Title: Effective field theory of the unstable top (quark)

Date: Mar 26, 2013 01:00 PM

URL: http://pirsa.org/13030090

Abstract: I introduce a new
nonrelativistic effective field theory which systematically accounts for the finite lifetime effects in production of unstable particles.
The theory is applied to the threshold production of top quark-antiquark pairs.

Effective Field Theory of Unstable Top

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Perimeter Institute, March 2013

Topics discussed

- Top-antitop threshold production
  - *brief introduction and review*

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- Top-antitop threshold production
  - *brief introduction and review*
- Top width effect:
  - *beyond the complex energy shift*
  - *effective theory of unstable particles "ρNRQCD"*
  - *unstable top production in NLO and NNLO*
  - *spurious divergences in ρNRQCD*

Why top threshold scan at a LC?

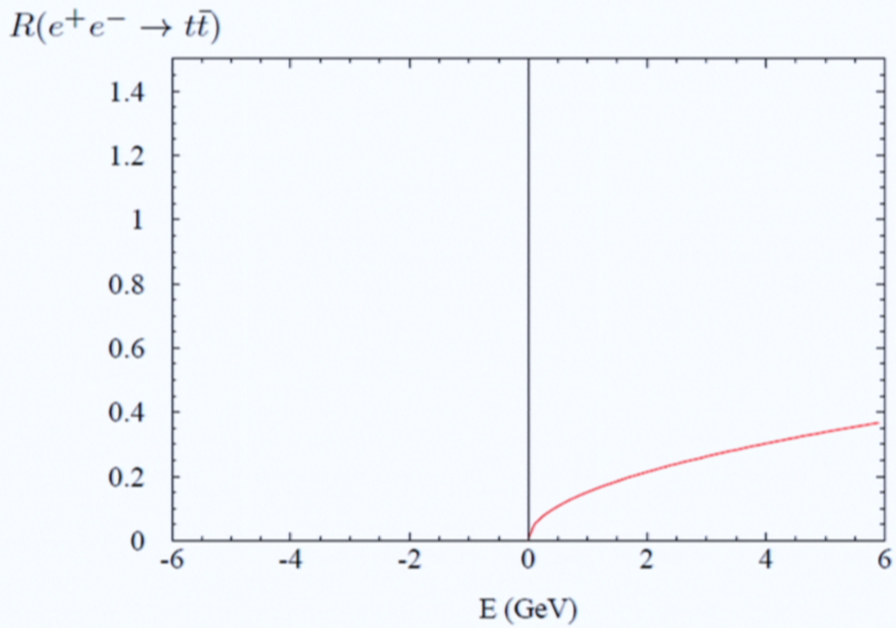
- Theory
  - *top quark width is a natural infrared cutoff*
  - ↪ *first principle QCD predictions*

- Experiment
  - *as clean as possible for a strongly interacting particle*

- Phenomenology
  - *most precise determination of top quark*

    mass, width, vector couplings

Born cross section



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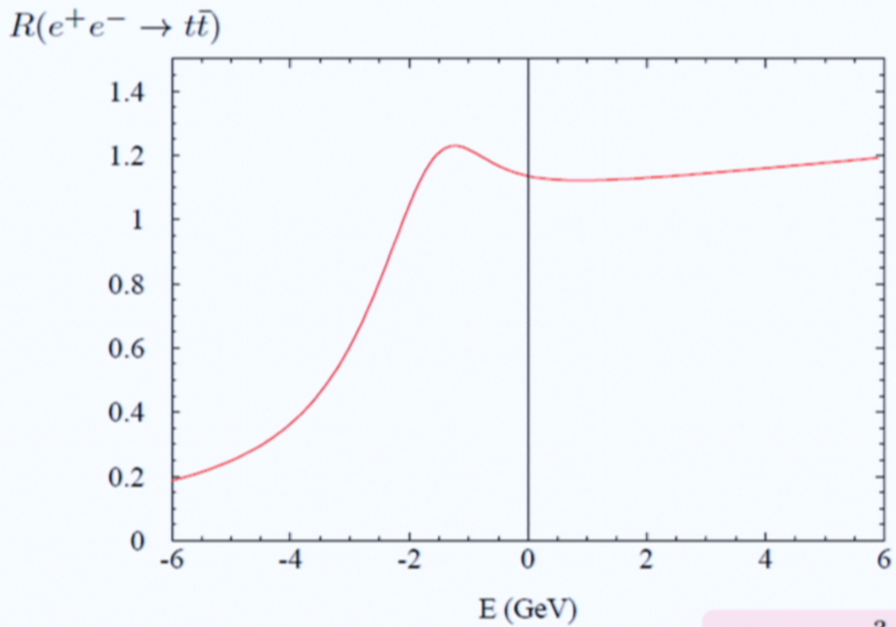
Coulomb and finite width effects



$$R_{\rm res} \sim \frac{\alpha_s^3}{m_t\Gamma_t}, \quad E_{\rm res} \sim \alpha_s^2 m_t$$

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Perturbation theory for heavy quarkonium

Pinnacle of modern effective field theory!

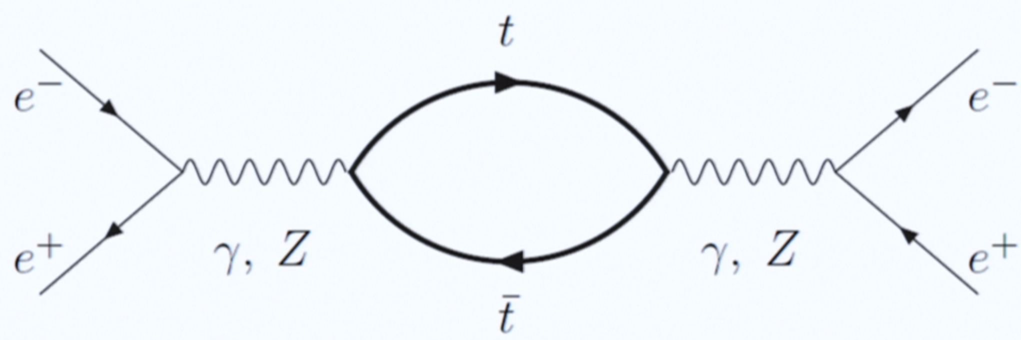
- Apparent slow convergence
- Possible reasons:
  - *Renormalons* $n!(\beta_0\alpha_s)^n$
  - *Threshold logs* $\alpha_s^n \ln^m \alpha_s$

Finite top lifetime

- Resonant approximation
  - *complex energy shift* $E \to E + i\Gamma_t$

    (V.Fadin, V.Khoze, JETP Lett. 46 (1987) 525)
  - *not consistent in pNRQCD beyond LO!*

Stable top



Optical theorem:

$$R_{res}^{Born} \sim \Im \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} \frac{1}{\boldsymbol{p}^2 - m_t E - i\epsilon} \sim \Im\sqrt{-E - i\epsilon},$$

On-shell top:

$$\Im\left[\frac{1}{\boldsymbol{p}^2 - m_t E - i\epsilon}\right] \sim \delta(\boldsymbol{p}^2 - m_t E),$$

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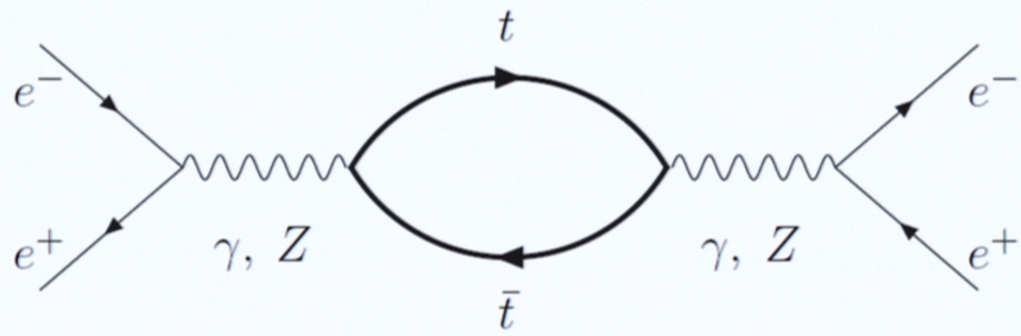

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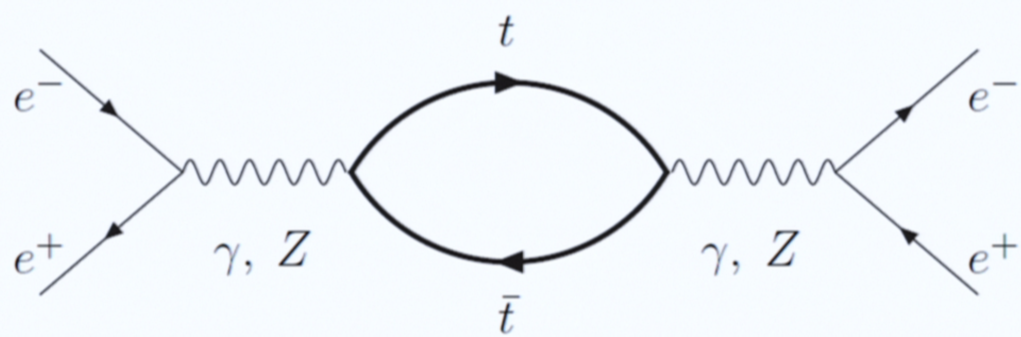

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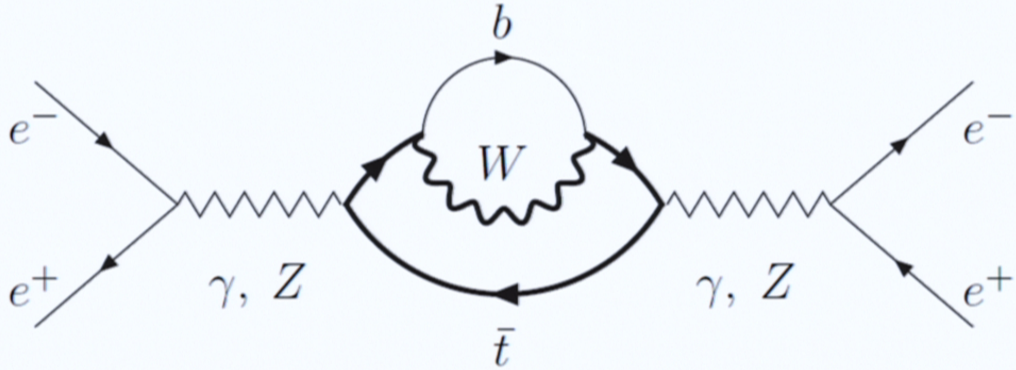

Imaginary part of mass operator:

here $\rho = 1 - M_W/m_t,\ z = (\boldsymbol{p}^2 - m_t E)/m_t^2 \ll 1$

$$\Im[\Sigma(z)] = \frac{\Gamma_t}{2} - \frac{\Gamma_t}{2}\left[\theta(z-\rho) + \left(\frac{2z}{\rho} - \frac{z^2}{\rho^2}\right)\theta(\rho - z) + \mathcal{O}(\rho, z)\right]$$

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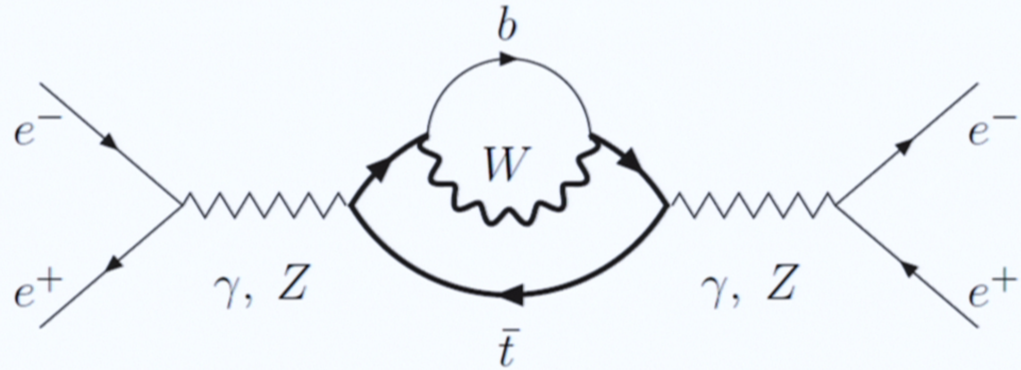

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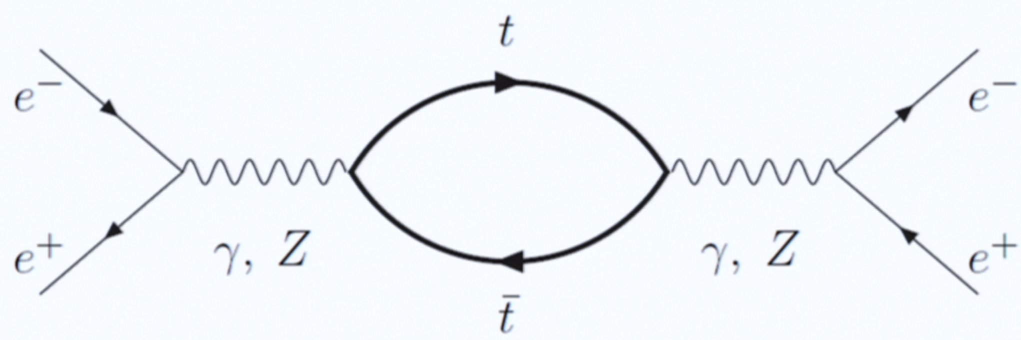

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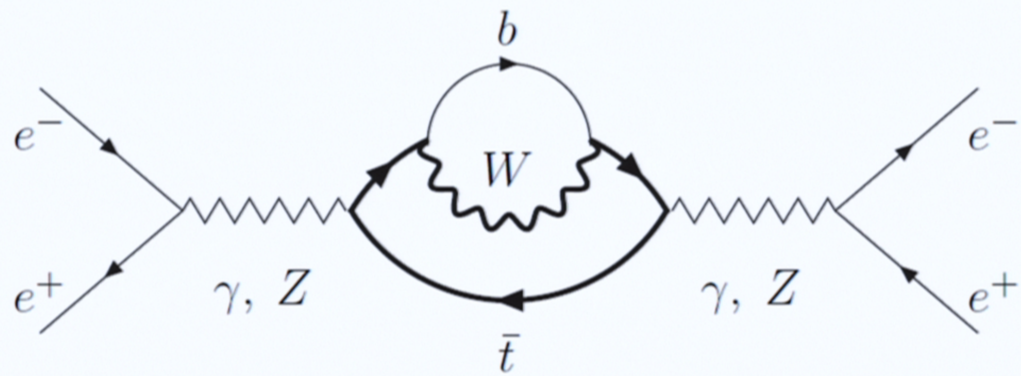

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Resonant contribution *Nonresonant contribution*

Resonant contribution

- Complex energy shift:
  - *Dyson resummation*

$$\frac{1}{\boldsymbol{p}^2 - m_t E - i\epsilon} \to \frac{1}{\boldsymbol{p}^2 - m_t E - i m_t \Gamma_t}$$

  - *Breit-Wigner resonance*

$$\delta(\boldsymbol{p}^2 - m_t E) \to \frac{1}{\pi} \frac{\Gamma_t}{(\boldsymbol{p}^2/m_t - E)^2 + \Gamma_t^2},$$

  - *Born cross section*

$$R_{res}^{Born} \sim \Im\left[\sqrt{-E - i\Gamma_t}\right]$$

- Invariant mass distribution:
  - $2\boldsymbol{p}^2 \approx m_t^2 - (p_W + p_b)^2$

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Nonresonant contribution

- On-shell t ⇨ on-shell W and b
  - *kinematical constraint* $M_W^2 < (p_W + p_b)^2 < m_t^2$
  - *natural cutoff on spatial momentum* $0 < \boldsymbol{p}^2 < \rho m_t^2$
  - $\Im[\Sigma] - \Gamma_t/2 \neq 0$ *for* $\boldsymbol{p}^2 \neq 0$ ⇨ *"nonresonant"*

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- Approximation $\rho \ll 1$
  - *nonrelativistic* t *and* W, *ultrarelativistic* b
  - *expansion in* ρ *similar to pNRQCD expansion in* $v^2 \sim E/m_t$
  - *actual value* $\rho = 0.53\ldots$

Nonrelativistic effective theory of unstable top

- Scales
  - *pNRQCD:*

    hard m_t *soft* vm_t *ultrasoft* $v^2 m_t$

  - *ρNRQCD:*

    hard m_t *ρ-soft* $\rho^{1/2} m_t$ *ρ-ultrasoft* ρm_t

Nonrelativistic effective theory of unstable top

- Scales
  - *pNRQCD:*

    hard m_t *soft* $v m_t$ *ultrasoft* $v^2 m_t$

  - ρ*NRQCD:*

    hard m_t ρ*-soft* $\rho^{1/2} m_t$ ρ*-ultrasoft* ρm_t

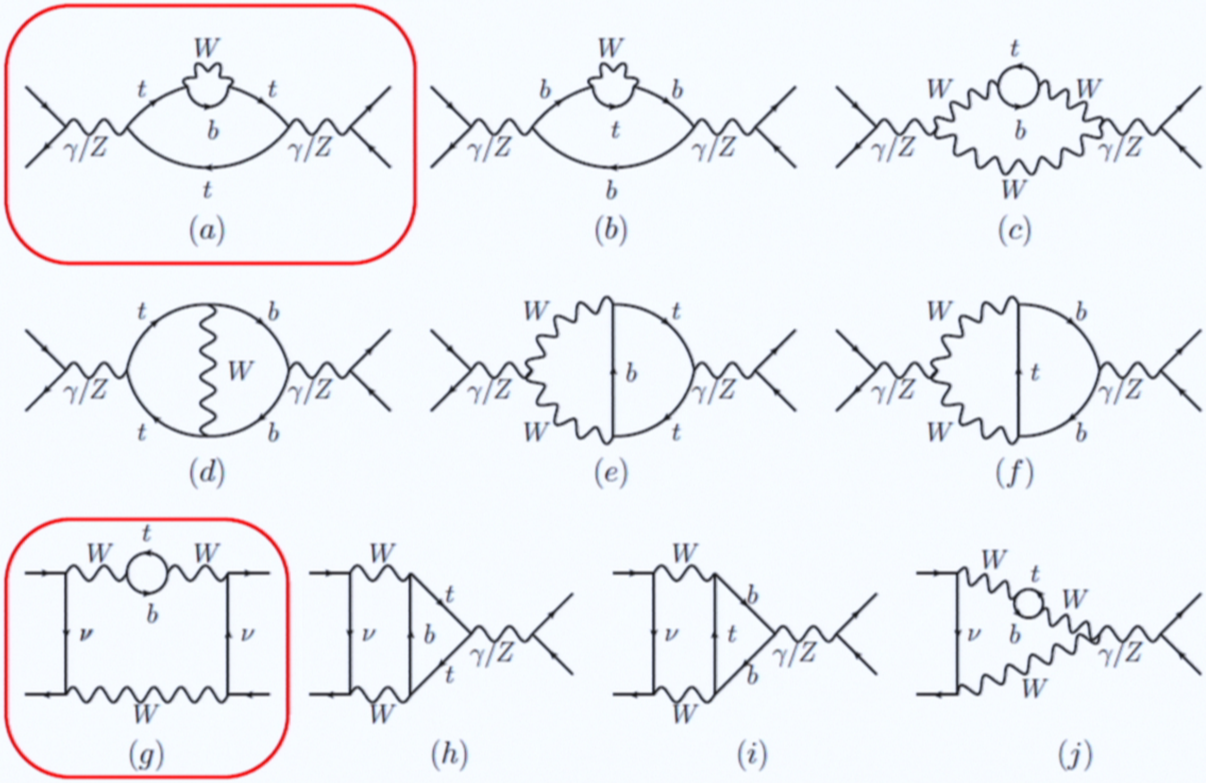
- Scale hierarchy and power counting
  - *pNRQCD scaling:* $\alpha_{ew}^{1/2} \sim \alpha_s \sim v \ll 1, \quad \Gamma_t/m_t \sim \alpha_{ew}$
  - *complimentary expansion in* ρ *with* $v \ll \rho^{1/2} \ll 1$
  - ρ*-Coulomb terms* $\alpha_s/\rho^{1/2} \ll 1$

NLO nonresonant contribution

- Power counting
  - *resonant contribution* $\Im\sqrt{-E-i\Gamma_t} \sim v$
  - *nonresonant contribution* $\Gamma_t \sim v^2$
- Calculation steps
  - *treat* $\Im[\Sigma] - \Gamma_t/2$ *as a perturbation*
  - *add all the two-loop diagrams with* t-W-b *cut*
  - *expand in* $E/\rho m_t$, *expand in* ρ ⇨ *single region left:* t *and* W *are* ρ*-potential,* b *is* ρ*-ultrasoft*

↪ *recover nonrelativistic propagators and vertices*

NLO diagrams



NLO result

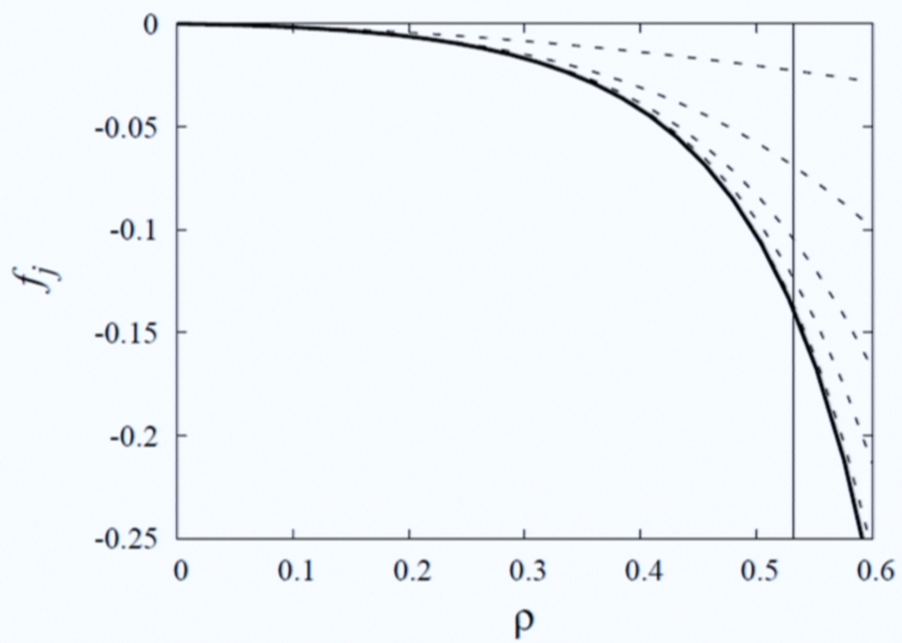
- leading term of ρ-expansion

$$R_{nr}^{NLO} = -\frac{24}{\pi\rho^{1/2}}\frac{\Gamma_t}{m_t}\left[\frac{4}{9} + \text{“}Z\text{”} - \frac{1}{\sin^4\theta_W}\left(\frac{17}{48} - \frac{9\sqrt{2}}{32}\ln\left(1+\sqrt{2}\right)\right)\right]$$

- Convergence?
  - *generally not bad*
  - *for some diagrams Padé is necessary*

Convergence

diagram "j"



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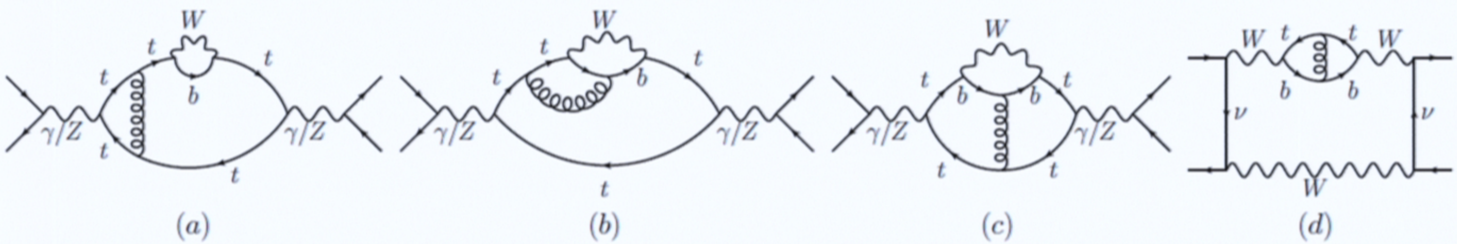
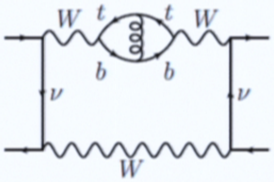
NNLO nonresonant contribution

ρ-leading diagrams



- Regions of gluon momentum
  - (a) and (c) - hard, potential, ρ-potential
  - (b) and (d) - hard, ρ-soft

NNLO nonresonant contribution



(d)

- Diagram (d):
  - *box momentum is ρ-potential*
  - *gluon momentum is hard or ρ-soft*

$$\mathrm{Im}[\Pi^{(1)}(z)] = \frac{C_F\alpha_s}{\pi}\left[\frac{9}{4}+\frac{1}{3}\pi^2-\frac{3}{2}\ln(2\rho)-\frac{3}{2}\ln\left(1-\frac{z}{\rho}\right)+\mathcal{O}(\rho,z)\right]\mathrm{Im}[\Pi^{(0)}(z)]$$

  - $\ln(\rho)$ *term is absorbed into corrected* Γ_t

$$\Gamma_t = \left[1+\frac{C_F\alpha_s}{\pi}\left(\frac{9}{4}-\frac{2}{3}\pi^2-\frac{3}{2}\ln(2\rho)+\mathcal{O}(\rho)\right)\right]\Gamma_t^{LO}$$

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$$R_{nr}^{N^2LO} = \frac{3C_F\alpha_s}{\pi^2\rho^{1/2}}\frac{\Gamma_t}{m_t}\left\{\left[\frac{4}{9}+\text{“}Z\text{”}\right]\left[\frac{\pi^2}{\rho^{1/2}}\left(3\ln\left(\sqrt{E^2+\Gamma_t^2}/\rho m_t\right)+\frac{3}{2}+6\ln 2\right)+(18+24\ln 2)\right]\right.$$
$$+\frac{1}{\sin^4\theta_W}\left[\frac{22}{3}+\frac{17\pi^2}{6}-\frac{17}{2}\ln 2+(2-3\pi^2+9\ln 2)\frac{3\sqrt{2}}{4}\ln\left(1+\sqrt{2}\right)\right.$$
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  - ρ-*Coulomb term* $\alpha_s/\rho^{1/2}$
  - *new type of logs* $\ln(E/\rho m_t) \sim \ln(v^2/\rho)$

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  - *systematically accounts for finite width effects in threshold top-antitop production*
  - *optimized for high-order calculations*
  - *solve the problem of the spurious divergences*
  - *conceptually clear and aesthetically appealing*