Title: Fractional Quantum Hall States on an infinite cylinder: characterizing topological order and quasiparticle forces using infinite DMRG.

Date: Feb 12, 2013 03:30 PM

URL: http://pirsa.org/13020129

Abstract: The density matrix renormalization group (DMRG), which has proved so successful in one dimension, has been making the push into higher dimensions, with the fractional quantum Hall (FQH) effect an important target. I'll briefly explain how the infinite DMRG algorithm can be adapted to find the degenerate ground states of a microscopic FQH Hamiltonian on an infinitely long cylinder, then focus on two applications. To characterize the topological order of the phase, I'll show that the bipartite entanglement spectrum of the ground state is sufficient to determine the quasiparticle charges, topological spins, quantum dimensions, chiral central charge, and Hall viscosity of the phase. Then I will show how to introduce localized quasiparticles of fixed topological charge. By pinning a pair of quasiparticles and dragging them into contact, we can directly measure the force curve of their interaction.

The Fractional Quantum Hall Effect on an Infinite Cylinder: characterizing topological order and quasiparticles using iDMRG

Mike Zaletel
The Moore Group, UC Berkeley

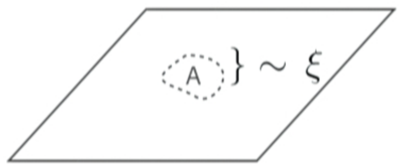
Frank Pollmann
PKS Dresden

Roger Mong
Caltech

arxiv: 1211.3733

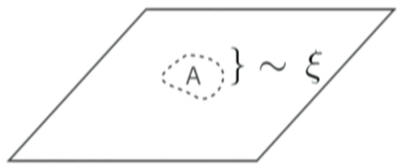
Perimeter Institute, Feb. 12, 2013

Symmetry Breaking



$\hat{\rho}_A$ contains everything that can be known about GS

Symmetry Breaking



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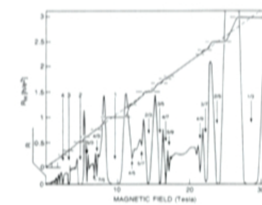
Topological order

Order can't be characterized by a local density operator $\hat{\rho}_A$

[Wen '90]



[Klitzing '80; Tsui, '82; Laughlin '83; Stormer '92; ...]

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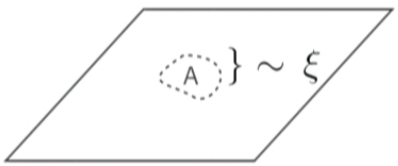

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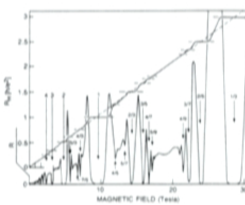


[Klitzing '80; Tsui, '82; Laughlin '83; Stormer '92; ...]

Q: What *non-local* (numerical) measurements can we make to characterize top. order?

Goal

$\hat{H}$

c_-

$\mathcal{S}_{ab}$ TQFT $\mathcal{T}_{ab}$

R^z_{xy}

UV: microscopic Hamiltonian

$$\hat{H} = \sum_{m,n} V_{mnkl} c^\dagger_m c^\dagger_n c_k c_l + \cdots$$

Goal

$\hat{H}$

c_- $\quad \mathcal{T}_{ab}$

$\mathcal{S}_{ab}$ **TQFT**

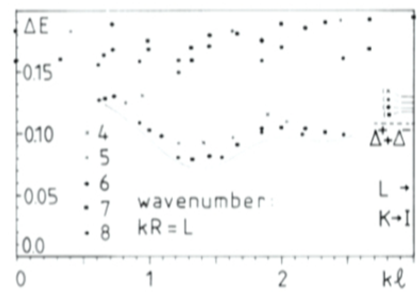
R^z_{xy}

RG

UV: microscopic Hamiltonian

$$\hat{H} = \sum_{m,n} V_{mnkl} c^\dagger_m c^\dagger_n c_k c_l + \cdots$$

RGB: energetics



[Haldane & Rezayi, '85]

IR: braiding and statistics

$$(S_z)_{xy} \stackrel{\text{def}}{=} \frac{1}{\mathcal{D}}$$

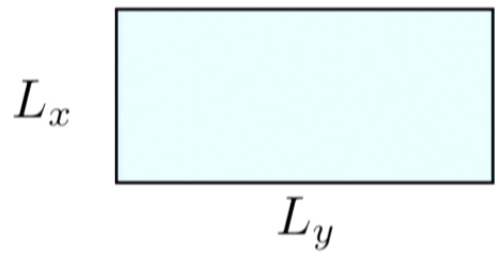
Outline

- Why iDMRG, basic concepts
- Quantum Hall Problem on the Cylinder
- The iDMRG algorithm (briefly)
- Measuring the modular T-matrix: Hall Viscosity, chiral central charge, and topological spin.
- Pinning anyons

Why iDMRG in 2D?

Reason 1: Complexity

Exact Diagonalization



$$\mathcal{O}(e^{\alpha L_x L_y})$$

[Yoshioka '83; Haldane '85]

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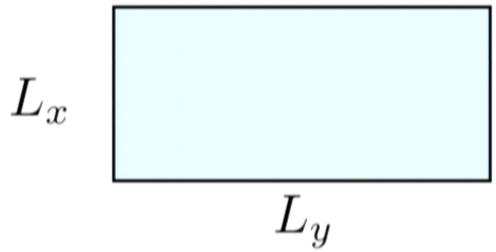
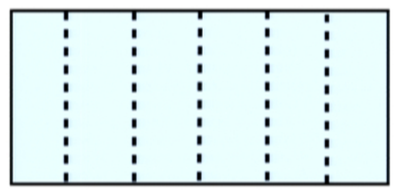


$\mathcal{O}(e^{\alpha L_x L_y})$

[Yoshioka '83; Haldane '85]

finite DMRG

[White '92]



$\mathcal{O}(L_x L_y e^{\alpha L_x})$

[Shibata '01; Bergholtz '03; Feiguin '08; Kovrizhin '10; Hu '12; Zhao '12;...]

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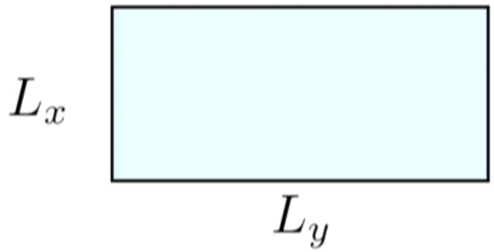

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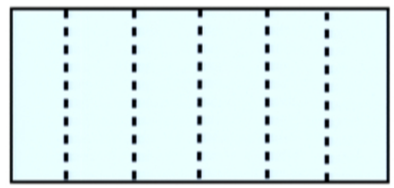

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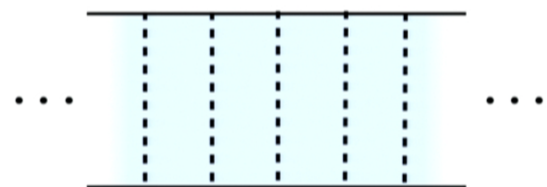
infinite DMRG

[McCulloch '08]



$\mathcal{O}(e^{\alpha L_x})$

N_e irrelevant

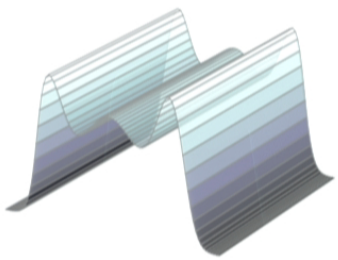
Not clear if chiral states have PEPS, while 'model' FQHE states have well behaved *analytic* expression as MPS [Zaletel&Mong '12; Estienne et al. '12]

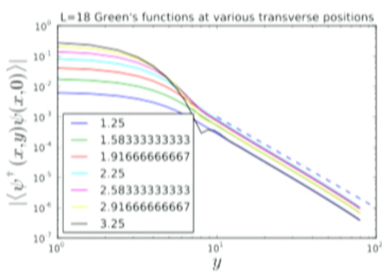
Why iDMRG in 2D?

Reason II: Geometry

quasiparticle 'defects'

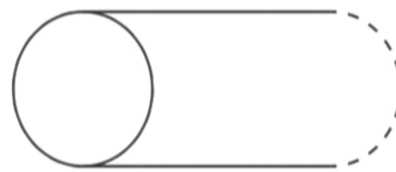
[this talk]





infinite edges

[with Joel Moore,
in progress]



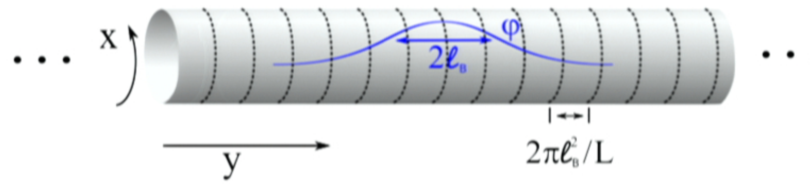
semi-infinite

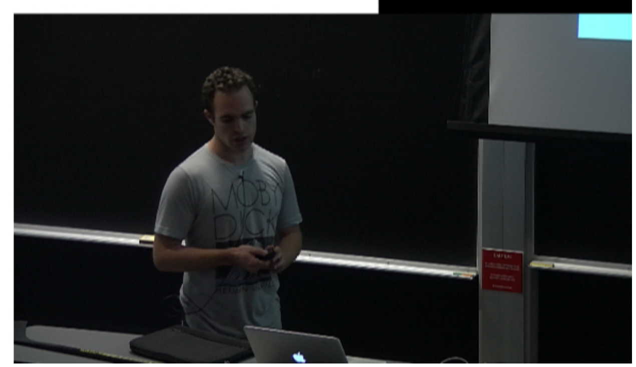
Why iDMRG in 2D?

Reason III: Topology

iDMRG : use infinite cylinder

[Lattice: Cincio & Vidal '12]





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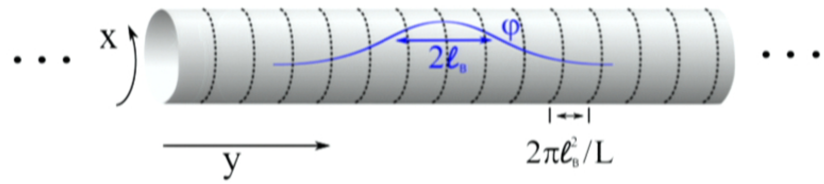

Topological Order ⟶ $\mathfrak{m}$ degenerate ground states on infinite cylinder

[Wen '90; Kitaev '05]

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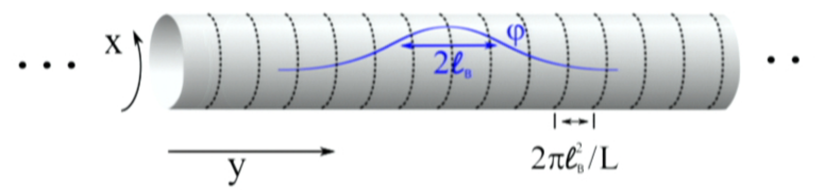


Topological Order $\longrightarrow$ $\mathfrak{m}$ degenerate ground states on infinite cylinder

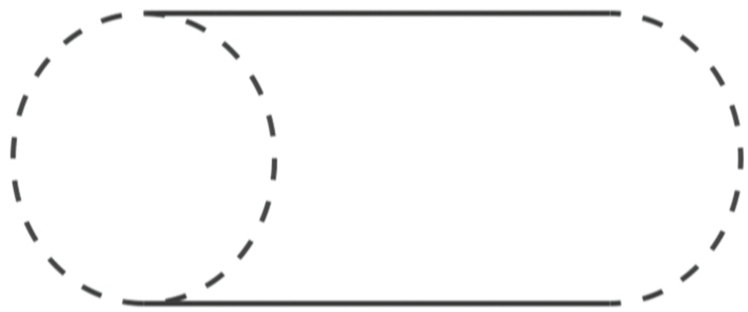
[Wen '90; Kitaev '05]

> The $\mathfrak{m}$ GS alone contain information about the quasiparticle excitations and braiding.

[Kitaev & Preskill '06; Levin & Wen '06; Zhang et al. '12; Grover et al. '12; Cincio & Vidal '12]

Why are GSs enough?

$$|\mathbb{1}\rangle =$$



[Kitaev&Preskill '06; Li&Haldane '08; Papic et al. 2011]

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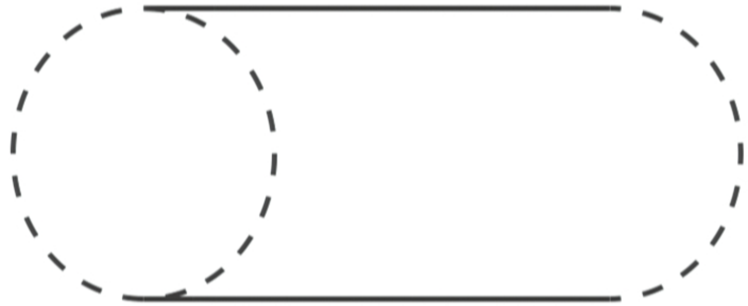

Anyons: $\{\mathbb{1}, a, b, \cdots\} = \{\mathbb{1}, \frac{e}{3}, -\frac{e}{3}\}$

[Kitaev&Preskill '06; Li&Ha

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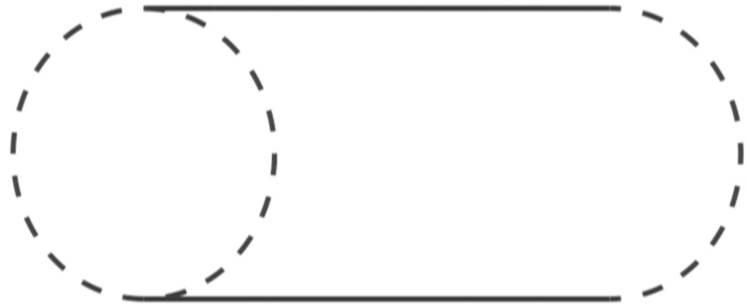

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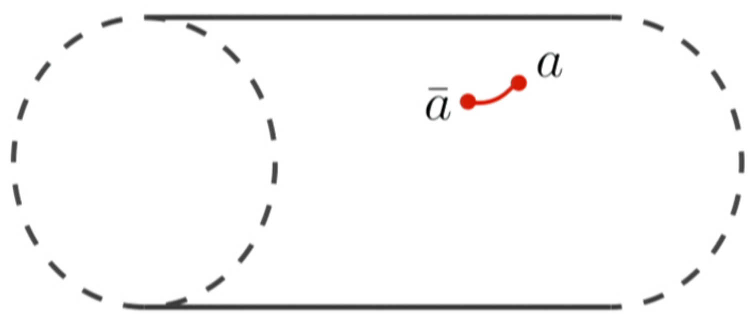

[Kitaev&Preskill '06; Li&Ha

Why are GSs enough?

Minimal Entanglement States (MES) [Zhang et al. '12]

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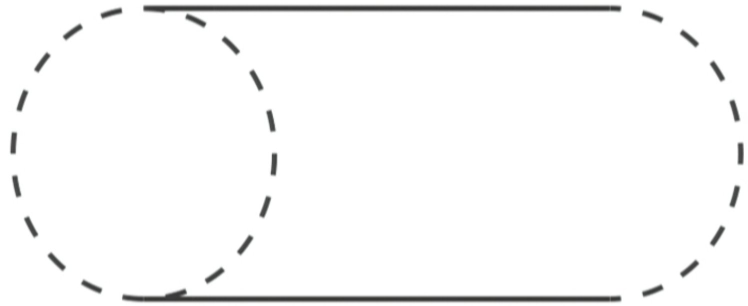

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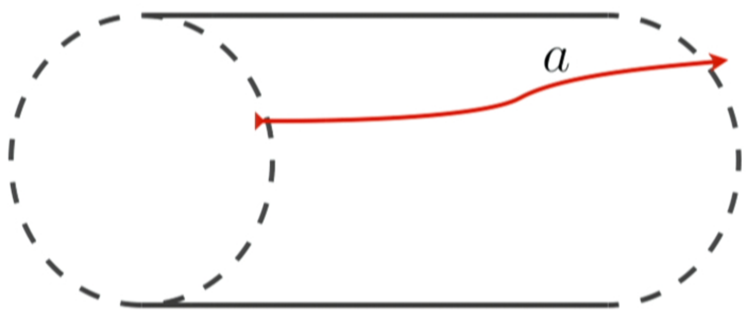

$$\langle \mathbb{1} \mid a \rangle = 0$$

[Kitaev&Preskill '06; Li&Ha

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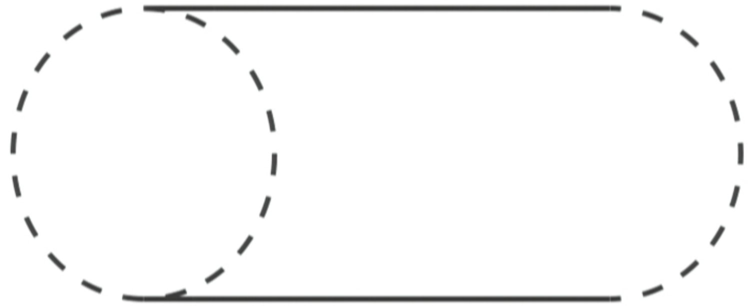

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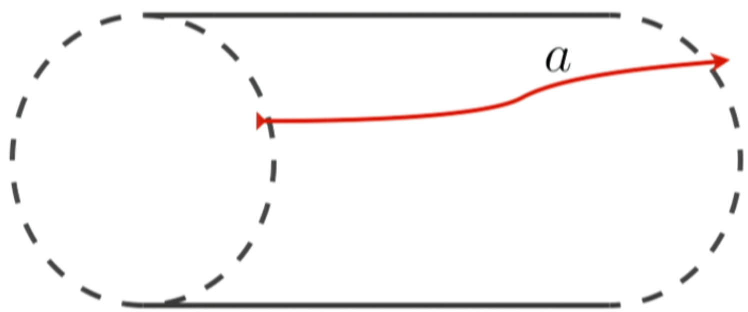

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$$E_E = -\log(p)$$

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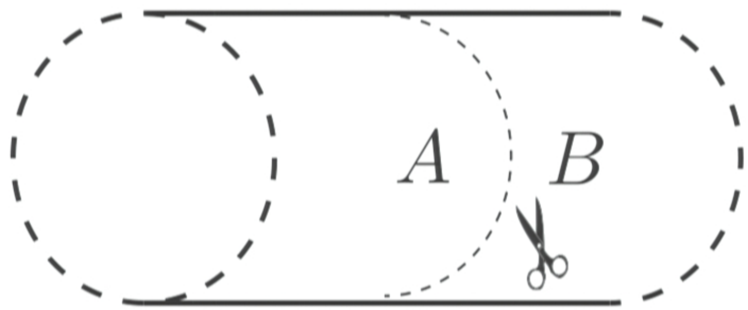

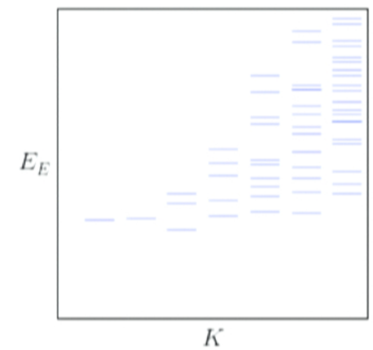

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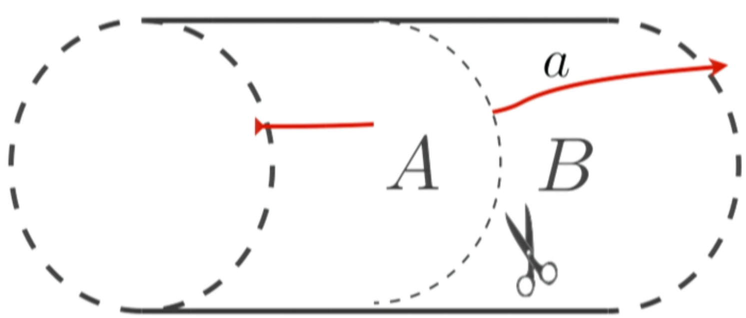

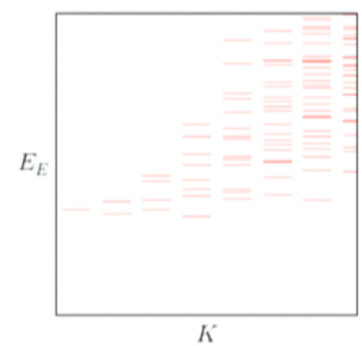

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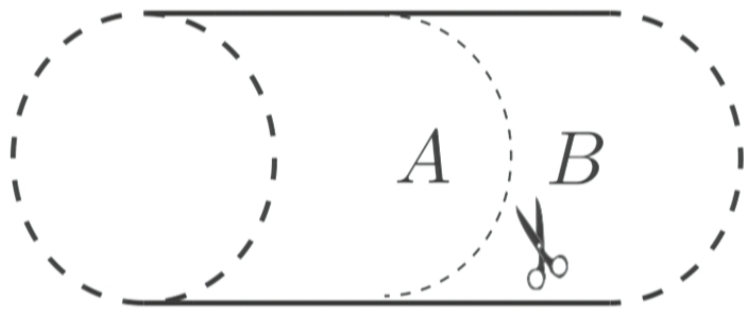

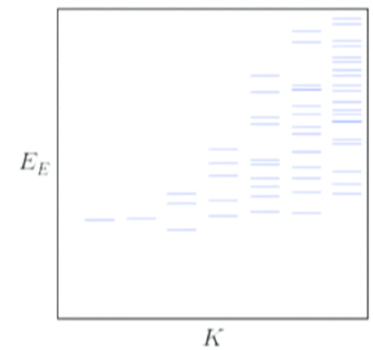

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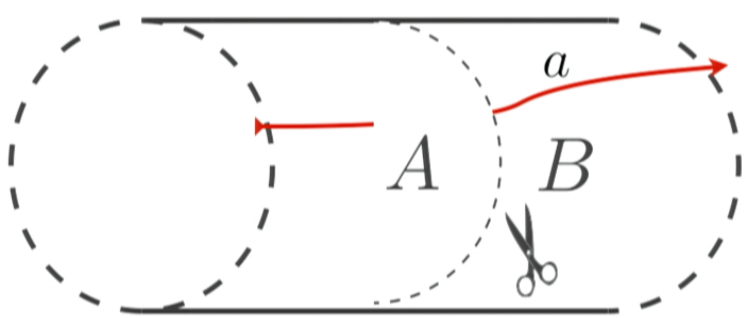
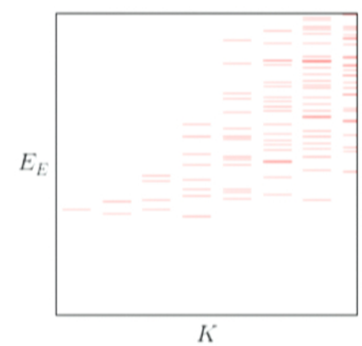




$$\langle \mathbb{1} \mid a \rangle = 0$$

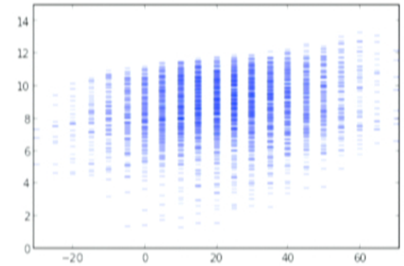
[Kitaev&Preskill '06; Li&Haldane '08; Papic et al. 2011]

Example: $S_E = \sum_i \log(p_i) p_i$

$$S_{E;a} = \alpha L - \gamma_a, \quad \gamma_a = \log \mathcal{D}/d_a$$

$\gamma_a > 0$: top. order $\qquad d_a > 1$: non-abelian anyon

[Kitaev&Preskill '06; Levin&Wen '06; Dong et al '08]



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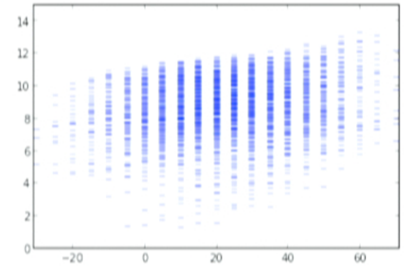

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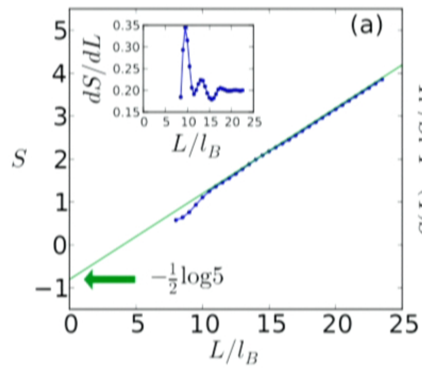
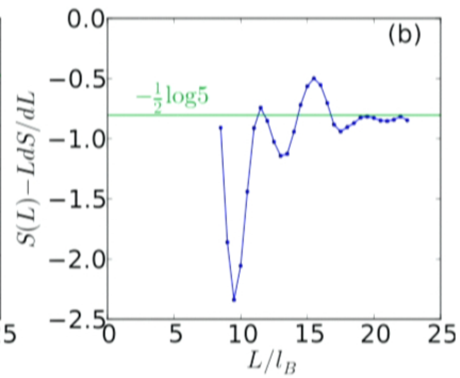
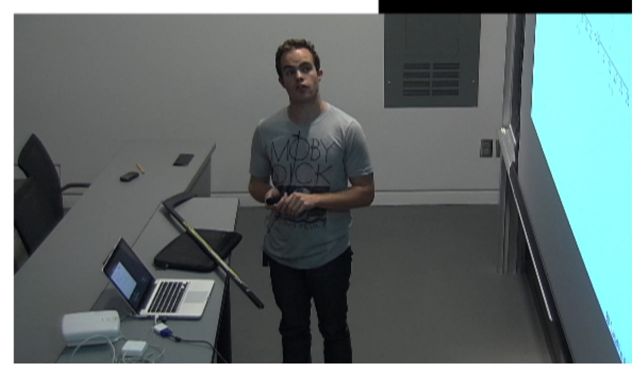
Hierarchy

$$\nu = \frac{2}{5}$$

$V_1, V_3 \neq 0$







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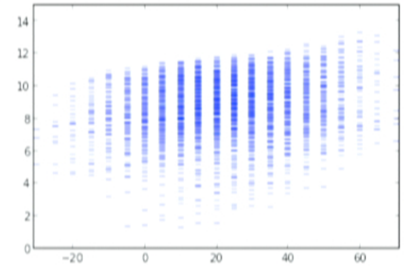

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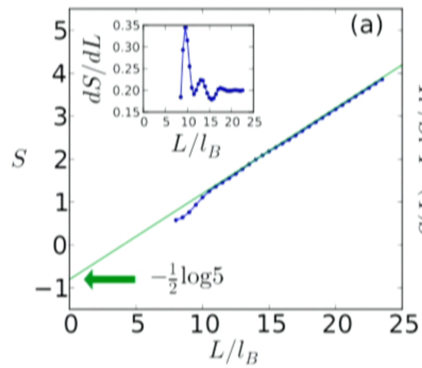

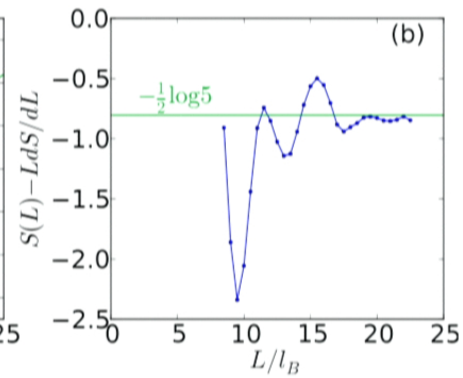

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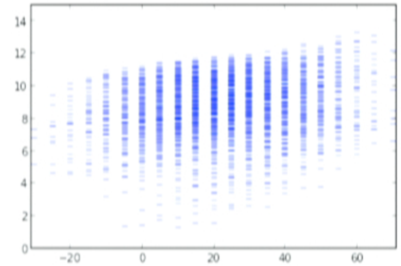

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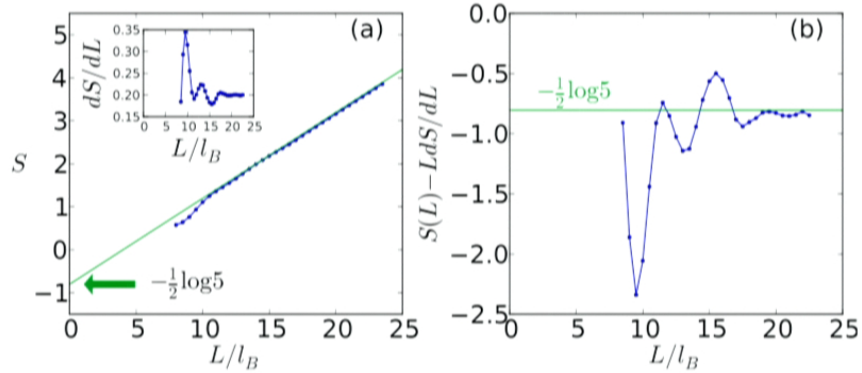
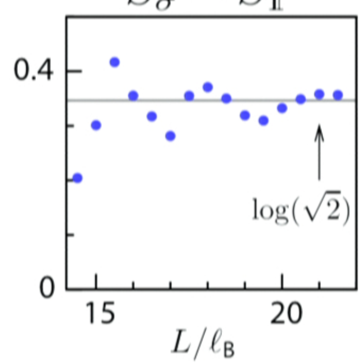


Moore Read

$$\nu = \frac{1}{2}$$

$V_1, V_3 \neq 0$



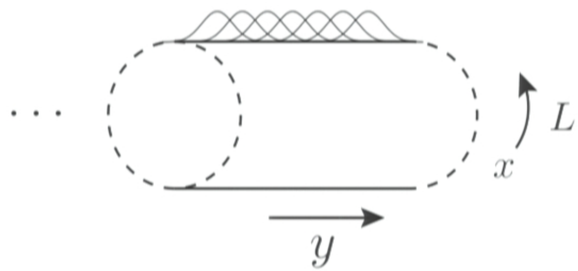
$$d_\sigma = 1.43 \approx \sqrt{2}$$

ED: [Laeuchli et al. '10]

Lowest Landau Level of Cylinder

infinitely degenerate: $H = \frac{1}{2}\hbar\omega_c$

$$\mathbf{A} = (-y, 0)$$



$$\phi_n(x,y) \propto e^{ik_n x - \frac{1}{2}(y-k_n)^2},$$
$$k_n = \frac{2\pi n}{L}$$

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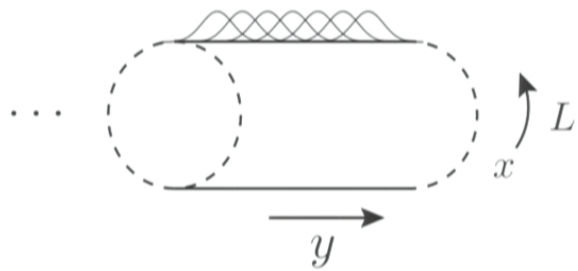

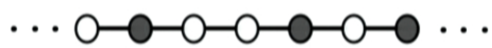

$$|0100101\rangle$$

$$\nu = \frac{\#\ \text{electrons}}{\#\ \text{orbitals}}$$

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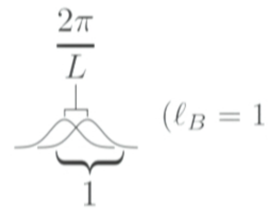
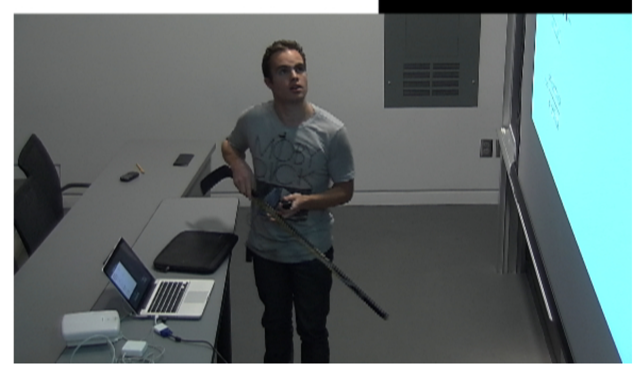
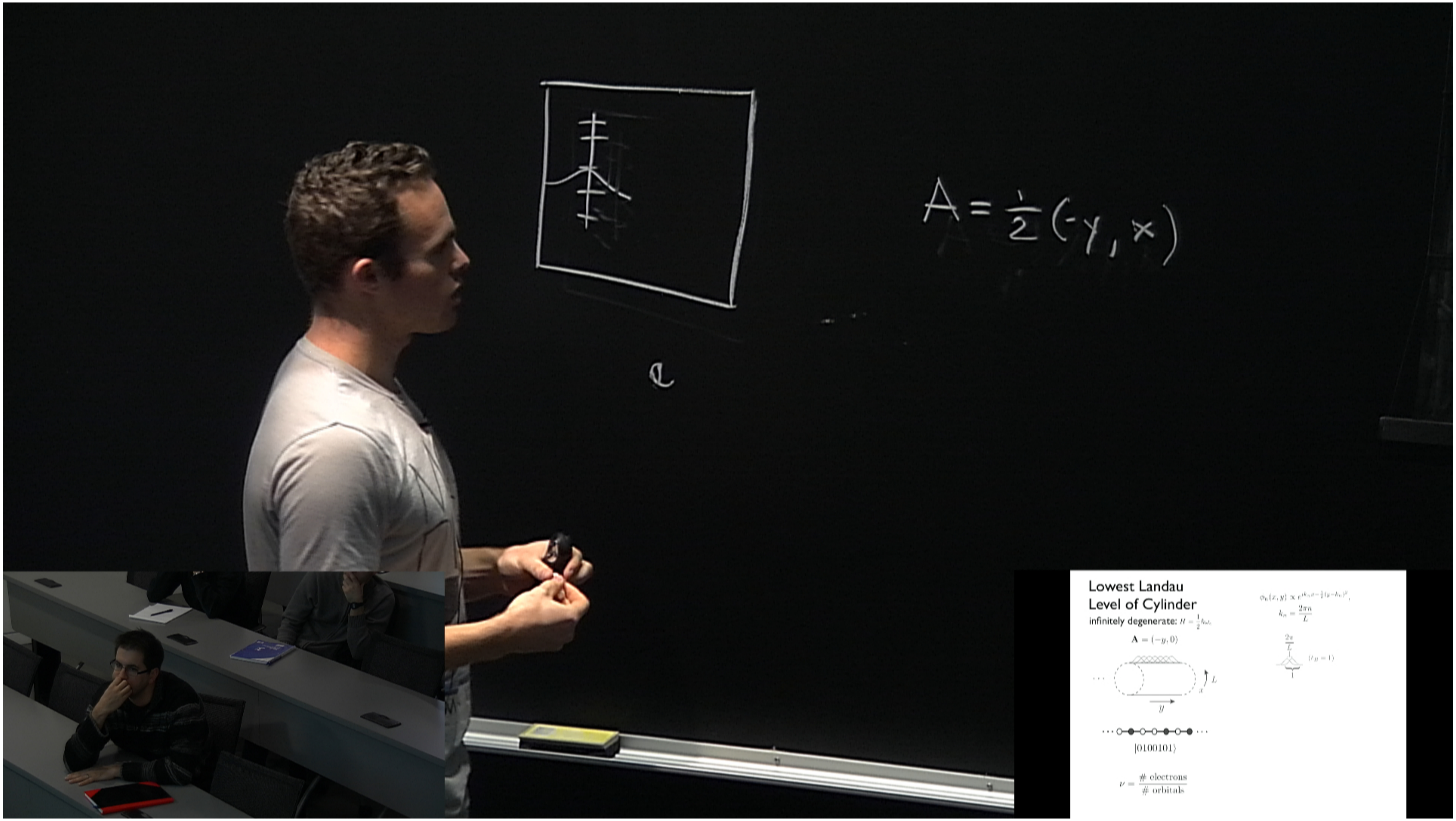
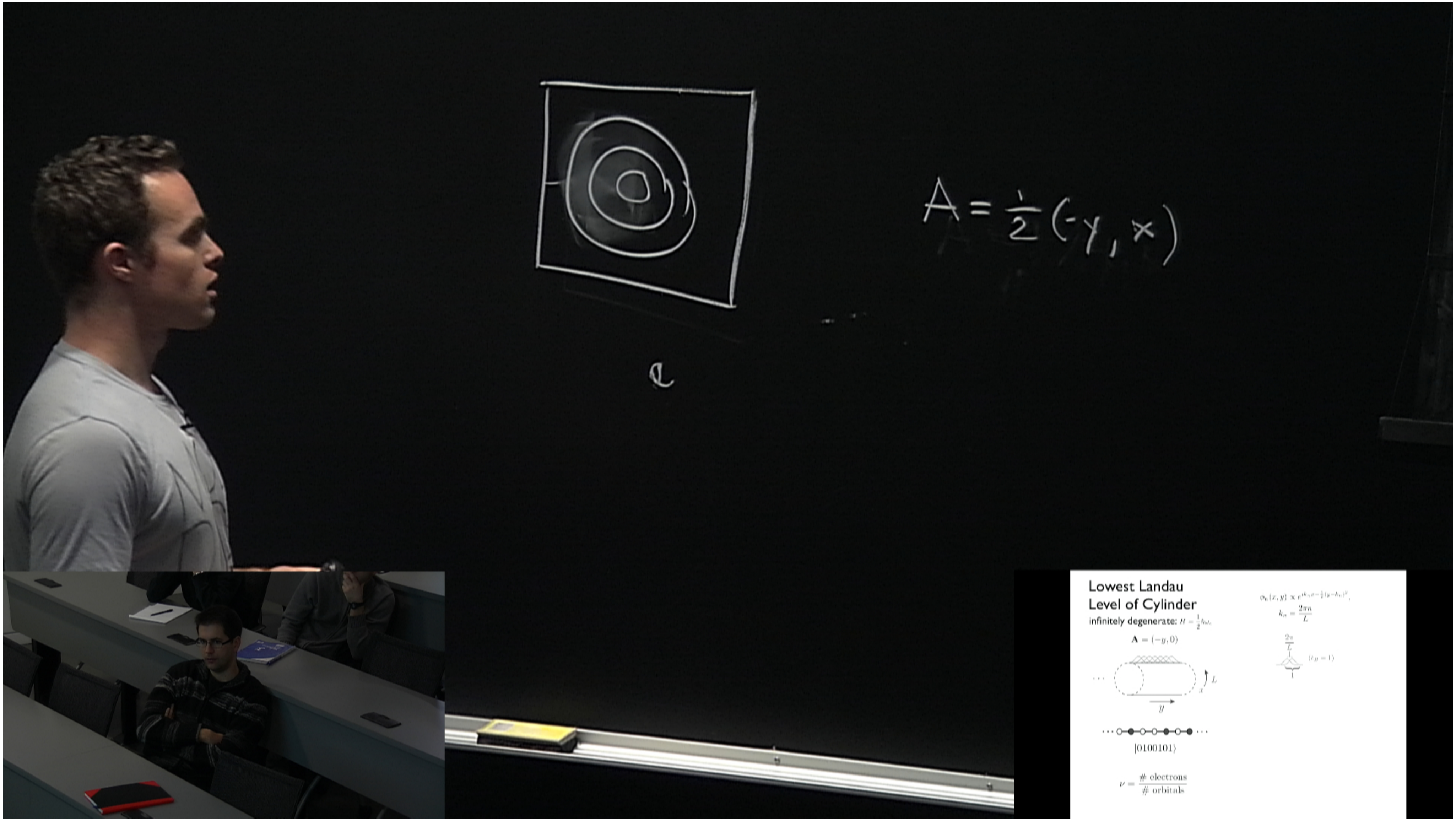






A = ½(-y, x)
Lowest Landau
Level of Cylinder
infinitely degenerate:
A = (−y, 0)
L
x
y
|0100101⟩
ν = # electrons / # orbitals
k_n = 2πn / L
2π / L



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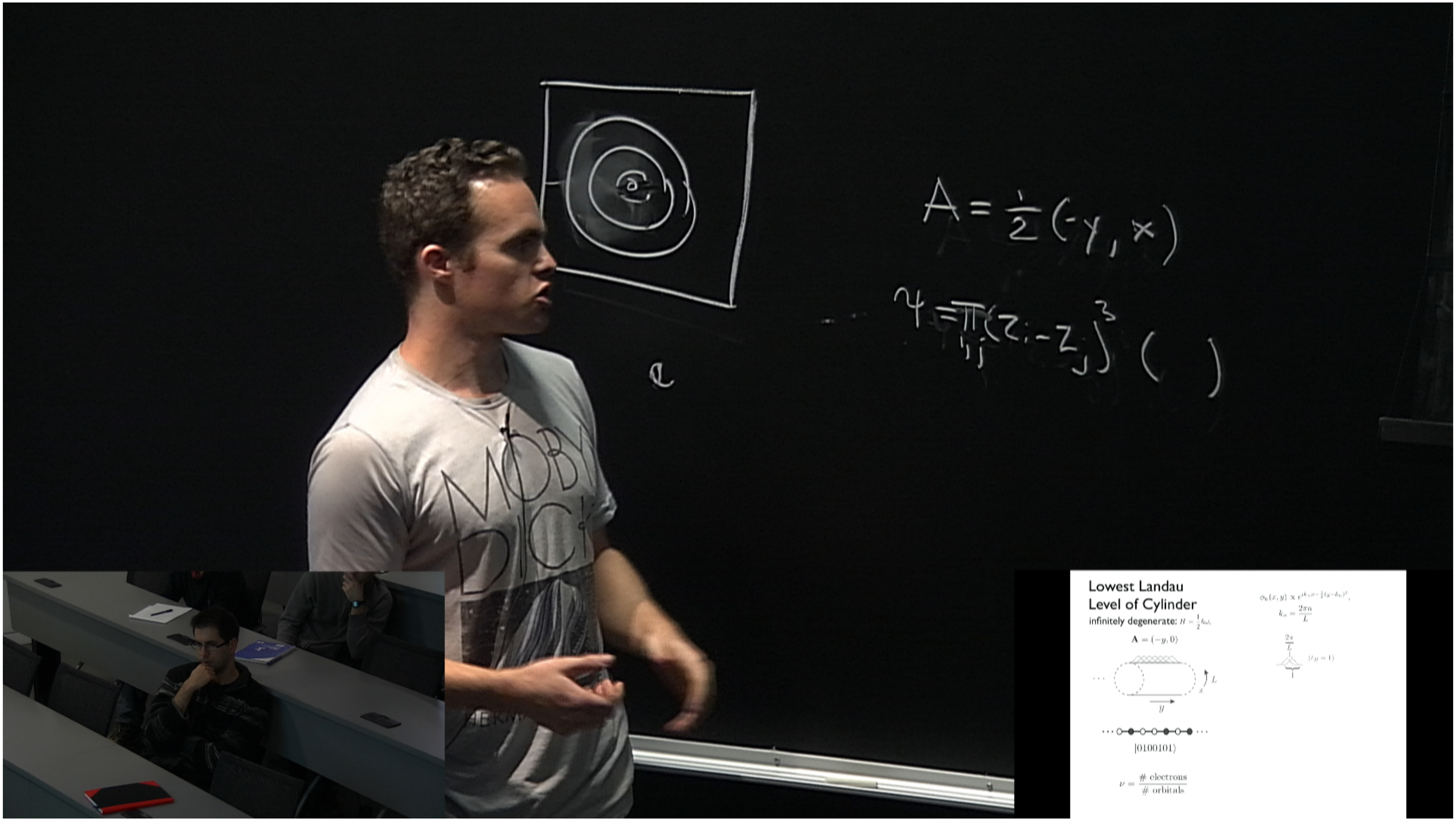

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Lowest Landau Level of Cylinder

infinitely degenerate: $H = \frac{1}{2}\hbar\omega_c$

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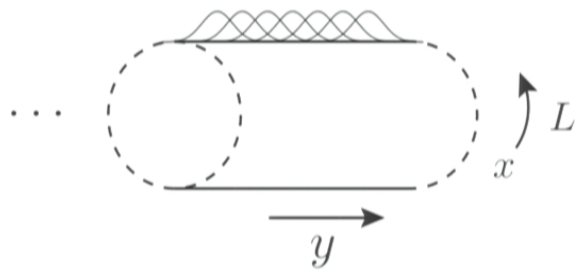

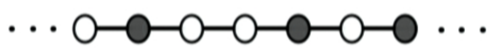

$$|0100101\rangle$$

$$\nu = \frac{\#\ \text{electrons}}{\#\ \text{orbitals}}$$

$$\phi_n(x, y) \propto e^{ik_n x - \frac{1}{2}(y-k_n)^2},$$

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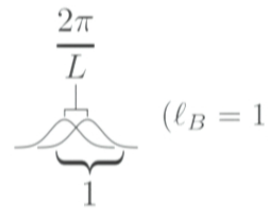

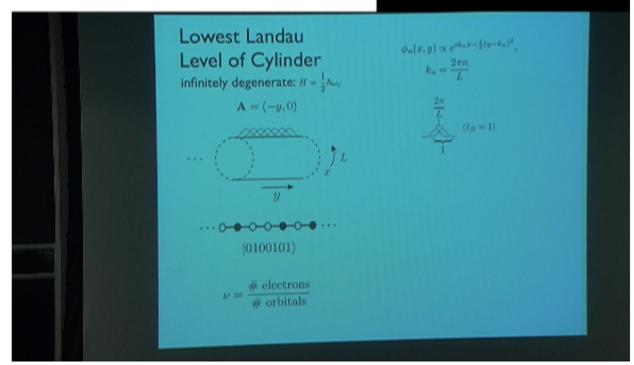

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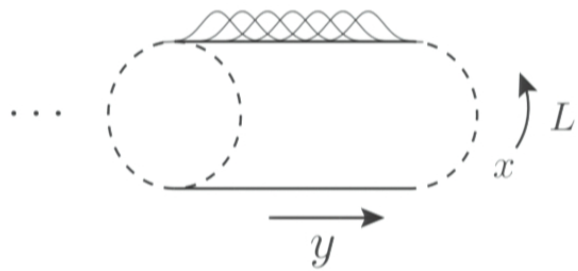

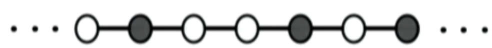

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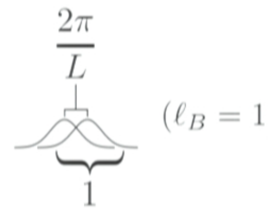


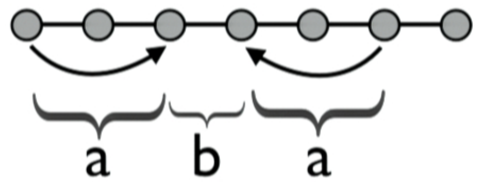
Symmetries:

$$\hat{C} = \sum_n \hat{N}_n \quad \text{(charge)}$$

$$\hat{K} = \sum_n n\,\hat{N}_n \quad (\ x \text{ momentum})$$

Interactions

$$\hat{K} = \sum_n n\,\hat{N}_n$$



[Haldane '83; Trugman & Kivelson '85; Haldane & Rezayi '94; Bergholtz et al.; '05, Seidel et al. '05]

Interactions

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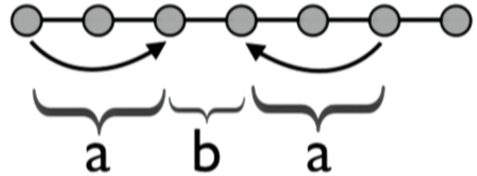

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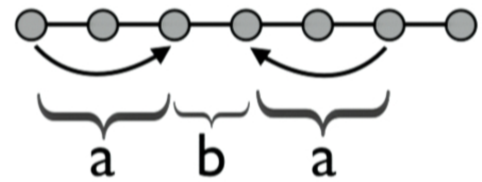

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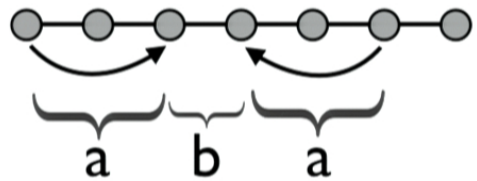

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$(\ell_B = 1) \rightarrow \quad a, b \sim \mathcal{O}(L)$

Fixing i, $\hat{H}$ contains $\mathcal{O}(L^2)$ terms (100 - 300).

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Lowest Landau Level of Cylinder

infinitely degenerate: $H = \frac{1}{2}\hbar\omega_c$

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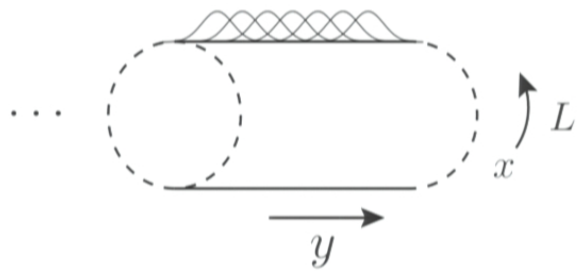

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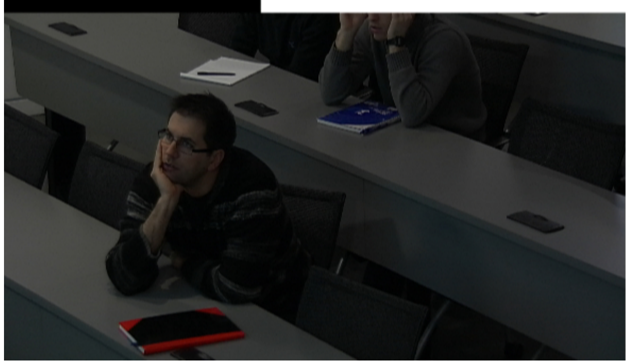

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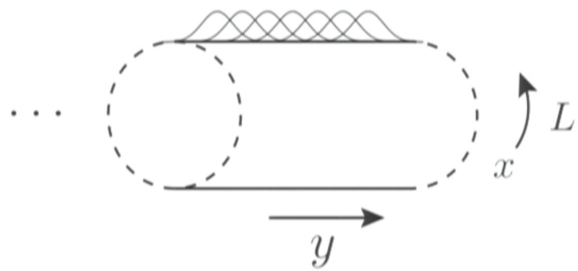

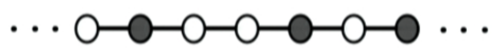

$|0100101\rangle$

electrons

orbitals

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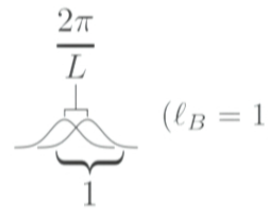

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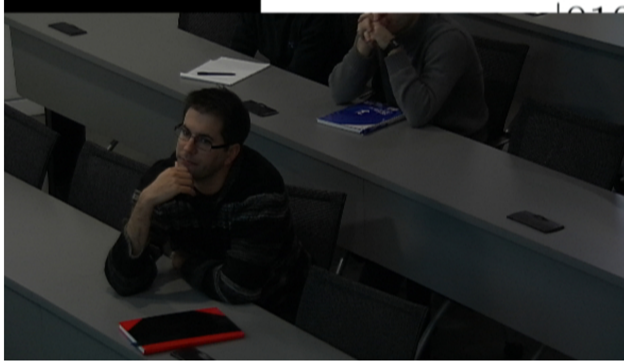

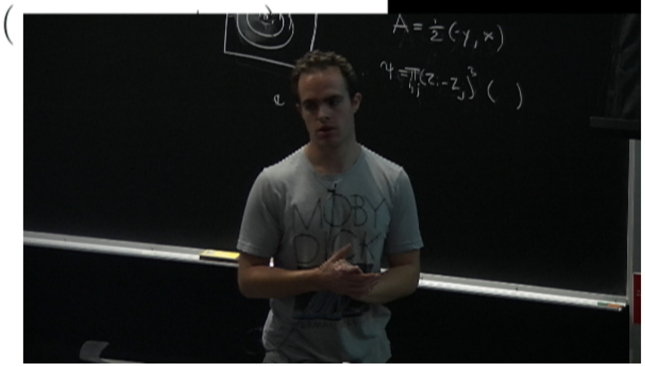

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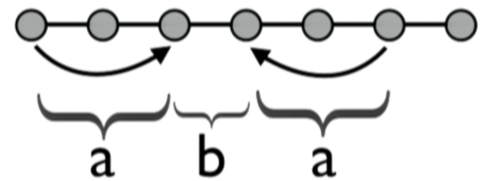

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$V(|r_i - r_j|) \longrightarrow$ `Haldane pseudopotentials' $V_m \longrightarrow V_{ab}$

[Haldane '83; Trugman & Kivelson '85; Haldane & Rezayi '94; Bergholtz et al.; '05, Seidel et al. '05]

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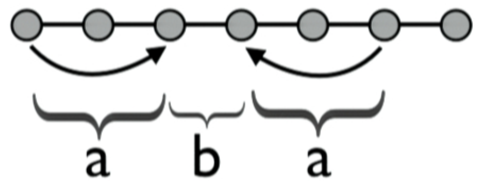

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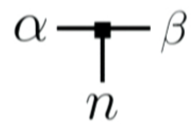
iDMRG: MPS and MPO

$B^n_{\alpha\beta}$ =

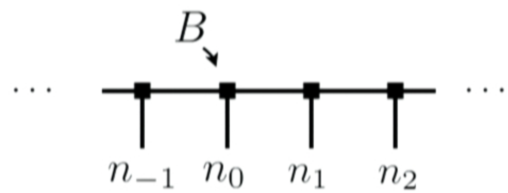


$\alpha, \beta \in \{1, 2, \cdots, \chi\}$

$$\boxed{\chi \sim e^{aS_E} \sim e^{aL_x}}$$

(area law)

$\Psi_{\cdots n_0 n_1 \cdots}$ =



[Fannes et al. '92]

'iMPS'

iDMRG: MPS and MPO

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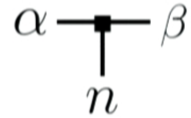

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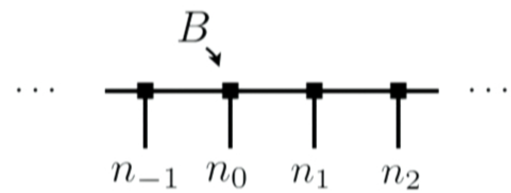
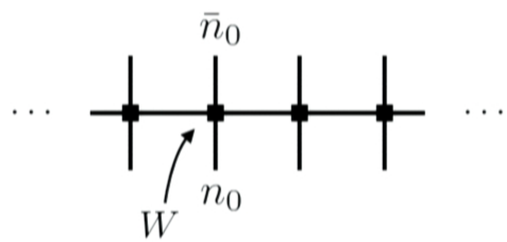


[Fannes et al. '92]

'iMPS'

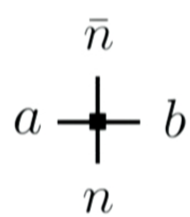
$H^{\cdots \bar{n}_0 \bar{n}_1 \cdots}_{\cdots n_0 n_1 \cdots}$ =



[McCulloch '07]

'MPO'

${W^{\bar{n}}}_{n\,ab}$ =



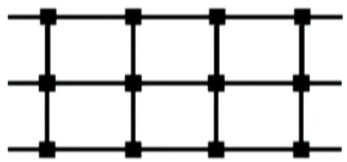
$a, b \in \{1, 2, \cdots, D\}$

$D \sim L_x^2$

(L_x^4 3-body)

iDMRG

Variationally minimize energy of iMPS

$$\langle\Psi|\, H \,|\Psi\rangle = \quad \cdots$$  $\cdots$

iDMRG

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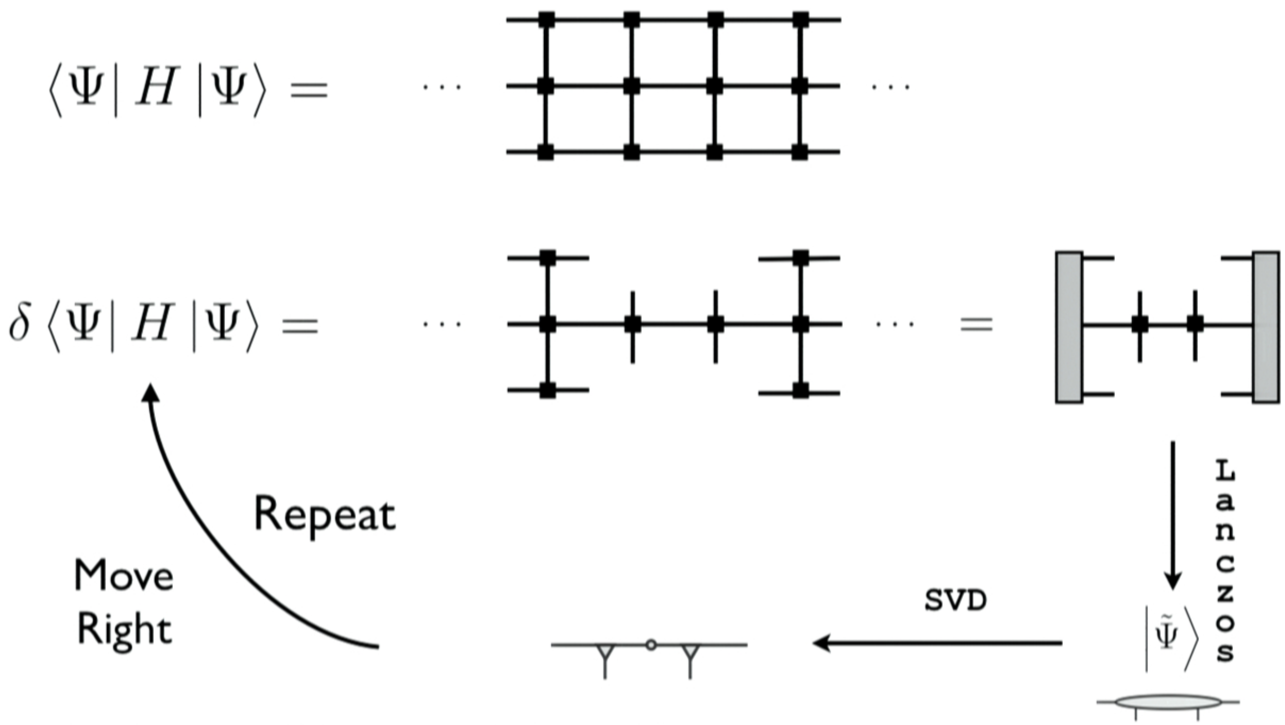
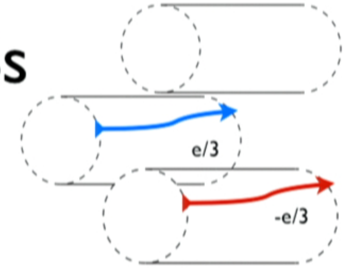


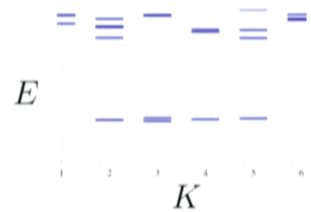
[White '92; McCulloch '08; Crosswhite et al. '08; Kjaell et al '12]

Obtaining the GSs



Issue 1: Does iDMRG produce MES, or a linear combination?

Cincio & Vidal '12: MES are eigenstates of infinite cylinder with energies split by $\mathcal{O}(e^{-bL_x/\xi})$



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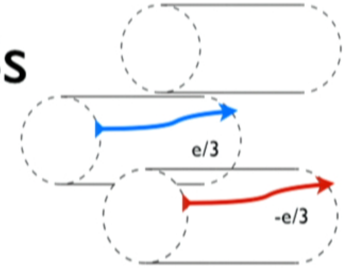

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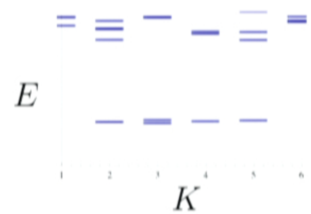
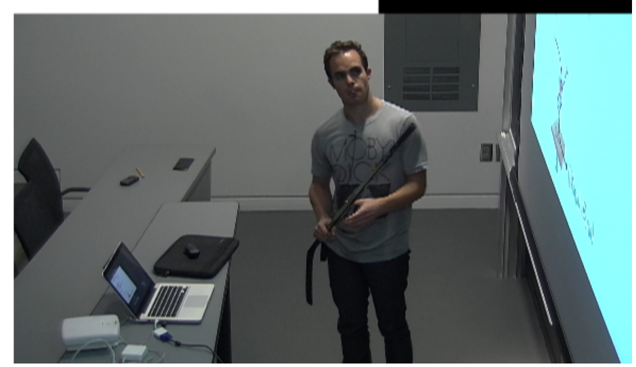


Issue II: How do we obtain all of them?

$\{E_{0110}\}$

$|\cdots 0110 \cdots\rangle$

$(|\Psi_{0110}\rangle \, , E_{0110})$



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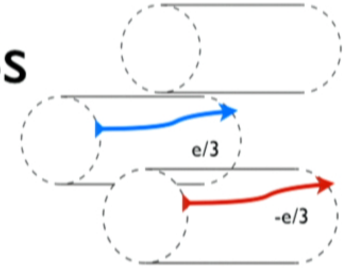

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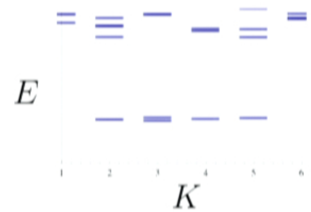

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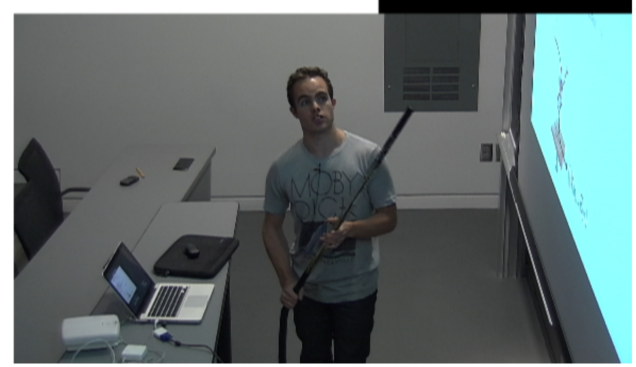
$\{E_{0110}, E_{0101}, \cdots\}$

$|\cdots 0101 \cdots\rangle$

$(|\Psi_{01}\rangle, E_{01})$



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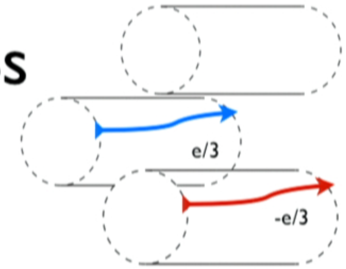
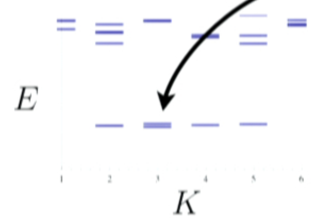


$\nu = \frac{3}{5}:\ k = 3$ Read-Rezayi

01110 vs 10101

Issue I: Does iDMRG produce MES, or a linear combination?



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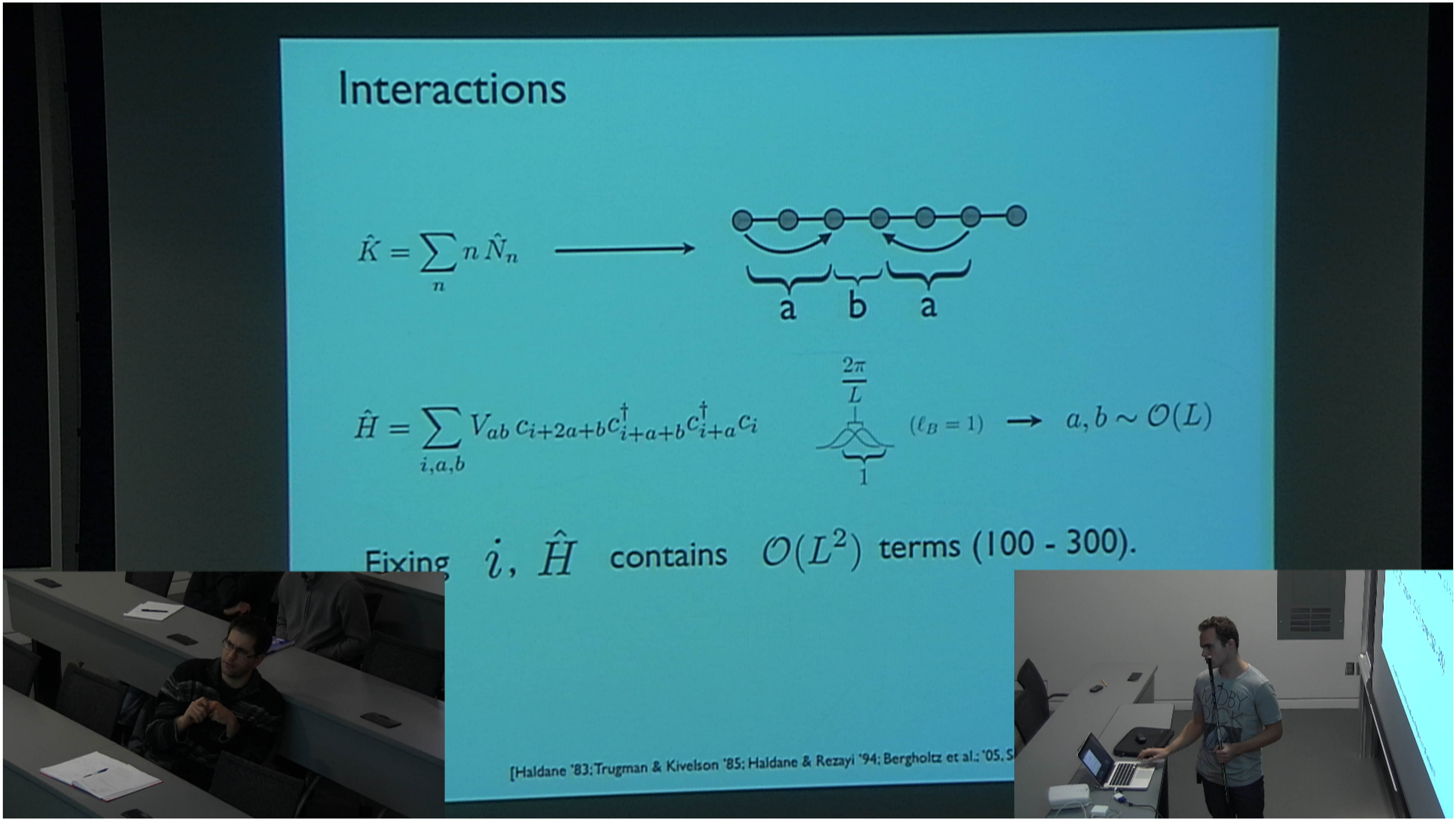
$|\cdots 0101 \cdots\rangle$

$(|\Psi_{01}\rangle\,, E_{01})$

[Wen & Wang '08; Bernevig '08]


Interactions
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a b a
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$\frac{2\pi}{L}$
1
$(\ell_B = 1)$ $\rightarrow$ $a, b \sim \mathcal{O}(L)$
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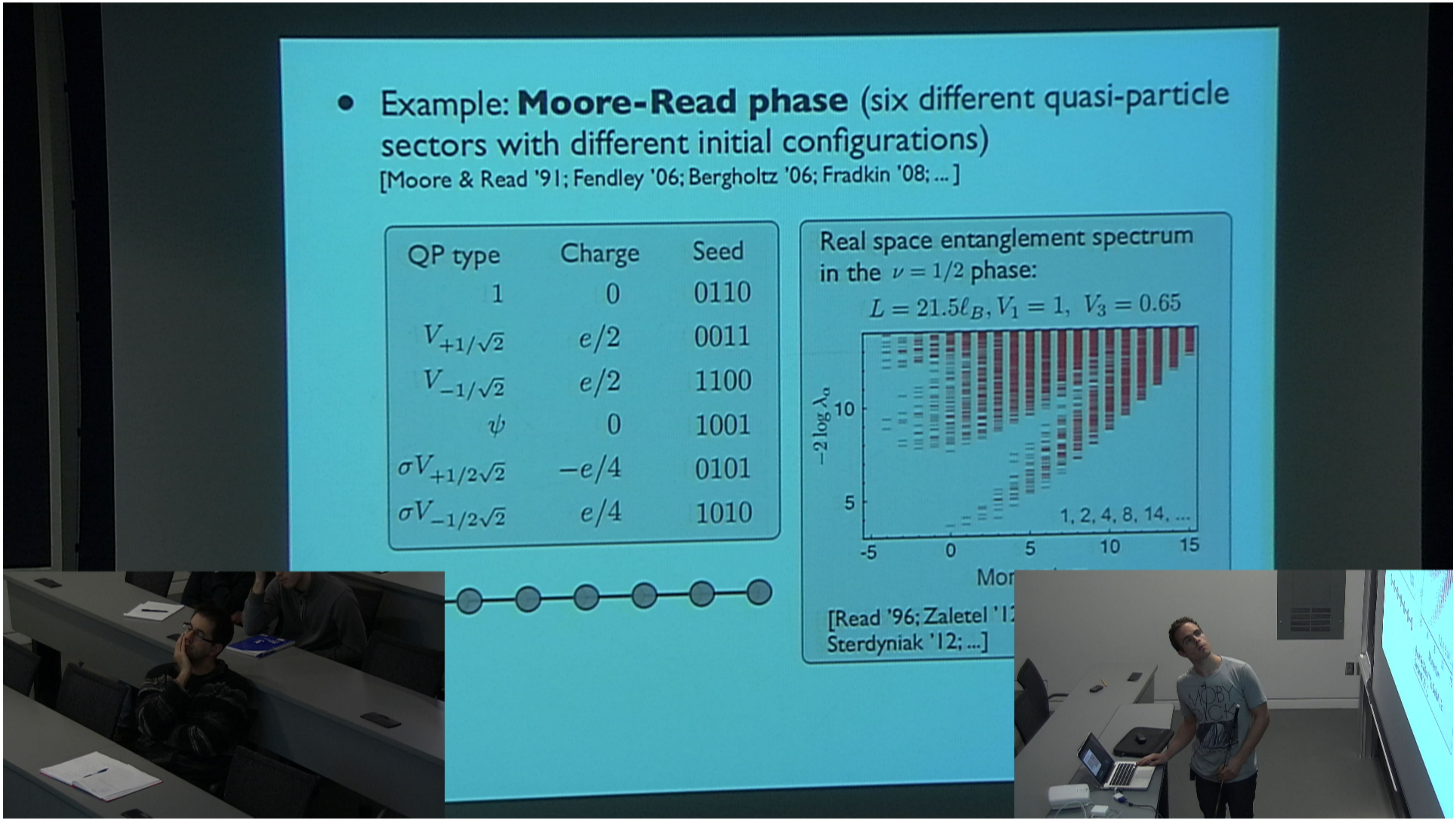
[Haldane '83; Trugman & Kivelson '85; Haldane & Rezayi '94; Bergholtz et al.; '05, S


Example: Moore-Read phase (six different quasi-particle sectors with different initial configurations)
[Moore & Read '91; Fendley '06; Bergholtz '06; Fradkin '08; ...]
QP type Charge Seed
1 0 0110
$V_{+1/\sqrt{2}}$ $e/2$ 0011
$V_{-1/\sqrt{2}}$ $e/2$ 1100
ψ 0 1001
$\sigma V_{+1/2\sqrt{2}}$ $-e/4$ 0101
$\sigma V_{-1/2\sqrt{2}}$ $e/4$ 1010
Real space entanglement spectrum in the $\nu = 1/2$ phase:
$L = 21.5\ell_B, V_1 = 1,\ V_3 = 0.65$
$-2\log\lambda_\alpha$
10
5
1, 2, 4, 8, 14, ...
-5
0
5
10
15
[Read '96; Zaletel '1
Sterdyniak '12; ...]

- Example: **Moore-Read phase** (six different quasi-particle sectors with different initial configurations)
[Moore & Read '91; Fendley '06; Bergholtz '06; Fradkin '08; ...]

QP type	Charge	Seed
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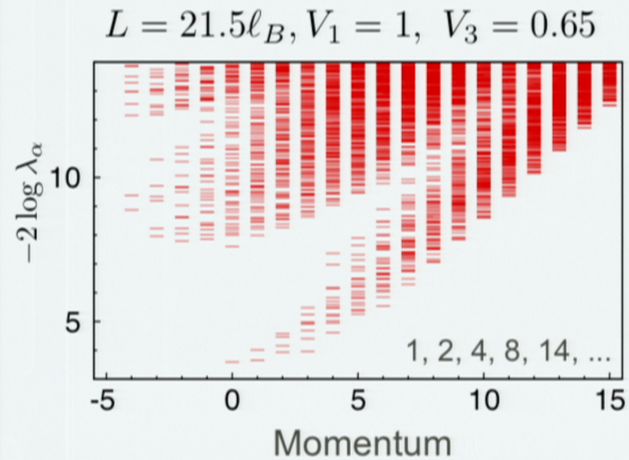
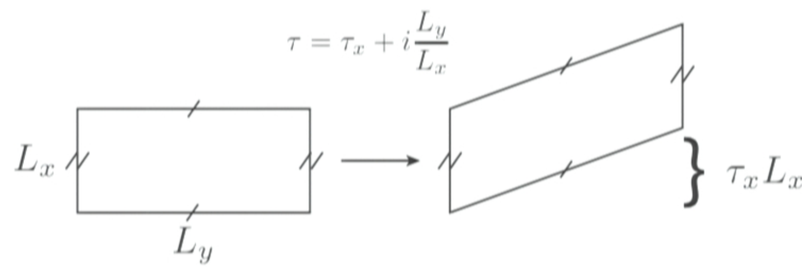
$V_{+1/\sqrt{2}}$	$e/2$	0011
$V_{-1/\sqrt{2}}$	$e/2$	1100
ψ	0	1001
$\sigma V_{+1/2\sqrt{2}}$	$-e/4$	0101
$\sigma V_{-1/2\sqrt{2}}$	$e/4$	1010

Real space entanglement spectrum in the $\nu = 1/2$ phase:



[Read '96; Zaletel '12; Dubail '12; Sterdyniak '12; ...]

the Modular T-matrix



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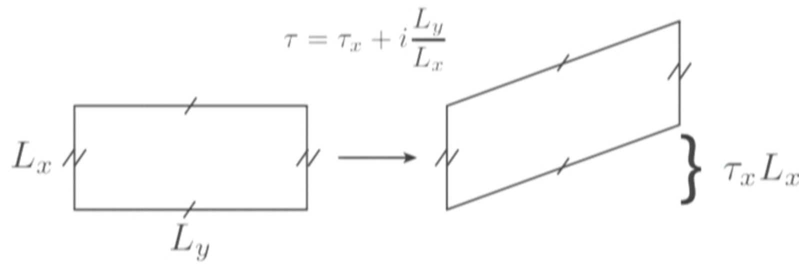
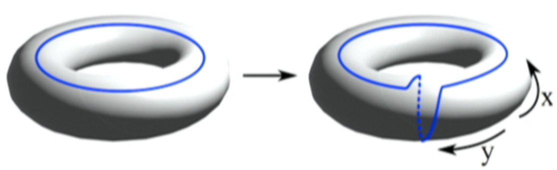


$T : \tau \rightarrow \tau + 1$



Non-abelian Berry Connection:

$$A_{ab}(\tau) = -i \langle a | \partial_\tau | b \rangle$$

T-matrix:

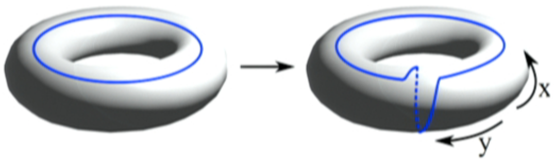
$$U_T = \mathcal{P} e^{i \oint d\tau A}$$

[Vakkuri & Wen, '95]

What do we learn?

$$T : \tau \to \tau + 1$$

$$U_T = \mathcal{P} e^{i \oint d\tau A} = ?$$



$$U_{T;ab} = \delta_{ab} \exp\left[2\pi i \left(h_a - \frac{c_-}{24} - \frac{\eta_H}{2\pi\hbar} L_x^2\right)\right]$$

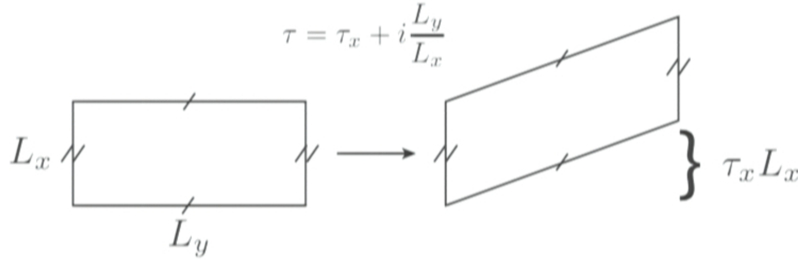
I. Topological Spin (h_a)

II. Chiral central charge (c_-)

III. Hall Viscosity (alias the 'shift') (η_H)

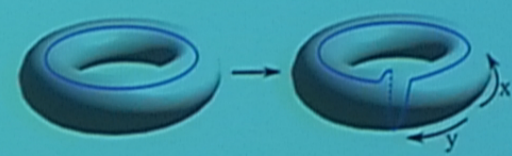
the Modular T-matrix

$$\tau = \tau_x + i\frac{L_y}{L_x}$$



What do we learn?

$T : \tau \to \tau + 1$



$U_T = \mathcal{P} e^{i \oint d\tau A} = ?$

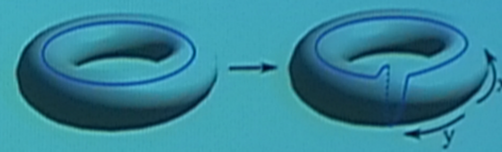
$$U_{T;ab} = \delta_{ab} \exp\left[2\pi i \left(h_a - \frac{c_-}{24} - \frac{\eta_H}{2\pi\hbar} L_x^2\right)\right]$$

I. Topological Spin (h_a)

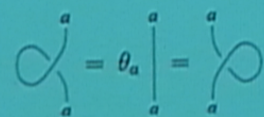
II. Chiral central charge (c_-)

III. Hall Viscosity (alias the 'shift') (η_H)

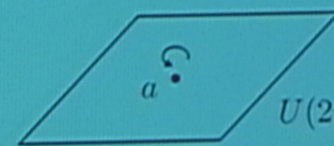
I. Topological Spin



$e^{2\pi i h_a}$



$\theta_a = e^{2\pi i h_a}$



$U(2\pi) = e^{2\pi i (h_a + \mathbb{Z})}$

II. Chiral Central Charge

$e^{-2\pi i \frac{c_-}{24}}$ (breaks time reversal)

$$c_- = c_R - c_L$$

$I_E = \frac{\pi c_-}{12} T^2$

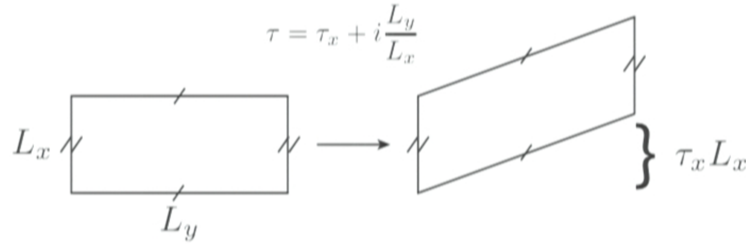
[Volovik '92; Kane&Fisher'97]

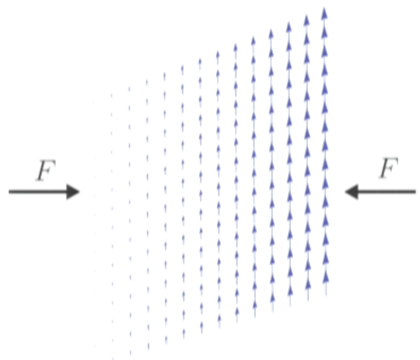
$$\mathcal{D}^{-1} \sum_a d_a^2 \theta_a = e^{2\pi c_-/8}$$

[Kitaev '08]

III. Hall Viscosity

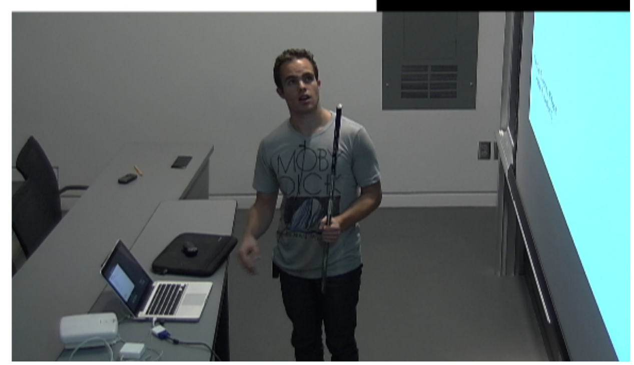
$e^{-i\eta_H L_x^2/\hbar}$ (breaks time reversal)





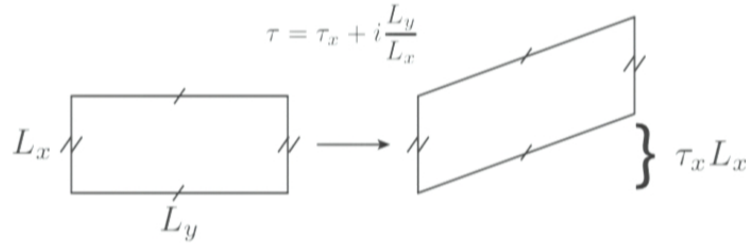
$$T_{xx} \sim -\eta_H \partial_x v_y$$

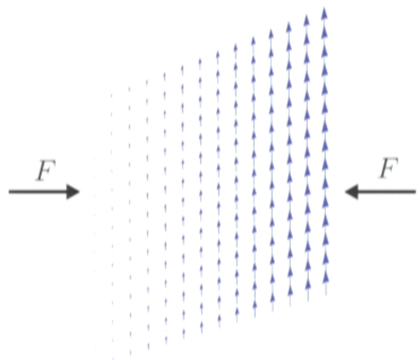
[Wen&Zee '92; Avron et al. '95; Read '09; Haldane '09; Hoyos&Son '12]



III. Hall Viscosity

$e^{-i\eta_H L_x^2/\hbar}$ (breaks time reversal)





Area

$$\eta_H \int dt\ \frac{d\tau}{dt}\frac{L_x}{L_y} \cdot L_x L_y = \eta_H L_x^2$$

Strain $(\partial_y v_x)$

$$T_{xx} \sim -\eta_H \partial_x v_y$$

$$\eta_H = \frac{1}{2}\bar{s}\bar{n}\hbar = \frac{1}{4}\mathcal{S}\bar{n}\hbar$$

[Wen&Zee '92; Avron et al. '95; Read '09; Haldane '09; Hoyos&Son '12]

charge

$$\mathbf{d} = + -$$

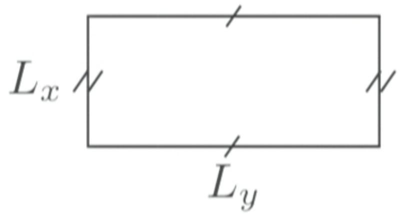
$$H = e\phi(dy) - e\phi(0)$$
$$= -\mathbf{d}.\mathbf{E}$$
$$\theta = e^{i\int dt\, \mathbf{E}.\mathbf{d}/\hbar}$$

momentum

$$s = \circlearrowleft = \downarrow \uparrow$$

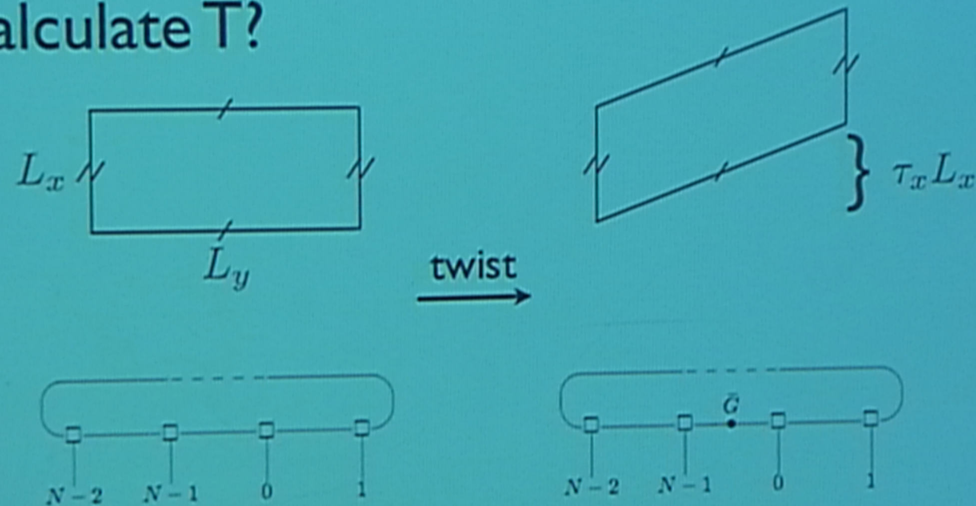
$$H = p_x v_x(dy) - p_x v_x(0)$$
$$= s\ \partial_y v_x$$
$$\theta = e^{-i\int dt\, s\partial_y v_x/\hbar}$$

How do we calculate T?



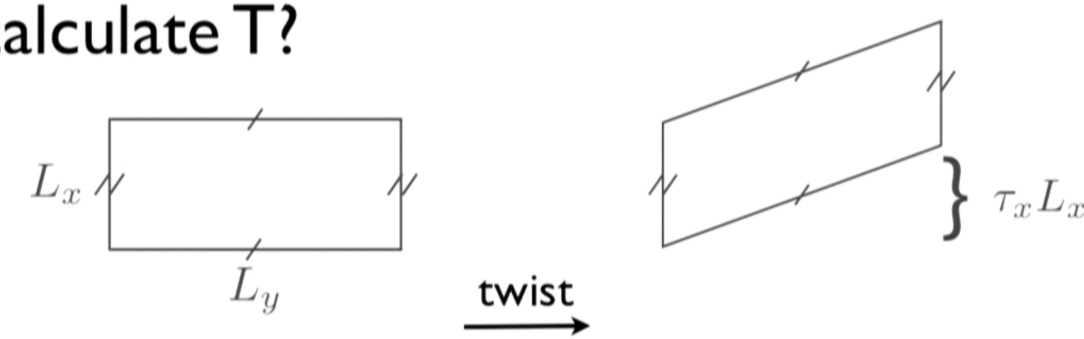
[Cincio&Vidal '12]

How do we calculate T?

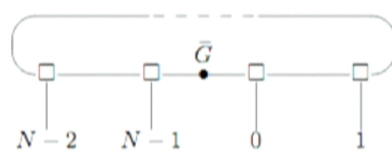


[Cincio&Vidal '12]

How do we calculate T?



[Cincio&Vidal '12]



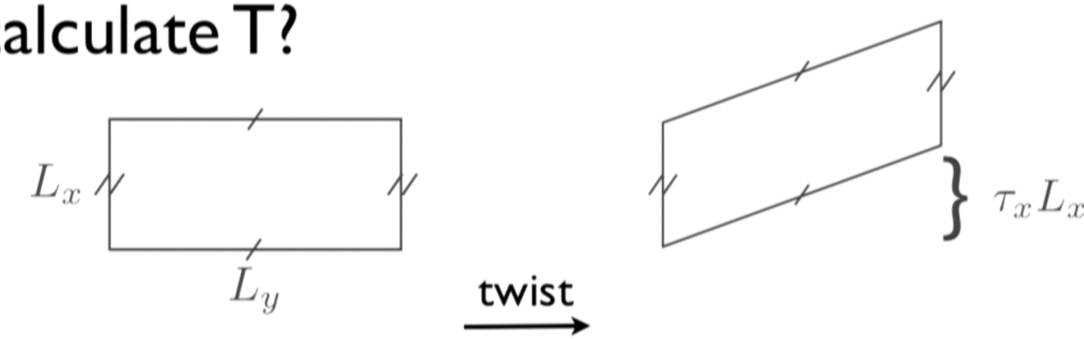
$$\hat{C} = \sum_n \hat{N}_n \quad \text{(charge)}$$

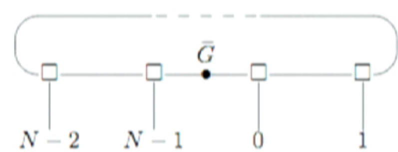
$$\hat{K} = \sum_n n\,\hat{N}_n \quad (\ x \text{ momentum})$$

$$\overline{G} = (-1)^{(N_e-1)\hat{C}} \exp\left[-2\pi i \tau_x \hat{K}\right]$$

Jordan-Wigner orbital momentum

How do we calculate T?





$\hat{C} = \sum_n \hat{N}_n$ (charge)

$\hat{K} = \sum_n n\,\hat{N}_n$ (x momentum)

[Cincio&Vidal '12]

$$\overline{G} = (-1)^{(N_e-1)\hat{C}} \exp\left[-2\pi i \tau_x \hat{K}\right]$$

Jordan-Wigner　　orbital momentum

$\{|a;\tau\rangle\}$ *without* re-running DMRG

Results.

$$
\begin{aligned}
U_{T;ab} &= \delta_{ab} \exp\left[2\pi i \left(h_a - \frac{c}{24} - \frac{\eta_H}{2\pi\hbar} L_x^2\right)\right] \\
&= \delta_{ab}\, e^{2\pi i\left(\overline{K}_{\bar{a}} - \langle\langle \overline{K} - \bar{n}\overline{C} \rangle\rangle - \nu/24 - \frac{\nu L_x^2}{16\pi^2 \ell_B^2}\right)} .
\end{aligned}
$$

can be calculate from orbital entanglement spectrum of iDMRG ground states

[Closely related lattice formulation: Tu, Zhang, Qi '13]

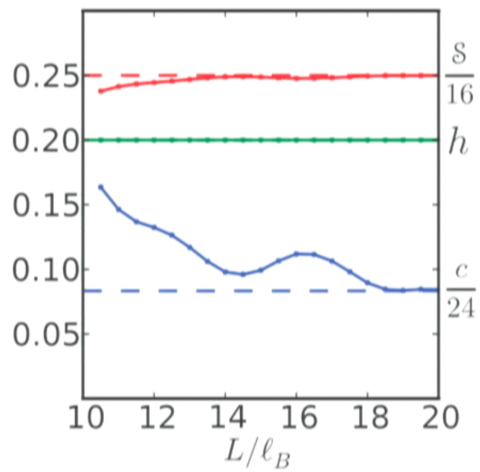
Results.

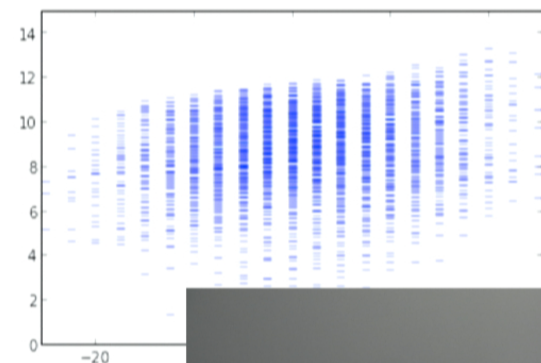
$$U_{T;ab} = \delta_{ab} \exp\left[2\pi i \left(h_a - \frac{c}{24} - \frac{\eta_H}{2\pi\hbar} L_x^2\right)\right]$$
$$= \delta_{ab}\, e^{2\pi i \left(\overline{K}_{\bar{a}} - \langle\langle \overline{K} - \bar{n}\overline{C}\rangle\rangle - \nu/24 - \frac{\nu L_x^2}{16\pi^2 \ell_B^2}\right)}.$$

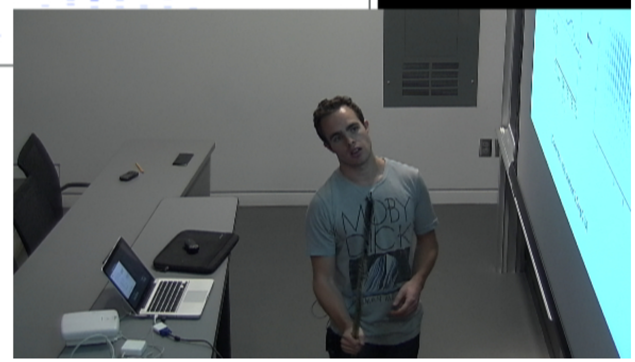
can be calculate from orbital entanglement spectrum of iDMRG ground states

$$\nu = \frac{2}{5}$$

$$V_1, V_3 \neq 0$$

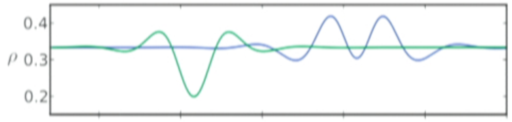


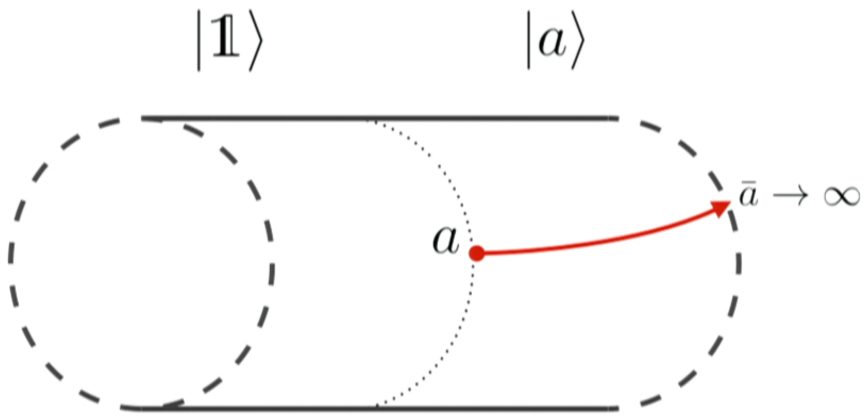


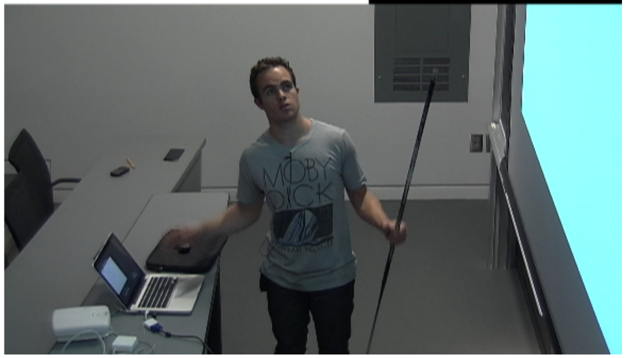


[Closely related lattice formulation: Tu, Zhang, Qi '13]

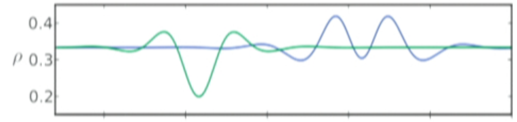
Anyons

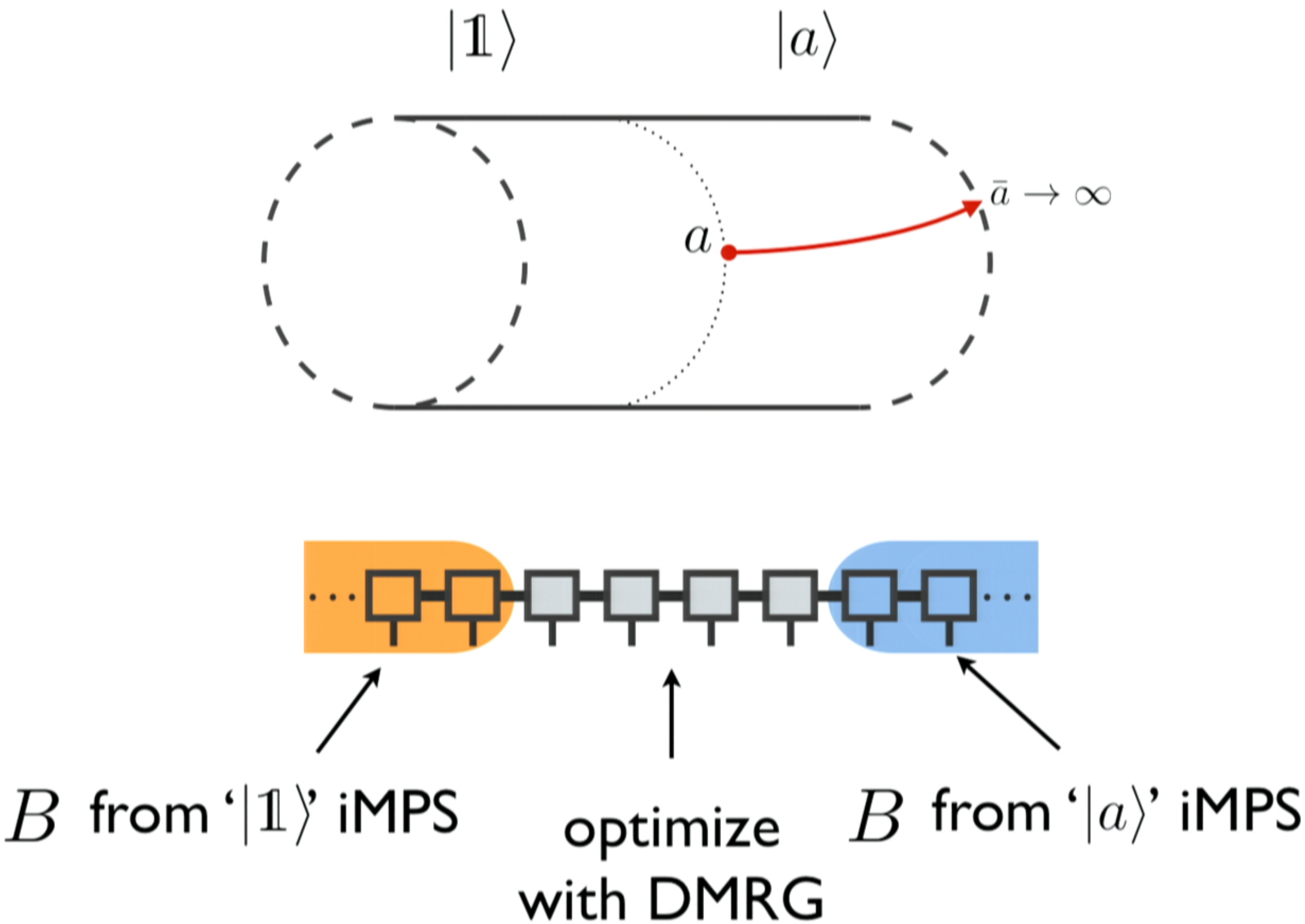






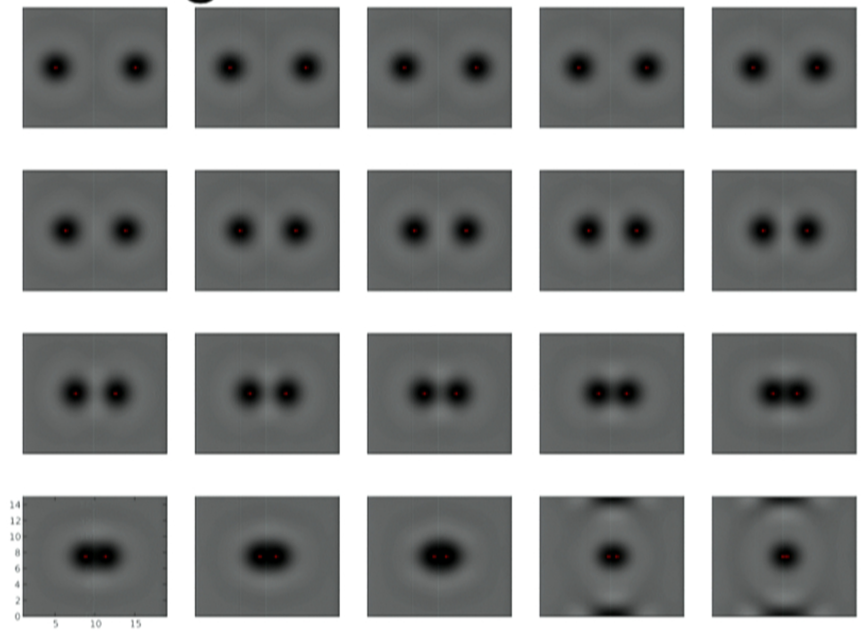
Anyons


0.4
ρ 0.3
0.2


$|\mathbb{1}\rangle$
$|a\rangle$
a
$\bar{a} \to \infty$
B from '$|\mathbb{1}\rangle$' iMPS
optimize with DMRG
B from '$|a\rangle$' iMPS

Pin and drag

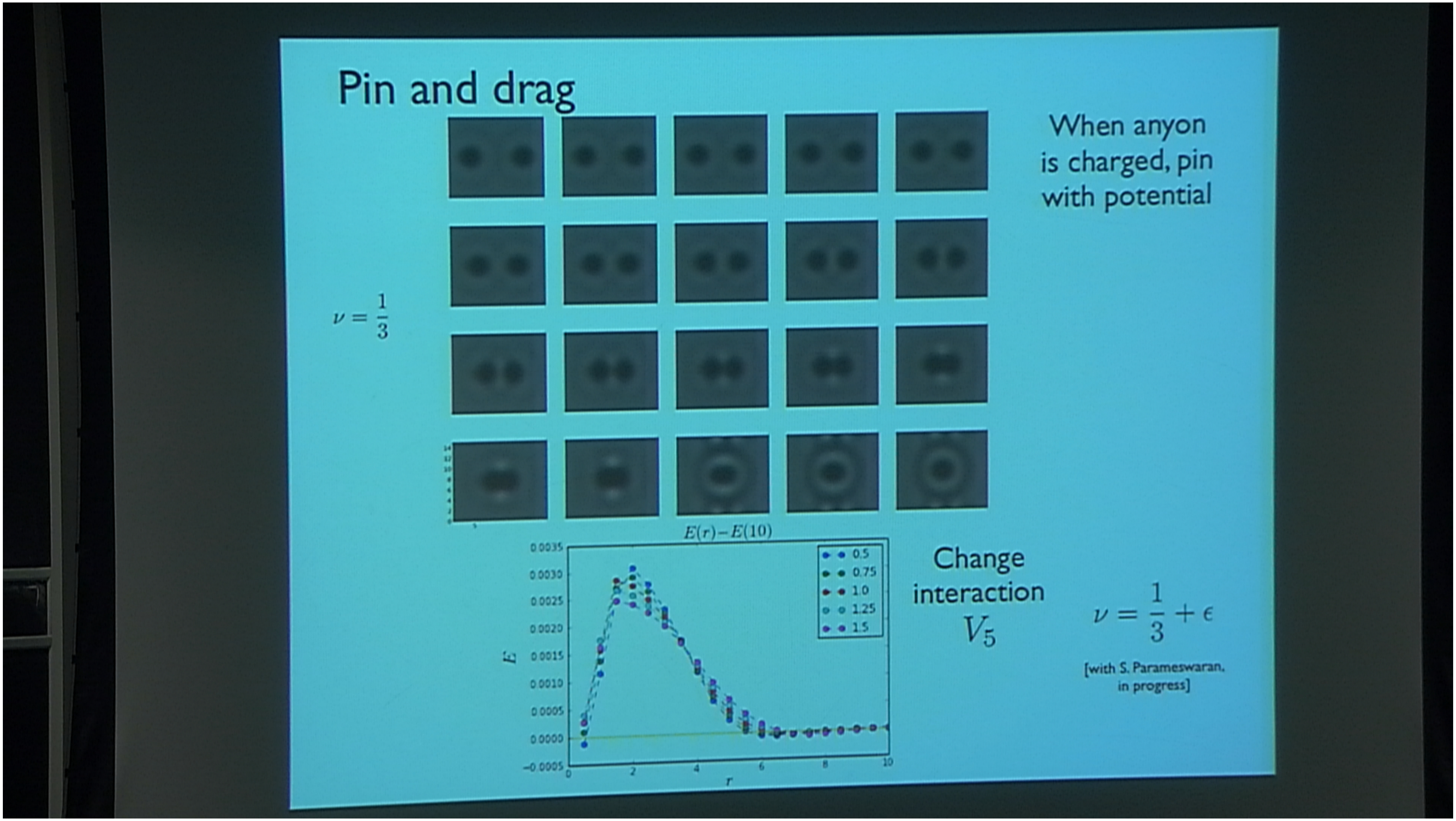
$\nu = \frac{1}{3}$

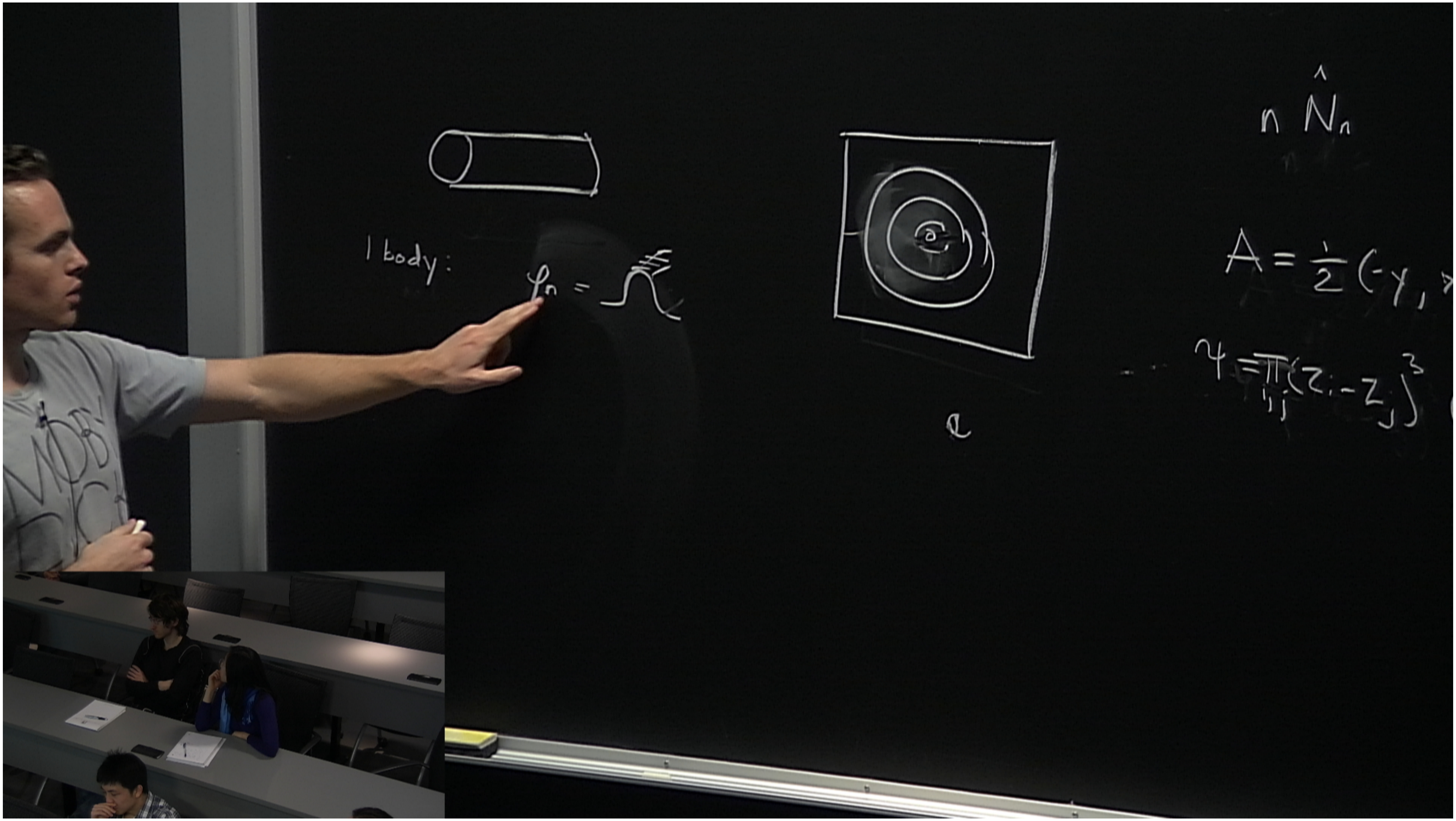


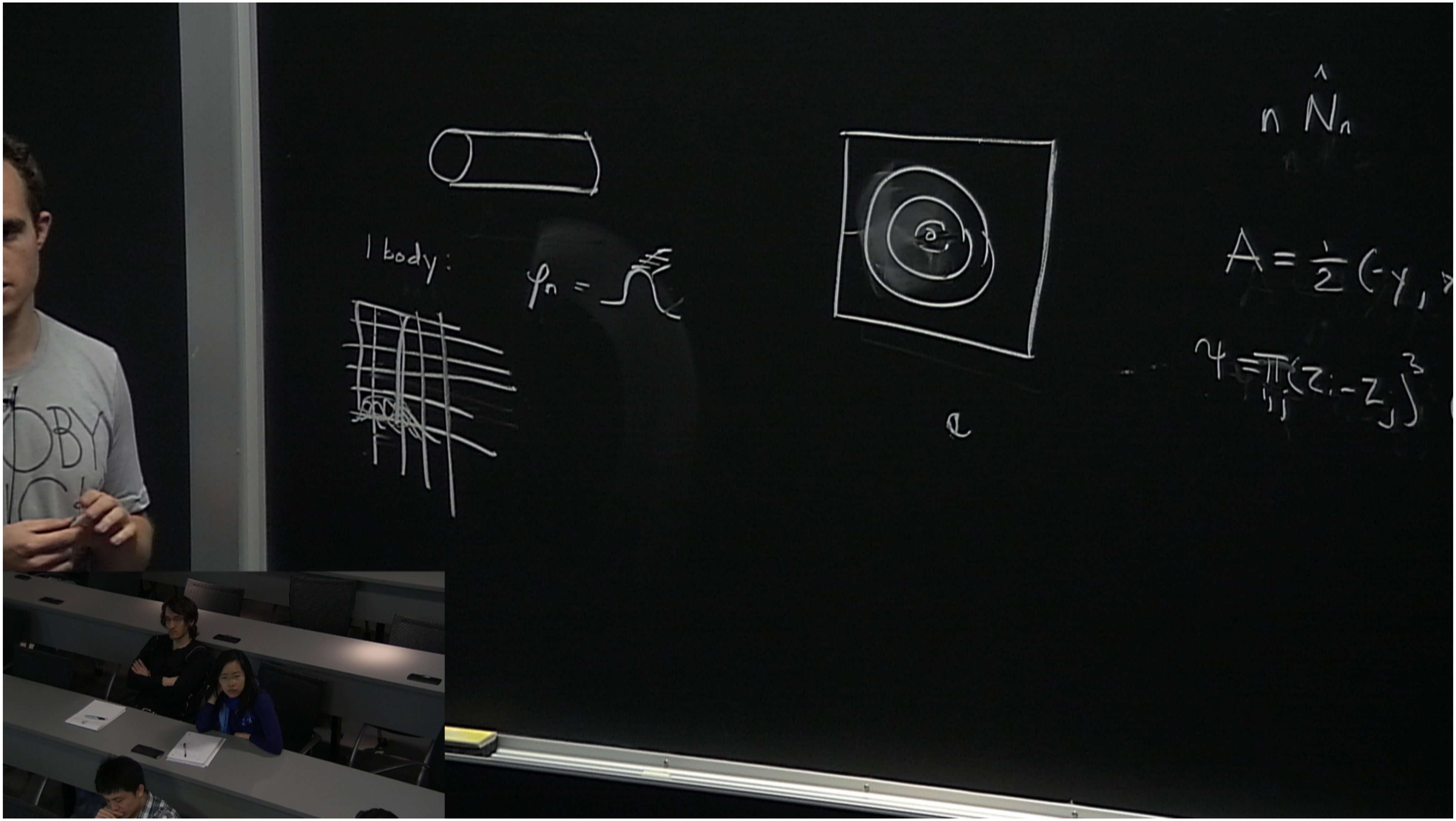
When anyon is charged, pin with potential

$$\nu = \frac{1}{3} + \epsilon$$

[with S. Parameswaran, in progress]


Pin and drag
When anyon is charged, pin with potential
$\nu = \frac{1}{3}$
$E(r) - E(10)$
0.5
0.75
1.0
1.25
1.5
E
r
Change interaction V_5
$\nu = \frac{1}{3} + \epsilon$
[with S. Parameswaran, in progress]


1 body:
$\varphi_n =$
$n \ \hat{N}_n$
$A = \frac{1}{2}(-y,$
$\psi = \prod_{i,j}(z_i - z_j)^3$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