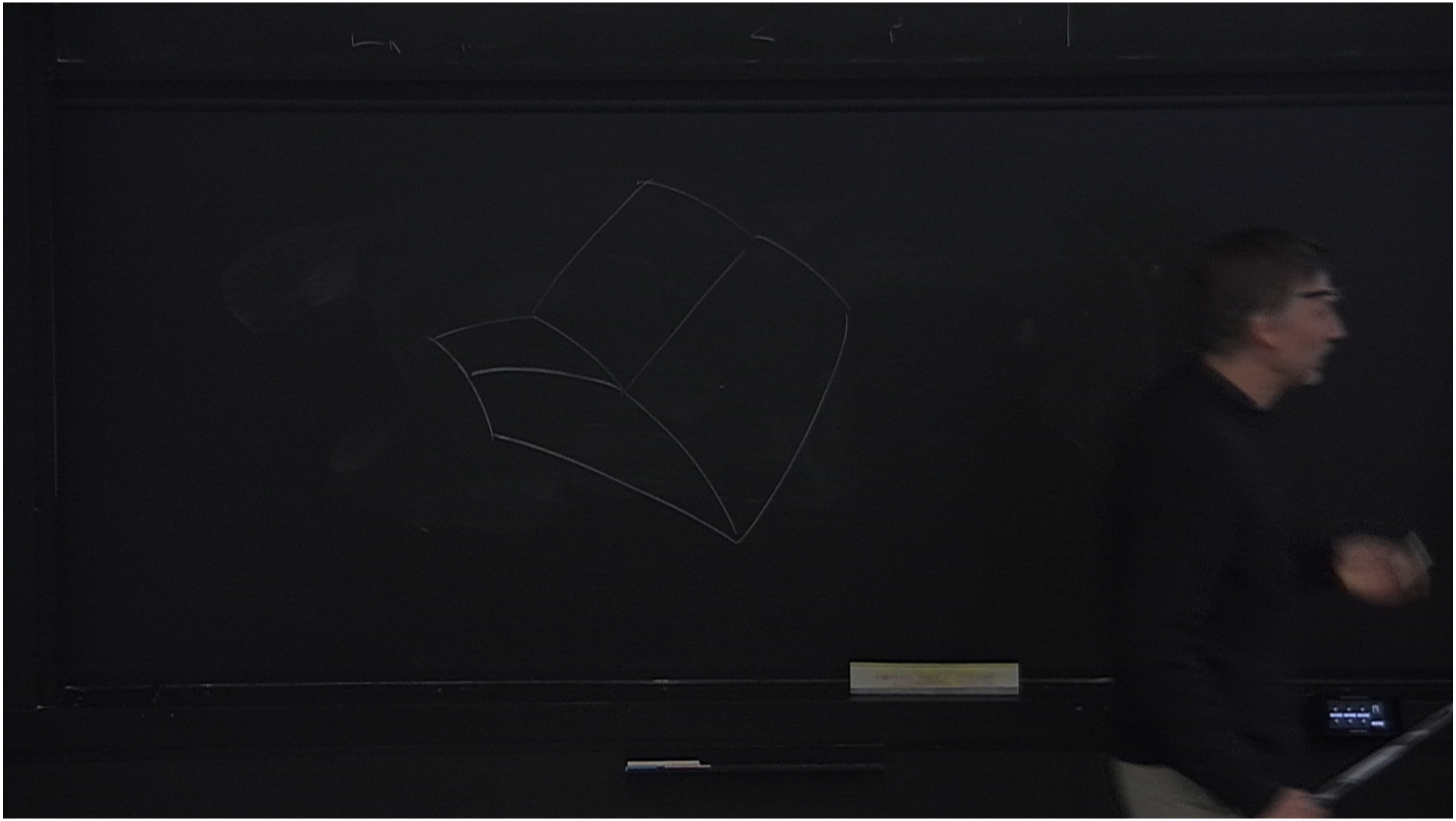


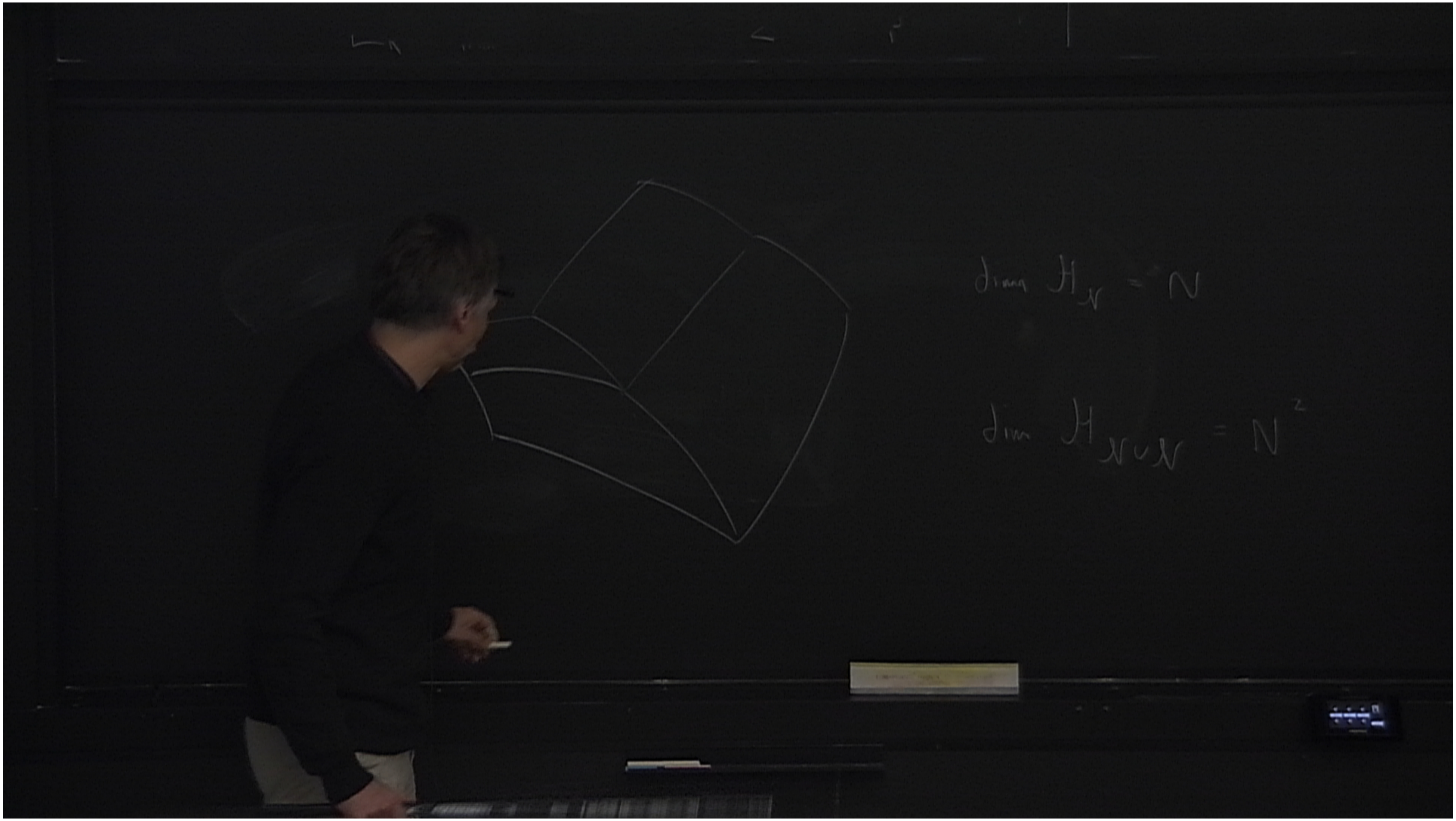








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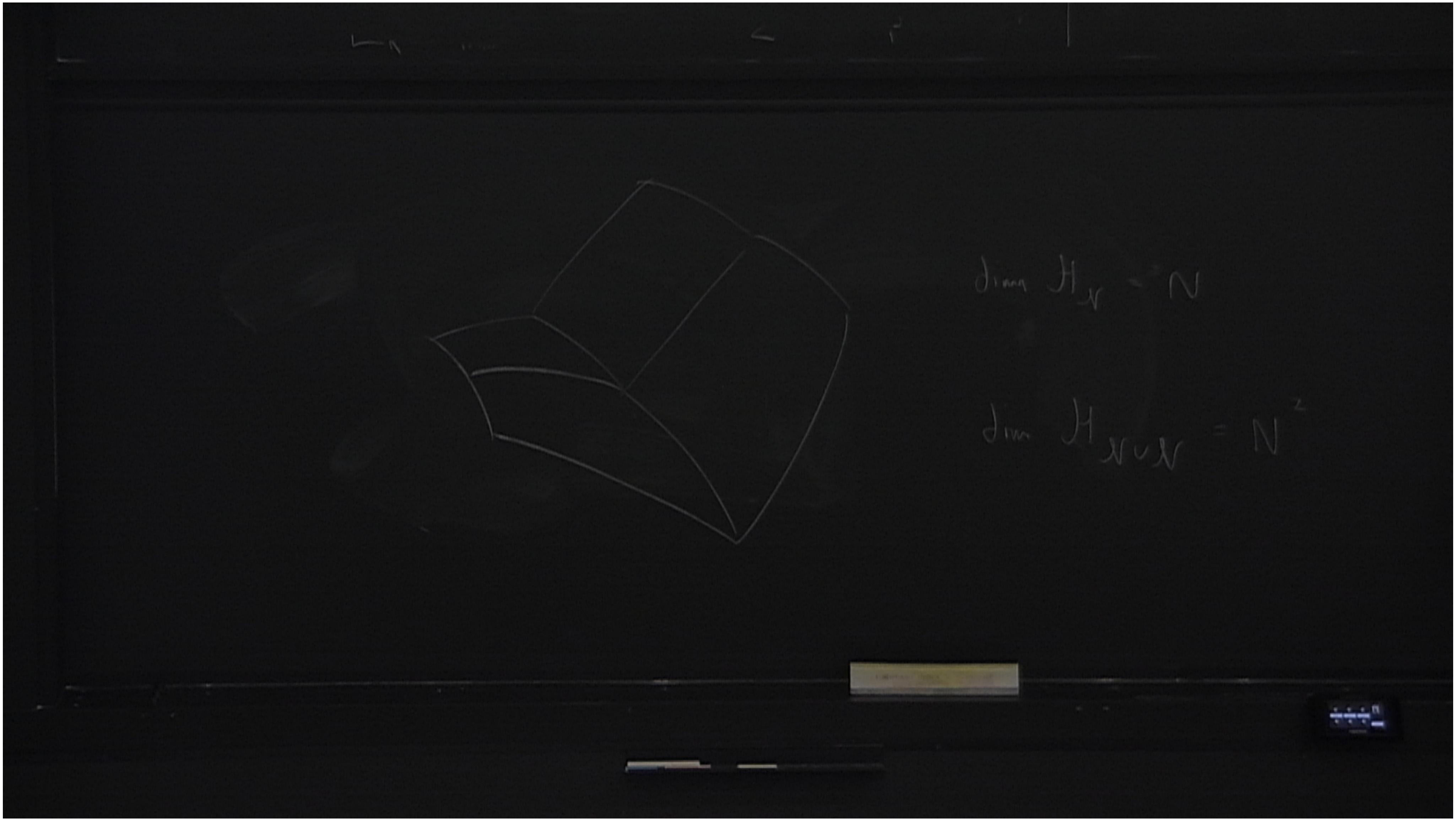
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- A somewhat better picture, purely in terms of initial data on $\mathcal{N}$:
  - Let θ be the expansion of the congruence of generators, and λ an affine parameter. Then

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} - 8\pi G T_{\lambda\lambda}.$$

    The shear σ will be ignored, it only makes the convergence of the generators faster, and we will assume that the null energy density $T_{\lambda\lambda}$ has a uniform value τ on $\mathcal{N}_R$ (and $0 = \theta = \lambda$ at S_0). Then $\theta = -2\sqrt{4\pi G\tau}\tan\sqrt{4\pi G\tau}\lambda$, and the generators form a caustic at

$$\lambda_{max} = \frac{\pi}{2}\frac{1}{\sqrt{4\pi G\tau}}.$$

    The value $\bar{\lambda}$ of λ where the generators of $\mathcal{N}_R$ are cut off must be less than λ_{max}.
  - Suppose a mode of the scalar field on $\mathcal{N}_R$ is exited with one quantum. Then $p_a = \hbar k_a$ and $p_a = \langle T_{a\lambda}\rangle\bar{\lambda}A_{S_0}f$, with $f < 1$. Thus

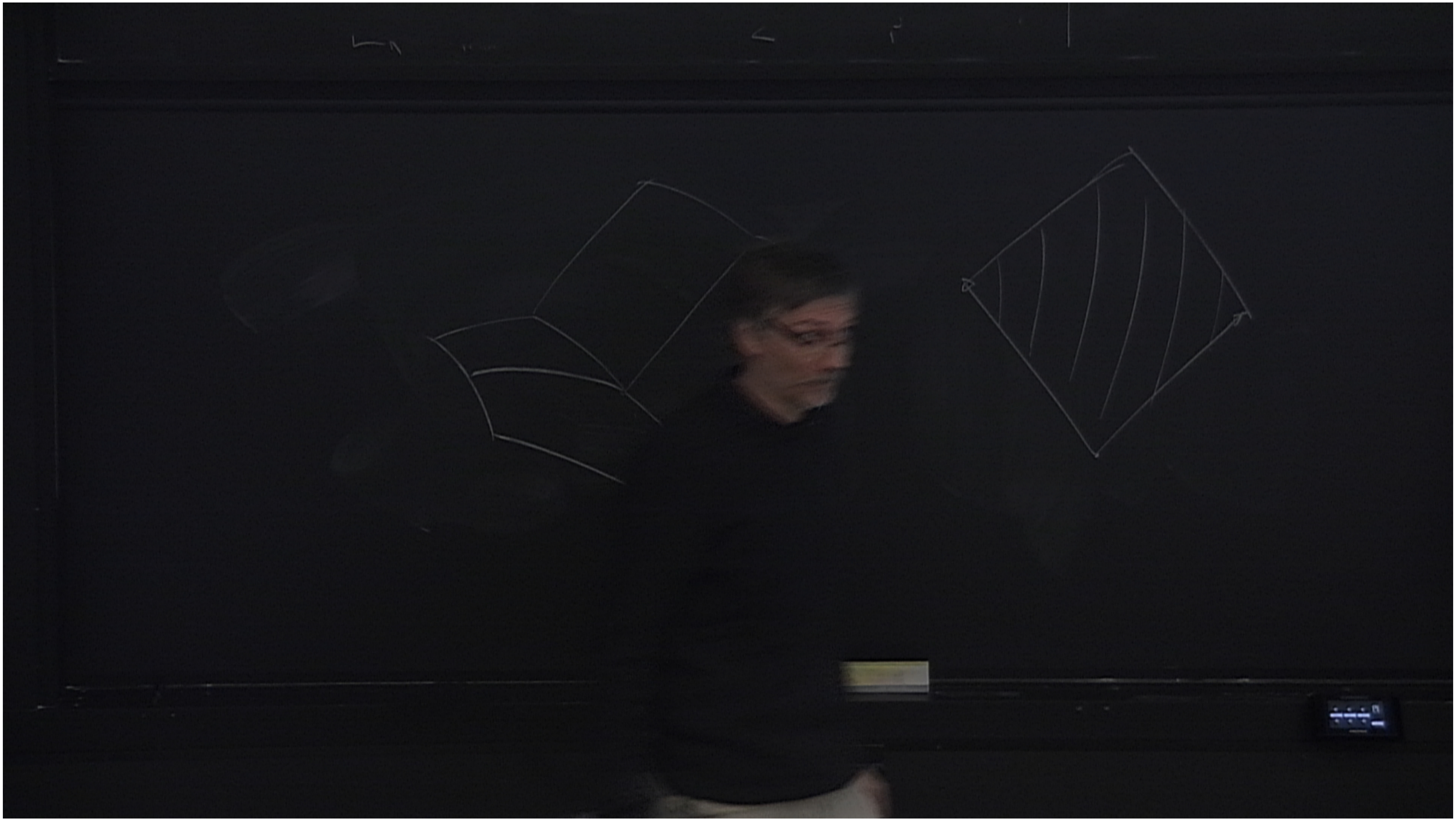
$$\tau = \langle T_{\lambda\lambda}\rangle = \hbar k_\lambda/(\bar{\lambda}A_{S_0}f) > \hbar 2\pi m/(\bar{\lambda}^2 A_{S_0})$$

    where m is the number of wavelengths of the mode along the generator.

- $\bar{\lambda} < \lambda_{max}$ then implies $m < A_{S_0}/(32G\hbar) = A_{S_0}/(32A_{Planck})$. If several modes m are occupied with n_m quanta in each then

$$\sum_m m n_m < A_{S_0}/(32A_{Planck}).$$

- If we apply the same reasoning to the other branch $\mathcal{N}_L$, and furthermore assume that $\bar{\lambda}_R \bar{\lambda}_L \partial_{\lambda_R} \cdot \partial_{\lambda_L} > A_{Planck}$ then only a finite subset of the Fock basis is allowed. Looks holographic!
- Can one do better using a proper quantum field theory?



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Fock quantization of Klein-Gordon field in curved spacetime

Work with Rodrigo Eyheralde.

- Standard procedure: Linear system $\Longrightarrow$ choose linear (real) canonical coordinates Q_i, P_i and require corresponding operators satisfy $[\hat{Q}_i, \hat{P}_j] = i\hbar\delta_{ij}\mathbf{1}$.
- Equivalently set $\hat{a}_i = 1/\sqrt{2\hbar}(\hat{Q}_i + i\hat{P}_i)$ and require $[\hat{a}_i, \hat{a}_j^\dagger] = i\hbar\delta_{ij}\mathbf{1}$.
- Define representation of operator algebra by requiering $\hat{a}_i|0\rangle = 0 \forall i$ for one state $|0\rangle$ and the rest of the Hilbert space = Fock space is the span of the vectors obtained by acting on $|0\rangle$ with a finite number of $\hat{a}^\dagger$s.
- But do not need a particular set of linear canonical coordinates to define Fock space. Q_i, P_i define a metric, $g = \sum_i (Q_i^2 + P_i^2)$, on phase space that makes these coordinates orthonormal. g and symplectic 2-form Ω define Fock quantization uniquely, modulo change of ON basis within each n-particle level.

A MODEST PROPOSAL

Here is a way to make a Fock quantization of data on $\mathcal{N}$:

- First use the standard flat spacetime quantization of the K-G field to quantize initial data on N a pair of intersecting null hyperplanes in Minkowski space.

$$N = \{x^- = 0, x^+ > 0\} \cup \{x^+ = 0, x^- > 0\}, \quad x^+ = x^0 + x^1, x^- = x^0 - x^1.$$

- Now import this quantization to $\mathcal{N}$ in curved spacetime:
  - The symplectic 2-form on $\mathcal{N}$ is

$$\begin{aligned}
\Omega_{\mathcal{N}}[\phi_1, \phi_2] &= \sum_{A=L,R} \int_{\mathcal{N}_A} (\phi_2 d\phi_1 - \phi_1 d\phi_2) \wedge \varepsilon && (1)\\
&= \sum_{A=L,R} \int_{\mathcal{N}_A} (\phi_2 \partial_s \phi_1 - \phi_1 \partial_s \phi_2) \rho ds d^2 x^\perp && (2)\\
&= \sum_{A=L,R} \int_{\mathcal{N}_A} (\varphi_2 \partial_s \varphi_1 - \varphi_1 \partial_s \varphi_2) ds d^2 x^\perp && (3)\\
&= \Omega_N[\varphi_1, \varphi_2] && (4)
\end{aligned}$$

where $\varphi = \sqrt{\rho}\phi$, ρ is the area density, $s, x^\perp$ ranges over $R_+ \times R^2$, and s is a function of ρ fixed [illegible] at ρ is a fixed function of s.

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where $\varphi = \sqrt{\rho}\phi$, ρ is the area density, $s, x^\perp$ ranges over $R_+ \times R^2$, and s is a function of ρ fixed once and for all, so that ρ is a fixed function of s.

A MODEST PROPOSAL

Here is a way to make a Fock quantization of data on $\mathcal{N}$:

- First use the standard flat spacetime quantization of the K-G field to quantize initial data on N a pair of intersecting null hyperplanes in Minkowski space.

$$N = \{x^- = 0, x^+ > 0\} \cup \{x^+ = 0, x^- > 0\}, \quad x^+ = x^0 + x^1, x^- = x^0 - x^1.$$

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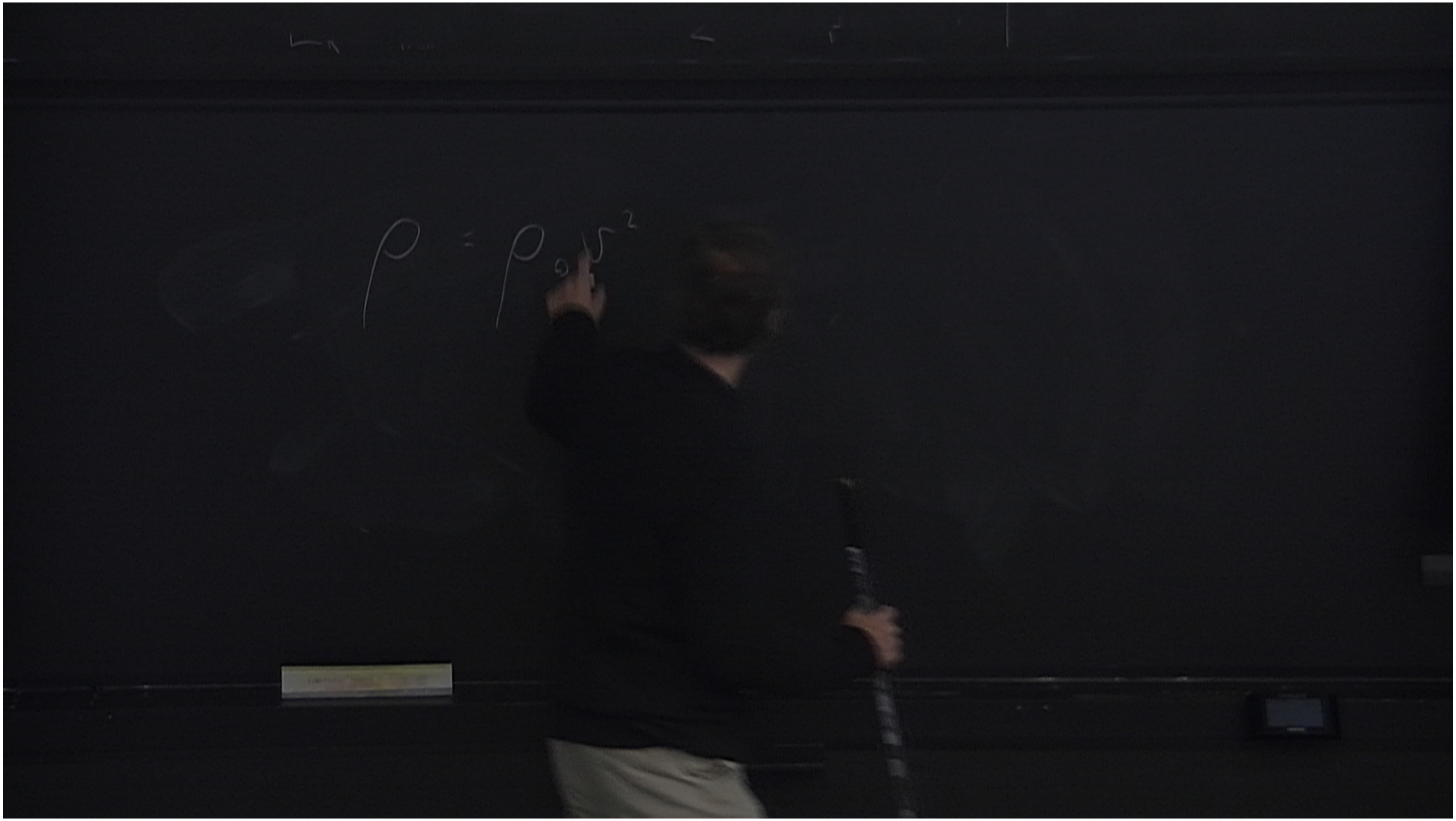
- One phase space metric compatible with Ω is $g_{\mathcal{N}}(\phi_1, \phi_2) = g_N(\varphi_1, \varphi_2)$.

- This defines the quantization. But it has the same Hilbert space of states as the flat spacetime theory, no matter how the scalar field affects the geometry. No holography!



MICROLOCAL SPECTRAL CONDITION

- What's wrong with this quantization?
- If there are several quantizations, how do I know which is the good one?
- Is the expectation value $\langle T_{ss} \rangle$ well defined in this quantum theory?

All these questions are answered by the microlocal spectral condition (μSC).
μSC:

- In Minkowski space field theory one demands that energy of all states be positive. More generally that $\langle \hat{P}^a \rangle$ lie within the future light cone.
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- μSC is a precise statement of that: Multiply distribution $\hat{\phi}(x)|0\rangle$ by a smooth test function of compact support to localize it. Take Fourier transform in your favourite coordinates. Fourier transform at ηk should fall off more rapidly than any inverse power of η as $\eta \to \infty$ except if k lies on the past light cone.
- Radzikowski 1996 showed that μSC is equivalent to requiering that the vacuum state is "Hadamard"
- Expectation values of $\hat{T}_{ss}$ defined on a dense subspace of Fock space if vacuum is Hadamard.
- Verch 1994 showed that Fock spaces with Hadamard vacua are indistinguishable via the expectation values of functions of the fields on an open spacetime domain of compact closure.
- Hadamard vacua are the good vacua.

INCONCLUSION

- Well, is the vacuum defined by our $g_{\mathcal{N}}$ Hadamard? Almost, but not quite.
- Conjecture: Holography follows as a consequence of the μSC.