Title: A CDT Hamiltonian from Horava-Lifshitz gravity

Date: Mar 14, 2013 02:30 PM

URL: http://pirsa.org/13030091

Abstract: Causal
Dynamical Triangulationsâ€• (CDT) is a lattice theory where aspects of quantum gravity can be studied. Two-dimensional CDT can be solved analytically and the continuum (quantum) Hamiltonian obtained.
In this talk I will show that this continuum Hamiltonian is the one obtained by quantizing two-dimensional projectable Horava-Lifshitz gravity.

A CDT Hamiltonian from Hořava-Lifshitz gravity

Yuki Sato

March 14th @ PI

J. Ambjørn, L. Glaser, Y. S. and Y. Watabiki, arXiv:1302.6359 [hep-th]

0. INTRODUCTION

Horava-Lifshitz gravity really looks like Causal Dynamical Triangulations (CDT).

Horava-Lifshitz gravity: Quantum gravity w/ anisotropic scaling (**foliation**)

↓

CDT: Lattice quantum gravity w/ a **foliation** ⟶ common feature

Moreover, both theories have several features in common in 4D:

(1) Behavior of the spectral dimension

P. Horava, Phys. Rev. Lett. 102, 161301 (2001)

(2) Structure of the phase diagram

J. Ambjorn, A. Gorlic, S. Jordan, J. Jurkiewicz and R. Loll, Phys. Lett B 690 (2010) 413

In our work, we have determined that

2D CDT is 2D Horava-Lifshitz quantum gravity!!

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Horava-Lifshitz gravity: Quantum gravity w/ anisotropic scaling (foliation)

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2D CDT is 2D Horava-Lifshitz quantum gravity!!

OUTLINE

1. 2D CAUSAL DYNAMICAL TRIANGULATION

2. 2D HORAVA-LIFSHITZ QUANTUM GRAVITY

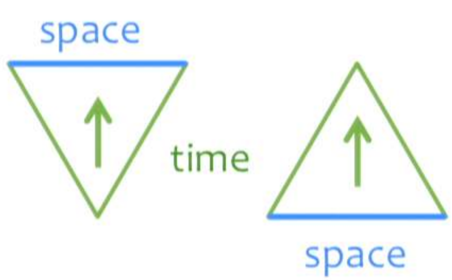
3. SUMMARY

1. 2D CAUSAL DYNAMICAL TRIANGULATION

Causal Dynamical Triangulation (CDT)

Path-integral in CDT = Sum over all triangulated geometries w/ causality

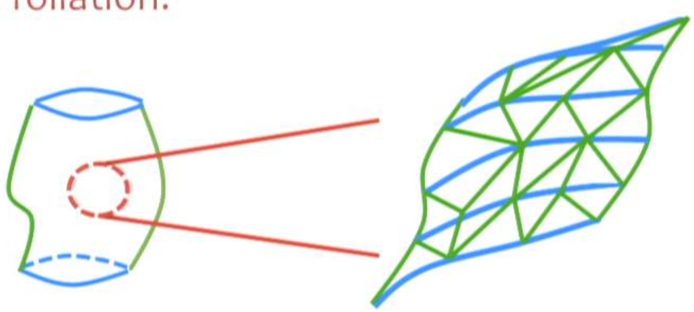
Building block: Lorentzian triangles



space $a_s^2 = \varepsilon^2$

time $a_t^2 = -\alpha\varepsilon^2$

Time foliation:



Sum over geometries:

fixed boundaries

1. 2D CAUSAL DYNAMICAL TRIANGULATION

Discrete action for CDT:

$$S_L(\Lambda; g) = -\Lambda \int d^2x \sqrt{-g}$$

↓ discretise

$$S_L(\lambda; T) = -\lambda \frac{\sqrt{4\alpha+1}}{4} n(T)$$

↓ rotation to Euclidian

$$S_E(\lambda; T) = \lambda \frac{\sqrt{4\alpha-1}}{4} n(T) \equiv \lambda n(T)$$

Vol (triangle)

$$\frac{\sqrt{4\alpha+1}}{4}\varepsilon^2$$

Bijection between Lorentzian & Euclidean

$$\alpha \rightarrow -\alpha$$
$$iS_L(\alpha) \rightarrow -S_E(\alpha)$$

Gauss-Bonnet's theorem (in 1848):

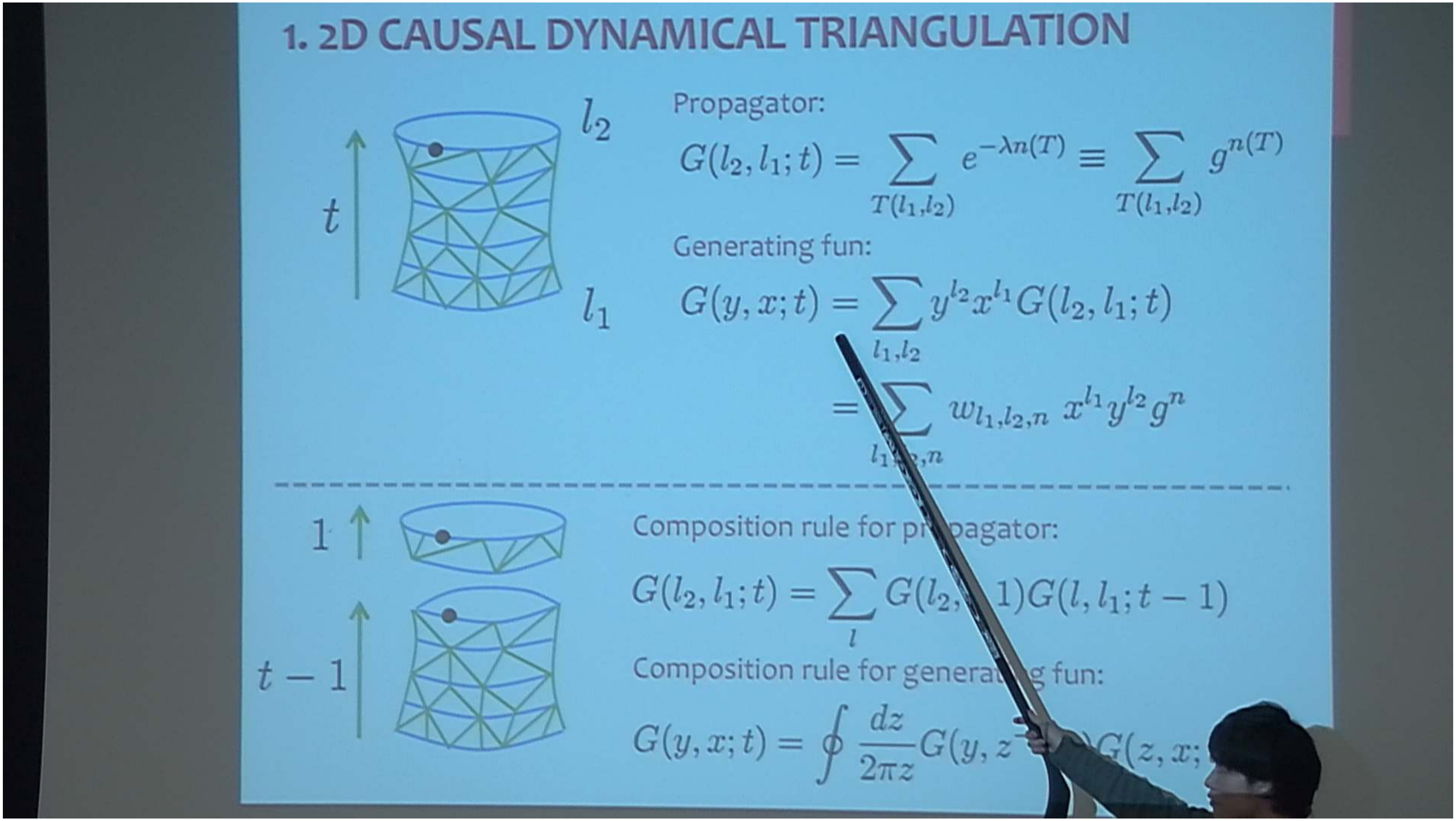
$$\frac{1}{4\pi}\int d^2x \, \sqrt{g}R = \chi = 2 - 2h$$

χ : Euler number

h : genus

$\chi = 2$

$\chi = 0$


1. 2D CAUSAL DYNAMICAL TRIANGULATION
t
l_2
l_1
Propagator:
$G(l_2, l_1; t) = \sum_{T(l_1,l_2)} e^{-\lambda n(T)} \equiv \sum_{T(l_1,l_2)} g^{n(T)}$
Generating fun:
$G(y, x; t) = \sum_{l_1,l_2} y^{l_2} x^{l_1} G(l_2, l_1; t)$
$w_{l_1,l_2,n}\ x^{l_1} y^{l_2} g^n$
1
$t-1$
Composition rule for pr
pagator:
$G(l_2, l_1; t) = \sum_{l} G(l_2,$
$1)G(l, l_1; t-1)$
Composition rule for genera
g fun:
$G(y, x; t) = \oint \frac{dz}{2\pi z} G(y, z$
$G(z, x;$

1. 2D CAUSAL DYNAMICAL TRIANGULATION

One step propagator:

 ~ gy  ~ gx

$$G(y,x;1) = \sum_{l_1,l_2,n} w_{l_1,l_2,n}\, x^{l_1} y^{l_2} g^n$$

$$= \frac{g^2 xy}{(1-gx)(1-gx-gy)}$$

Composition rule:

$$G(y,x;t) = \oint \frac{dz}{2\pi z} G(y,z^{-1};1) G(z,x;t-1)$$

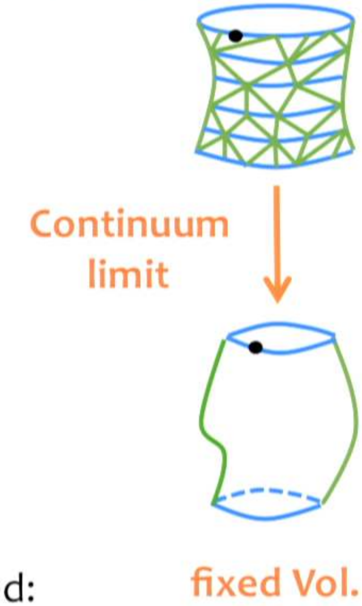
$$= \frac{gy}{1-gy} G\left(\frac{g}{1-gy}, x; t-1\right)$$ Important one-step recursion

Non-analytic at critical values:

$$(g_c, x_c, y_c) = (1/2, 1, 1)$$

1. 2D CAUSAL DYNAMICAL TRIANGULATION

To obtain non-trivial continuum limit,
one needs to do the followings simultaneously:



(1) Fine-tuning coupling constants to their critical values:

$$(g, x, y) \to (g_c, x_c, y_c) = (1/2, 1, 1)$$

(triangle) --> infinity

(2) Shrinking the lattice spacing to zero:

$$\varepsilon \to 0$$

under the physical macroscopic loops and ``time'' are fixed:

$$L_1 := \varepsilon l_1 \quad L_2 := \varepsilon l_2, \quad T := \varepsilon t$$

Things above can be done by the following renormalisation:

$$g = g_c e^{-\varepsilon^2 \Lambda}, \quad x = x_c e^{-\varepsilon X}, \quad y = y_c e^{-\varepsilon Y}$$

$$G(Y, X; T) = \lim_{\varepsilon \to 0} \varepsilon \, G(y, x; t)$$

2. 2D HORAVA-LIFSHITZ QUANTUM GRAVITY

4D example:

Einstein (covariant) gravity in ADM form:

$$S_{\rm ADM} = \frac{1}{\kappa}\int dt d^3x\ \sqrt{h}N(K_{ij}K^{ij} - K^2 + R - 2\Lambda) \qquad [\kappa] = -2$$

[Diff] $t \to t + \xi^0(t, x^i), \quad x^i \to x^i + \xi^i(t, x^i)$

Horava-Lifshitz (anisotropic) gravity: $t \to b^z t, \quad x^i \to b x^i$ **Power-counting renormalisable**

$$S_{\rm HL} = \frac{1}{\kappa}\int dt d^3x\ \sqrt{h}N(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}[h_{ij}]) \qquad [\kappa] = z - 3$$

[Foliation preserving Diff] $t \to t + \xi^0(t), \quad x^i \to x^i + \xi^i(t, x^i)$

$$\mathcal{V}[h_{ij}] = \sigma + \zeta R + \underline{\alpha R^2 + \beta R^{ij}R_{ij} + \gamma R \Delta R + \cdots}$$

↓

Higher spatial curvature invariants determined by "z"

2. 2D HORAVA-LIFSHITZ QUANTUM GRAVITY

2D projectable Horava-Lifshitz gravity:

$$S_{\rm HL} = \int dt\ dx\ N\gamma\left[(1-\lambda)K^2 - 2\Lambda\right]$$

projectable lapse

$$N = N(t)$$

where

$$\gamma := \sqrt{h} \qquad \& \qquad K = \frac{1}{N}\left(\frac{1}{\gamma}\partial_0\gamma - \frac{1}{\gamma^2}\partial_1 N_1 + \frac{N_1}{\gamma^3}\partial_1\gamma\right)$$

Move onto the canonical formalism to define conjugate momentum:

$$\pi^\gamma = \frac{\partial\mathcal{L}}{\partial(\partial_0\gamma)} = 2(1-\lambda)K, \qquad \{\gamma(x,t), \pi^\gamma(y,t)\} = \delta(x-y)$$

Hamiltonian can be found as

$$H = \int dx\ \left[N\mathcal{H} + N_1\mathcal{H}^1\right]$$

Hamiltonian constr. Diff constr.

$$\mathcal{H} = \gamma\frac{(\pi^\gamma)^2}{4(1-\lambda)} + 2\Lambda\gamma, \quad \mathcal{H}^1 = -\frac{\partial_1\pi^\gamma}{\gamma}$$

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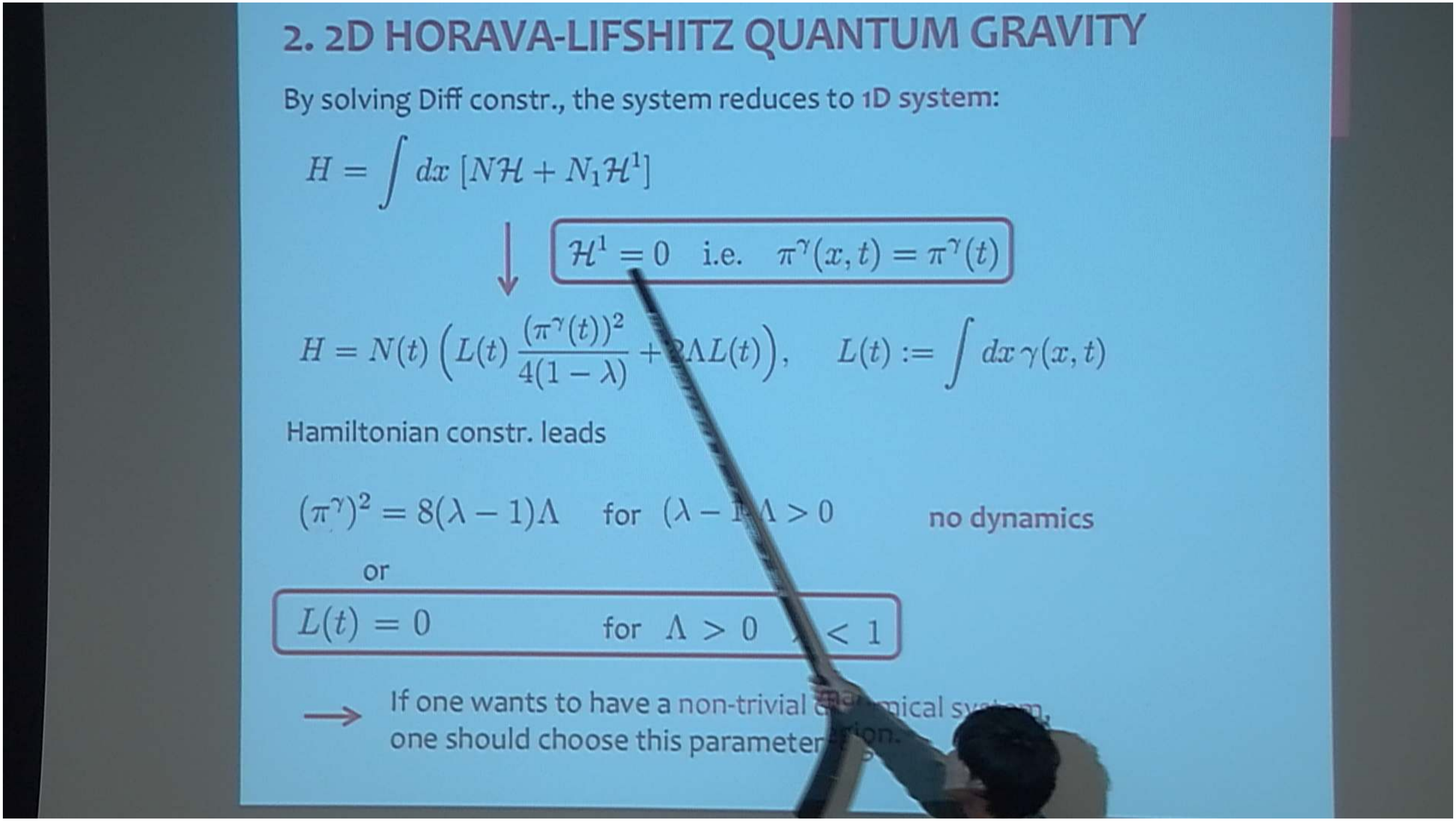
2. 2D HORAVA-LIFSHITZ QUANTUM GRAVITY
By solving Diff constr., the system reduces to 1D system:
$H = \int dx \, [N\mathcal{H} + N_1 \mathcal{H}^1]$
$\mathcal{H}^1 = 0$ i.e. $\pi^\gamma(x,t) = \pi^\gamma(t)$
$H = N(t) \left(L(t) \frac{(\pi^\gamma(t))^2}{4(1-\lambda)} + 2\Lambda L(t) \right), \quad L(t) := \int dx \, \gamma(x,t)$
Hamiltonian constr. leads
$(\pi^\gamma)^2 = 8(\lambda - 1)\Lambda$ for $(\lambda -$
no dynamics
or
$L(t) = 0$ for $\Lambda > 0$
< 1
If one wants to have a non-trivial
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one should choose this parameter

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or

$$\boxed{L(t) = 0 \qquad \text{for} \quad \Lambda > 0 \quad \lambda < 1}$$

$\longrightarrow$ If one wants to have a non-trivial dynamical system, one should choose this parameter region.

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$(\pi^\gamma)^2 = 8(\lambda - 1)\Lambda$ for $(\lambda - 1)\Lambda > 0$ **no dynamics**

or

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→ If one wants to have a non-trivial dynamical system, one should choose this parameter region.

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Diff constr.

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2. 2D HORAVA-LIFSHITZ QUANTUM GRAVITY

By sitting on the situation that

$$\Lambda > 0 \quad \lambda < 1$$

the classical dynamics of L(t) can be determined by the following action:

$$S = \int dt \left(\frac{\dot{L}^2}{4N(t)L(t)} - \tilde{\Lambda} N(t) L(t) \right), \qquad \tilde{\Lambda} = \frac{\Lambda}{2(1-\lambda)}$$

Here we have the physical quantities:

proper time

$$t_p(t) := \int_0^t dt' N(t')$$

&

macroscopic loop

$$L(t)$$

It's a legitimate question to ask for the following quantum amplitude:
(after a rotation to Euclidean signature)

$$G(L_2, L_1; T) \;=\; \langle L_2 | \, \mathrm{e}^{-T\hat{H}} \, | L_1 \rangle \qquad \text{where} \qquad \int_0^1 dt\, N(t) = T.$$

2. 2D HORAVA-LIFSHITZ QUANTUM GRAVITY

We quantise the system using the path-integral:

$$G(L_2, L_1; T) = \int \frac{\mathcal{D}N(t)}{\mathrm{Diff}[0,1]} \int \mathcal{D}L(t)\, \mathrm{e}^{-S_E[N(t),L(t)]}$$

where

$$S_E = \int dt \left(\frac{\dot{L}^2}{4N(t)L(t)} + \tilde{\Lambda} N(t) L(t) \right)$$

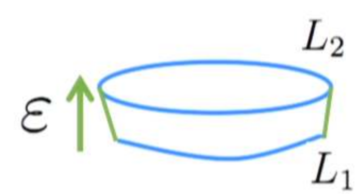
and we integrate all functions:

$$N:\ t \to N(t), \quad t \in [0,1], \qquad \int_0^1 dt\, N(t) = T,$$

$$L:\ t \to L(t), \quad t \in [0,1], \quad L(0) = L_1, \quad L(1) = L_2.$$

To compute this path-integral, it is enough to know the one step propagator:

$$G(L_2, L_1; \varepsilon) = \exp\left(-\frac{(L_2 - L_1)^2}{4\varepsilon L_2} - \varepsilon \tilde{\Lambda} L_2 \right)$$



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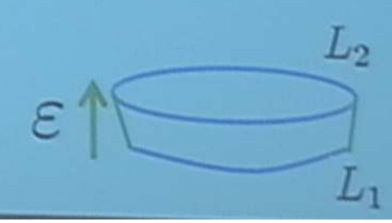

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2. 2D HORAVA-LIFSHITZ QUANTUM GRAVITY

Using the one-step propagator, we compute the following equation:

$$\int_0^\infty \left[\frac{(L_1)^a dL_1}{A}\right] G(L_2, L_1; \varepsilon)\psi(L_1) = \psi(L_2) - \varepsilon(\hat{H}\psi)(L_2) + \mathcal{O}(\varepsilon^{3/2}),$$

The left-hand side is

$$(\text{lhs}) = \int_0^\infty \left[\frac{(L_1)^a dL_1}{A}\right] \exp\left(-\frac{(L_2 - L_1)^2}{4\varepsilon L_2} - \varepsilon\tilde{\Lambda}L_2\right)\psi(L_1)$$

$$= \frac{(L_2)^a\sqrt{\varepsilon L_2 \pi}}{A}\left[\psi(L_2) - \varepsilon\left(-L_2\frac{\partial^2}{\partial L_2^2} - 2a\frac{\partial}{\partial L_2} - \frac{a(a-1)}{L_2} + \tilde{\Lambda}L_2\right)\psi(L_2) + \mathcal{O}(\varepsilon^{3/2})\right]$$

If we set the normalisation as $A = (L_2)^a\sqrt{\varepsilon L_2 \pi}$, we can read off the quantum Hamiltonian:

$$\hat{H} = -L_2\frac{\partial^2}{\partial L_2^2} - 2a\frac{\partial}{\partial L_2} - \frac{a(a-1)}{L_2} + \tilde{\Lambda}L_2$$

2. 2D HORAVA-LIFSHITZ QUANTUM GRAVITY

Quantum Hamiltonian:

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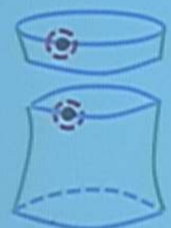
From this, we find

$$a = 0 \leftrightarrow \underline{dL} \leftrightarrow \hat{H} = -L \frac{\partial^2}{\partial L^2} + \tilde{\Lambda} L \longrightarrow$$

CDT Hamiltonian w/ **marked** loop



$$G(L_2, L_1; T_1 + T_2) = \int_0^\infty \underline{dL}\, G(L_2, L; T_2)\, G(L, L_1; T_1)$$

$$a = 1 \leftrightarrow \underline{LdL} \leftrightarrow \hat{H} = -L \frac{\partial^2}{\partial L^2} - 2 \frac{\partial}{\partial L} + \tilde{\Lambda} L \longrightarrow$$

CDT Hamiltonian w/ **unmarked** loop

$$G(L_2, L_1; T_1 + T_2) = \int_0^\infty \underline{dL}\, G(L_2, L; T_2)\, \underline{L}\, G(L, L_1; T_1)$$

3. SUMMARY

2D CDT turns out to be the 2D projectable Horava-Lifshitz quantum gravity,

$$S_{\mathrm{HL}} = \int dt\, dx\, N\gamma \left[(1-\lambda)K^2 - 2\Lambda\right]$$

where $N = N(t)$

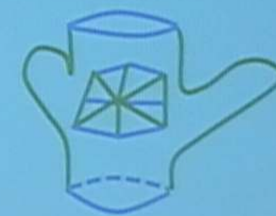
$\Lambda > 0 \quad \lambda < 1$

DT

Integrate out baby universes*

CDT

Generalised CDT



continuum

?

Liouville gravity

Infinite outgrowth of baby universes

Projectable Horava-Lifshitz gravity

No baby universe

Non-projectable Horava-Lifshitz gravity?

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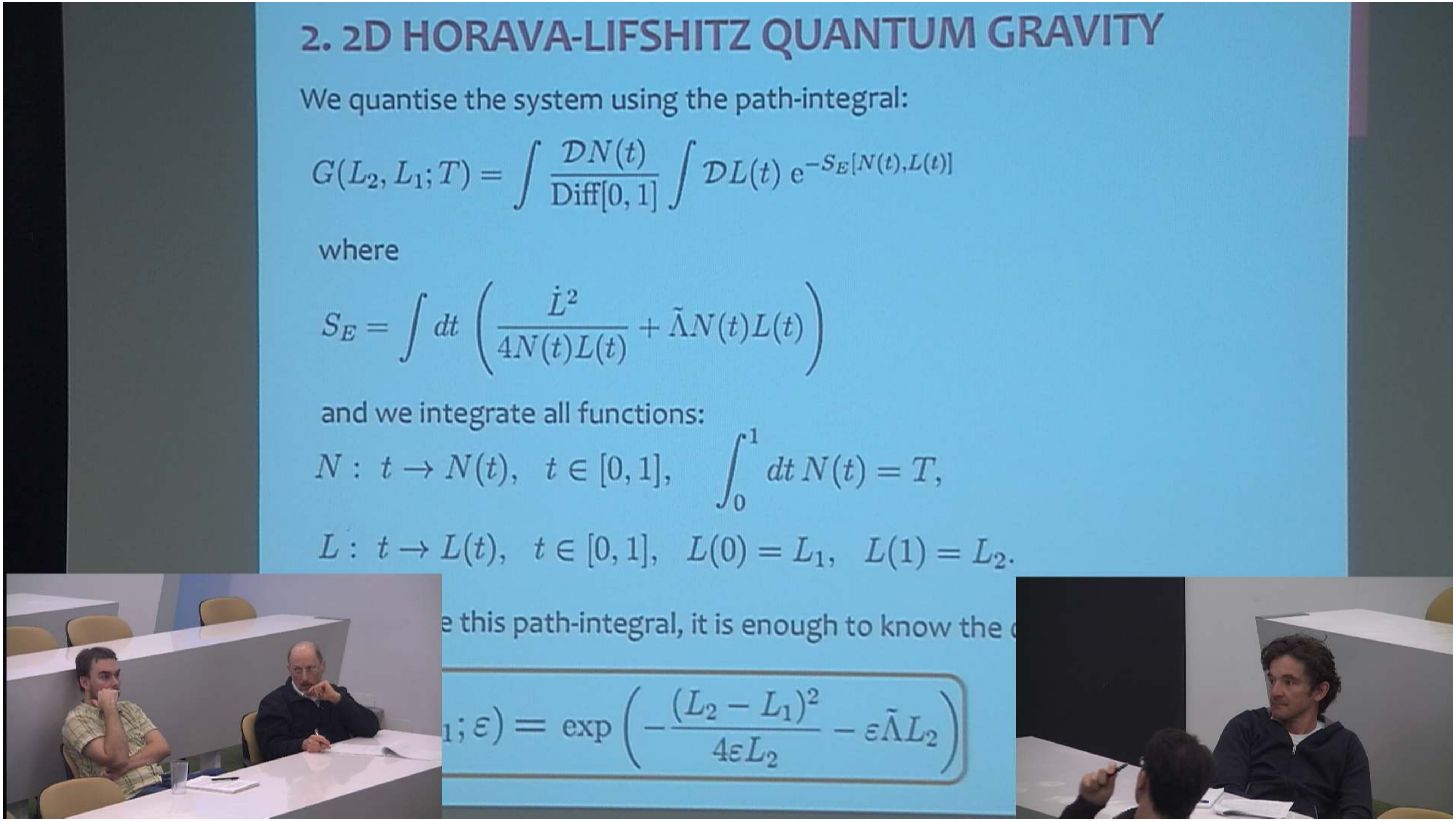
* J. Ambjorn, J. Correia, C. Kristjansen and R. Loll, Phys. Lett. B 475 (200)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where
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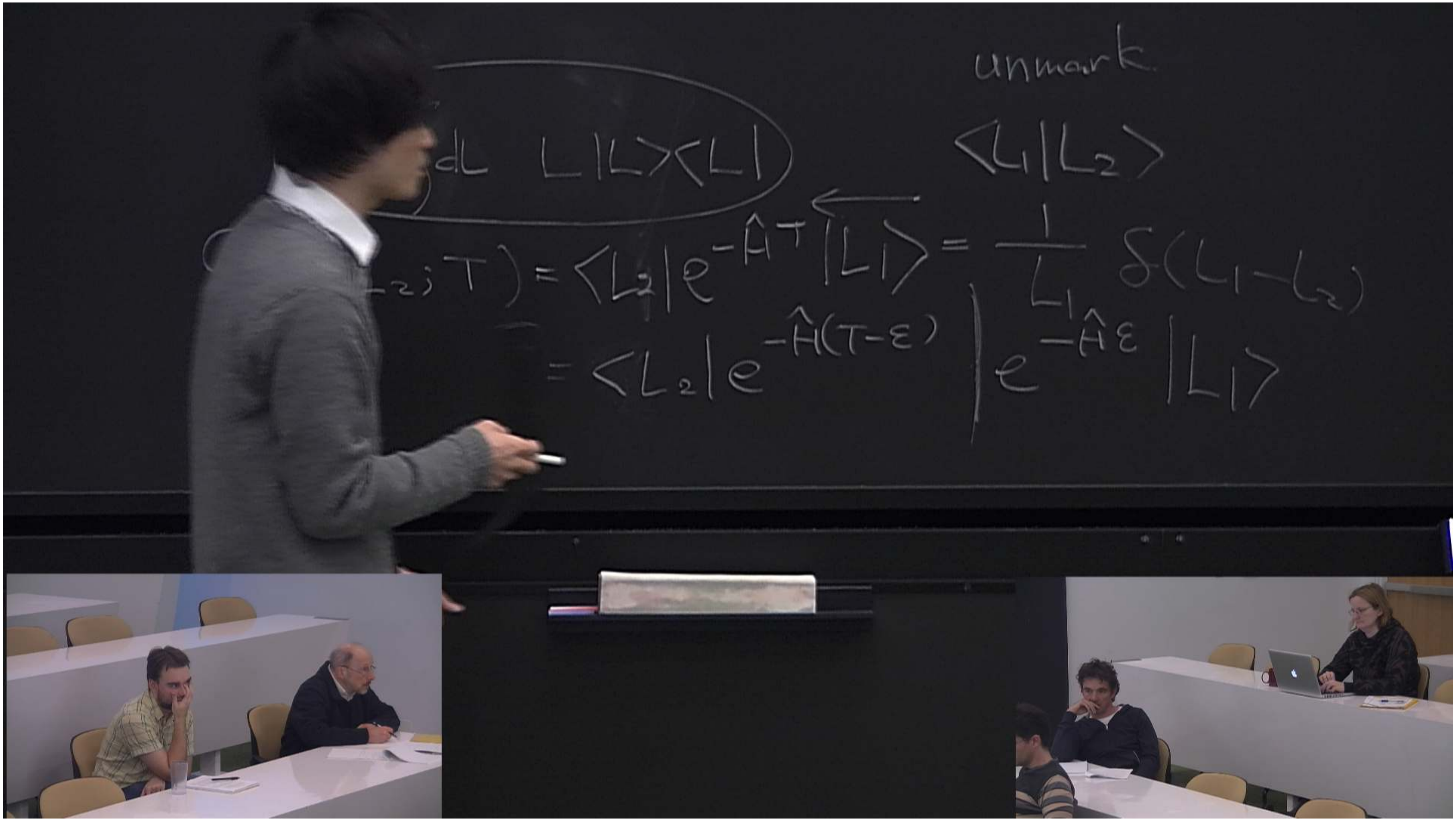
e this path-integral, it is enough to know the
$_1; \varepsilon) = \exp\left(-\frac{(L_2 - L_1)^2}{4\varepsilon L_2} - \varepsilon \tilde{\Lambda} L_2 \right)$


unmark
dL L|L⟩⟨L|
⟨L1|L2⟩
;T) = ⟨L2|e^{-ĤT}|L1⟩ = 1/L1 δ(L1−L2)
= ⟨L2|e^{-Ĥ(T−ε)}|e^{-Ĥε}|L1⟩

unmark

$$\langle L_1 | L_2 \rangle$$

$$1 = \int dL \; L \, |L\rangle\langle L|$$

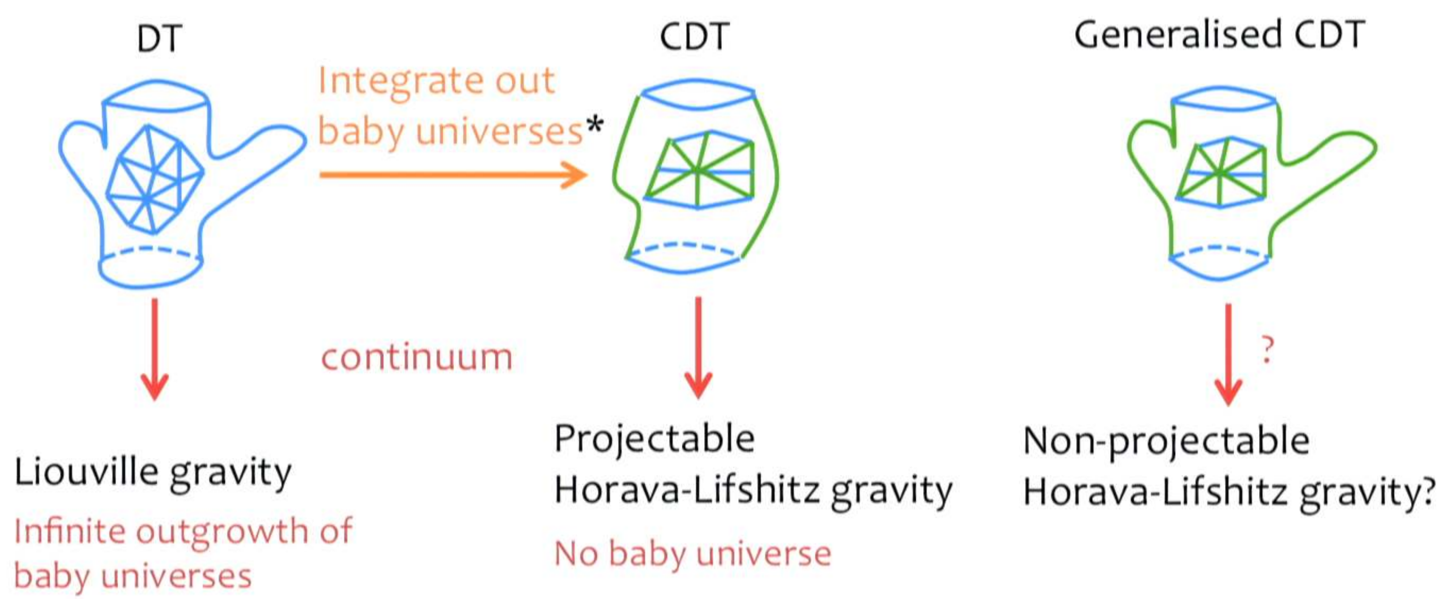
$$G(L_1, L_2; T) = \langle L_2 | e^{-\hat{H}T} | L_1 \rangle = \frac{1}{L_1}\,\delta(L_1 - L_2)$$

$$= \langle L_2 | e^{-\hat{H}(T-\varepsilon)} \, \Big| \, e^{-\hat{H}\varepsilon} | L_1 \rangle$$

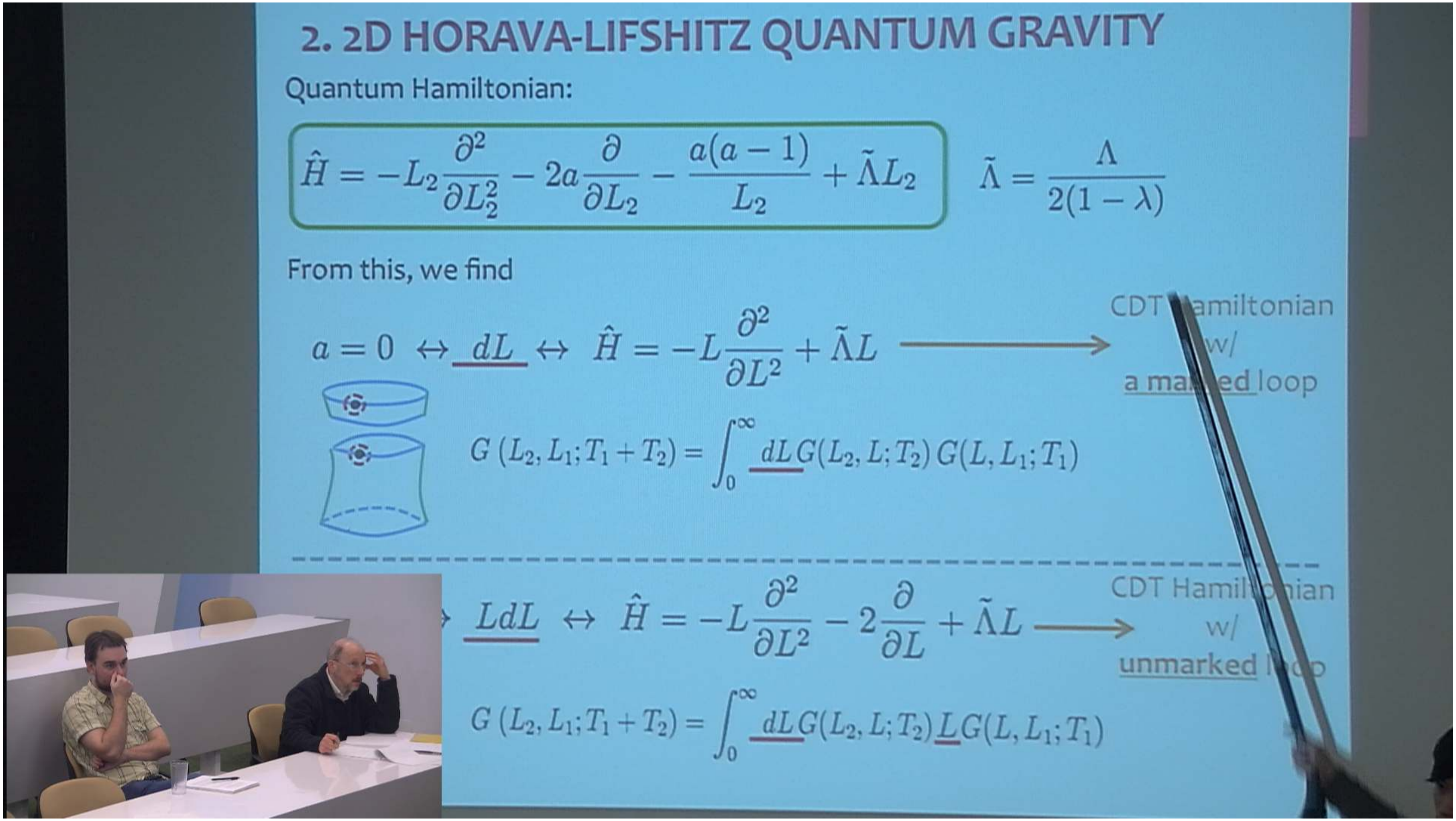
3. SUMMARY

2D CDT turns out to be the 2D projectable Horava-Lifshitz quantum gravity,

$$S_{\mathrm{HL}} = \int dt\, dx\, N\gamma \left[(1-\lambda)K^2 - 2\Lambda\right] \quad \text{where} \quad N = N(t), \quad \Lambda > 0 \quad \lambda < 1$$



* J. Ambjorn, J. Correia, C. Kristjansen and R. Loll, Phys. Lett. B **475** (200)


2. 2D HORAVA-LIFSHITZ QUANTUM GRAVITY
Quantum Hamiltonian:
$\hat{H} = -L_2 \frac{\partial^2}{\partial L_2^2} - 2a \frac{\partial}{\partial L_2} - \frac{a(a-1)}{L_2} + \tilde{\Lambda} L_2$
$\tilde{\Lambda} = \frac{\Lambda}{2(1-\lambda)}$
From this, we find
$a = 0 \leftrightarrow dL \leftrightarrow \hat{H} = -L \frac{\partial^2}{\partial L^2} + \tilde{\Lambda} L$
CDT amiltonian
w/
a ma ed loop
$G(L_2, L_1; T_1 + T_2) = \int_0^\infty dL\, G(L_2, L; T_2)\, G(L, L_1; T_1)$
$LdL \leftrightarrow \hat{H} = -L \frac{\partial^2}{\partial L^2} - 2 \frac{\partial}{\partial L} + \tilde{\Lambda} L$
CDT Hamil nian
w/
unmarked l p
$G(L_2, L_1; T_1 + T_2) = \int_0^\infty dL\, G(L_2, L; T_2)\, L\, G(L, L_1; T_1)$