Title: Beyond Massive Gravity

Date: Feb 25, 2013 11:00 AM

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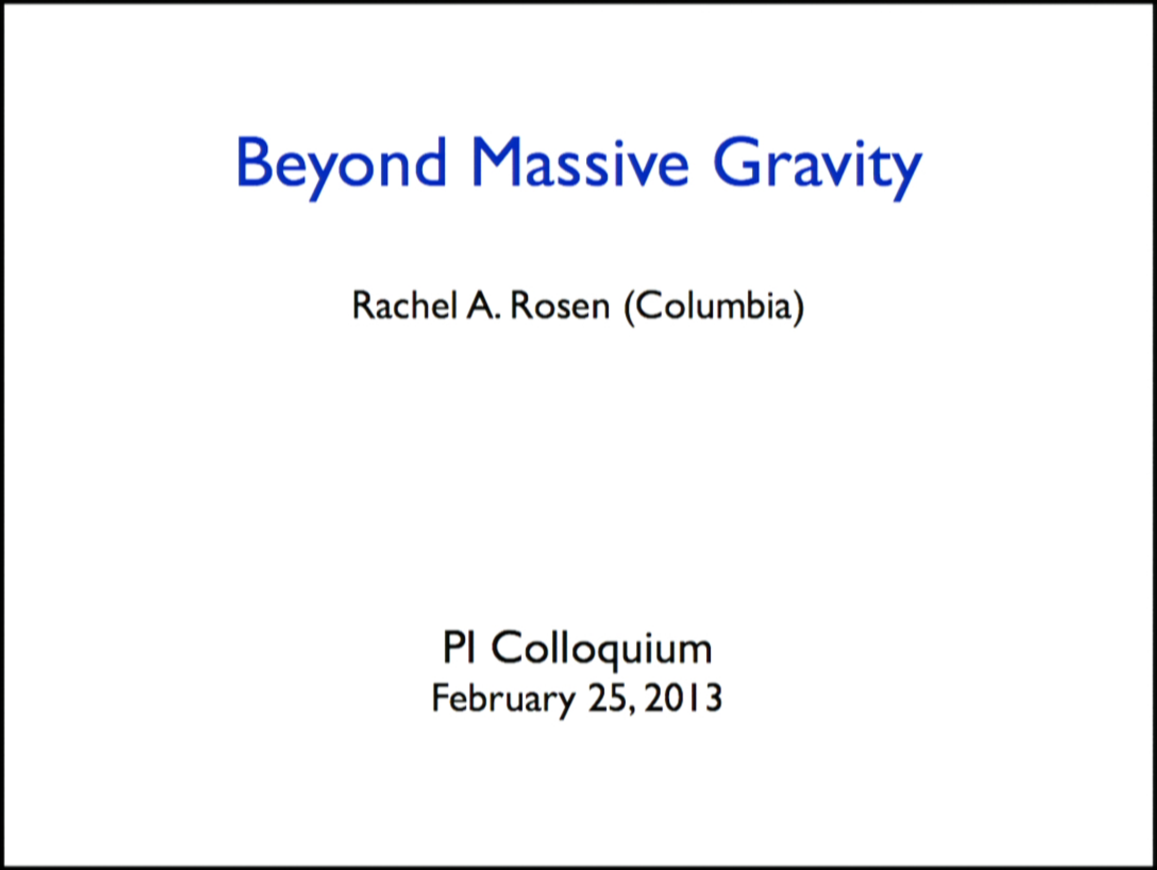
Abstract: In the last few years there has been a burst of progress in the field of massive gravity. The construction of consistent theories in which the graviton has a small mass has in turn led to the development a new family of compelling, consistent low-energy modifications of General Relativity. These theories improve our understanding of the interplay between gravity and particle physics and provide new approaches to solving the cosmological constant problem. In this talk I will review these recent developments. I will discuss the search among these new theories for a symmetry that has the potential to address the cosmological constant problem.



Beyond Massive Gravity

Rachel A. Rosen (Columbia)

PI Colloquium
February 25, 2013

What can fundamental physics tell us about macroscopic phenomena?

- Quantum field theory for astrophysics
  (G. Gabadadze)
- Effective field theory for hydrodynamics
  (A. Nicolis, R. Penco)
- Particle physics for gravity
  (S.F. Hassan, K. Hinterbichler, C. de Rham, A.J. Tolley)

A field-theorist's approach to gravity:

- Particles are characterized by their mass and spin
- Forces are mediated by the exchange of bosons

A field-theorist's approach to gravity:

- spin-0 $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + g\phi T$

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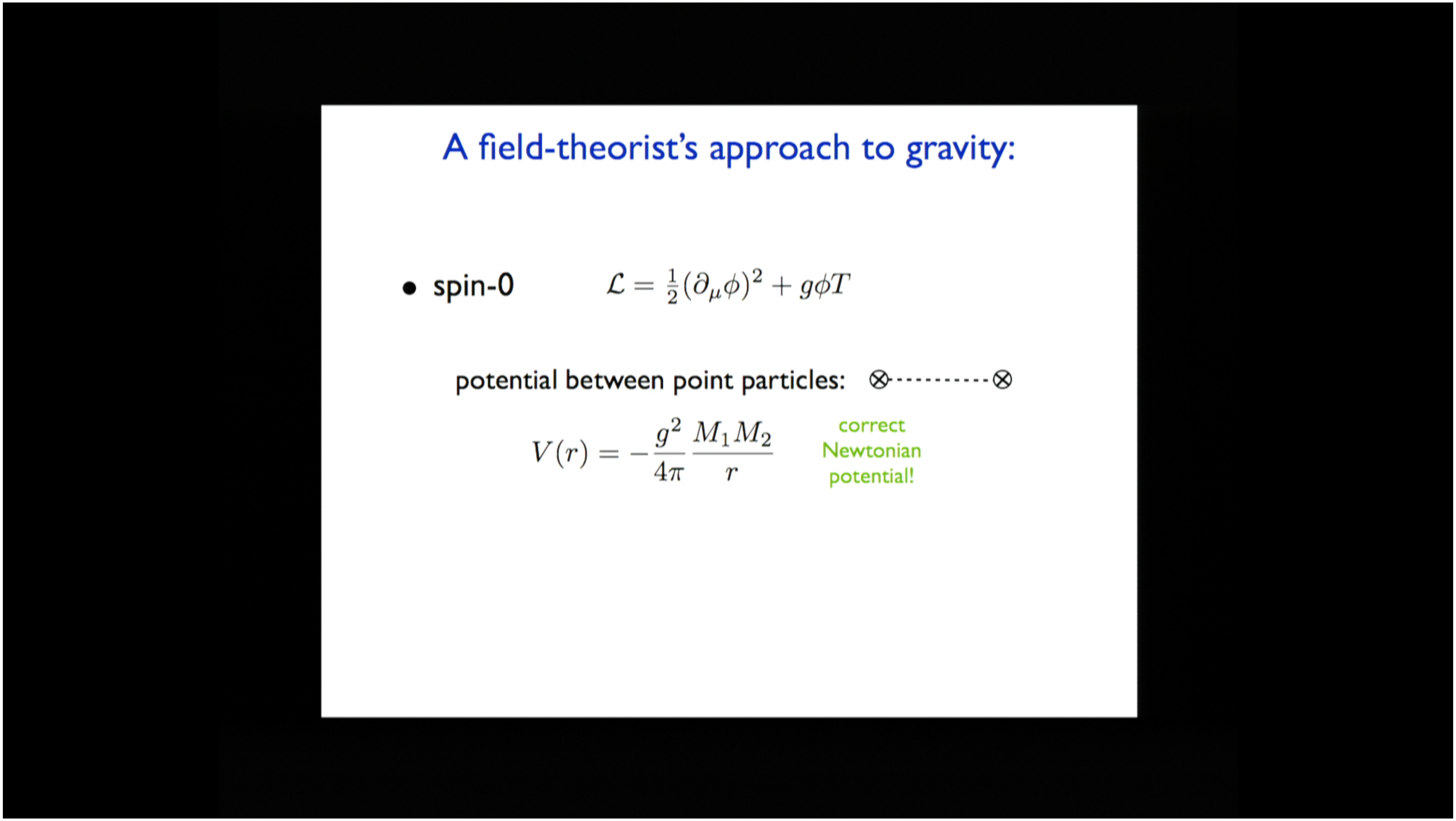

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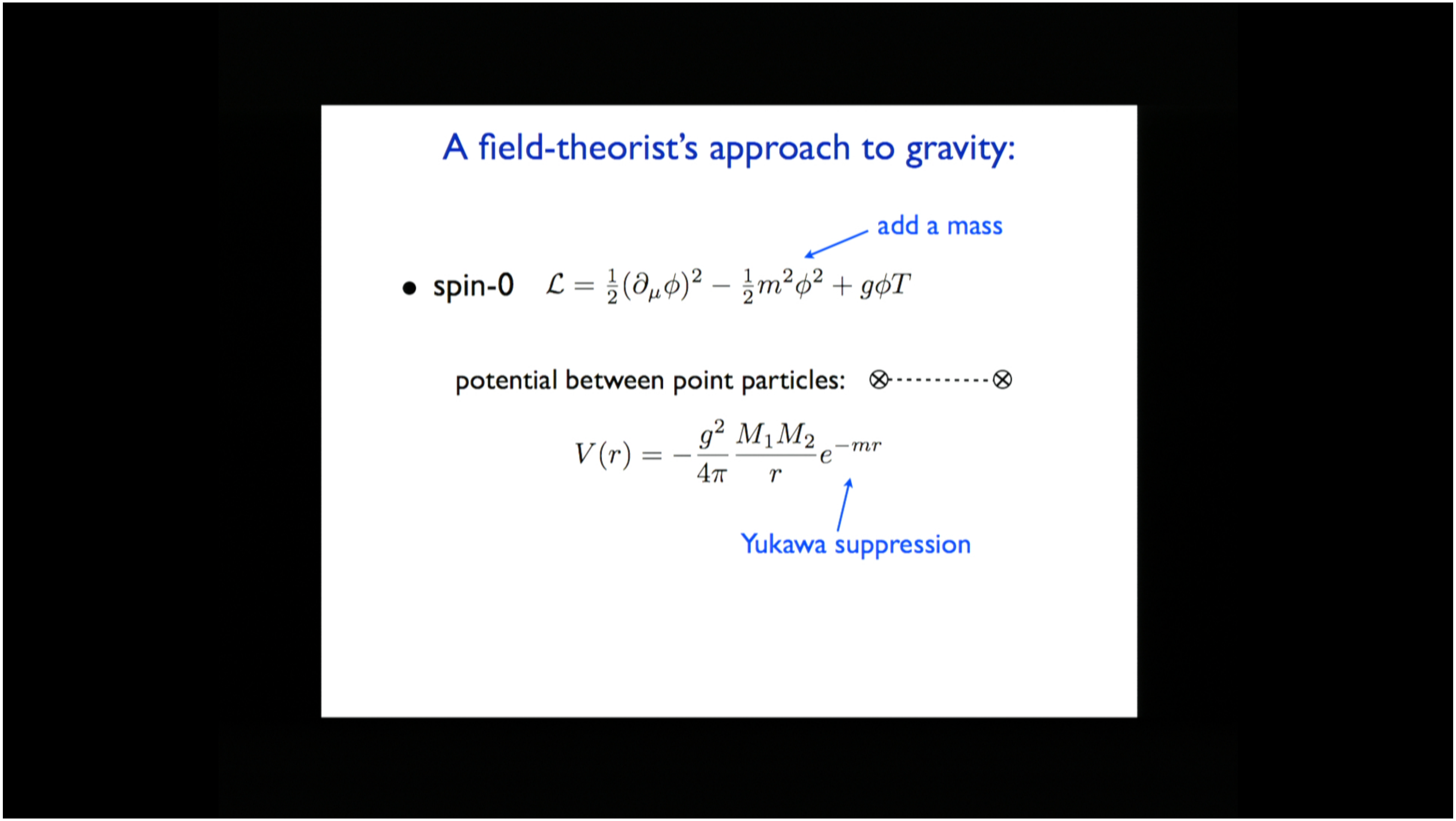
- spin-0 $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + g\phi T$

potential between point particles: ⊗- - - - - - - - - - -⊗

$$V(r) = -\frac{g^2}{4\pi}\frac{M_1 M_2}{r}$$

correct
Newtonian
potential!



A field-theorist's approach to gravity:

add a mass

- **spin-0** $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 + g\phi T$

potential between point particles: $\otimes$- - - - - - - - - - -$\otimes$

$$V(r) = -\frac{g^2}{4\pi}\frac{M_1 M_2}{r}e^{-mr}$$

Yukawa suppression

A field-theorist's approach to gravity:

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NASA-ESA

no bending of light ⇒ FAILS

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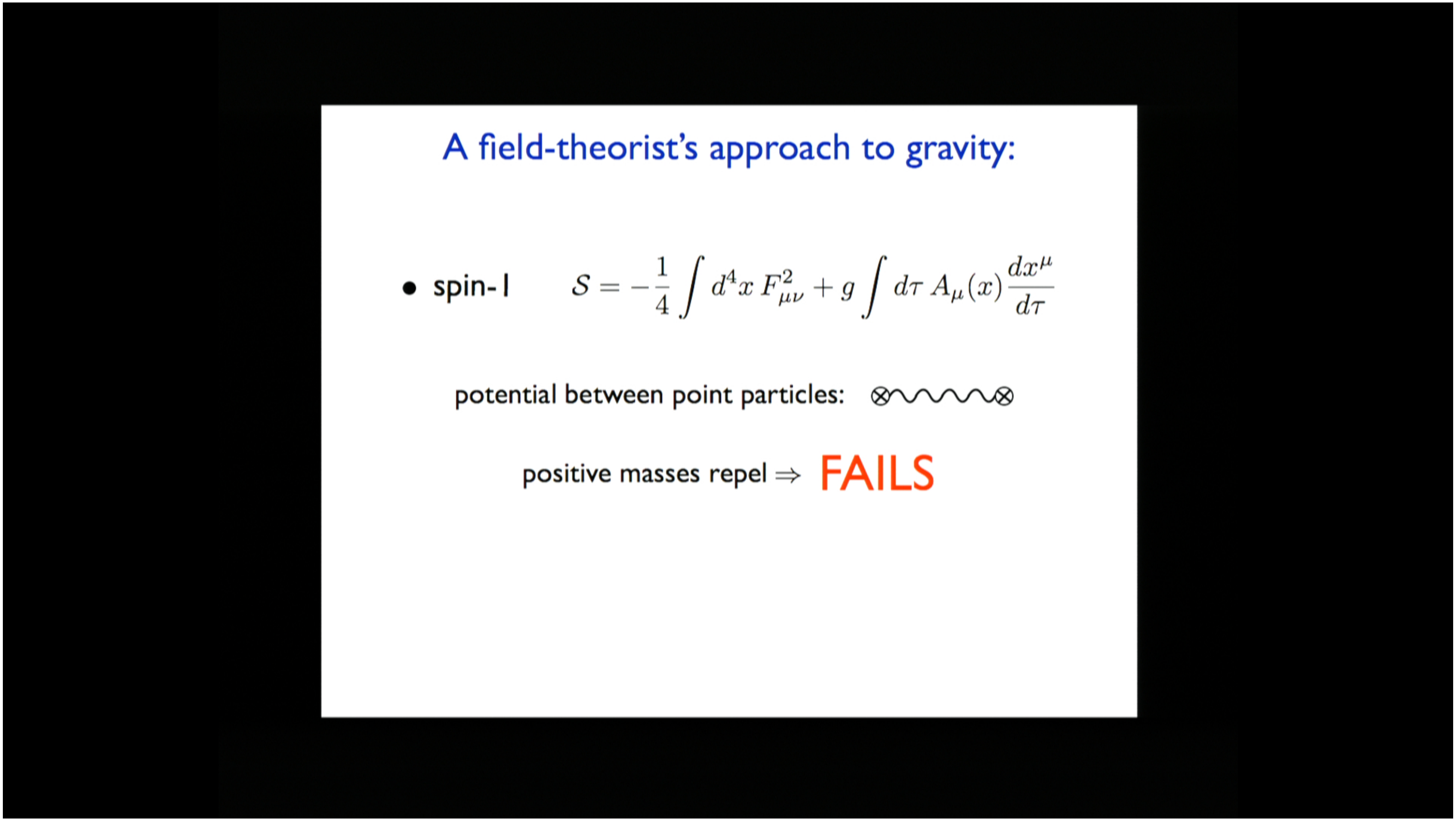
A field-theorist's approach to gravity:

- spin-1 $\mathcal{S} = -\frac{1}{4}\int d^4x\, F_{\mu\nu}^2 + g\int d\tau\, A_\mu(x)\,\frac{dx^\mu}{d\tau}$



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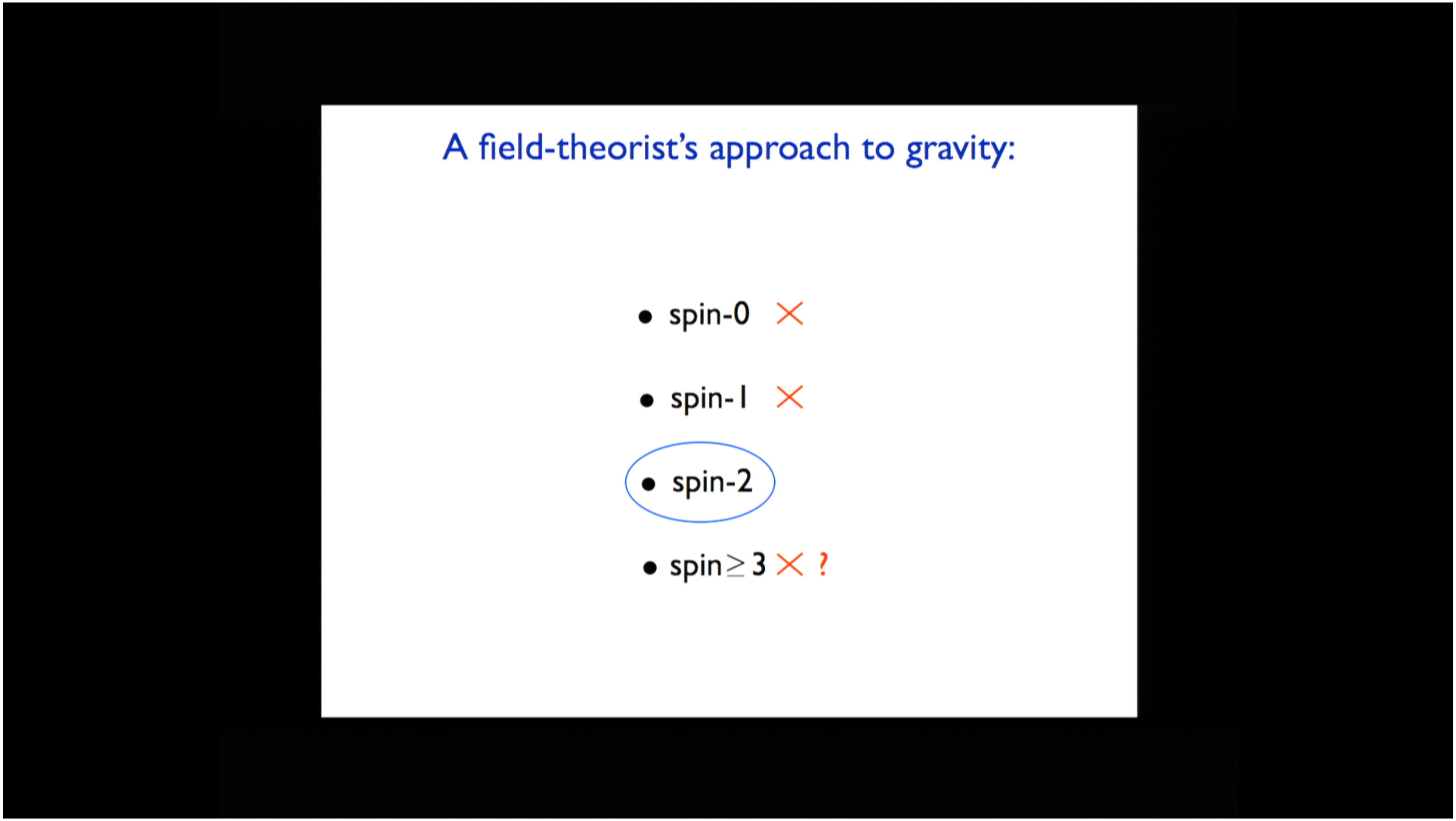
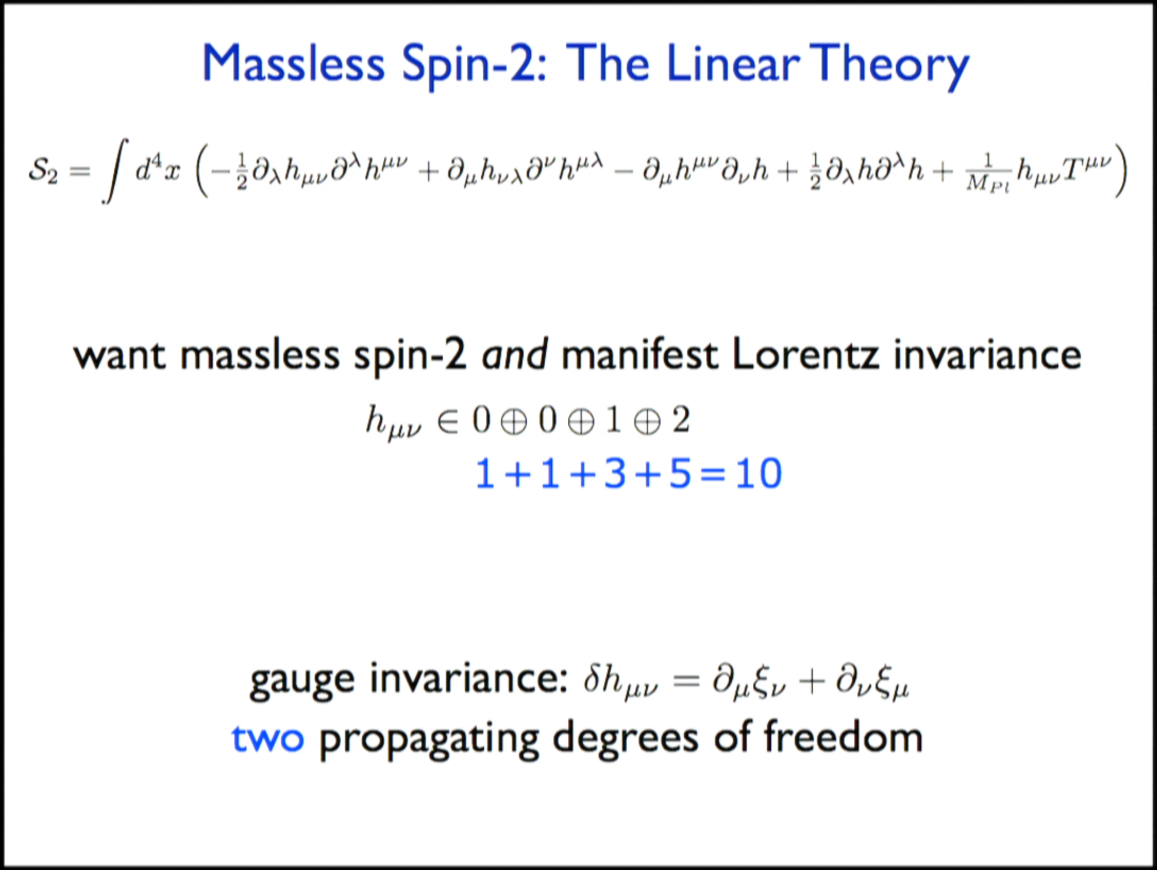
potential between point particles: $\otimes\!\!\sim\!\!\sim\!\!\sim\!\!\sim\!\!\otimes$

positive masses repel $\Rightarrow$ FAILS



A field-theorist's approach to gravity:

- spin-0 ✗
- spin-1 ✗
- spin-2
- spin$\geq$3 ✗ ?



Massless Spin-2: The Linear Theory

$$\mathcal{S}_2 = \int d^4x \left(-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\lambda h \partial^\lambda h + \tfrac{1}{M_{Pl}} h_{\mu\nu} T^{\mu\nu} \right)$$

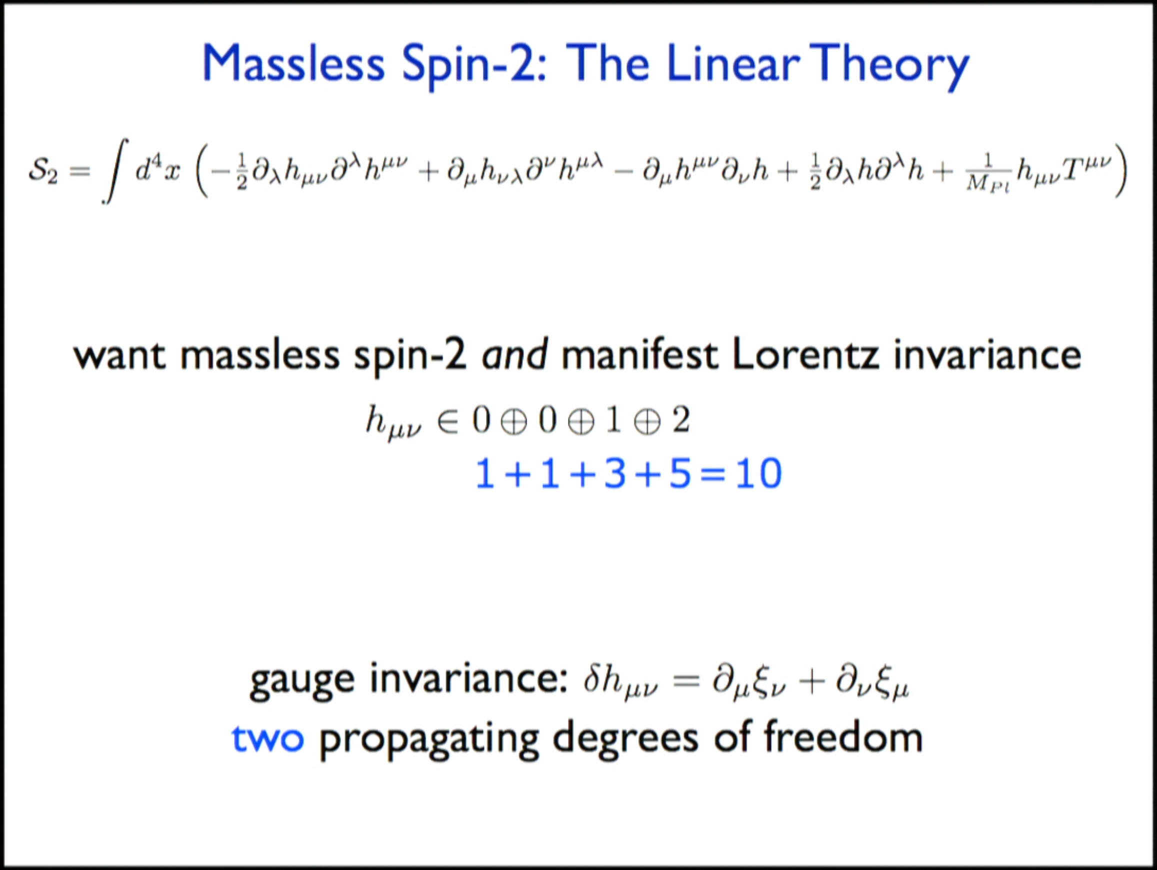
want massless spin-2 *and* manifest Lorentz invariance

$$h_{\mu\nu} \in 0 \oplus 0 \oplus 1 \oplus 2$$

$1+1+3+5=10$

gauge invariance: $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$

two propagating degrees of freedom



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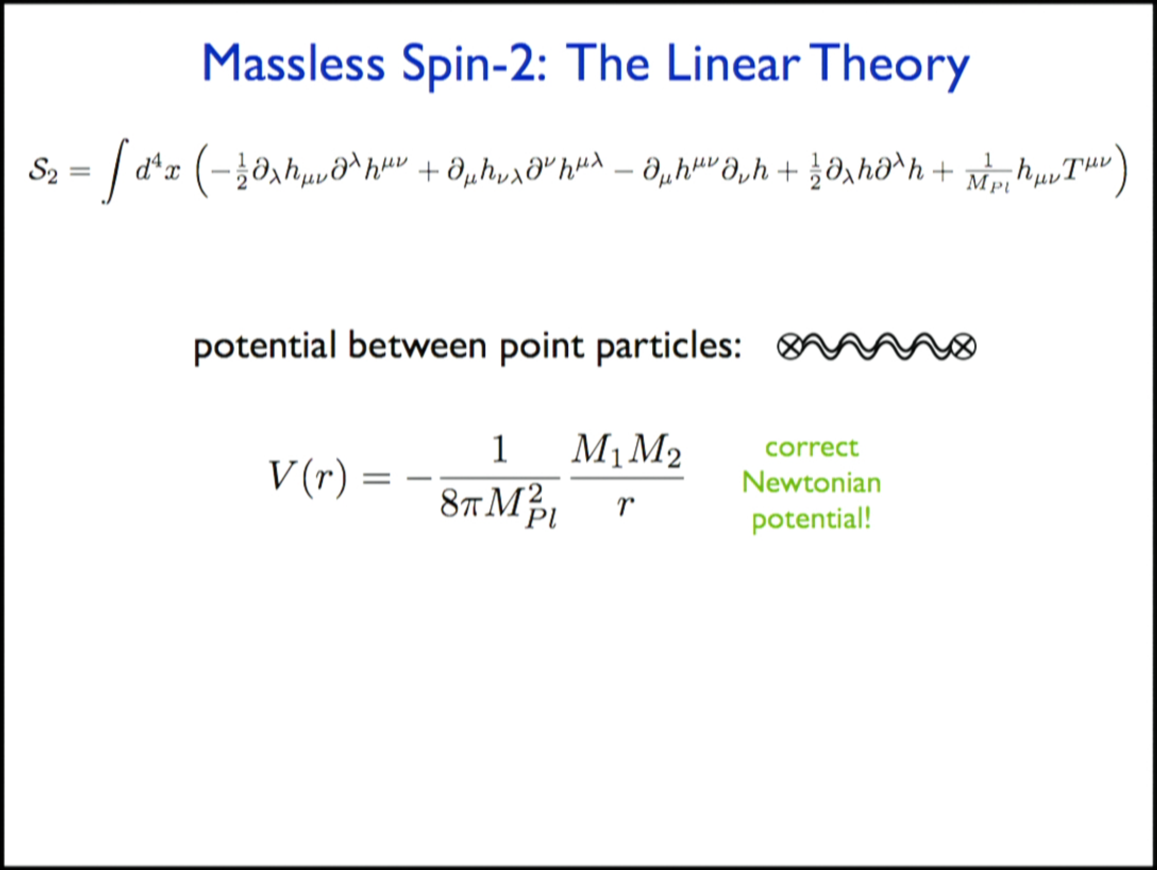

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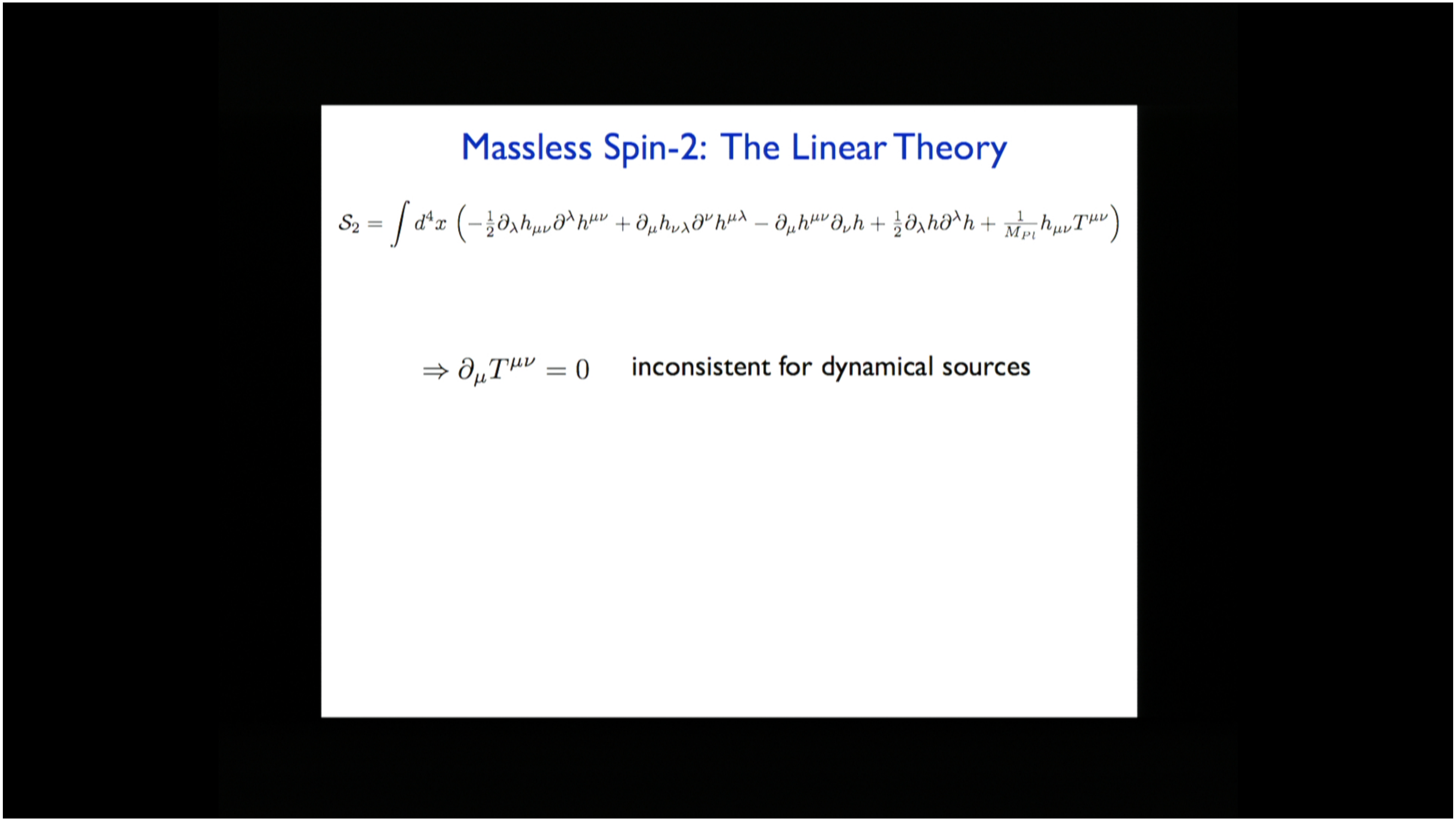
NASA/ESA

bending of light ✓

(+ hosts of other observational tests...)



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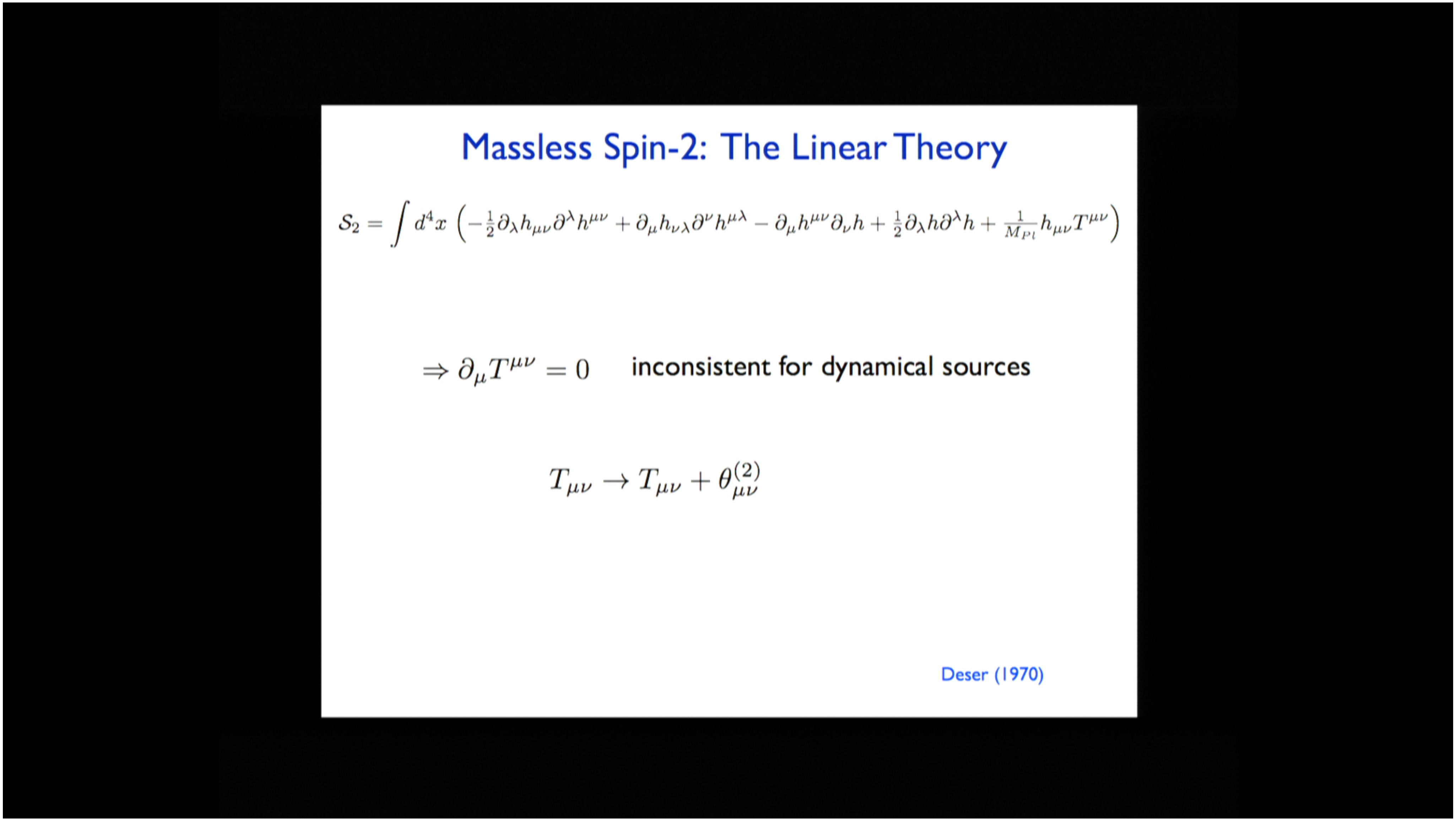
$\Rightarrow \partial_\mu T^{\mu\nu} = 0$ inconsistent for dynamical sources



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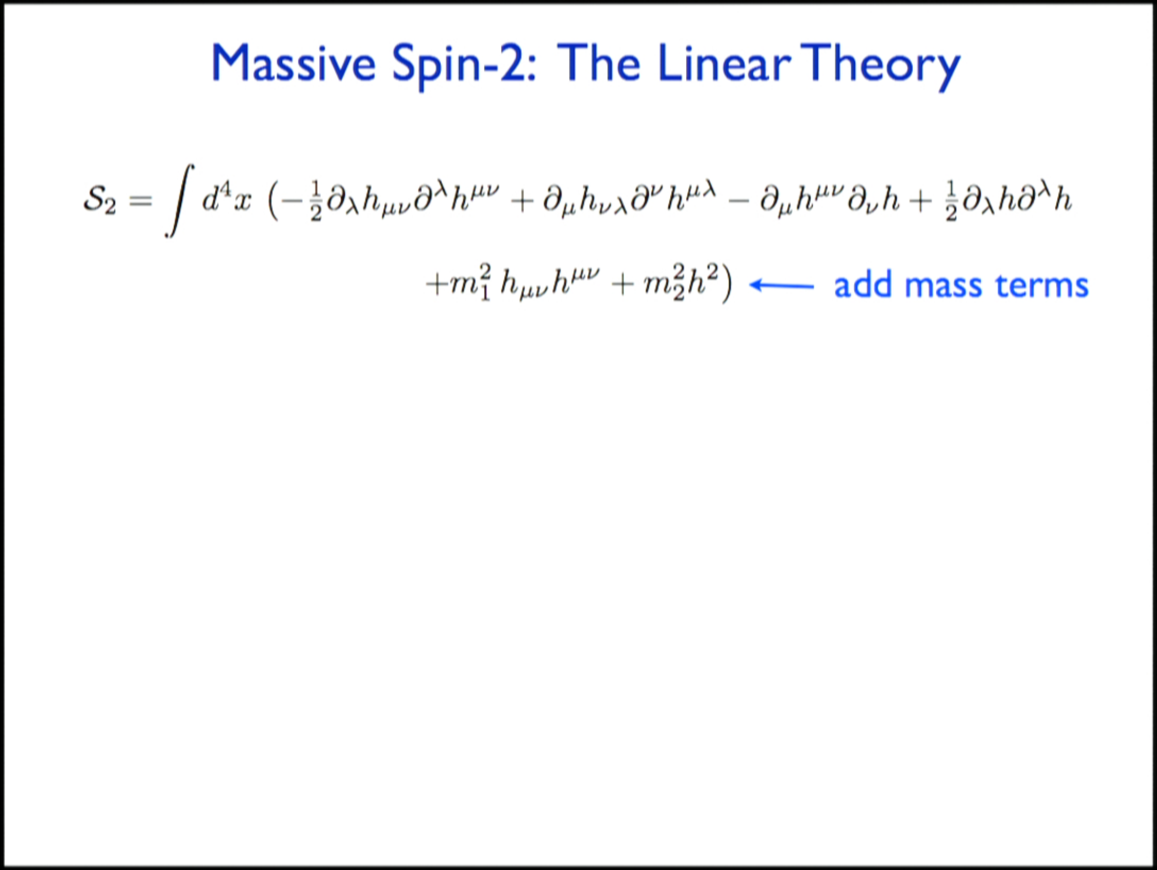
$$T_{\mu\nu} \to T_{\mu\nu} + \theta^{(2)}_{\mu\nu}$$

Deser (1970)



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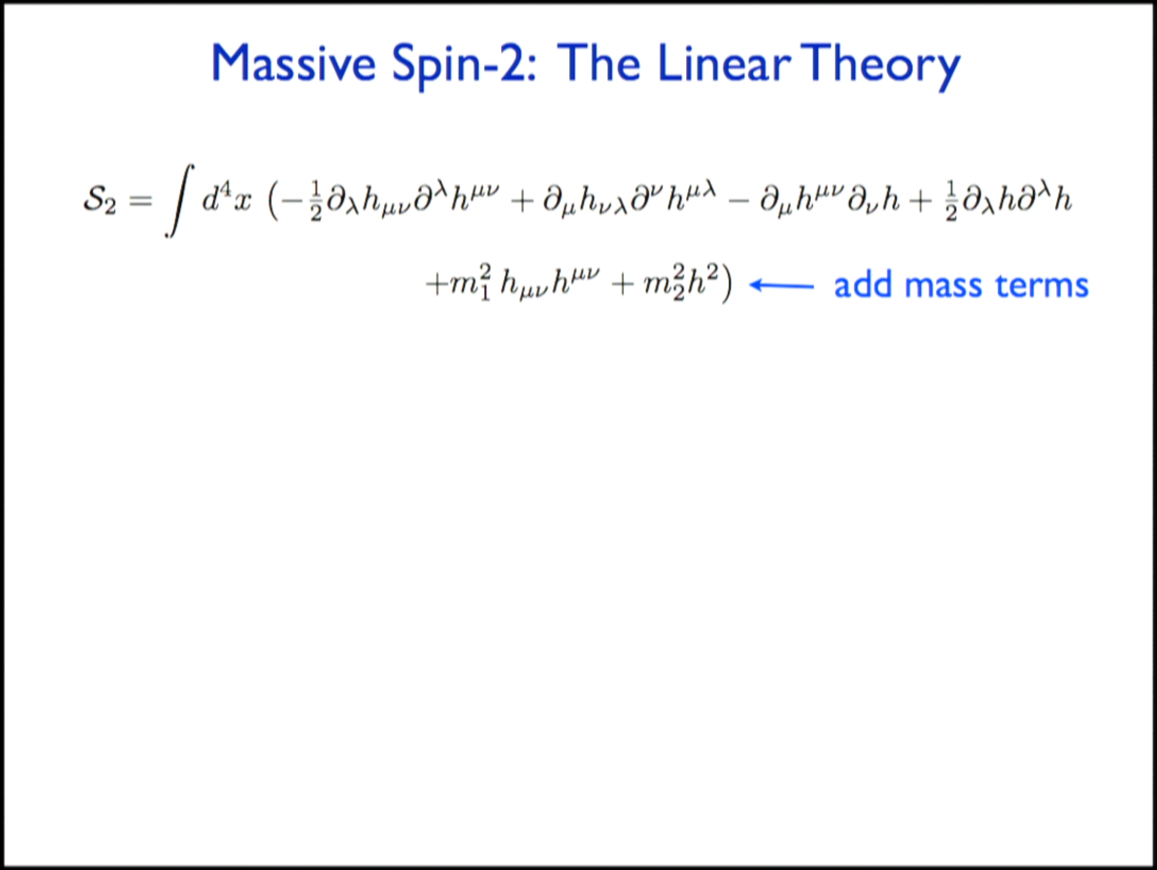
$$\left. + m_1^2 \, h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2\right) \longleftarrow \text{add mass terms}$$



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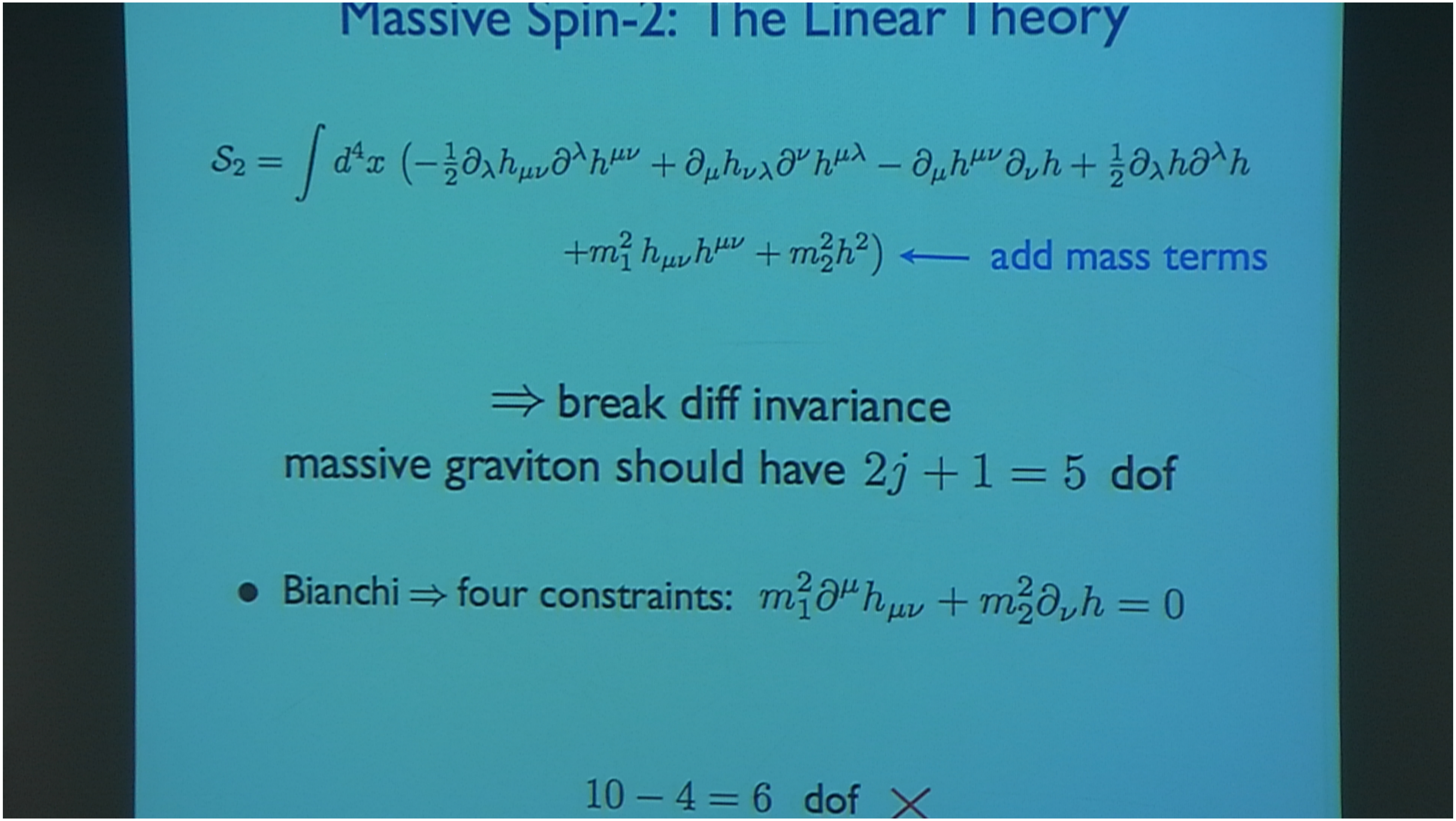

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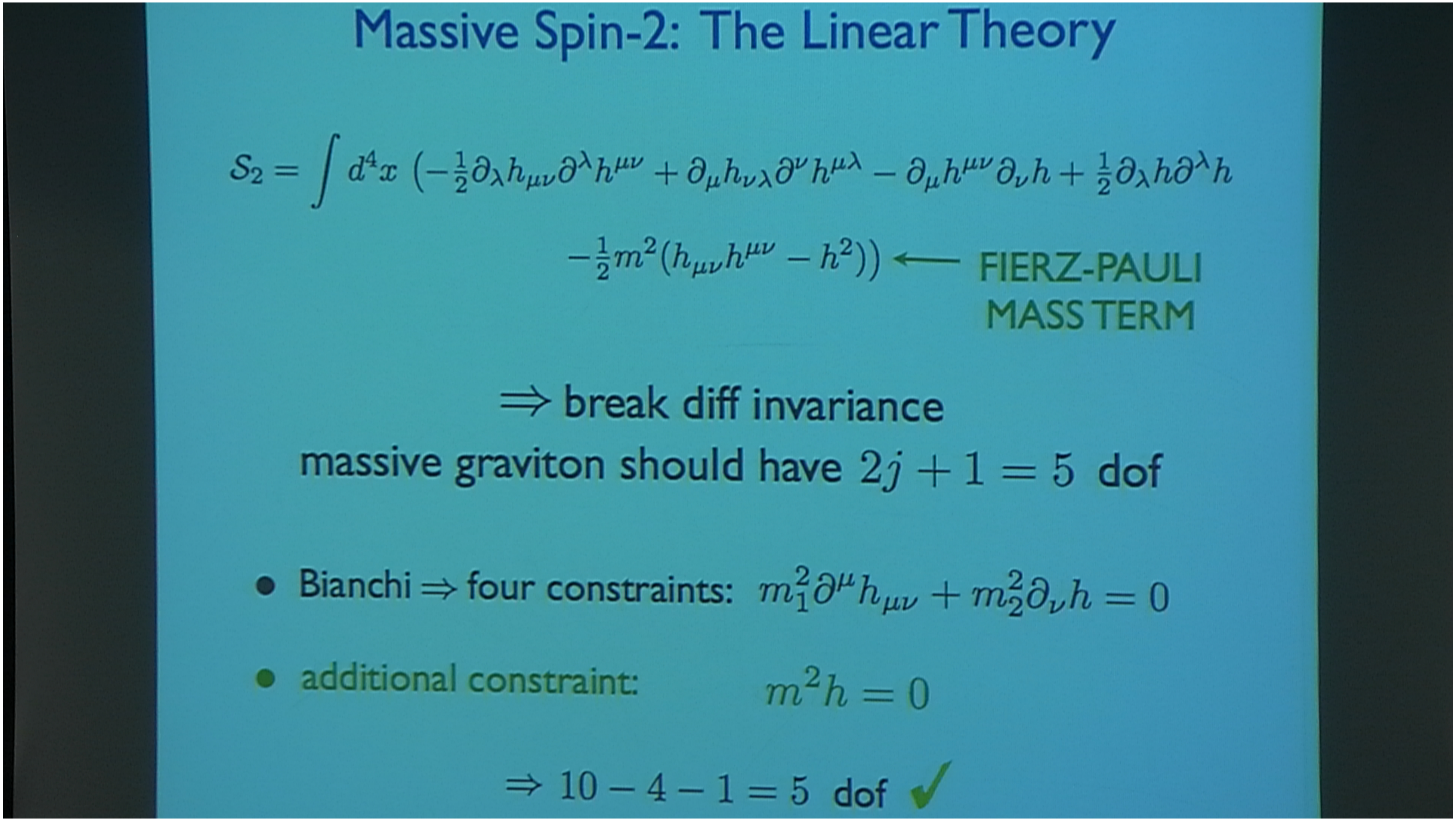
⇒ break diff invariance

massive graviton should have $2j+1=5$ dof

- Bianchi ⇒ four constraints: $m_1^2\partial^\mu h_{\mu\nu} + m_2^2\partial_\nu h = 0$

$10-4=6$ dof ×



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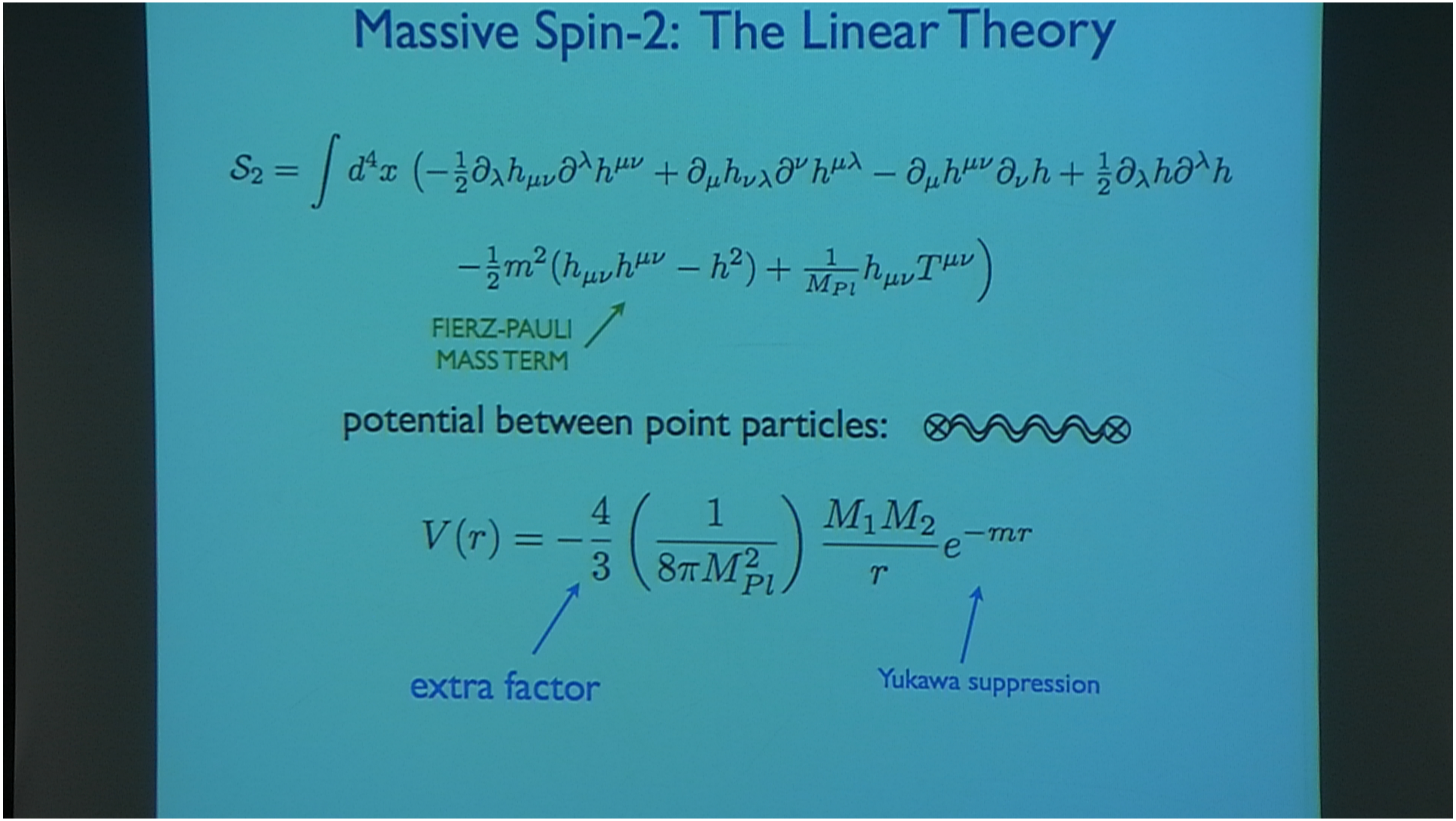
$$\left. -\tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2)\right) \longleftarrow \text{FIERZ-PAULI MASS TERM}$$

$\Rightarrow$ break diff invariance

massive graviton should have $2j+1=5$ dof

- Bianchi $\Rightarrow$ four constraints: $m_1^2\partial^\mu h_{\mu\nu} + m_2^2\partial_\nu h = 0$
- additional constraint: $m^2 h = 0$

$\Rightarrow 10-4-1=5$ dof ✓



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$$-\tfrac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - h^2) + \tfrac{1}{M_{Pl}} h_{\mu\nu}T^{\mu\nu}\Big)$$

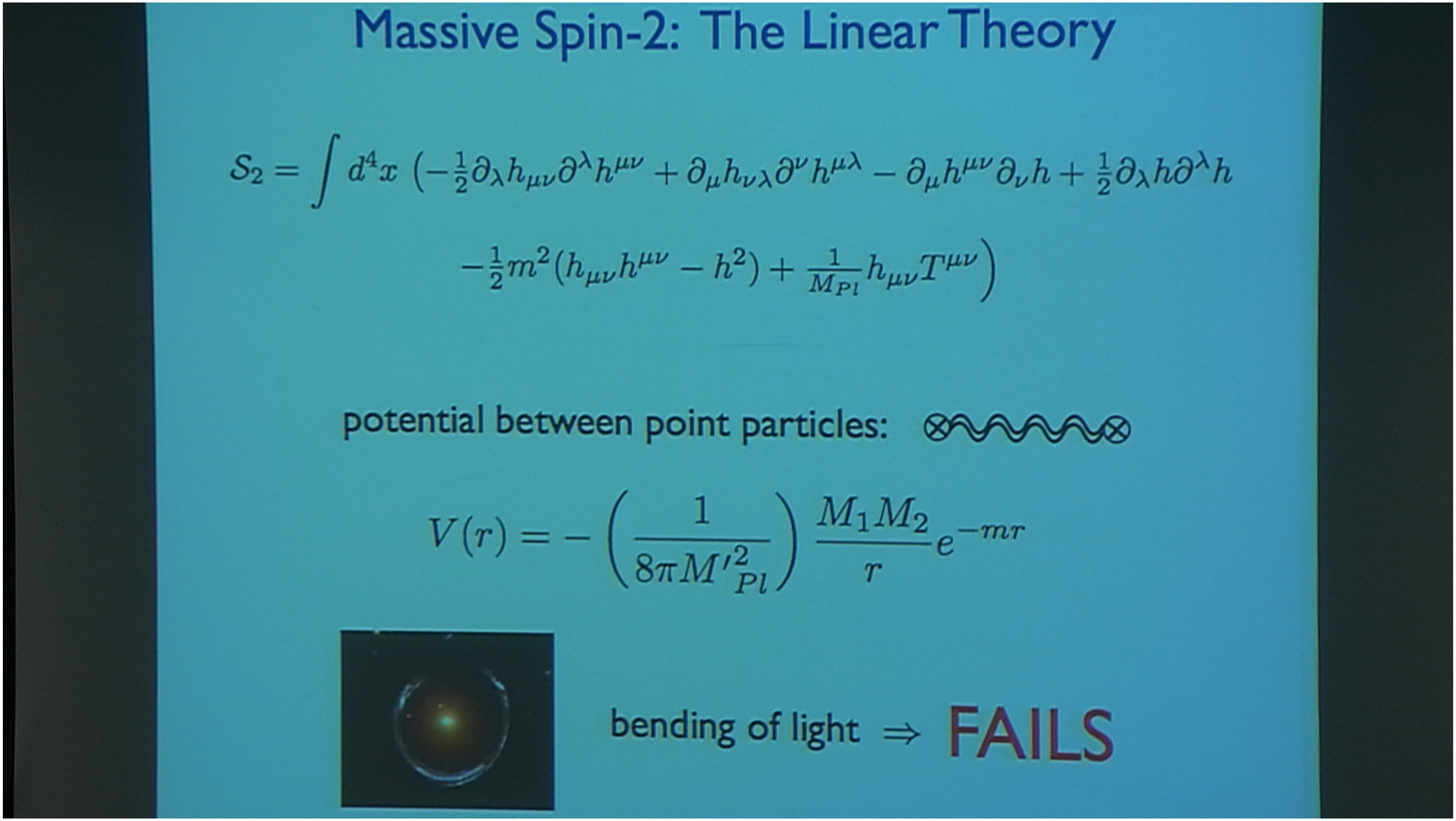
FIERZ-PAULI
MASS TERM

potential between point particles: ⊗〰〰〰⊗

$$V(r) = -\frac{4}{3}\left(\frac{1}{8\pi M_{Pl}^2}\right)\frac{M_1 M_2}{r}e^{-mr}$$

extra factor

Yukawa suppression



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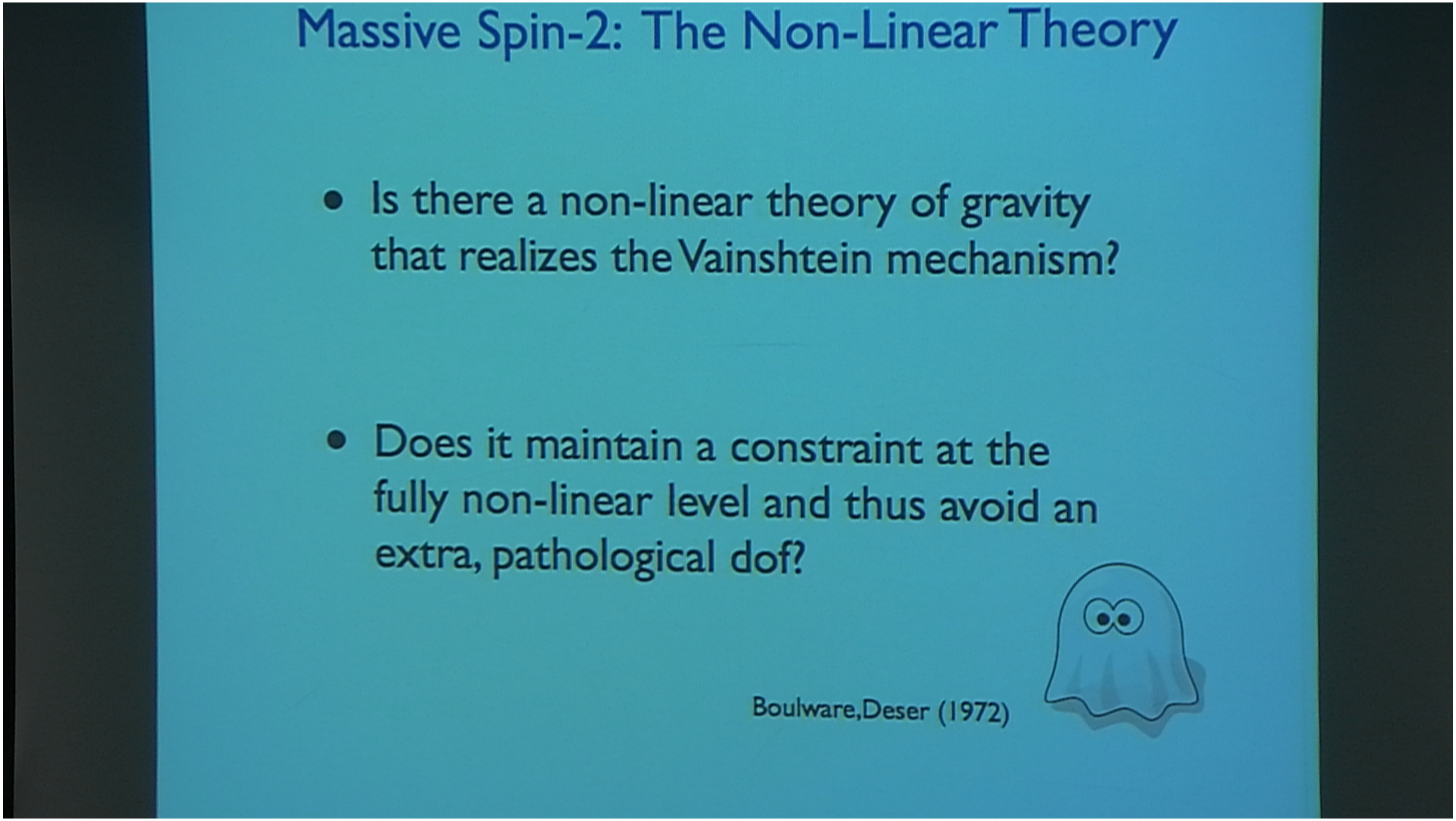
potential between point particles:

$$V(r) = -\left(\frac{1}{8\pi M'^2_{Pl}}\right)\frac{M_1 M_2}{r}e^{-mr}$$

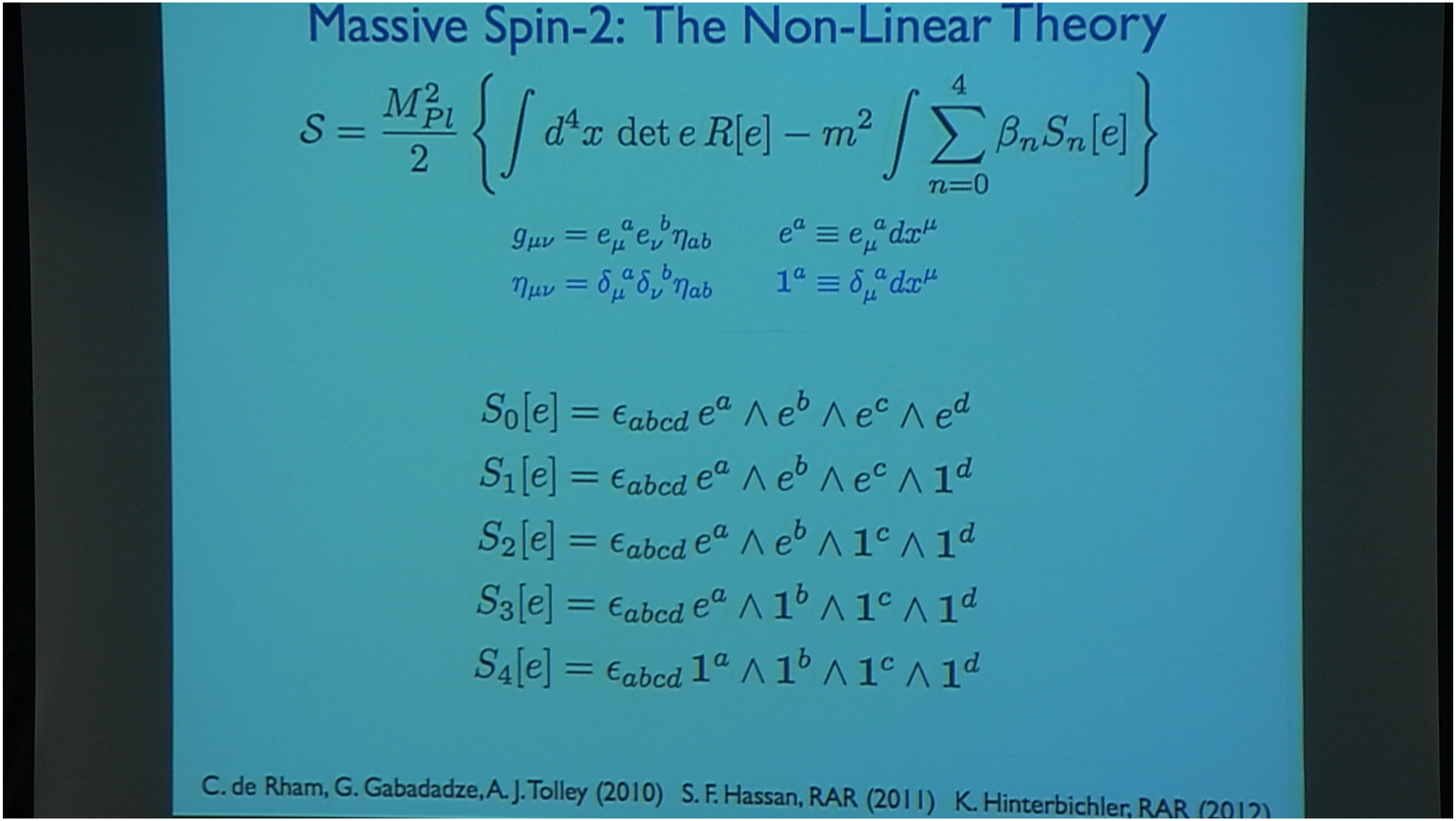
bending of light ⇒ FAILS



Massive Spin-2: The Non-Linear Theory

- Is there a non-linear theory of gravity that realizes the Vainshtein mechanism?
- Does it maintain a constraint at the fully non-linear level and thus avoid an extra, pathological dof?

Boulware,Deser (1972)



Massive Spin-2: The Non-Linear Theory

$$\mathcal{S} = \frac{M_{Pl}^2}{2}\left\{\int d^4x \,\det e\, R[e] - m^2 \int \sum_{n=0}^{4} \beta_n S_n[e]\right\}$$

$$g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab} \qquad e^a \equiv e_\mu{}^a dx^\mu$$

$$\eta_{\mu\nu} = \delta_\mu{}^a \delta_\nu{}^b \eta_{ab} \qquad \mathbf{1}^a \equiv \delta_\mu{}^a dx^\mu$$

$$S_0[e] = \epsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge e^d$$

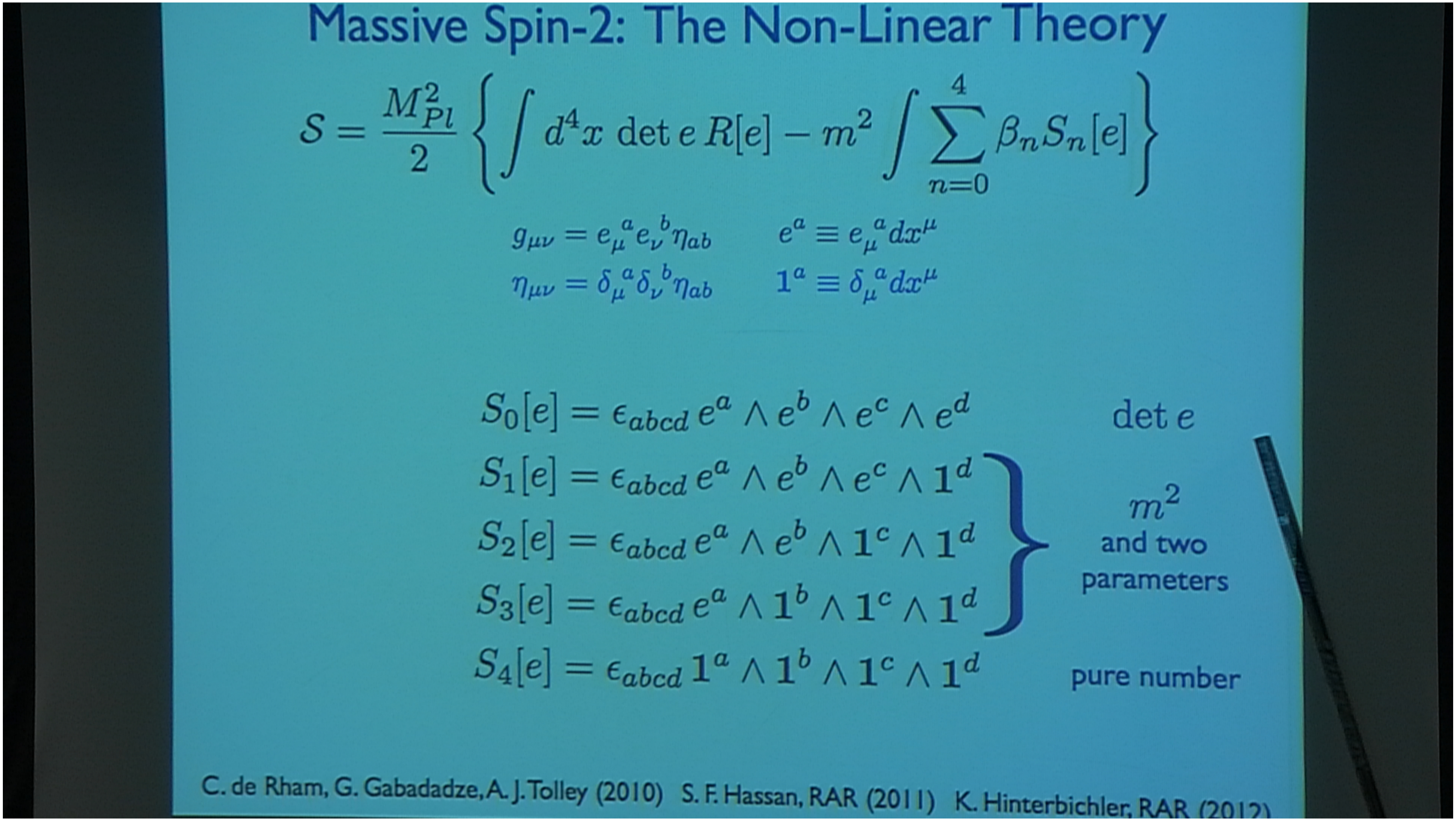
$$S_1[e] = \epsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge \mathbf{1}^d$$

$$S_2[e] = \epsilon_{abcd}\, e^a \wedge e^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

$$S_3[e] = \epsilon_{abcd}\, e^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

$$S_4[e] = \epsilon_{abcd}\, \mathbf{1}^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d$$

C. de Rham, G. Gabadadze, A. J. Tolley (2010) S. F. Hassan, RAR (2011) K. Hinterbichler, RAR (2012)



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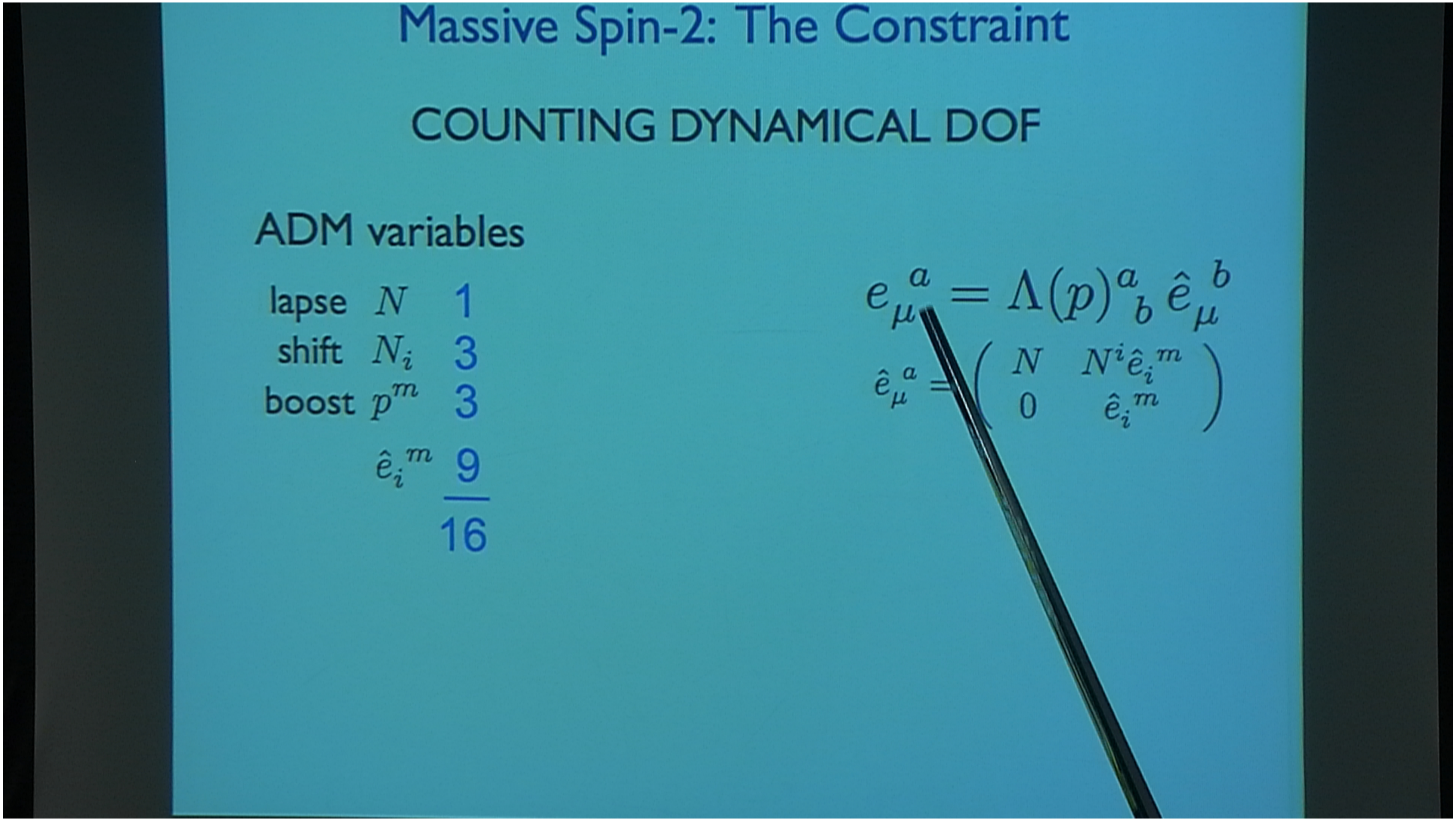
$$\left.\begin{aligned} S_1[e] &= \epsilon_{abcd}\, e^a \wedge e^b \wedge e^c \wedge \mathbf{1}^d \\ S_2[e] &= \epsilon_{abcd}\, e^a \wedge e^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d \\ S_3[e] &= \epsilon_{abcd}\, e^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d \end{aligned}\right\} \; m^2 \text{ and two parameters}$$

$$S_4[e] = \epsilon_{abcd}\, \mathbf{1}^a \wedge \mathbf{1}^b \wedge \mathbf{1}^c \wedge \mathbf{1}^d \qquad \text{pure number}$$

C. de Rham, G. Gabadadze, A. J. Tolley (2010)   S. F. Hassan, RAR (2011)   K. Hinterbichler, RAR (2012)



Massive Spin-2: The Constraint

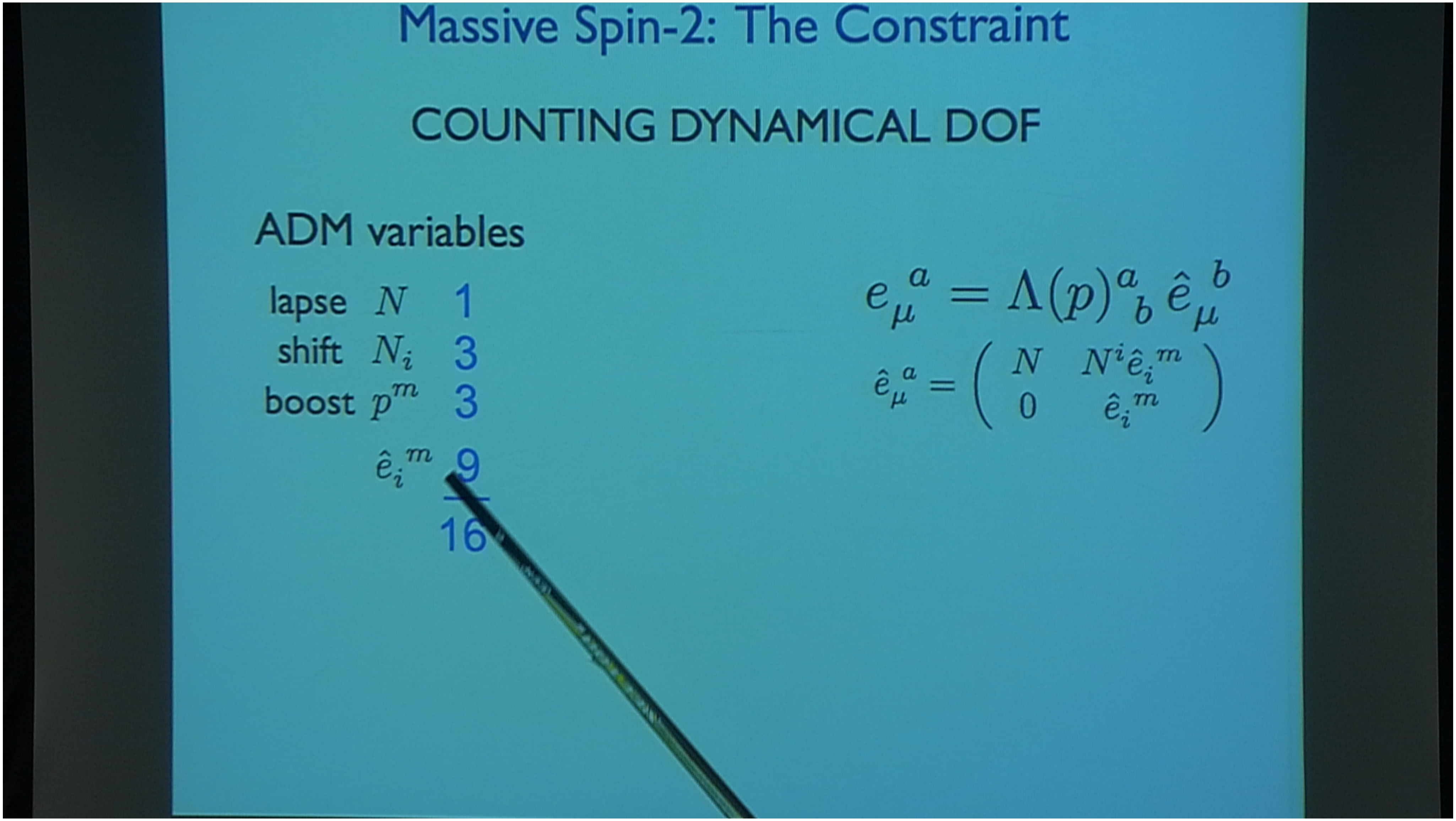
COUNTING DYNAMICAL DOF

ADM variables

lapse	N	1
shift	N_i	3
boost	p^m	3
	$\hat{e}_i{}^m$	9
		16

$$e_\mu{}^a = \Lambda(p)^a{}_b\, \hat{e}_\mu{}^b$$

$$\hat{e}_\mu{}^a = \begin{pmatrix} N & N^i \hat{e}_i{}^m \\ 0 & \hat{e}_i{}^m \end{pmatrix}$$



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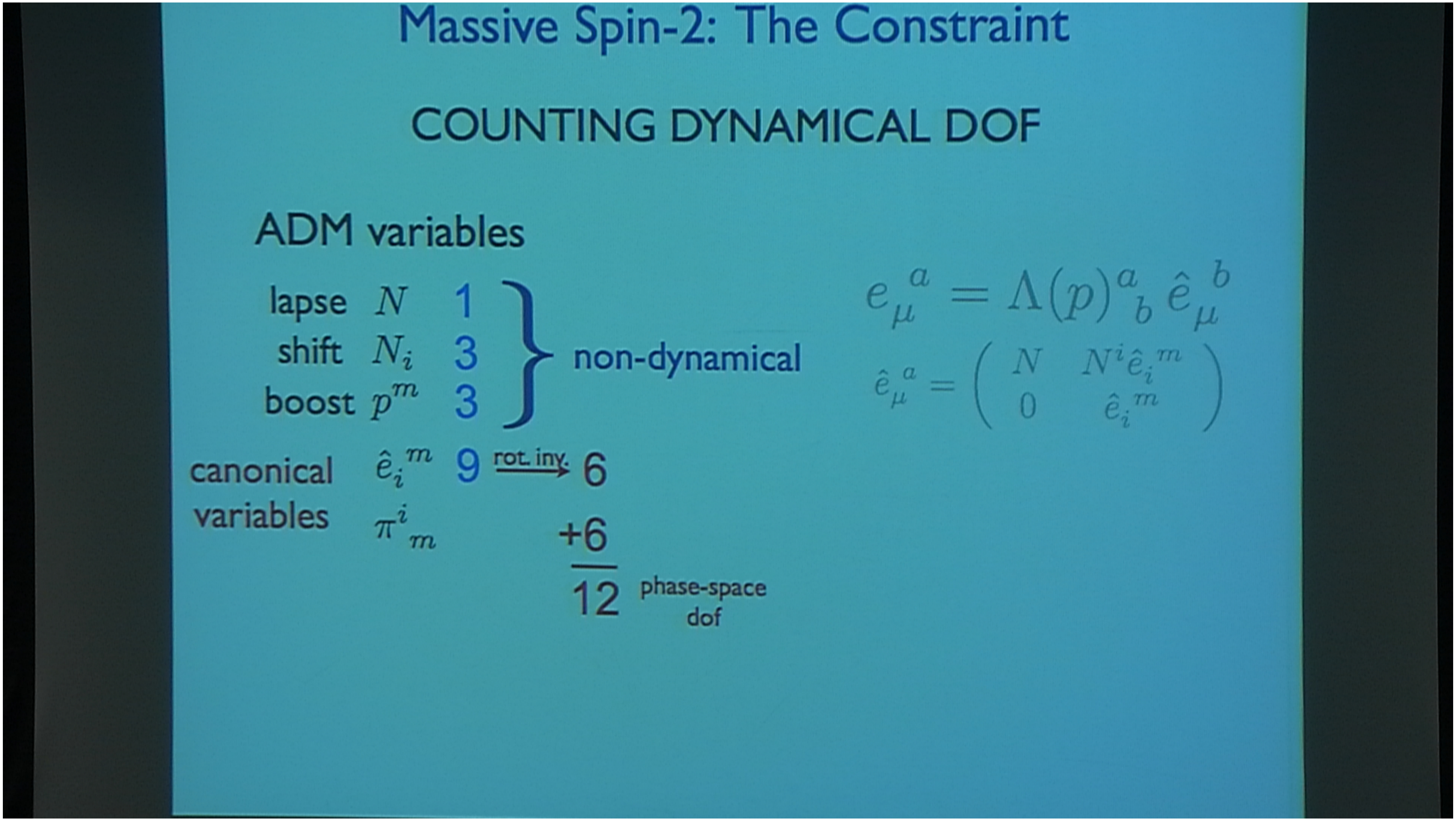

Massive Spin-2: The Constraint

COUNTING DYNAMICAL DOF

ADM variables

$$\left.\begin{array}{rll} \text{lapse} & N & 1 \\ \text{shift} & N_i & 3 \\ \text{boost} & p^m & 3 \end{array}\right\} \text{non-dynamical}$$

$$e_\mu{}^a = \Lambda(p)^a{}_b \, \hat{e}_\mu{}^b$$

$$\hat{e}_\mu{}^a = \begin{pmatrix} N & N^i \hat{e}_i{}^m \\ 0 & \hat{e}_i{}^m \end{pmatrix}$$

$$\begin{array}{lll} \text{canonical} & \hat{e}_i{}^m & 9 \xrightarrow{\text{rot. inv.}} 6 \\ \text{variables} & \pi^i{}_m & \quad\quad\quad \underline{+6} \\ & & \quad\quad\quad 12 \ \text{phase-space dof} \end{array}$$

Massive Spin-2: The Constraint

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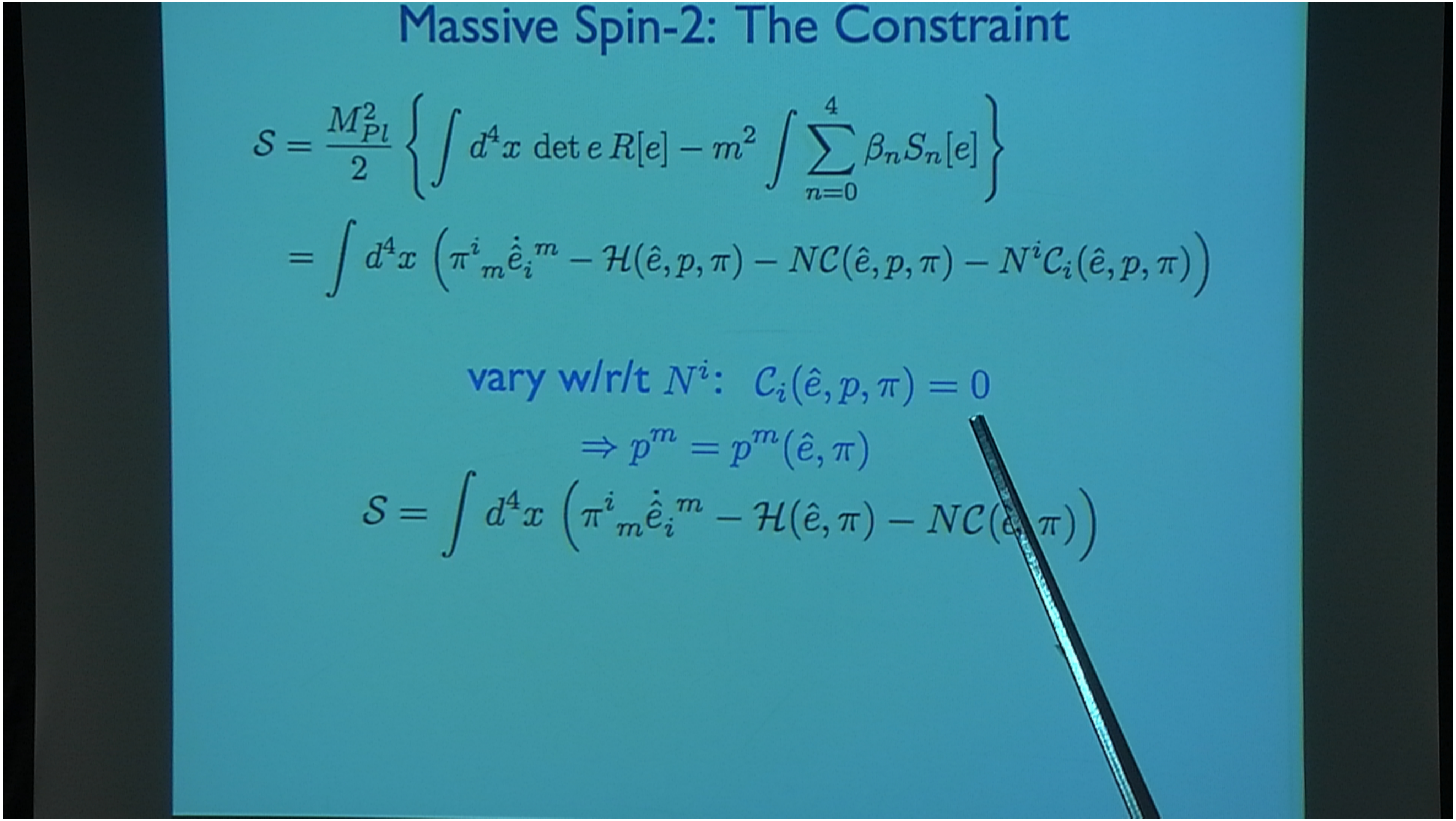
$$= \int d^4x \left(\pi^i{}_m \dot{\hat{e}}_i{}^m - \mathcal{H}(\hat{e},p,\pi) - N\mathcal{C}(\hat{e},p,\pi) - N^i\mathcal{C}_i(\hat{e},p,\pi)\right)$$



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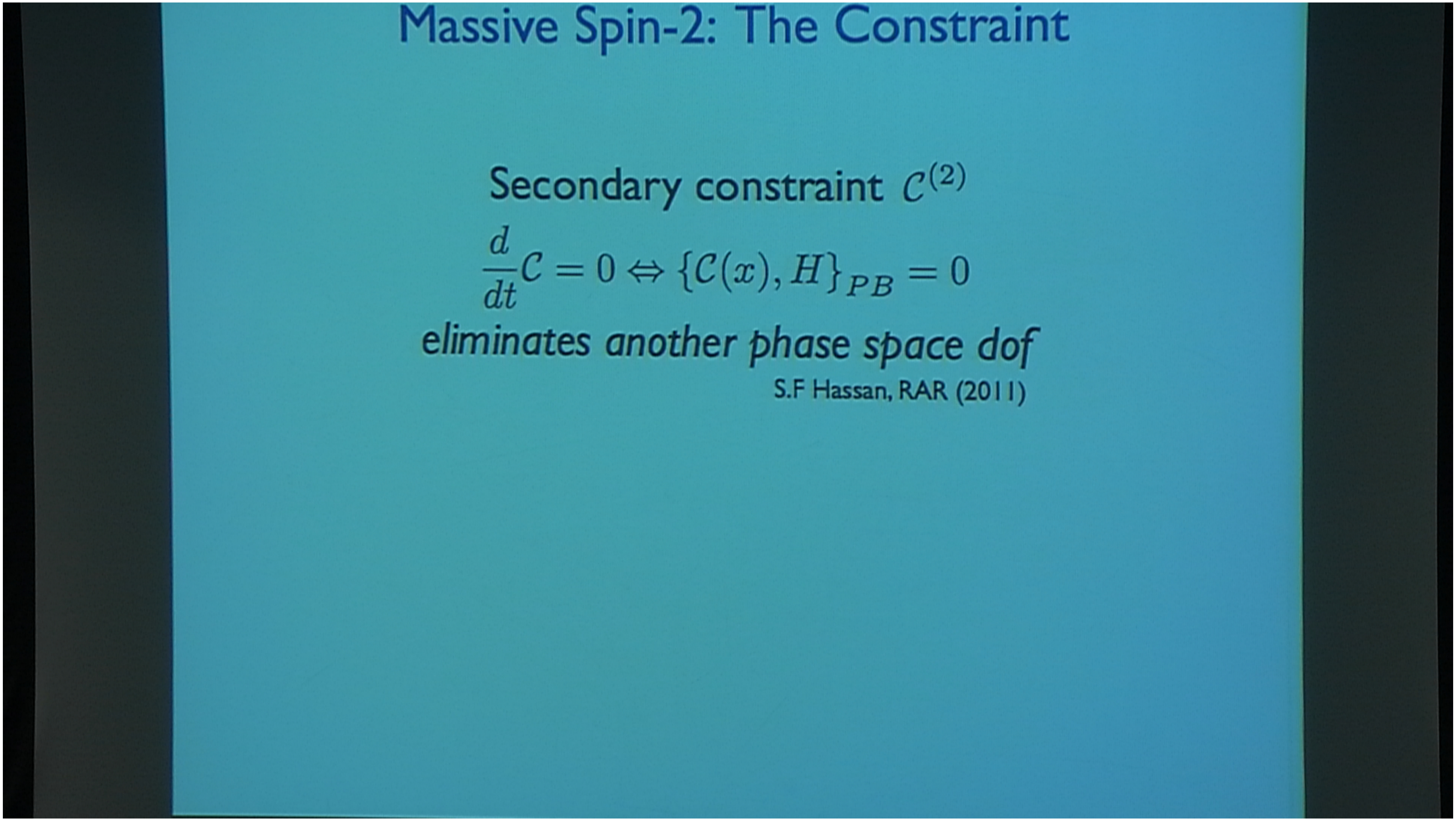
vary w/r/t N^i: $\mathcal{C}_i(\hat{e},p,\pi) = 0$

$$\Rightarrow p^m = p^m(\hat{e},\pi)$$

$$\mathcal{S} = \int d^4x \left(\pi^i{}_m \dot{\hat{e}}_i{}^m - \mathcal{H}(\hat{e},\pi) - N\mathcal{C}(\hat{e},\pi)\right)$$



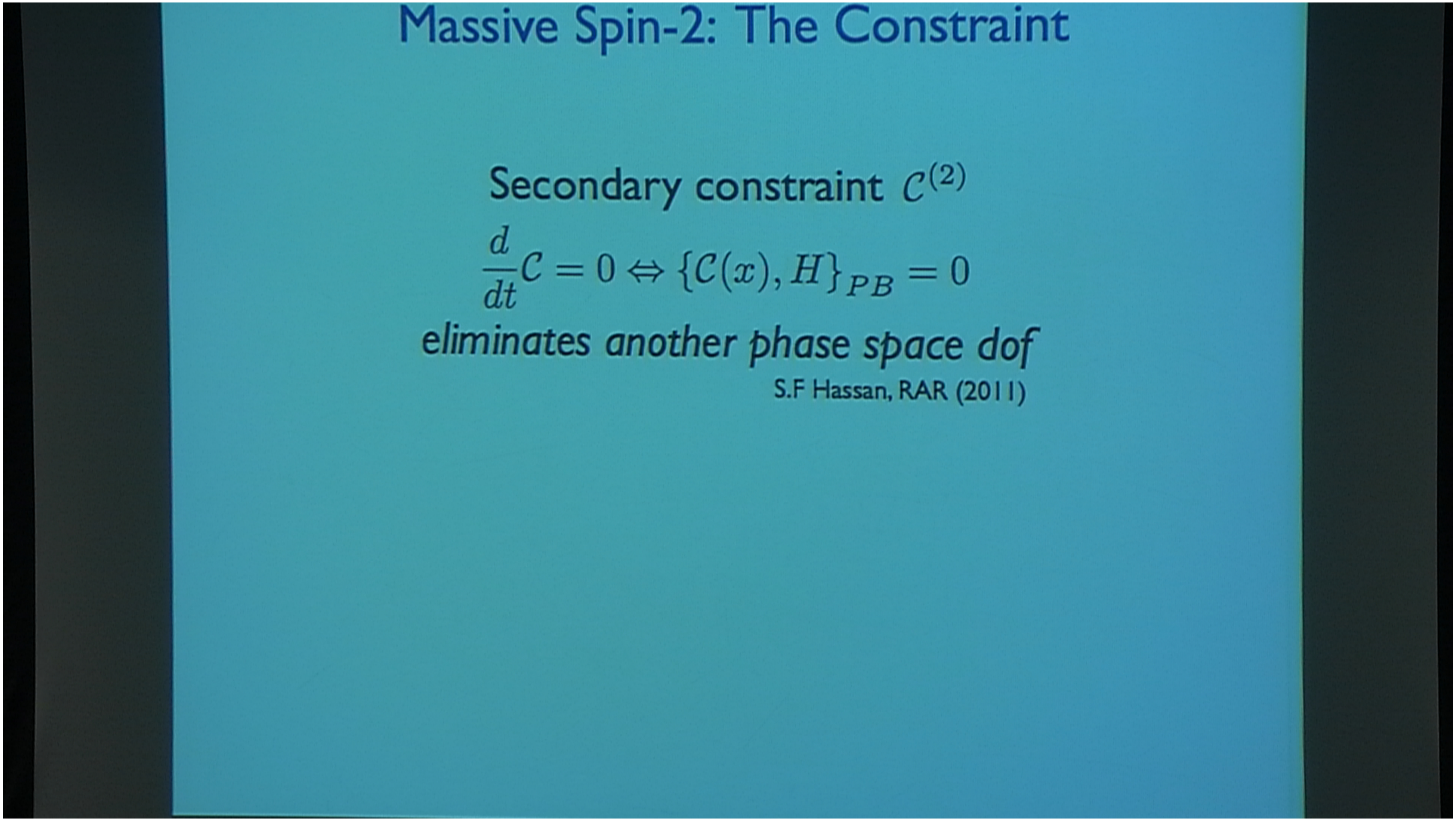
Massive Spin-2: The Constraint

Secondary constraint $\mathcal{C}^{(2)}$

$$\frac{d}{dt}\mathcal{C} = 0 \Leftrightarrow \{\mathcal{C}(x), H\}_{PB} = 0$$

eliminates another phase space dof

S.F Hassan, RAR (2011)



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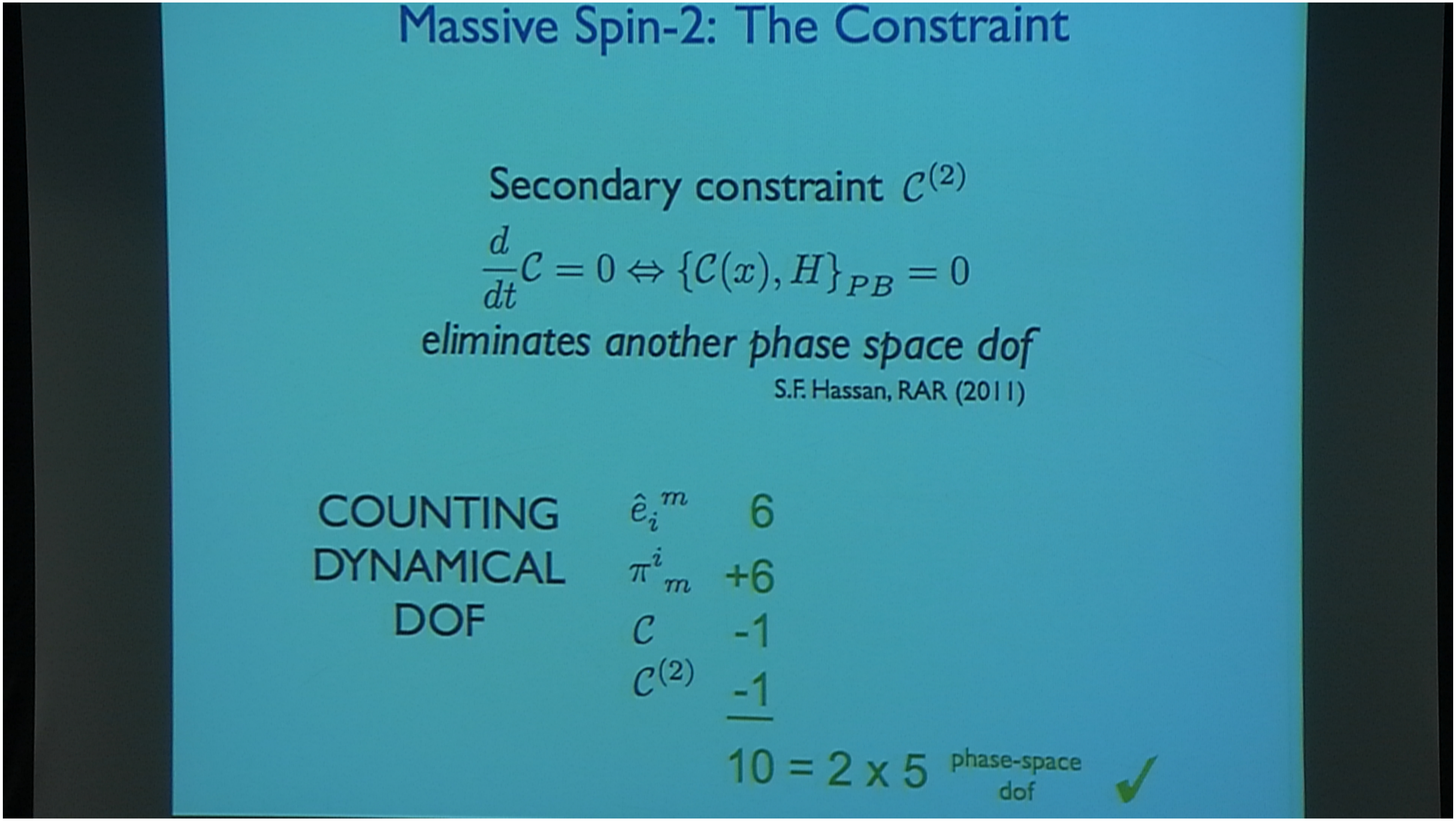

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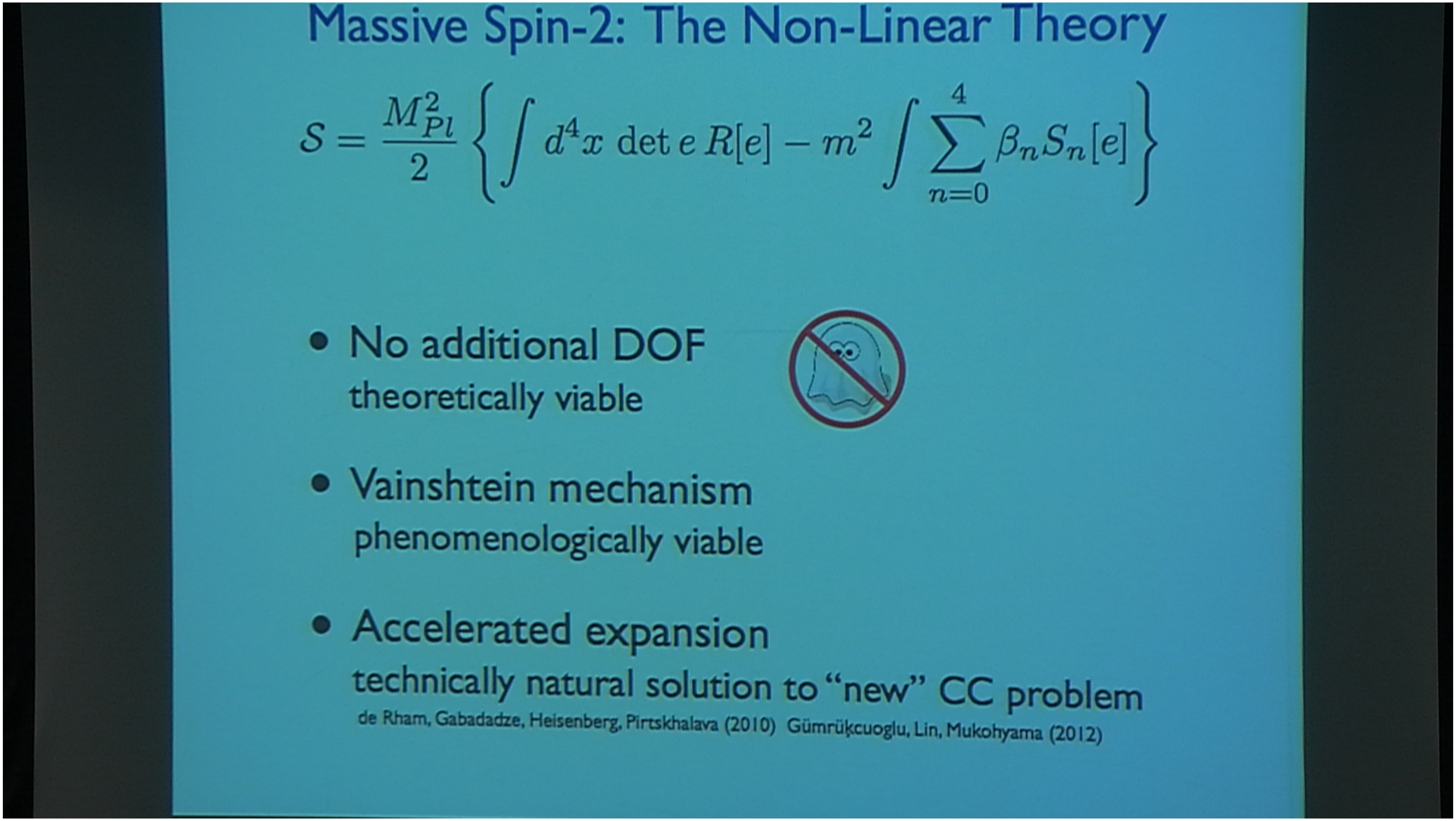
S.F. Hassan, RAR (2011)

COUNTING
DYNAMICAL
DOF

$\hat{e}_i{}^m$	6
$\pi^i{}_m$	+6
$\mathcal{C}$	-1
$\mathcal{C}^{(2)}$	-1

10 = 2 x 5 phase-space dof ✓

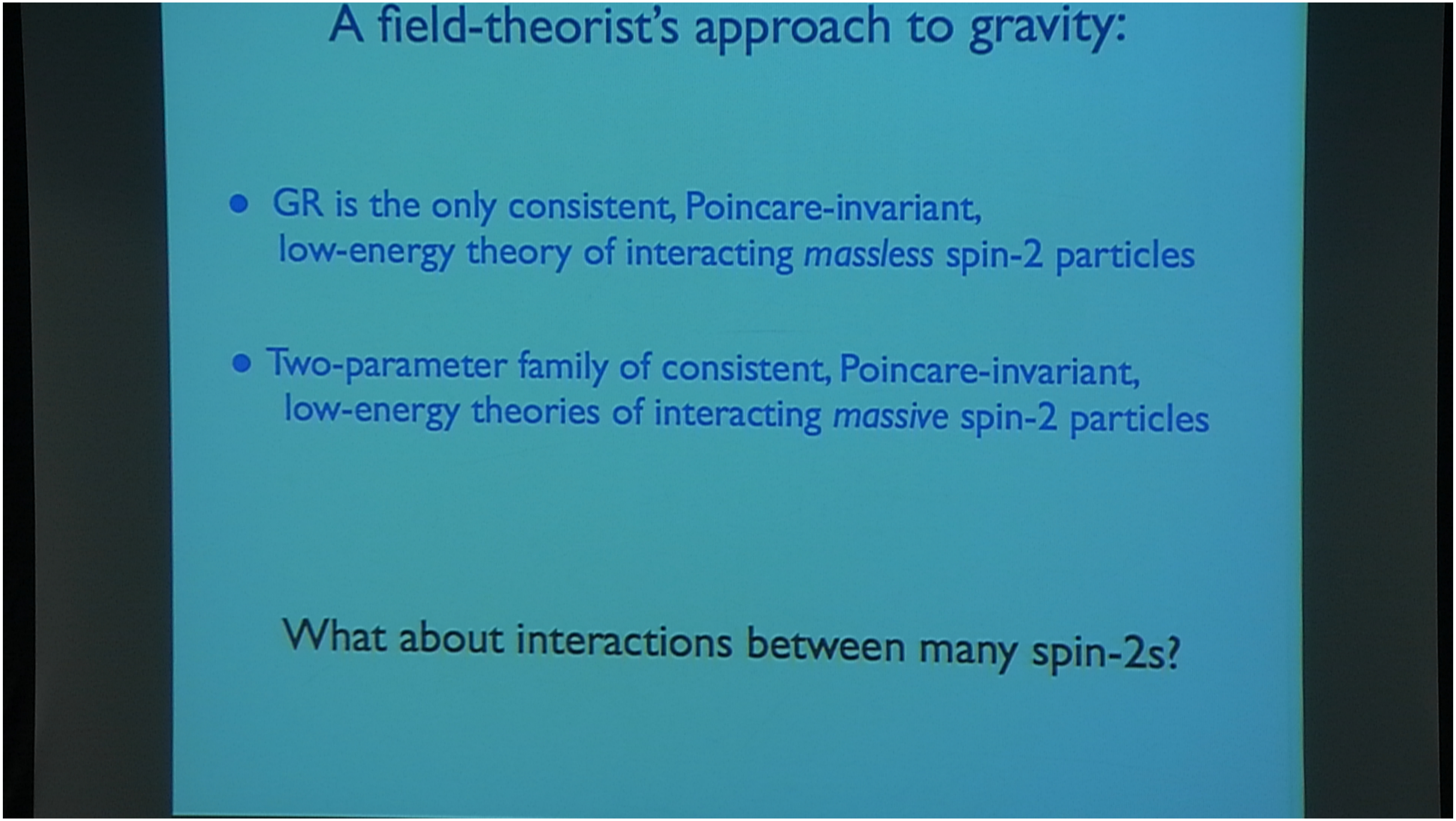


Massive Spin-2: The Non-Linear Theory

$$\mathcal{S} = \frac{M_{Pl}^2}{2}\left\{\int d^4x \, \det e \, R[e] - m^2 \int \sum_{n=0}^{4} \beta_n S_n[e]\right\}$$

- No additional DOF
  theoretically viable
- Vainshtein mechanism
  phenomenologically viable
- Accelerated expansion
  technically natural solution to "new" CC problem

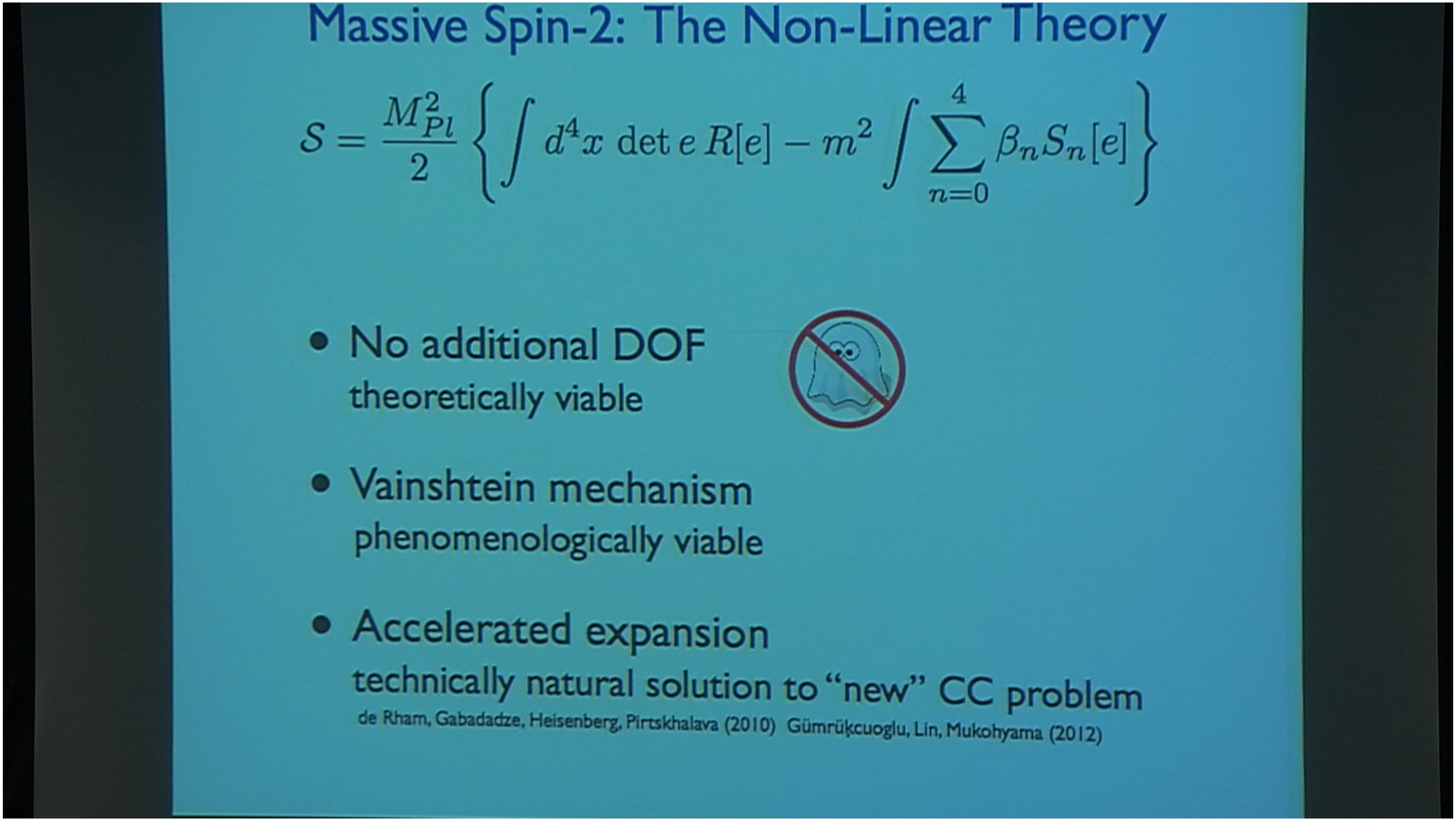
de Rham, Gabadadze, Heisenberg, Pirtskhalava (2010) Gümrükcuoglu, Lin, Mukohyama (2012)



A field-theorist's approach to gravity:

- GR is the only consistent, Poincare-invariant, low-energy theory of interacting *massless* spin-2 particles
- Two-parameter family of consistent, Poincare-invariant, low-energy theories of interacting *massive* spin-2 particles

What about interactions between many spin-2s?



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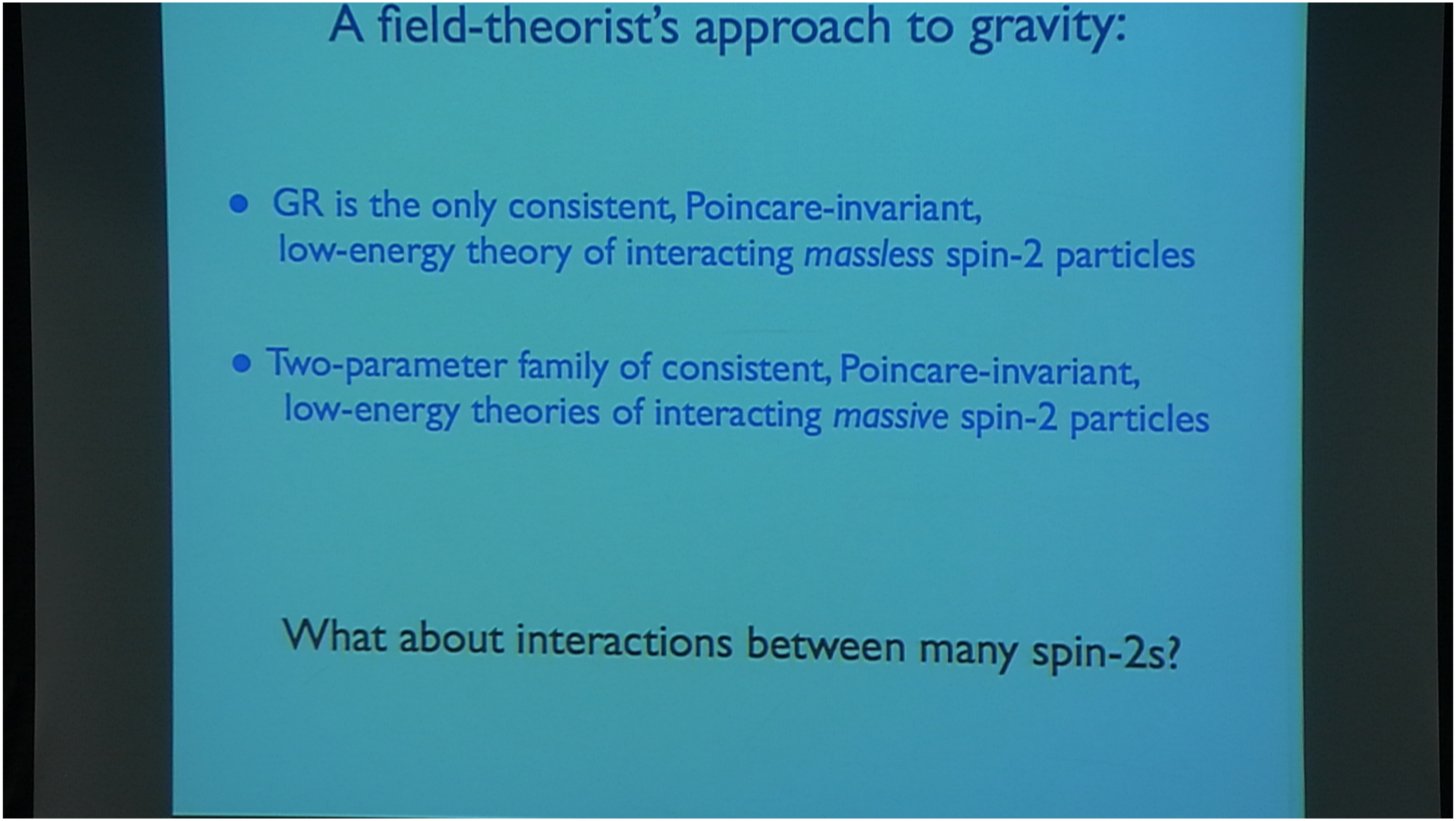

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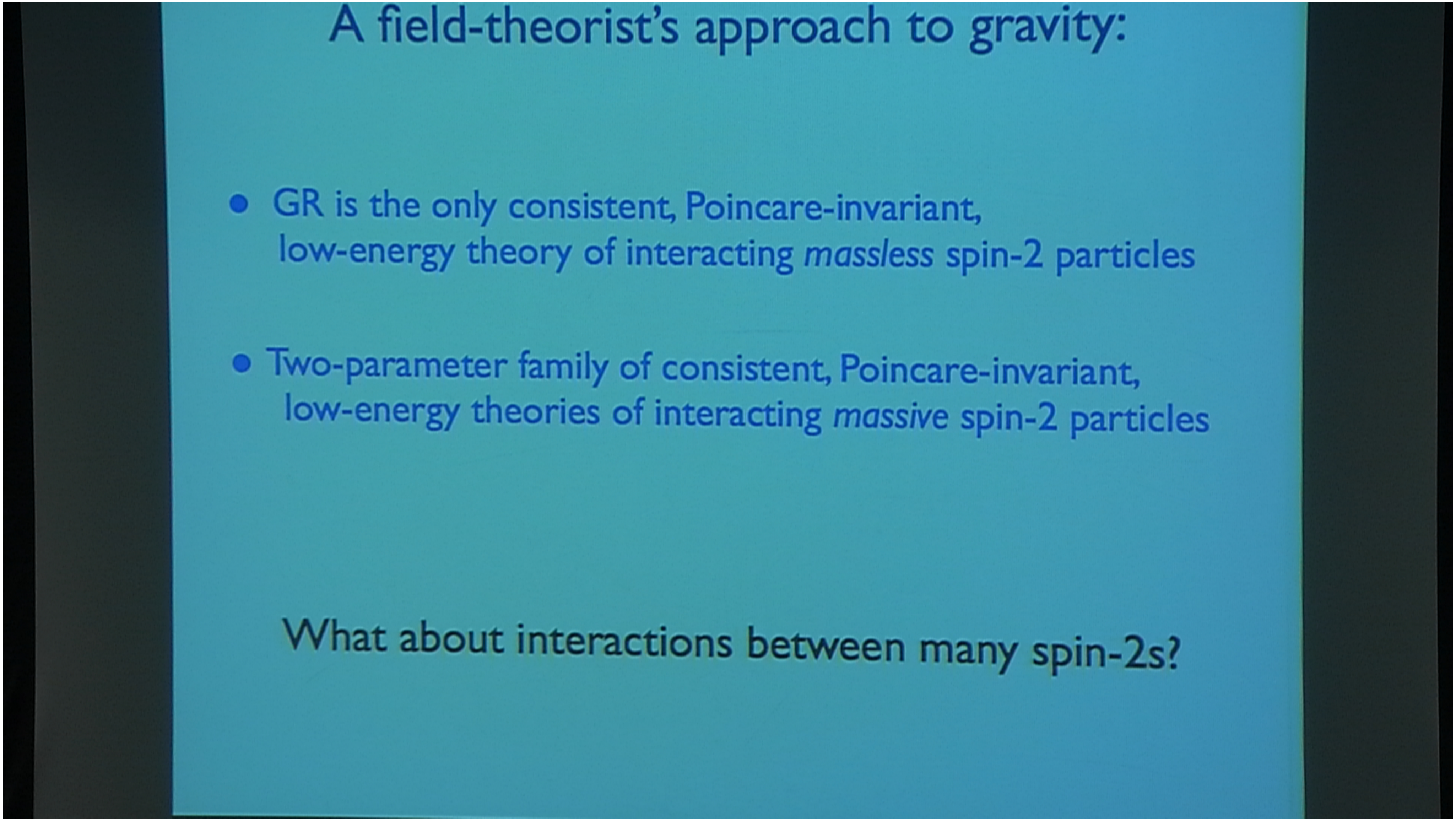

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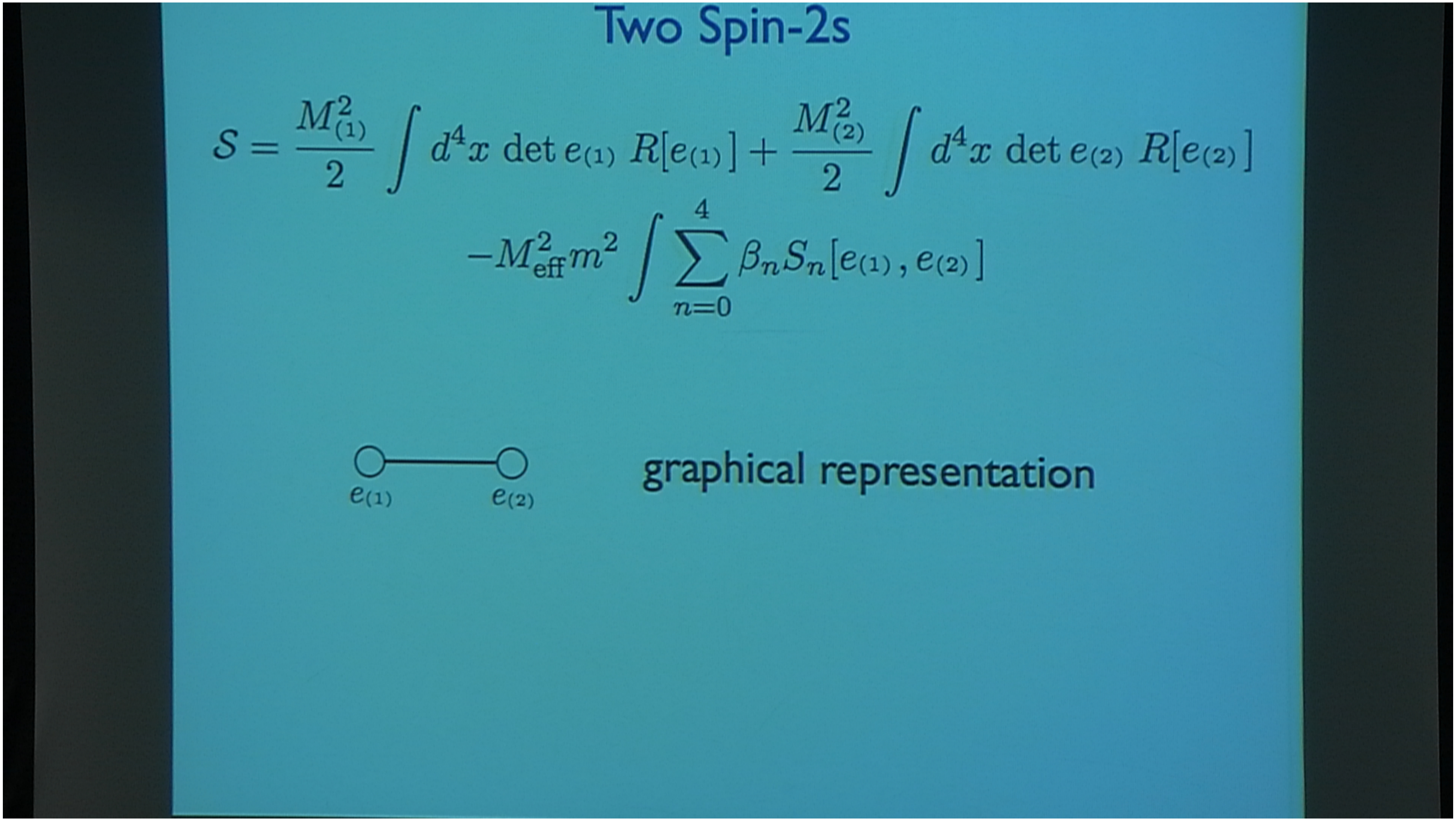
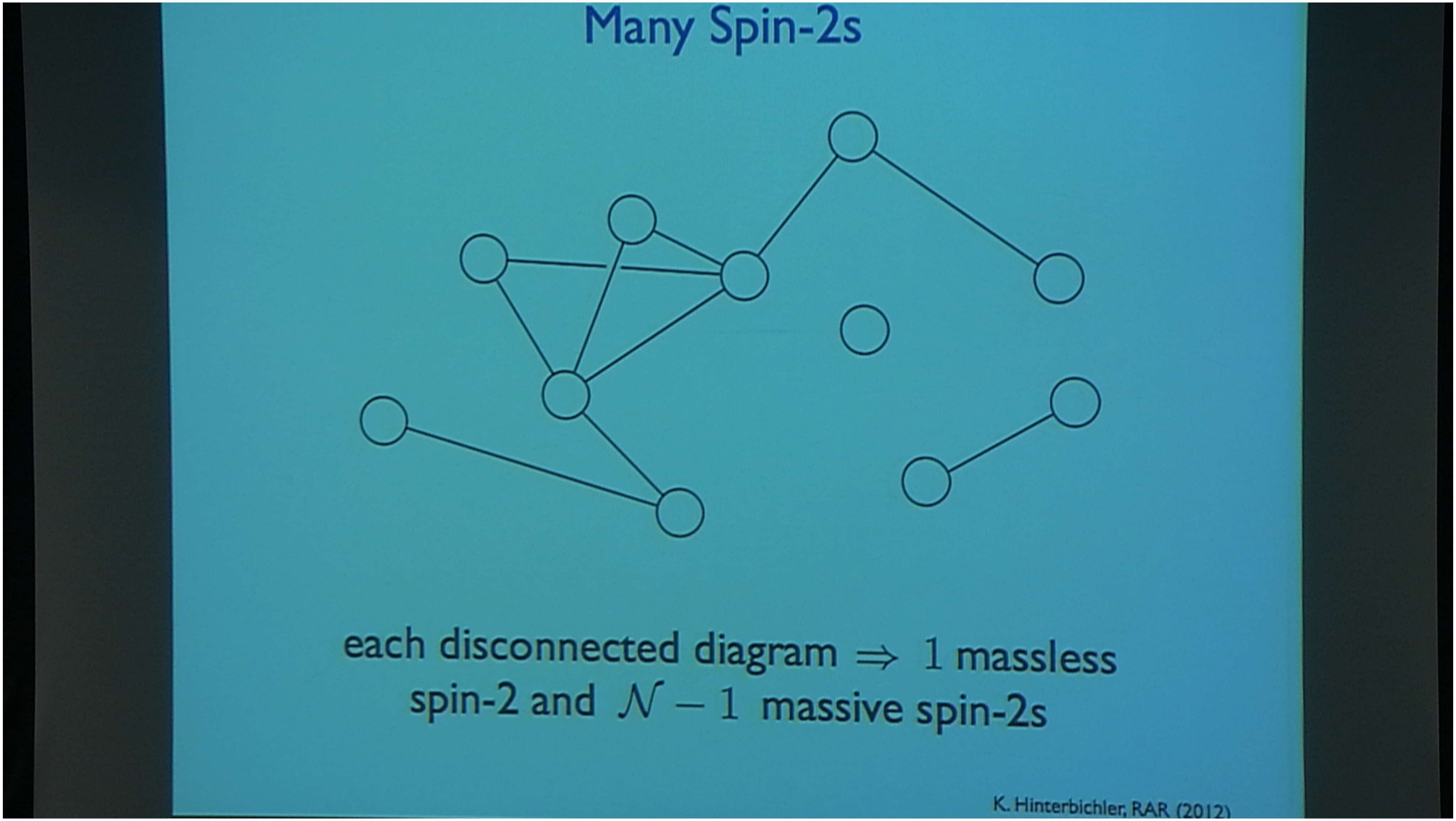


Two Spin-2s

$$\mathcal{S} = \frac{M_{(1)}^2}{2} \int d^4x \, \det e_{(1)} \, R[e_{(1)}] + \frac{M_{(2)}^2}{2} \int d^4x \, \det e_{(2)} \, R[e_{(2)}]$$

$$- M_{\text{eff}}^2 m^2 \int \sum_{n=0}^{4} \beta_n S_n[e_{(1)}, e_{(2)}]$$

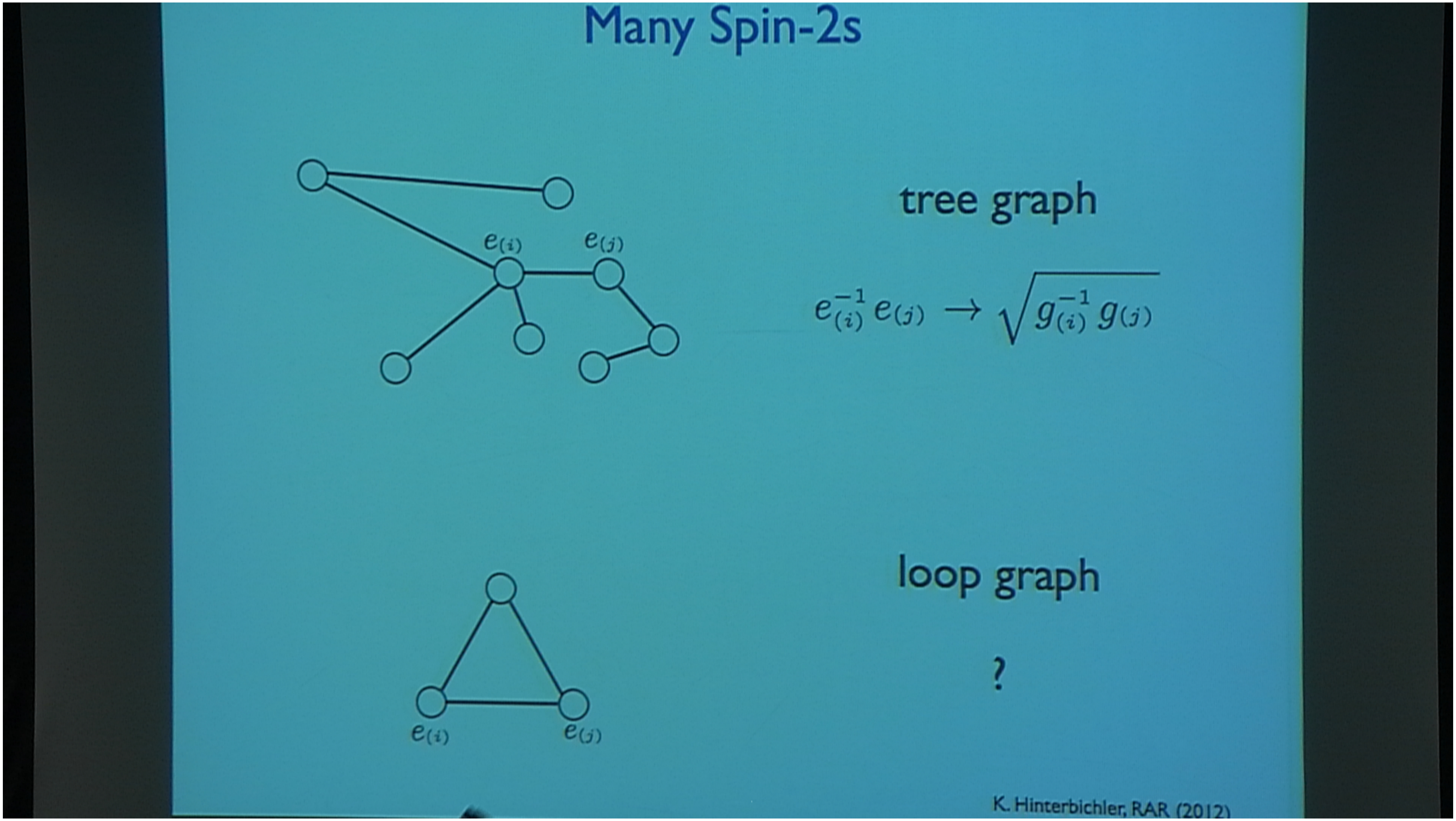
graphical representation



Many Spin-2s

each disconnected diagram $\Rightarrow$ 1 massless spin-2 and $\mathcal{N}-1$ massive spin-2s

K. Hinterbichler, RAR (2012)



Many Spin-2s

tree graph

$$e_{(i)}^{-1}\, e_{(j)} \to \sqrt{g_{(i)}^{-1}\, g_{(j)}}$$

loop graph

?

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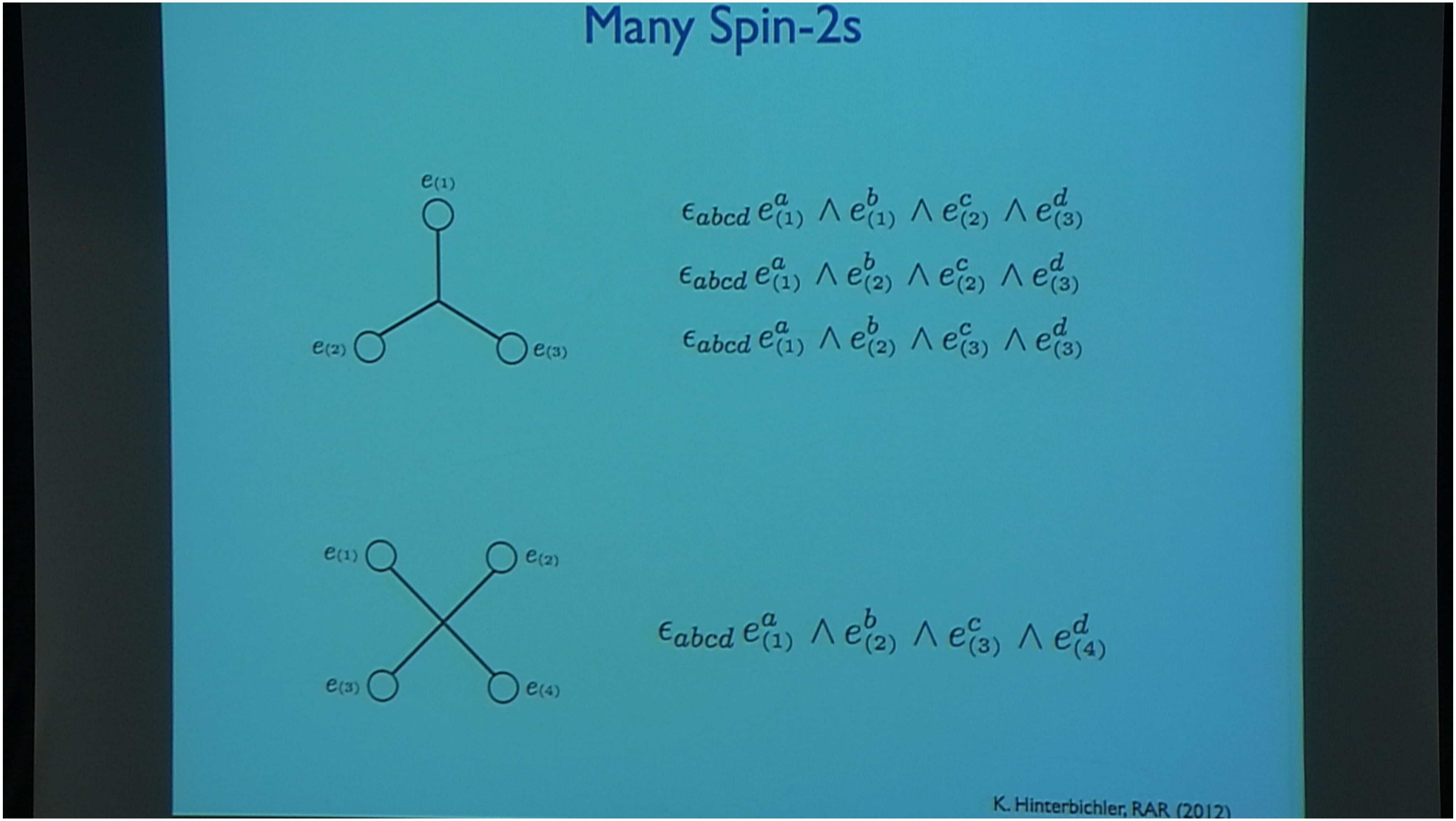

Many Spin-2s

$$\epsilon_{abcd}\, e^a_{(1)} \wedge e^b_{(1)} \wedge e^c_{(2)} \wedge e^d_{(3)}$$

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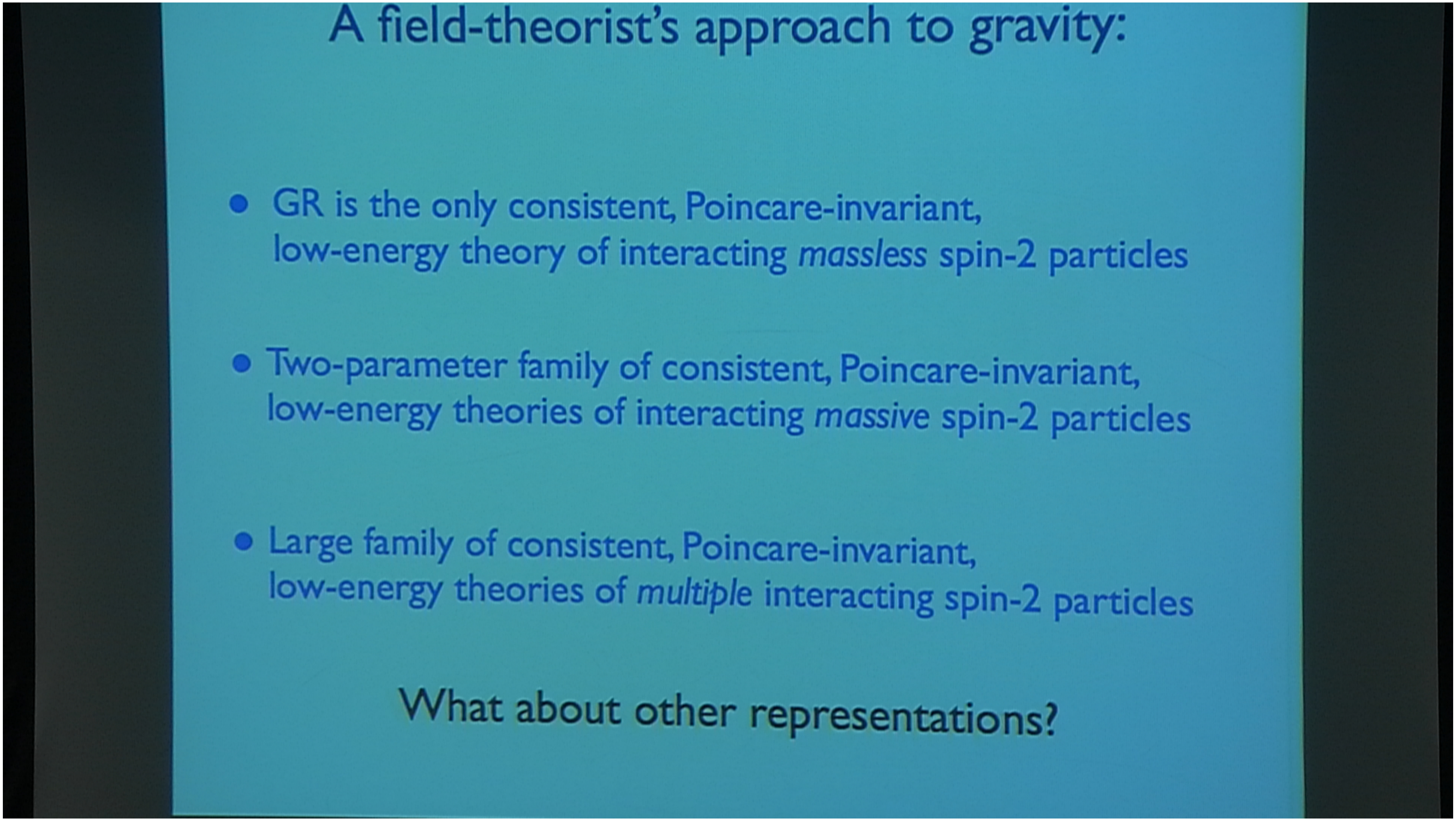

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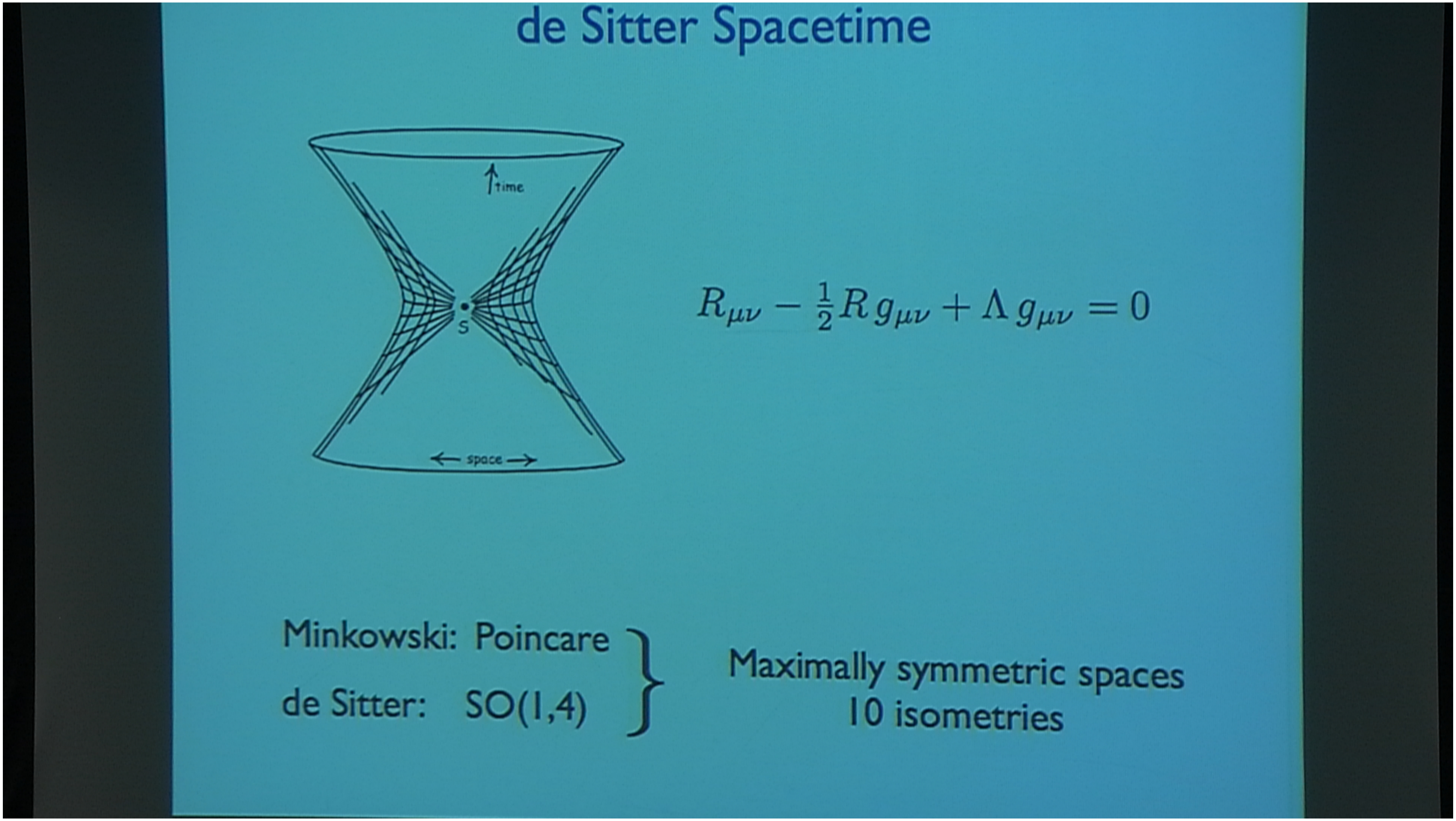
What about other representations?



de Sitter Spacetime

$$R_{\mu\nu} - \frac{1}{2}R\,g_{\mu\nu} + \Lambda\,g_{\mu\nu} = 0$$

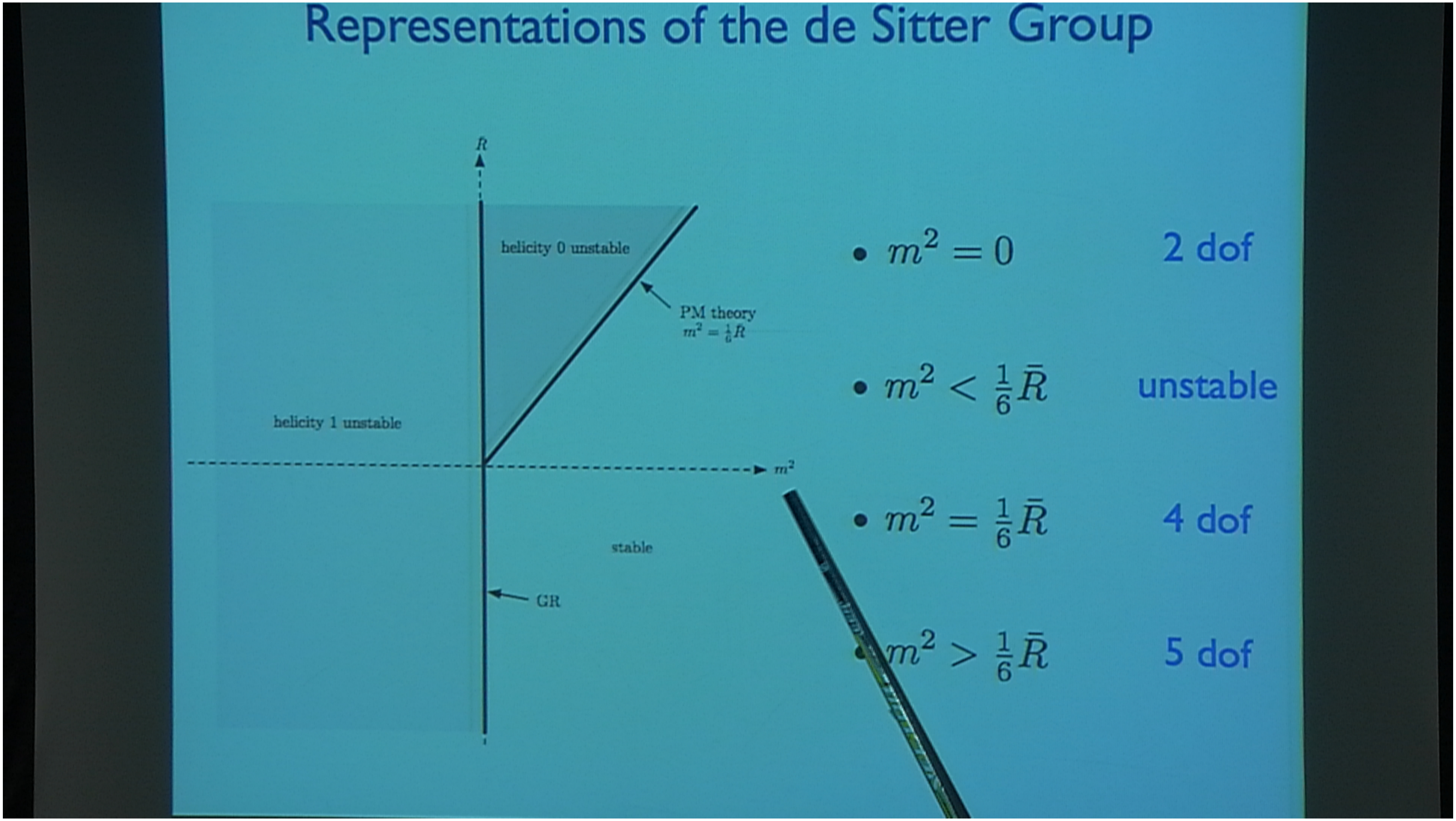
Minkowski: Poincare

de Sitter: SO(1,4)

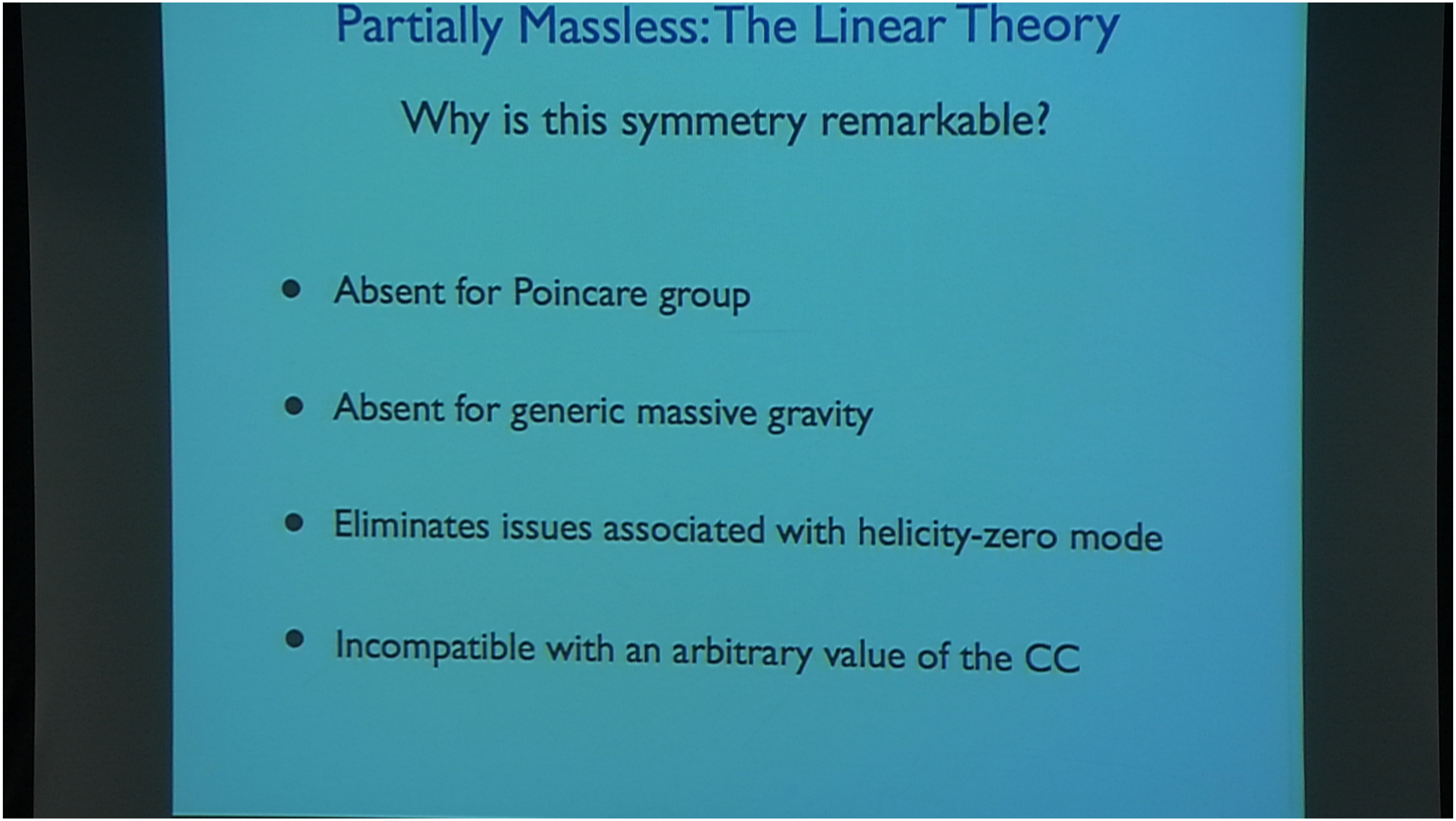
Maximally symmetric spaces

10 isometries

Representations of the de Sitter Group



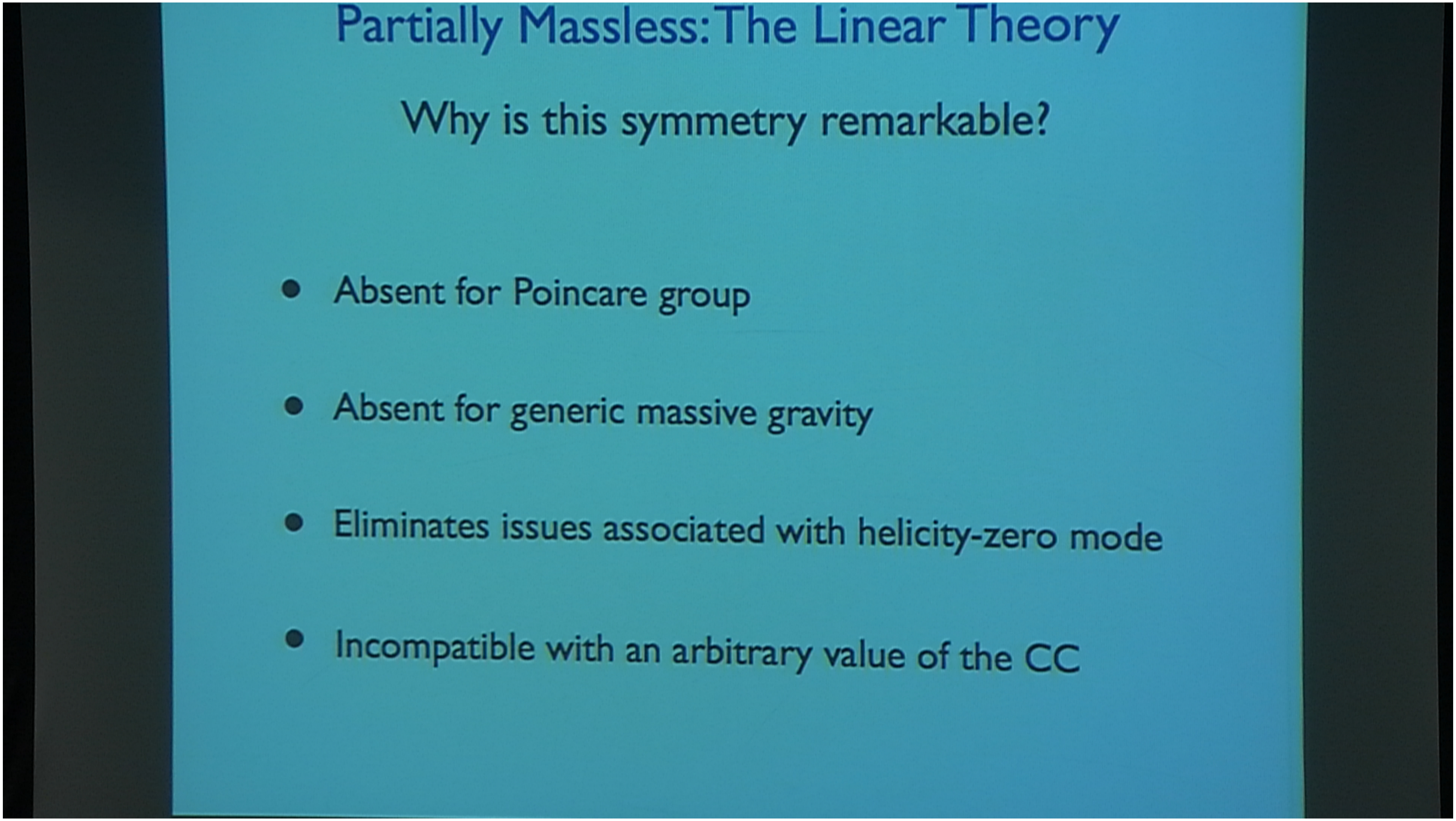
- $m^2 = 0$	2 dof
- $m^2 < \frac{1}{6}\bar{R}$	unstable
- $m^2 = \frac{1}{6}\bar{R}$	4 dof
- $m^2 > \frac{1}{6}\bar{R}$	5 dof



Partially Massless: The Linear Theory

Why is this symmetry remarkable?

- Absent for Poincare group
- Absent for generic massive gravity
- Eliminates issues associated with helicity-zero mode
- Incompatible with an arbitrary value of the CC



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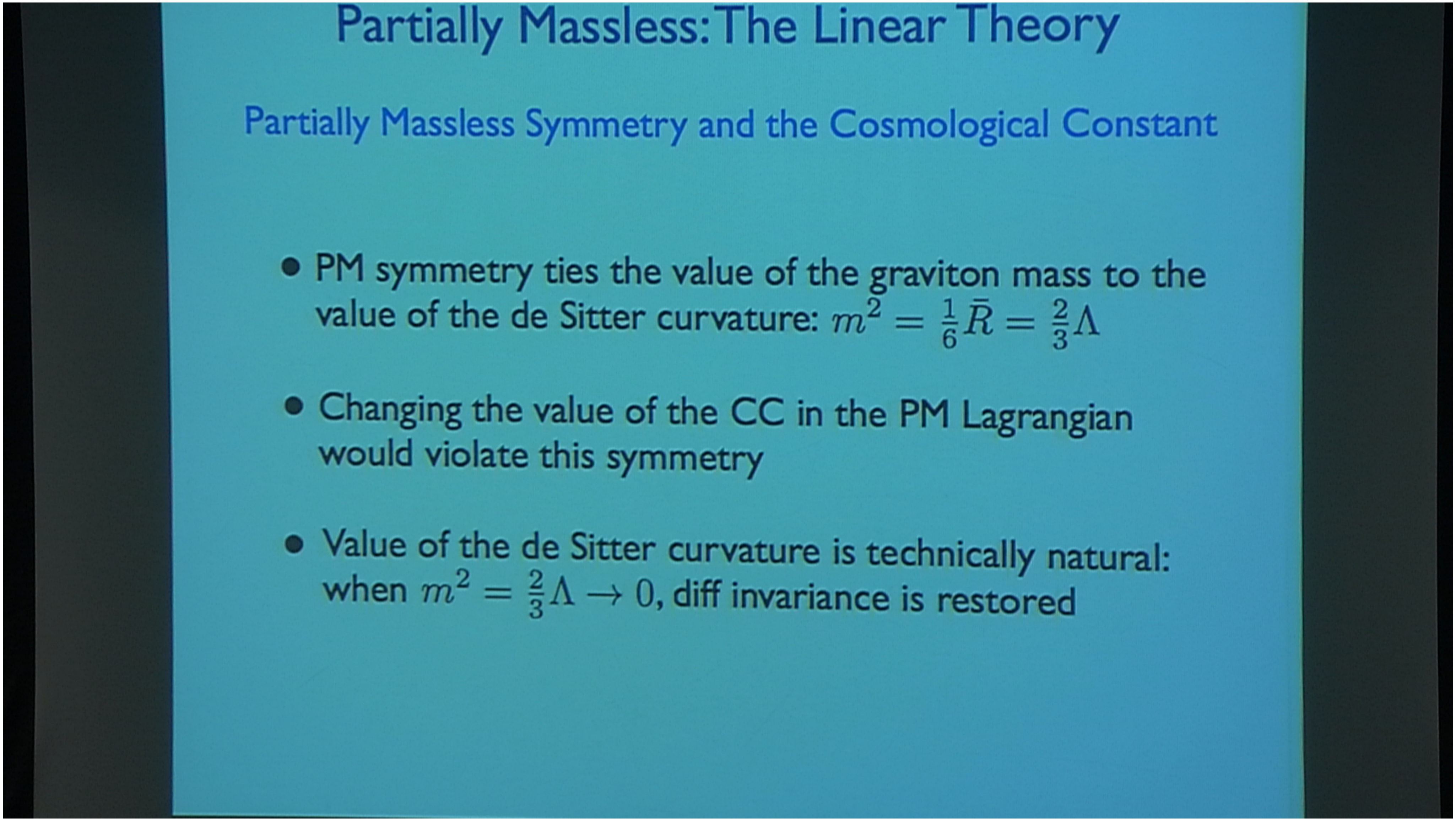

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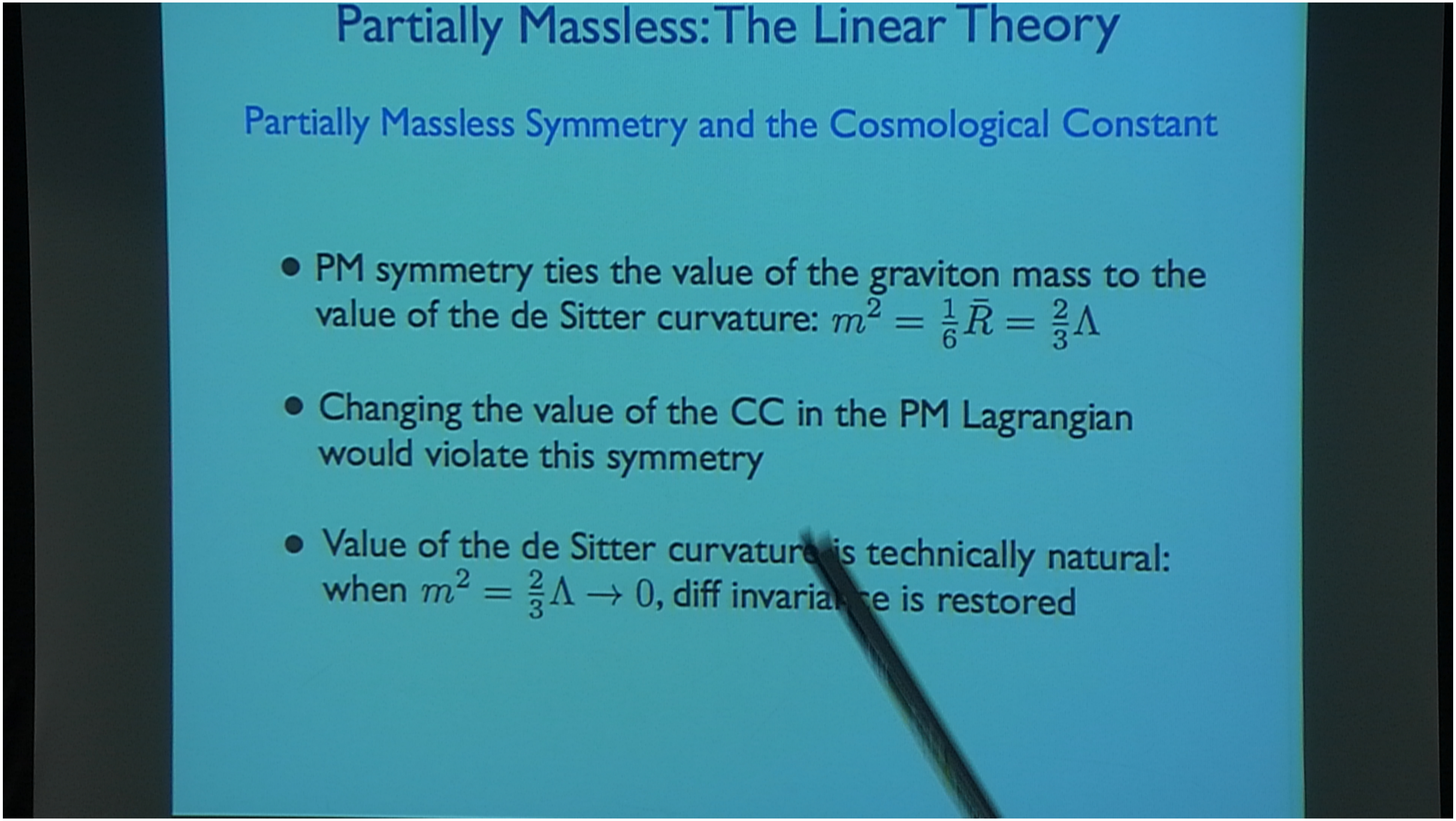
Partially Massless Symmetry and the Cosmological Constant

- PM symmetry ties the value of the graviton mass to the value of the de Sitter curvature: $m^2 = \frac{1}{6}\bar{R} = \frac{2}{3}\Lambda$
- Changing the value of the CC in the PM Lagrangian would violate this symmetry
- Value of the de Sitter curvature is technically natural: when $m^2 = \frac{2}{3}\Lambda \to 0$, diff invariance is restored



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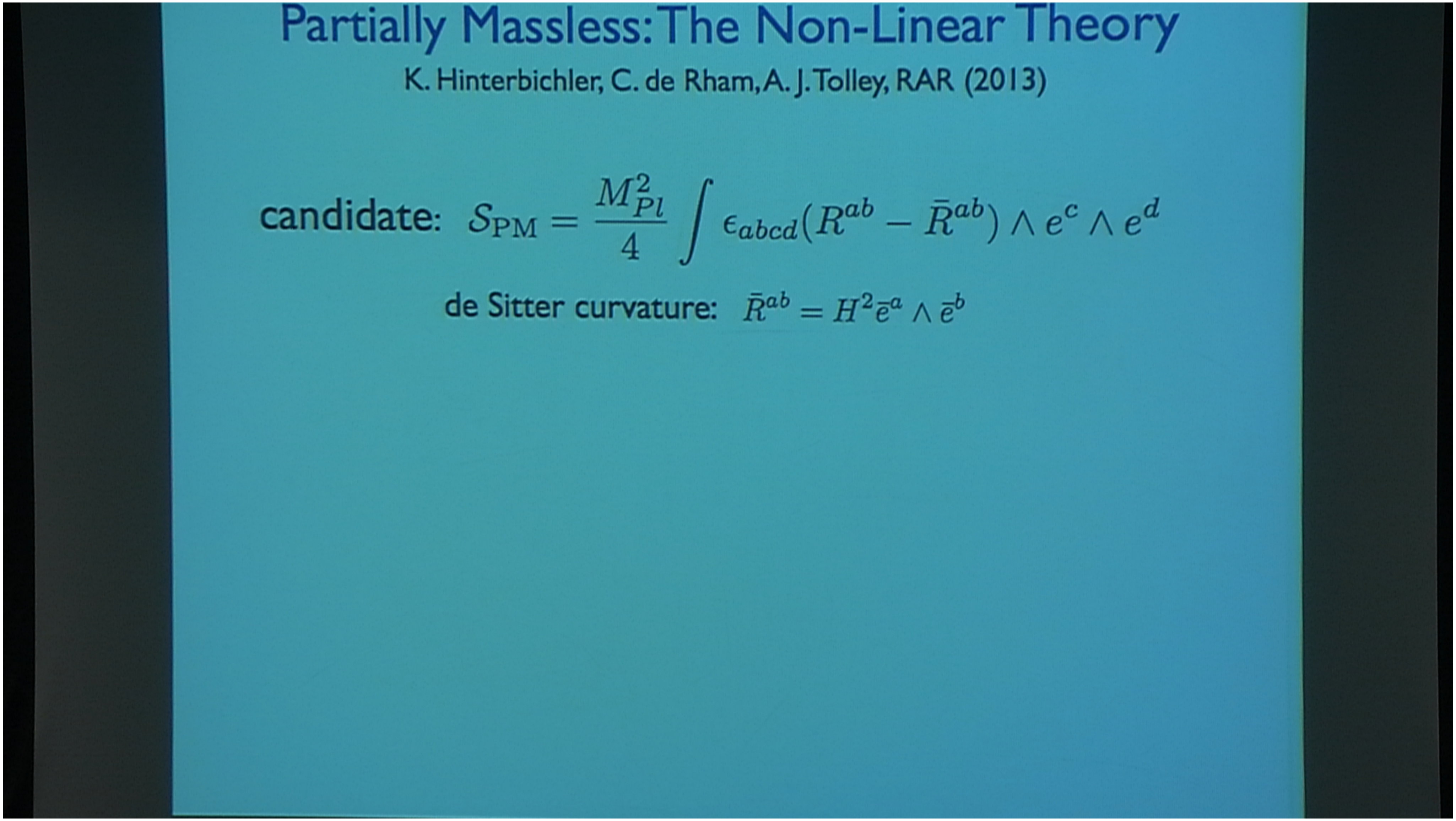
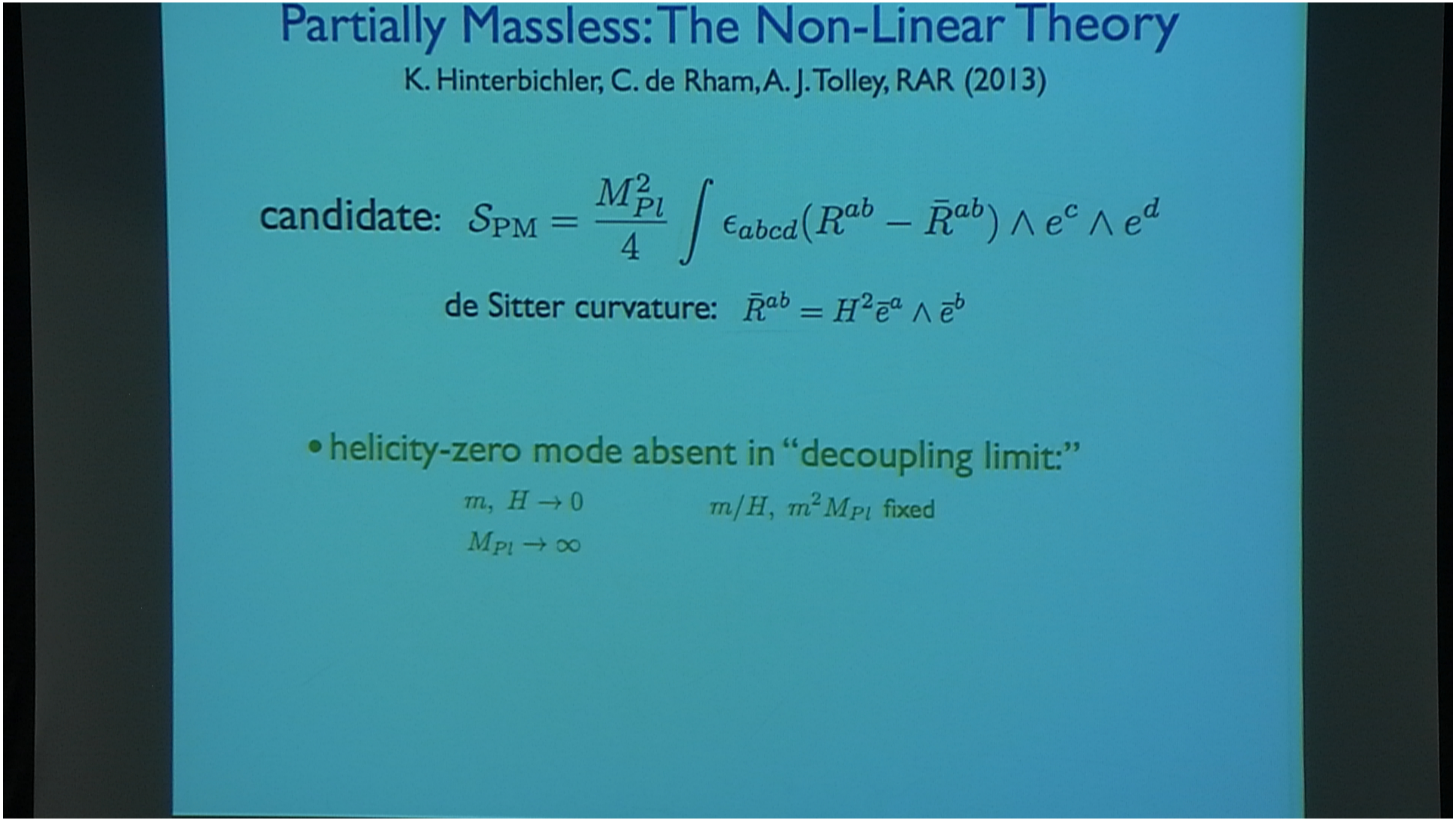


Partially Massless: The Non-Linear Theory

K. Hinterbichler, C. de Rham, A. J. Tolley, RAR (2013)

candidate: $S_{\rm PM} = \frac{M_{Pl}^2}{4} \int \epsilon_{abcd} (R^{ab} - \bar{R}^{ab}) \wedge e^c \wedge e^d$

de Sitter curvature: $\bar{R}^{ab} = H^2 \bar{e}^a \wedge \bar{e}^b$



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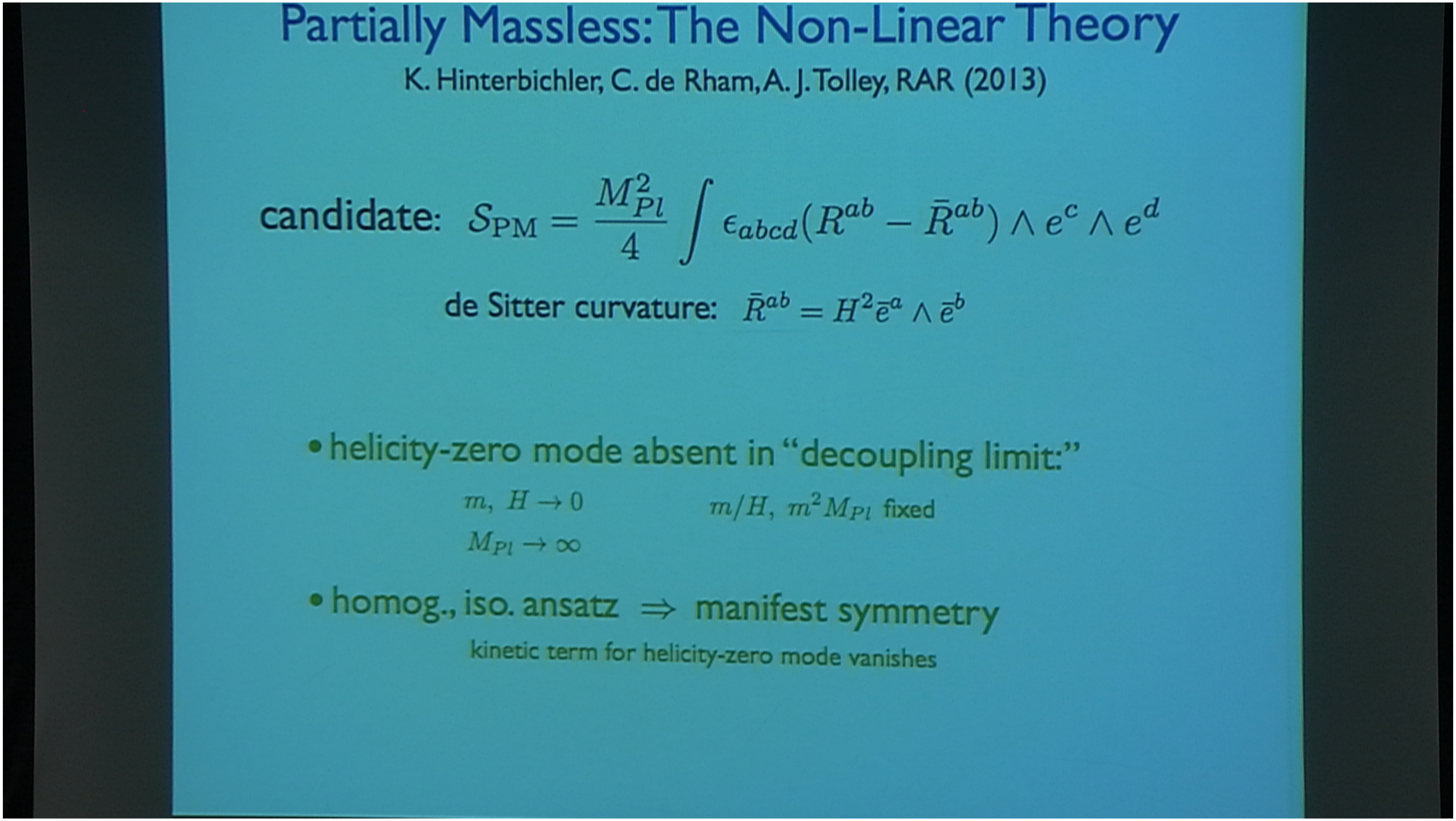

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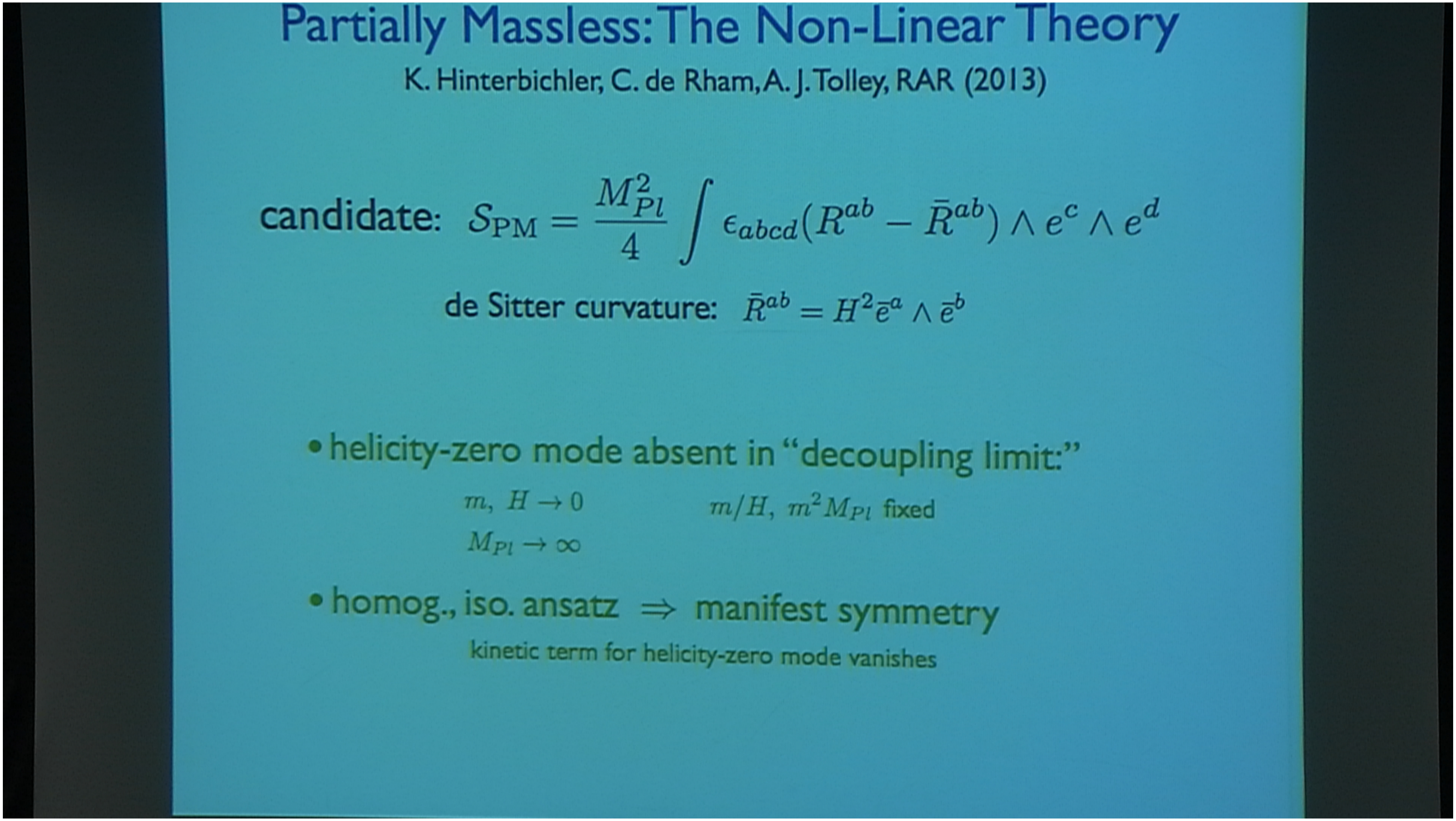

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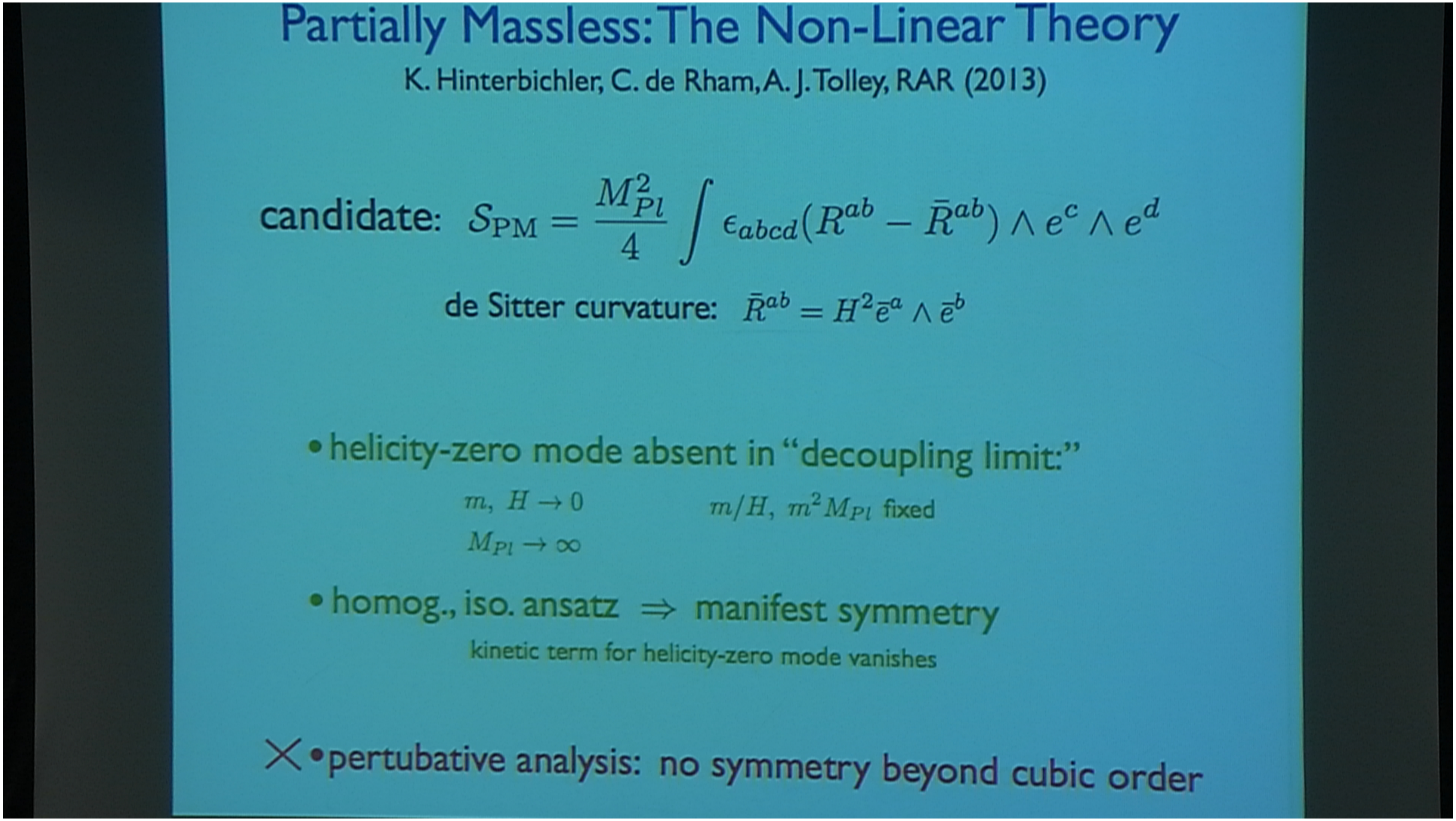

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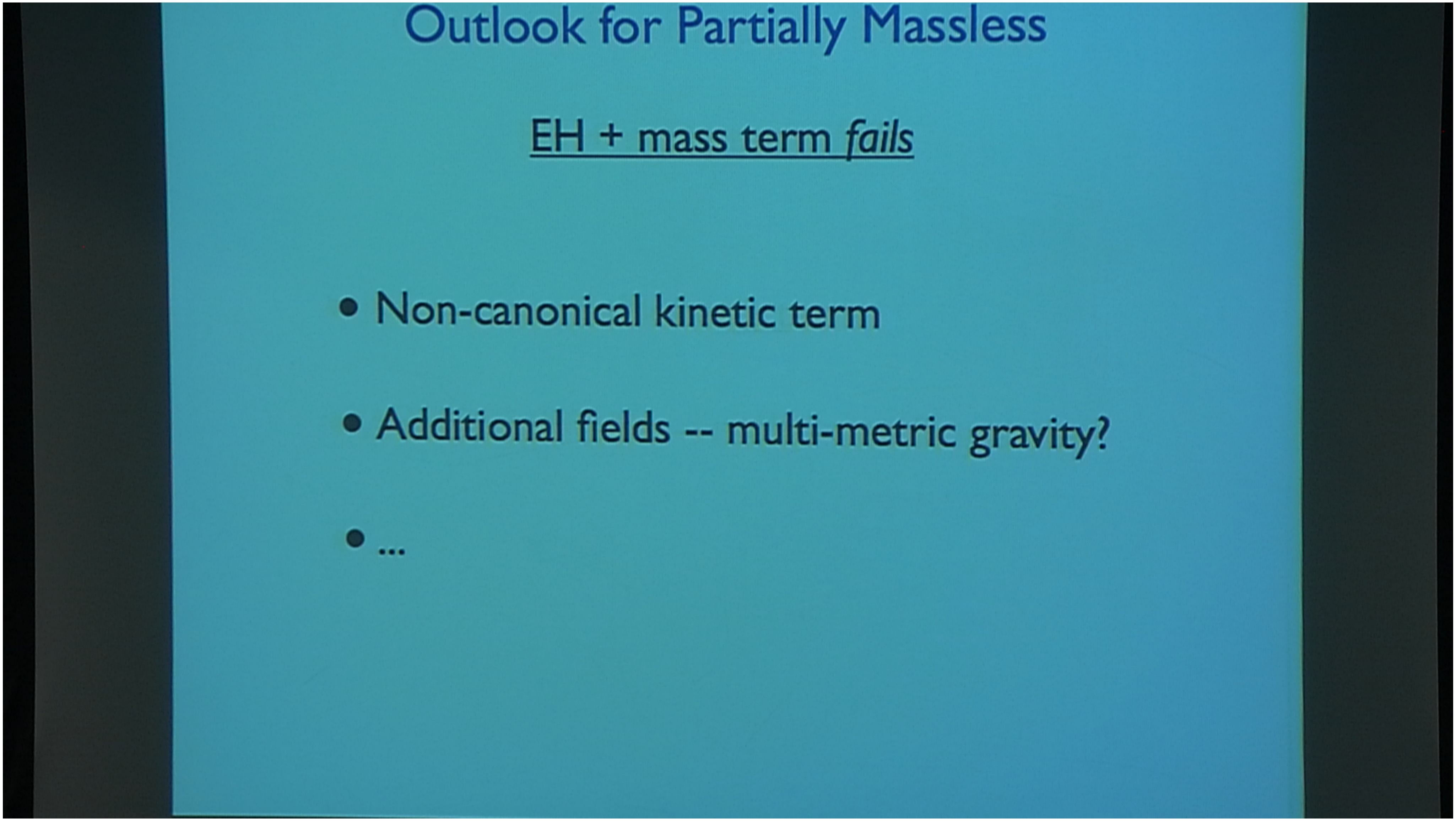
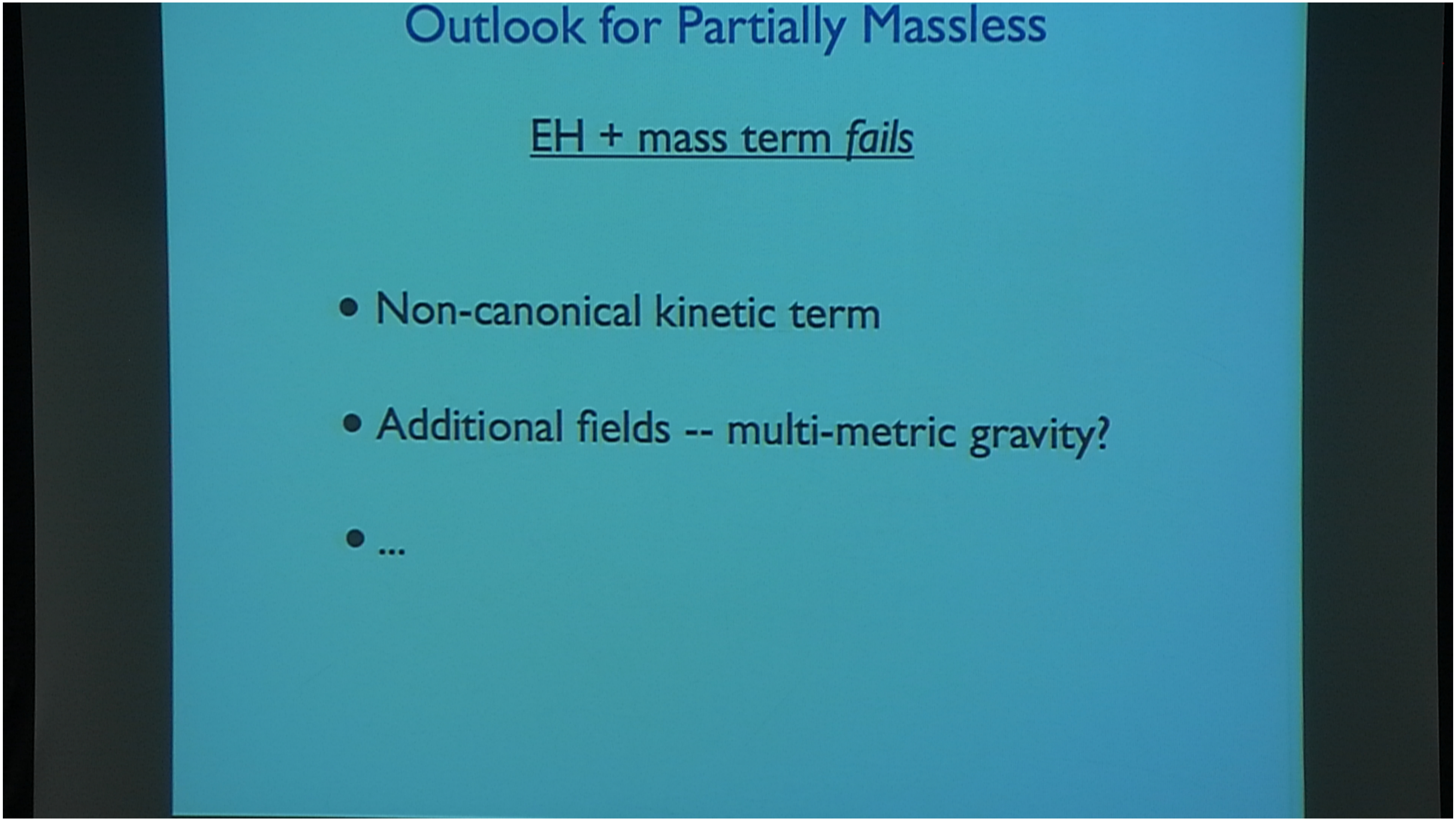
✗ • pertubative analysis: no symmetry beyond cubic order



Outlook for Partially Massless

EH + mass term *fails*

- Non-canonical kinetic term
- Additional fields -- multi-metric gravity?
- ...



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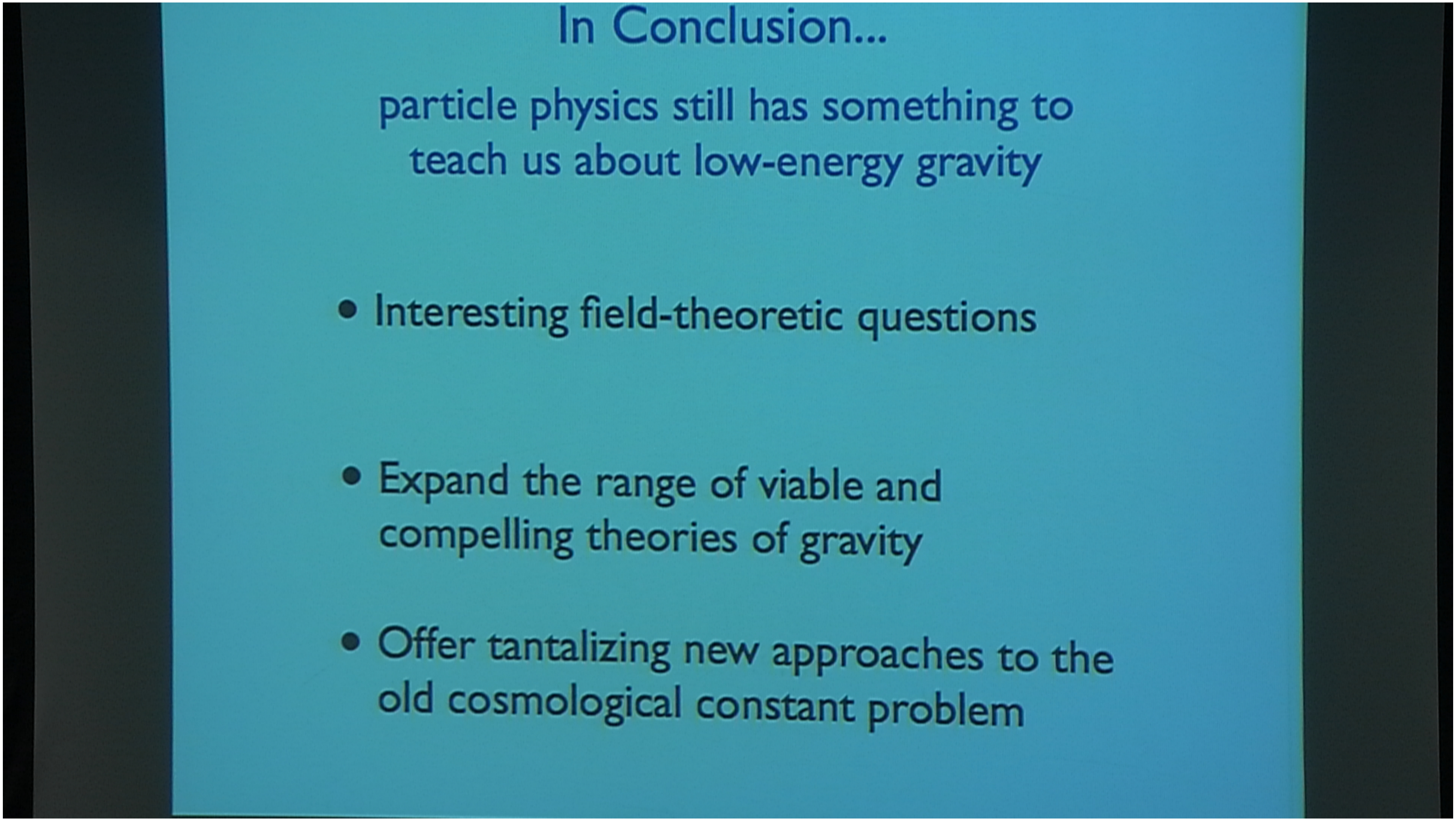

In Conclusion...

particle physics still has something to teach us about low-energy gravity

- Interesting field-theoretic questions
- Expand the range of viable and compelling theories of gravity
- Offer tantalizing new approaches to the old cosmological constant problem