

Title: Advanced General Relativity - Lecture 15

Date: Mar 12, 2013 05:00 PM

URL: <http://pirsa.org/13030013>

Abstract:

$$\partial\phi, \quad \nabla_a, \quad \frac{d}{ds}$$

Christoffel Symbol

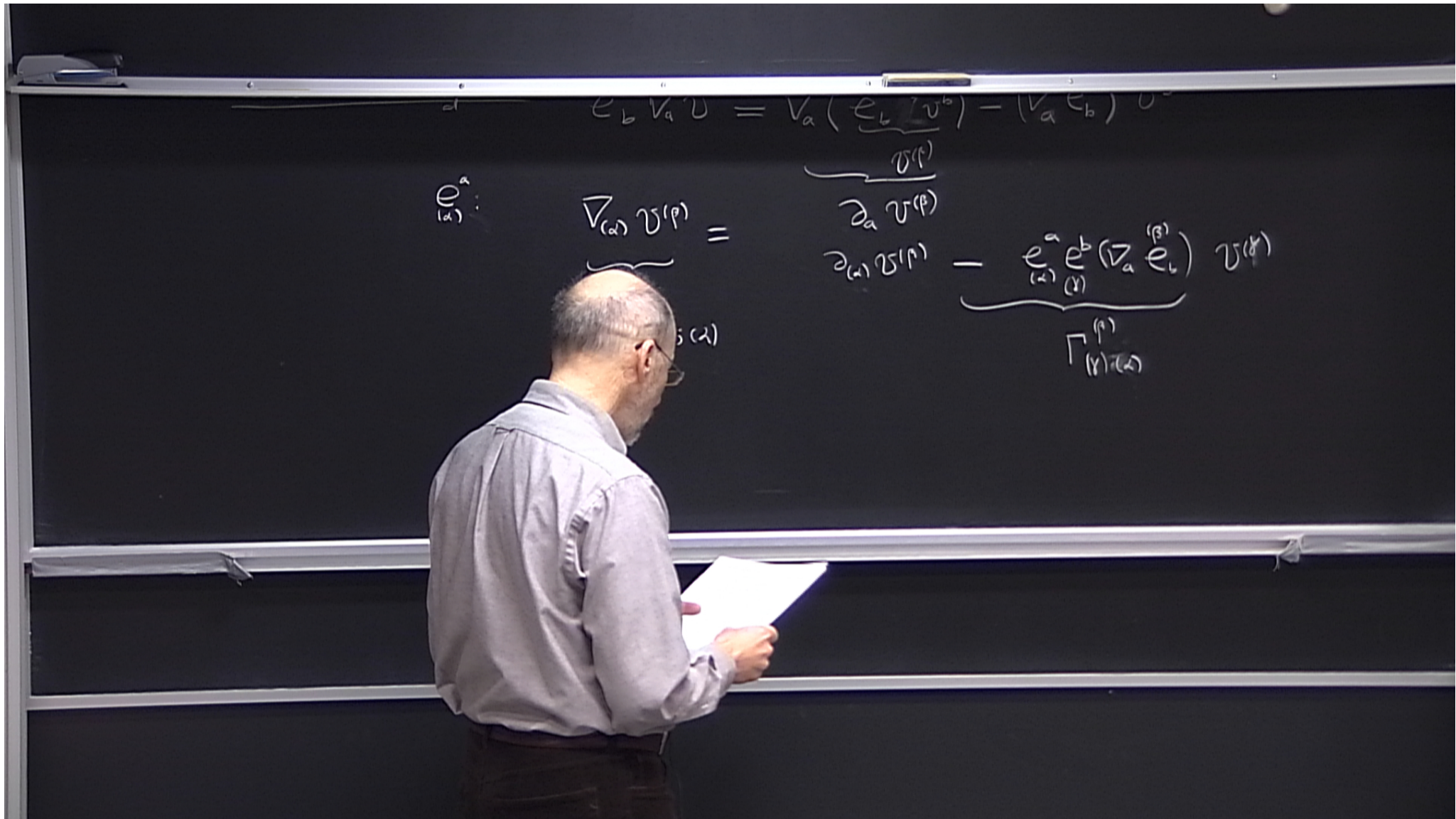
$$\nabla_a v^b$$

$\partial\phi, \nabla_a, \frac{d}{dt}$

Christoffel Symbol

$\nabla_a v^b$ $\frac{d}{dt} e_b$

$$\nabla_a v^b = \nabla_a \left(\underbrace{\frac{d}{dt} v^b}_{\partial_a v^b} \right) - \left(\nabla_a \frac{d}{dt} e_b \right) v^b$$



Christoffel Symbol

$$e_b^{(\beta)} \nabla_a v^b = \nabla_a \left(\underbrace{e_b^{(\beta)}}_{v^{(\beta)}} \right) - (\nabla_a e_b^{(\beta)}) v^b$$

$e_{(\alpha)}^a$

$$\underbrace{\nabla_{(\alpha)} v^{(\beta)}}_{v^{(\beta)}_{;(\alpha)}} = \underbrace{\partial_a v^{(\beta)}}_{v^{(\beta)}} - \underbrace{e_{(\alpha)}^a e_b^{(\beta)} (\nabla_a e_b^{(\beta)})}_{\Gamma_{(\alpha)(\gamma)}^{(\beta)}} v^{(\gamma)}$$

Christoffel Symbol

$$e_b^{(p)} \nabla_a v^b = \nabla_a (\underbrace{e_b^{(p)}}_{v^{(p)}}) - (\nabla_a e_b^{(p)}) v^b$$

$e_{(a)}^a$

$$\underbrace{\nabla_{(a)} v^{(p)}}_{v^{(p)}_{; (a)}} =$$

$$\underbrace{\partial_a v^{(p)}}_{v^{(p)}} - \underbrace{e_{(a)}^a e_b^{(p)} (\nabla_a e_b^{(p)})}_{v^{(p)}_{; (a)}} v^{(p)}$$

Christoffel Symbol

$$e_b^{(p)} \nabla_a v^b = \nabla_a \left(\underbrace{e_b^{(p)}}_{v^{(p)}} \right) - (\nabla_a e_b^{(p)}) v^b$$

$e_{(a)}^a$

$$\underbrace{\nabla_{(a)} v^{(p)}}_{v^{(p)}} = \underbrace{\partial_a v^{(p)}}_{\partial_{(a)} v^{(p)}} - \underbrace{e_{(a)}^a e_b^{(p)} (\nabla_a e_b^{(p)})}_{\Gamma_{(a)(b)}^{(p)}} v^{(p)}$$

$$\Gamma_{(a)(b)}^{(p)} = e_{(a)}^a (\nabla_a e_b^{(p)}) e_{(b)}^b$$

Christoffel Symbol

$$e_b^{(p)} \nabla_a v^b = \nabla_a \left(\underbrace{e_b^{(p)}}_{v^{(p)}} \right) - (\nabla_a e_b^{(p)}) v^b$$

$$e_{(a)}^a$$

$$\nabla_{(a)} v^{(p)}$$

$$v$$

$$\frac{\partial v^{(p)}}{\partial x^a}$$

$$T_2$$

$$- e_{(a)}^a e_{(b)}^b (\nabla_a e_b^{(p)}) v^{(p)}$$

$$\Gamma_{(b)(a)}^{(p)} = e_{(a)}^a (\nabla_a e_b^{(p)}) e_{(b)}^b$$

Christoffel Symbol

$$e_b^{(p)} \nabla_a v^b = \nabla_a \left(\underbrace{e_b^{(p)}}_{v^{(p)}} v^b \right) - (\nabla_a e_b^{(p)}) v^b$$

$e_{(a)}^a$

$$\underbrace{\nabla_{(a)} v^{(p)}}_{v^{(p)}_{; (a)}} =$$

$$\frac{\partial_a v^{(p)}}{\partial_{(a)} v^{(p)}} \left[e_b^{(p)} (\nabla_a e_b^{(p)}) \right] v^{(p)}$$

$$\text{"T}_2\text{M"} \quad \Gamma_{(a)(b)}^{(p)} = \boxed{e_{(a)}^a (\nabla_a e_b^{(p)}) e_{(b)}^b}$$

coord. chart $\Rightarrow$ $\Gamma_{\beta\gamma}^{\alpha}$ from basis e associated to coord chart?
same as $\delta \int dt$?

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same as $\delta \int dt$?

1

$$\frac{d^2 x^\alpha}{d\tau^2} + \Gamma_{\beta\gamma}^\alpha \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau} = 0$$

CAUTION
DO NOT TOUCH THE BOARD OR THE BOARDER
IF YOU ARE NOT A TEACHER OR A STUDENT
OF THE COURSE

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CAUTION
DO NOT TOUCH THE BOARD OR THE BOARDER. BOARDER IS THE PROPERTY OF THE BOARD.
IF IT IS DAMAGED OR LOST, IT WILL BE REPAIRED OR REPLACED AT THE USER'S EXPENSE.
UNIVERSITY OF MICHIGAN

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UNIVERSITY OF MICHIGAN

coord. chart $\Rightarrow$ $\Gamma_{\beta\gamma}^{\alpha}$ from basis e associate to coord chart?

same as $\delta \int dt$?

Note: $\Gamma_{\alpha\beta}^{\gamma} = \Gamma_{\beta\alpha}^{\gamma}$

coord. chart $\Rightarrow$ $\Gamma_{\beta\gamma}^{\alpha}$ from basis e associate to coord chart?

same as $\delta \int dt$?

Note: $\Gamma_{\alpha\beta}^{\gamma} = \Gamma_{\beta\alpha}^{\gamma}$ $\forall \alpha$

coord. chart $\Rightarrow$ $\Gamma_{\beta\gamma}^{\alpha}$ from basis e associated to coord chart?

same as $\delta \int dt$?

$$\Gamma_{\alpha\beta}^{\gamma} = \Gamma_{\beta\alpha}^{\gamma}$$

$$e_{(a)}^{\alpha} e_{(b)}^{\beta} \underbrace{\nabla_a \nabla_b x^{(p)}}_{\text{symmetric scalar}}$$

coord. chart $\Rightarrow$ $\Gamma_{\beta\gamma}^\alpha$ from basis e associate to coord chart?

same as $\delta \int dt$?

Note: $\Gamma_{\alpha\beta}^\gamma = \Gamma_{\beta\alpha}^\gamma$

$$\nabla_a g_{ab} = 0$$

$$\nabla_{\mu} g_{\mu\nu} = 0$$

symmetric tensor

$$e^a_{\mu} \nabla_b x^\mu \quad \checkmark$$

coord. chart $\Rightarrow$ $\Gamma_{\beta\gamma}^{\alpha}$ from basis e associated to coord chart?

same as $\delta \int dt$?

Note: $\Gamma_{\alpha\beta}^{\gamma} = \Gamma_{\beta\alpha}^{\gamma}$

$e_{(a)}^{\alpha} e_{(b)}^{\beta} \underbrace{\nabla_{\alpha} \nabla_{\beta} x^{(p)}}_{\text{symmetric scalar}} \checkmark$

$\nabla_{(a)} \partial^{(p)} =$

$$\nabla_a g_{ab} = 0$$

$$\nabla_{(a)} g_{(b)(p)} = 0$$

coord. chart $\Rightarrow$ $\Gamma_{\beta\gamma}^\alpha$ from basis e associated to coord chart?

same as $\delta \int dt$?

Note: $\Gamma_{\alpha\beta}^\gamma = \Gamma_{\beta\alpha}^\gamma$

$e_{(\alpha)}^a e_{(\beta)}^b \underbrace{\nabla_a \nabla_b x^{(\rho)}}_{\text{symmetric scalar}} \checkmark$

$$\nabla_a g_{ab} = 0$$

$$\nabla_{(\alpha)} g_{(\beta)(\rho)} = 0$$

$$\nabla_{(\alpha)} U^{(\rho)} = \partial_{(\alpha)} U^{(\rho)} + \Gamma_{(\alpha)(\gamma)}^{(\rho)} U^{(\gamma)}$$

coord. chart $\Rightarrow \Gamma_{\beta\gamma}^\alpha$ from basis e associated to coord chart?

same as $\delta \int dt$?

Note: $\Gamma_{\alpha\beta}^\gamma = \Gamma_{\beta\alpha}^\gamma$

$e_{(\alpha)}^a e_{(\beta)}^b \underbrace{\nabla_a \nabla_b x^{(\gamma)}}_{\text{symmetric scalar}} \checkmark$

$$\nabla_a g_{ab} = 0$$

$$\nabla_{(\alpha)} g_{(\beta)(\gamma)} = 0$$

$$\nabla_{(\alpha)} v^{(\beta)} = \partial_{(\alpha)} v^{(\beta)} + \Gamma_{(\gamma)(\alpha)}^{(\beta)} v^{(\gamma)}$$

$$\nabla_{(\alpha)} v_{(\beta)} = \partial_{(\alpha)} v_{(\beta)} - \Gamma_{(\beta)(\alpha)}^{(\gamma)} v_{(\gamma)}$$

coord. chart $\Rightarrow \Gamma_{\beta\gamma}^\alpha$ from basis e associated to coord chart?

same as $\delta \int dt$?

Note: $\Gamma_{\alpha\beta}^\gamma = \Gamma_{\beta\alpha}^\gamma$

$e_{(\alpha)}^a e_{(\beta)}^b \underbrace{\nabla_a \nabla_b x^{(\gamma)}}_{\text{symmetric scalar}}$

$$\nabla_a g_{ab} = 0$$

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$$\nabla_{(\alpha)} v^{(\beta)} = \partial_{(\alpha)} v^{(\beta)} + \Gamma_{(\alpha)(\gamma)}^{(\beta)} v^{(\gamma)}$$

$$\nabla_{(\alpha)} v_{(\beta)} = \partial_{(\alpha)} v_{(\beta)} - \Gamma_{(\beta)(\gamma)}^{(\alpha)} v_{(\gamma)}$$

$$\partial_{(\alpha)} T^{...}$$

$$\nabla_a g_{bc} = 0 \quad \nabla_{(a} g_{b)(c)} = 0$$

$$\Gamma^{(old)} = \Gamma^{(new)}$$

Acceleration curve

$$u^a = \frac{dx^a}{d\tau}$$

$$\nabla_{(a} v_{b)} = \partial_{(a} v_{b)} - \Gamma_{(b)(a)}^{(c)} v_{(c)}$$

$$\partial_{(a} T^{...}$$

$$\nabla_a g_{bc} = 0$$

$$\nabla_{(a} g_{b)(c)} = 0$$

$$\Gamma^{(old)} = \Gamma^{(new)}$$

$$\nabla_{(a} \nabla_{b)} \nabla_{c)} = \partial_{(a} \nabla_{b)} \nabla_{c)} - \Gamma_{(a}^{(b)} \nabla_{c)} \nabla_{d)} \nabla_{e)}$$

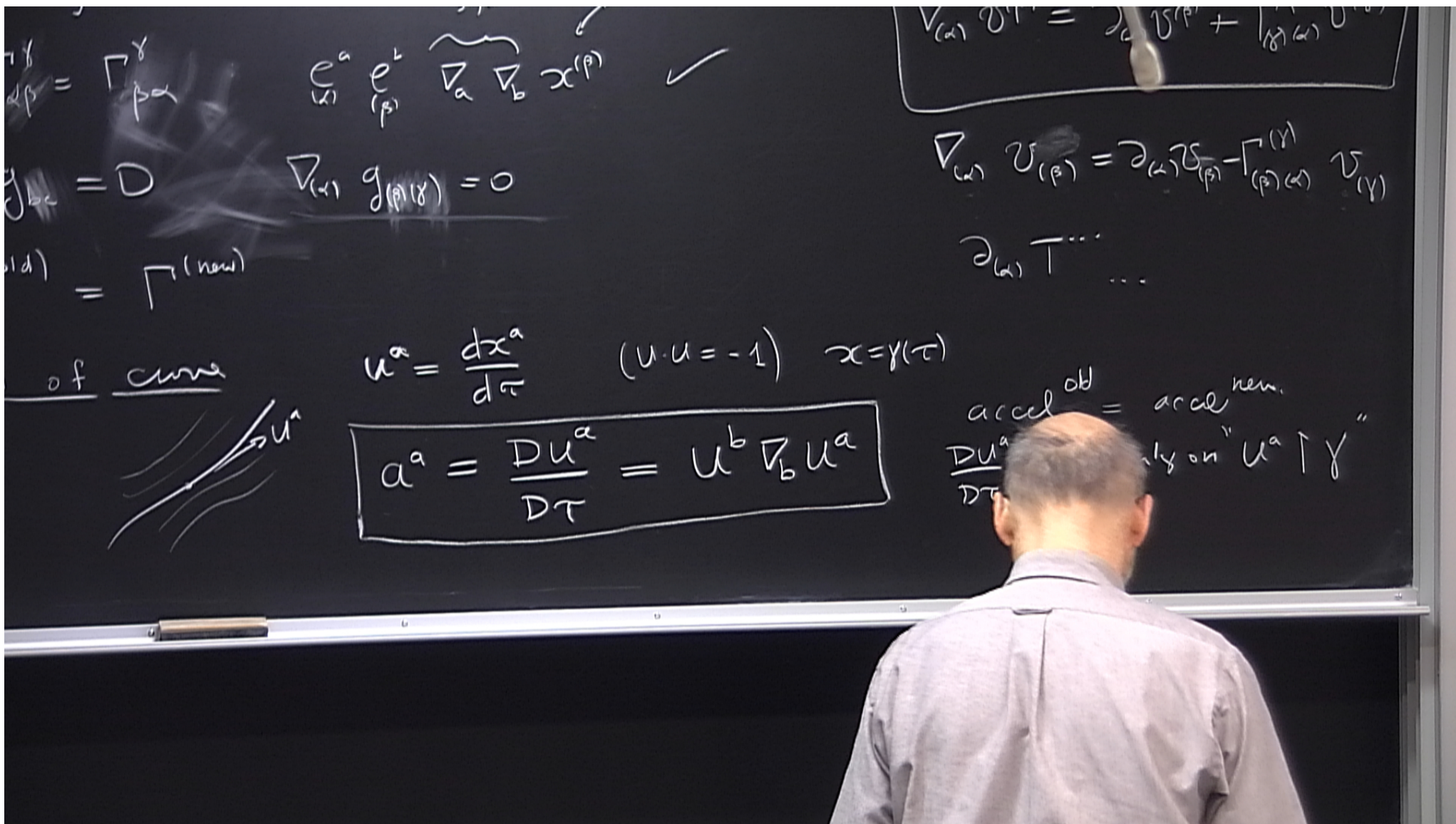
$$\partial_{(a} T^{...}$$

Acceleration of curve

$$u^a = \frac{dx^a}{d\tau}$$

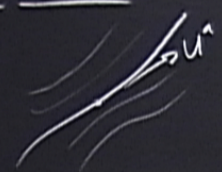
$$(u \cdot u = -1) \quad x = \gamma(\tau)$$

$$a^a = \frac{Du^a}{D\tau} = \nabla_b u^a$$



$$\Gamma^{(old)} = \Gamma^{(new)}$$

Acceleration of curve



$$u^a = \frac{dx^a}{d\tau}$$

$$(u \cdot u = -1) \quad x = \gamma(\tau)$$

$$a^a = \frac{Du^a}{D\tau} = u^b \nabla_b u^a$$

$accel^{old} = accel^{new}$
 $\frac{Du^a}{D\tau}$ depends only on " $u^a \upharpoonright \gamma$ "

proof
inertial frame. $\nabla_{(u)} = \partial_{(u)}$
or from (#)

Geodesic as straightest curve (2nd def)

Def γ geodesic $\Leftrightarrow \text{acc} = 0$

$$\Gamma^\lambda \equiv \frac{d^2 x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

$$-dt^2 + d\vec{x} \cdot d\vec{x}$$



$$\Lambda^\alpha \equiv \frac{d^2 x^\alpha}{d\tau^2} + \Gamma^\alpha_{\beta\gamma} \frac{dx^\beta}{d\tau} \frac{dx^\gamma}{d\tau} = 0$$

$$ds^2 = -dt^2 + d\vec{x} \cdot d\vec{x}$$



$$\Gamma^\lambda_{\mu\nu} \equiv \frac{d^2 x^\lambda}{d\tau^2} + \Gamma^\lambda_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = 0$$

$$ds^2 = -dt^2 + d\vec{x} \cdot d\vec{x}$$

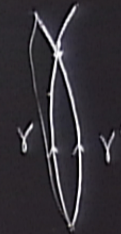
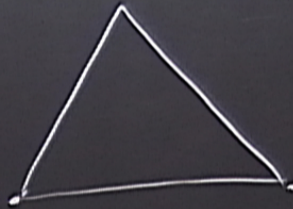


CAUTION
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$$A^\mu \equiv \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda} \frac{dx^\nu}{d\tau} \frac{dx^\lambda}{d\tau} = 0$$

$$ds^2 = -dt^2 + d\vec{x} \cdot d\vec{x}$$



proof

inertial frame: $\nabla_{\dot{\gamma}} = \partial_{\dot{\gamma}}$
or from (#)

Geodesic as straightest curve (2nd def)

Def γ geodesic $\Leftrightarrow$ accel. $= 0$ (i.f. λ appropriate)

$$\begin{aligned}\gamma: \mathbb{R} &\rightarrow M \\ \gamma &\rightarrow \gamma(\lambda)\end{aligned}$$

If $\frac{D}{D\lambda} \frac{d\gamma(\lambda)}{d\lambda} = 0$ Then γ is geod. and
 λ is affine parameter.

Claim: λ aff. p. Then $a\lambda + b$ also. = p.

proof

inertial frame: $\nabla_{\dot{\gamma}} = \partial_{\dot{\gamma}}$
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Geodesic as straightest curve (2nd def)

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Claim: λ aff. p. Then $a\lambda + b$ also = p.

Claim τ is affine param if γ timelike

$$\dot{x}^a = \frac{dx^a}{d\lambda} \quad \frac{D\dot{x}^a}{D\lambda} = 0 \Rightarrow \frac{D}{D\lambda} (g_{ab} \dot{x}^a \dot{x}^b)$$

CAUTION

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Claim τ is affine param if γ timelike

$$\dot{x}^a = \frac{dx^a}{d\lambda} \quad \frac{D\dot{x}^a}{D\lambda} = 0 \Rightarrow \frac{D}{D\lambda} (g_{ab} \dot{x}^a \dot{x}^b) = (\cancel{\dot{x}^c} \cancel{\partial_c} g_{ab}) \dot{x}^a \dot{x}^b + 2 g_{ab} \dot{x}^a \frac{D}{D\lambda} \dot{x}^b = 0$$

Claim τ is affine param if γ timelike

$$\dot{x}^a = \frac{dx^a}{d\lambda} \quad \frac{D\dot{x}^a}{D\lambda} = 0 \Rightarrow \frac{D}{D\lambda} (g_{ab} \dot{x}^a \dot{x}^b) = \cancel{(\dot{x}^c \partial_c g_{ab})} \dot{x}^a \dot{x}^b + 2 g_{ab} \dot{x}^a \frac{D}{D\lambda} \dot{x}^b = 0$$

$\left\{ \begin{array}{l} \text{timelike remains timelike} \\ \text{null} \quad \quad \quad \text{null} \end{array} \right\}$

timelike

$g_{ab} dx^a dx^b$

CAUTION

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Claim τ is affine param if γ timelike

$$\dot{x}^a = \frac{dx^a}{d\lambda} \quad \frac{D\dot{x}^a}{D\lambda} = 0 \Rightarrow \frac{D}{D\lambda} (g_{ab} \dot{x}^a \dot{x}^b) = (\cancel{\dot{x}^c} \cancel{\partial_c} g_{ab}) \dot{x}^a \dot{x}^b + 2 g_{ab} \dot{x}^a \frac{D}{D\lambda} \dot{x}^b = 0$$

timelike remain timelike
null " null

$$\text{timelike } g_{ab} \dot{x}^a \dot{x}^b = g_{ab} \frac{dx^a}{d\lambda} \frac{dx^b}{d\lambda} = -c^2$$

$$+ d\tau^2 = +c^2 d\lambda^2$$

$$d\tau = c d\lambda$$

$$\tau = c\lambda + b$$

τ affine

coord. chart $dx^a \rightarrow E_a^{\mu} dx^a = dx^{\mu} \quad \dot{x}^a \rightarrow \frac{dx^{\mu}}{d\lambda}$

$$a^a = \frac{D}{D\lambda} \underbrace{\left(\frac{dx^a}{d\lambda} \right)}_{\dot{x}^a} = \dot{x}^b \nabla_b \dot{x}^a \rightarrow \underbrace{\dot{x}^{\mu}}_{\frac{dx^{\mu}}{d\lambda}} \left[\partial_{\mu} \dot{x}^a + \Gamma_{(\mu)(\rho)}^{(a)} \dot{x}^{(\rho)} \right]$$

$$\frac{d}{d\lambda} \left(\frac{dx^{\mu}}{d\lambda} \right) + \Gamma_{(\mu)(\rho)}^{(a)} \frac{dx^{\rho}}{d\lambda} \frac{dx^{\mu}}{d\lambda} = a^{(a)}$$

CAUTION

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Coord. chart $dx^a \rightarrow \tilde{e}_a dx^a = dx^{(a)} \quad \dot{x}^a \rightarrow \frac{dx^{(a)}}{d\lambda}$

$$a^a = \frac{D}{D\lambda} \left(\frac{dx^a}{d\lambda} \right) = \dot{x}^b \nabla_b \dot{x}^a \rightarrow \frac{\dot{x}^{(a)}}{\frac{dx^{(a)}}{d\lambda}} \left[\partial_{(a)} \dot{x}^{(a)} + \Gamma_{(a)(b)}^{(a)} \dot{x}^{(b)} \right]$$

$$\frac{d}{d\lambda} \left(\frac{dx^{(a)}}{d\lambda} \right) + \Gamma_{(a)(b)}^{(a)} \frac{dx^{(b)}}{d\lambda} \frac{dx^{(a)}}{d\lambda} = a^{(a)}$$

Acceleration of curve

$$u^a = \frac{dx^a}{d\tau}$$

$$(u \cdot u = -1) \quad x = \gamma(\tau)$$

$$a^a = \frac{Du^a}{D\tau} = u^b \nabla_b u^a$$

accel^{old} = accel^{new}.
 $\frac{Du^a}{D\tau}$ depends only on " $u^a \upharpoonright \gamma$ "

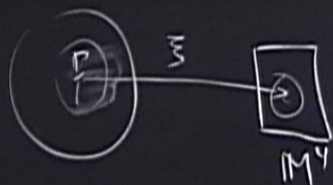
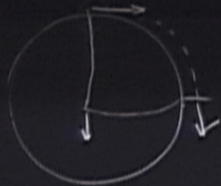
$\gamma \in \gamma$ geodesic $\Leftrightarrow$ accel = 0 (if λ appropriate)

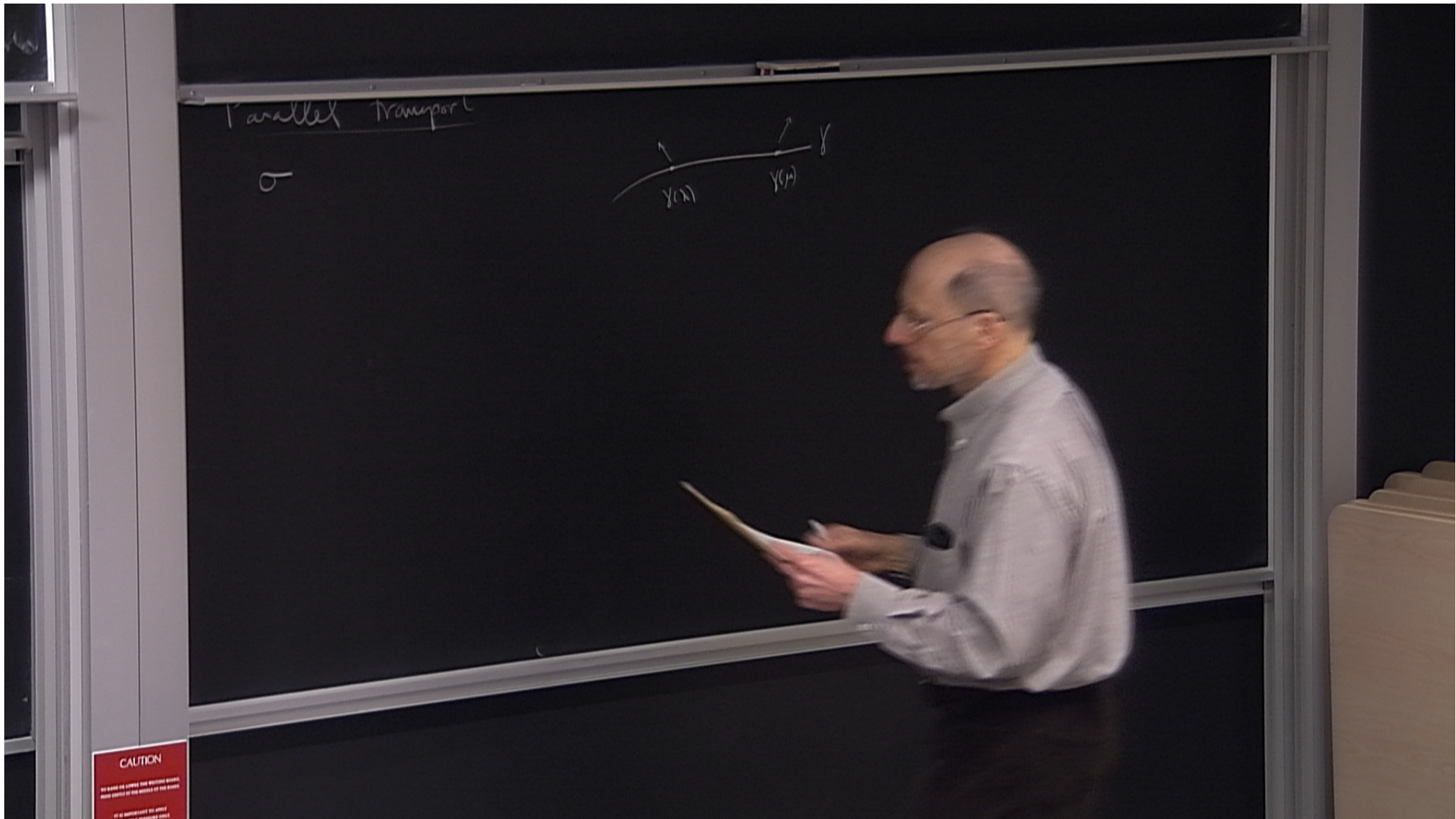
$$\gamma: \mathbb{R} \rightarrow M$$

$$\gamma \rightarrow \gamma(\lambda)$$

If $\frac{D}{D\lambda} \frac{d\gamma(\lambda)}{d\lambda} = 0$ Then γ is geod. and
 λ is affine parametr.

clm: λ aff. p. Then $a\lambda + b$ also. = p.



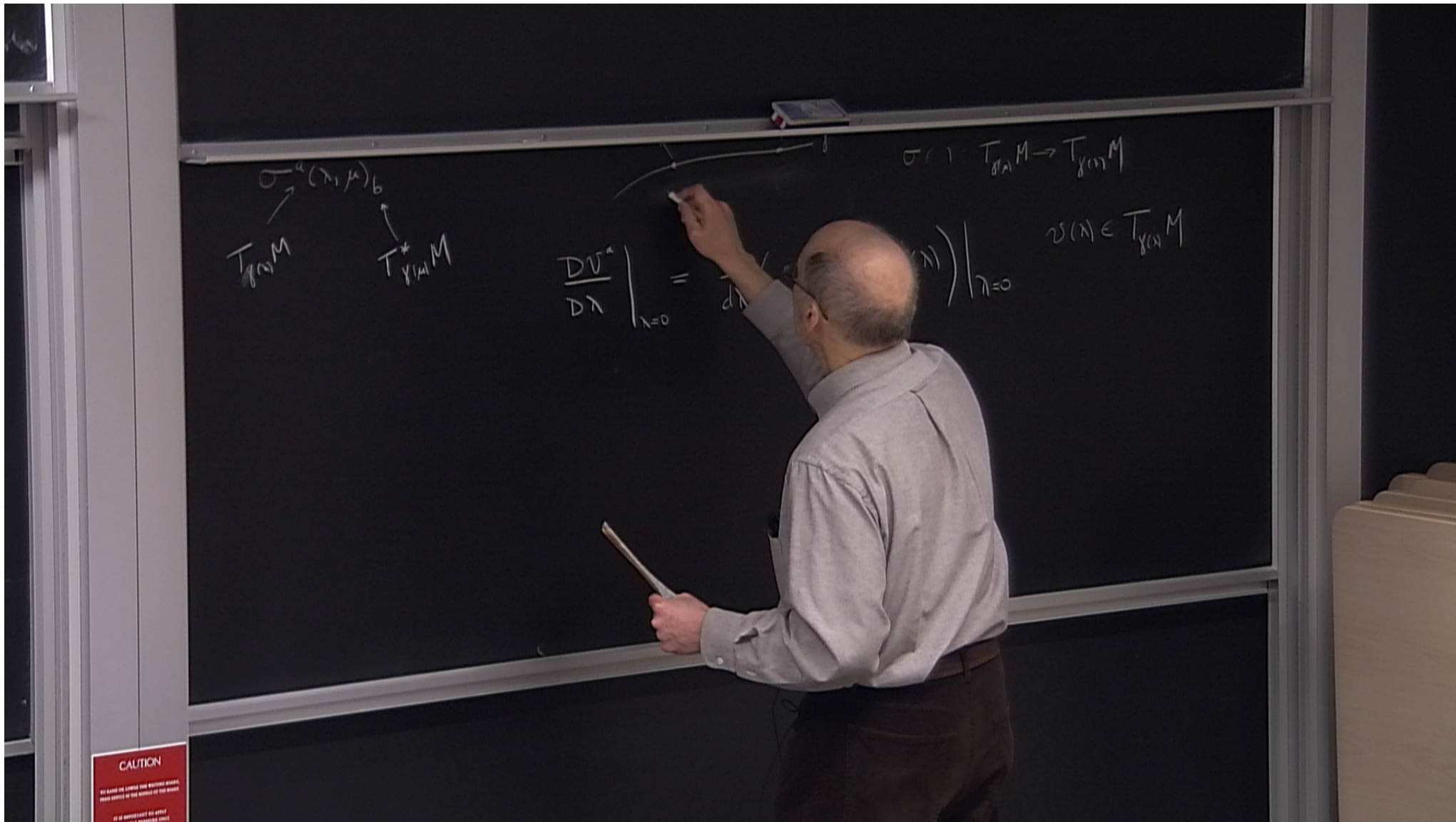


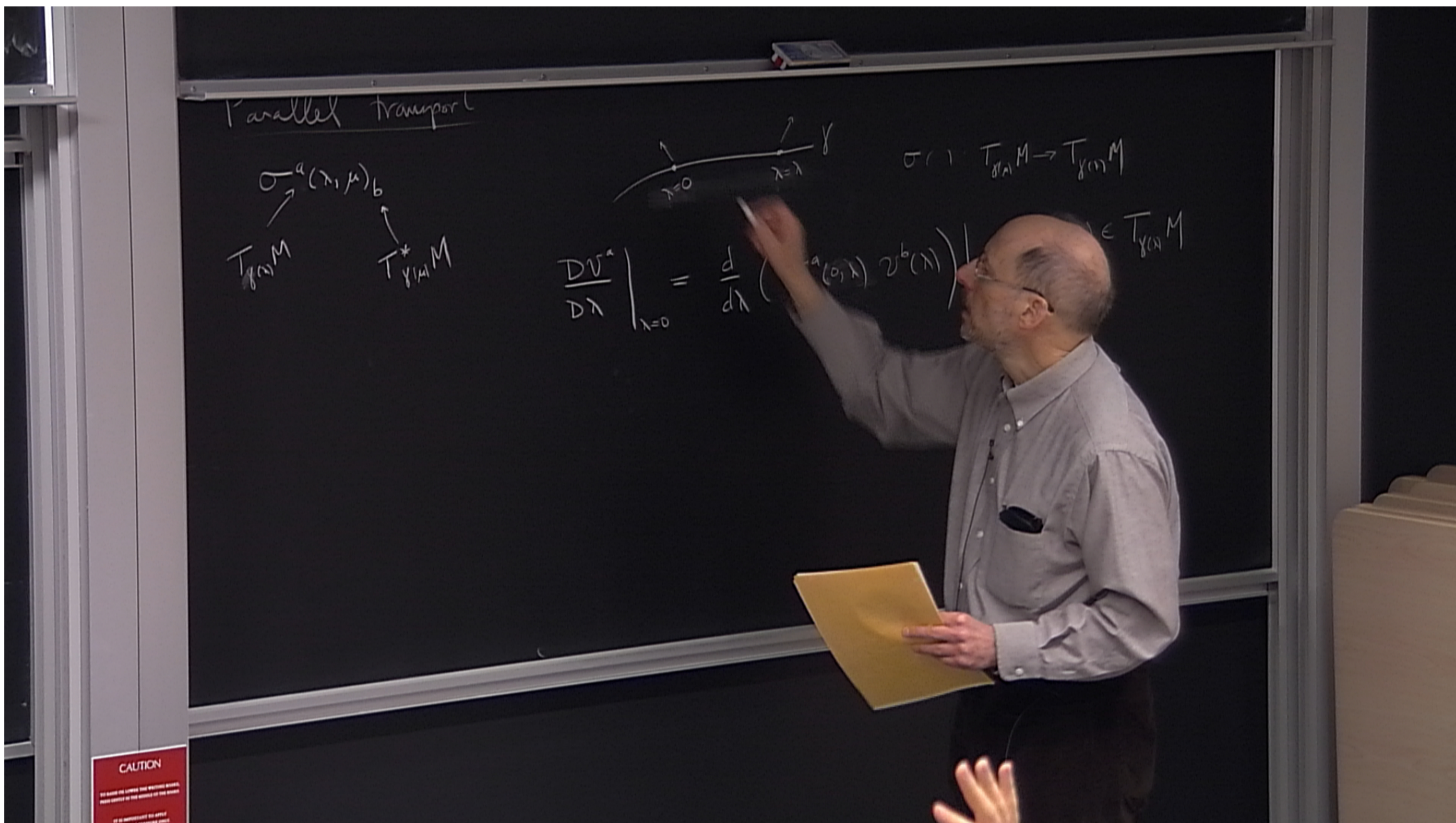
$$\begin{array}{c} \sigma^a(\gamma, \mu)_b \\ \nearrow \quad \nwarrow \\ T_{\gamma(n)}M \quad T_{\gamma(n)}^*M \end{array}$$

$$\begin{array}{c} \gamma(n) \quad \gamma(n) \end{array}$$

$$\sigma : T_{\gamma(n)}M \rightarrow T_{\gamma(n)}M$$

CAUTION
DO NOT TOUCH THE BOARD WHEN IT IS HOT
DO NOT TOUCH THE BOARD WHEN IT IS HOT





Parallel transport

$$\sigma^a(\lambda, \mu)_b$$

$\nearrow T_{\gamma(\mu)}M$
 $\nwarrow T_{\gamma(\lambda)}^*M$

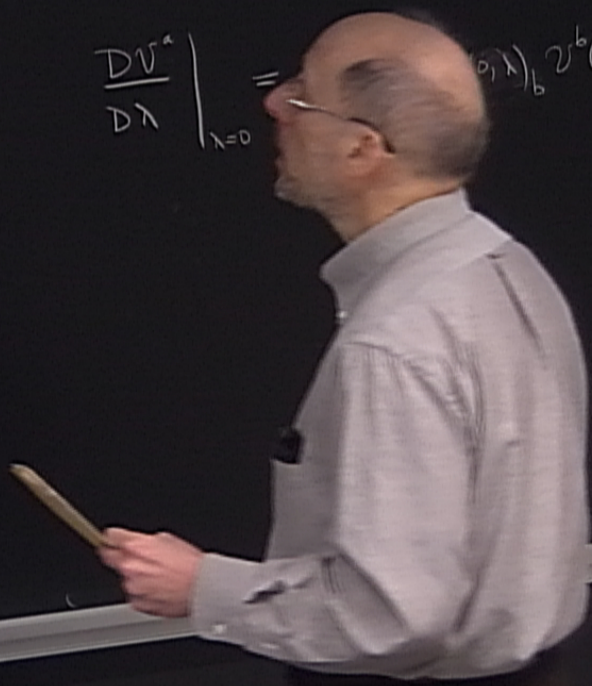


$$\sigma^a : T_{\gamma(\lambda)}M \rightarrow T_{\gamma(\mu)}M$$

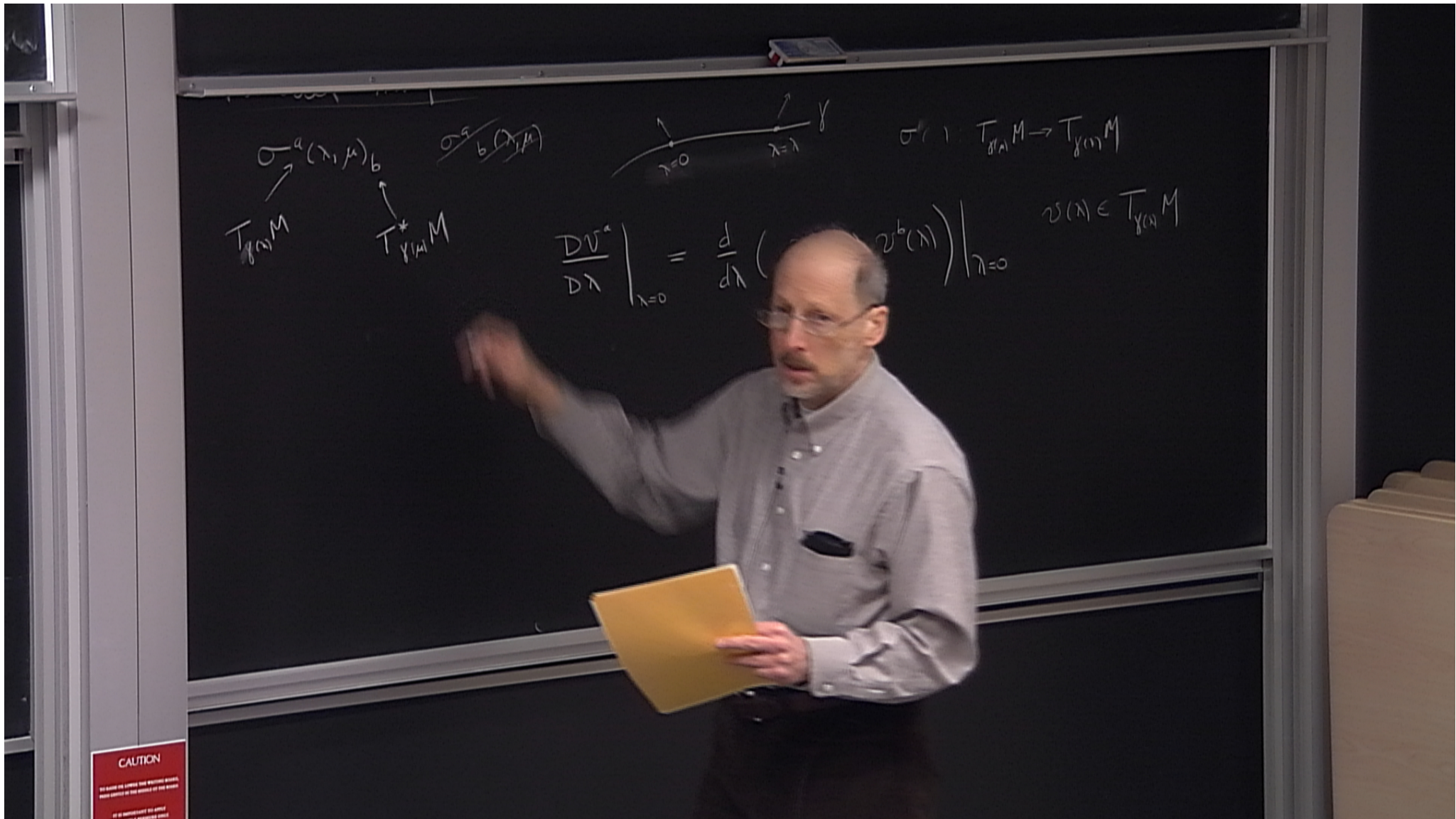
$$\left. \frac{DV^a}{d\lambda} \right|_{\lambda=0}$$

$$\left. \sigma^a(\lambda)_b v^b(\lambda) \right|_{\lambda=0}$$

$$v(\lambda) \in T_{\gamma(\lambda)}M$$



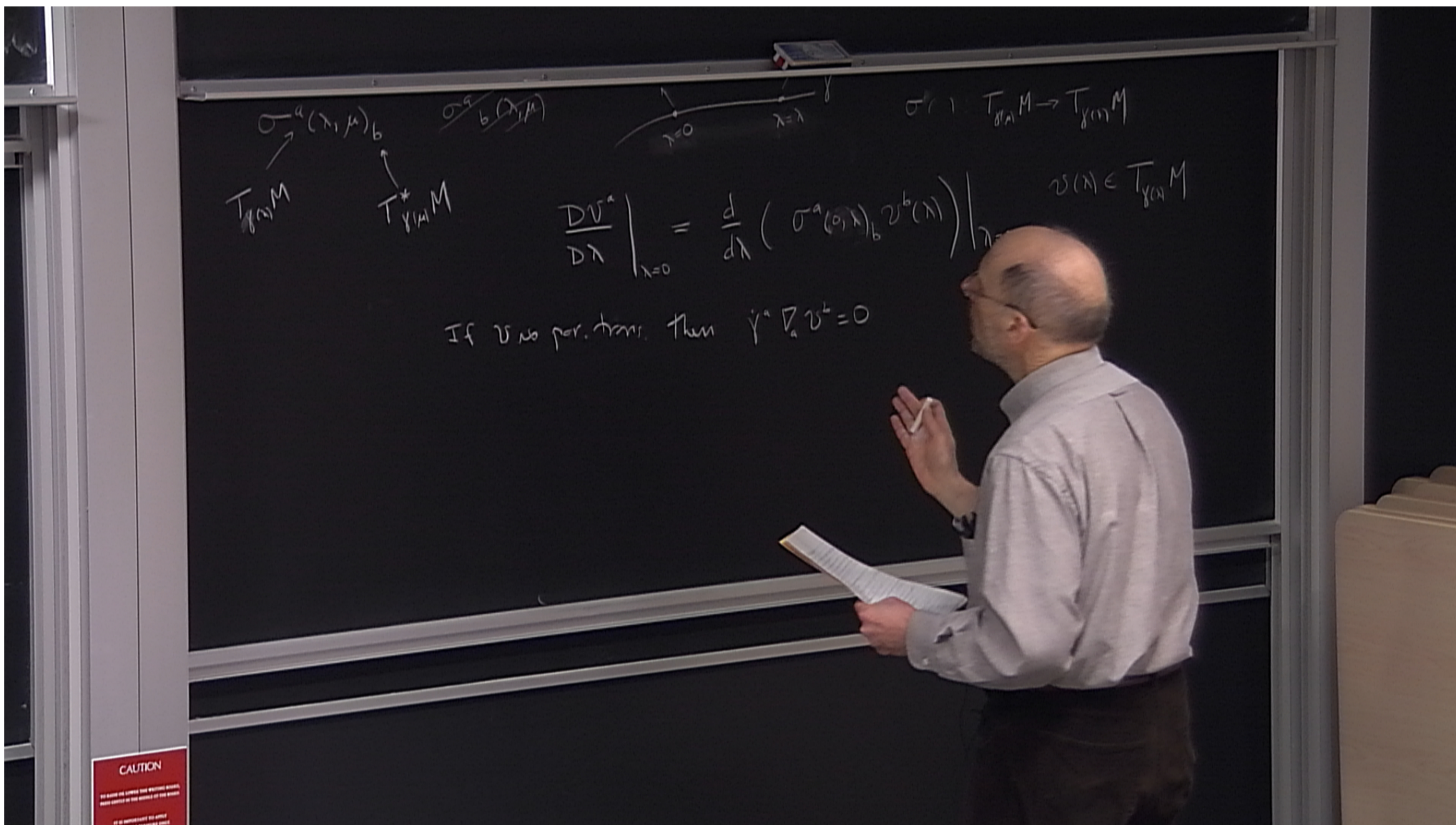
CAUTION
DO NOT RE-ENTER THE WAITING ROOM
FROM THE BACK OF THE ROOM
DO NOT RE-ENTER THE WAITING ROOM
FROM THE BACK OF THE ROOM



$$\begin{array}{c}
 \sigma^a(\lambda, \mu)_b \\
 \nearrow T_{\gamma(\mu)} M \\
 \nwarrow T_{\gamma(\lambda)}^* M
 \end{array}
 \quad
 \sigma^a_b(\lambda, \mu)
 \quad
 \begin{array}{c}
 \gamma \\
 \nearrow \lambda=0 \\
 \nwarrow \lambda=\lambda
 \end{array}
 \quad
 \sigma^a(\lambda) : T_{\gamma(\lambda)} M \rightarrow T_{\gamma(\mu)} M$$

$$\left. \frac{DV^a}{d\lambda} \right|_{\lambda=0} = \left. \frac{d}{d\lambda} \left(\sigma^a(\lambda, \lambda)_b v^b(\lambda) \right) \right|_{\lambda=0} \quad v(\lambda) \in T_{\gamma(\lambda)} M$$

If v no par. trans. then



Parallel transport

$$\begin{array}{ccc} \sigma^a(\lambda, \mu)_b & \sigma^a_b(\lambda, \mu) \\ \nearrow & \nwarrow \\ T_{\gamma(\lambda)} M & T_{\gamma(\mu)}^* M \end{array}$$



$$\sigma^a_b : T_{\gamma(\lambda)} M \rightarrow T_{\gamma(\mu)} M$$

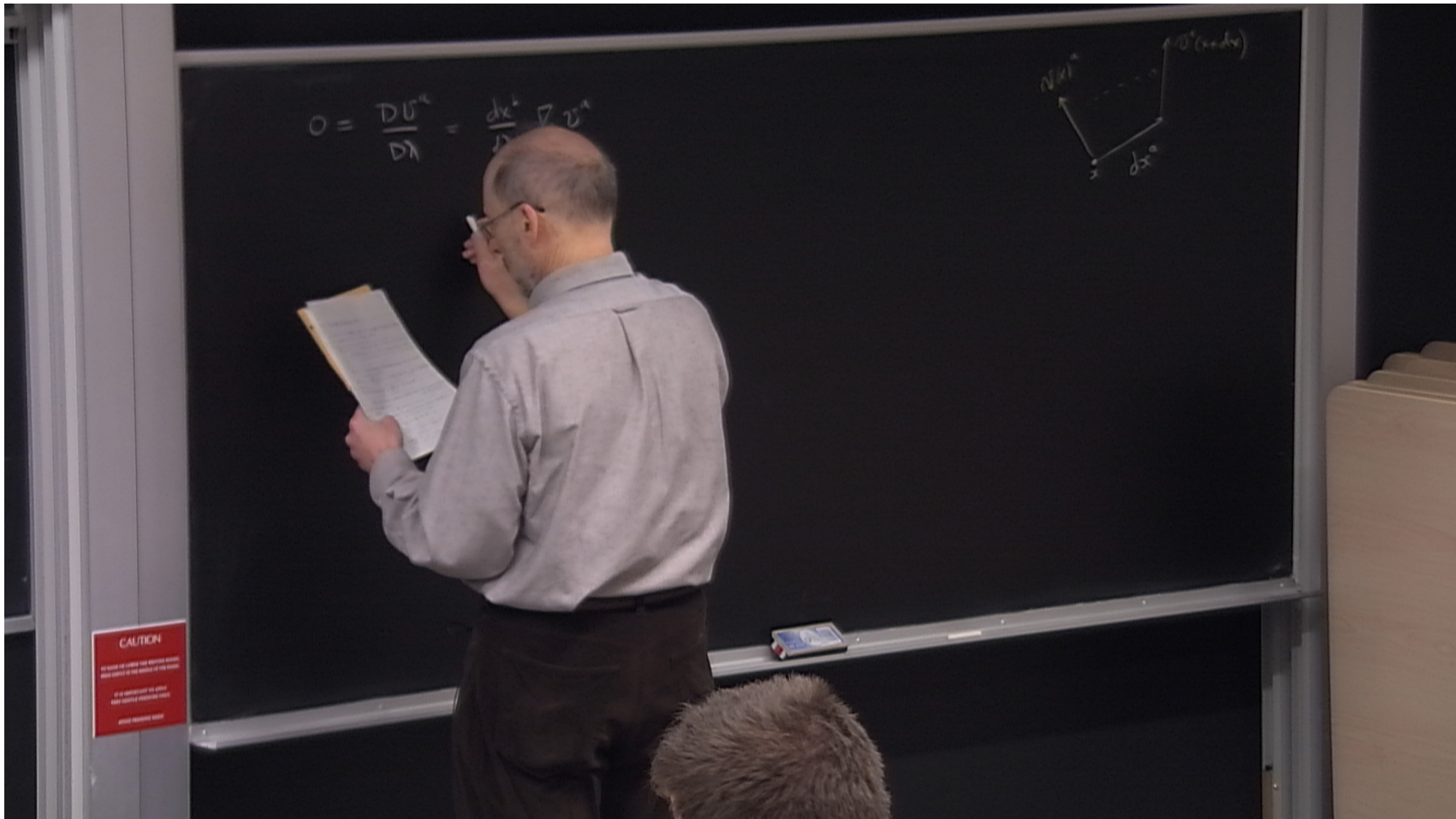
$$\left. \frac{DV^a}{d\lambda} \right|_{\lambda=0} = \left. \frac{d}{d\lambda} \left(\sigma^a_b(\lambda, \lambda)_b V^b(\lambda) \right) \right|_{\lambda=0}$$

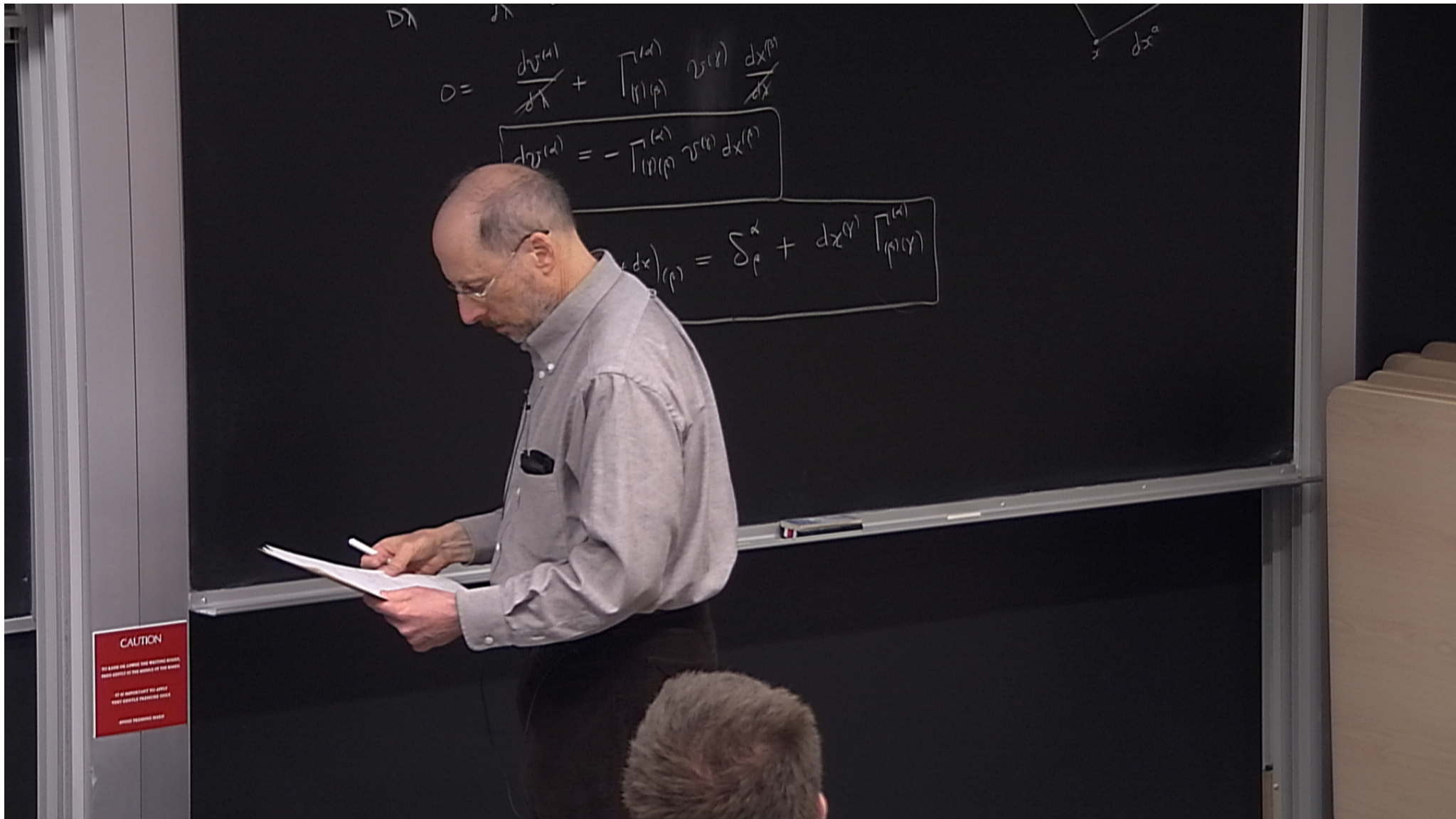
$$V(\lambda) \in T_{\gamma(\lambda)} M$$

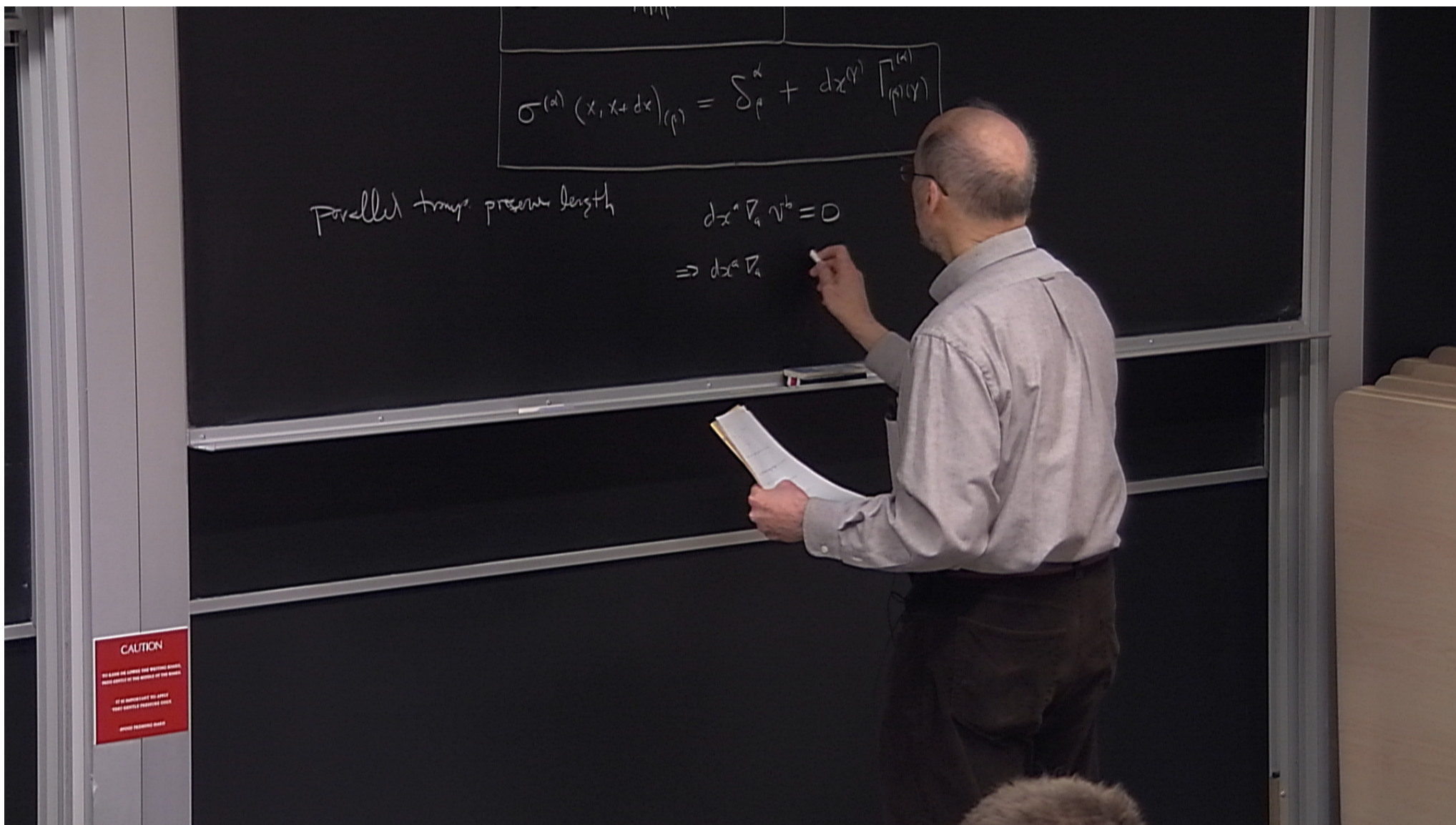
If V is par. trans. then $\dot{\gamma}^a \nabla_a V^b = 0$

→ Holonomy

$$\boxed{\dot{\gamma}^a \nabla_a V^b = \frac{DV^b}{d\lambda}}$$







$$\sigma^{(x)}(x, x+dx)_{(p)} = \delta_p^x + dx^{(y)} \Gamma_{(p)(y)}^{(x)}$$

Parallel transport preserve length

$$\Rightarrow dx^a \nabla_a v_b = 0 \quad (\text{since } \nabla_a g_{bc} = 0)$$

CAUTION
DO NOT STAND ON CORNERS OR REACHING DOWN
DO NOT ENTER AT THE REAR OF THE STAGE
IF AN EMERGENCY DO STAY
WHERE YOU ARE SEATED
STAY SEATED PLEASE

$$0 = \frac{Dv^a}{D\lambda} = \frac{dx^b}{d\lambda} \nabla_b v^a$$

$$0 = \frac{dv^a}{d\lambda} + \Gamma_{(b)(c)}^{(a)} v^c \frac{dx^b}{d\lambda}$$

$$dv^a = -\Gamma_{(b)(c)}^{(a)} v^c dx^b$$

forward transport
 $x \rightarrow x+dx$

$$v^a(x, x+dx)_{(p)} = v^a_{(p)} + dx^b \Gamma_{(b)(c)}^{(a)} v^c_{(p)}$$

$$v^a_{(p)} \rightarrow v^a_{(p)} + dx^b \Gamma_{(b)(c)}^{(a)} v^c_{(p)}$$

Parallel transport preserves length
 & angles

$$dx^a \nabla_a v^b \Rightarrow dx^a \nabla_a (g_{bc} v^c)$$

