Title: Thermodynamical Property of Entanglement Entropy for Excited States and Holographic Local Quench

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Abstract: In this talk, we will describe our recent work. Recently, we focus on the thermodynamical property and time dependence of entanglement entropy. Using holography, we found that the entanglement entropy for a very small subsystem obeys a property which is analogous to the first law of thermodynamics when we excite the system. In relativistic setups, its effective temperature is proportional to the inverse of the subsystem size. This provides a universal relationship between the energy and the amount of quantum information. Moreover, we will propose a new holographic model of local quench and describe some results which we got by using this model.

This talk is based on arXiv:1212.1164 [hep-th] and arXiv:1302.5703 [hep-th].

Thermodynamical Property of Entanglement Entropy for Excited States and Holographic Local Quench

Masahiro Nozaki

Yukawa Institute for Theoretical Physics (YITP),Kyoto University

1. Based on arXiv:1212.1164 (Phys. Rev. Lett. 110, 091602 (2013))
   with Jyotirmoy Bhattacharya (Kavli IPMU,Tokyo) , Tadashi Takayanagi (YITP, Kyoto)
   and Tomonori Ugajin (Kavli IPMU,Tokyo)

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Introduction

In equilibrium system, entropy is well-defined by thermal entropy.

On the other hand, entropy is not well-defined in non-equilibrium system.

In condensed matter physics, entanglement entropy is considered as one of the candidates for the entropy.

Therefore, it is important to know the property of the entanglement entropy.

The question appears.

Question

Are there the fundamental laws (like the laws of thermodynamics) which entanglement entropy (EE) should obey ?

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At high energy , EE obeys the law analogous to

first law of thermodynamics.

$$dU = TdS \quad \Longleftrightarrow \quad dE_A = T_{\mathrm{ent}} dS_A .$$

Our result

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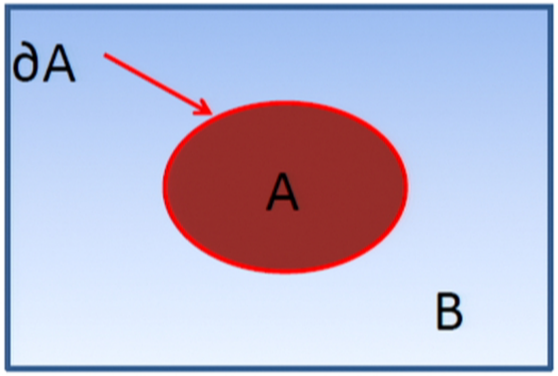
1. Entanglement Entropy
2. Holographic Entanglement Entropy
3. A Fundamental Law for Entanglement Entropy
4. Holographic Local Quench
5. Conclusions and Future works

1. Entanglement Entropy (EE)

- Definition of Entanglement Entropy

We divide the total Hilbert space into A and B: $H_{tot} = H_A \otimes H_B$.
The reduced density matrix ρ_A is defined by $\rho_A \equiv Tr_B \rho_{tot}$
This means the degrees of freedom in B are traced out.
The entanglement entropy is defined by von Neumann entropy S_A.

$$S_A = -Tr_A \rho_A \log \rho_A$$



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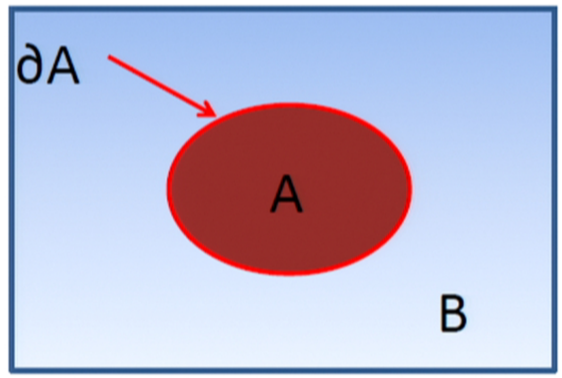

on a certain time slice

Example

For a product state: $|\Psi\rangle = \frac{1}{2}\left(|\uparrow\rangle_A + |\downarrow\rangle_A\right) \otimes \left(|\downarrow\rangle_B + |\uparrow\rangle_B\right)$

⇒Reduced density matrix : $\rho_A = \frac{1}{2}\left(|\uparrow\rangle_A + |\downarrow\rangle_A\right)\left(\langle\uparrow|_A + \langle\downarrow|_A\right)$

⇒Entanglement entropy : $S_A = 0$

For an entangled state: $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A \otimes |\downarrow\rangle_B + |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right)$

⇒Reduced density matrix : $\rho_A = \frac{1}{2}\left(|\uparrow\rangle_A \langle\uparrow|_A\right) + \frac{1}{2}\left(|\downarrow\rangle_A \langle\downarrow|_A\right)$

⇒Entanglement Entropy : $S_A = \log 2$

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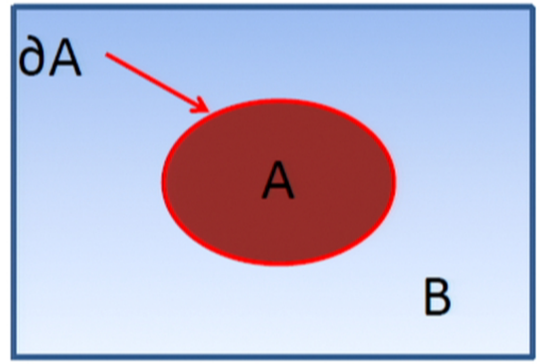
In general ,

Entangled states (not-product state) have the entanglement entropy.
S_A measures the quantum entanglement.

Properties 1

- For a pure state, entanglement entropy satisfies

$$S_A = S_B .$$

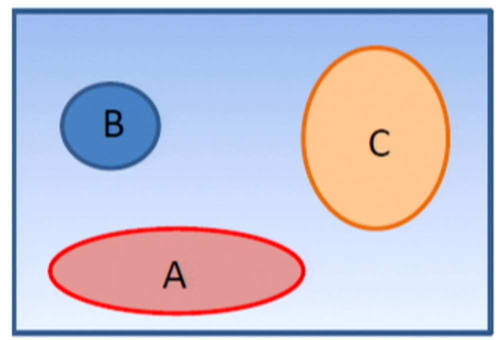


- For a mixed state (thermal state etc.), entanglement entropy satisfies

$$S_A \neq S_B .$$

- Strong Subadditivity
  Entanglement entropy necessarily satisfies

$$\begin{aligned} S_{A+B+C} + S_B &\leq S_{A+B} + S_{B+C} \\ S_A + S_C &\leq S_{A+B} + S_{B+C} . \end{aligned}$$



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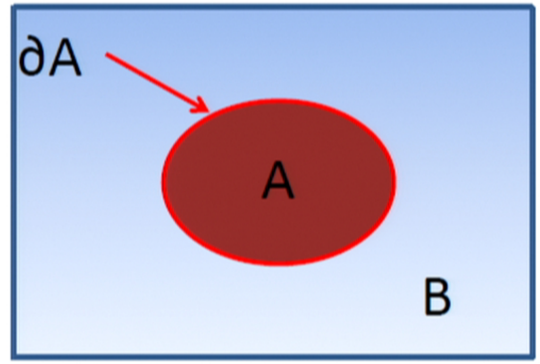

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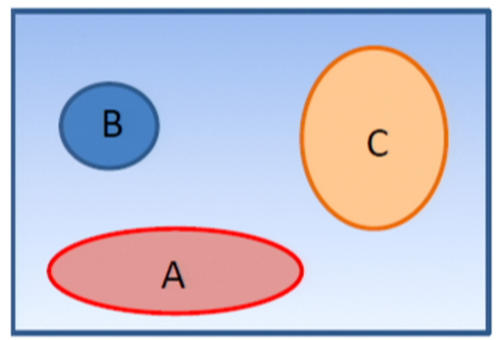


Properties 2

Entanglement entropy can be defined in QFT.

However, EE gets divergent and depends on a cut off ε.

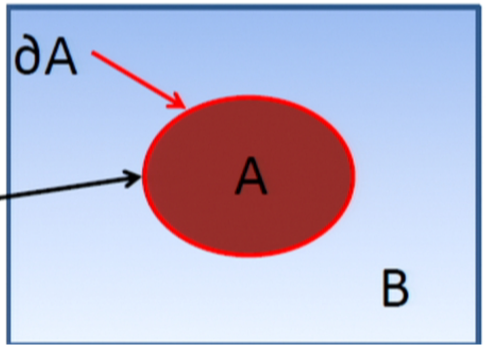
- Area law : in a d dimensional QFT, the leading divergent term of entanglement entropy for its ground state satisfies

$$S_A \sim \frac{\mathrm{Area}(\partial A)}{\epsilon^{d-2}}.$$

In the d=2 case, this divergence is replaced with a log divergence.

This property means:

Most strongly entangled



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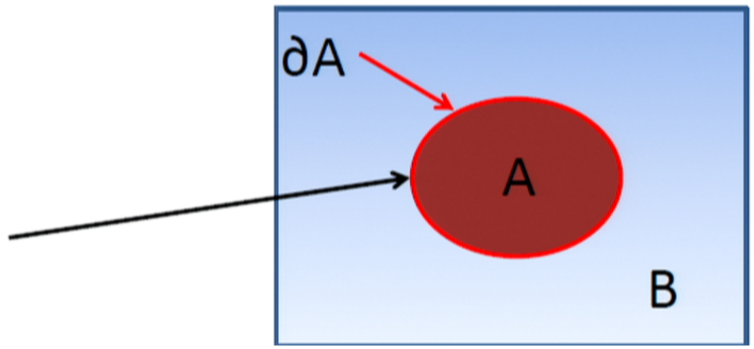


Properties 3

- The cut off independent parts of the entanglement entropy are universal quantities. For example, if subsystem A is an interval with width l in 2d CFT, the entanglement entropy is given by

$$S_A = \frac{c}{3} \log \frac{l}{\epsilon}.$$

- The usages of the entanglement entropy are as follows:

  (i) A quantum order parameter in condensed matter physics.

  (ii) A generalization of black hole entropy in string theory.

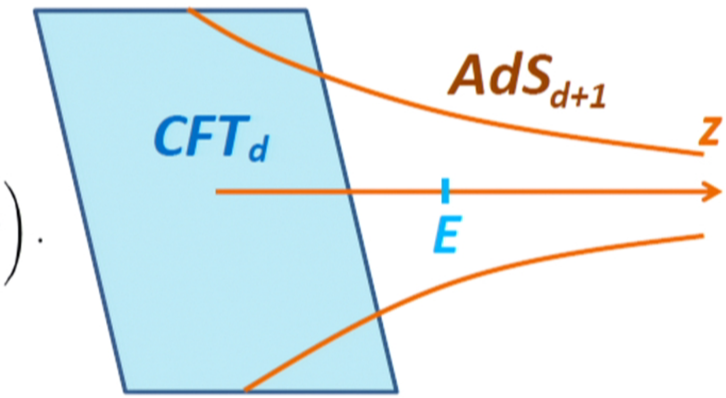
2. Holographic Entanglement Entropy

- AdS/CFT correspondence

  1. AdS/CFT correspondence says that the gravity on AdS spacetime is equivalent to a CFT on its boundary.

  2. Moreover, the AdS/CFT correspondence says that in AdS spacetime the physics at $z^{-1} = E$ corresponds to the physics of the state at an energy scale E in CFT.

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}\left(-dt^2 + \sum_i dx^i dx^i\right).$$



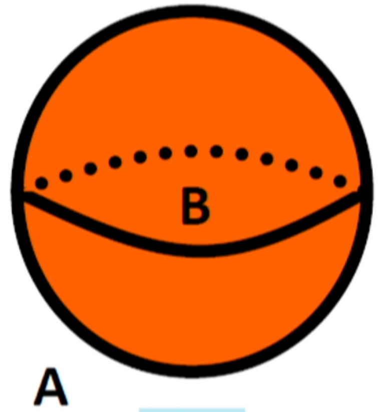
Holographic Entanglement Entropy

The black hole entropy S_{BH} is

$$S_{BH} = \frac{\mathcal{A}}{4G_N}.$$

$\mathcal{A}$ is the area of the black hole horizon.
G_N is the Newton constant.

Generalize



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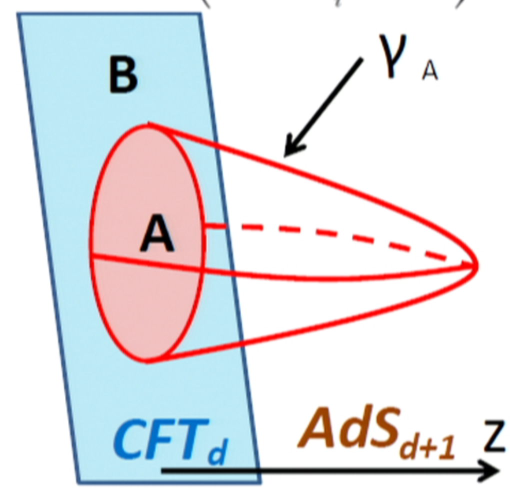

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Holographic Entanglement Entropy

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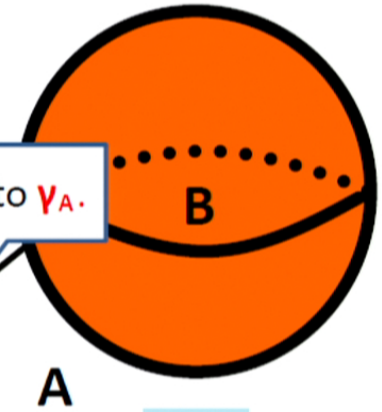
$\mathcal{A}$ is the area of the black hole horizon.
G_N is the Newton constant.

The horizon is generalized to γ_A.



Generalized

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Generalize

When we divide the boundary into A and B at a certain time, the holographic entanglement entropy (HEE) is given by

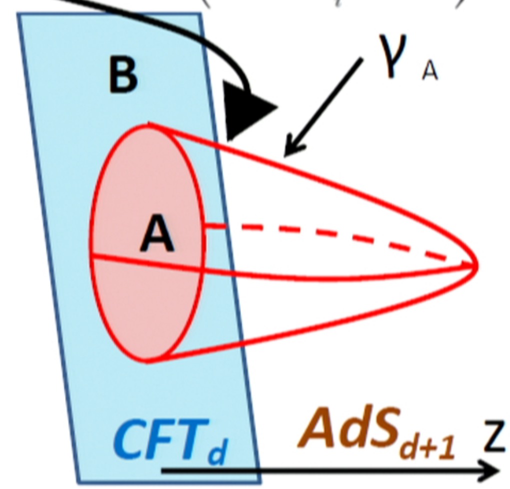
This is the definition of HEE .

$$S_A = \frac{\mathrm{Area}(\gamma_A)}{4G}.$$

•The area of γ_A is minimum.

= γ_A is the minimal surface.

$(\partial\gamma_A = \partial A)$



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Properties 1

- HEE satisfies Area Law :

  In AdS $_{d+1}$, we expand HEE(S_A) with respect to ε.

  The leading divergent term of HEE is

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- HEE also satisfies Strong Subadditivity :

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Properties 2

- General Behavior of HEE in AdS$_{d+1}$

$$S_A = p_1 \left(\frac{l}{\epsilon}\right)^{d-2} + p_2 \left(\frac{l}{\epsilon}\right)^{d-4} + \cdots$$

$$+ \begin{cases} p_{d-2}\left(\frac{l}{\epsilon}\right) + p_d, & d: \text{odd} \\ p_{d-3}\left(\frac{l}{\epsilon}\right)^2 + \tilde{c}\log\left(\frac{l}{\epsilon}\right), & d: \text{even}. \end{cases}$$

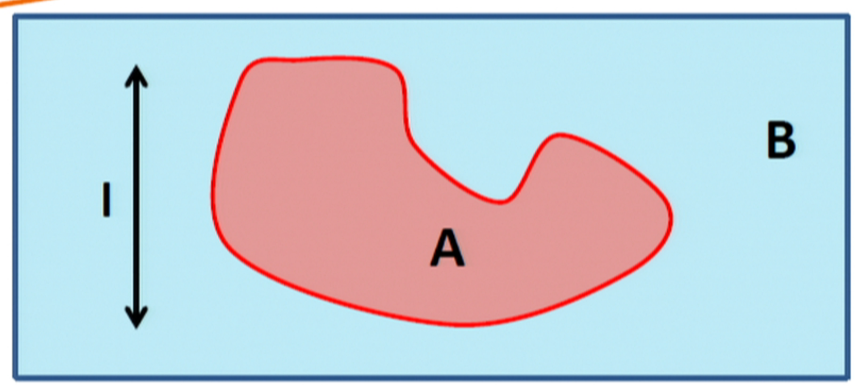
Area law div.

A universal quantity which characterizes odd dim. CFT.

⇩

F-theorem in 3 dim. CFTs

Central Charges



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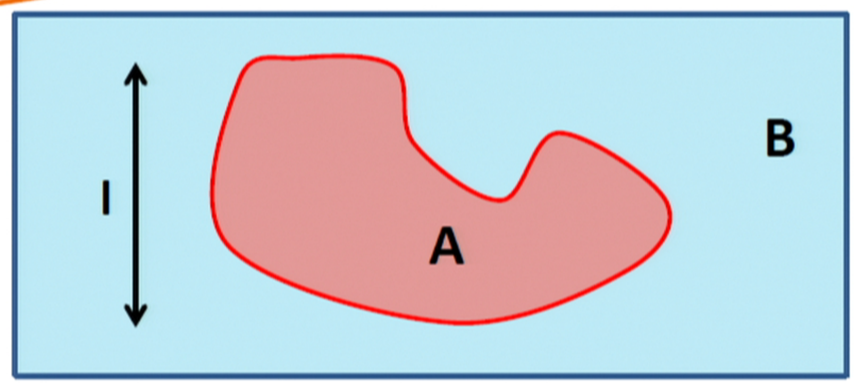
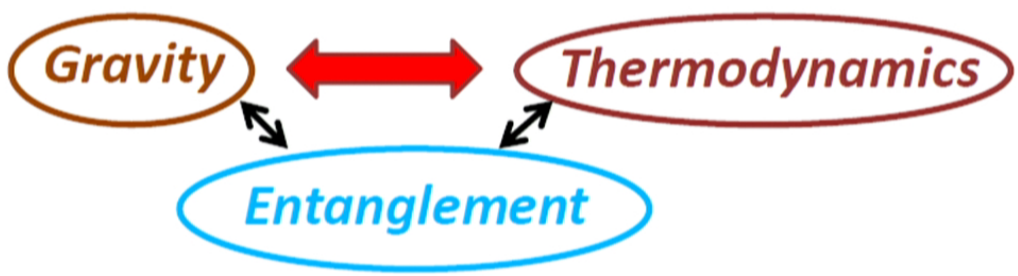


3. Fundamental Law for Entanglement Entropy

Thermal entropy obeys the laws of thermodynamics.

Are there the fundamental laws which entanglement entropy (EE) should obey ?



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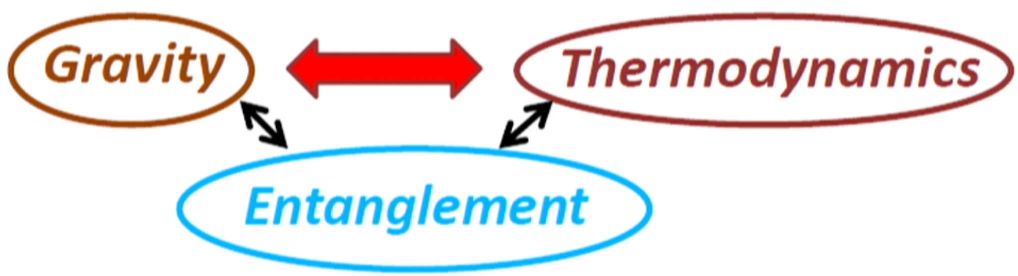


Setup

- ***Field theory side*** :

  In CFT_d, we excite the system somehow

  (thermalization or producing massive particles and etc.).

  ⇒ Total system becomes non-equilibrium in general.

- ***Gravity dual*** :

  On AdS_{d+1} background, we deform the IR region (large z region).

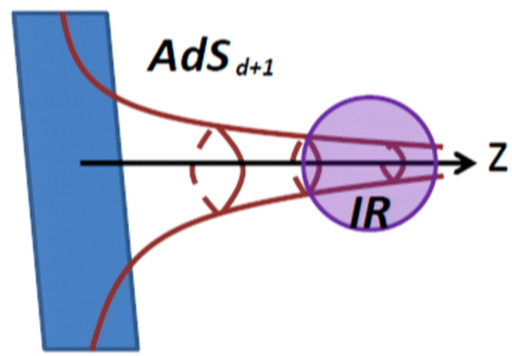
  (The IR geometry changes to BH or star or etc.)

Metric:

$$ds^2 = \frac{R^2}{z^2}\left[-f(z)dt^2 + g(z)dz^2 + \sum_{i=1}^{d-1}(dx_i)^2\right]$$

$Z \to 0$

$$g(z) \simeq 1/f(z) \simeq 1 + mz^d$$



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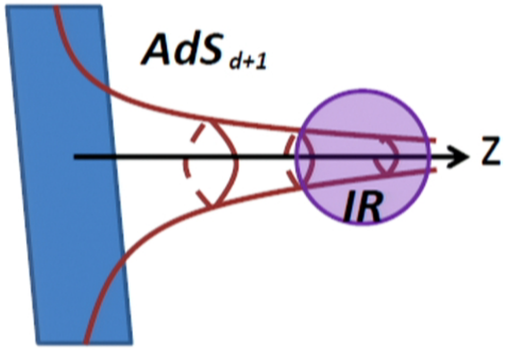


Setup

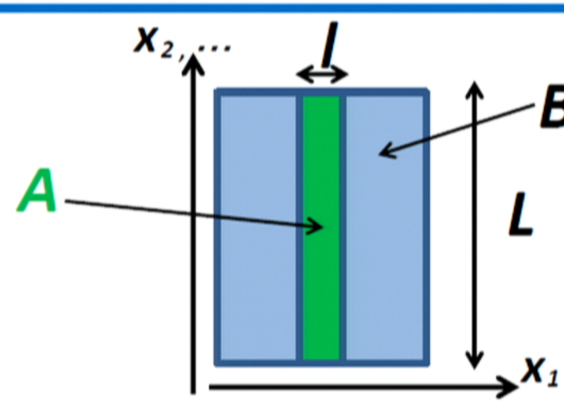
- We divide the boundary into A and B at certain time.
  The subsystem A is a strip with width *l*.

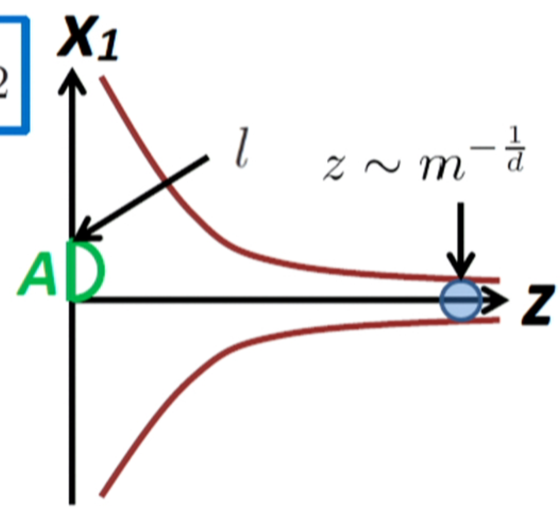
Assumption

This width *l* satisfies $ml^d << 1$.

Subsystem A

$0 < x_1 < l, \; -L/2 < x_{2,3,\cdots,d-1} < L/2$





Relation between ΔS_A and ΔE_A

The energy of subsystem A :

$$\Delta T_{tt} = \frac{(d-1)\, R^{d-1} m}{16\pi G_N} \Rightarrow \Delta E_A = \int dx^{d-1} \Delta T_{tt} = \frac{(d-1) m l L^{d-2} R^{d-1}}{16\pi G_N}.$$

The relation between ΔS_A and ΔE_A :
(ΔS_A is the change of HEE. by deformation.)

$$\Delta E_A = \left(\frac{\sqrt{\pi}\,\Gamma\left(\frac{1}{2(d-1)}\right)^2 \Gamma\left(\frac{1}{d-1}\right)}{2(d^2-1)\Gamma\left(\frac{1}{2}+\frac{1}{d-1}\right)\Gamma\left(\frac{d}{2(d-1)}\right)^2} \right)^{-1} \cdot l^{-1} \cdot \Delta S_A \quad \sim \quad dU = TdS$$

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- ***At $ml^d << 1$, this is the relation between amount of the quantum information and the energy in a subsystem.***
- ***The effective temperature T_{ent} in the subsystem is proportional to the inverse of the subsystem's size l^{-1} .***

Round Ball Case

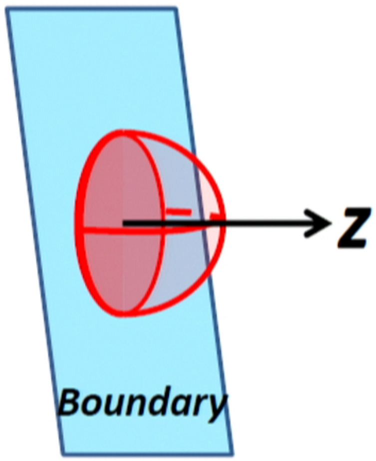
The energy of subsystem A : $\Delta E_A = \frac{\pi^{\frac{d-1}{2}} R^{d-1}}{8\pi G_N \Gamma\left(\frac{d-1}{2}\right)} \cdot \left(ml^{d-1}\right).$

The change of HEE. : $\Delta S_A = \frac{\pi^{\frac{d-1}{2}}}{4(d+1)\Gamma\left(\frac{d-1}{2}\right)} \cdot \frac{R^{d-1}}{G_N} \cdot ml^d.$

The relation between ΔS_A and ΔE_A : $\Delta E_A = \left(\frac{2\pi l}{d+1}\right)^{-1} \cdot \Delta S_A$

At $ml^d << 1$, in the round ball case, HEE. also satisfies the same relation:

$$\Delta E_A = T_{\text{ent}} \cdot \Delta S_A.$$



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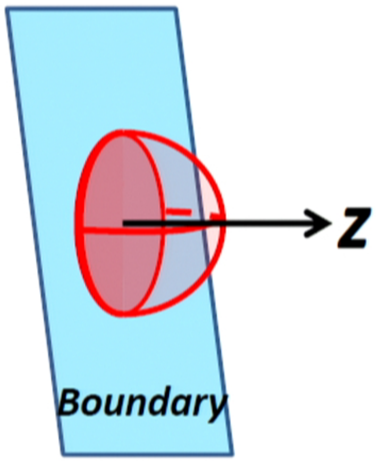

The effective temperature T_{ent}

Interval case : $T_{\mathrm{ent}} = \left(\frac{\sqrt{\pi}\,\Gamma\left(\frac{1}{2(d-1)}\right)^2 \Gamma\left(\frac{1}{d-1}\right)}{2(d^2-1)\Gamma\left(\frac{1}{2}+\frac{1}{d-1}\right)\Gamma\left(\frac{d}{2(d-1)}\right)^2} \right)^{-1} \cdot l^{-1}$

Round ball case : $T_{\mathrm{ent}} = \frac{d+1}{2\pi} \cdot l^{-1}$

⇒ The effective temperature ***does not*** depend on ***G_N ,R (N, λ)***.
It does not depend on ***the detail of the theories***.

The effective temperature $T_{\rm ent}$

Interval case : $T_{\rm ent} = \left(\frac{\sqrt{\pi}\,\Gamma\left(\frac{1}{2(d-1)}\right)^2 \Gamma\left(\frac{1}{d-1}\right)}{2(d^2-1)\Gamma\left(\frac{1}{2}+\frac{1}{d-1}\right)\Gamma\left(\frac{d}{2(d-1)}\right)^2} \right)^{-1} \cdot l^{-1}$

Round ball case : $T_{\rm ent} = \frac{d+1}{2\pi} \cdot l^{-1}$

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This constant depends on only the shape of subsystem A.

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At $ml^d << 1$, this relationship is also ***universal***.

The interpretation of $ml^d << 1$ in the field theoretic language

In the gravity side ,

$$ml^d << 1 \Rightarrow T_{tt} \cdot l^d << R^{d-1}/G_N$$

In a strongly coupled large N gauge theory,

$$T_{tt} \cdot l^d << O(N^2).$$

In a strongly coupled large N gauge theory, when *l* satisfies this constraint , EE and energy obey

$$\Delta E_A = T_{\mathrm{ent}} \cdot \Delta S_A.$$

The results from CFT₂ at Finite Temperature

In CFT at finite temperature $T = \beta^{-1}$, the EE of an interval with width ***l*** is

$$S_A = \frac{c}{3} \log \left(\frac{\beta}{\pi \epsilon} \sinh \frac{\pi l}{\beta} \right).$$

When we expand the EE with respect to l/β in the limit $l << \beta$ (= $ml^2 \ll 1$),

$$S_A = \frac{c}{3} \log \left(\frac{l}{\epsilon} \right) + \frac{c\pi^2 T^2 l^2}{18} + \mathcal{O}\left(T^4 l^4\right).$$

(The EE for its ground state is $S_A^{ground} = \frac{c}{3} \log \left(\frac{l}{\epsilon} \right)$.)

The difference ($\Delta S_A \equiv S_A - S_A^{ground}$) is

$$\Delta S_A = \frac{c\pi^2 T^2 l^2}{18} = \frac{Rml^2}{48 G_N} = \frac{\pi}{3} l \cdot \Delta E_A.$$

($m = (2\pi T)^2, c = \frac{3R}{2G_N}, \Delta E_A = \frac{mlR}{16\pi G_N}$)

The results from CFT_2

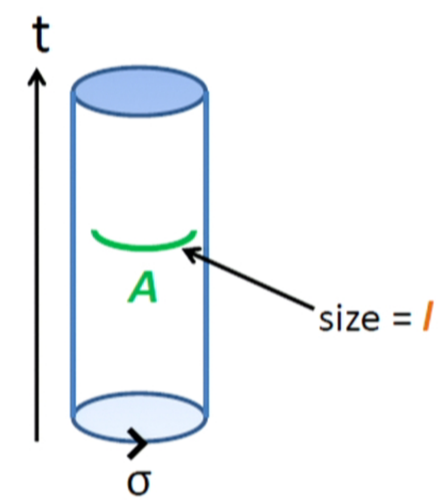
(This result dose not depend on the calculation on gravity side.)

In field theory side, we calculate the EE for the primary state with conformal weight $(h, \bar{h})$.

This total system is a circle.

We divide the total system into A and B.

The size of the subsystem A is ***l*** .



In the limit $l \ll 1$, we expand EE with respect to ***l*** , the EE is

$$\Delta S = S_A^{total} - S_A^{ground} \sim \frac{2\pi^2}{3}(h+\bar{h})\left(\frac{l}{2\pi}\right)^2 = \frac{\pi}{3} l \cdot \Delta E_A \, .$$

(The EE for its ground state is S_A^{ground} , $\Delta E_A = \frac{(h+\bar{h})}{2\pi} \cdot l$.)

[F. C. Alcaraz-M. I. Berganza-G. Sierra 11]

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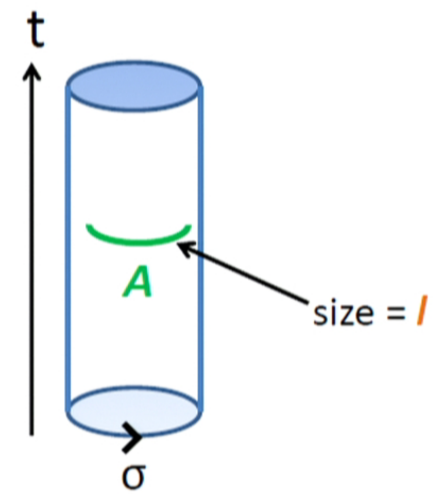

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Suggestion

$T_{\rm ent}$ is also universal ***in any*** CFT_2 .

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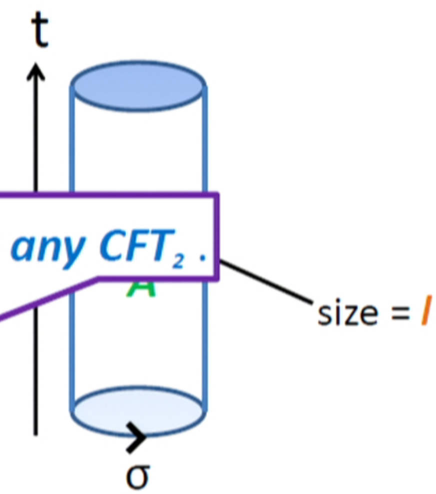

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Holographic Local Quench

Based on arXiv:1302.5703
with Tadashi Takayanagi (YITP, Kyoto)
and Tokiro Numasawa (YITP, Kyoto)

- Motivations:

  1. In the time-dependent case,
     The relationship, $\Delta E_A = T_{\rm ent} \cdot \Delta S_A$ holds ?

  2. We would like to construct gravity dual to local quantum quench.
     And we would like to study the behavior of local quantum quench in detail.

→ ***Non-equilibrium physics***

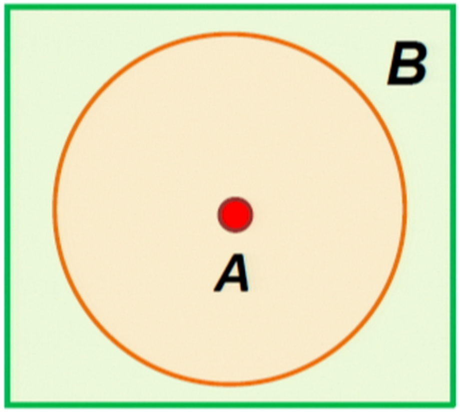
Thermodynamical relation

For d=2,3,4 case, we derive the thermodynamical relation in the limit $l \ll \sqrt{\alpha^2 + t^2}$

$$\Delta E_A = T_{\rm ent} \cdot \Delta S_A,$$
$$T_{\rm ent} = \frac{d+1}{2\pi} \cdot l^{-1}.$$

In time-dependent case, the thermodynamical relation (effective temperature) also holds.

We expect that this thermodynamical relation (effective temperature $T_{\rm ent}$) holds in any dimensional time-dependent case.

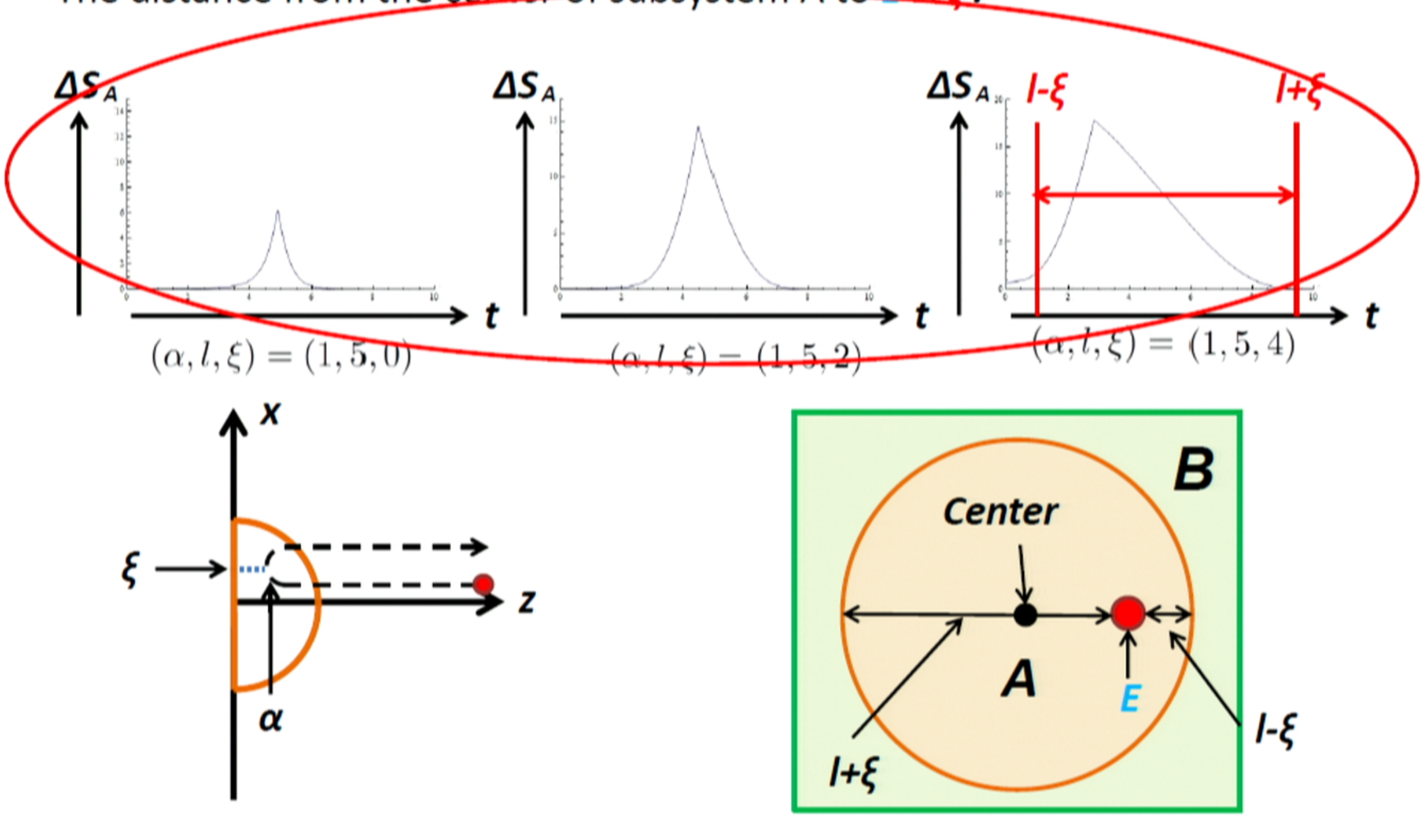


Case 2

We studied the behavior of the entanglement entropy in AdS_5 .

E is the point where we add the energy.

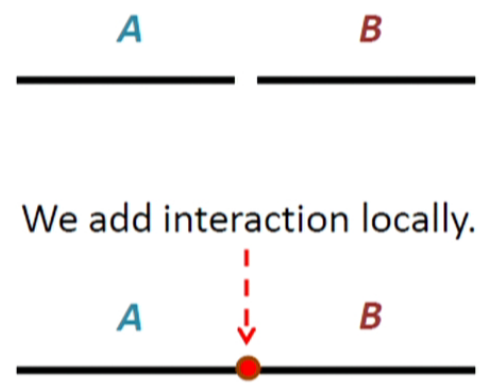
The distance from the center of subsystem A to *E* is ξ .



Local Quantum Quench

We prepare the two independent systems of each other.

We join these and add a new local interaction between them at certain time.



This total system is excited.

We study the time-evolution of entanglement by using the entanglement entropy.

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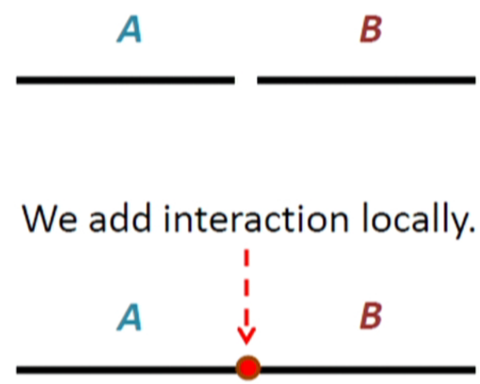

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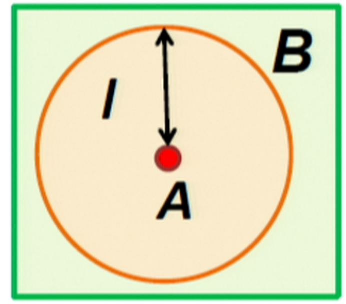
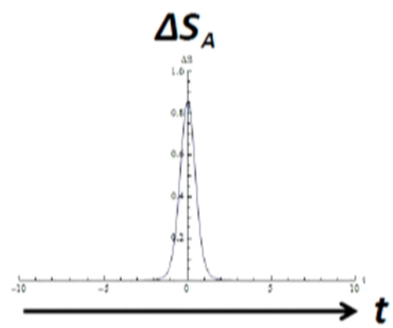


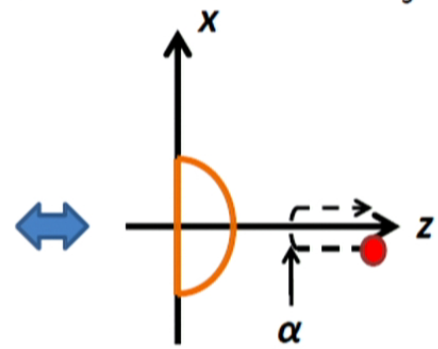
We studied the behavior of the entanglement entropy at two regions in AdS_5 .

Region 1 : $l << \alpha$

The peak appears at $t = 0$.

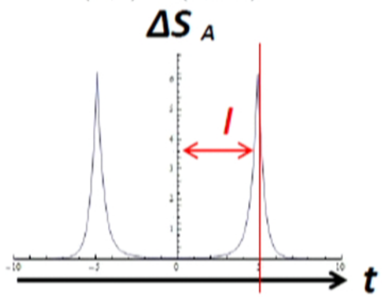


$(\alpha, l) = (1, 0.5)$

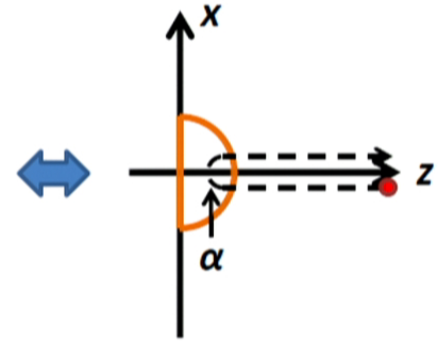


Region 2 : $l \gtrsim \alpha$

The peak appears at $t \sim l$.



$(\alpha, l) = (1, 5)$



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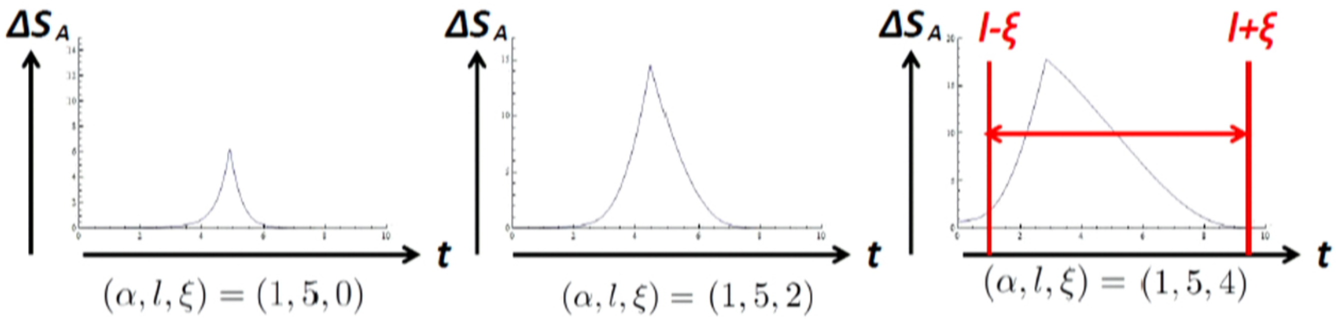
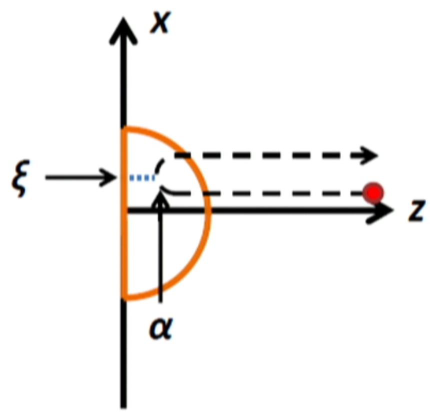
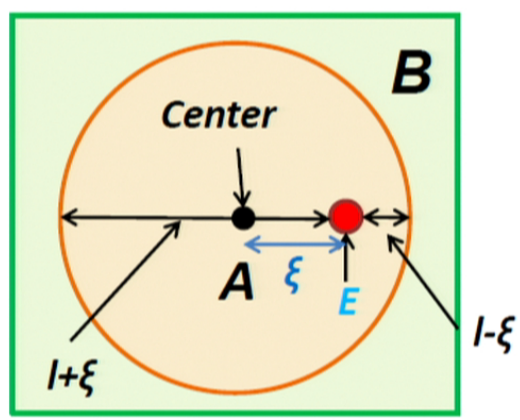




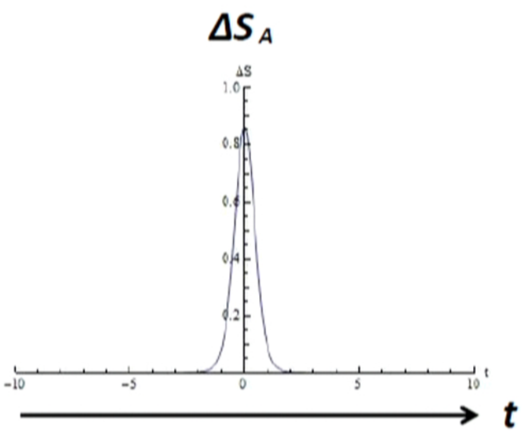


The interpretation of these behavior

This thermodynamical relation

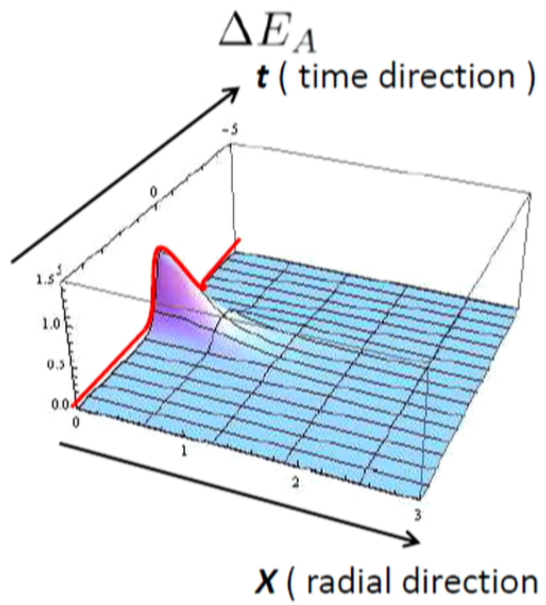
$$\Delta E_A = T_{\rm ent} \cdot \Delta S_A$$

says that



$(\alpha, l) = (1, 0.5)$

$\propto$

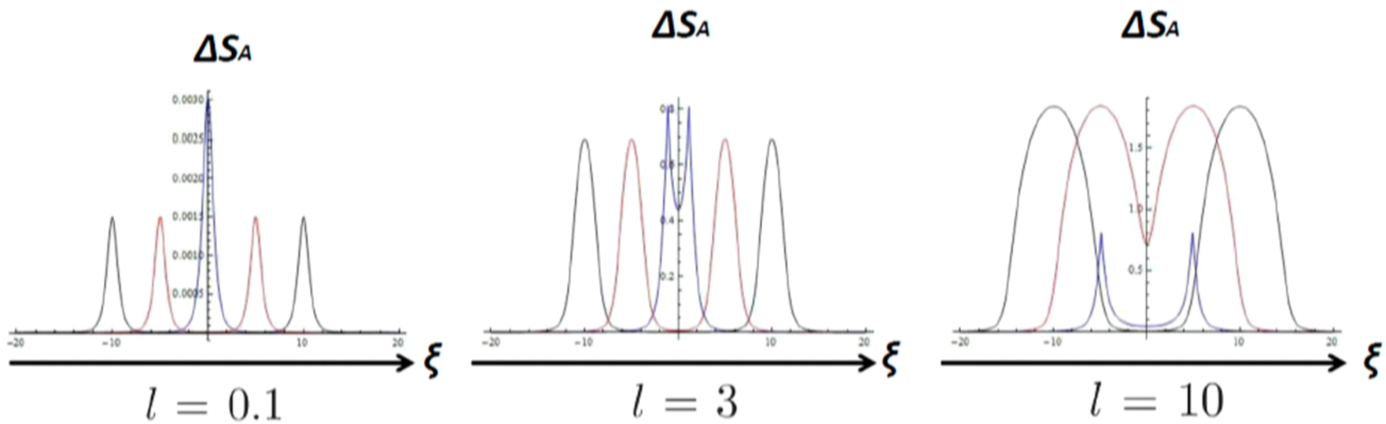


.

Exact result

Blue line : t=0
Red line : t=5
Black line : t=10

$\alpha = 1$



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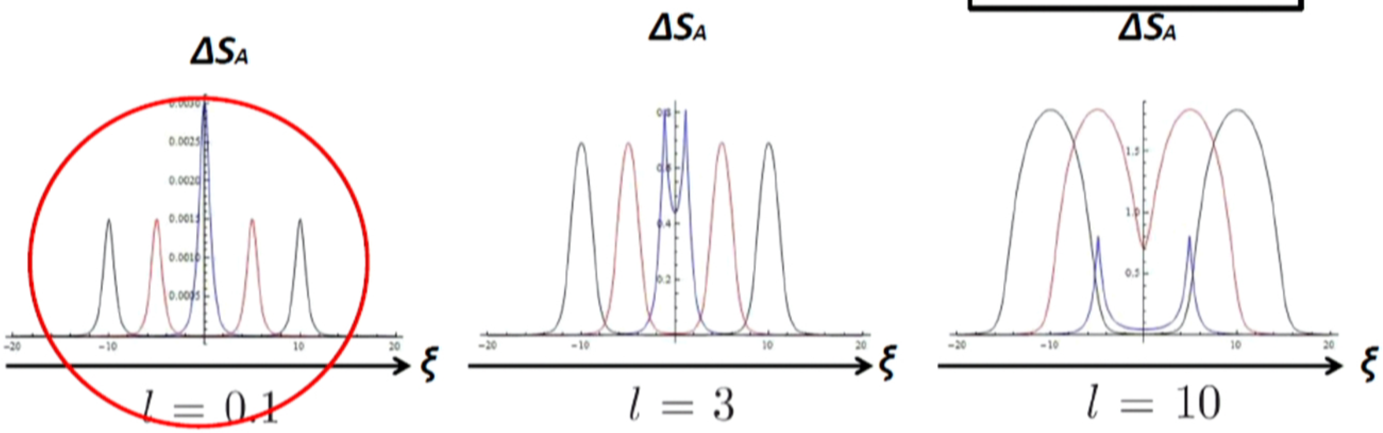


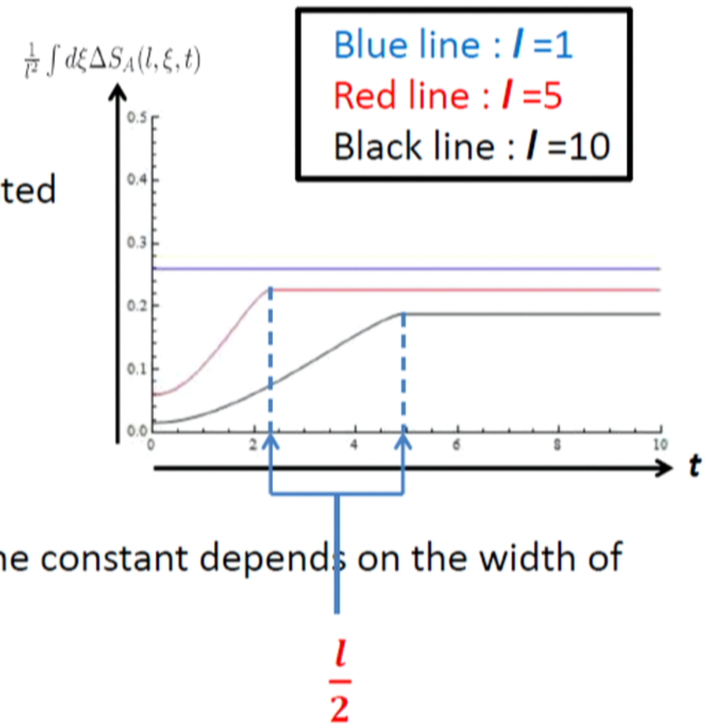
In this region ($l \ll \sqrt{\alpha^2 + t^2}$), the amount of quantum information seems to be conserved at each time.

In this region ($l \ll \sqrt{\alpha^2 + t^2}$), the thermodynamical relation , $\Delta E_A = T_{\rm ent} \cdot \Delta S_A$ holds.

The energy is ***conserved***. ➡ ***It is true.***

Exact result

We introduce a new quantity $\frac{1}{l^2}\int d\xi \Delta S_A(l,\xi,t)$.
$\frac{1}{l^2}\int d\xi \Delta S_A(l,\xi,t)$ is the quantity which is integrated with respect to ξ.



This quantity approaches to the constant at the late time.

The time which this quantity reaches at the constant depends on the width of the interval.

The saturation time is $\frac{l}{2}$.

We introduce the new quantity (***Entanglement Density***) to describe this behavior.

Entanglement Density

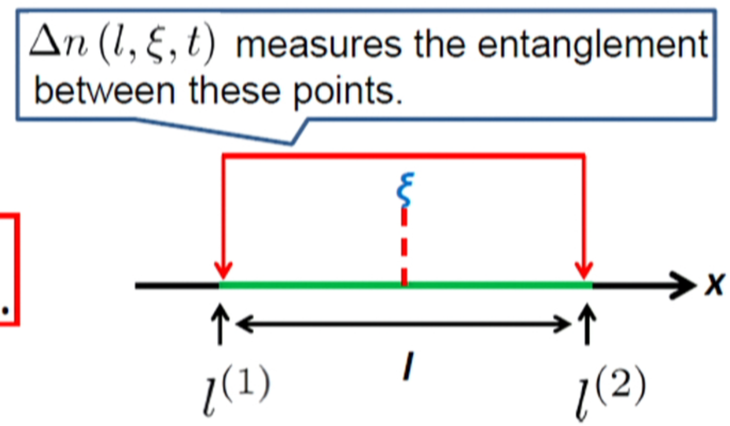
We introduce the ***Entanglement Density*** as the quantity which measures the entanglement between two points.

We assume the entanglement between two bodies only contributes to the entanglement between two points.

So, this quantity counts the number of entanglement pair between two points.

This quantity is defined by

$$\Delta n\,(l,\xi,t) = \frac{1}{2}\cdot\frac{\partial^2 S_A}{\partial l^{(1)}\partial l^{(2)}}.$$



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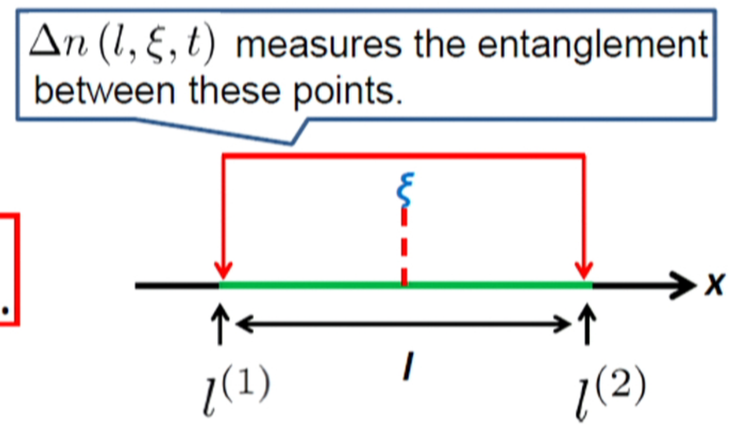
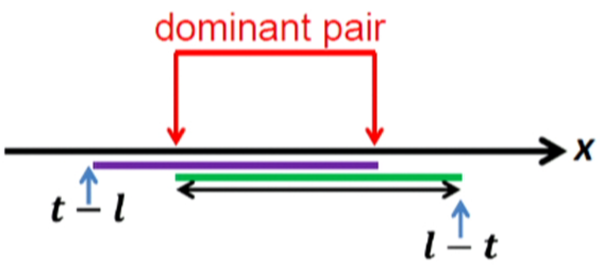


We found that the entanglement density between $l^{(1)} = -\sqrt{t^2+\alpha^2}$ and $l^{(2)} = \sqrt{t^2+\alpha^2}$ is dominant. (= ***dominant entangled pair***)

At t< $\frac{l}{2}$, the distance between dominant pair is less than the width of the interval.
Then, the intervals which center is $t - \frac{l}{2} < \xi < -t + \frac{l}{2}$ include the both points of dominant pair.
These intervals do not contribute to $\frac{1}{l^2}\int d\xi \Delta S_A(l,\xi,t)$.



At t> $\frac{l}{2}$, the distance between dominant pair is larger than the width of the interval.
The number of intervals which contribute to $\frac{1}{l^2}\int d\xi \Delta S_A(l,\xi,t)$ is saturated.