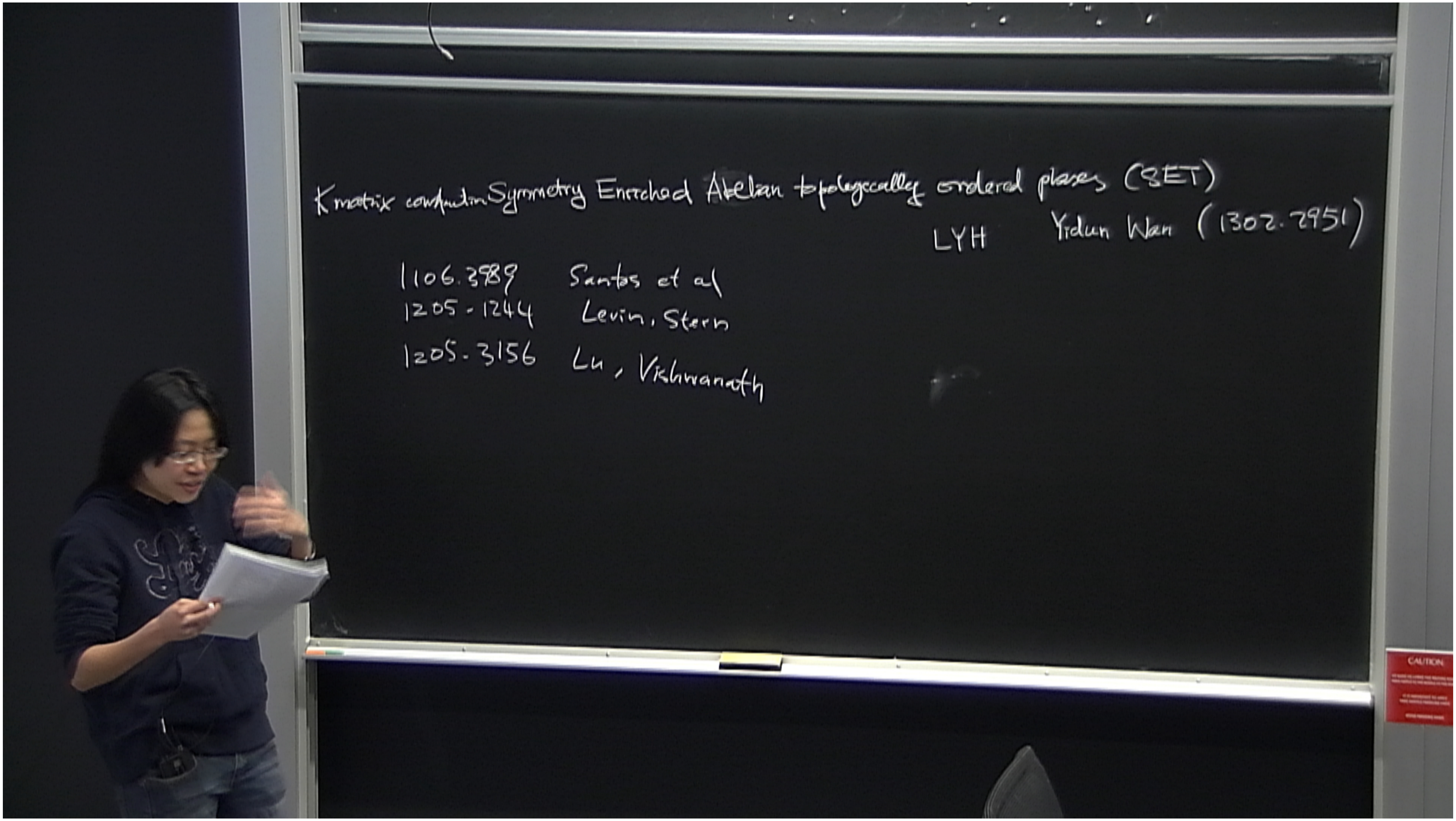
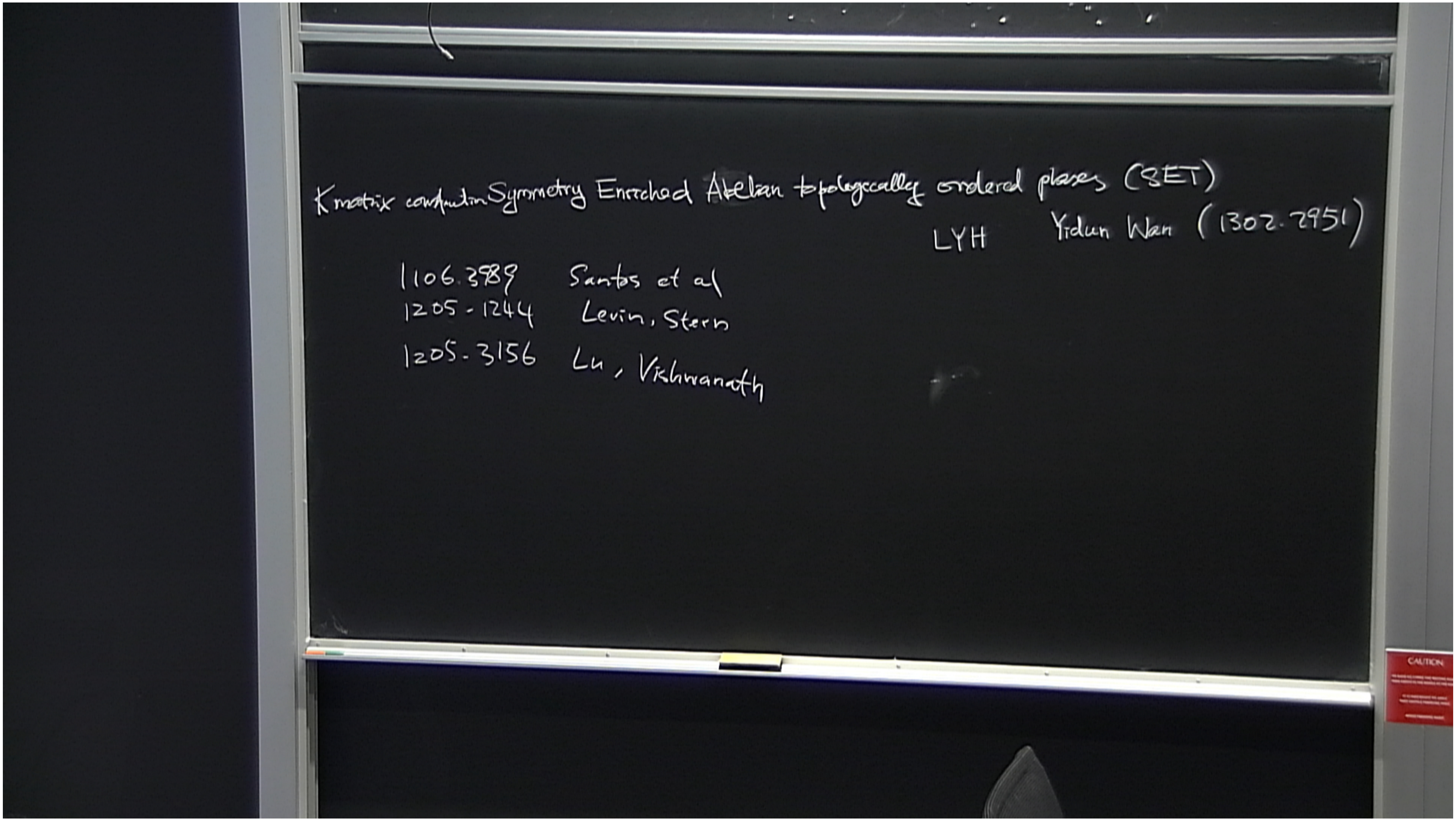
Title: A K matrix construction of symmetry enriched phases

Date: Feb 20, 2013 10:30 AM

URL: http://www.pirsa.org/13020140

Abstract: <span>We construct in the K matrix formalism concrete examples of symmetry enriched topological phases, namely intrinsically topological phases with global symmetries. We focus on the Abelian and non-chiral topological phases and demonstrate by our examples how the interplay between the global symmetry and the fusion algebra of the anyons of a topologically ordered system determines the existence of gapless edge modes protected by the symmetry and that a (quasi)-group structure can be defined among these phases. Our examples include phases that display charge fractionalization and more exotic non-local anyon exchange under global symmetry that correspond to general group extensions of the global symmetry group.</span>


K matrix construction Symmetry Enriched Abelian topologically ordered phases (SET)
LYH Yidun Wan (1302.2951)
1106.3989 Santos et al
1205-1244 Levin, Stern
1205.3156 Lu, Vishwanath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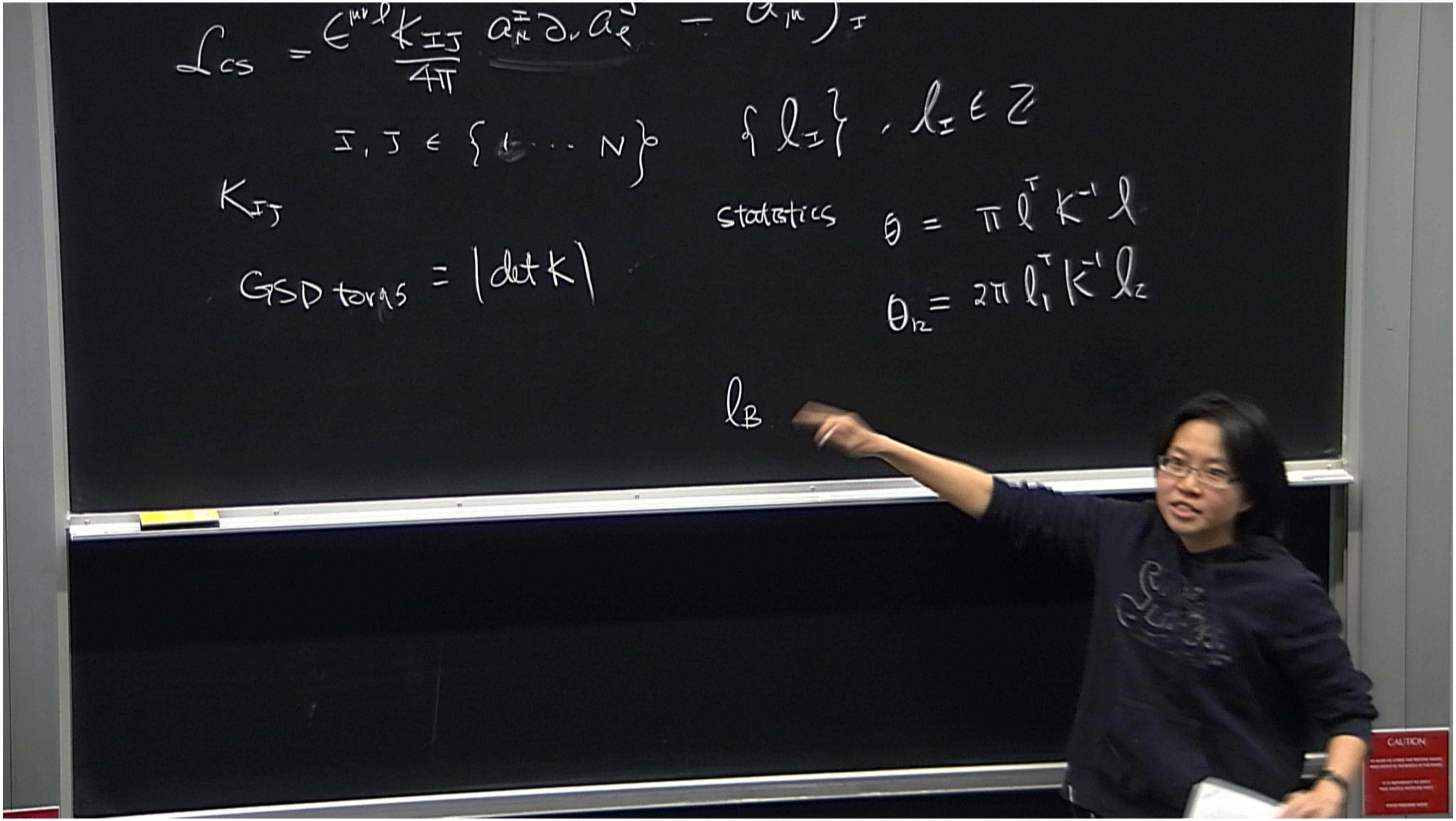
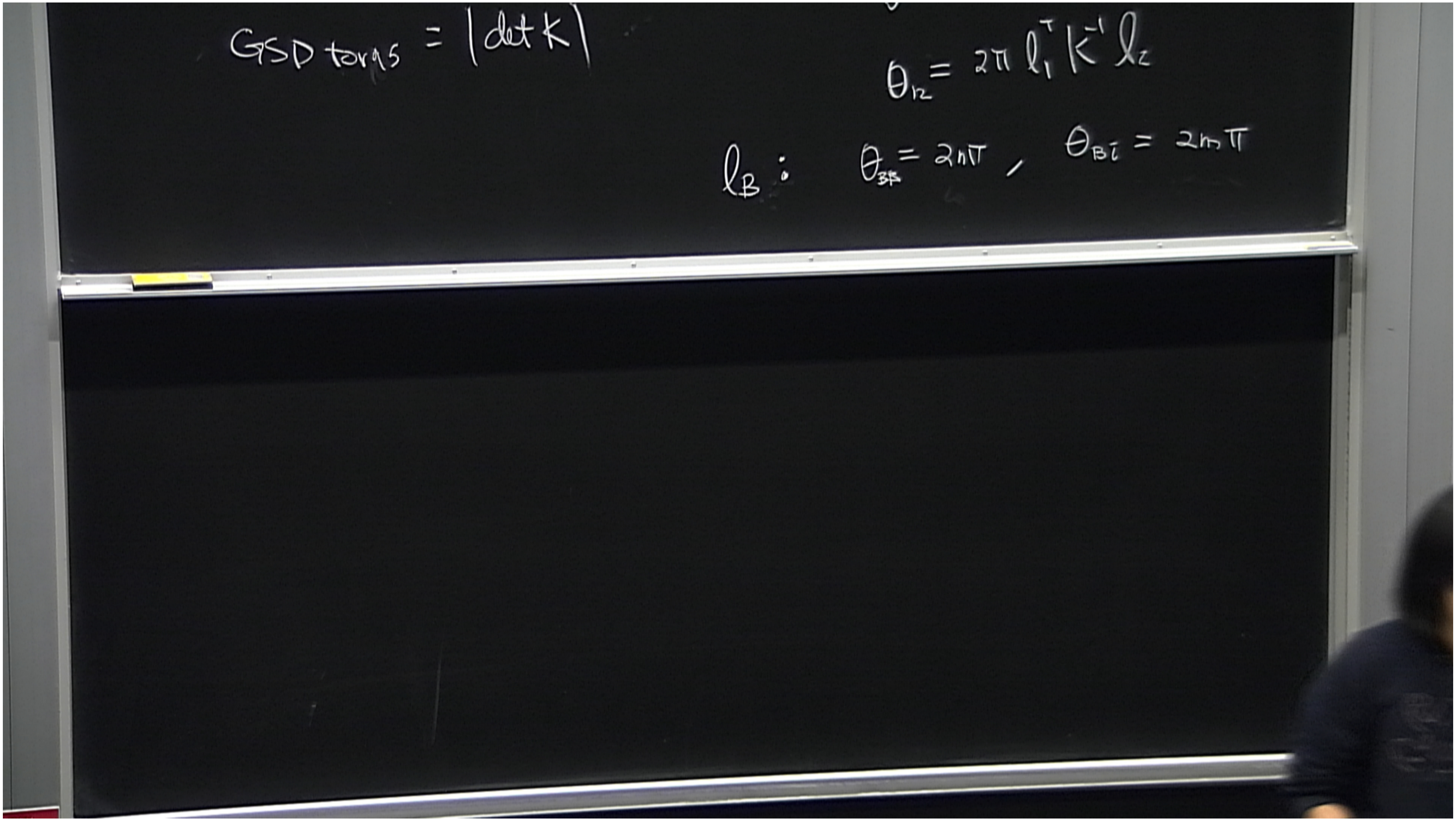
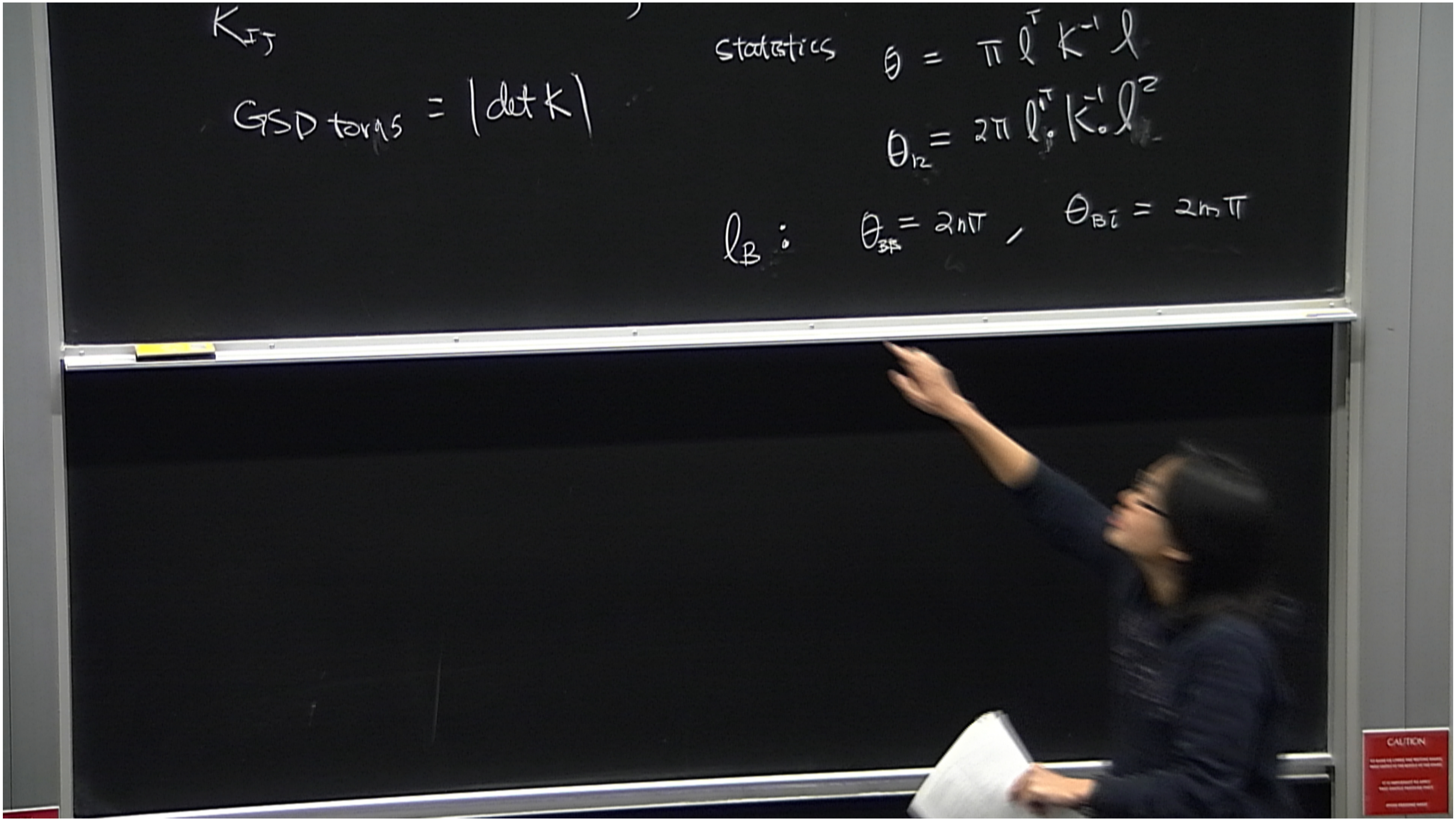
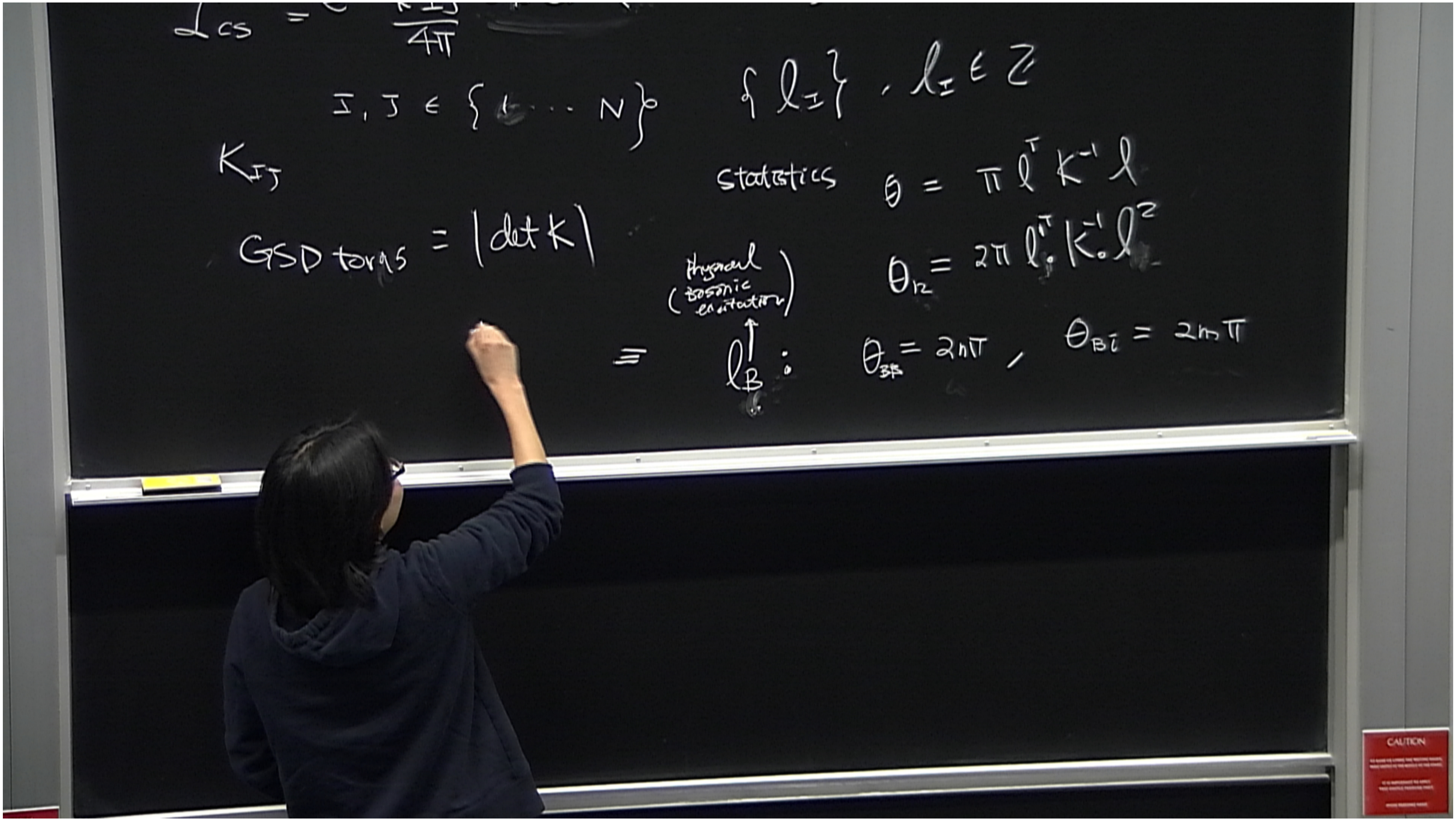
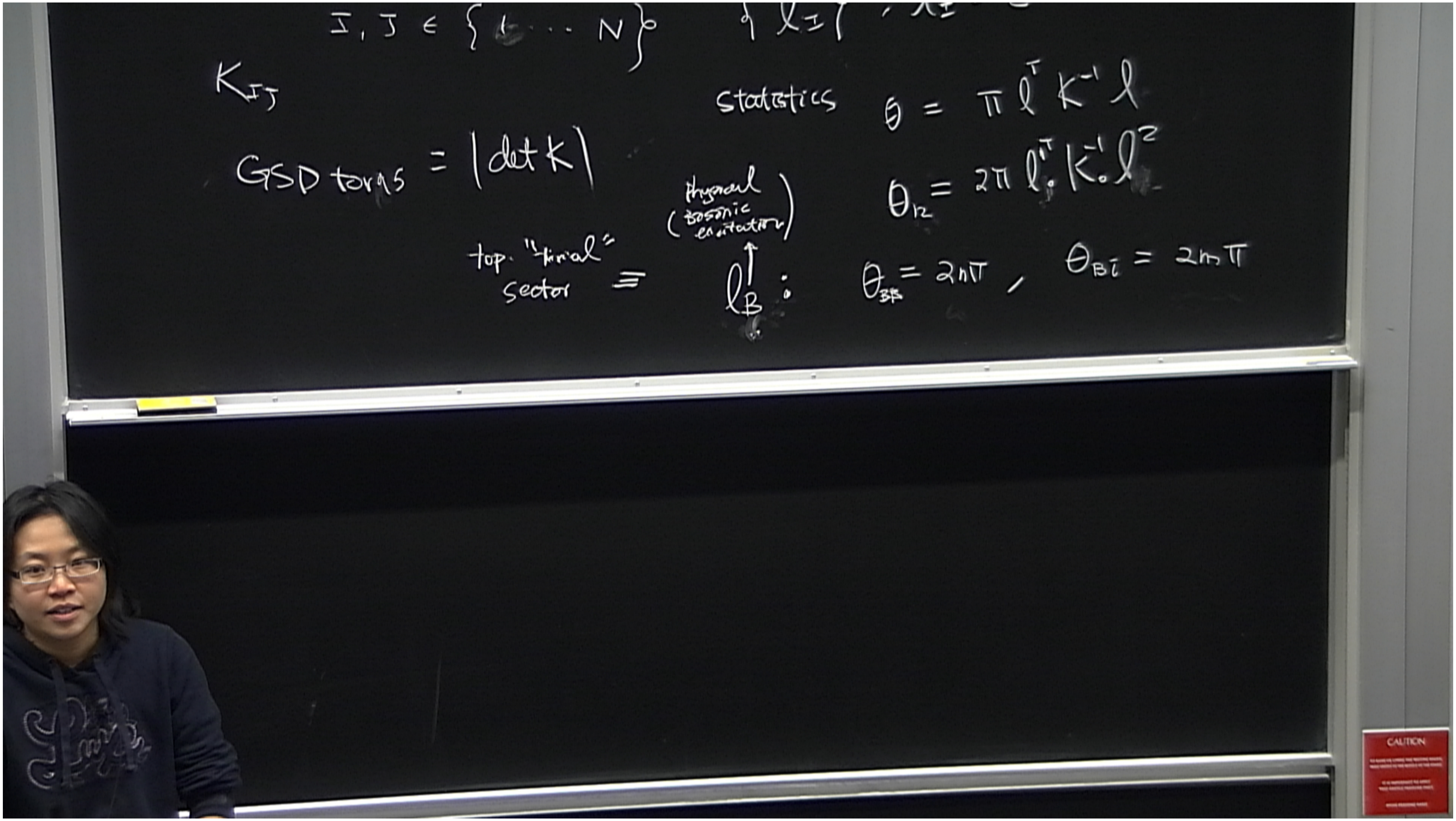

I, J ∈ {1 … N}
{l_I}, l_I ∈ Z
K_IJ
statistics θ = π l^T K^-1 l
GSD torus = |det K|
θ_12 = 2π l_1^T K^-1 l_2
l_B

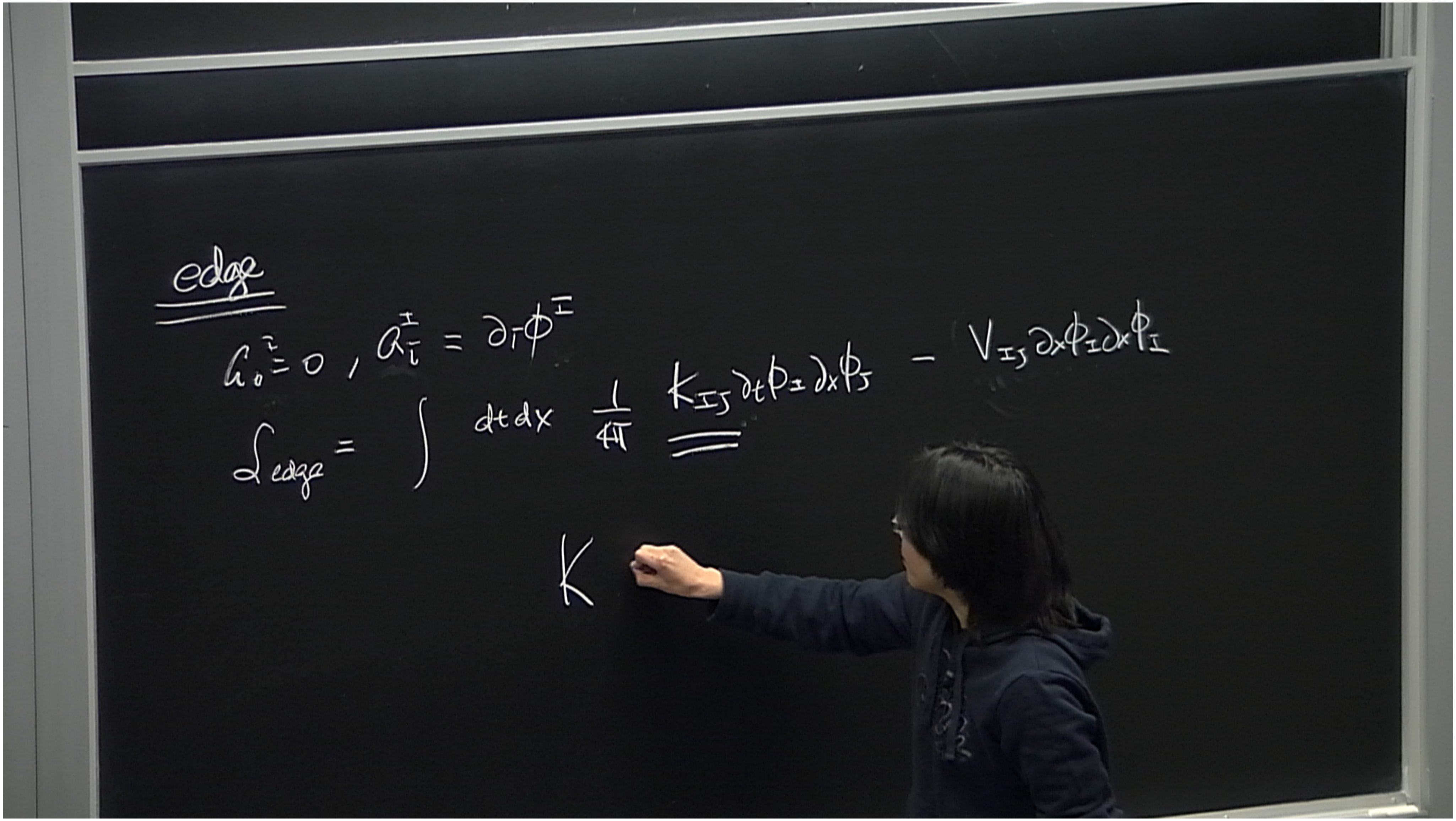

GSD torus = |det K|
θ12 = 2π l1T K-1 l2
lB : θBB = 2nπ , θBi = 2mπ

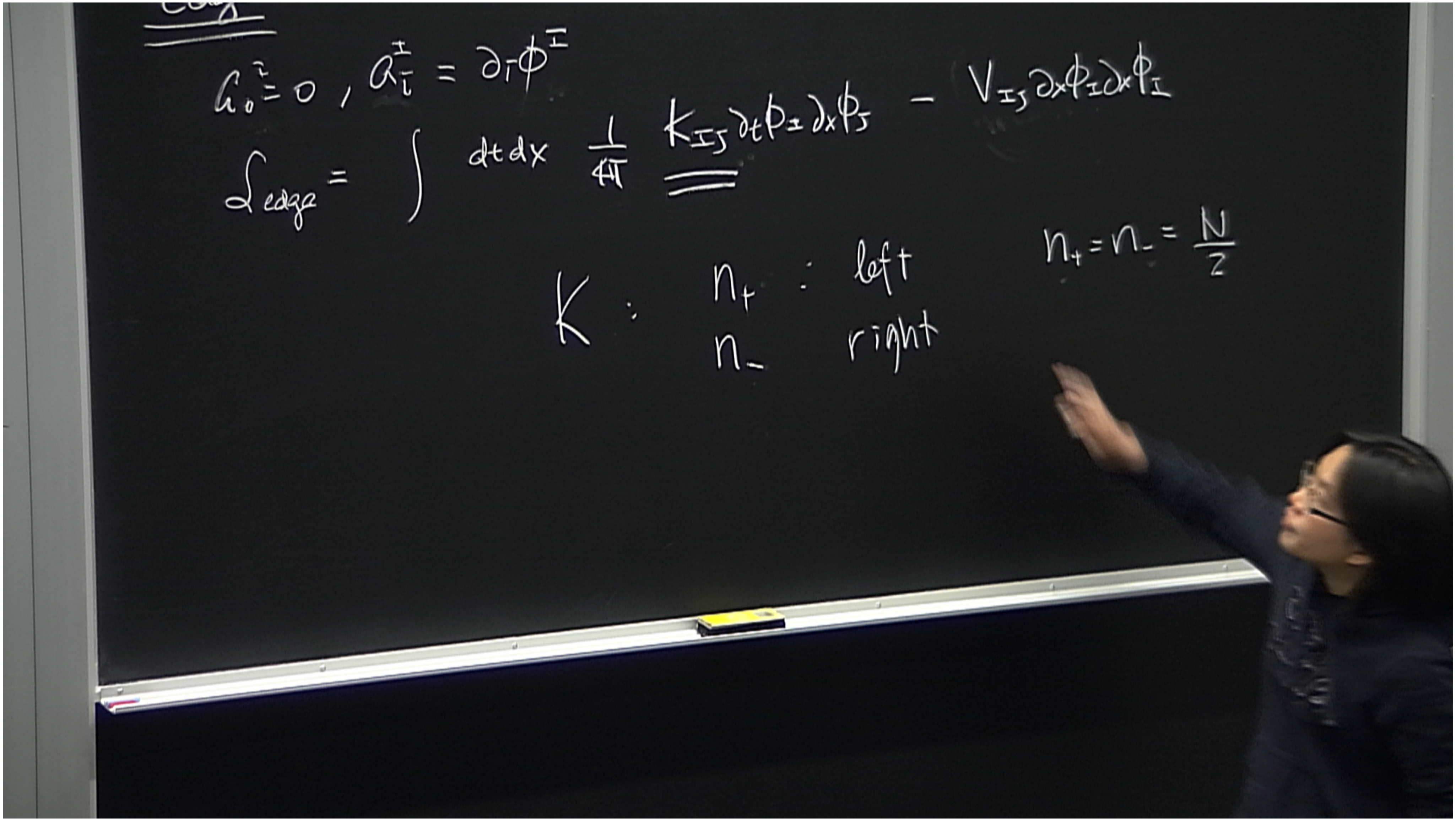

$K_{IJ}$
statistics $\theta = \pi l^T K^{-1} l$
GSD torus $= |\det K|$
$\theta_{12} = 2\pi l_1^T K^{-1} l_2$
$l_B$ : $\theta_{BB} = 2n\pi$ , $\theta_{Bi} = 2m\pi$

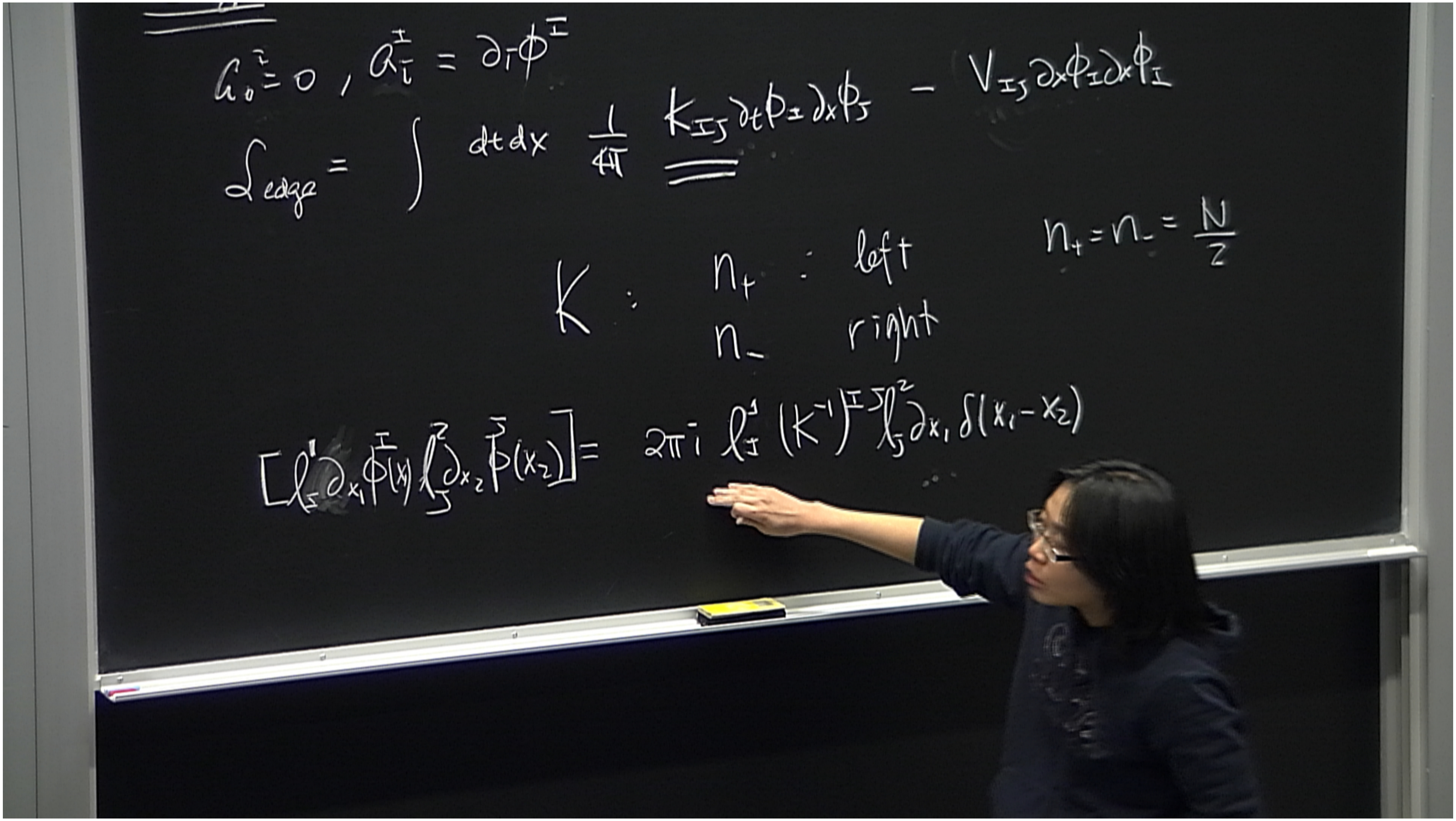

I, J ∈ {1 ... N}
{l_I}, l_I ∈ Z
K_IJ
statistics θ = π l^T K^-1 l
GSD torus = |det K|
θ_12 = 2π l_1^T K^-1 l_2
(physical bosonic excitation)
≡ l_B : θ_BB = 2nπ , θ_Bi = 2mπ

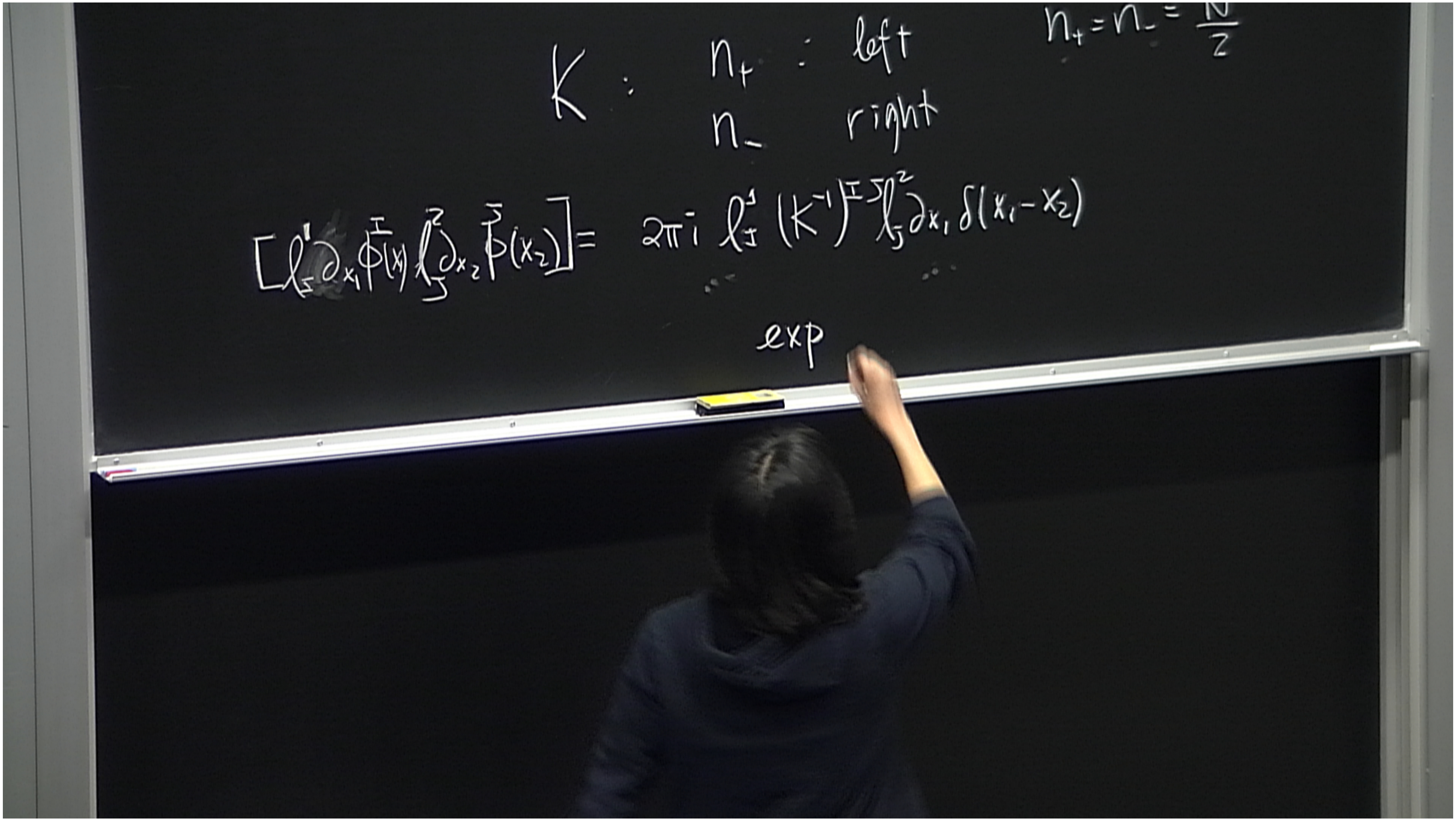


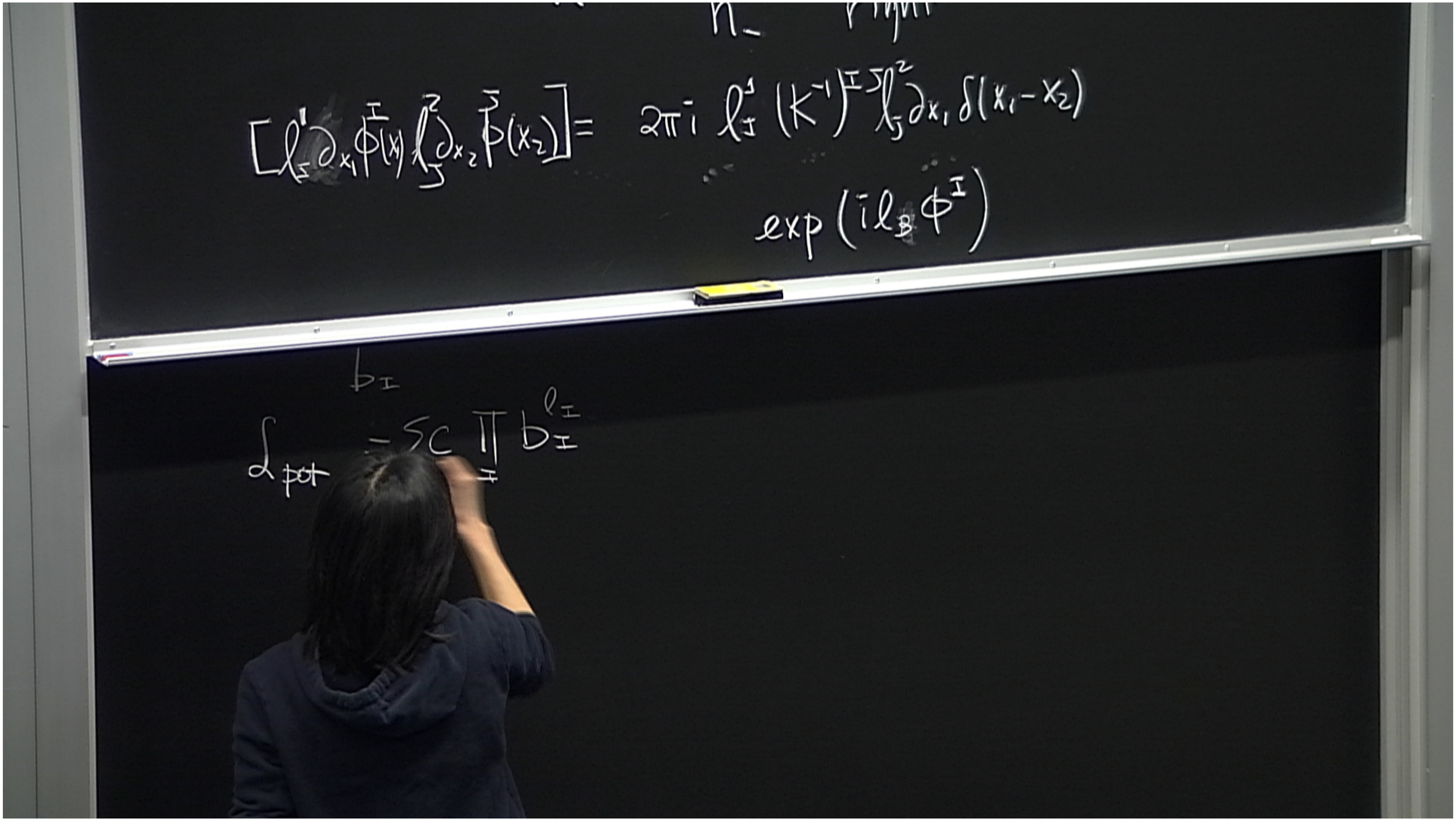
I, J ∈ {1 … N}
K_IJ
GSD torus = |det K|
statistics θ = π l^T K^-1 l
θ_12 = 2π l_1^T K^-1 l_2
physical bosonic excitation
top. "trivial" sector ≡ l_B : θ_BB = 2nπ , θ_Bi = 2mπ
CAUTION

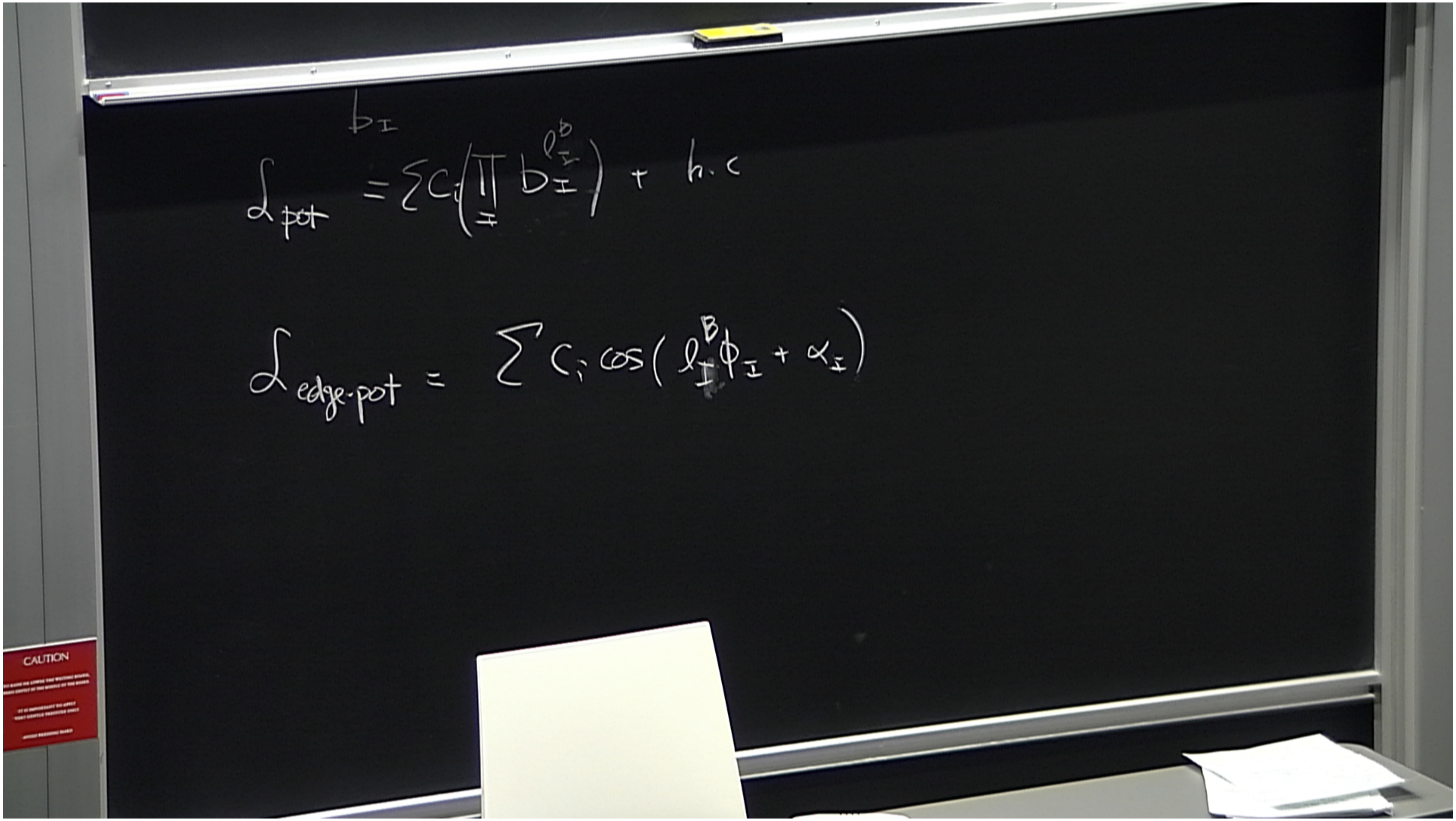

edge
$a_0^I = 0$ , $a_i^I = \partial_i \phi^I$
$\mathcal{L}_{edge} = \int dt\, dx \, \frac{1}{4\pi} K_{IJ} \partial_t \phi_I \partial_x \phi_J - V_{IJ} \partial_x \phi_I \partial_x \phi_J$
K

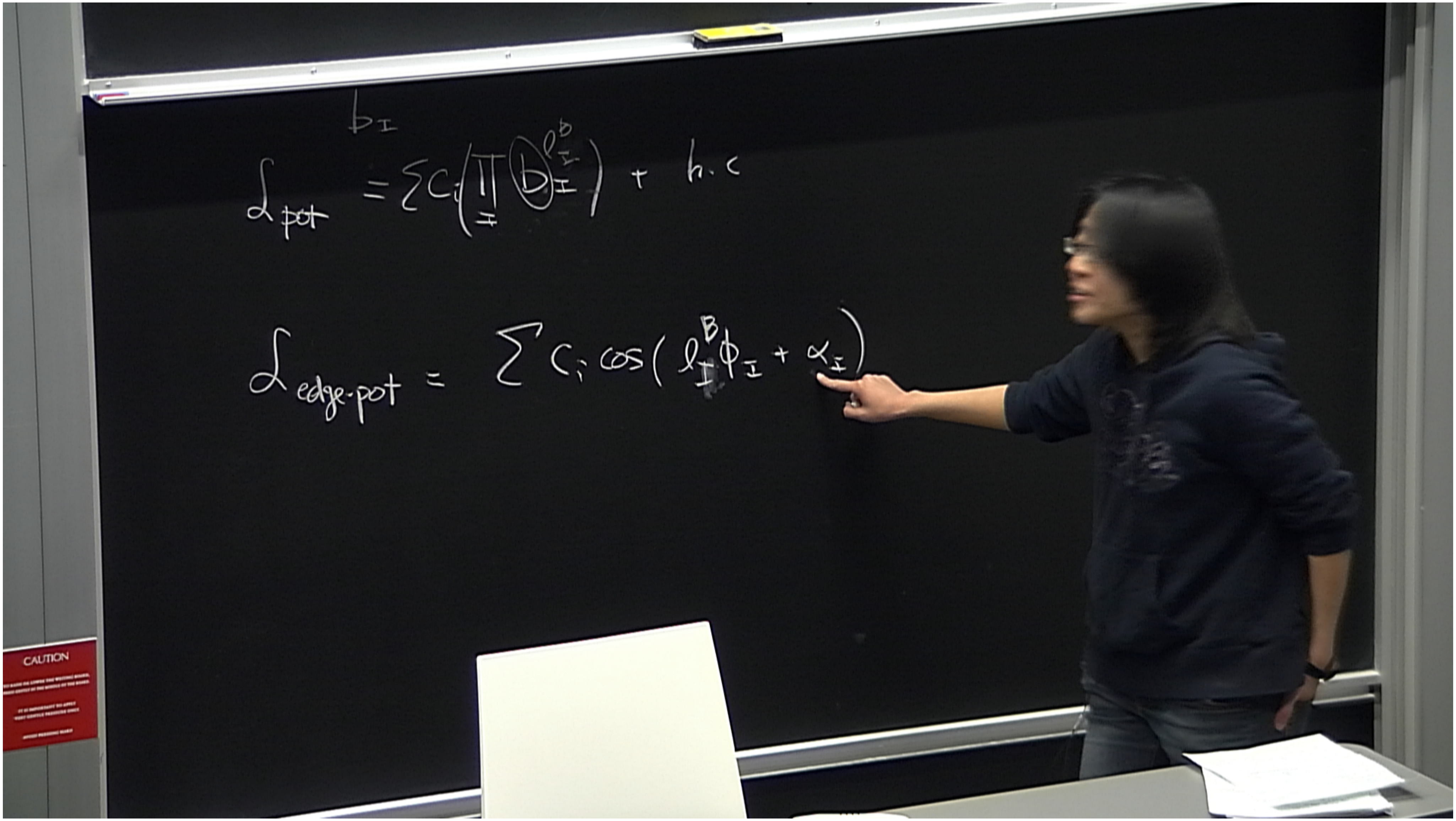

$a_0^I = 0$ , $a_i^I = \partial_i \phi^I$
$\mathcal{L}_{edge} = \int dt\,dx \frac{1}{4\pi} K_{IJ} \partial_t \phi_I \partial_x \phi_J - V_{IJ} \partial_x \phi_I \partial_x \phi_J$
$K$ : $n_+$ : left
$n_-$ right
$n_+ = n_- = \frac{N}{2}$


edge
dt dx
K : n+ : left
n- right
n+ = n- = N/2


K : n+ : left
n- right
n+ = n- = N/2
[l¹_I ∂x₁ φ^I(x₁), l²_J ∂x₂ φ^J(x₂)] = 2πi l¹_I (K⁻¹)^IJ l²_J ∂x₁ δ(x₁ − x₂)
exp


[l^1_I ∂x1 φ^I(x1) l^2_J ∂x2 φ^J(x2)] = 2πi l^1_I (K^-1)^IJ l^2_J ∂x1 δ(x1 - x2)
exp (i l_I φ^I)
b_I
L_pot = Σ C ∏_I b_I^(l_I)


$b_I$
$\mathcal{L}_{pot} = \sum C_i \left( \prod_I b_I^{l_I^B} \right) + h.c$
$\mathcal{L}_{edge\text{-}pot} = \sum C_i \cos( l_I^B \phi_I + \alpha_I )$
CAUTION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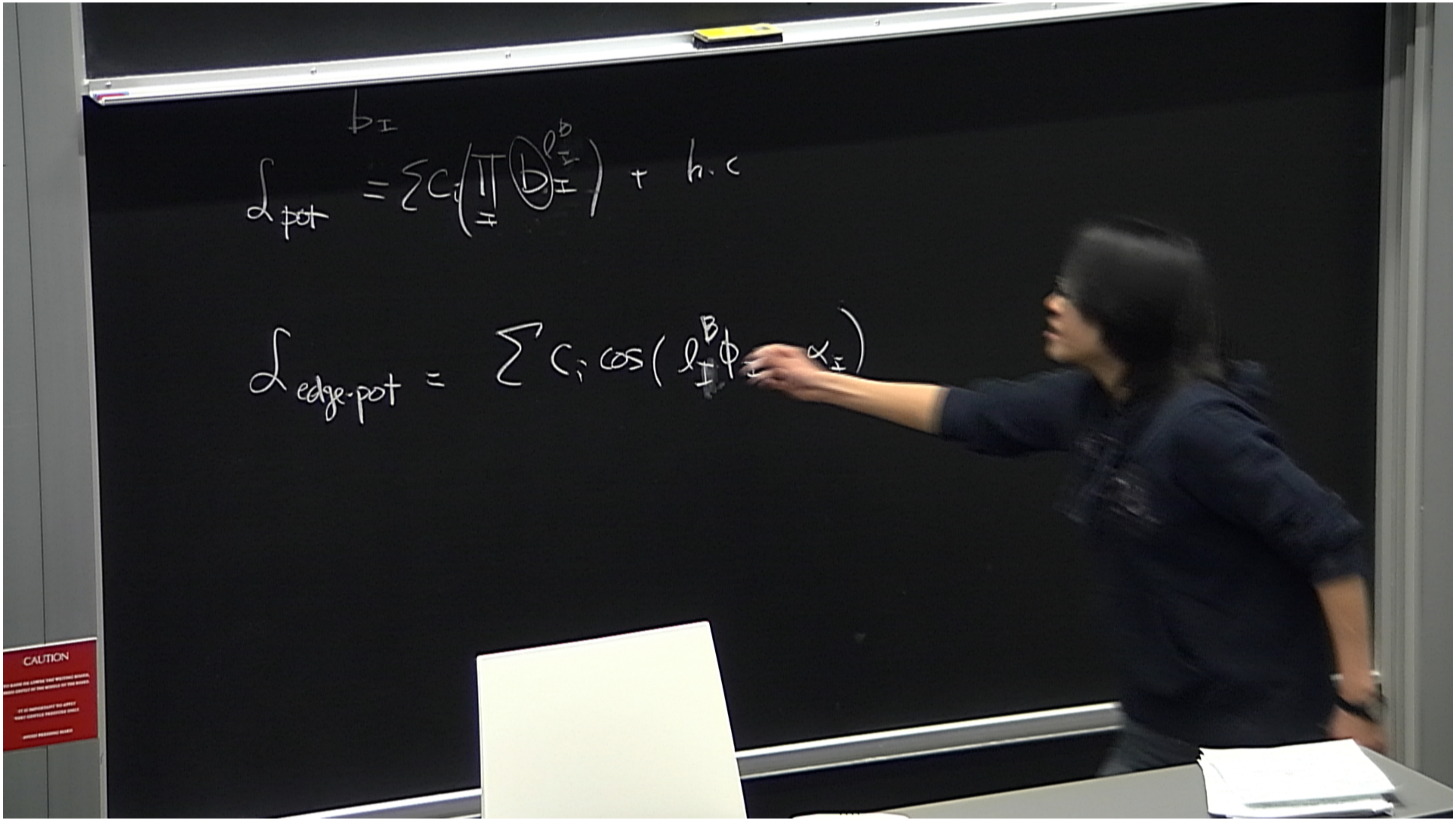
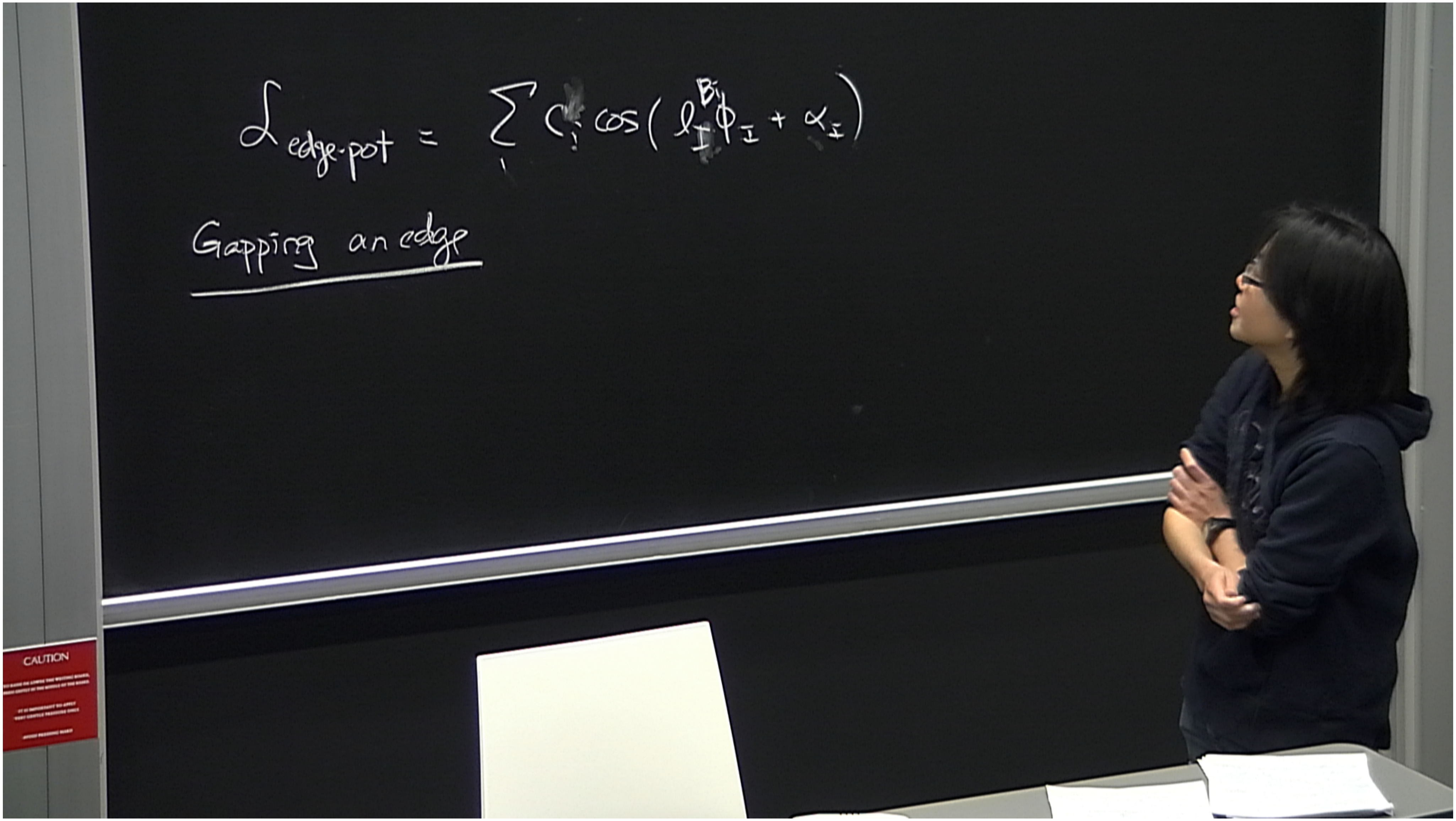
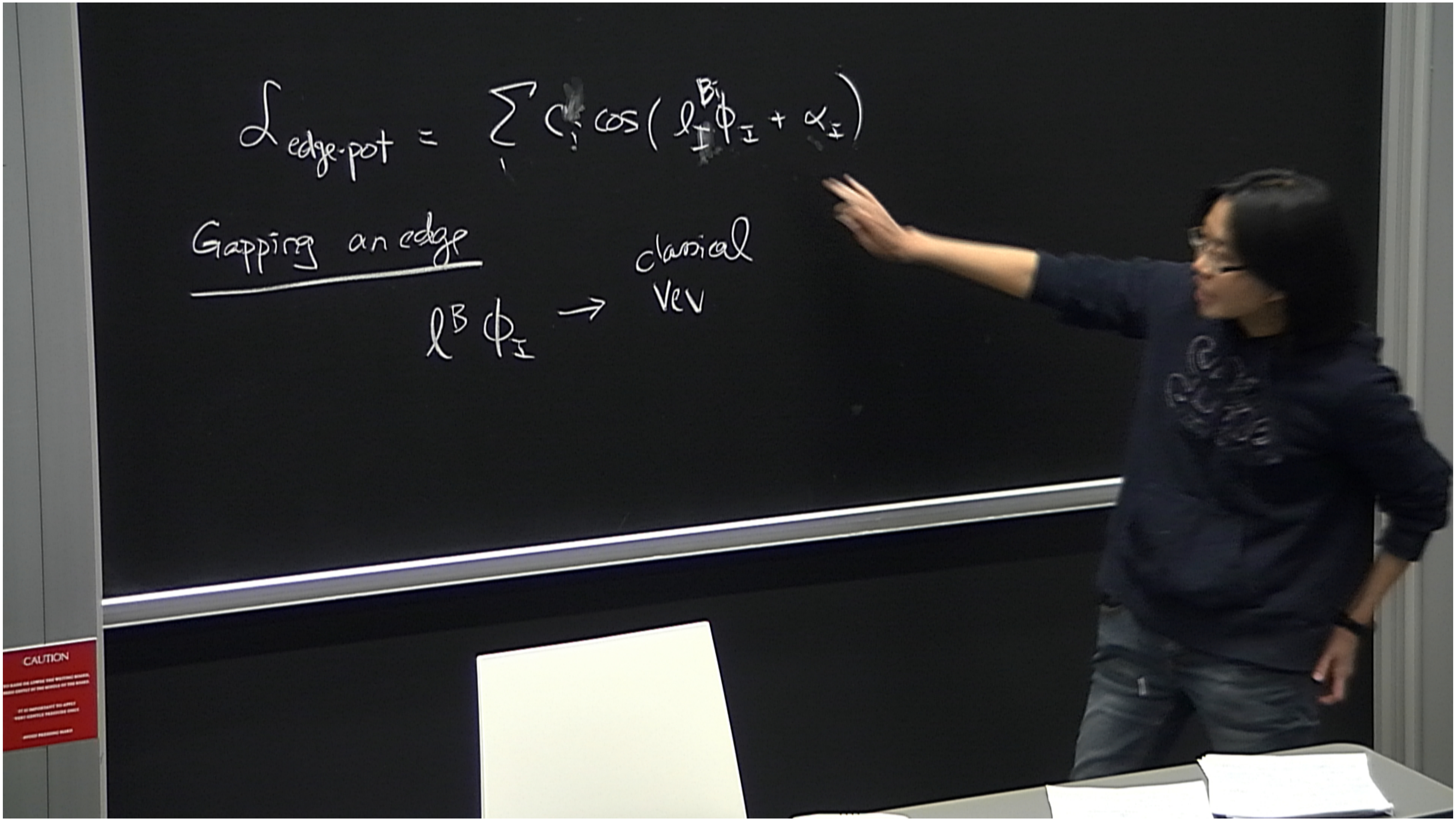
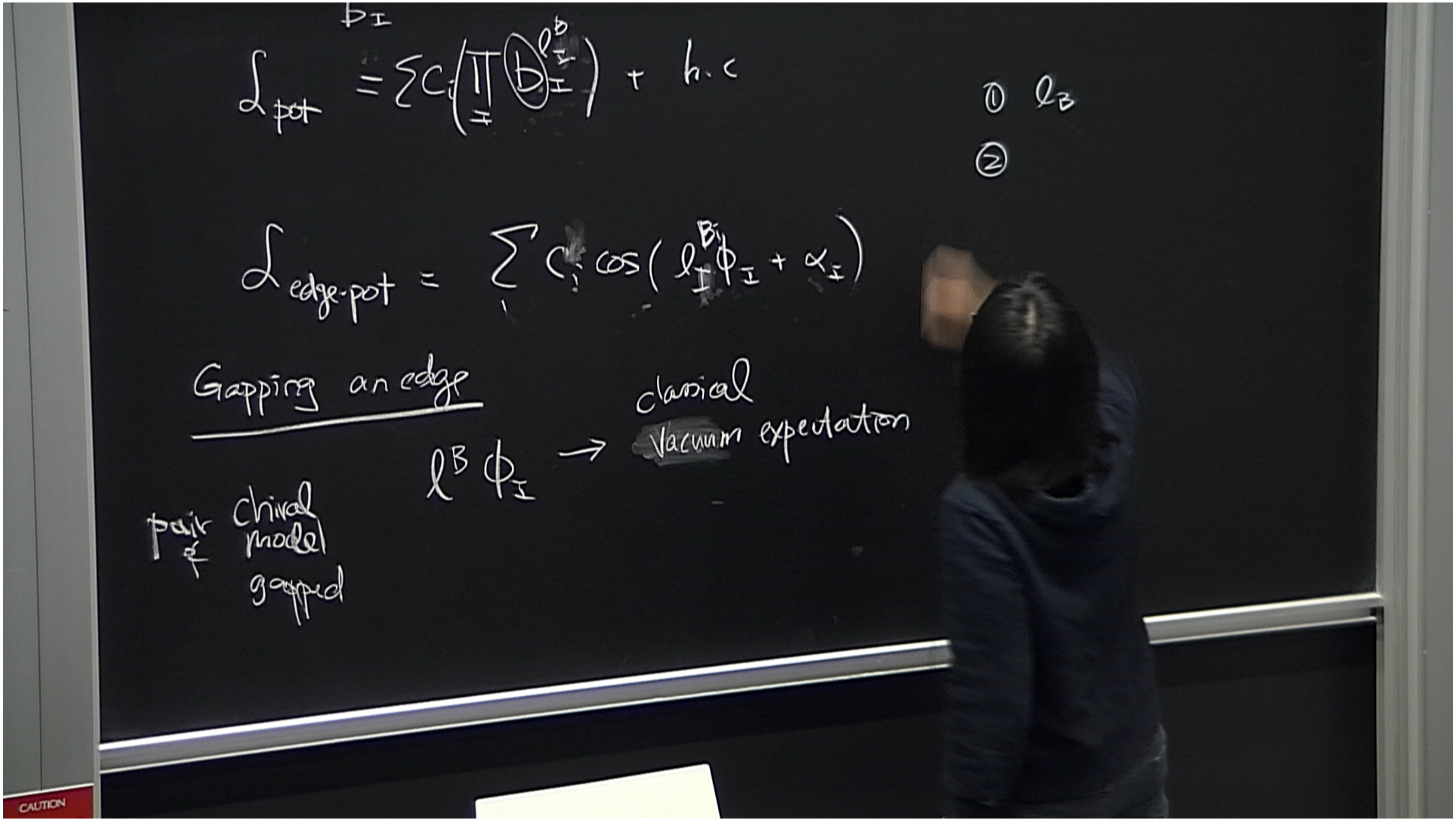
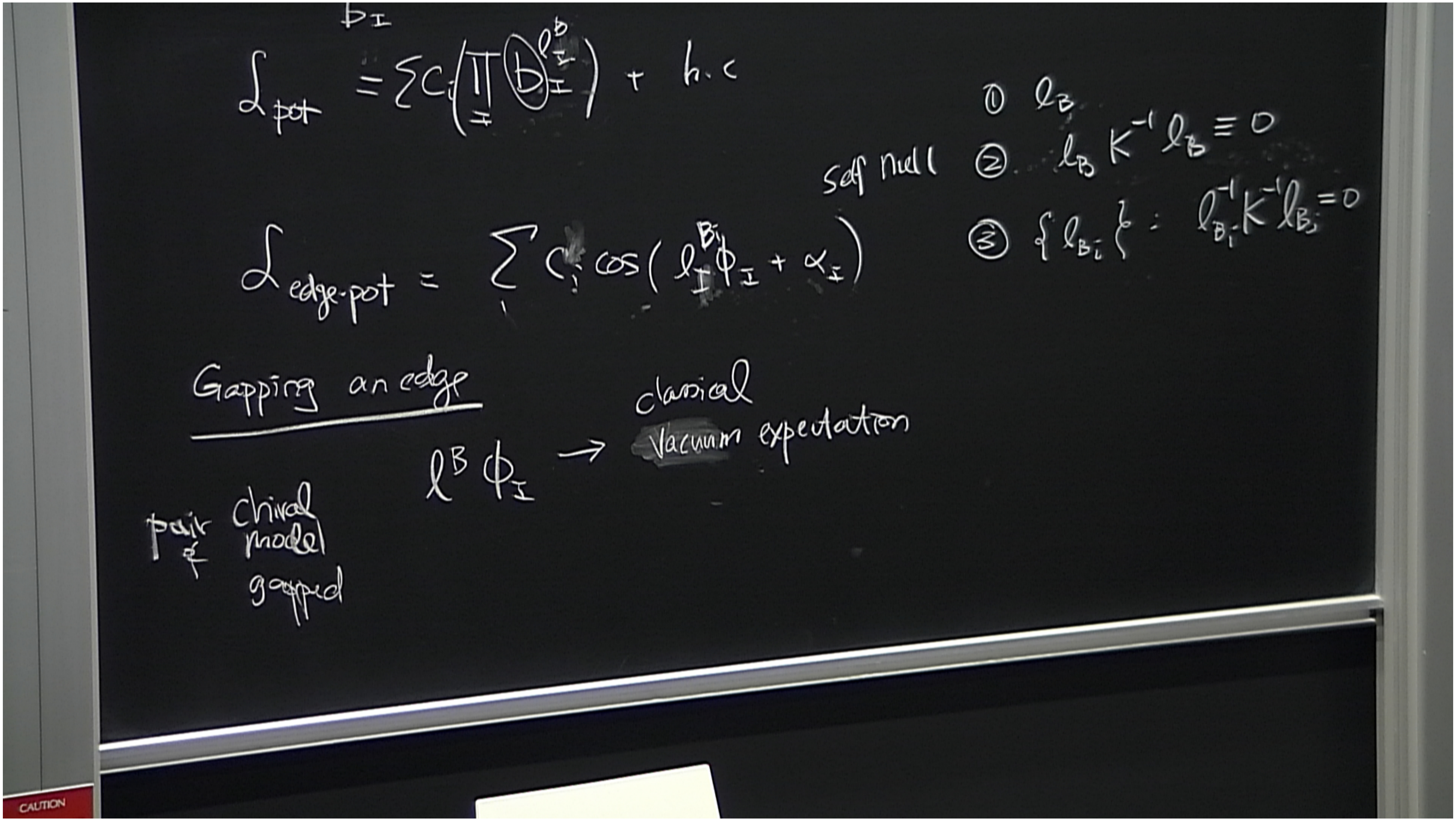
CAUTION


$b_I$
$\mathcal{L}_{pot} = \sum C_i \left( \prod_I b_I^{l_I^b} \right) + h.c$
$\mathcal{L}_{edge\text{-}pot} = \sum C_i \cos( l_I^B \phi_I - \alpha_I )$
CAUTION

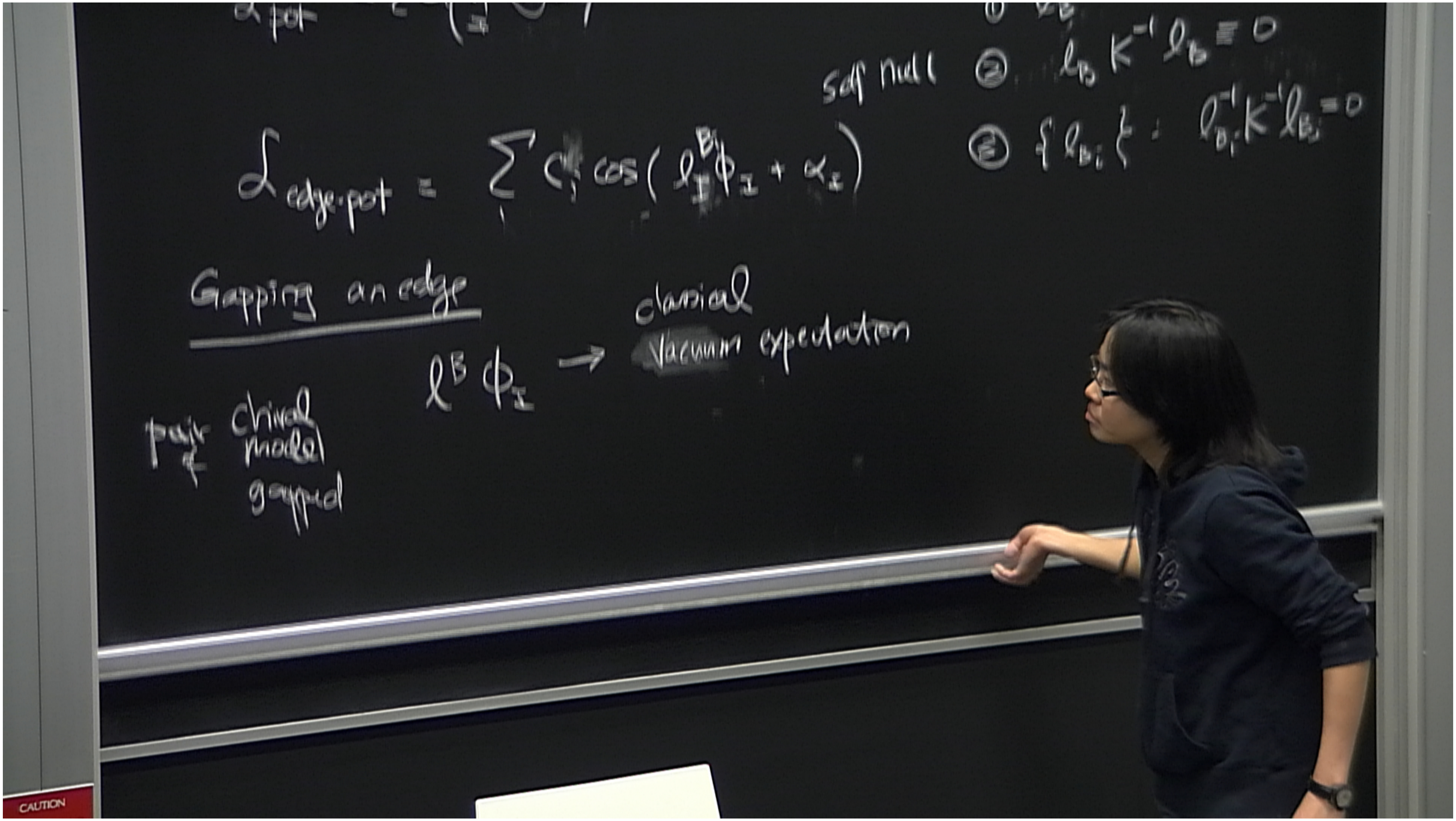

$\mathcal{L}_{edge\text{-}pot} = \sum_i C_i \cos( l_I^{B_i} \phi_I + \alpha_I )$
Gapping an edge
CAUTION


$\mathcal{L}_{edge\text{-}pot} = \sum_i C_i \cos(l_I^{B_i} \phi_I + \alpha_I)$
Gapping an edge
$l^B \phi_I \rightarrow$
classical
vev
CAUTION


$b_I$
$\mathcal{L}_{pot} = \sum C_i \left( \prod_I b_I^{l_I^b} \right) + h.c$
① $l_B$
②
$\mathcal{L}_{edge\text{-}pot} = \sum_i C_i \cos( l_I^{B_i} \phi_I + \alpha_I )$
Gapping an edge
$l^B \phi_I \rightarrow$ classical vacuum expectation
pair of chiral model gapped



$b_I$
$\mathcal{L}_{pot} = \sum C_i \left( \prod_I b_I^{l_I^B} \right) + h.c$
① $l_B$
self null ② $l_B K^{-1} l_B \equiv 0$
③ $\{ l_{B_i} \}$ : $l_{B_i}^{-1} K^{-1} l_{B_j} = 0$
$\mathcal{L}_{edge\text{-}pot} = \sum_i C_i \cos( l_I^{B_i} \phi_I + \alpha_I )$
Gapping an edge
classical
$l^B \phi_I \rightarrow$ vacuum expectation
pair chiral model gapped
CAUTION


self null
Gapping an edge
classical
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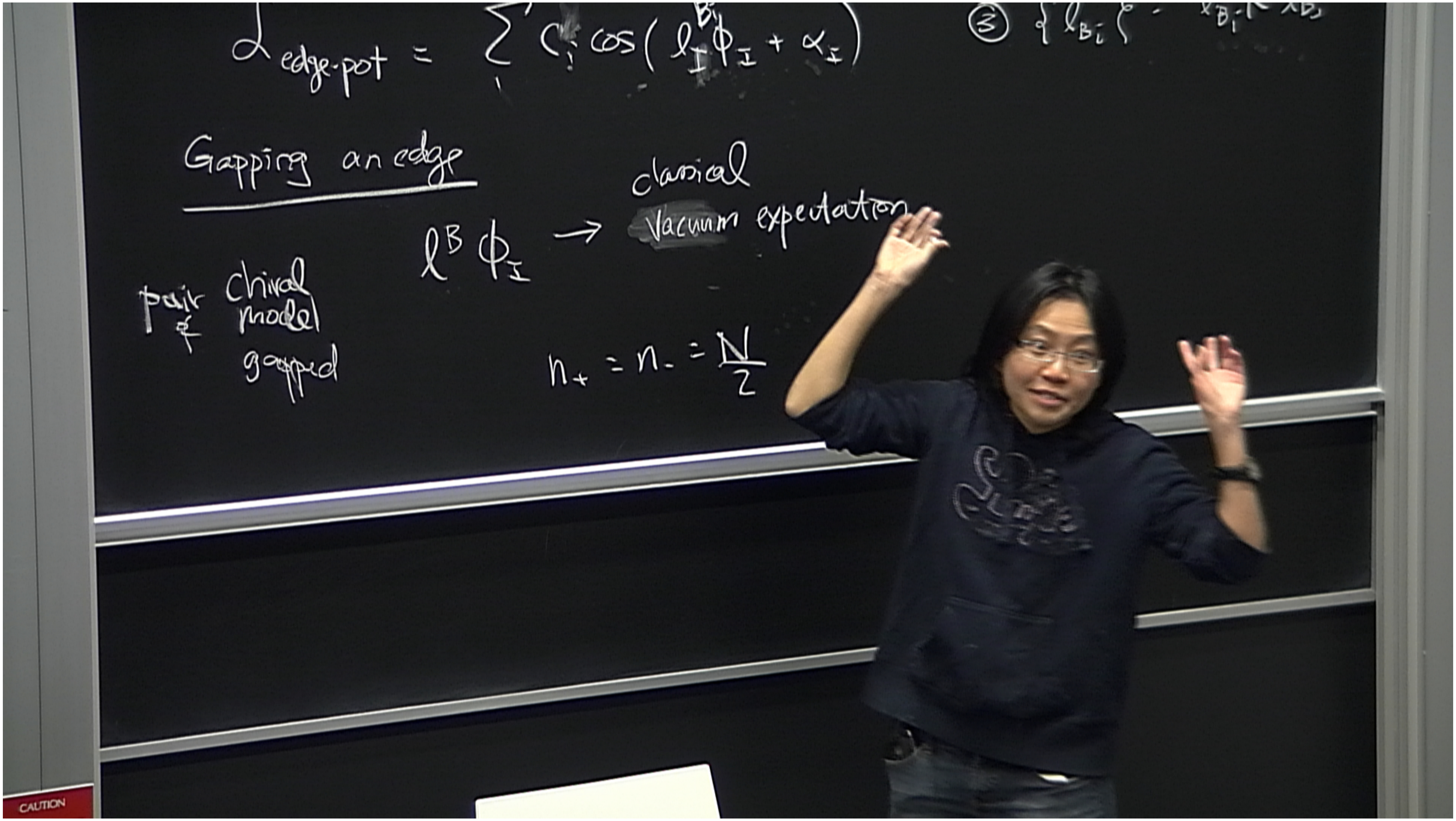
Gapping an edge
classical
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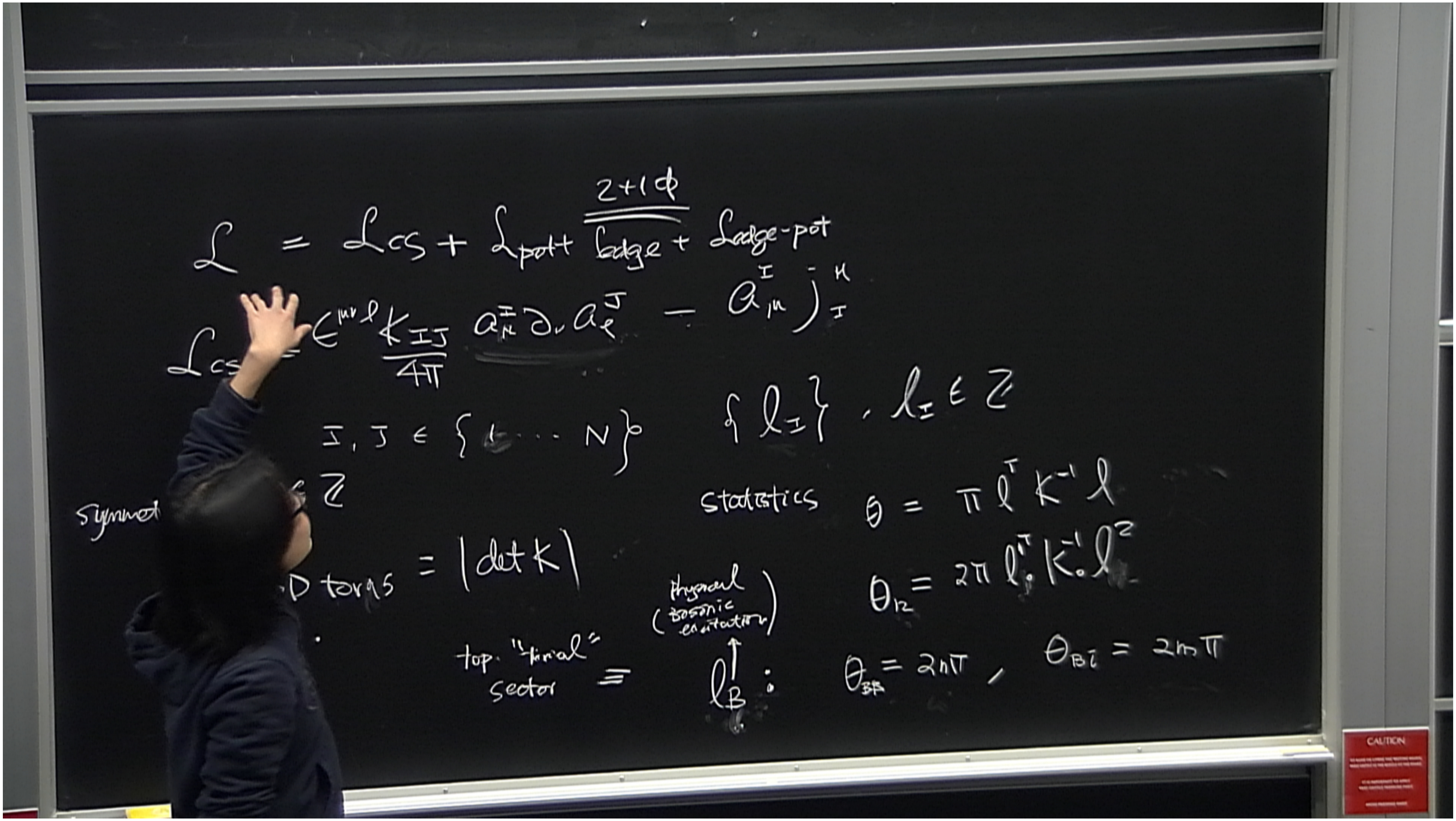
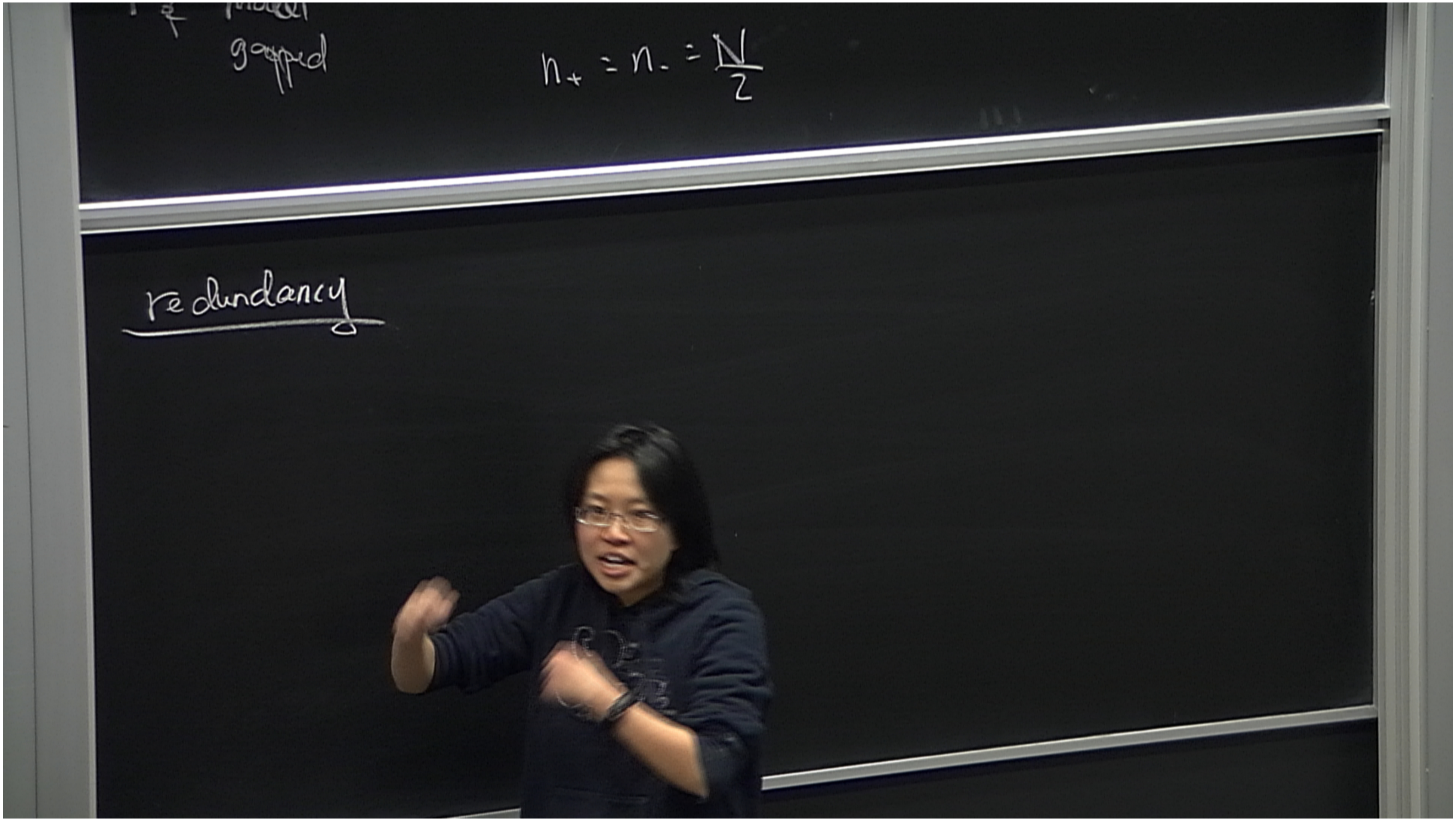
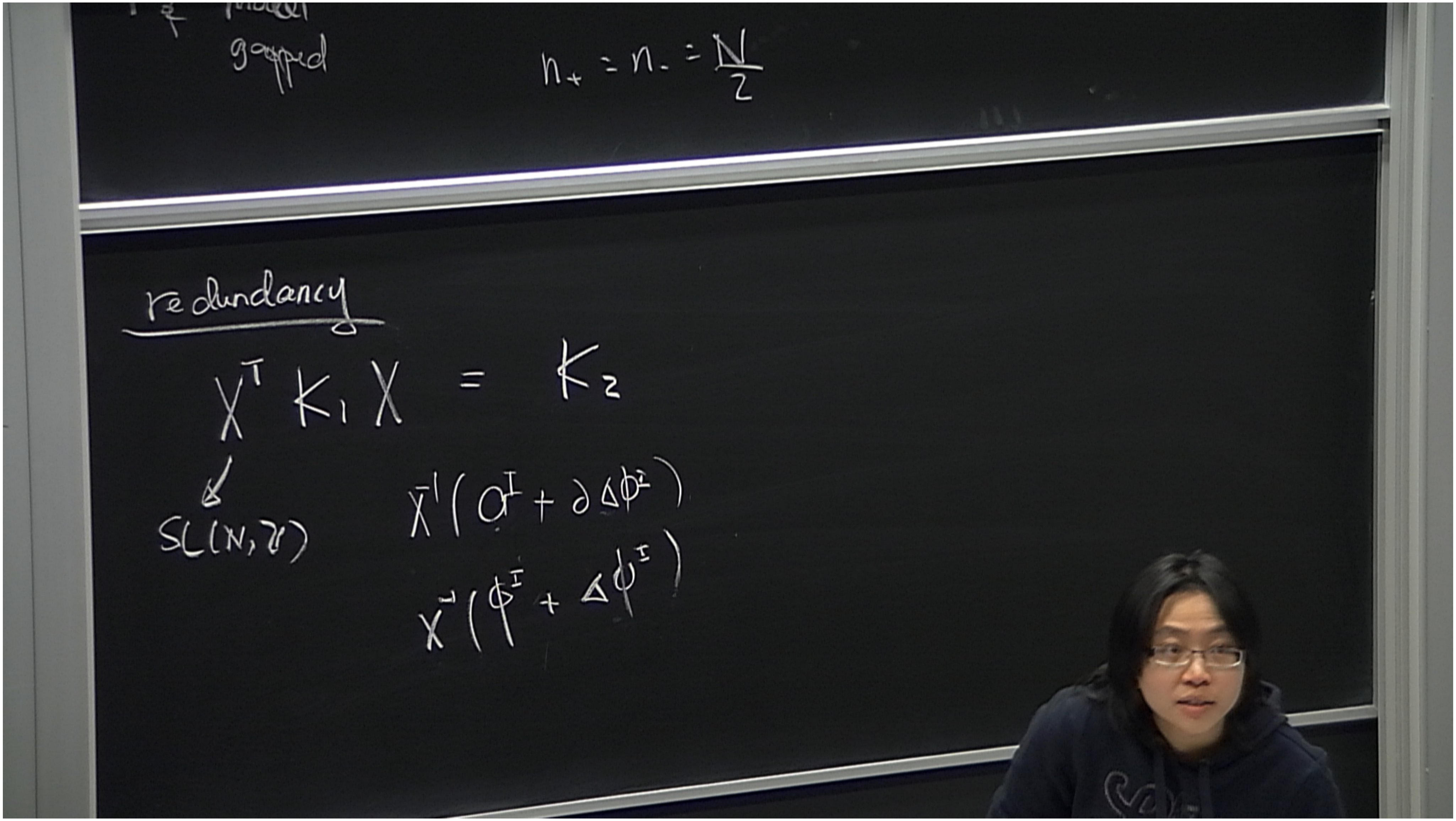
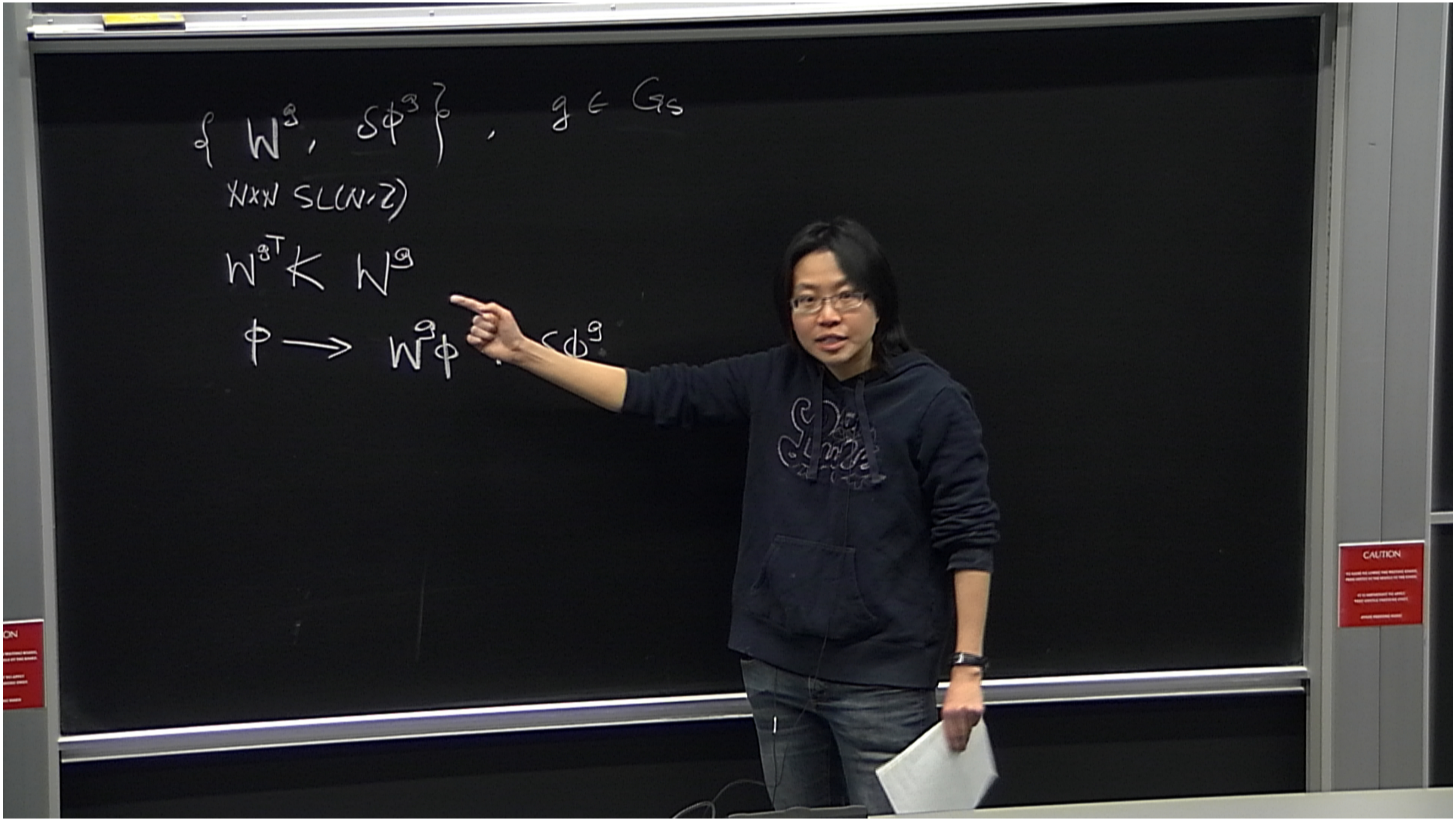
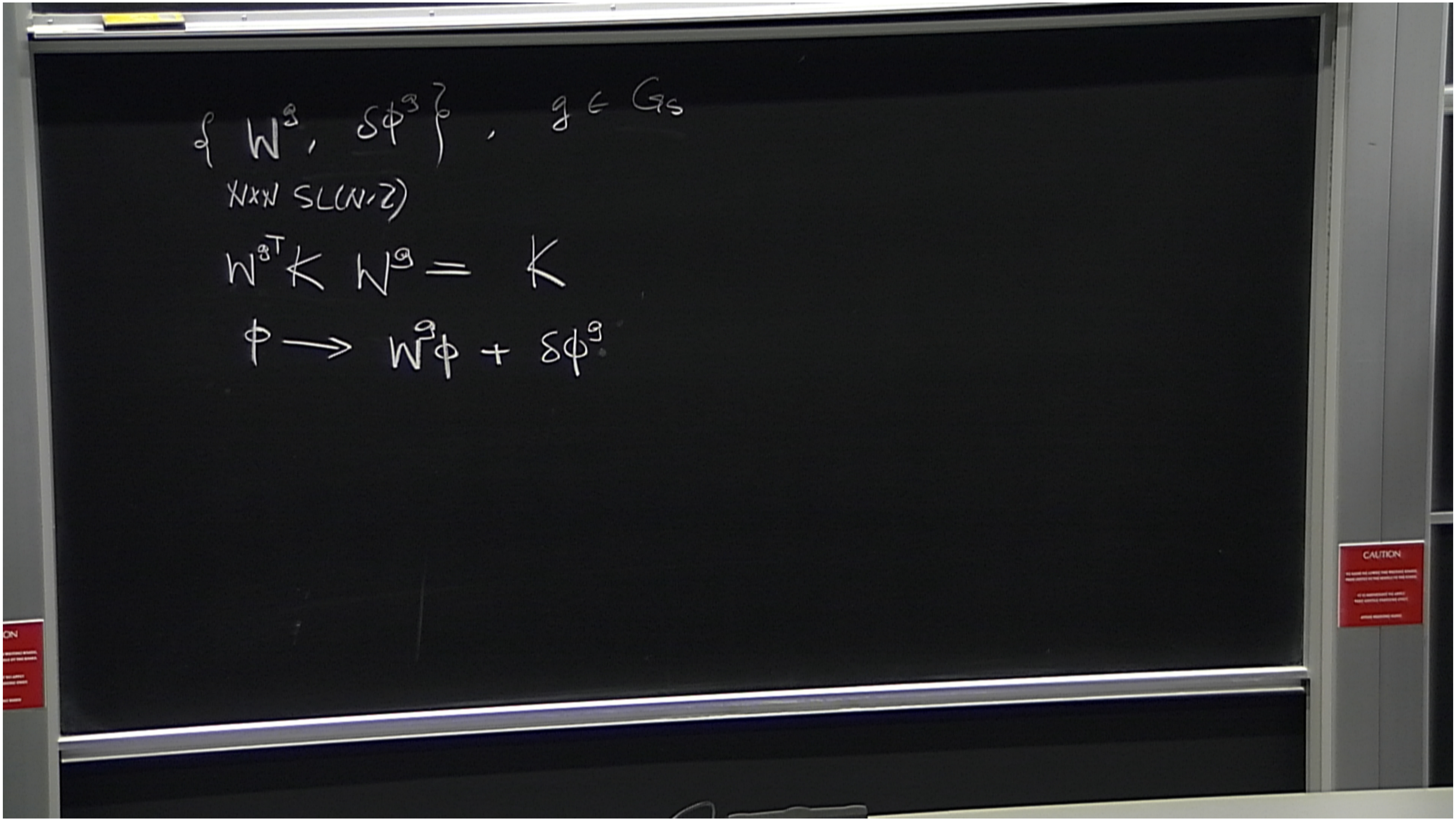
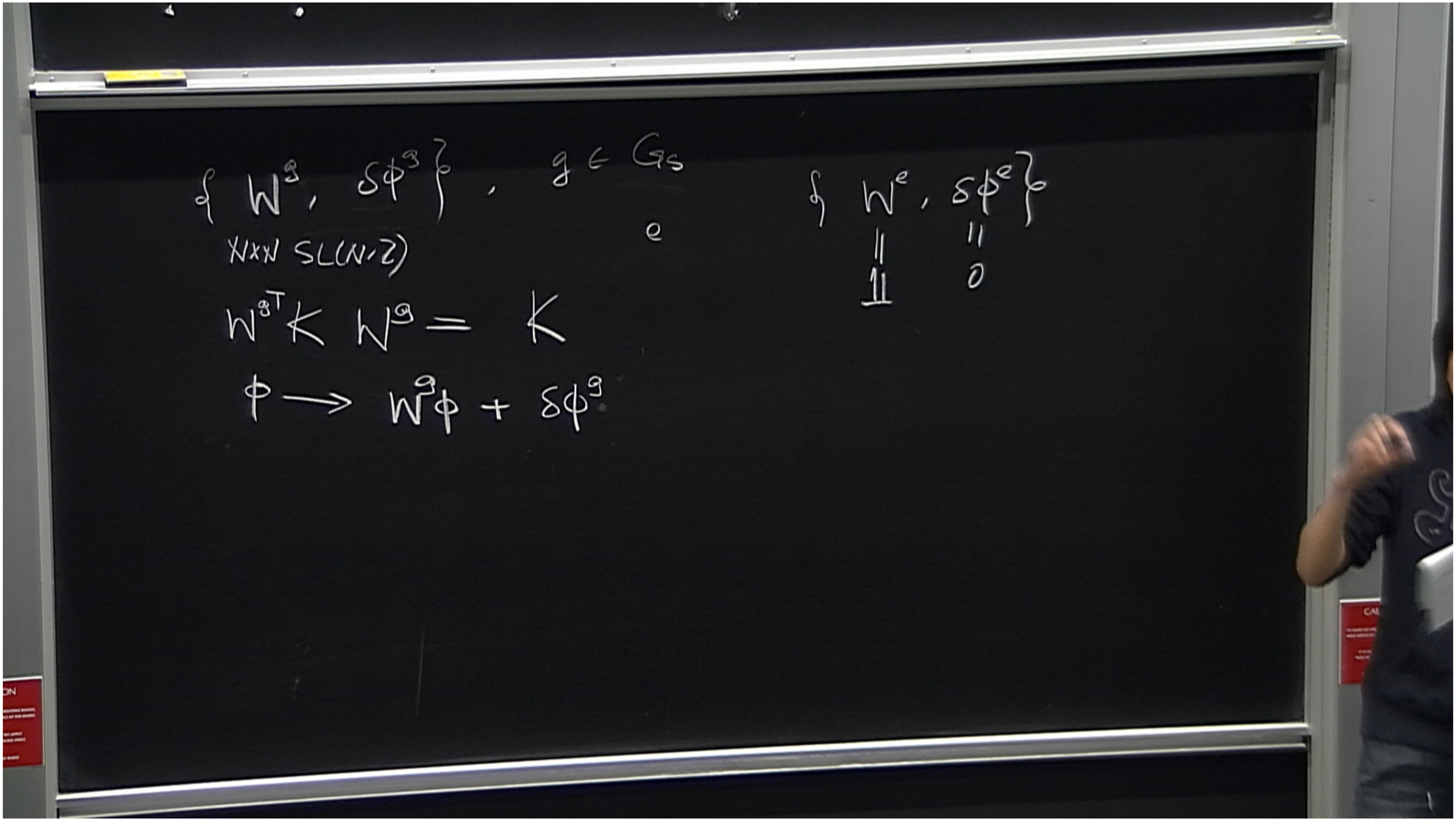

2+1 φ
L = Lcs + Lpot + Ledge + Ledge-pot
I, J ∈ {1 … N}
{l_I}, l_I ∈ Z
statistics
θ = π l^T K^-1 l
= |det K|
physical (bosonic excitation)
top. "trivial" sector
l_B :
θ_BB = 2nπ ,
θ_Bi = 2mπ
CAUTION

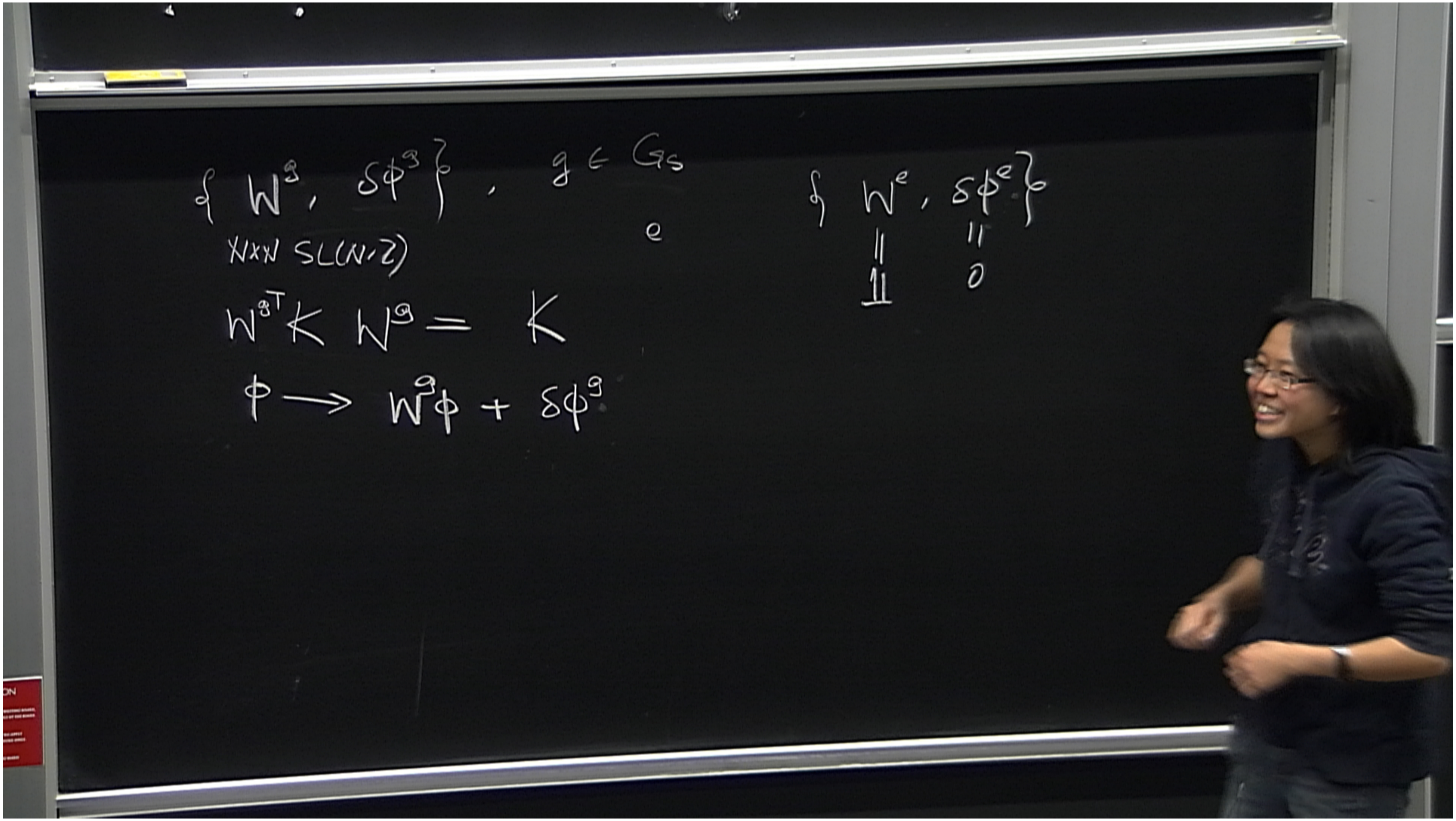

gapped
n+ = n- = N/2
redundancy

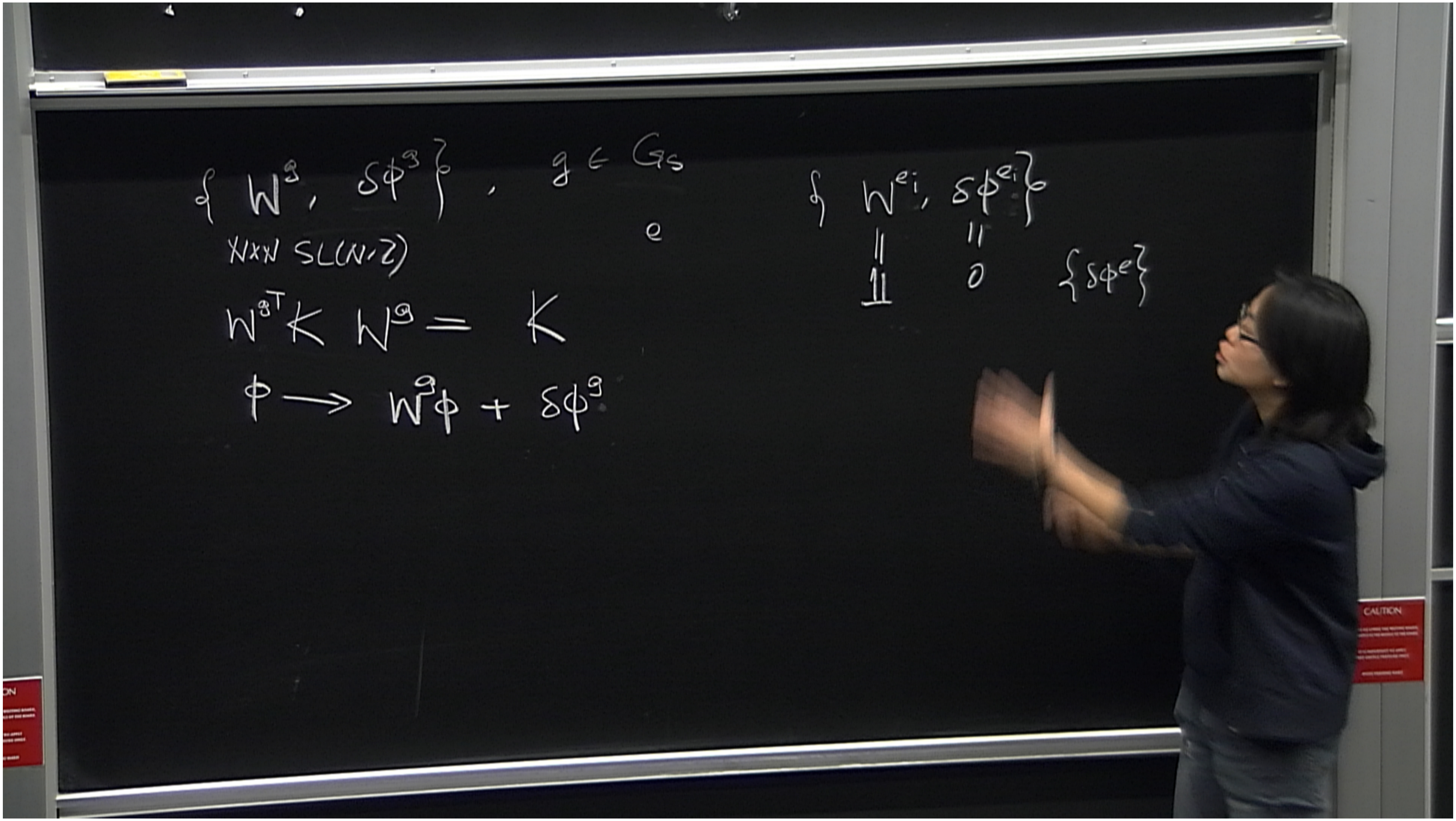

gapped
n₊ = n₋ = N/2
redundancy
X^T K_1 X = K_2
SL(N,Z)
X^{-1}(a^I + ∂Δφ^I)
X^{-1}(φ^I + Δφ^I)

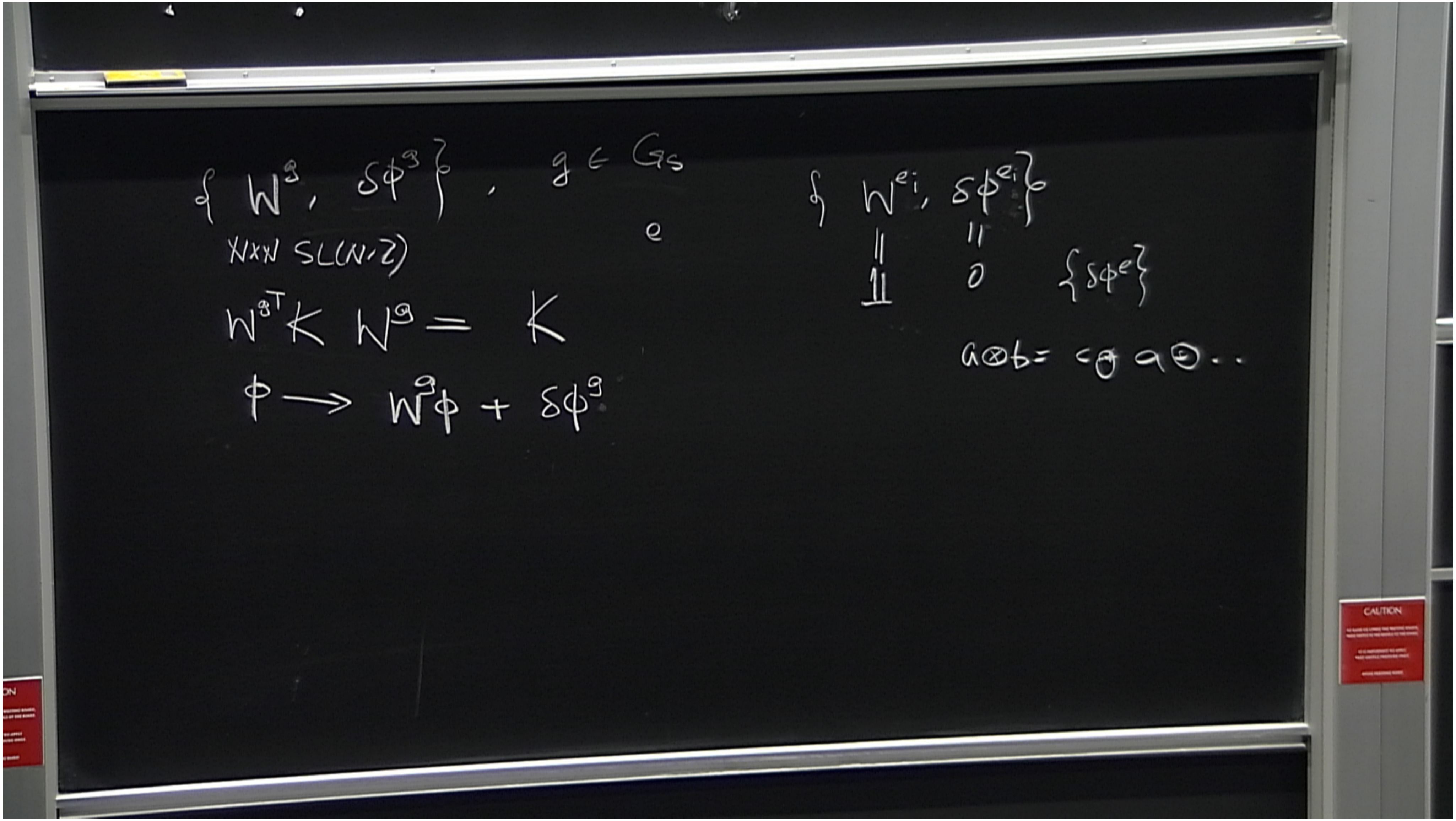

{ W^g , δφ^g } , g ∈ Gs
N×N SL(N,Z)
W^gT K W^g
φ ⟶ W^g φ
δφ^g
CAUTION

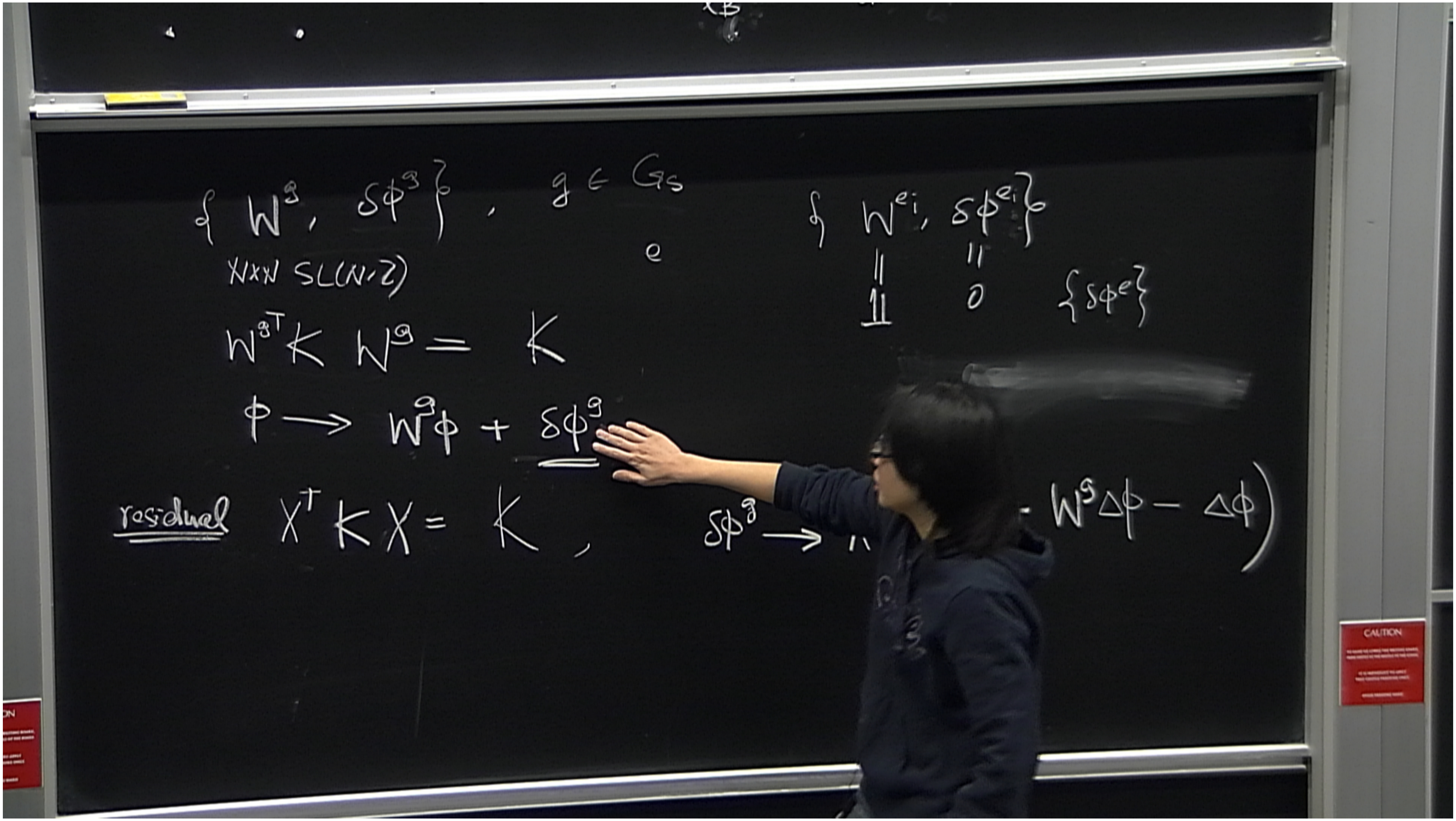

$\{ W^g , \delta\phi^g \}$ , $g \in G_s$
$N \times N$ $SL(N,Z)$
$W^{g^T} K W^g = K$
$\phi \longrightarrow W^g \phi + \delta\phi^g$

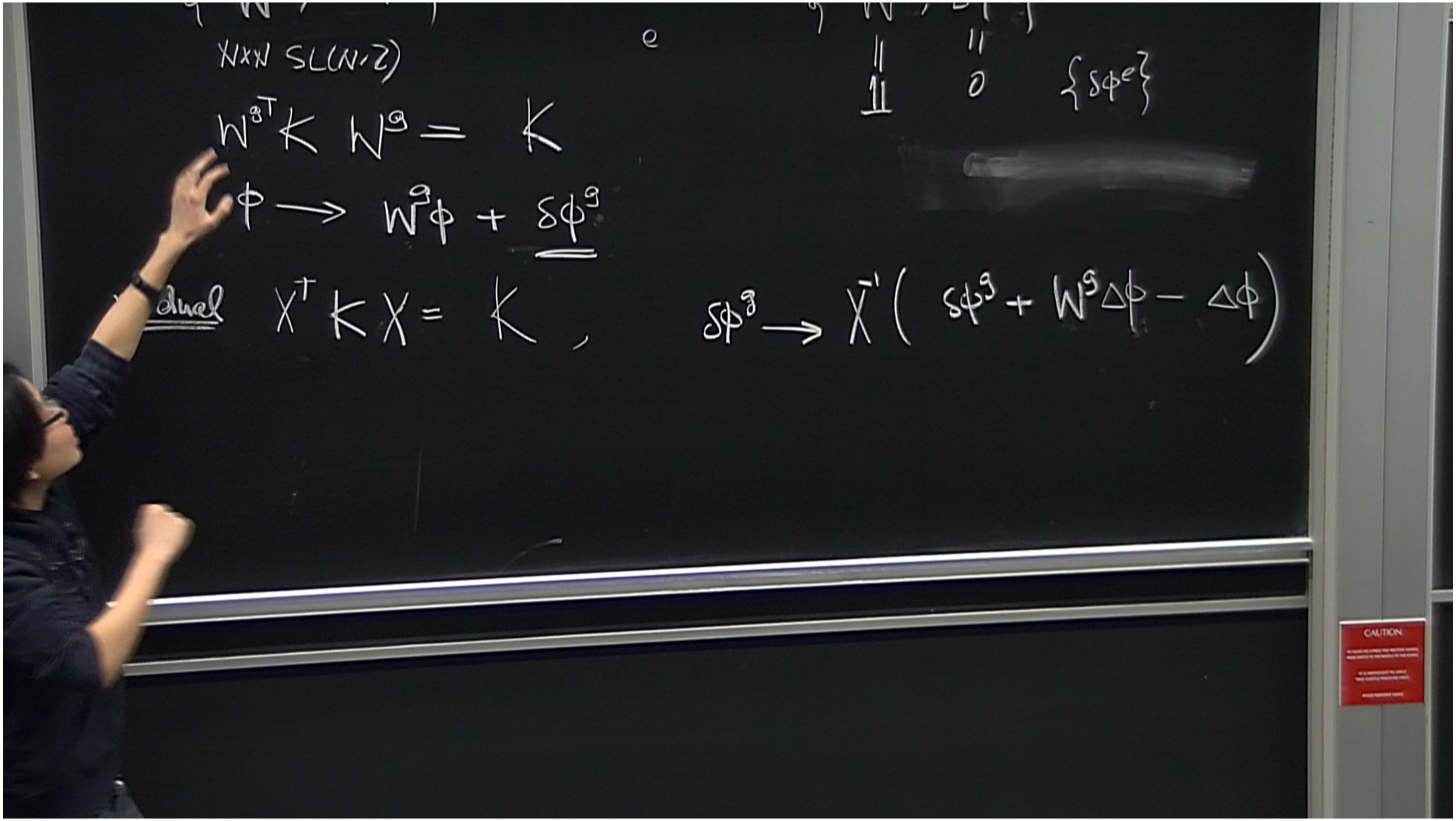

{W^g, δφ^g}, g ∈ G_s
N×N SL(N,Z)
W^{gT} K W^g = K
φ → W^g φ + δφ^g
e
{W^e, δφ^e}
= 1, = 0

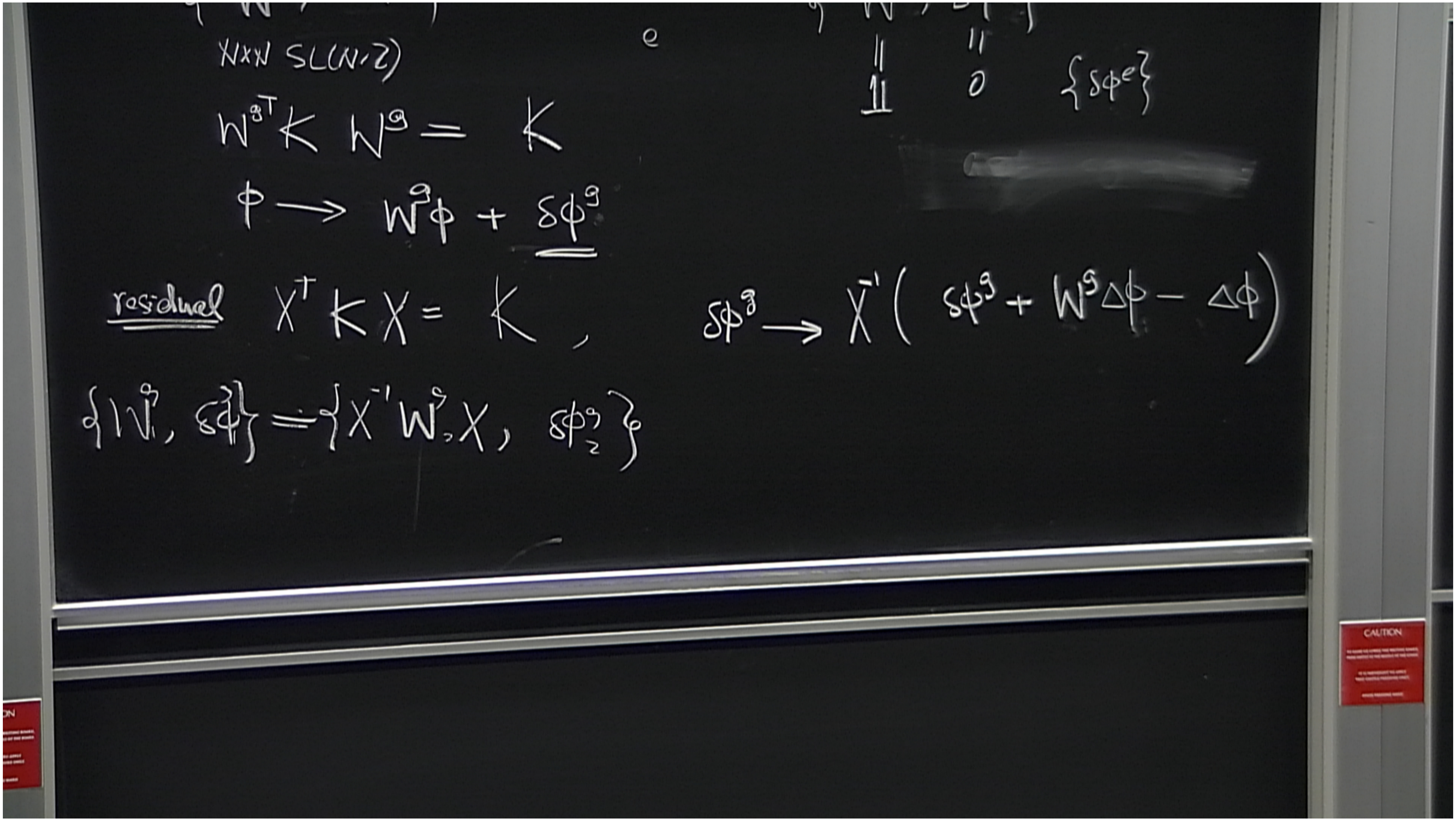

{ $W^g$, $\delta\phi^g$ } , $g \in G_s$
N×N SL(N,Z)
e
{ $W^e$, $\delta\phi^e$ }
= =
1 0
$W^{gT} K W^g = K$
$\phi \longrightarrow W^g \phi + \delta\phi^g$

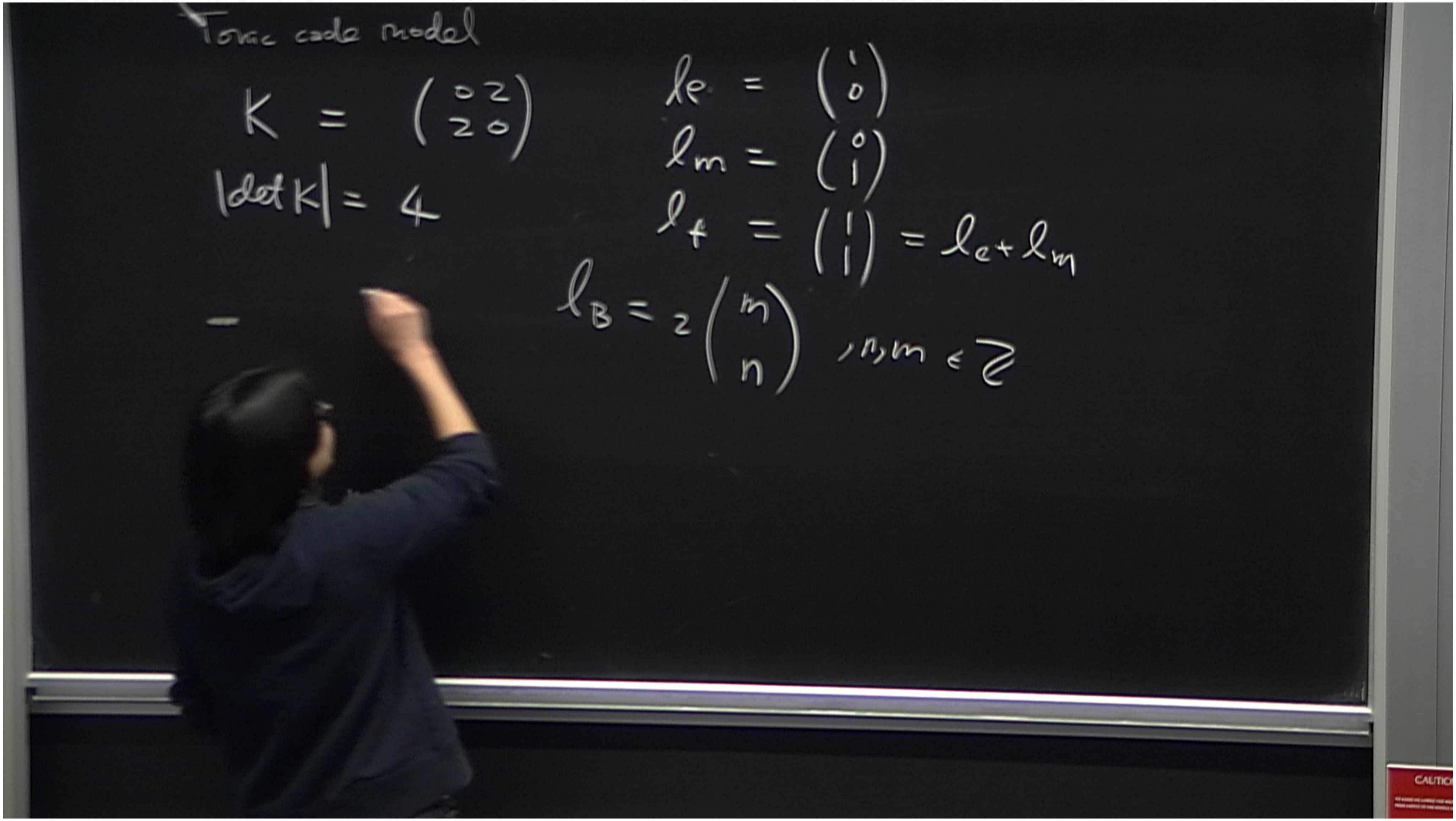

{ W^g , δφ^g } , g ∈ G_s
e
N×N SL(N,Z)
W^{gT} K W^g = K
φ → W^g φ + δφ^g
{ W^{e_i}, δφ^{e_i} }
11 0
{δφ^e}
CAUTION


{ W^g , δφ^g } , g ∈ G_s
N×N SL(N,Z)
e
W^{gT} K W^g = K
φ → W^g φ + δφ^g
{ W^{e_i} , δφ^{e_i} }
1  0  {δφ^e}
a⊗b =


{ W^g , δφ^g }, g ∈ G_s
N×N SL(N,Z)
e
{ W^{e_i}, δφ^{e_i} }
11 0
{δφ^e}
W^{gT} K W^g = K
φ ⟶ W^g φ + δφ^g
residual X^T K X = K ,
δφ^g ⟶
W^g Δφ − Δφ)
CAUTION


$N \times N$ $SL(N,\mathbb{Z})$
$\{\delta\phi^e\}$
$W^{g^T} K W^g = K$
$\phi \longrightarrow W^g \phi + \delta\phi^g$
dual $X^T K X = K$,
$\delta\phi^g \longrightarrow X^{-1}\left( \delta\phi^g + W^g \Delta\phi - \Delta\phi \right)$
CAUTION


e
1 0 {δφ^e}
NxN SL(N,Z)
W^{gT} K W^g = K
φ → W^g φ + δφ^g
residual X^T K X = K, δφ^g → X^{-1}( δφ^g + W^g Δφ − Δφ)
{W_1^g, δφ_1^g} = {X^{-1} W_2^g X, δφ_2^g}


Toric code model
$K = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$
$|\det K| = 4$
$l_e = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$l_m = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$l_f = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = l_e + l_m$
$l_B = 2\begin{pmatrix} m \\ n \end{pmatrix}, n, m \in \mathbb{Z}$

$K = \begin{pmatrix} \cdot & \cdot \\ 2 & 0 \end{pmatrix}$

$|\det K| = 4$

$N = 2$

$\frac{N}{2} = 1$

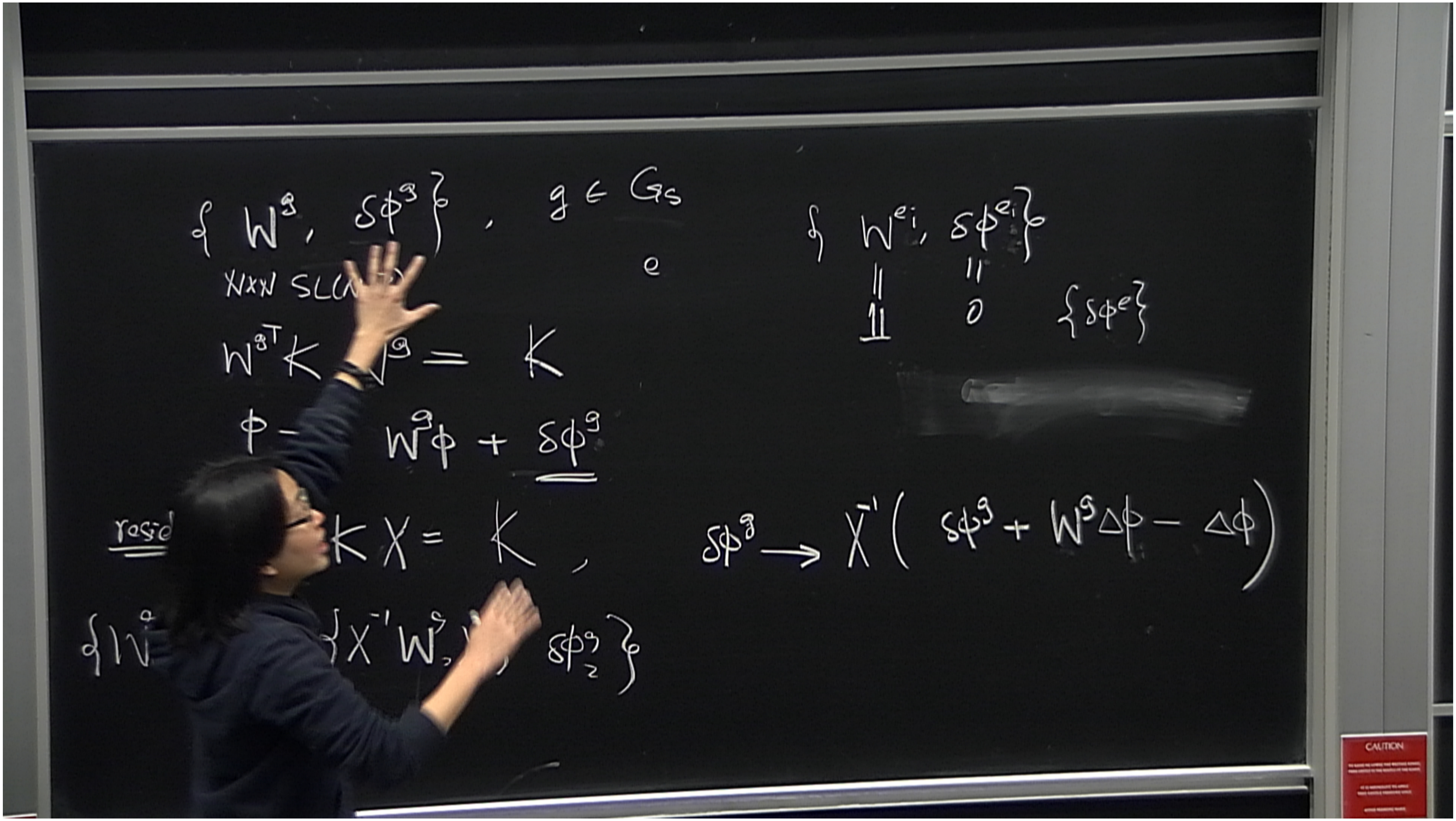
$l_m = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

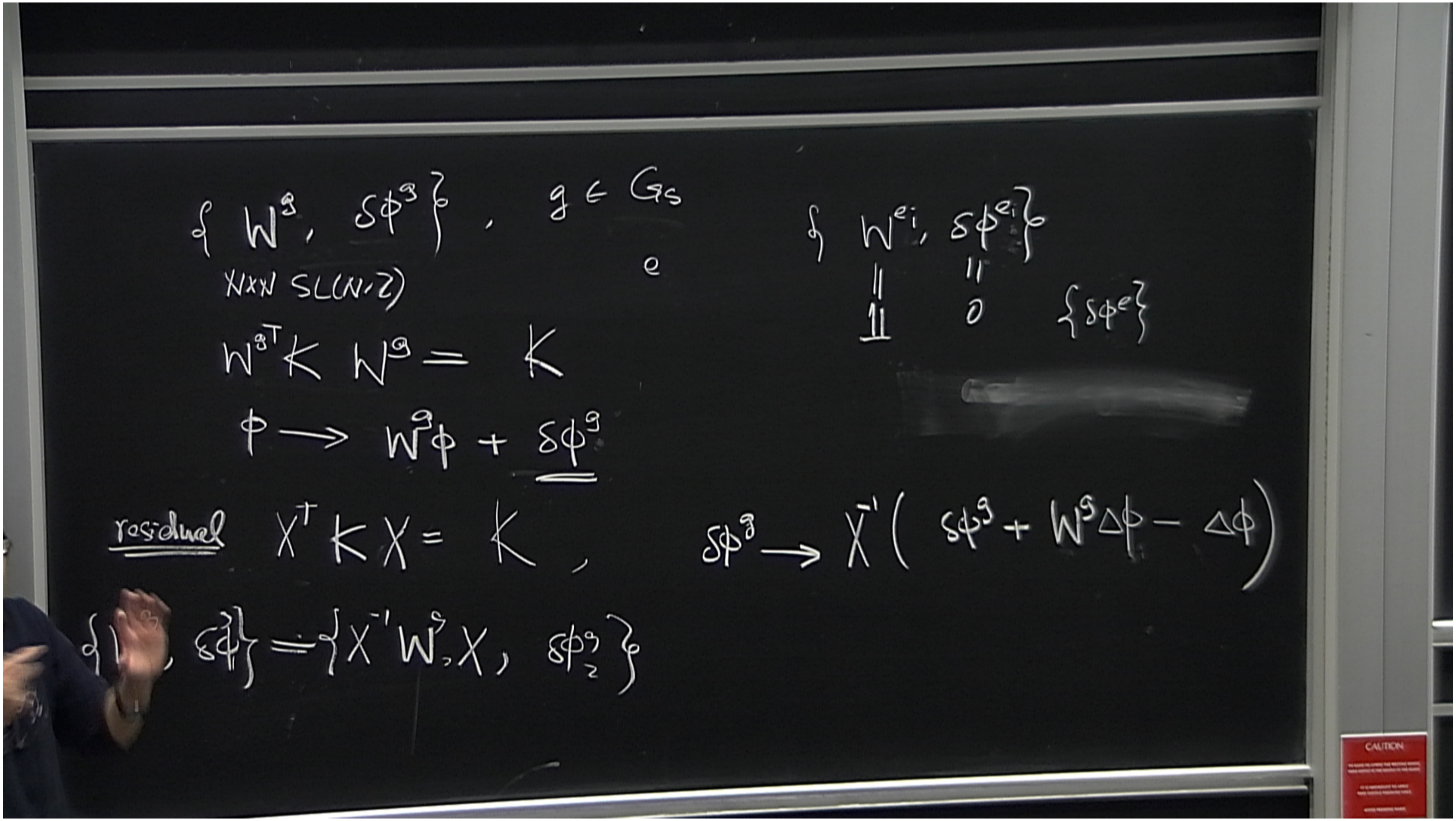
$l_f = \begin{pmatrix} 1 \\ 1 \end{pmatrix} = l_e + l_m$

$l_B = 2 \begin{pmatrix} m \\ n \end{pmatrix}, \; n, m \in \mathbb{Z}$

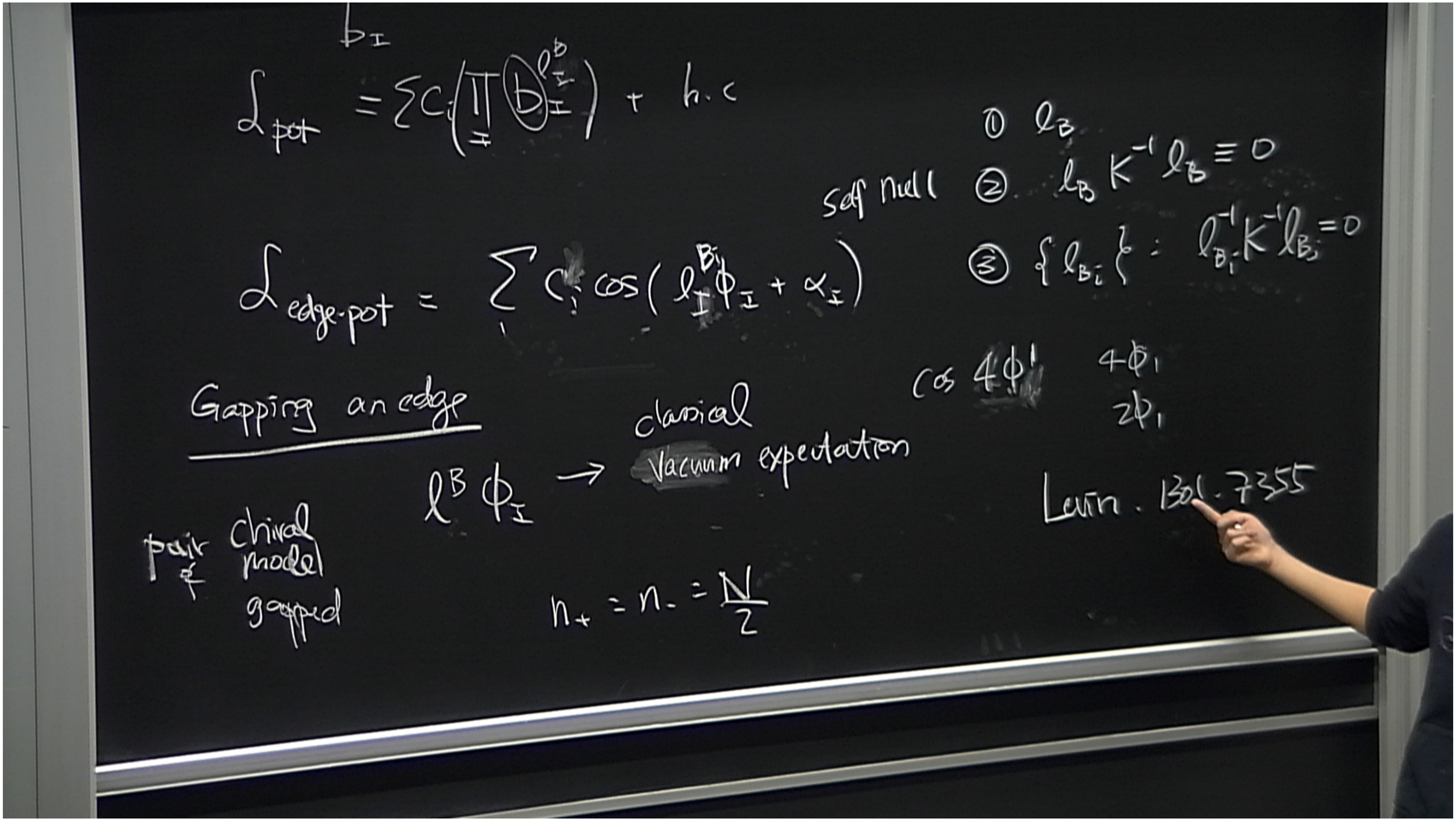
$l_{B^{e_1}} = 2 \begin{pmatrix} m \\ 0 \end{pmatrix}$

$l_{B^{e_2}} = 2 \begin{pmatrix} 0 \\ m \end{pmatrix}$


{ W^g , δφ^g } , g ∈ G_S
e
W^gT K W^g = K
φ → W^g φ + δφ^g
K X = K
{ W^ei , δφ^ei }
{ δφ^e }
δφ^g → X^-1 ( δφ^g + W^g Δφ − Δφ )



$\{ W^g, \delta\phi^g \}$ , $g \in G_s$
$e$
$N \times N$ $SL(N,\mathbb{Z})$
$\{ W^{e_i}, \delta\phi^{e_i} \}$
$= \mathbb{1}$ $= 0$ $\{\delta\phi^e\}$
$W^{g^T} K W^g = K$
$\phi \longrightarrow W^g\phi + \delta\phi^g$
residual $X^T K X = K$ ,
$\delta\phi^g \longrightarrow X^{-1}( \delta\phi^g + W^g\Delta\phi - \Delta\phi )$
$\delta\phi \} = \{ X^{-1} W^g X, \delta\phi^g_2 \}$


Gapping an edge
self null
classical
vacuum expectation
pair chiral model gapped
Levin . 1301 . 7355

$b_I$

$$\mathcal{L}_{pot} = \sum C_i \left( \prod_I b_I^{l_I^B} \right) + h.c$$

① $l_B$

self null ② $l_B K^{-1} l_B \equiv 0$

③ $\{l_{B_i}\}$ : $l_{B_i}^{-1} K^{-1} l_{B_j} = 0$

$$\mathcal{L}_{edge\text{-}pot} = \sum_i C_i \cos\left( l_I^{B_i} \phi_I + \alpha_I \right)$$

$\cos 4\phi_1$ $\quad 4\phi_1$

$2\phi_1$

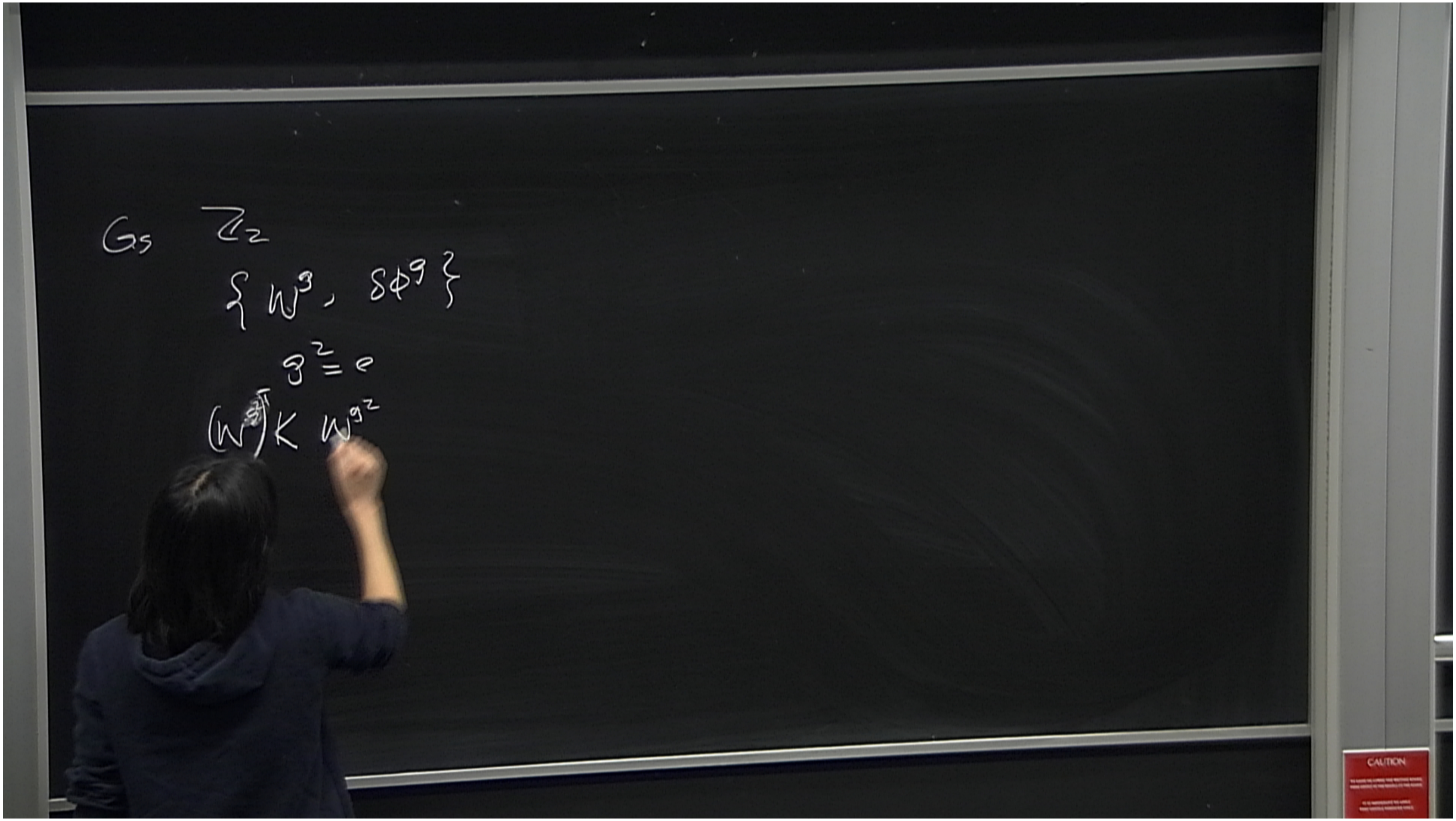
## Gapping an edge

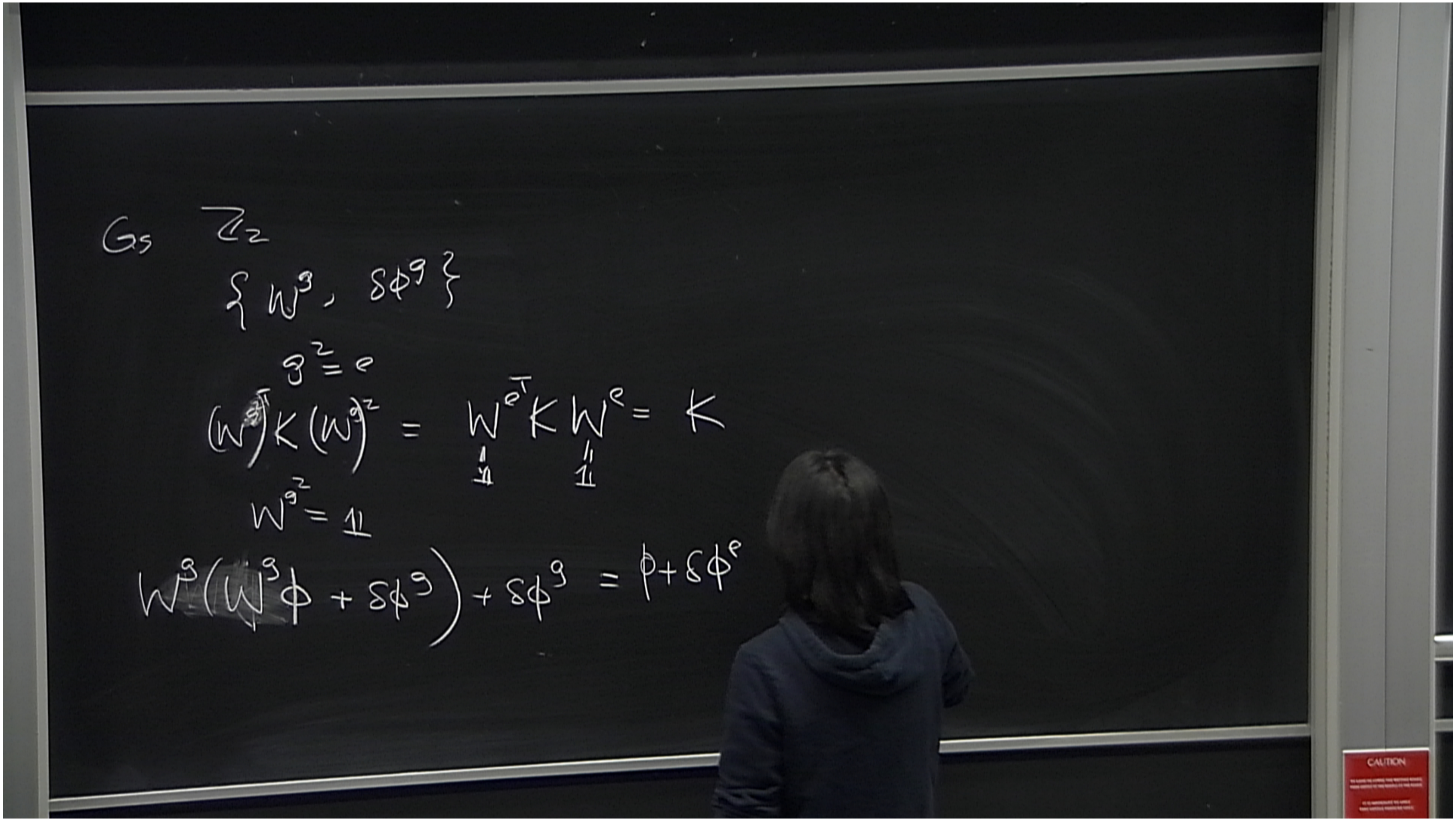
$l^B \phi_I \rightarrow$ classical vacuum expectation

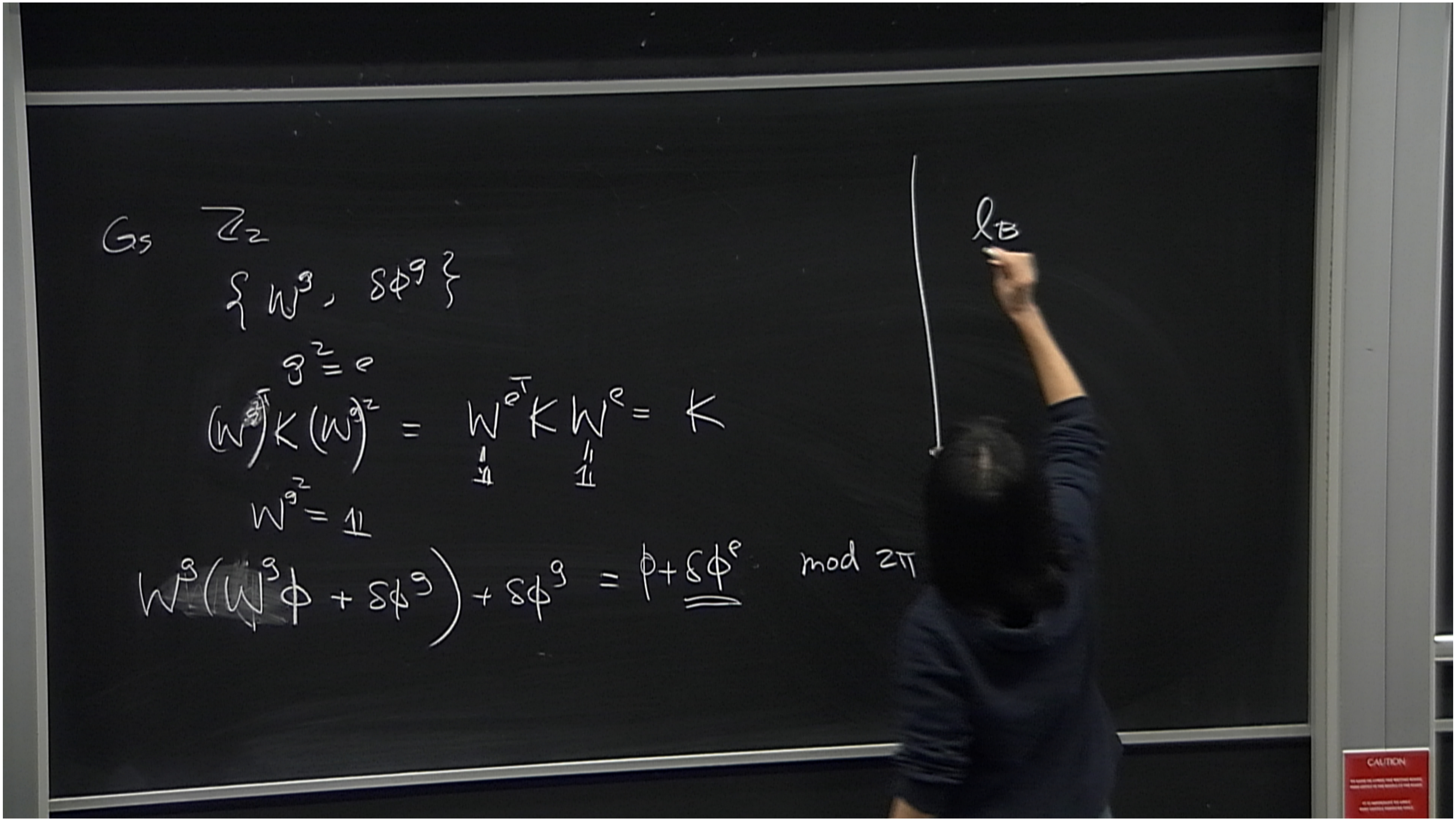
Levin. 1301.7355

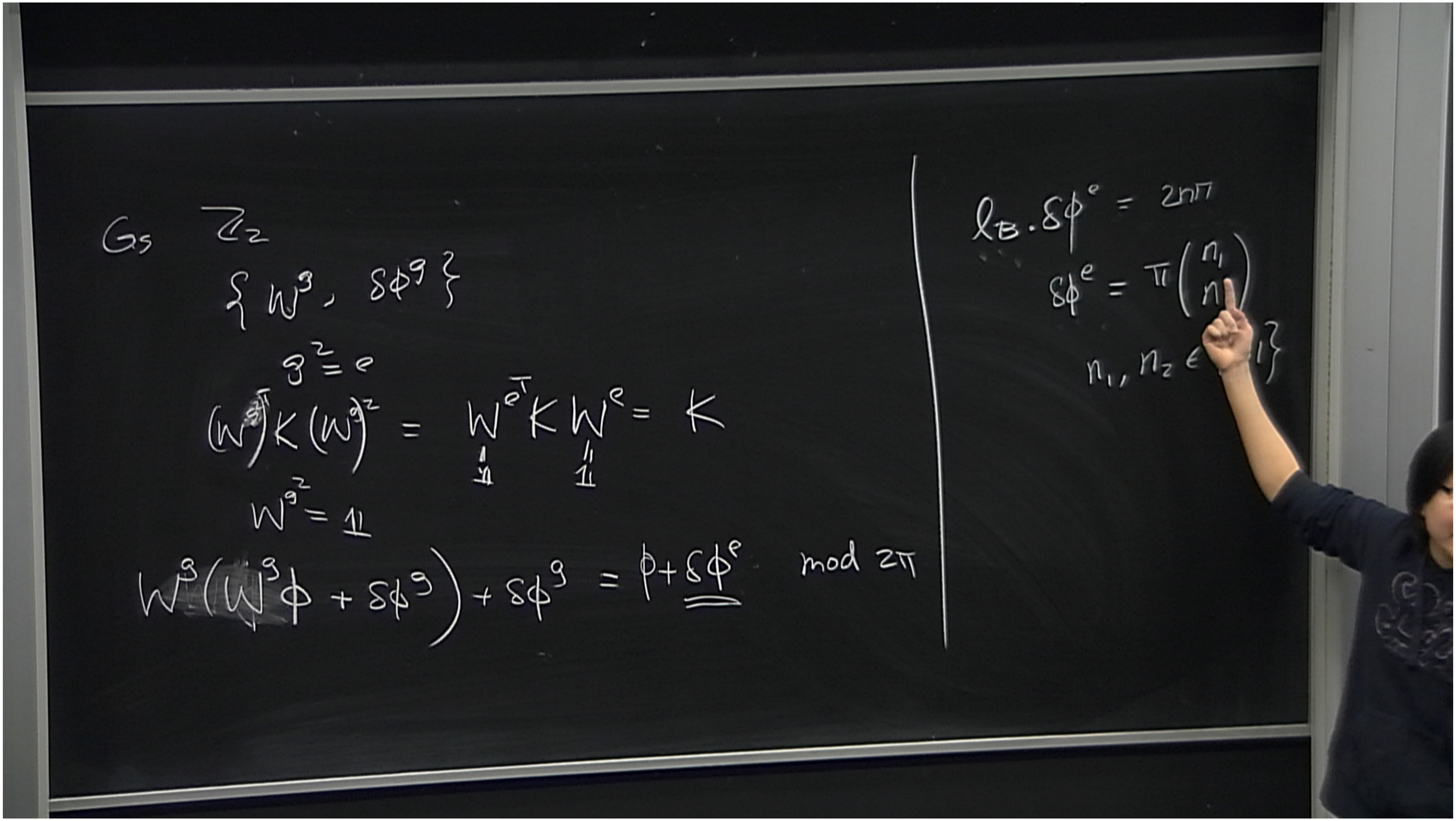
pair of chiral model gapped

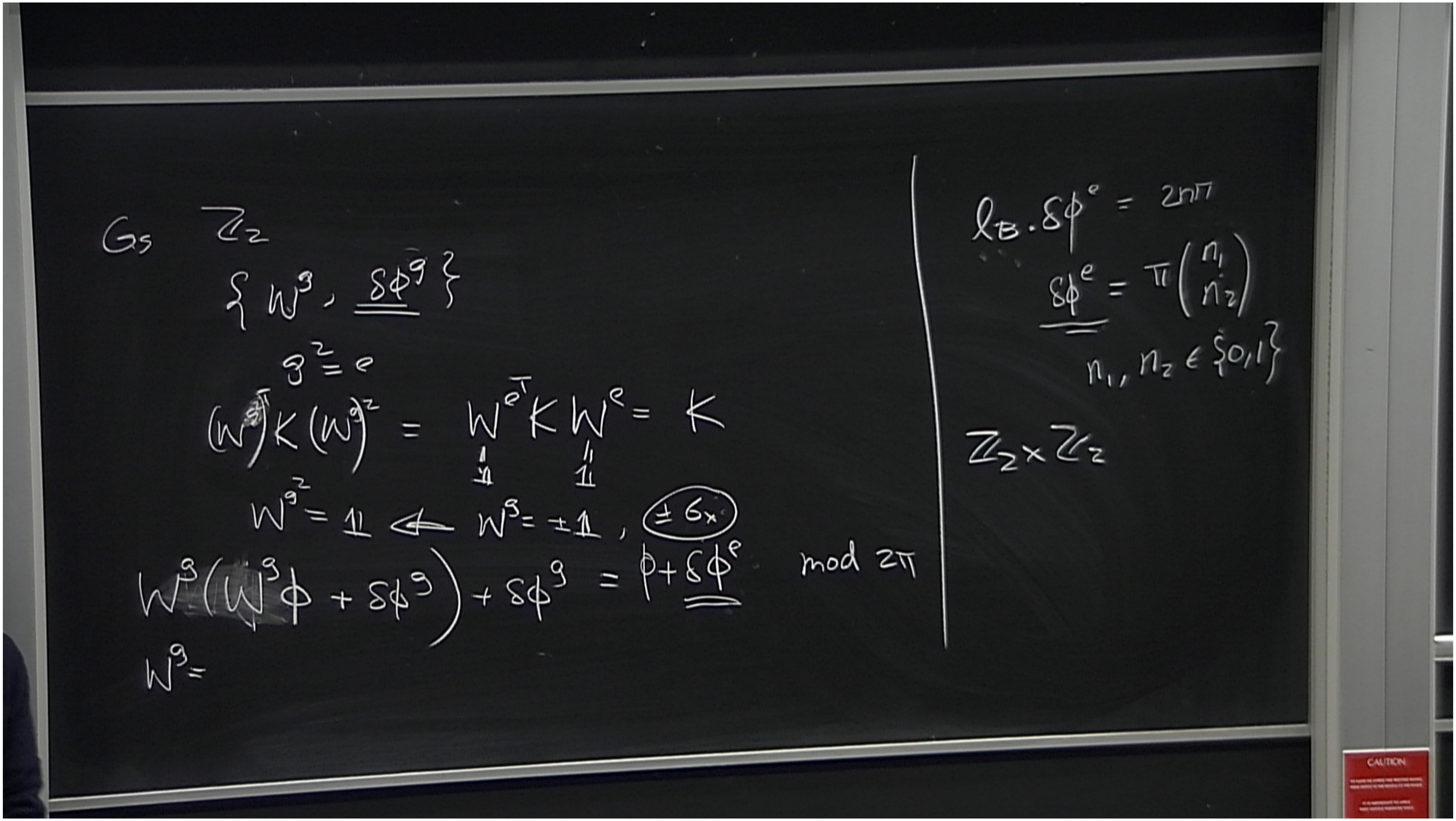
$$n_+ = n_- = \frac{N}{2}$$

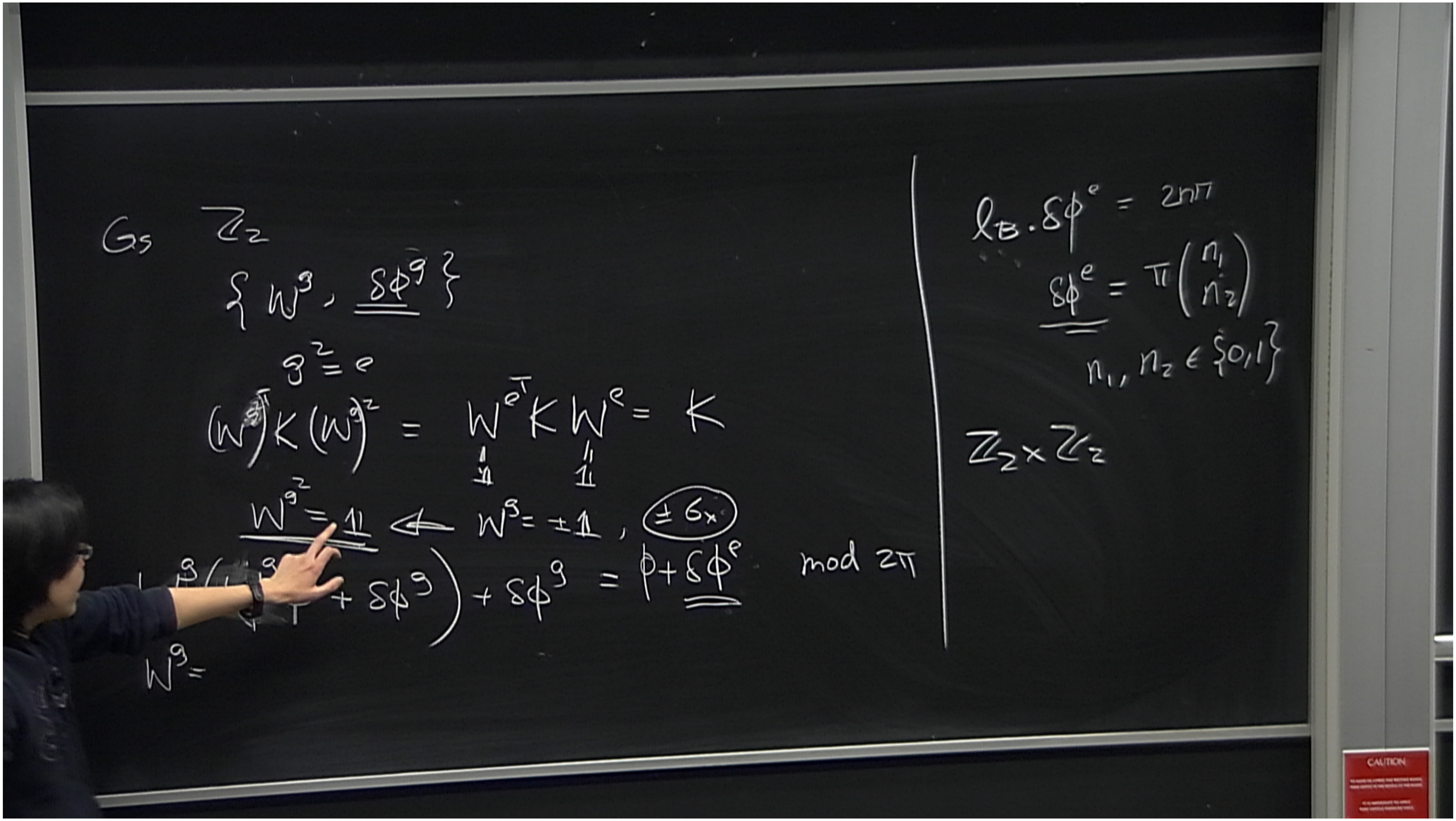

$G_5$ $\mathbb{Z}_2$
$\{ W^g , \delta\phi^g \}$
$g^2 = e$
$(W^{g^{-1}})\, K\, W^{g^2}$

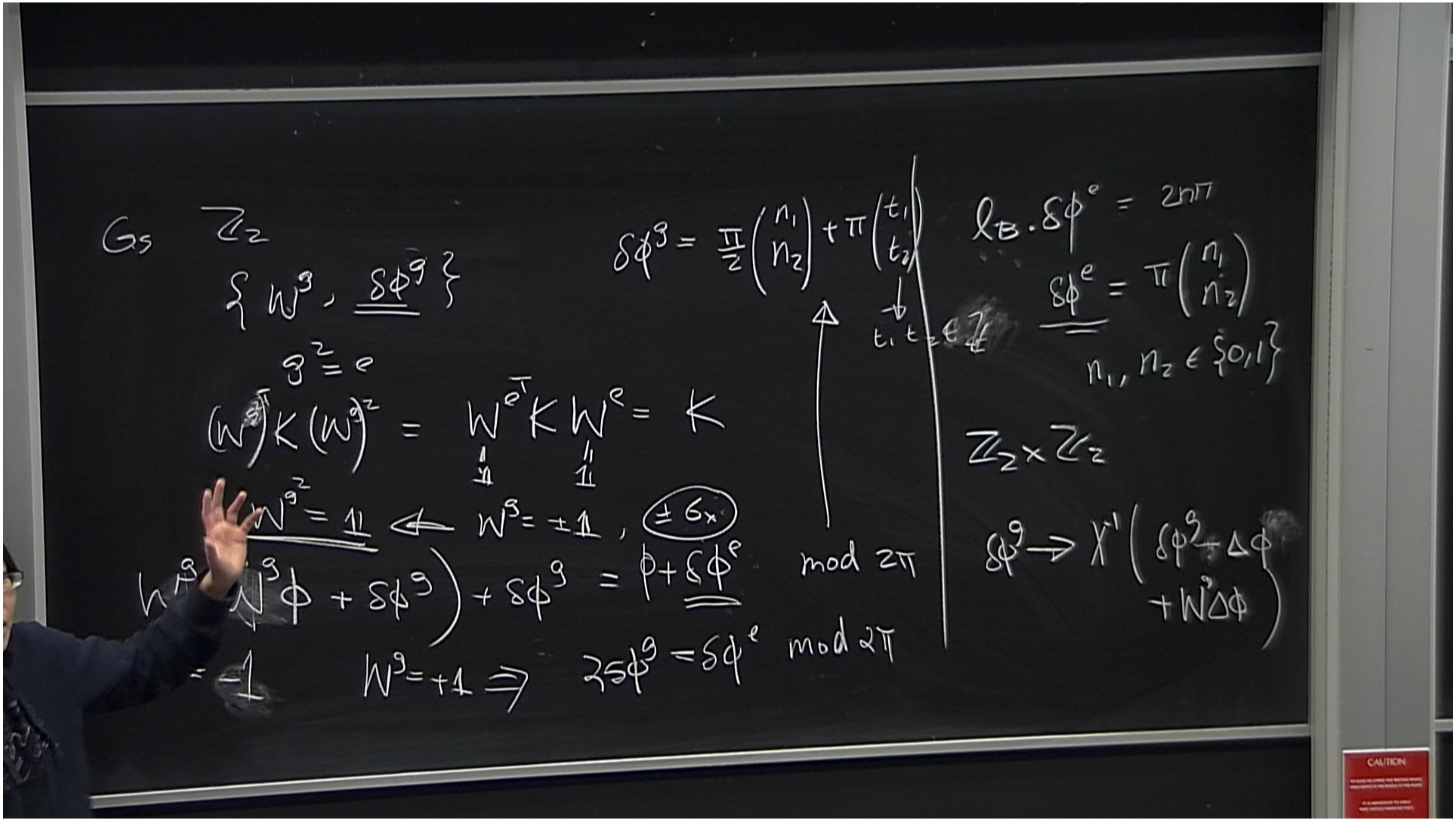

$G_5$ $\mathbb{Z}_2$
$\{W^g, \delta\phi^g\}$
$g^2 = e$
$(W^g)^T K (W^g) = W^{eT} K W^e = K$
$W^{g^2} = \mathbb{1}$
$W^g(W^g\phi + \delta\phi^g) + \delta\phi^g = \phi + \delta\phi^e$
CAUTION

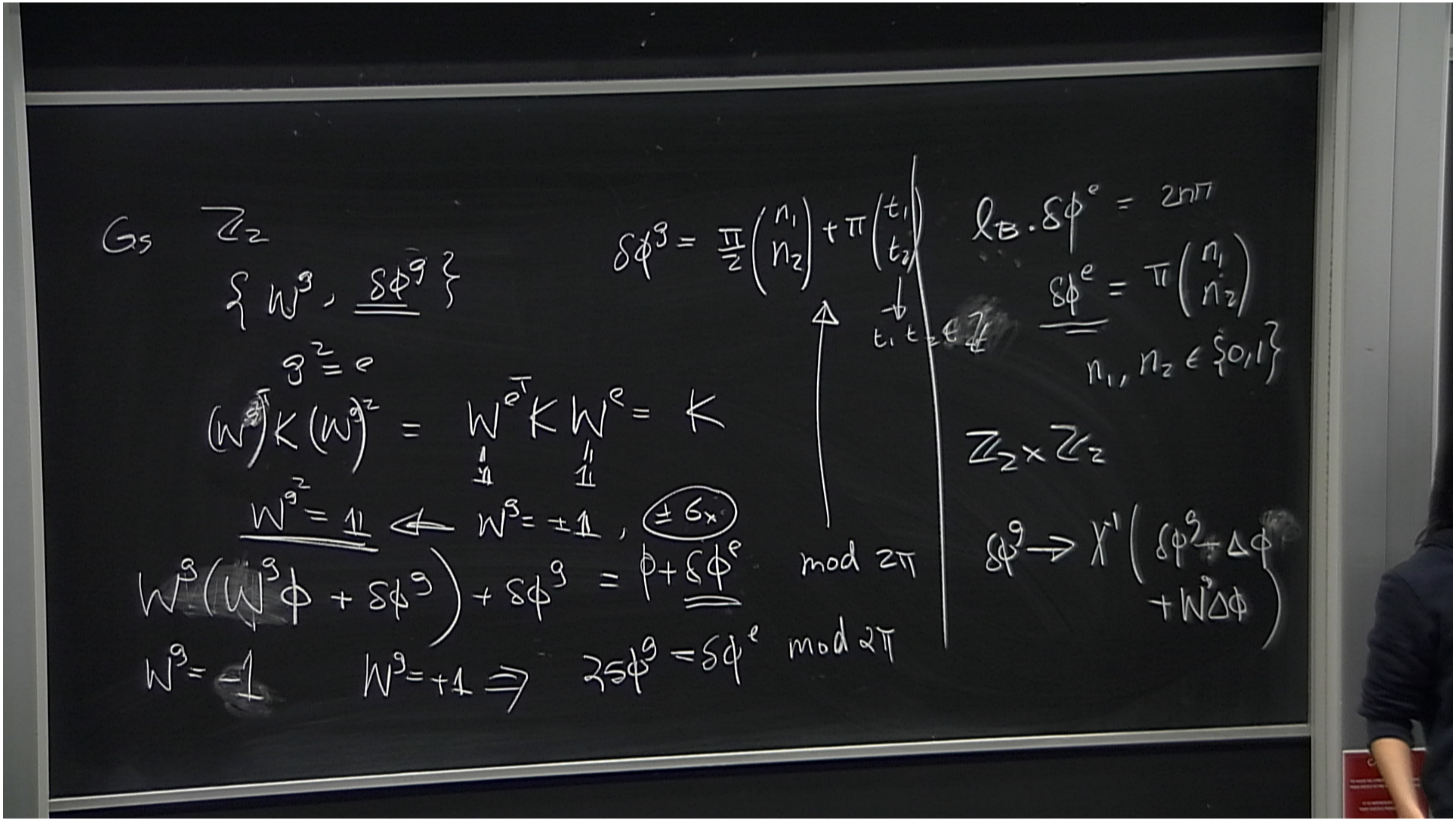

G5 Z2
{W^g, δφ^g}
g^2 = e
W^{g^2} = 1
mod 2π
l_B
CAUTION


G5 Z2
{W^g, δφ^g}
g^2 = e
(W^g)^T K (W^g) = W^{eT} K W^e = K
W^{g^2} = 11
W^g(W^g φ + δφ^g) + δφ^g = φ + δφ^e mod 2π
l_B · δφ^e = 2nπ
δφ^e = π(n1, n2)
n1, n2 ∈

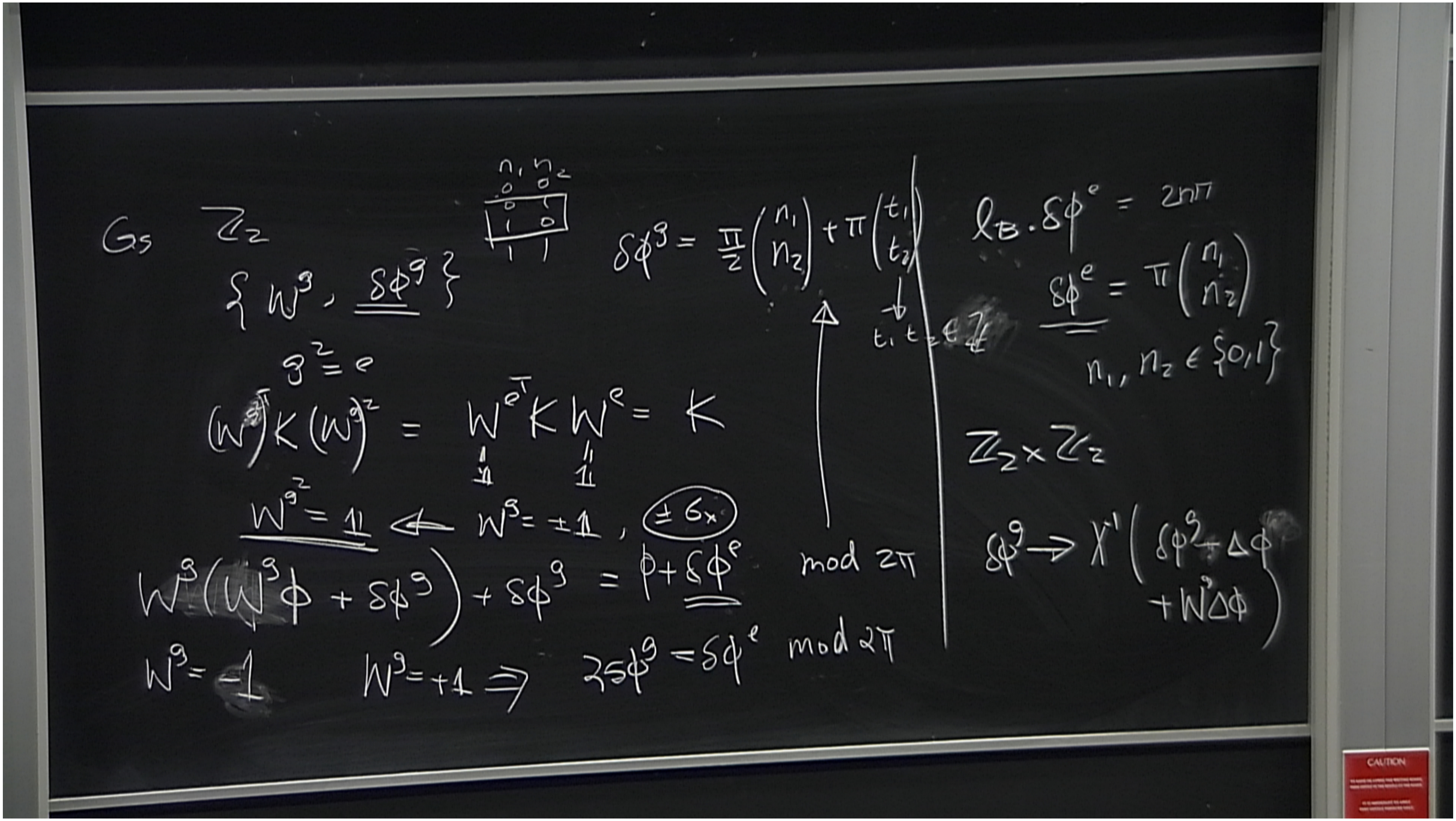

Gs Z2
{ W^g , δφ^g }
g^2 = e
(W^g)^T K (W^g)^2 = W^eT K W^e = K
W^g^2 = 1 ⇐ W^g = ±1 , ±σx
W^g(W^g φ + δφ^g) + δφ^g = φ + δφ^e mod 2π
W^g =
l_B . δφ^e = 2nπ
δφ^e = π (n1 n2)
n1, n2 ∈ {0,1}
Z2 x Z2
CAUTION

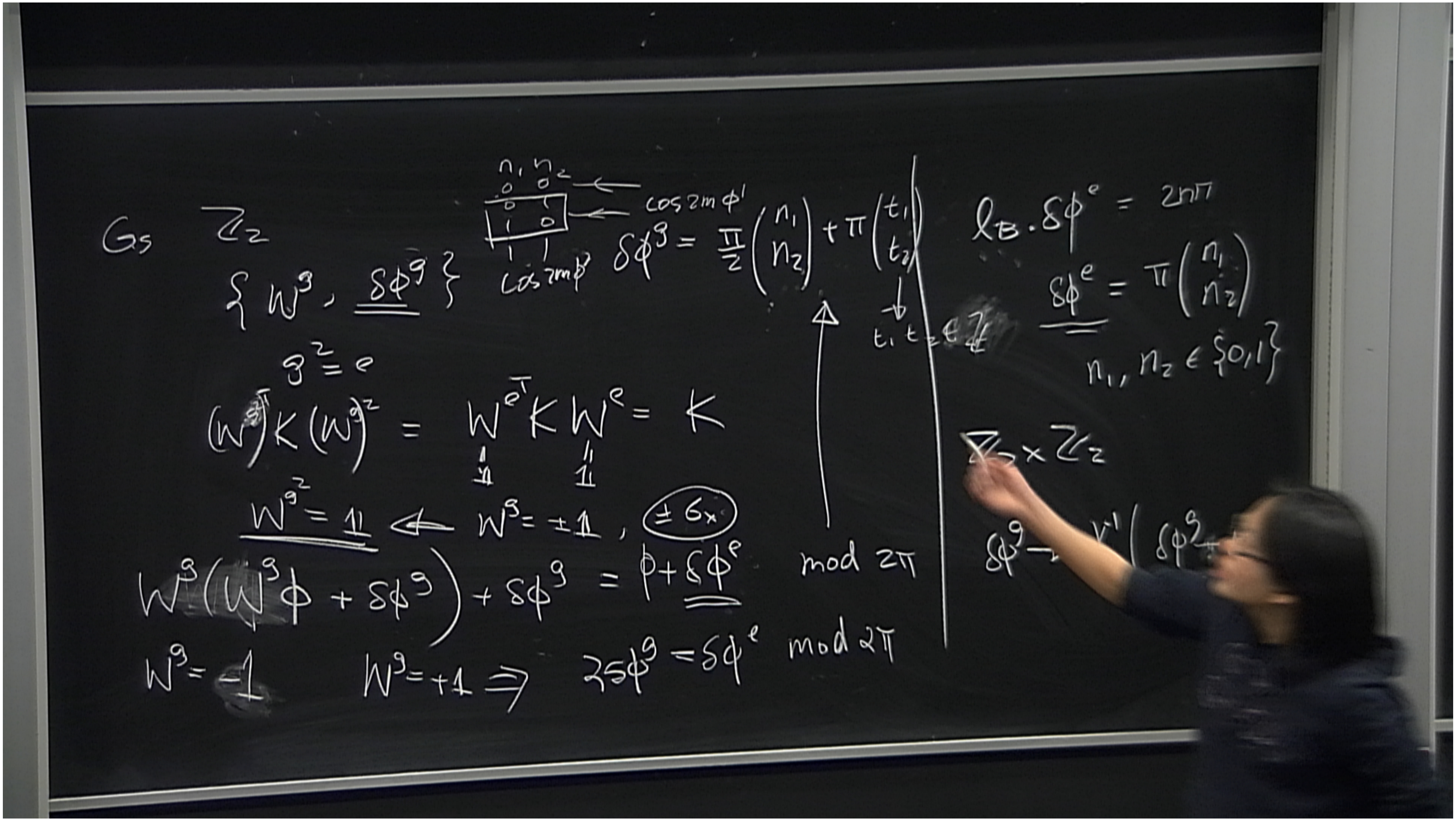

Gs Z2
{W^g, δφ^g}
g^2 = e
W^e K W^e = K
W^g = ±1 , ±σx
mod 2π
lB · δφ^e = 2nπ
δφ^e = π (n1 n2)
n1, n2 ∈ {0,1}
Z2 × Z2

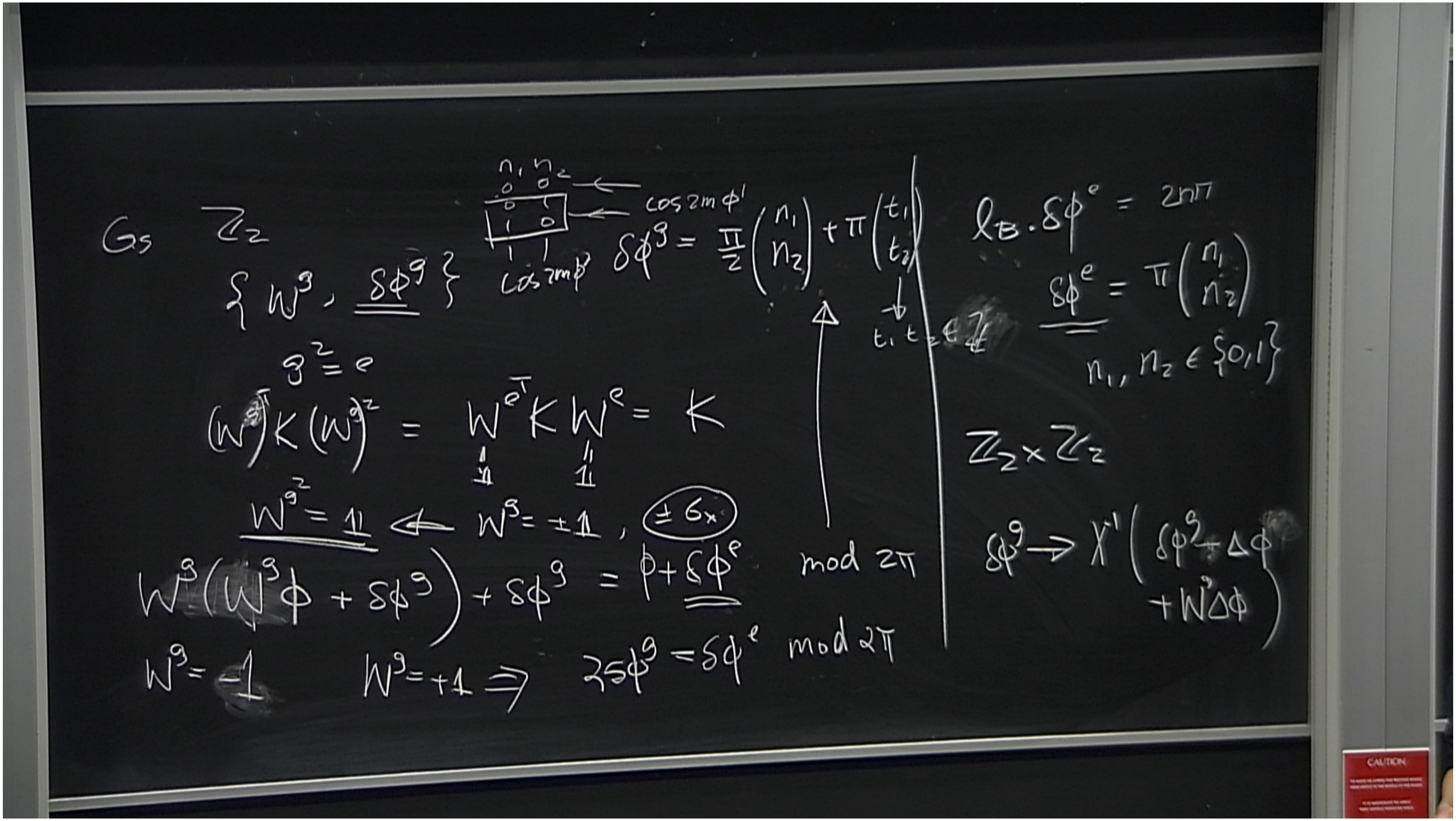

G_s Z_2
{W^g, δφ^g}
g^2 = e
W^{g^2} = 1 ⇐ W^g = ±1, ±σ_x
mod 2π
W^g = +1 ⇒ 2δφ^g = δφ^e mod 2π
δφ^g = π/2 (n_1, n_2) + π (t_1, t_2)
t_1, t_2 ∈ Z
l_B · δφ^e = 2nπ
δφ^e = π (n_1, n_2)
n_1, n_2 ∈ {0,1}
Z_2 × Z_2

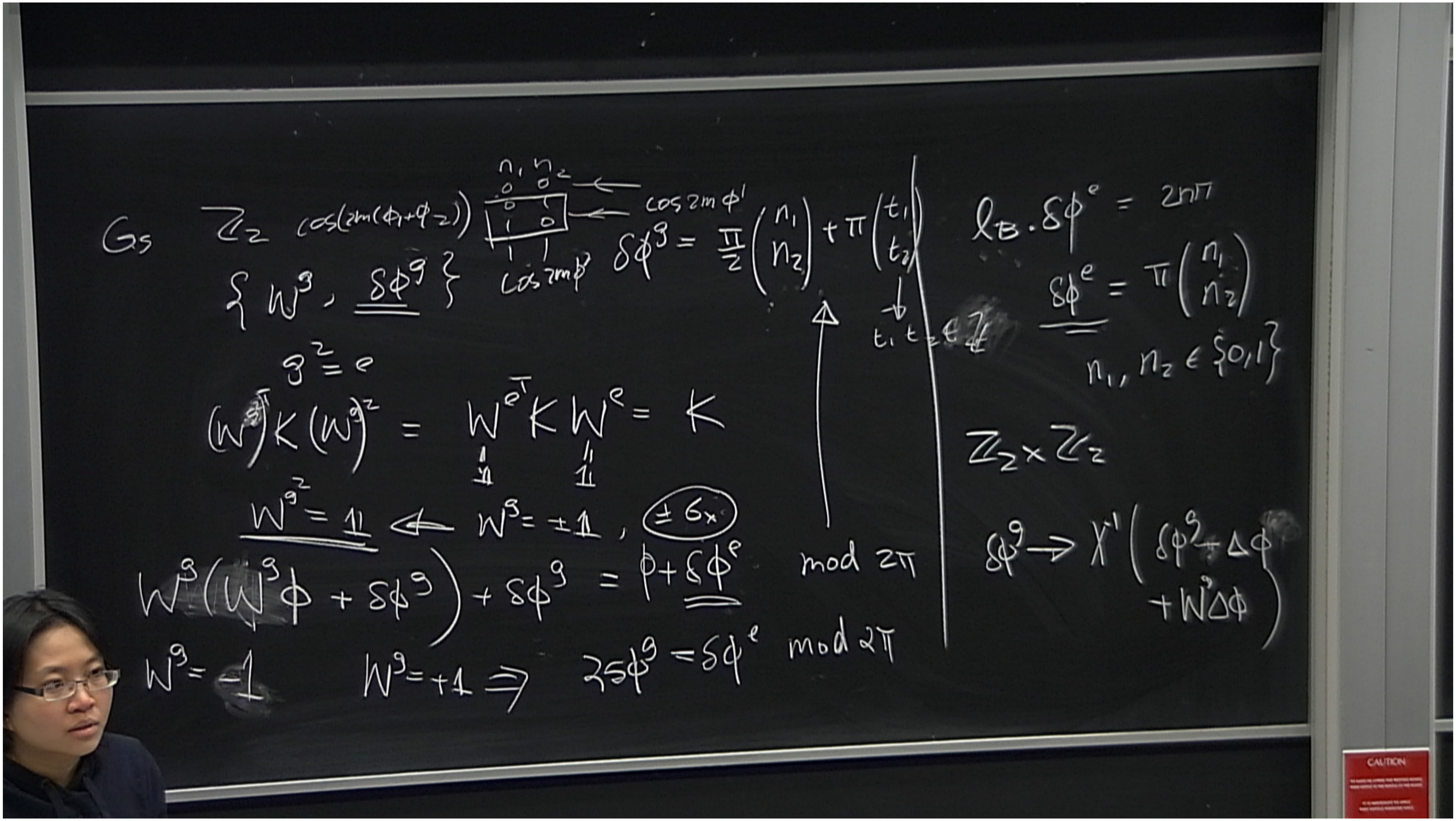


$G_s$ $\mathbb{Z}_2$
$\{ W^g, \delta\phi^g \}$
$g^2 = e$
$(W^g)^T K (W^g)^2 = W^{e^T} K W^e = K$
$W^{g^2} = 1$ ⟸ $W^g = \pm 1, \pm\sigma_x$
$W^g(W^g\phi + \delta\phi^g) + \delta\phi^g = \phi + \delta\phi^e$ mod $2\pi$
$W^g = -1$ $W^g = +1 \Rightarrow 2\delta\phi^g = \delta\phi^e$ mod $2\pi$
$\delta\phi^g = \frac{\pi}{2}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix} + \pi\begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$
$t_1, t_2 \in \mathbb{Z}$
$l_B \cdot \delta\phi^e = 2n\pi$
$\delta\phi^e = \pi\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$
$n_1, n_2 \in \{0,1\}$
$\mathbb{Z}_2 \times \mathbb{Z}_2$
$\delta\phi^g \to X^1(\delta\phi^g + \Delta\phi + W^g\Delta\phi)$


Gs Z2
{ W^g , δφ^g }
g^2 = e
(W^g)^T K (W^g)^2 = W^{eT} K W^e = K
W^{g^2} = 1 ⇐ W^g = ±1 , ±σx
W^g (W^g φ + δφ^g) + δφ^g = φ + δφ^e mod 2π
W^g = -1   W^g = +1 ⇒ 2δφ^g = δφ^e mod 2π
δφ^g = π/2 (n1, n2) + π (t1, t2)
t1, t2 ∈ Z
l_B · δφ^e = 2nπ
δφ^e = π (n1, n2)
n1, n2 ∈ {0,1}
Z2 × Z2
δφ^g → X' (δφ^g + Δφ + W^g Δφ)


G_5 Z_2
{W^g, δφ^g}
g^2 = e
W^{g^2} = 1
mod 2π
W^g = +1 ⇒ 2δφ^g = δφ^e mod 2π
l_B · δφ^e = 2nπ
n_1, n_2 ∈ {0,1}
Z_2 × Z_2


Gs Z2
cos 2m φ'
mod 2π
mod 2π
Z2 × Z2
n1, n2 ∈ {0,1}


Gs Z2 cos(2m(φ1+φ2))
cos 2m φ'
cos 2m φ
{ W^g , δφ^g }
δφ^g = π/2 (n1 n2) + π (t1 t2)
t1 t2 ∈ Z
g^2 = e
W^{g^2} = 1
W^g = ±1 , ±σx
mod 2π
W^g = -1
W^g = +1 ⇒ 2δφ^g = δφ^e mod 2π
l_B · δφ^e = 2nπ
δφ^e = π (n1 n2)
n1, n2 ∈ {0,1}
Z2 × Z2