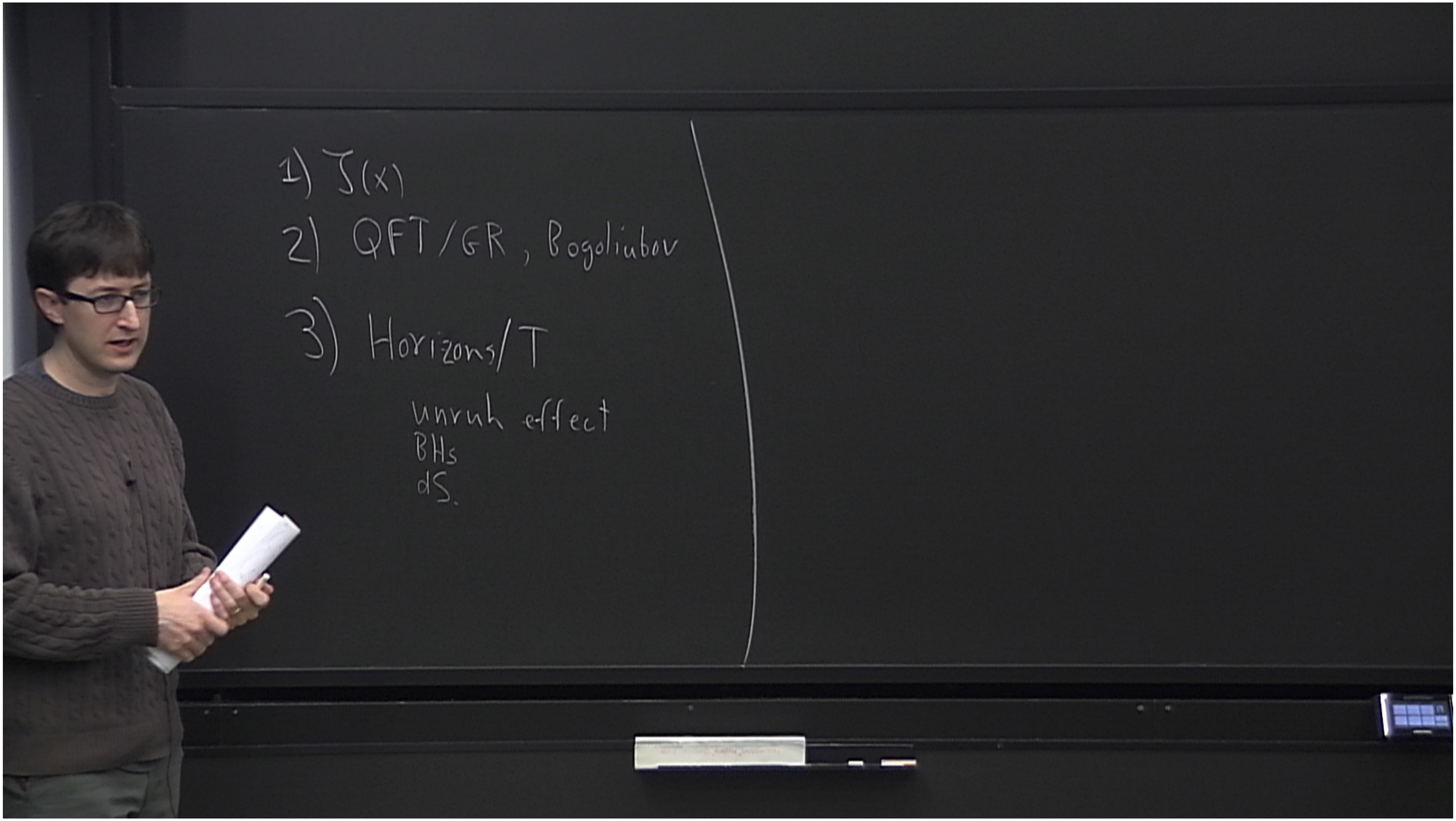
Title: 12/13 PSI - Cosmology Review Lecture 14

Date: Mar 08, 2013 11:30 AM

URL: http://pirsa.org/13030045

Abstract:


1) J(x)
2) QFT/GR, Bogoliubov
3) Horizons/T
unruh effect
BHs
dS.

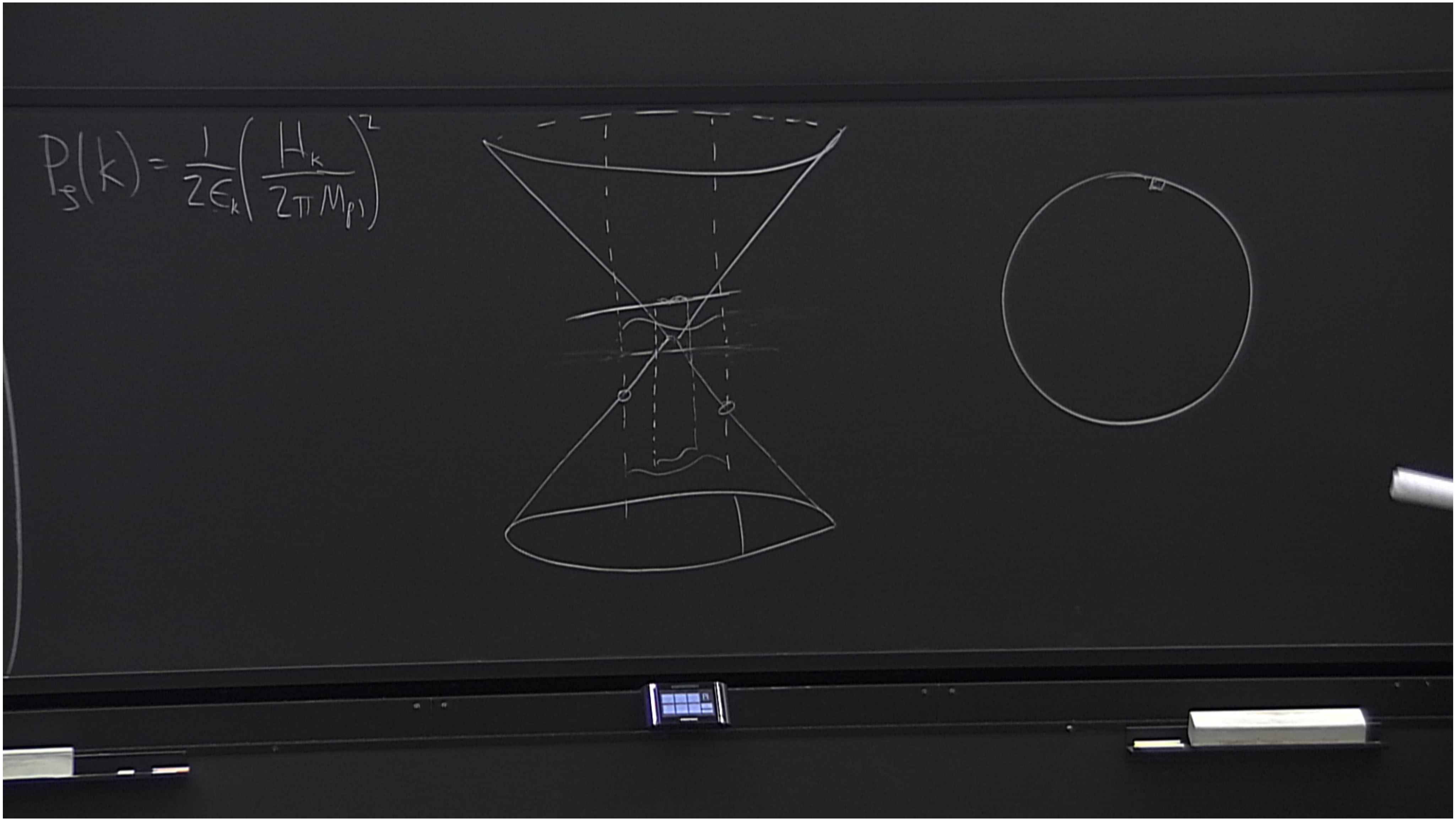
1) $\zeta(x)$

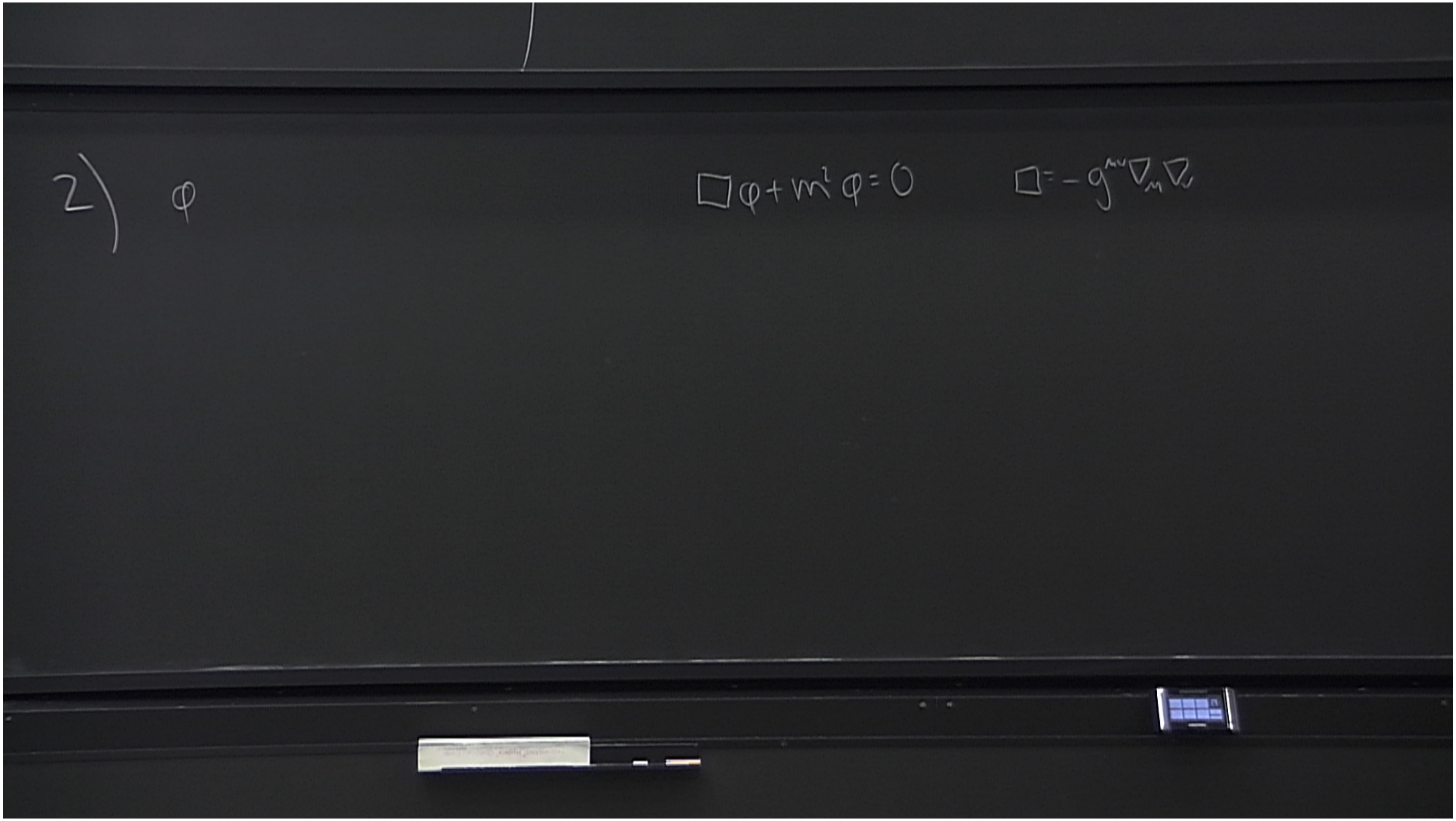
2) QFT / GR , Bogoliubov

3) Horizons / T

unruh effect
BHs
dS.

$$P_\zeta(k) = \frac{1}{2\epsilon}\left(\frac{H}{2\pi M_{pl}}\right)^2$$


$P_\zeta(k) = \frac{1}{2\epsilon_k}\left(\frac{H_k}{2\pi M_{pl}}\right)^2$


2) φ
$\Box\varphi + m^2\varphi = 0$
$\Box = -g^{\mu\nu}\nabla_\mu\nabla_\nu$

$$\Box\varphi + m^2\varphi = 0 \qquad \Box = -g^{\mu\nu}\nabla_\mu\nabla_\nu$$

$$\frac{\partial^2\varphi_{\vec{k}}}{\partial t^2} + \vec{k}^2\varphi_{\vec{k}} + m^2\varphi_{\vec{k}} = 0$$

$$\varphi_{i} = \varphi_{\vec{k}_i} e^{i\vec{k}\cdot\vec{x}} \qquad \varphi_{k_i} = e^{-i\omega_i t} \qquad \omega_i = \sqrt{m^2 + \vec{k}_i^2}$$

$$\varphi$$

$$\Box \varphi + m^2 \varphi = 0 \qquad \Box = -g^{\mu\nu} \nabla_\mu \nabla_\nu$$

$$\frac{\partial^2 \varphi_{\vec{k}}}{\partial t^2} + \vec{k}^2 \varphi_{\vec{k}} + m^2 \varphi_{\vec{k}} = 0$$

$$\varphi_i = \varphi_{\vec{k}_i} e^{i \vec{k} \vec{x}} \qquad \varphi_{k_i} = e^{-i\omega_i t} \qquad \omega_i = \sqrt{m^2 + \vec{k}_i^2} \qquad \varphi_i = e^{i(\vec{k}_i \vec{x} - \omega_i t)}$$

$$\varphi = \sum_i \left[\quad \varphi_i \right.$$

$$\varphi$$

$$\sqrt{m^2 + \vec{k}_i^2}$$

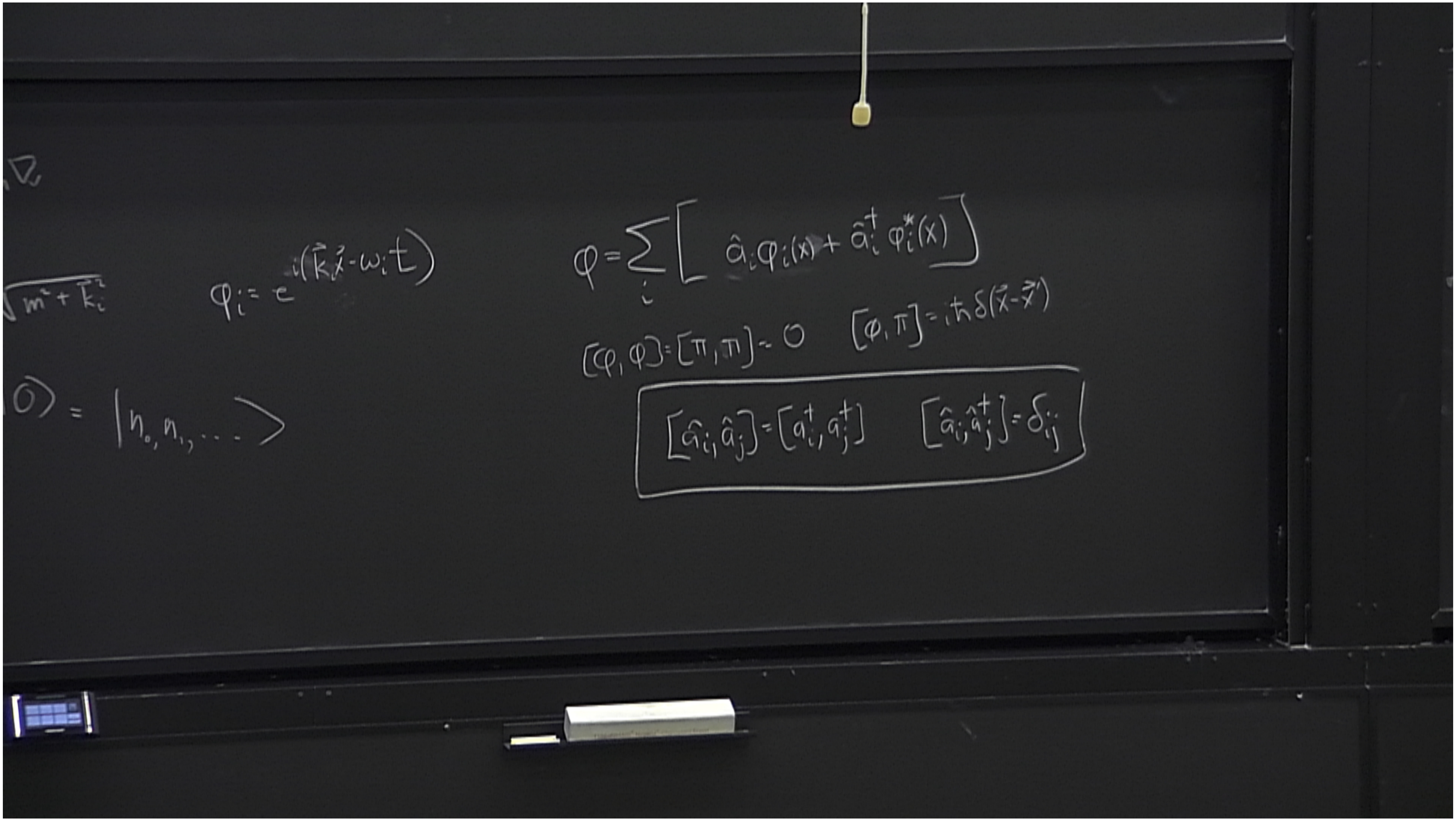
$$\varphi_i = e^{i(\vec{k}_i \vec{x} - \omega_i t)}$$

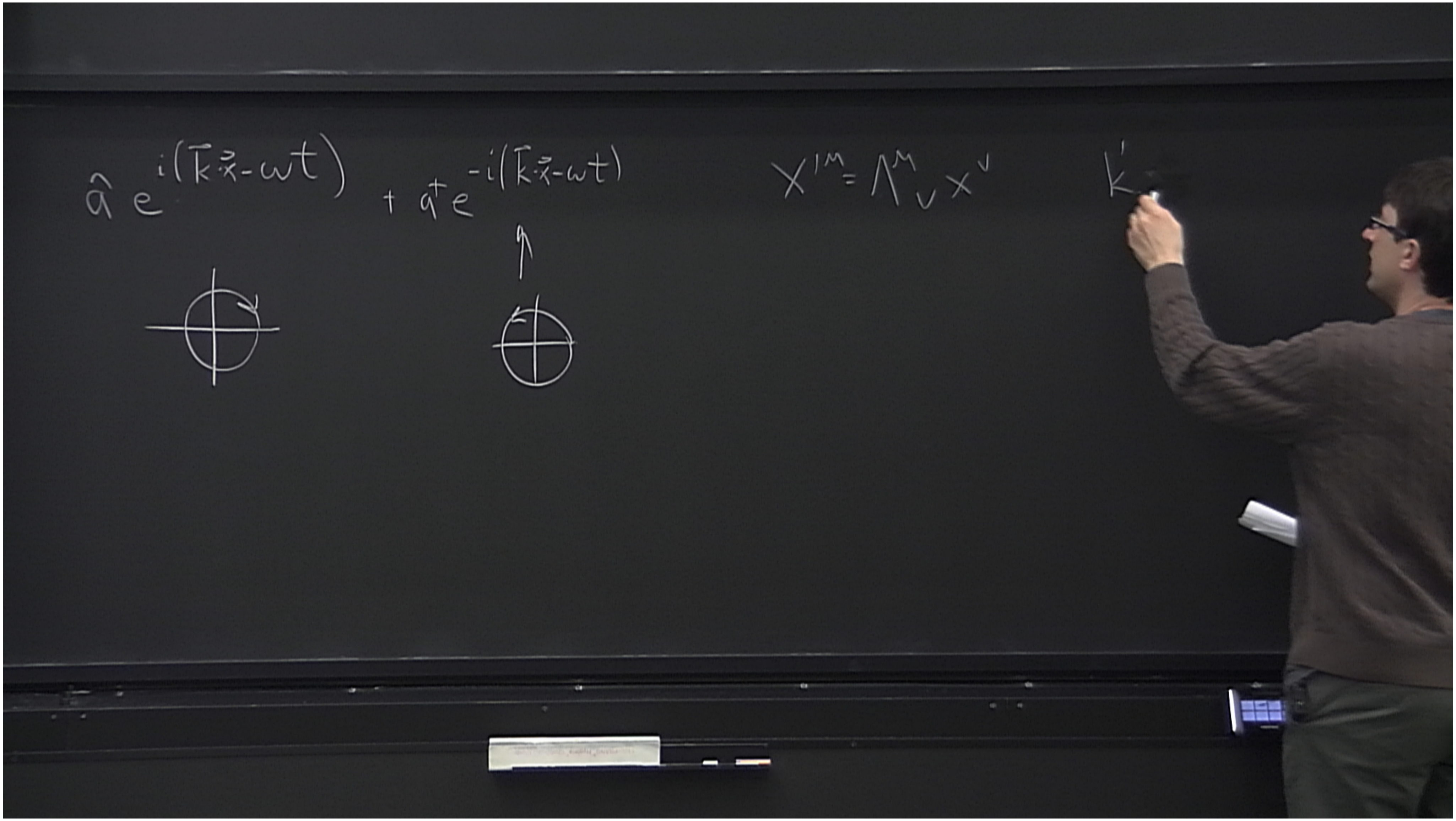
$$\varphi = \sum_i \left[\hat{a}_i \varphi_i(x) + \hat{a}_i^\dagger \varphi_i^*(x) \right]$$

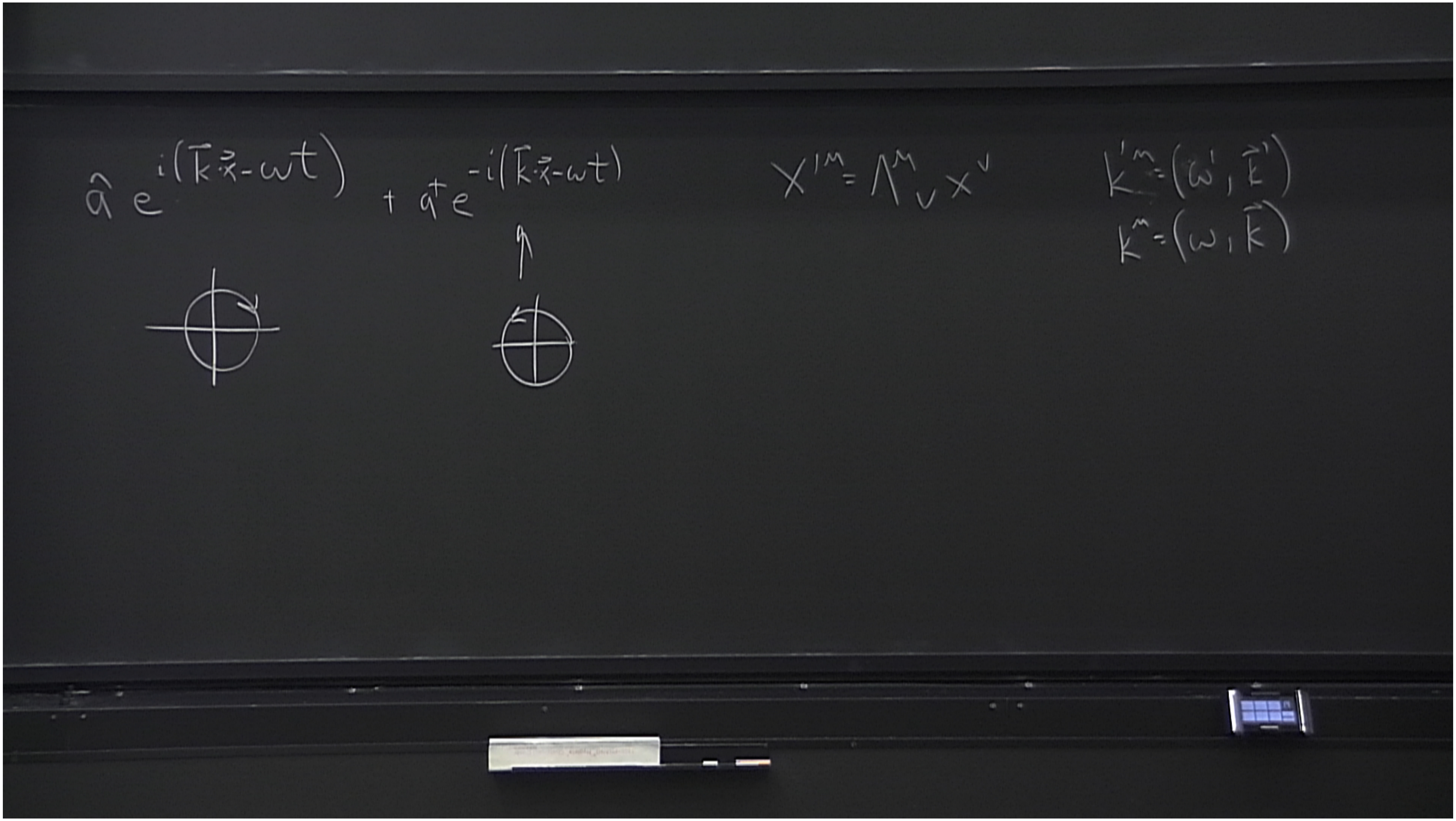
$$[\varphi, \varphi] = [\pi, \pi] = 0 \qquad [\varphi, \pi] = i\hbar\,\delta(\vec{x} - \vec{x}')$$

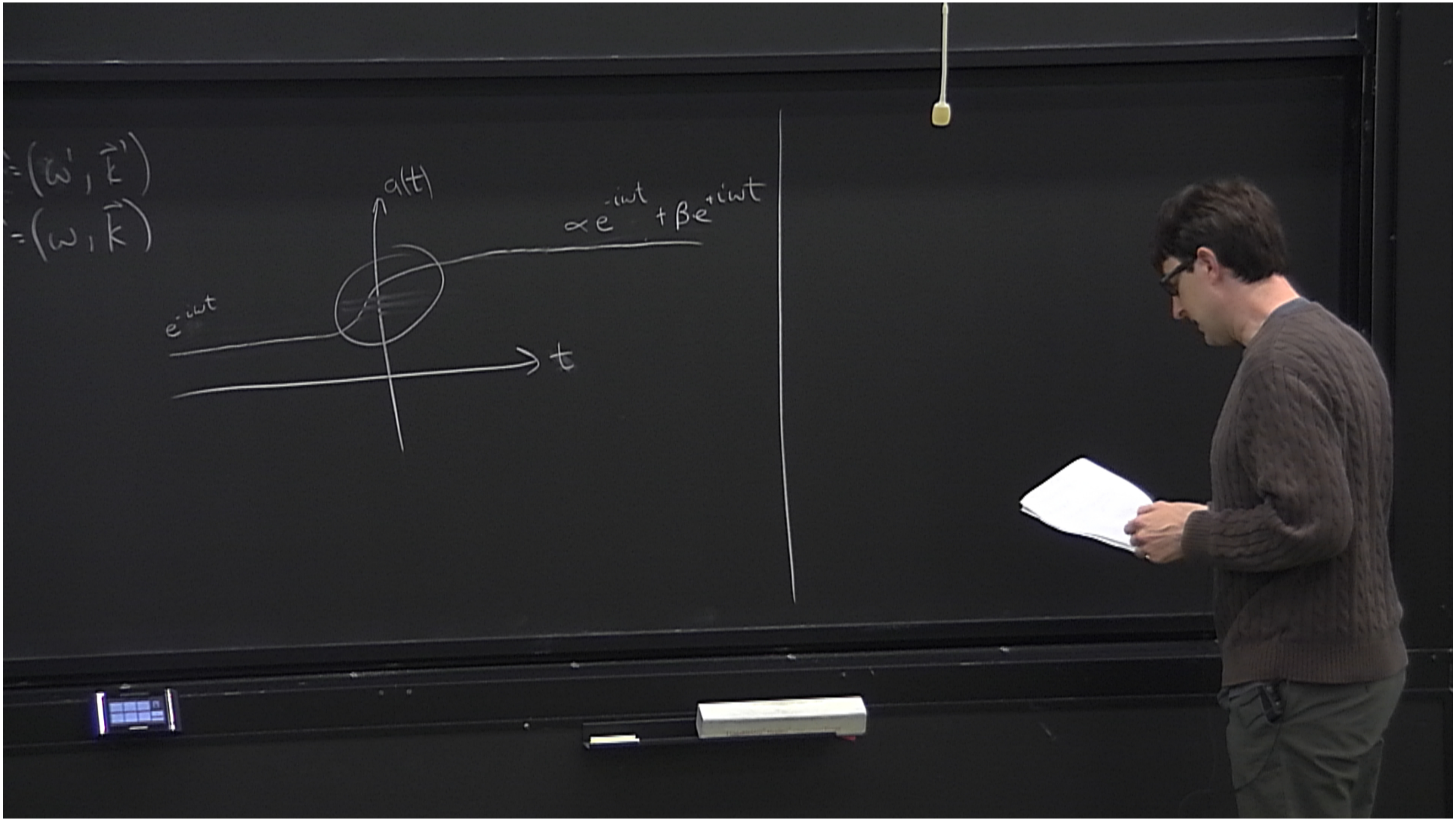
$$\boxed{[\hat{a}_i, \hat{a}_j] = [a_i^\dagger, a_j^\dagger] \qquad [\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}}$$

$$|0\rangle = |n_0, n_1, \ldots\rangle$$

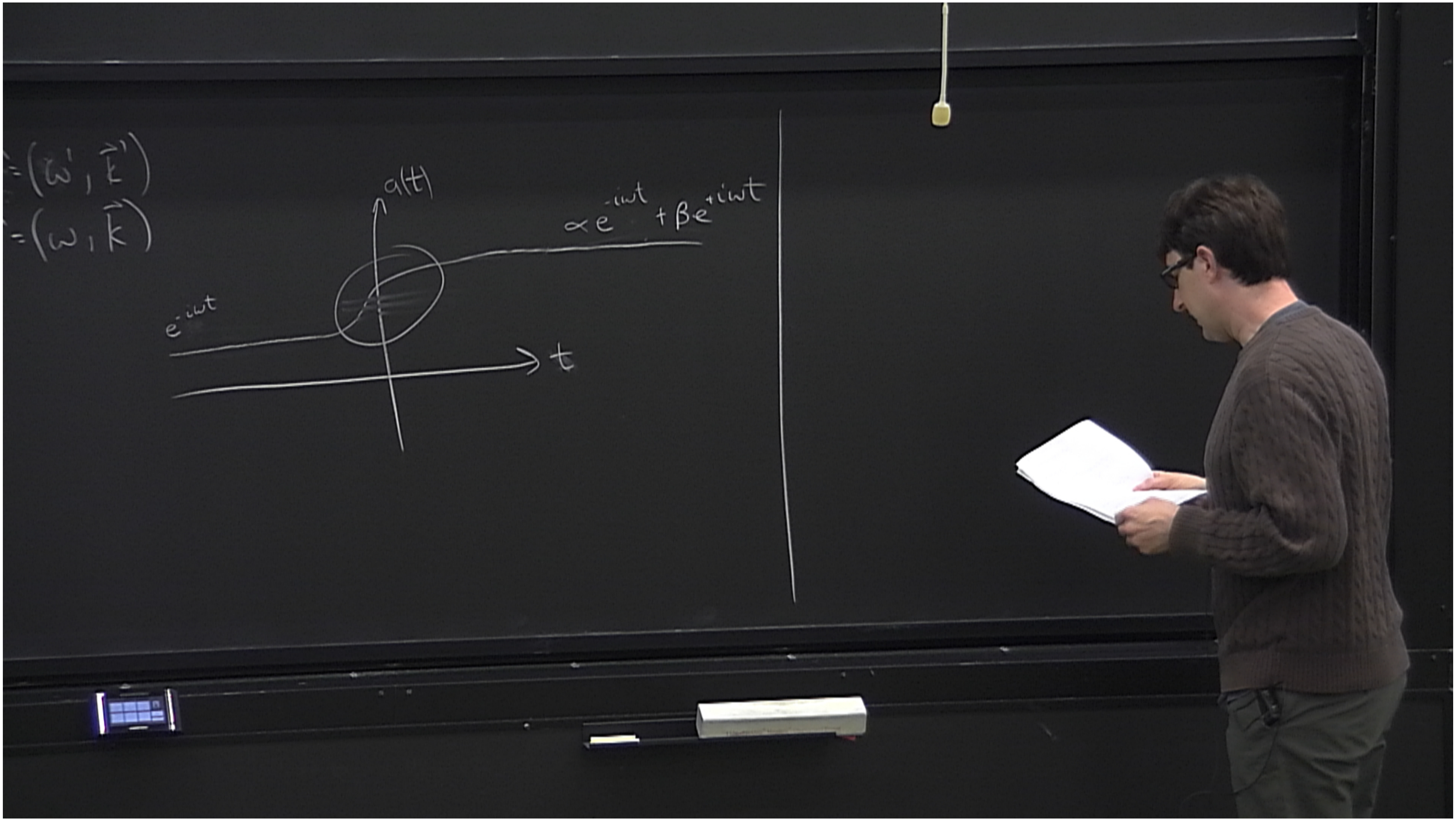

$\sqrt{m^2+\vec{k}_i^2}$
$\varphi_i = e^{i(\vec{k}_i\vec{x}-\omega_i t)}$
$\varphi = \sum_i \left[\hat{a}_i \varphi_i(x) + \hat{a}_i^\dagger \varphi_i^*(x) \right]$
$[\varphi,\varphi]=[\pi,\pi]=0$
$[\varphi,\pi]=i\hbar\delta(\vec{x}-\vec{x}')$
$[\hat{a}_i,\hat{a}_j]=[a_i^\dagger,a_j^\dagger]$
$[\hat{a}_i,\hat{a}_j^\dagger]=\delta_{ij}$
$0\rangle = |n_0,n_1,\ldots\rangle$


$\hat{a}\, e^{i(\vec{k}\cdot\vec{x}-\omega t)} + a^{+} e^{-i(\vec{k}\cdot\vec{x}-\omega t)}$
$x'^{\mu} = \Lambda^{\mu}{}_{\nu}\, x^{\nu}$
k'


$\hat{a}\, e^{i(\vec{k}\cdot\vec{x}-\omega t)} + a^{\dagger} e^{-i(\vec{k}\cdot\vec{x}-\omega t)}$
$x'^{\mu} = \Lambda^{\mu}{}_{\nu}\, x^{\nu}$
$k'^{\mu} = (\omega', \vec{k}')$
$k^{\mu} = (\omega, \vec{k})$



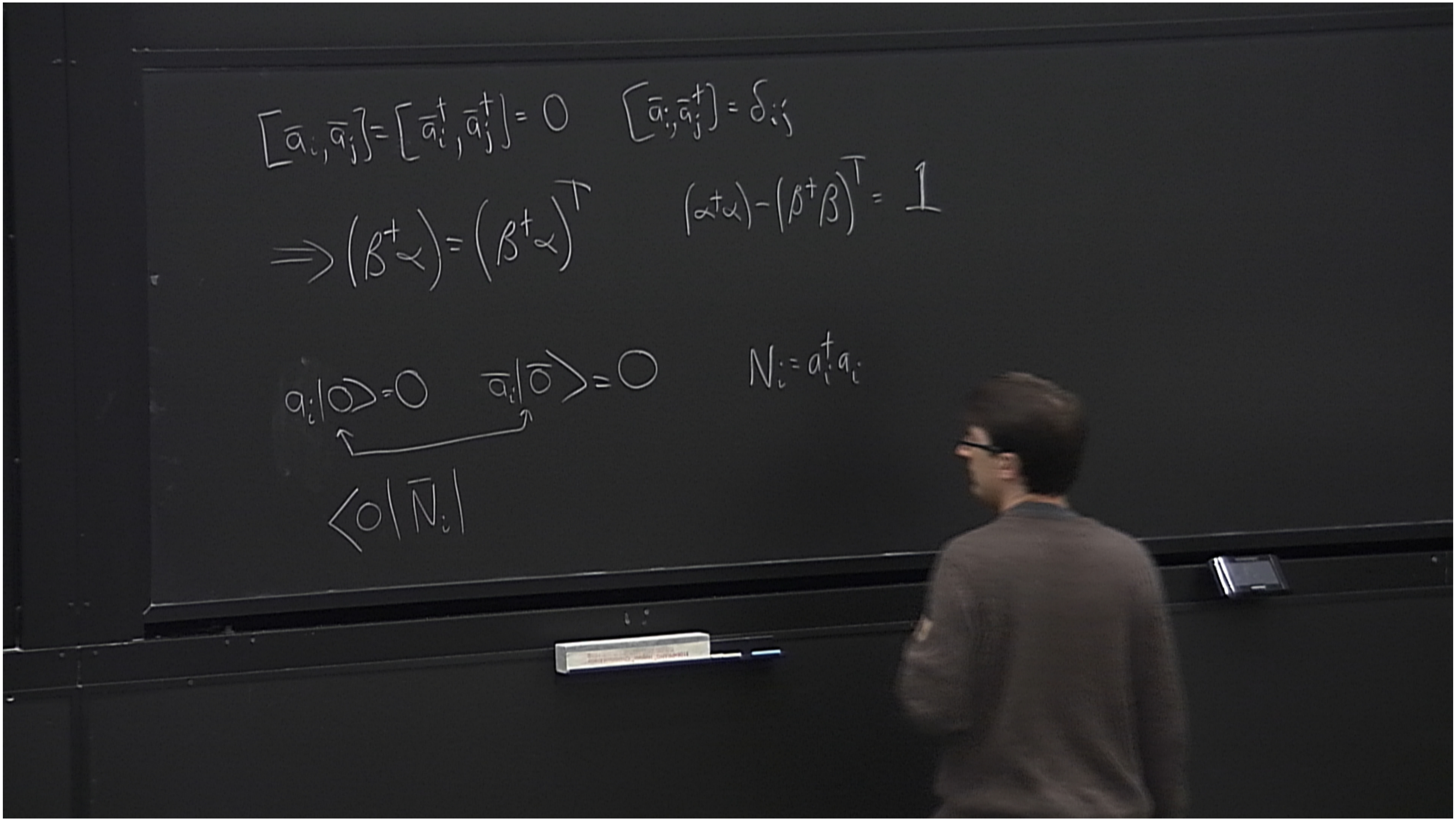
=(ω', k')
=(ω, k)
a(t)
αe^{-iωt} + βe^{+iωt}
e^{-iωt}
t



$= (\omega', \vec{k}')$
$= (\omega, \vec{k})$
$a(t)$
$\alpha e^{-i\omega t} + \beta e^{+i\omega t}$
$e^{-i\omega t}$
t

$$[\bar{a}_i, \bar{a}_j] = [\bar{a}_i^\dagger, \bar{a}_j^\dagger] = 0 \qquad [\bar{a}_i, \bar{a}_j^\dagger] = \delta_{ij}$$

$$\Rightarrow (\beta^\dagger \alpha) = (\beta^\dagger \alpha)^T \qquad (\alpha^\dagger \alpha) - (\beta^\dagger \beta)^T = \mathbb{1}$$


$[\bar{a}_i, \bar{a}_j] = [\bar{a}_i^\dagger, \bar{a}_j^\dagger] = 0$
$[\bar{a}_i, \bar{a}_j^\dagger] = \delta_{ij}$
$\Rightarrow (\beta^\dagger \alpha) = (\beta^\dagger \alpha)^T$
$(\alpha^\dagger \alpha) - (\beta^\dagger \beta)^T = \mathbb{1}$
$a_i|0\rangle = 0$
$\bar{a}_i|\bar{0}\rangle = 0$
$N_i = a_i^\dagger a_i$
$\langle 0|\bar{N}_i|$

$$[\bar{a}_i, \bar{a}_j] = [\bar{a}_i^\dagger, \bar{a}_j^\dagger] = 0 \qquad [\bar{a}_i, \bar{a}_j^\dagger] = \delta_{ij}$$

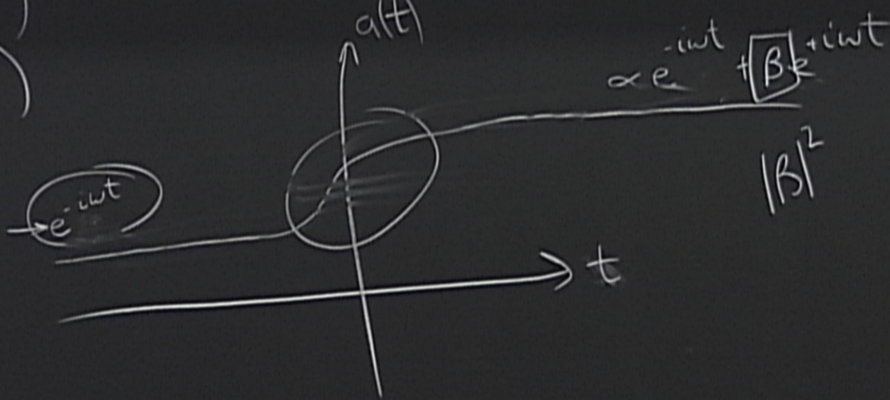
$$\Rightarrow (\beta^\dagger \alpha) = (\beta^\dagger \alpha)^T \qquad (\alpha^\dagger \alpha) - (\beta^\dagger \beta)^T = 1$$

$$\bar{N}_i = \bar{a}_i^\dagger \bar{a}_i$$

$$a_i|0\rangle = 0 \qquad \bar{a}_i|\bar{0}\rangle = 0 \qquad N_i = a_i^\dagger a_i \qquad \langle 0|N_i|0\rangle = 0$$

$$\langle 0|\bar{N}_i|0\rangle = (\beta^\dagger \beta)_{ii} \neq 0$$

$$k'^{\mu} = (\omega', \vec{k}')$$
$$k^{\mu} = (\omega, \vec{k})$$



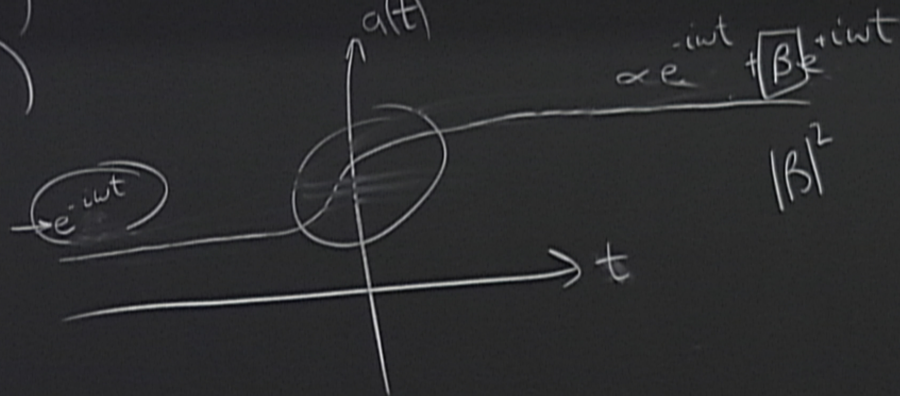
$$\varphi(x) = \sum_i \left[a_i u_i(x) + a_i^\dagger u_i^*(x) \right]$$
$$= \sum_i \left[\bar{a}_i \bar{u}_i(x) + \bar{a}_i^\dagger \bar{u}_i^*(x) \right]$$

$$u_i = \alpha_{ij} \bar{u}_j + \beta_{ij} \bar{u}_j^*$$

$$\sum_i \Big[\overbrace{\left(a_i \alpha_{ij} + a_i^\dagger \beta_{ij}^* \right)}^{\bar{a}_i} \bar{u}_j + \underbrace{\left(a_i \beta_{ij} + a_i^\dagger \alpha_{ij}^* \right)}_{\bar{a}_i^\dagger} u_j^* \Big]$$

$(\omega', \vec{k}')$

$(\omega, \vec{k})$

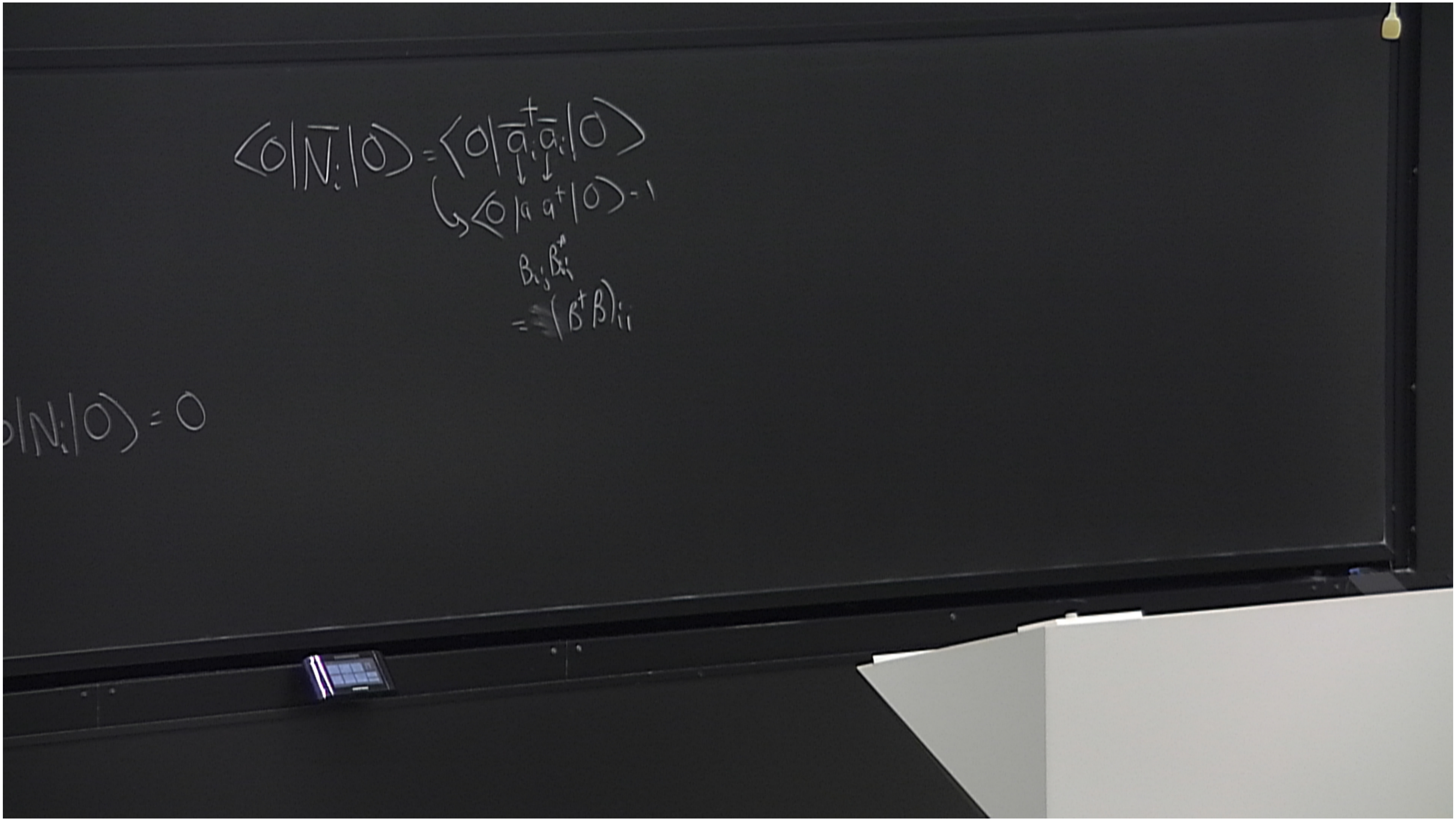


$$\varphi(x) = \sum_i \left[a_i u_i(x) + a_i^\dagger u_i^*(x) \right]$$

$$= \sum_i \left[\bar{a}_i \bar{u}_i(x) + \bar{a}_i^\dagger \bar{u}_i^*(x) \right]$$

$$u_i = \alpha_{ij} \bar{u}_j + \beta_{ij} \bar{u}_j^*$$

$$\sum_i \Big[\overbrace{\left(a_i \alpha_{ij} + a_i^\dagger \beta_{ij}^* \right)}^{\bar{a}_i} \bar{u}_j + \underbrace{\left(a_i \beta_{ij} + a_i^\dagger \alpha_{ij}^* \right)}_{\bar{a}_i^\dagger} u_j^* \Big]$$


⟨0|N̄ᵢ|0⟩ = ⟨0|āᵢ† āᵢ|0⟩
↳ ⟨0|a a†|0⟩ - 1
Bᵢⱼ Bᵢⱼ*
= (B†B)ᵢᵢ
0|Nᵢ|0⟩ = 0

$\beta = \frac{1}{kT}$

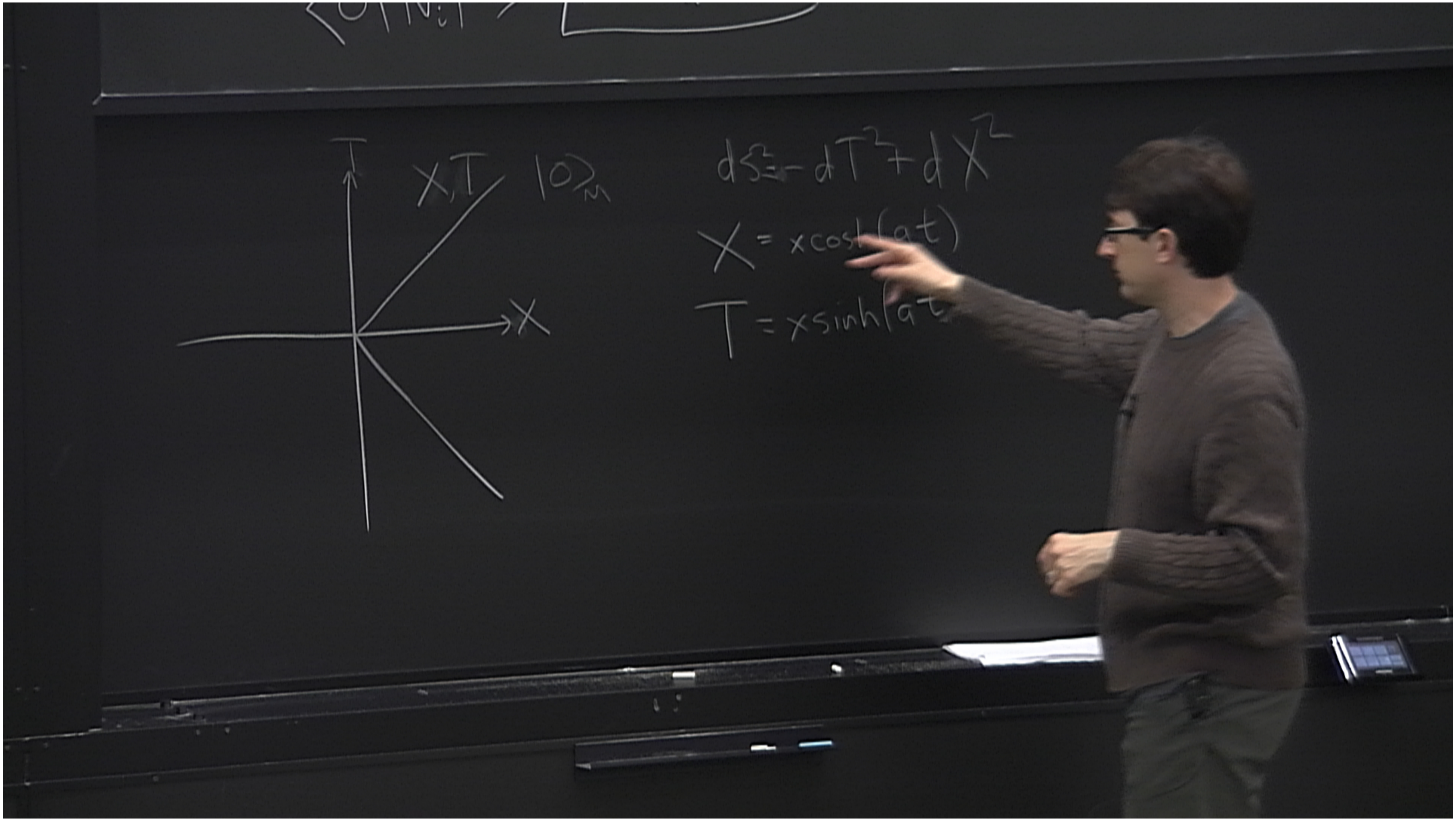
$$\langle A \rangle = \mathrm{Tr}[\rho A(t)]$$

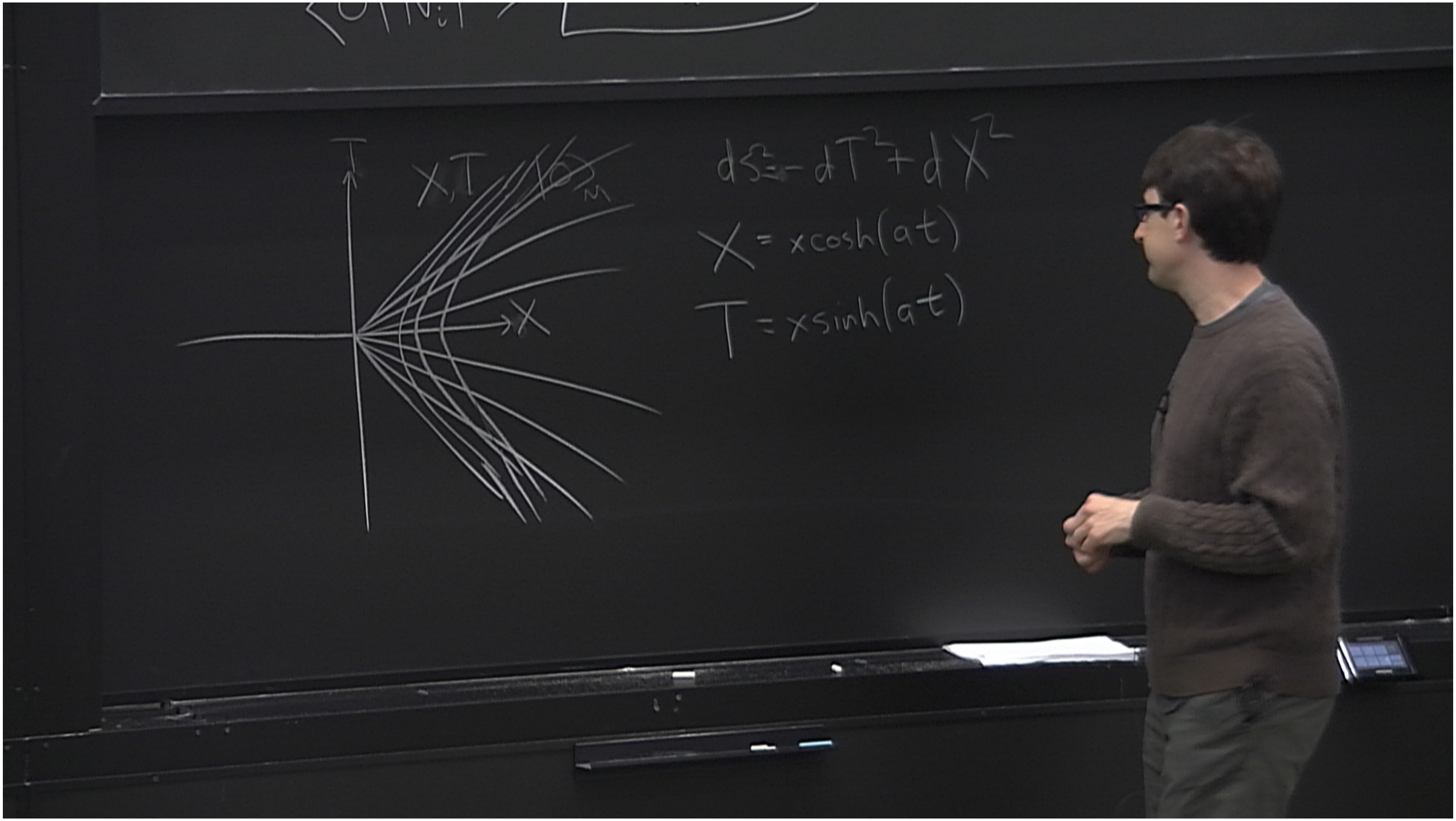
$$\langle A(t_0 - i\beta) \rangle = \langle A(t_0) \rangle$$

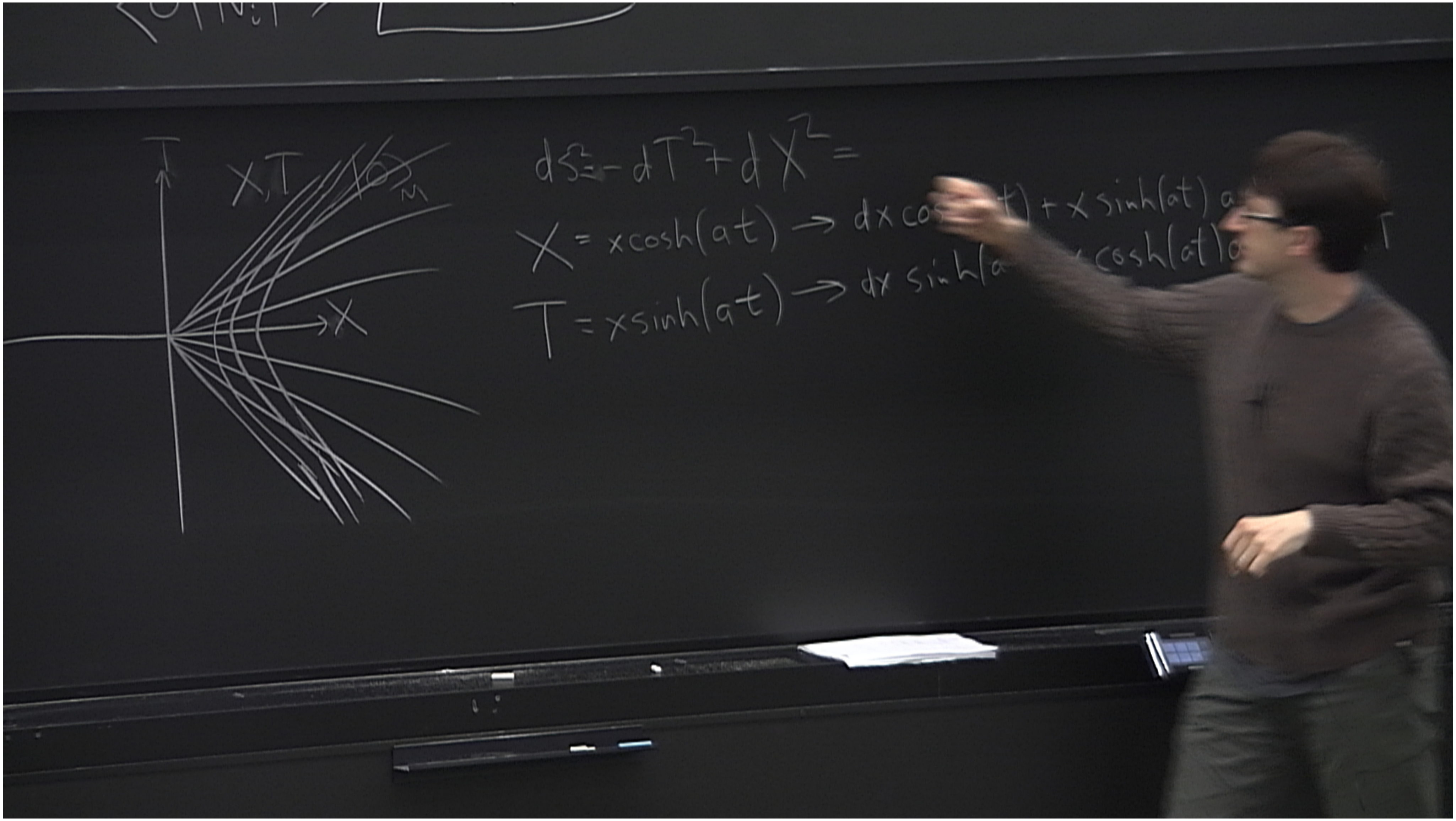
$t = i\beta$

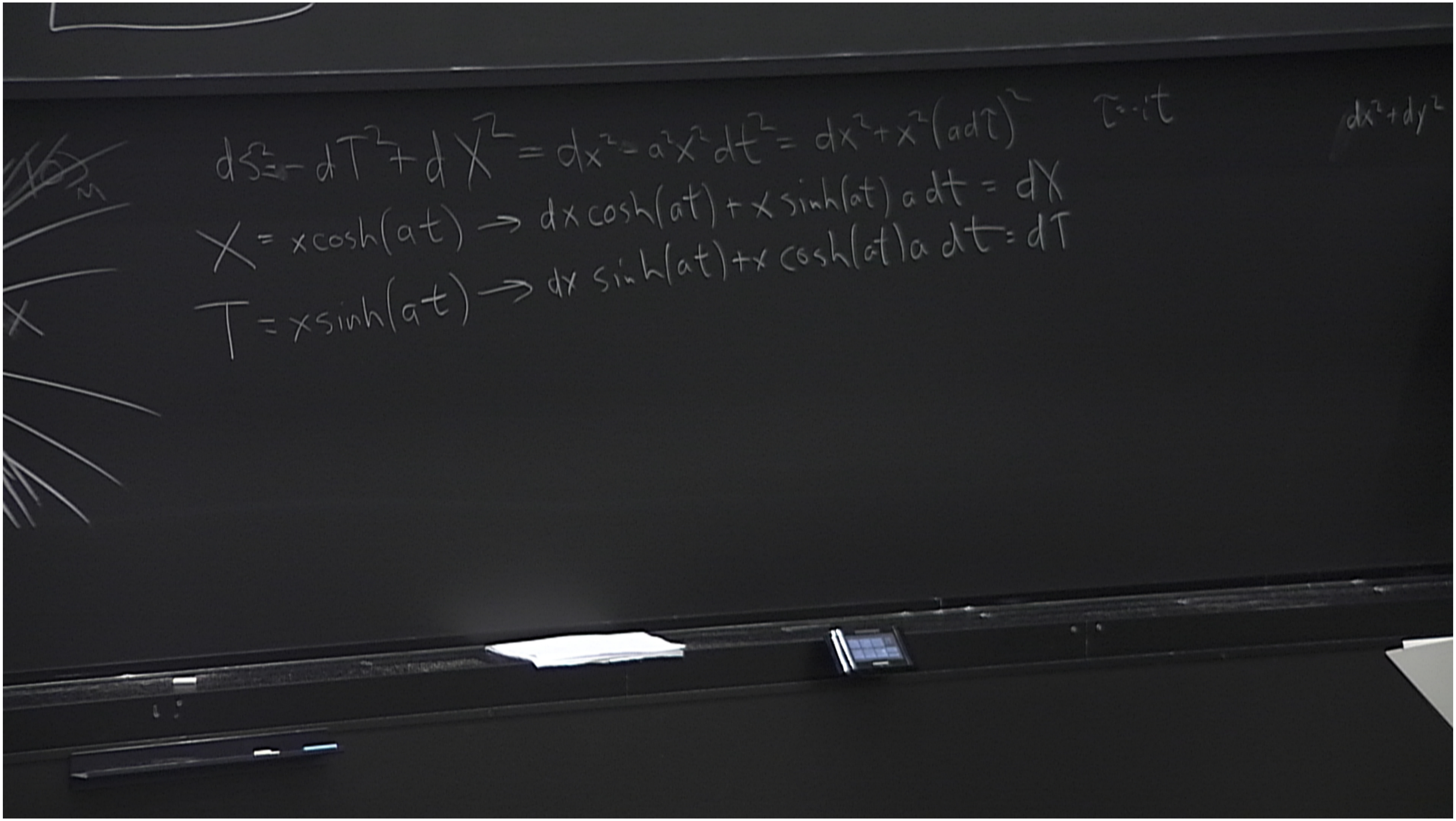
$$= \frac{1}{Z} \mathrm{Tr}[e^{-\beta H} e^{iHt} A_{t_0} e^{-iHt}]$$

$$= \frac{1}{Z} \mathrm{Tr}[A_{t_0} e^{-\beta H}] = \mathrm{Tr}[\rho A_{t_0}$$


ds² = -dT² + dX²
T
X
X,T
|0⟩M


T
X, T
X
$ds^2 = -dT^2 + dX^2$
$X = x\cosh(at)$
$T = x\sinh(at)$


$ds^2 = -dT^2 + dX^2 =$
$X = x\cosh(at) \to dx\cos$
$T = x\sinh(at) \to dx\sinh($


$ds^2 = -dT^2 + dX^2 = dx^2 - a^2x^2dt^2 = dx^2 + x^2(ad\tau)^2$
$\tau = it$
$dx^2 + dy^2$
$X = x\cosh(at) \rightarrow dx\cosh(at) + x\sinh(at)\,a\,dt = dX$
$T = x\sinh(at) \rightarrow dx\sinh(at) + x\cosh(at)\,a\,dt = dT$