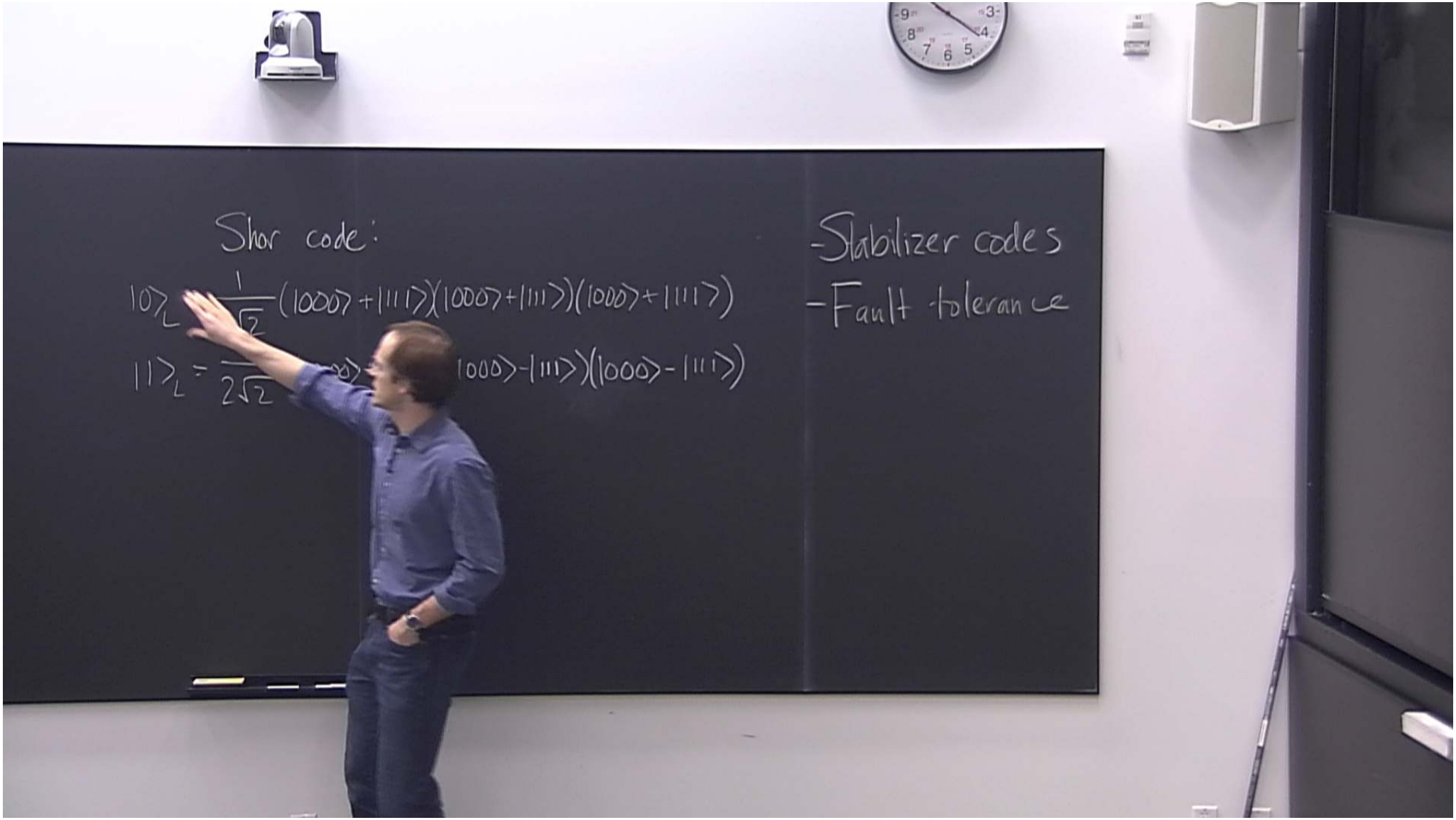
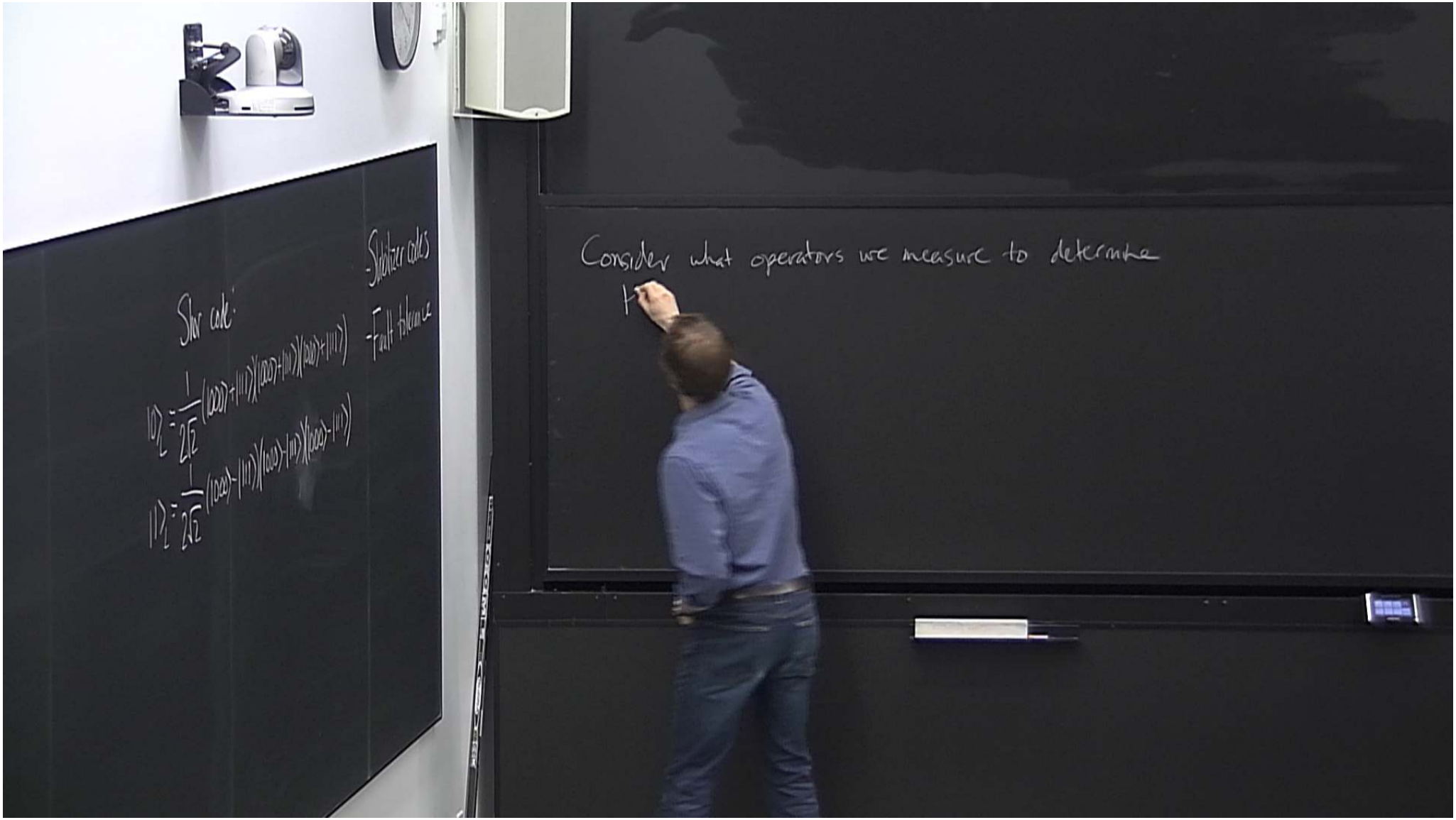
Title: 12/13 PSI - Quantum Information Review Lecture 13

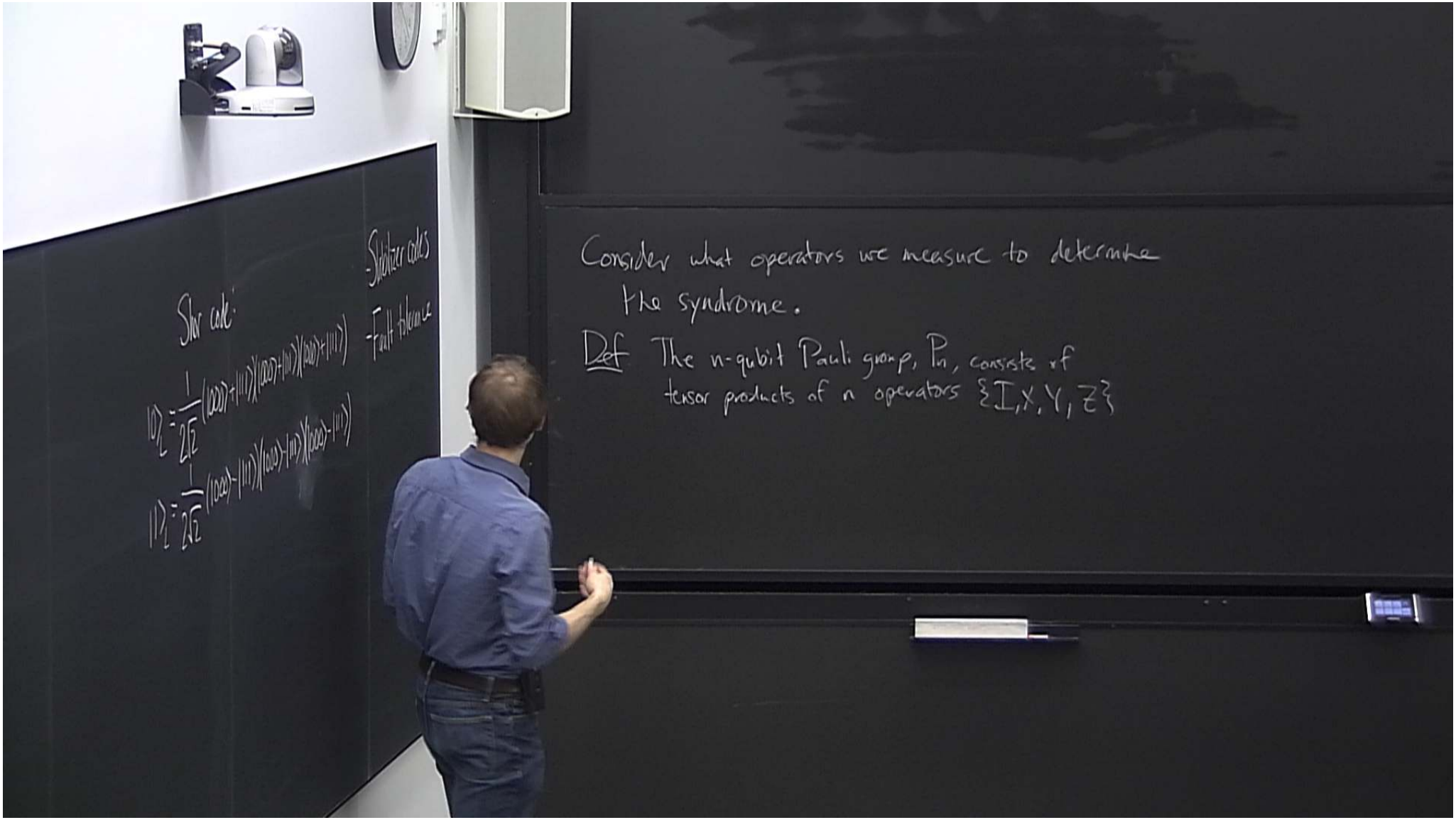
Date: Mar 07, 2013 10:15 AM

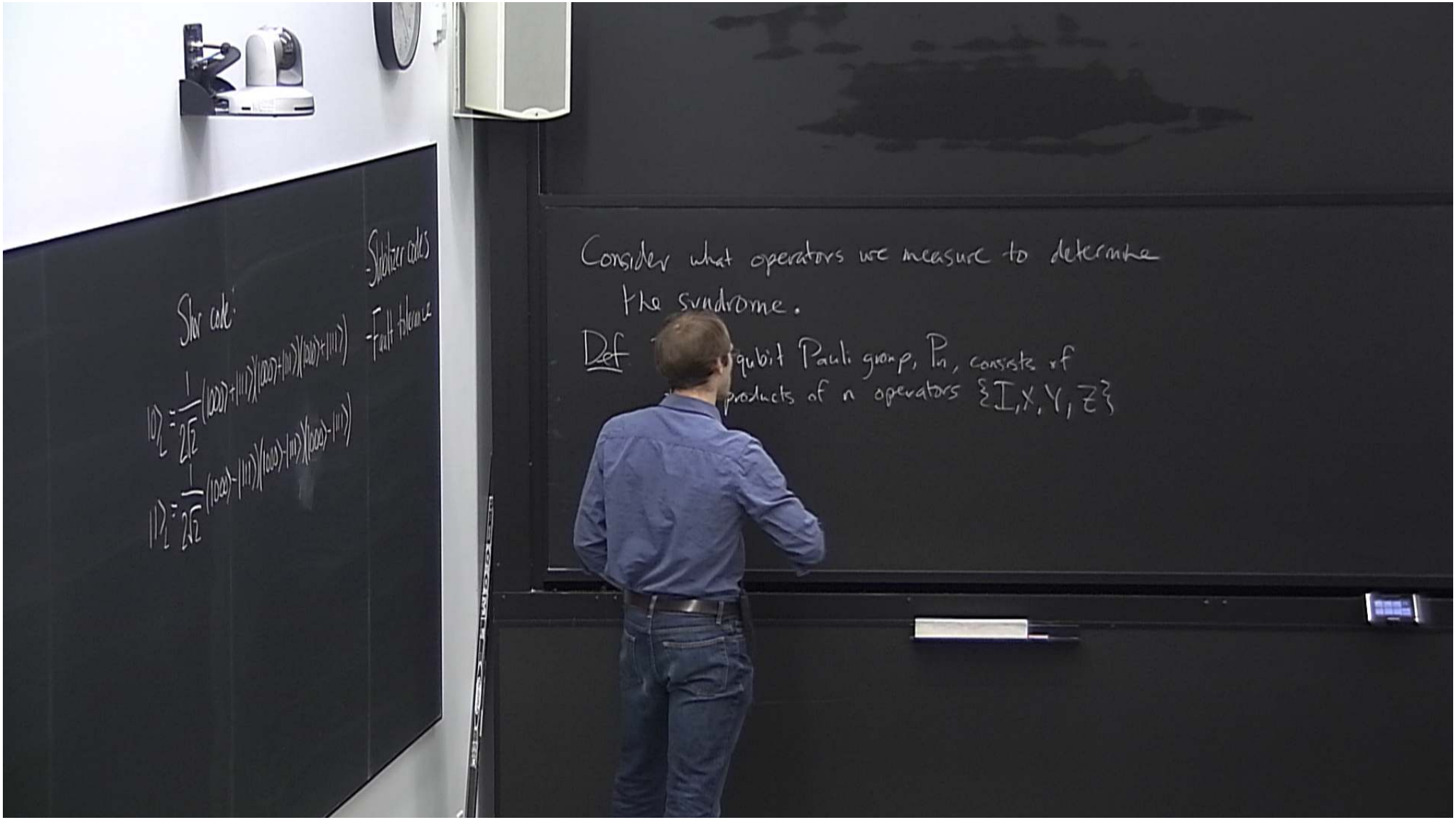
URL: http://www.pirsa.org/13030052

Abstract:


Shor code:
$|0\rangle_L = \frac{1}{2\sqrt{2}}(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)(|000\rangle+|111\rangle)$
$|1\rangle_L = \frac{1}{2\sqrt{2}}(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)(|000\rangle-|111\rangle)$
-Stabilizer codes
-Fault tolerance


Shor code:
-Stabilizer codes
-Fault tolerance
Consider what operators we measure to determine


Shor code:
- Stabilizer codes
- Fault-tolerance
Consider what operators we measure to determine the syndrome.
Def The n-qubit Pauli group, P_n, consists of tensor products of n operators $\{I, X, Y, Z\}$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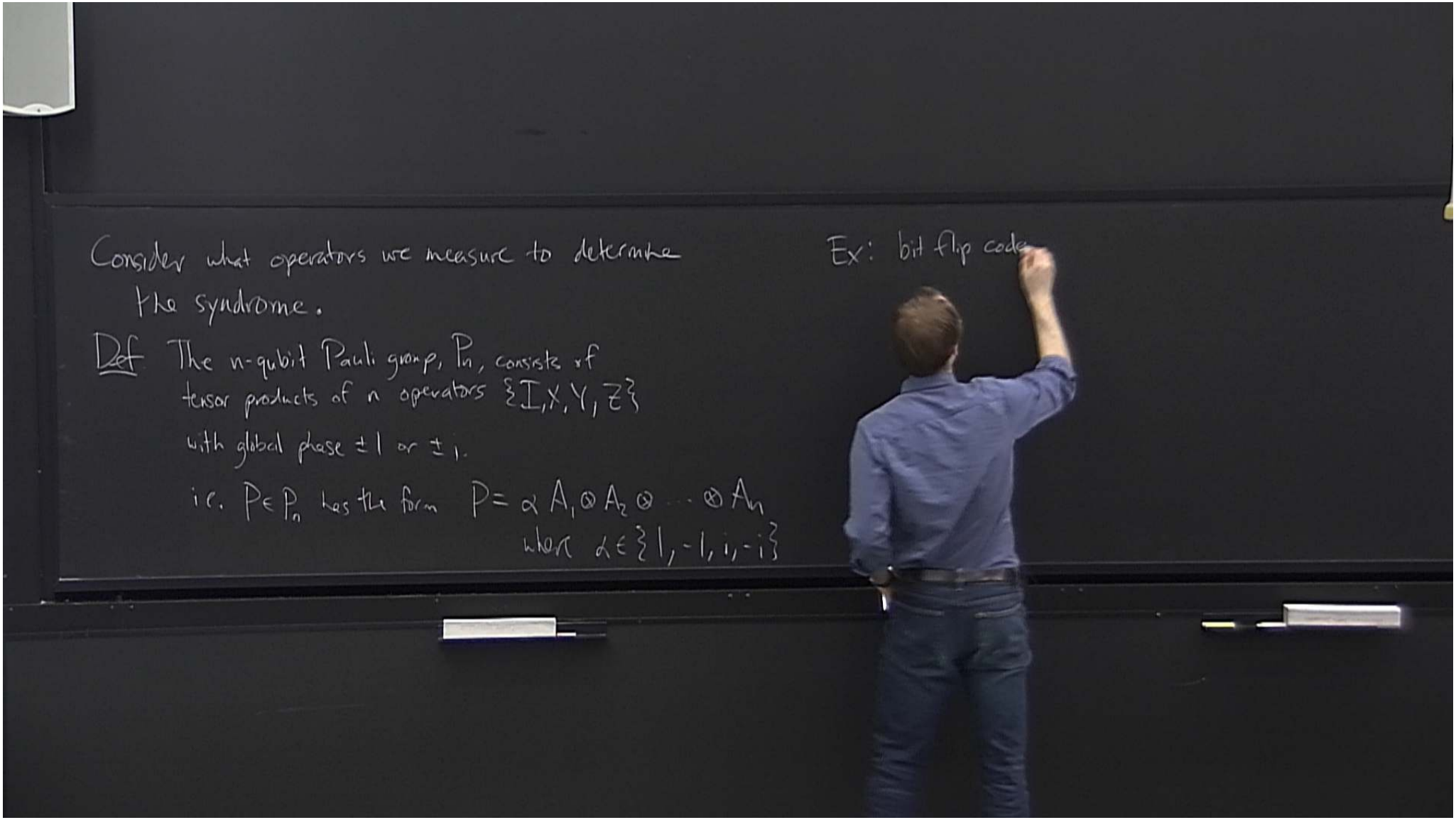
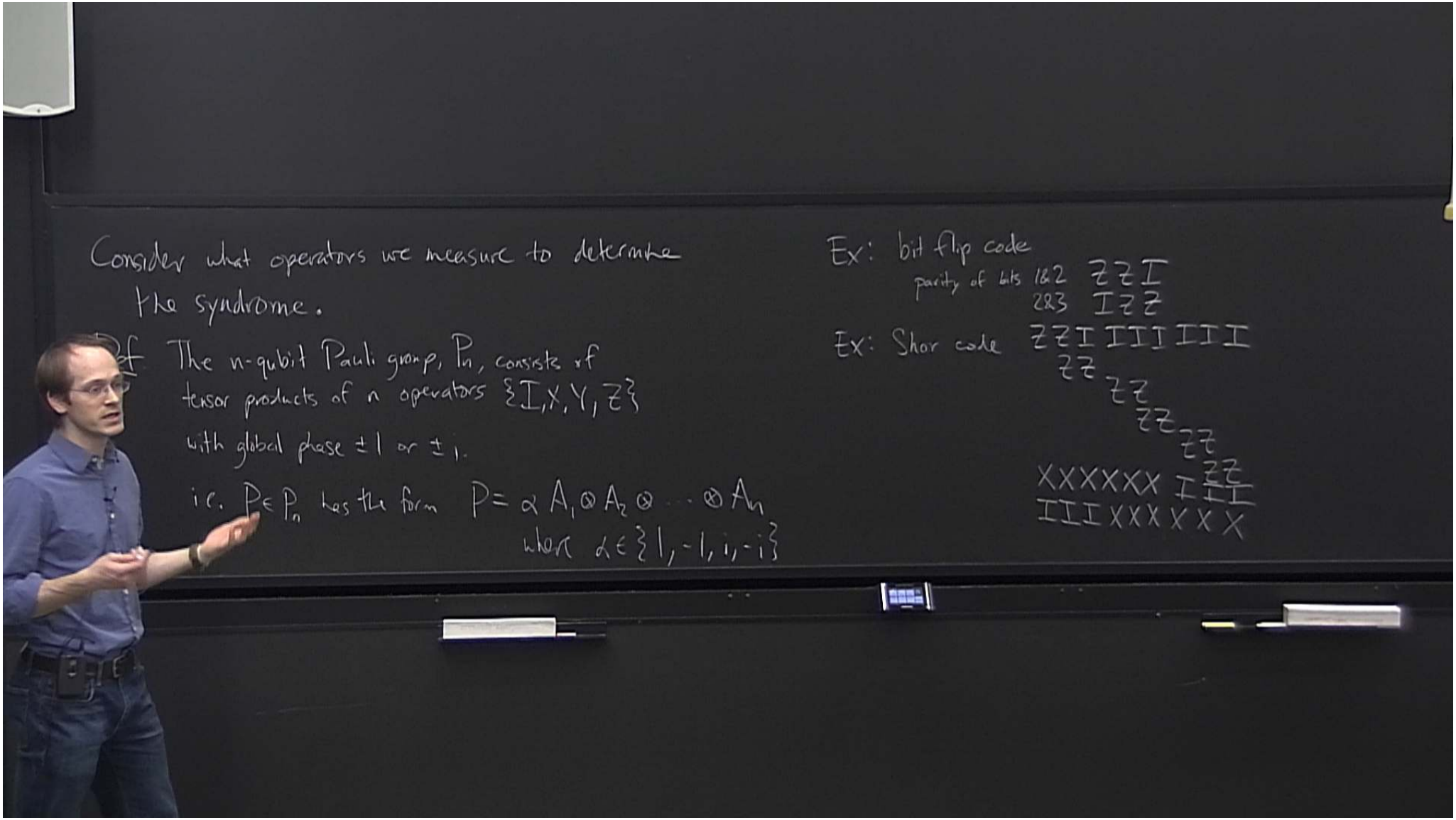

Consider what operators we measure to determine the syndrome.
Def The n-qubit Pauli group, P_n, consists of tensor products of n operators $\{I, X, Y, Z\}$ with global phase ± 1 or $\pm i$.
i.e. $P \in P_n$ has the form $P = \alpha A_1 \otimes A_2 \otimes \cdots \otimes A_n$ where $\alpha \in \{1, -1, i, -i\}$
Ex: bit flip code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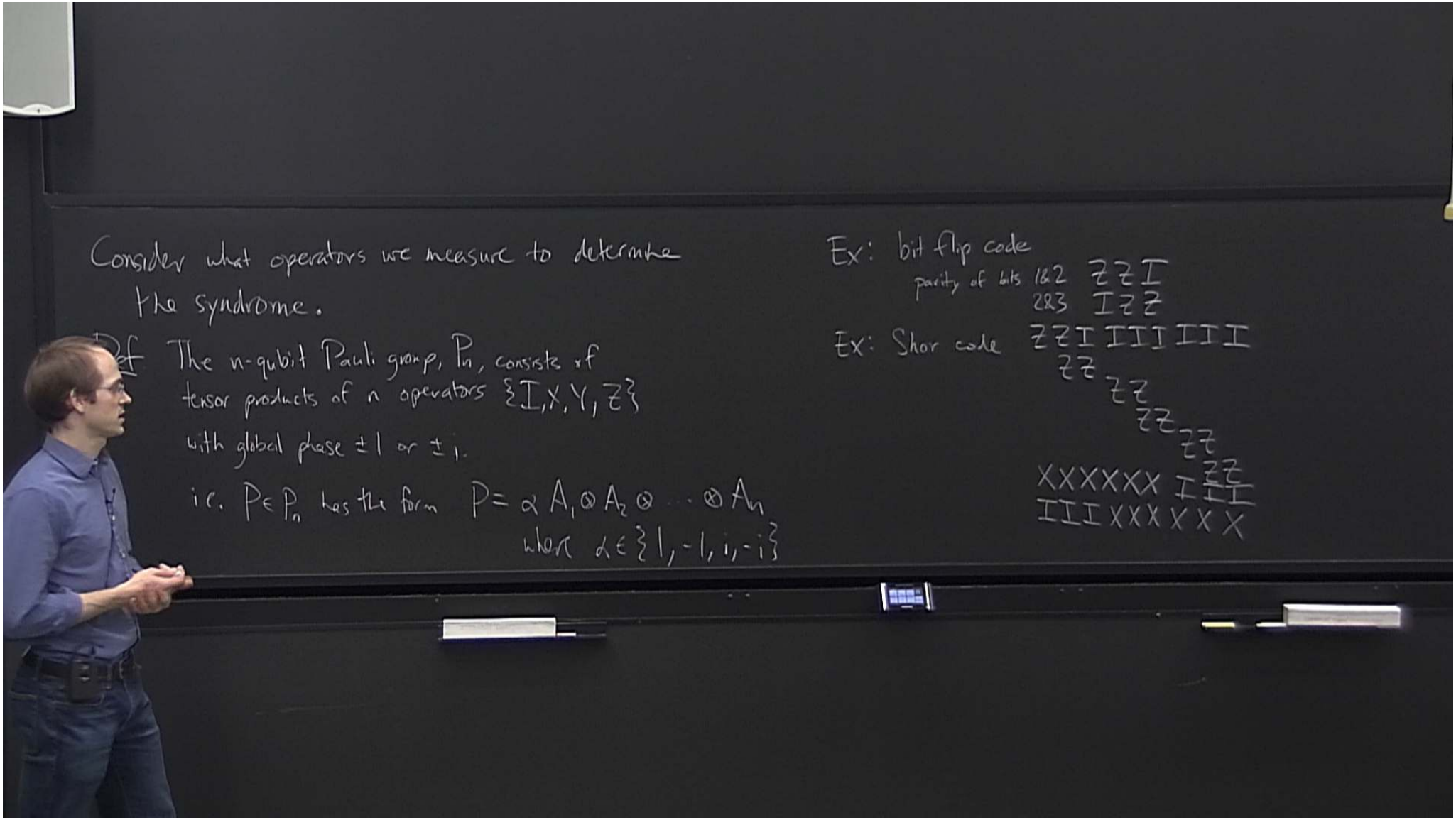
Ex: bit flip code
parity of bits 1&2 ZZI
2&3 IZZ
Ex: Shor code ZZIIIIIII
ZZ
ZZ
ZZ
ZZ
ZZ
XXXXXX III
IIIXXXXXX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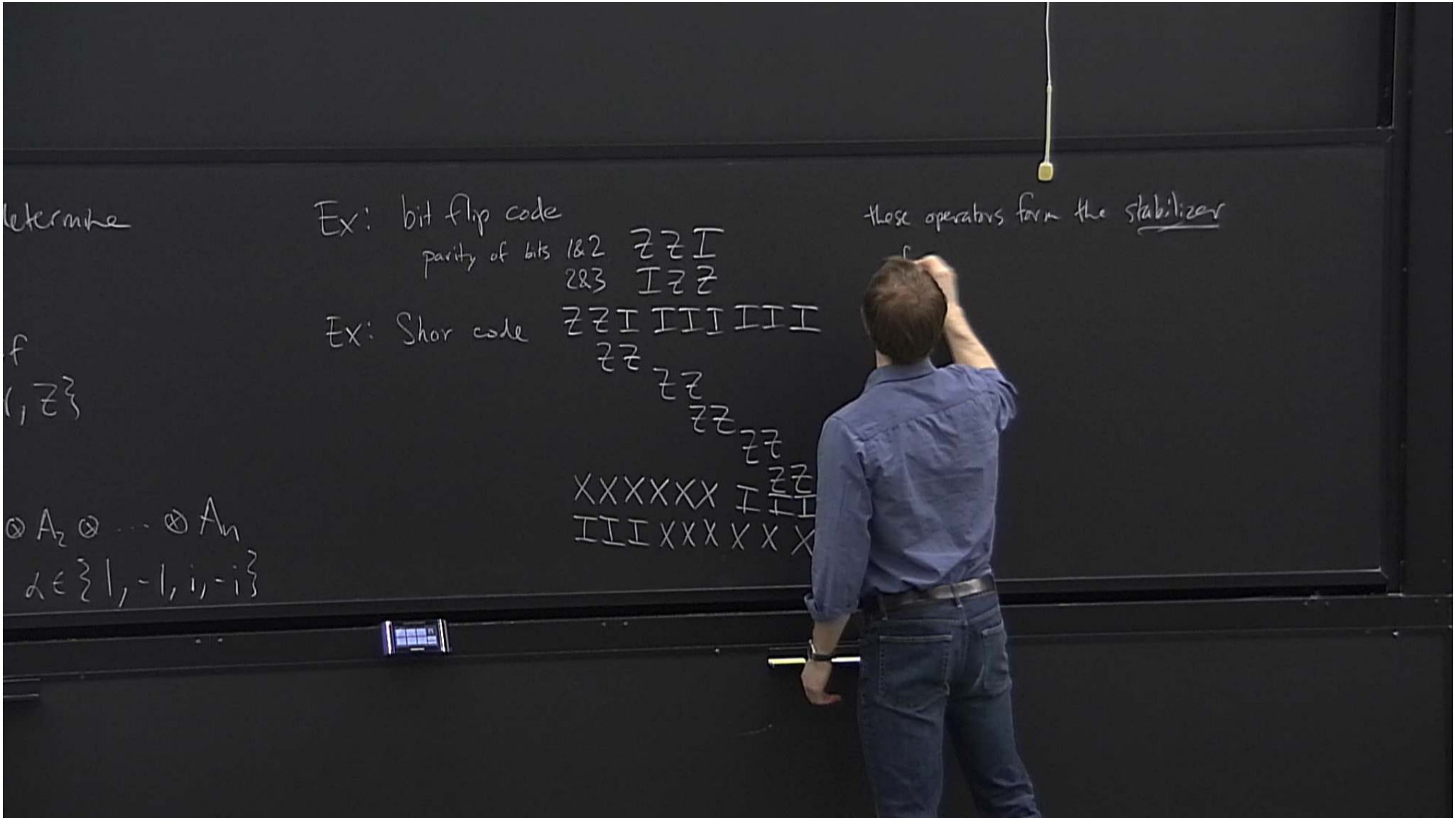
Ex: bit flip code
parity of bits 1&2 ZZI
2&3 IZZ
those operators form the stabilizer
Ex: Shor code ZZIIIIIII
ZZ
ZZ
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XXXXXX I
III XXXXXX

i.e. $P \in P_n$ has the form $P = \alpha A_1 \otimes A_2 \otimes \cdots \otimes A_n$
where $\alpha \in \{1, -1, i, -i\}$

XXXXXX I
III XXX X X

Note: $\dim T(S) =$

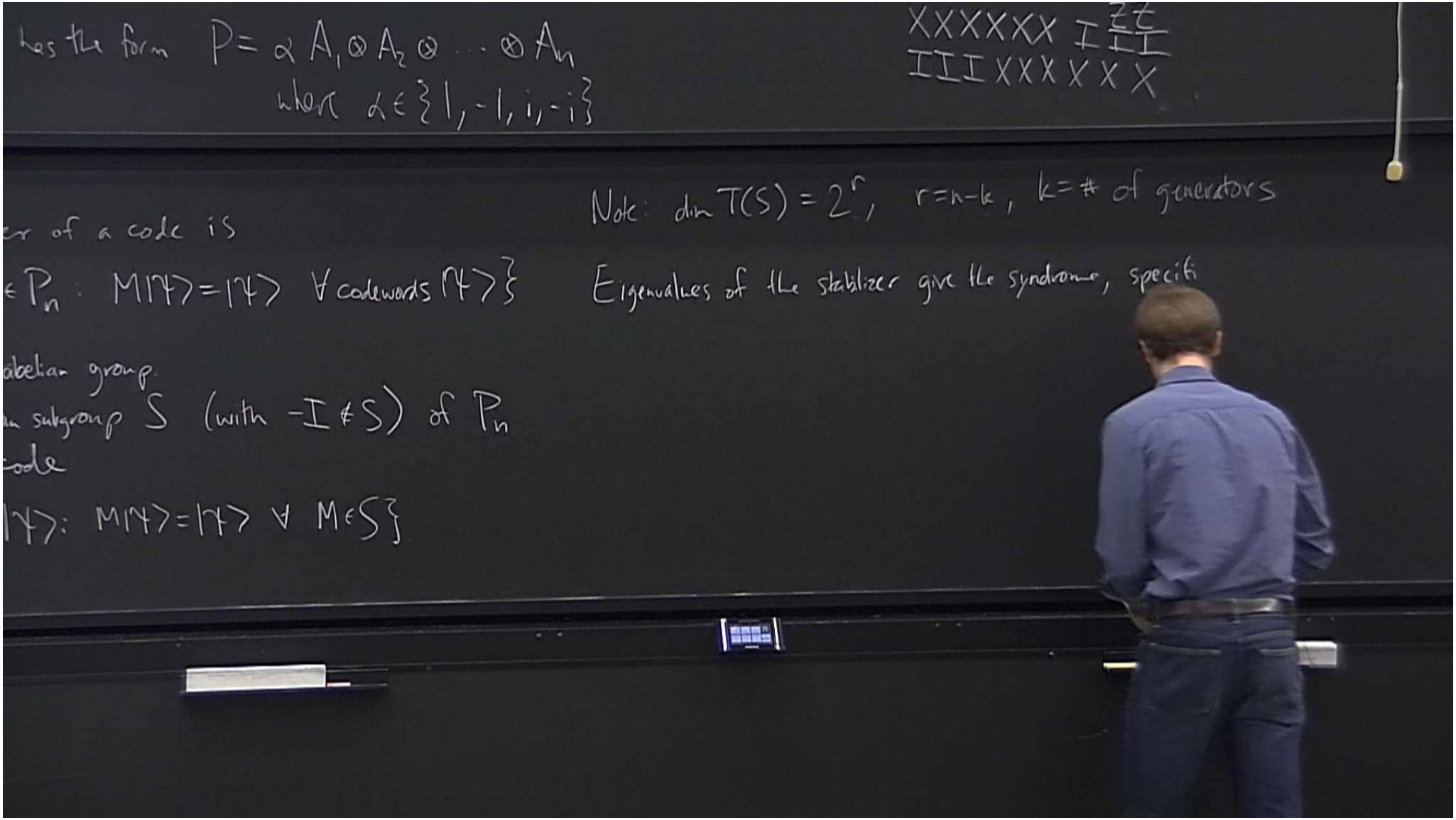
Def. The stabilizer of a code is

$$S = \{ M \in P_n : M|\psi\rangle = |\psi\rangle \ \forall \text{ codewords } |\psi\rangle \}$$

This is an abelian group.

In fact any abelian subgroup S (with $-I \notin S$) of P_n specifies a code

$$T(S) = \{ |\psi\rangle : M|\psi\rangle = |\psi\rangle \ \forall \ M \in S \}$$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XXXXXX III
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$\in P_n$: $M|\psi\rangle = |\psi\rangle$ $\forall$ codewords $|\psi\rangle\}$
Note: $\dim T(S) = 2^r$, $r = n-k$, $k = \#$ of generators
Eigenvalues of the stabilizer give the syndrome, specifi
abelian group
n subgroup S (with $-I \notin S$) of P_n
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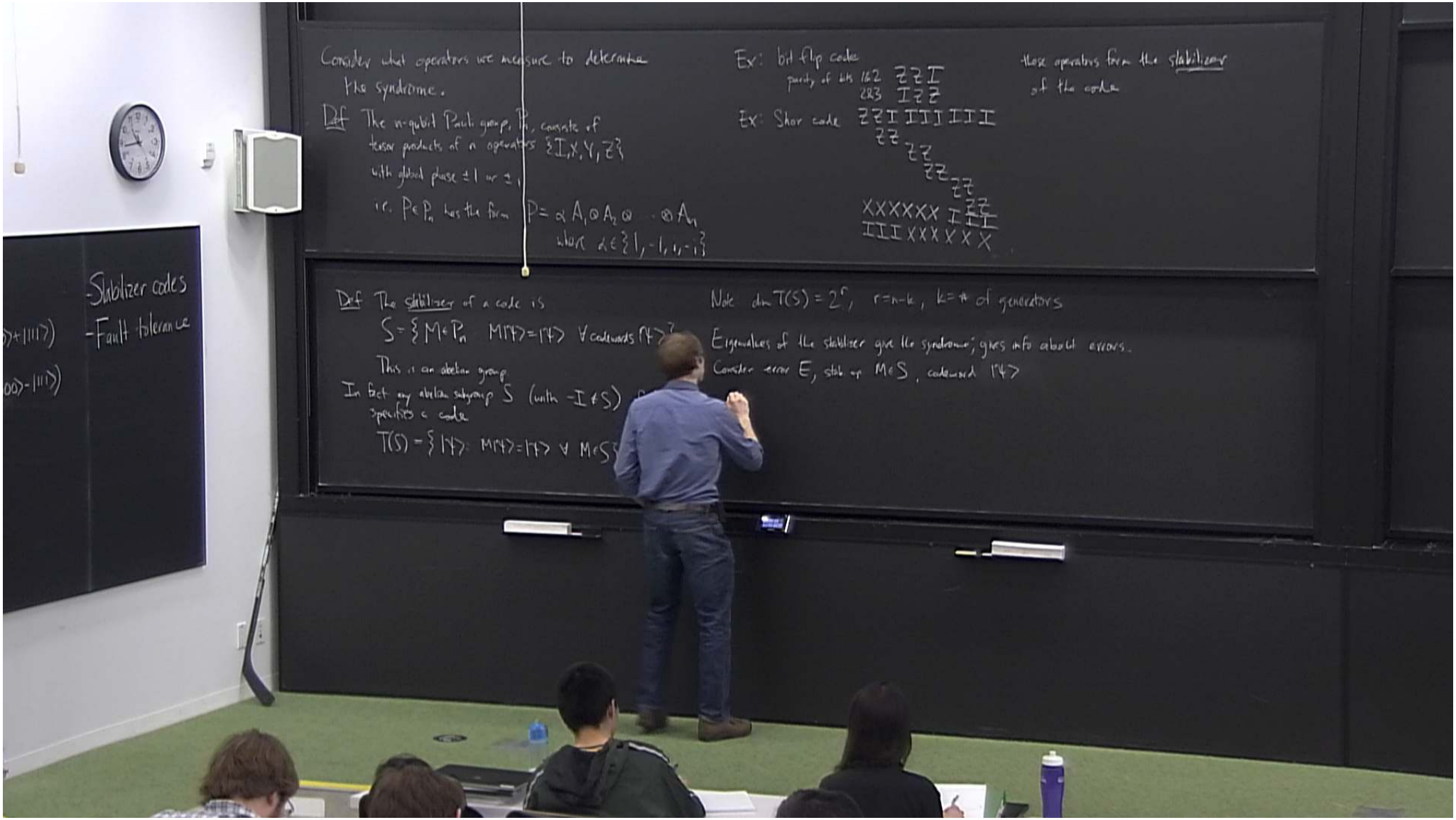
Eigenvalues of the stabilizer give the syndrome; gives info about errors.


Stabilizer codes
- Fault tolerance
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these operators form the stabilizer of the code
Def The stabilizer of a code is
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This is an abelian group.
In fact any abelian subgroup S (with $-I \notin S$) specifies a code
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Eigenvalues of the stabilizer give the syndrome; gives info about errors.
Consider error E, stab op $M \in S$, codeword $|\psi\rangle$

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$XXXXXX\ III$
$III\ XXX\ X\ X\ X$

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de

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Eigenvalues of the stabilizer give the syndrome; gives info about errors.
Consider error E, stab. or $M \in S$, codeword $|\psi\rangle$
- If $[E, M] = 0$ $M(E|\psi\rangle) = EM|\psi\rangle$

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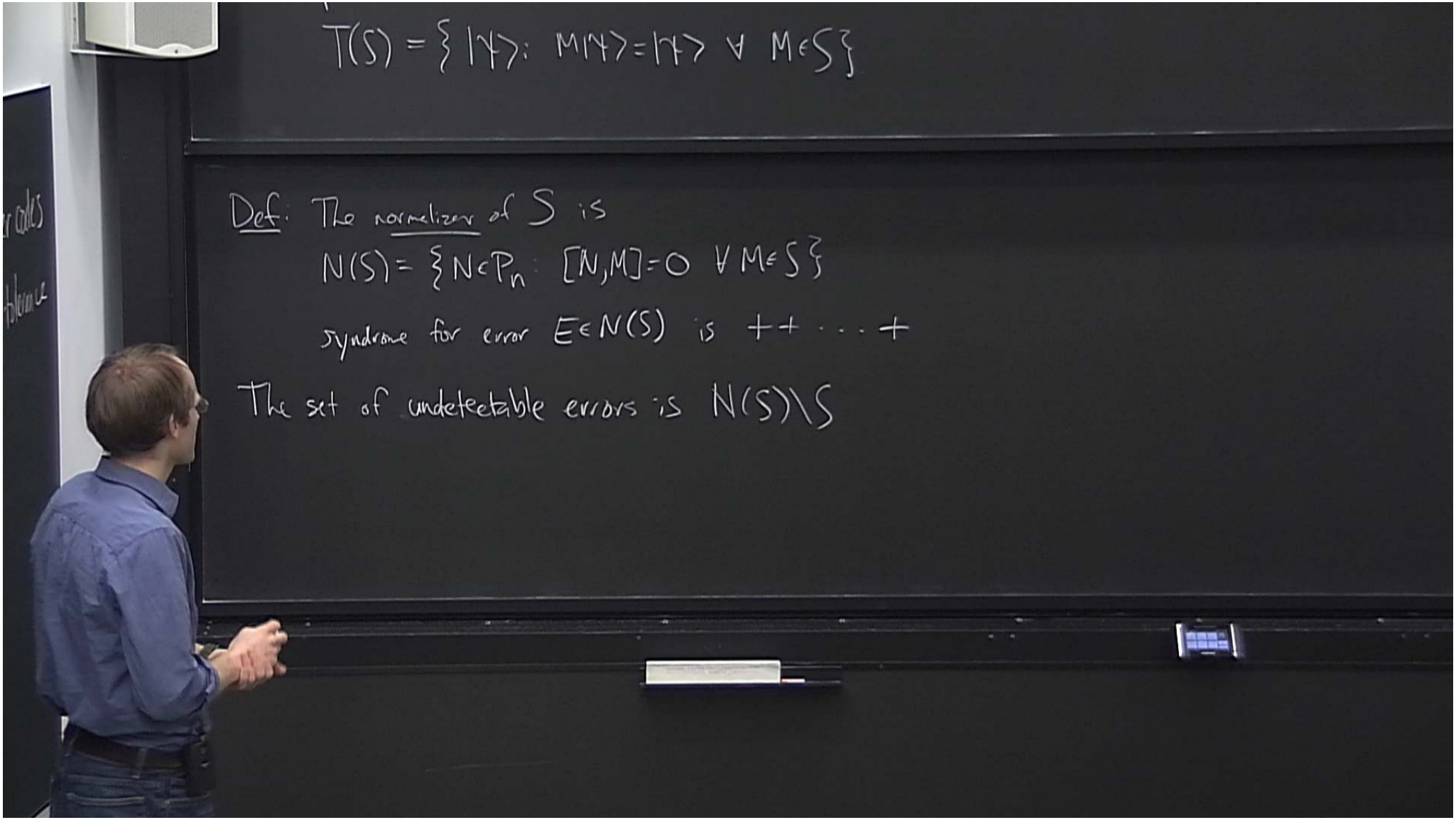

T(S) = { |ψ⟩ : M|ψ⟩ = |ψ⟩ ∀ M∈S }
Def: The normalizer of S is
N(S) = { N∈P_n : [N,M] = 0 ∀ M∈S }
syndrome for error E∈N(S) is ++ ... +
The set of undetectable errors is N(S)\S

$T(S) = \{ |\psi\rangle : M|\psi\rangle = |\psi\rangle \ \forall \ M \in S \}$

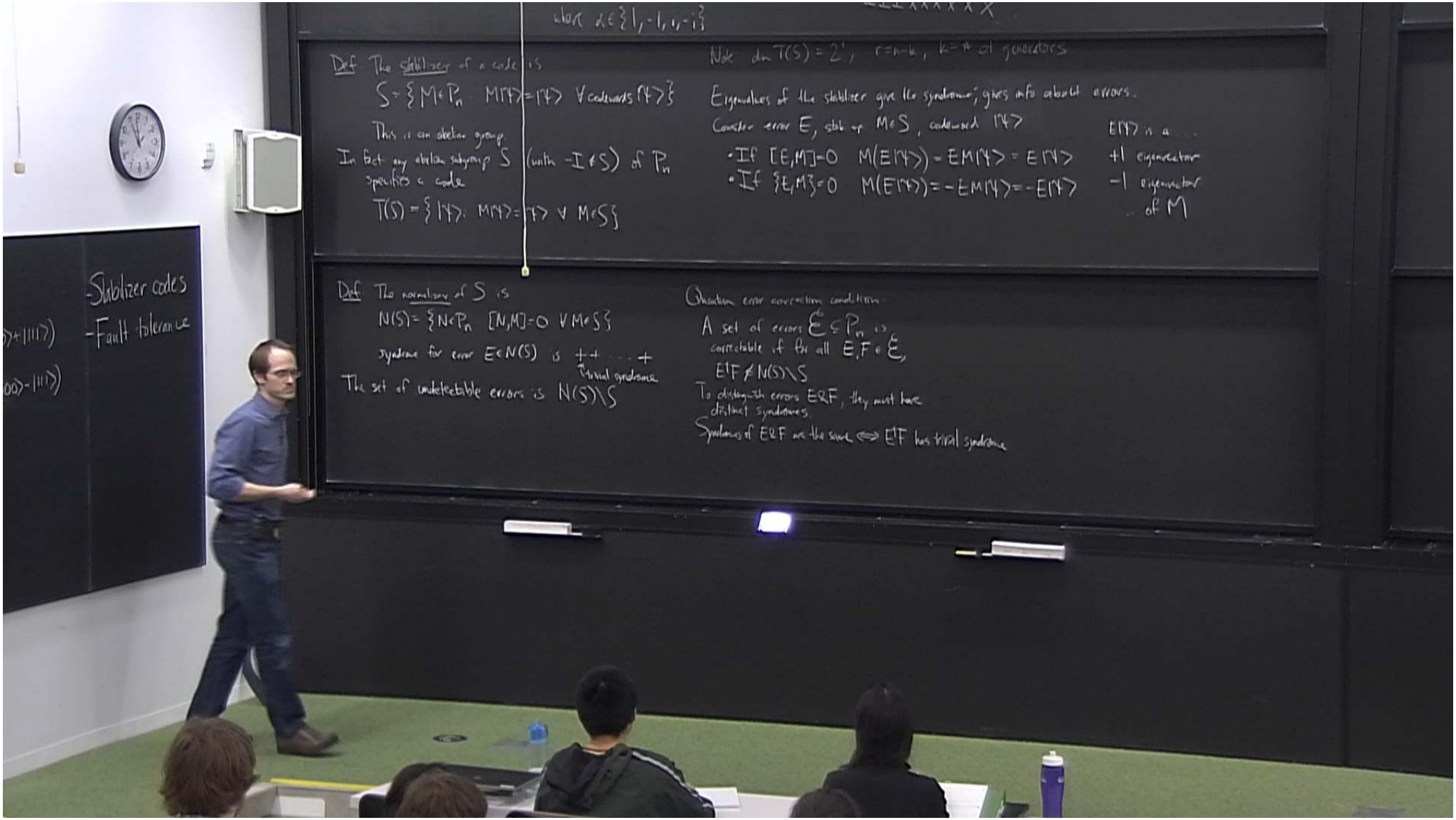
Def: The normalizer of S is

$N(S) = \{ N \in P_n : [N, M] = 0 \ \forall M \in S \}$

syndrome for error $E \in N(S)$ is $+ + \cdots +$

The set of undetectable errors is $N(S) \setminus S$

Quantum error correction condition: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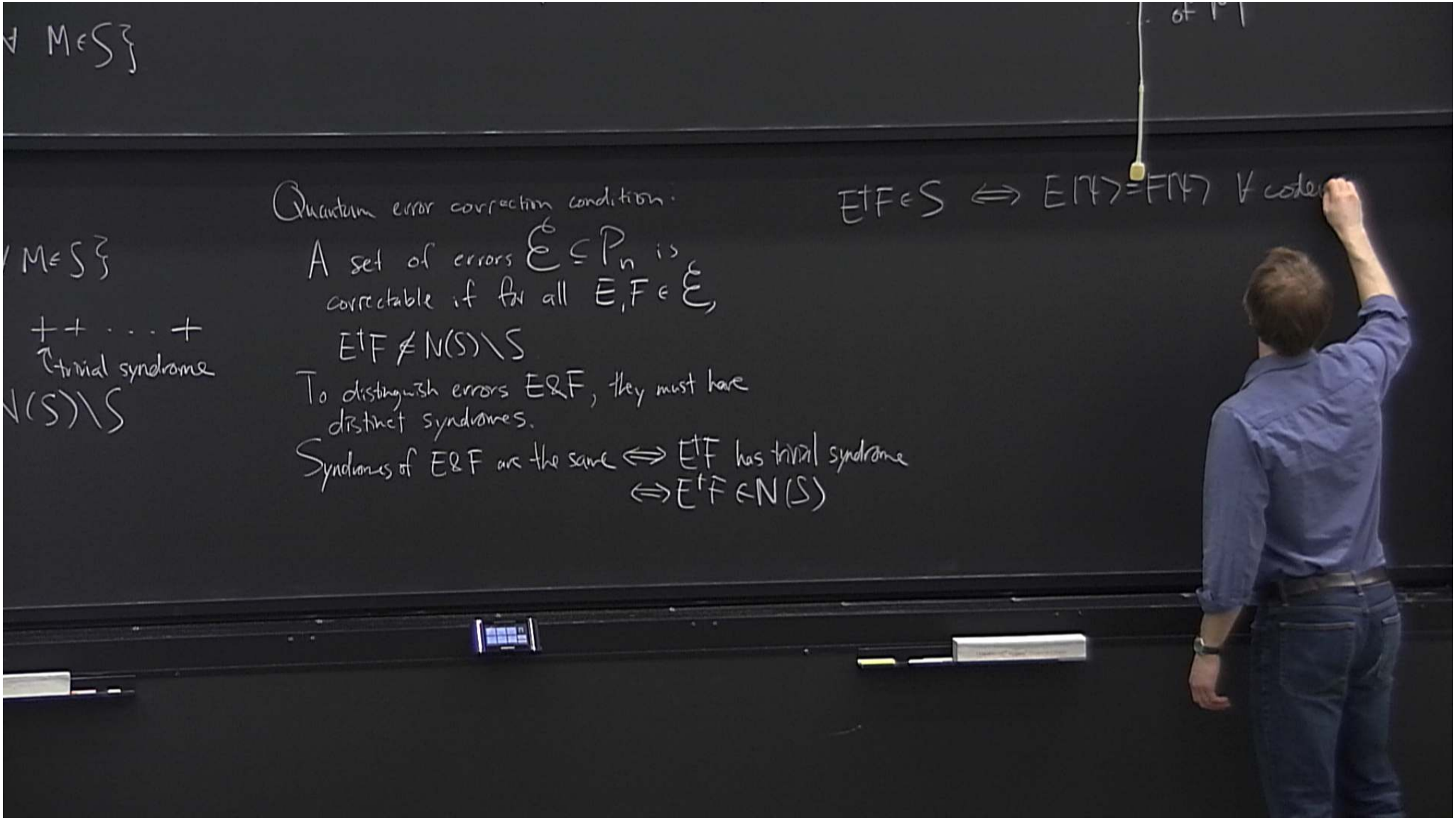
Note dim T(S) = 2^r, r = n-k, k = # of generators
Eigenvalues of the stabilizer give the syndrome; gives info about errors.
Consider error E, stab op M ∈ S, codeword |ψ⟩
If [E,M]=0 M(E|ψ⟩) = EM|ψ⟩ = E|ψ⟩
If {E,M}=0 M(E|ψ⟩) = -EM|ψ⟩ = -E|ψ⟩
E|ψ⟩ is a +1 eigenvector / -1 eigenvector of M
Def The normalizer of S is
N(S) = {N ∈ P_n : [N,M]=0 ∀ M ∈ S}
syndrome for error E ∈ N(S) is ++...+ (trivial syndrome)
The set of undetectable errors is N(S)\S
Quantum error correction condition:
A set of errors 𝓔 ⊆ P_n is correctable if for all E,F ∈ 𝓔,
E†F ∉ N(S)\S
To distinguish errors E ≠ F, they must have distinct syndromes.
Syndromes of E & F are the same ⟺ E†F has trivial syndrome


∀ M∈S}
Quantum error correction condition:
A set of errors $\mathcal{E} \subseteq P_n$ is correctable if ∀ all $E, F \in \mathcal{E}$,
$E^\dagger F \notin N(S) \setminus S$
To distinguish errors E&F, they must have distinct syndromes.
Syndromes of E&F are the same ⟺ $E^\dagger F$ has trivial syndrome
⟺ $E^\dagger F \in N(S)$
$E^\dagger F \in S \iff E|\psi\rangle = F|\psi\rangle$ ∀ code
M∈S}
+ + . . . +
trivial syndrome
N(S)\S

$\forall M \in S\}$

$\forall M \in S\}$

$+ + \cdots +$
↑ trivial syndrome

$N(S) \setminus S$

Quantum error correction condition:

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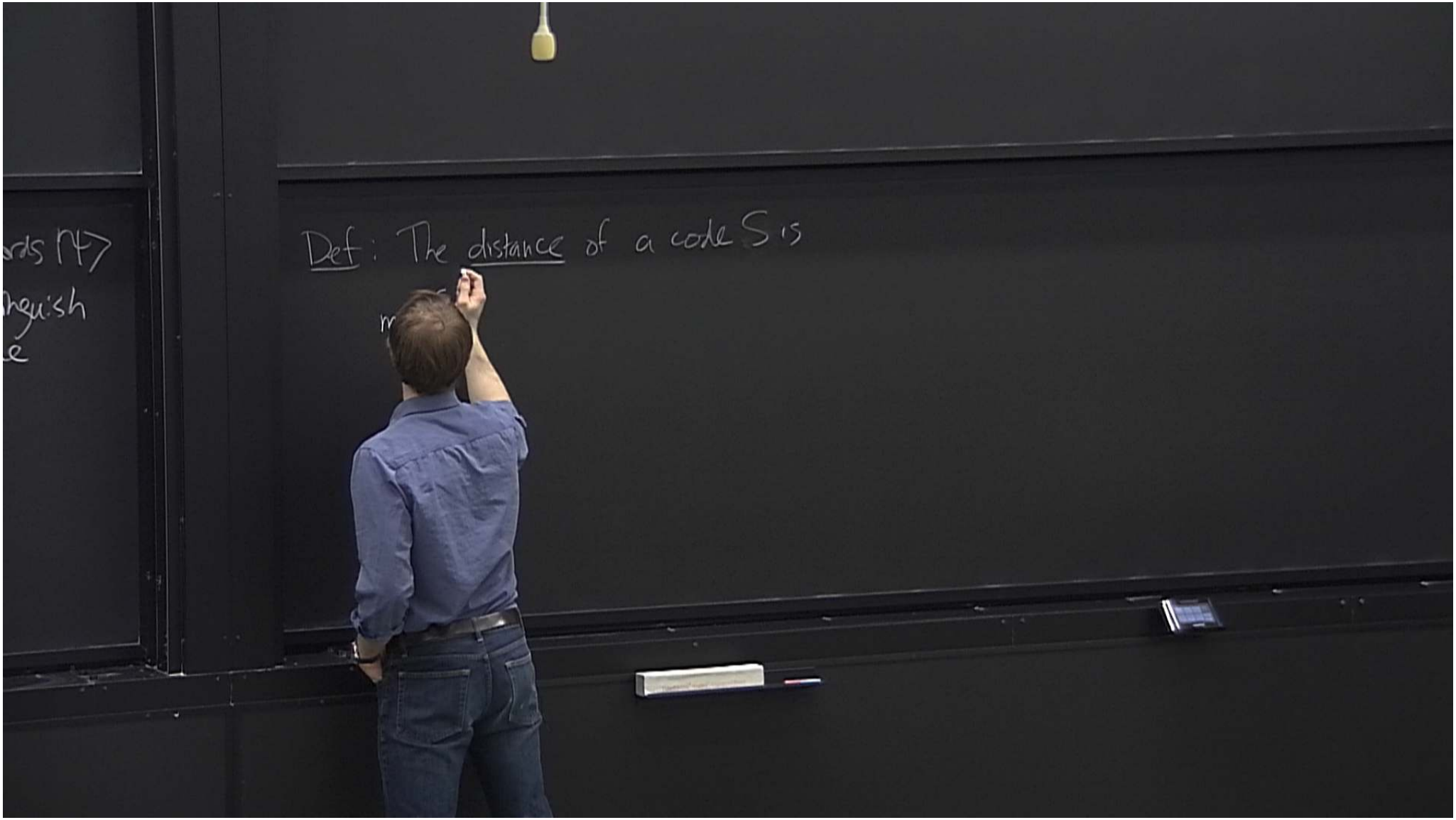
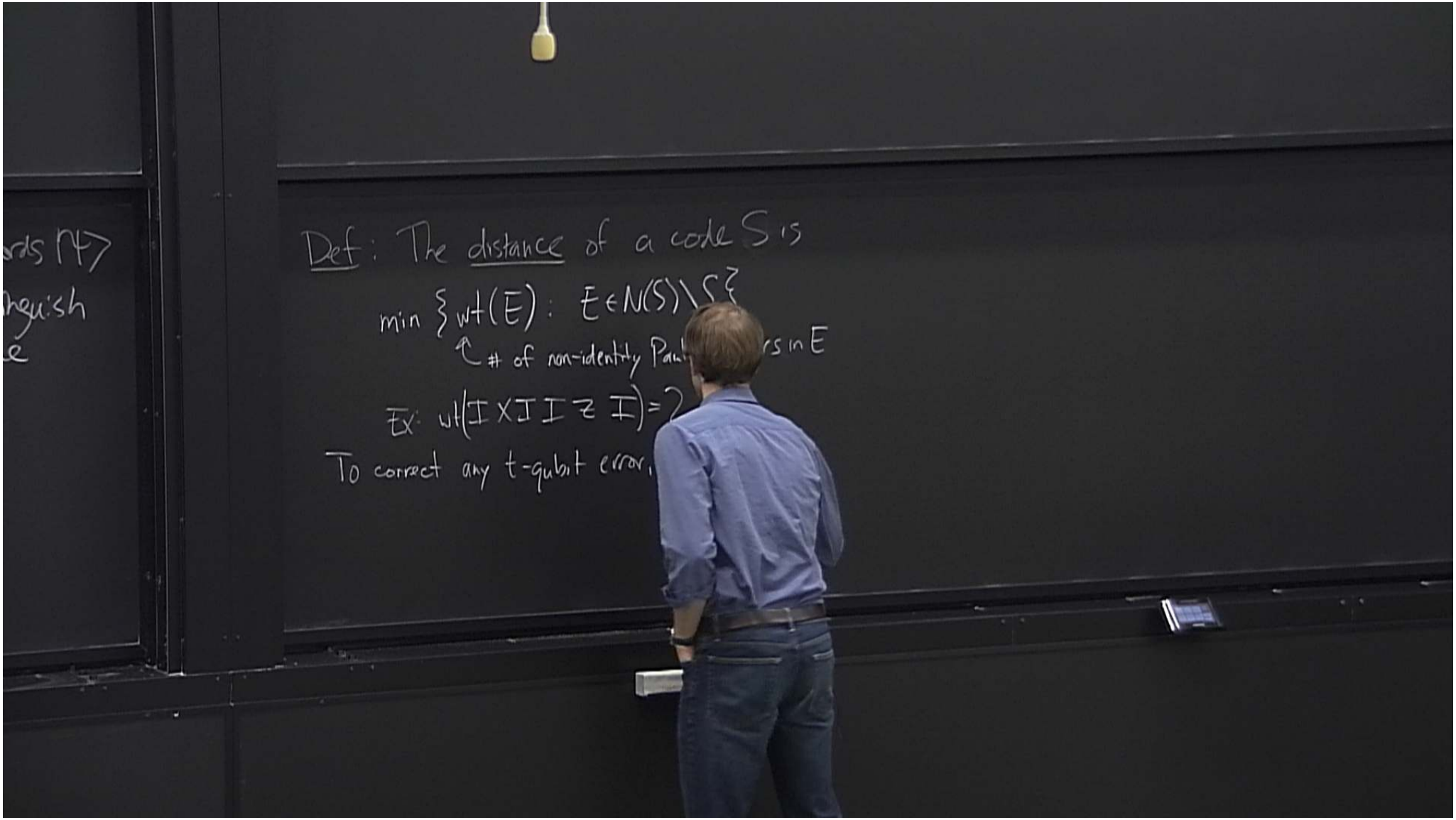
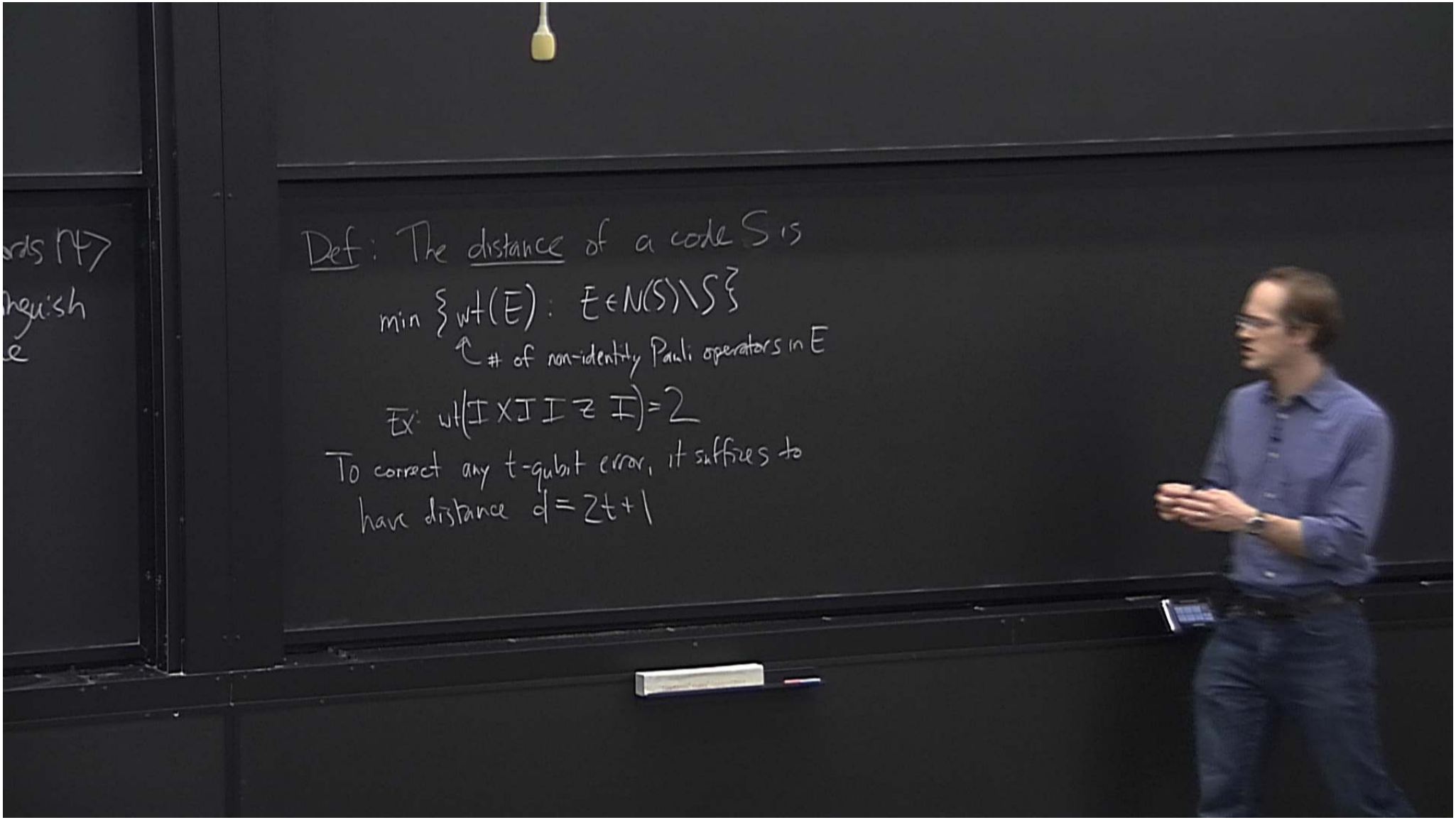
To distinguish errors E & F, they must have distinct syndromes.

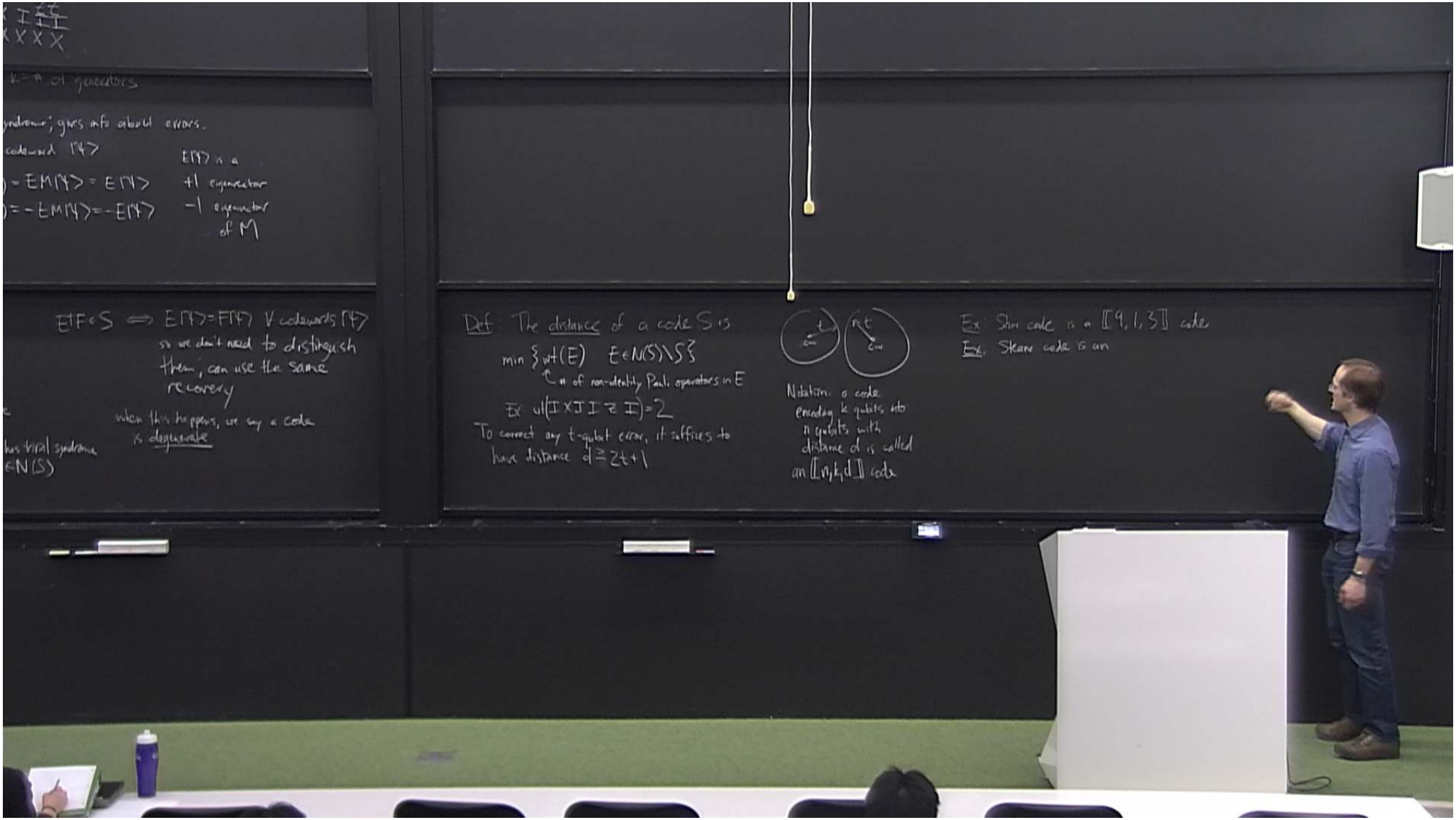
Syndromes of E & F are the same $\Leftrightarrow E^\dagger F$ has trivial syndrome
$\Leftrightarrow E^\dagger F \in N(S)$

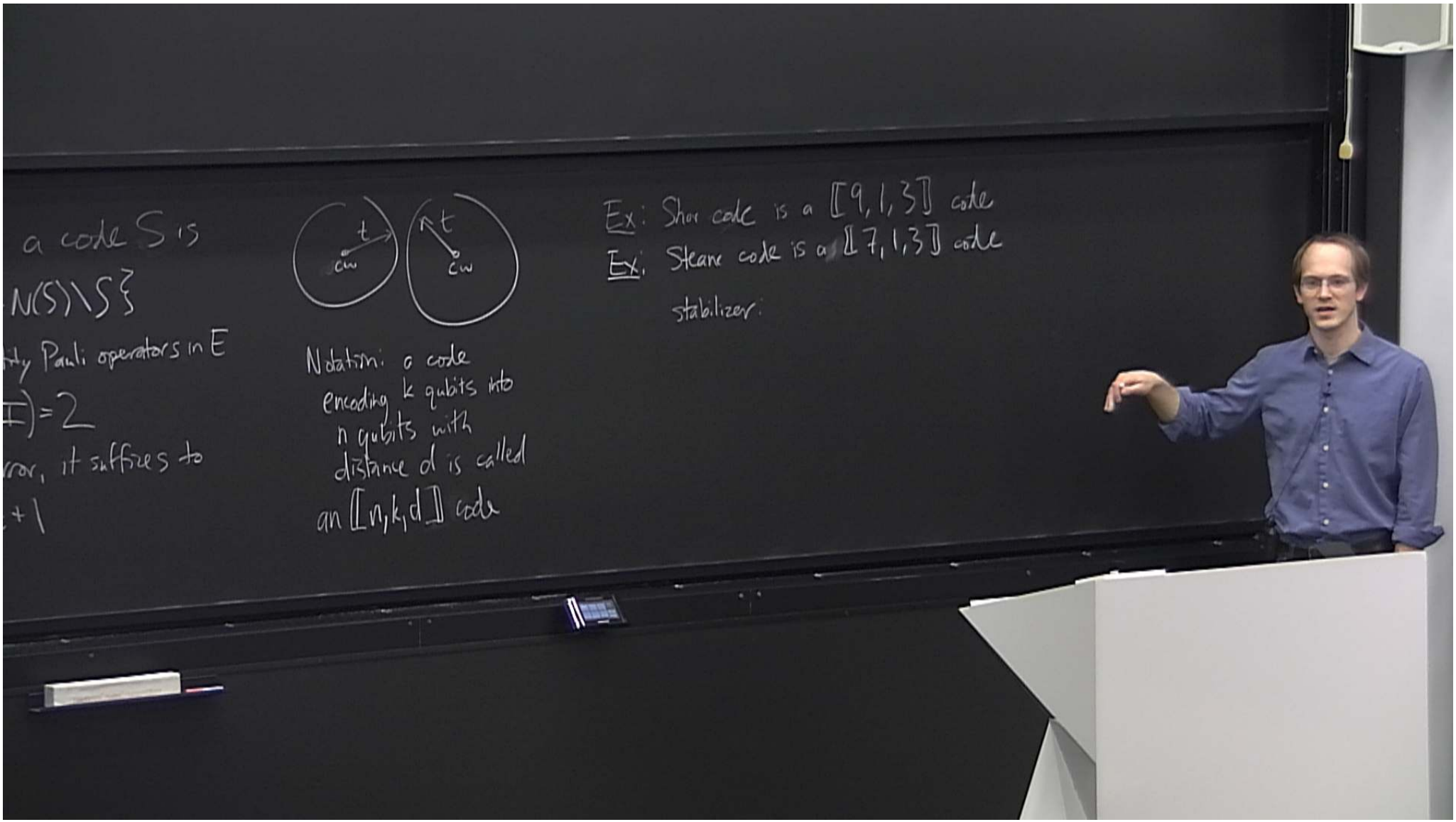
$E^\dagger F \in S \Leftrightarrow E|\psi\rangle = F|\psi\rangle \ \forall$ codew


Def: The distance of a code S is


Def: The distance of a code S is
min { wt(E) : E ∈ N(S) \ S }
# of non-identity Pau rs in E
Ex: wt(I X I I Z I) = ?
To correct any t-qubit error,


Def: The distance of a code S is
min { wt(E) : E ∈ N(S) \ S }
# of non-identity Pauli operators in E
Ex: wt(I X I I Z I) = 2
To correct any t-qubit error, it suffices to
have distance d = 2t + 1


k = # of generators
syndrome; gives info about errors.
codeword |ψ⟩
E|ψ⟩ is a
+1 eigenvector
−1 eigenvector of M
E|ψ⟩ = E|ψ⟩
= −E|ψ⟩ = −E|ψ⟩
E|ψ⟩=F|ψ⟩ ∀ codewords |ψ⟩
so we don't need to distinguish them, can use the same recovery
when this happens, we say a code is degenerate
has trivial syndrome
∈ N(S)
Def: The distance of a code S is
min { wt(E) E∈N(S)\S }
↳ # of non-identity Pauli operators in E
Ex wt(I X I Z I) = 2
To correct any t-qubit error, it suffices to have distance d ≥ 2t+1
t
cw
t
cw
Notation: a code encoding k qubits into n qubits with distance d is called an [[n,k,d]] code
Ex Shor code is a [[9,1,3]] code
Ex Steane code is an


a code S is
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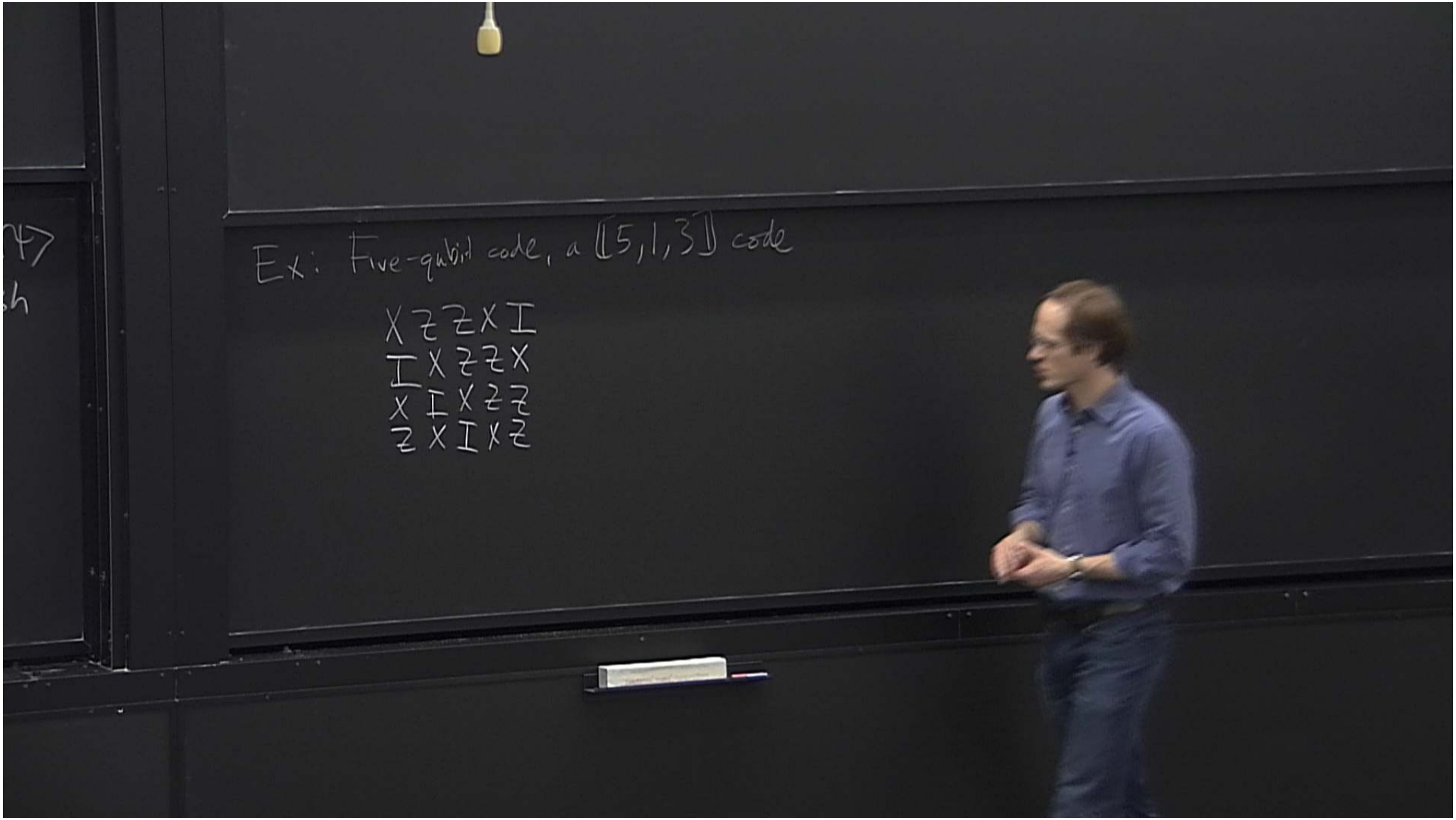
Pauli operators in E
I)=2
rror, it suffices to
+1
t
cw
t
cw
Notation: a code
encoding k qubits into
n qubits with
distance d is called
an [[n,k,d]] code
Ex: Shor code is a [[9,1,3]] code
Ex: Steane code is a [[7,1,3]] code
stabilizer:


Ex: Five-qubit code, a [[5,1,3]] code
X Z Z X I
I X Z Z X
X I X Z Z
Z X I X Z