Title: Looking for spinon deconfinement in two dimensions

Date: Feb 05, 2013 03:30 PM

URL: http://www.pirsa.org/13020127

Abstract: We have used a recently proposed quantum Monte Carlo algorithm [1] to study spinons (emergent S = 1/2 excitations) in 2D Resonating-Valence-Bond (RVB) spin liquids and in a J-Q model hosting a Neel â€“ Valence Bond Solid (VBS) phase transition at zero temperature [2]. We confirm that spinons are well defined quasi-particles with finite intrinsic size in the RVB spin liquid. The distance distribution between two spinons shows signatures of deconfinement.

However, at the Neelâ€“VBS transition, we found that the size of a single spinon is significantly greater than the bound-state in VBS, which indicates that spinons are â€œsoftâ€• and shrink when bound state is formed. Both spinon size and confinement length diverge as the critical point is approached. We have also compared spinon statistics in J-Q model with bilayer Heisenberg model and 1D spin chain. We conclude that the spinon deconfinement is marginal in the lowest-energy state in the spin-1 sector, due to very weak attractive spinon interactions. Deconfinement in the vicinity of the critical point should occur at higher energies.

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Looking for Spinon Deconfinement in Two Dimensions
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Feb 5, 2013
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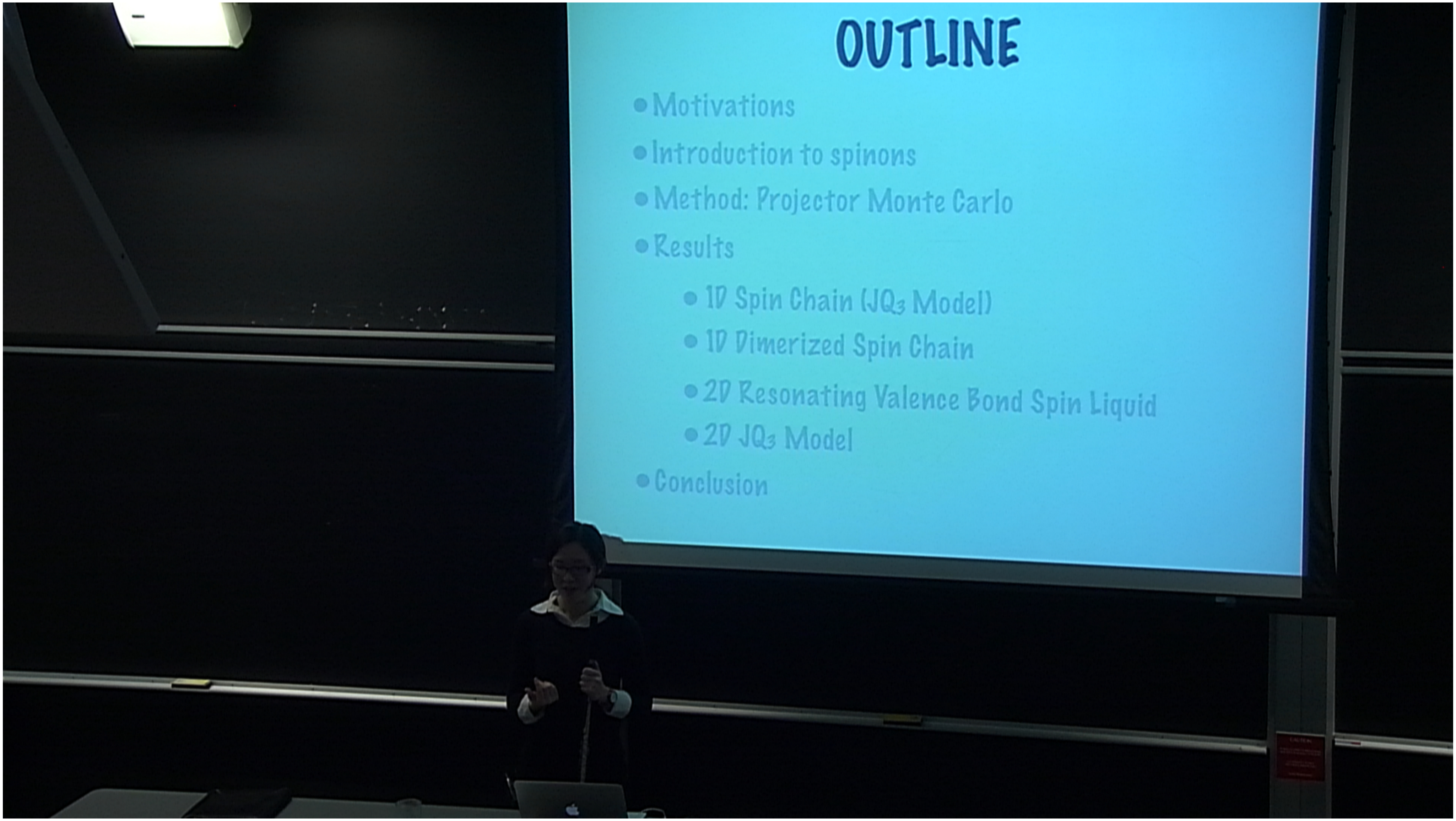
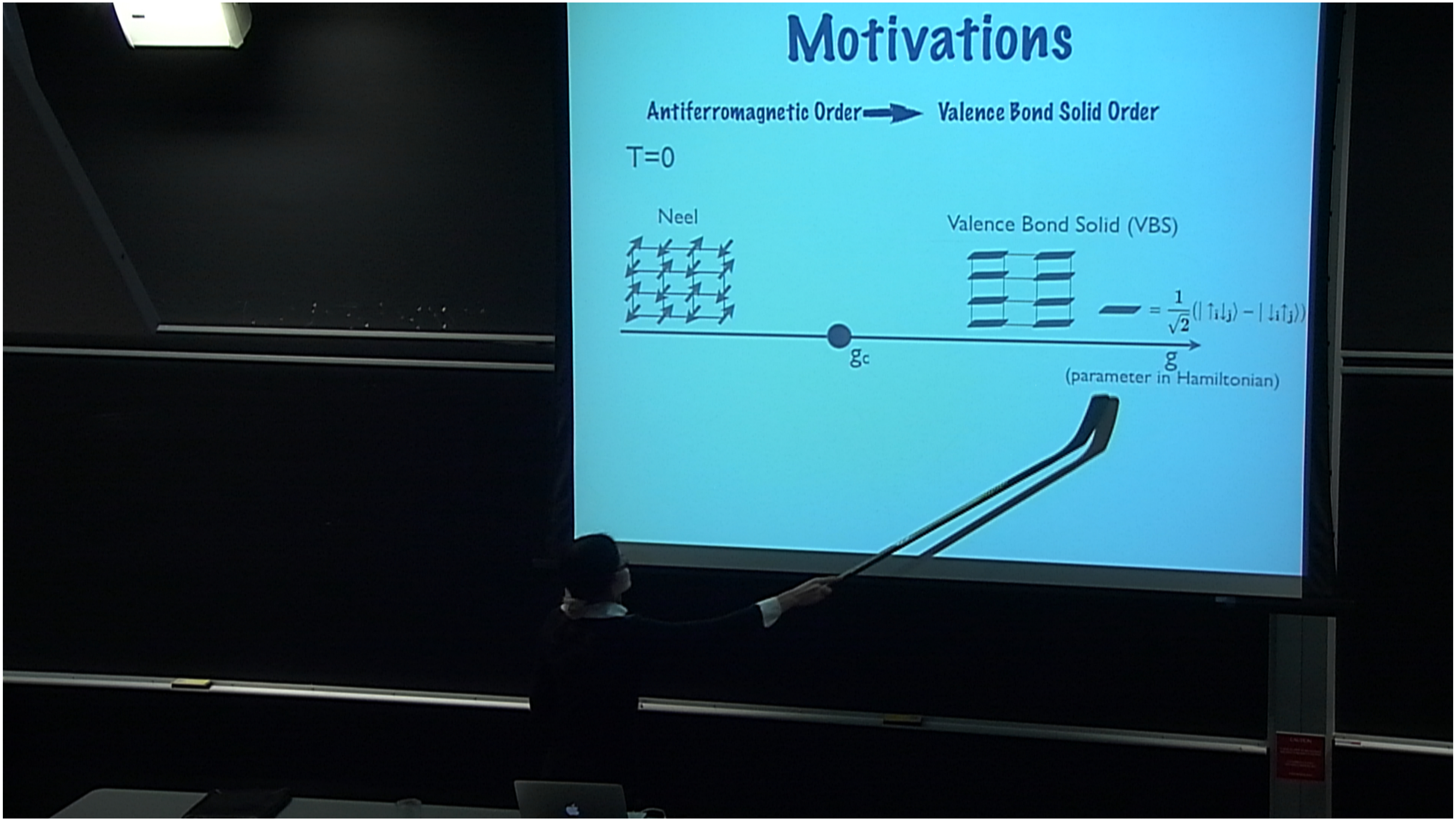

OUTLINE
Motivations
Introduction to spinons
Method: Projector Monte Carlo
Results
1D Spin Chain (JQ3 Model)
1D Dimerized Spin Chain
2D Resonating Valence Bond Spin Liquid
2D JQ3 Model
Conclusion

Motivations
Antiferromagnetic Order → Valence Bond Solid Order
T=0
Neel
Valence Bond Solid (VBS)
$= \frac{1}{\sqrt{2}}(|\uparrow_i\downarrow_j\rangle - |\downarrow_i\uparrow_j\rangle)$
g_c
g
(parameter in Hamiltonian)



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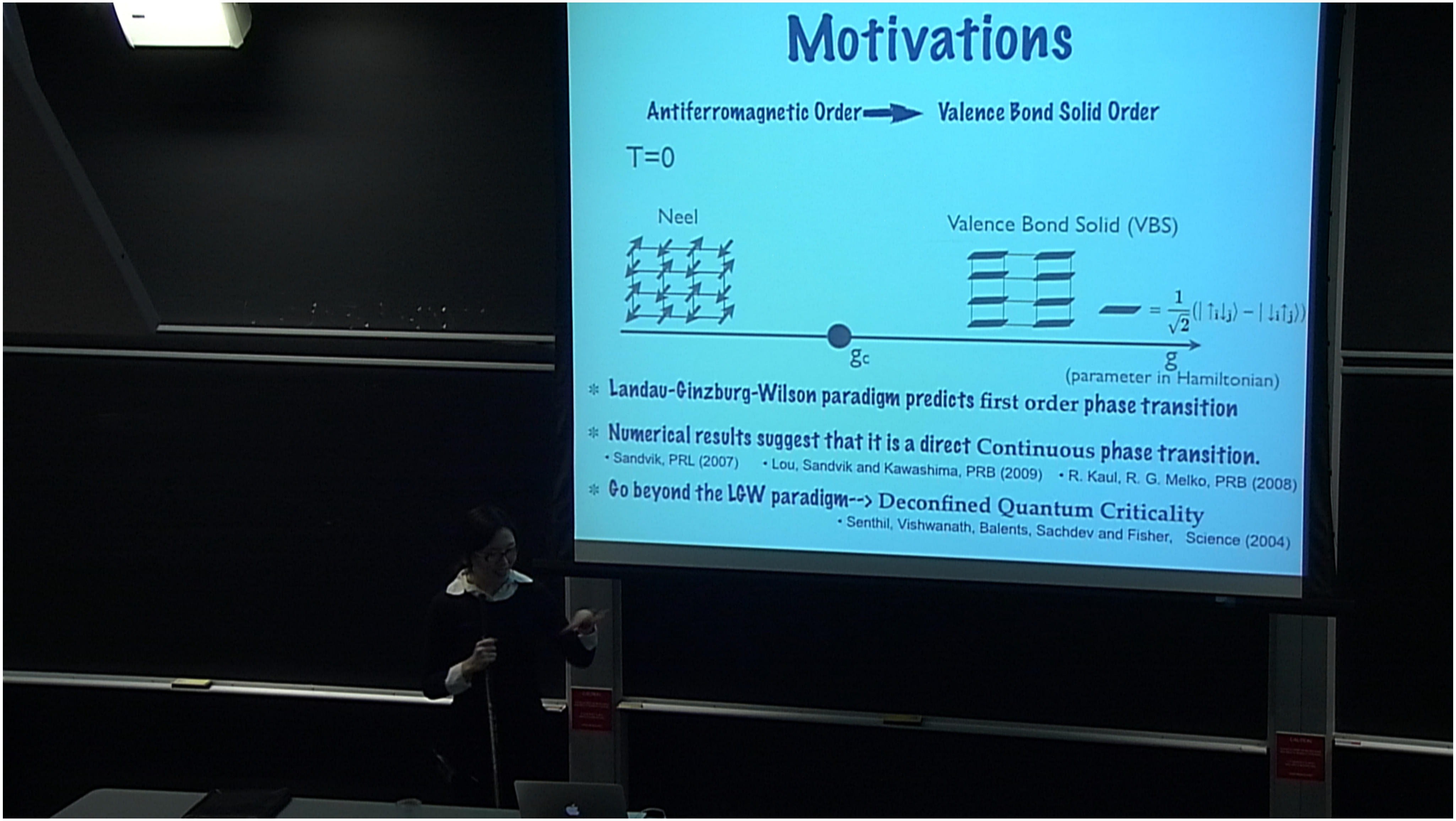
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Landau-Ginzburg-Wilson paradigm predicts first order phase transition
Numerical results suggest that it is a direct Continuous phase transition.
• Sandvik, PRL (2007) • Lou, Sandvik and Kawashima, PRB (2009) • R. Kaul, R. G. Melko, PRB (2008)
Go beyond the LGW paradigm--> Deconfined Quantum Criticality
• Senthil, Vishwanath, Balents, Sachdev and Fisher, Science (2004)

Motivations

Antiferromagnetic Order ⟶ Valence Bond Solid Order

T=0

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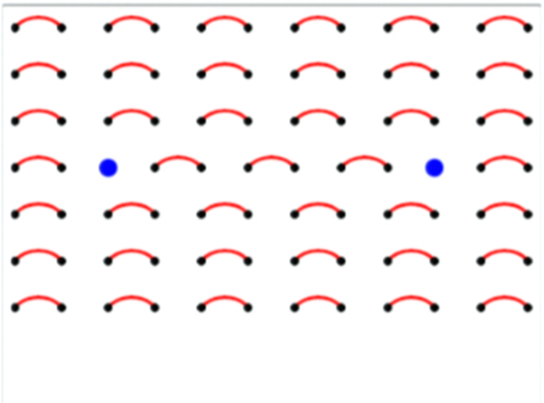
What are spinons?

Elementary S=1 Excitations in Valence Bond Solid States

What are spinons?



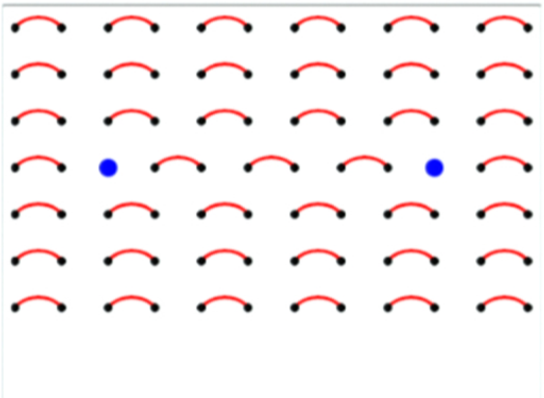
Elementary S=1 Excitations in Valence Bond Solid States

- Spinon Size
- Interactions between spinons

$\frown = \frac{1}{\sqrt{2}}(|\uparrow_i\downarrow_j\rangle - |\downarrow_i\uparrow_j\rangle)$

unpaired spin disturbs the background

What are spinons?



Elementary S=1 Excitations in Valence Bond Solid States

- Spinon Size
- Interactions between spinons

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unpaired spin disturbs the background

$$|\psi_{1/2}\rangle = \sum_r |singlet(N-1)\rangle \otimes |\uparrow_r\rangle = \sum_r |\phi(r)\rangle$$

If spinons are quasi-particles, it should have *finite* size

JQ Models

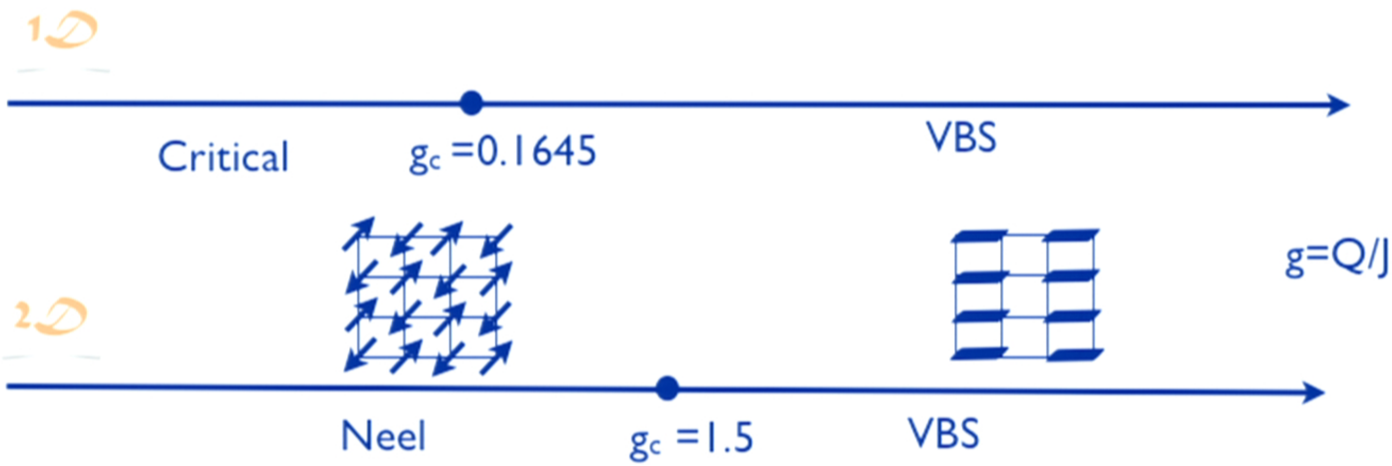
Sandvik, PRL (2007)

$$H = -\sum_{i=1}^{N} (J C_{i,i+1} + Q_3 C_{i,i+1} C_{i+2,i+3} C_{i+4,i+5})$$

J:

Q:

$$C_{i,j} = 1/4 - \mathbf{S_i} \cdot \mathbf{S_j}$$



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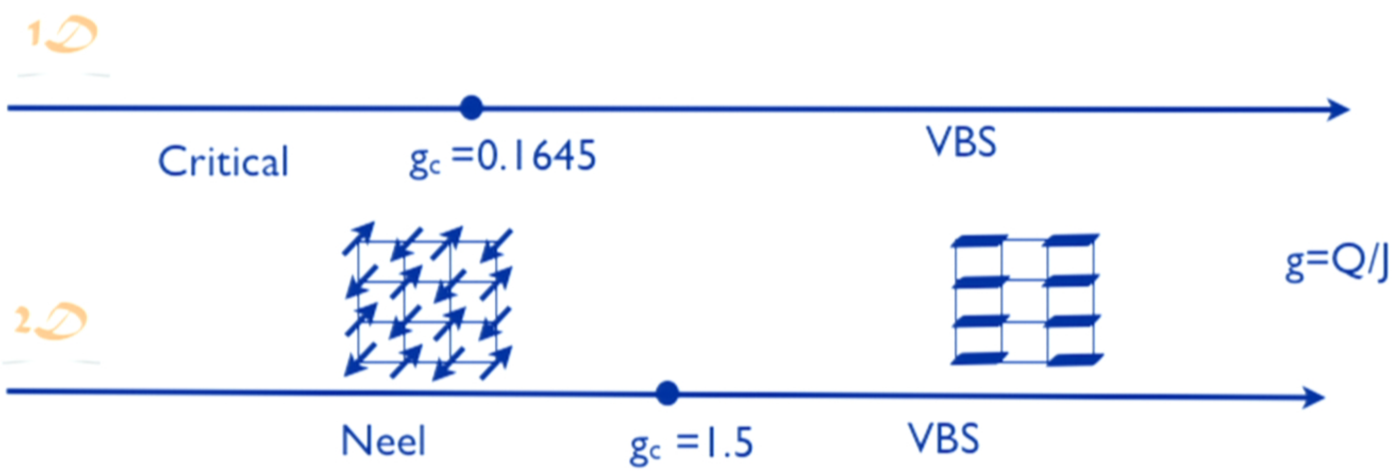

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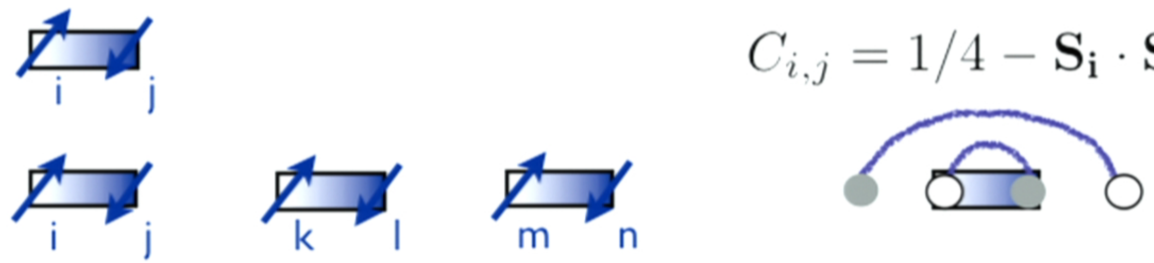


1D

Critical — $g_c = 0.1645$ — VBS

2D

Neel — $g_c = 1.5$ — VBS

g=Q/J

Method: Projector Monte Carlo

Sandvik and Evertz, PRB (2010)

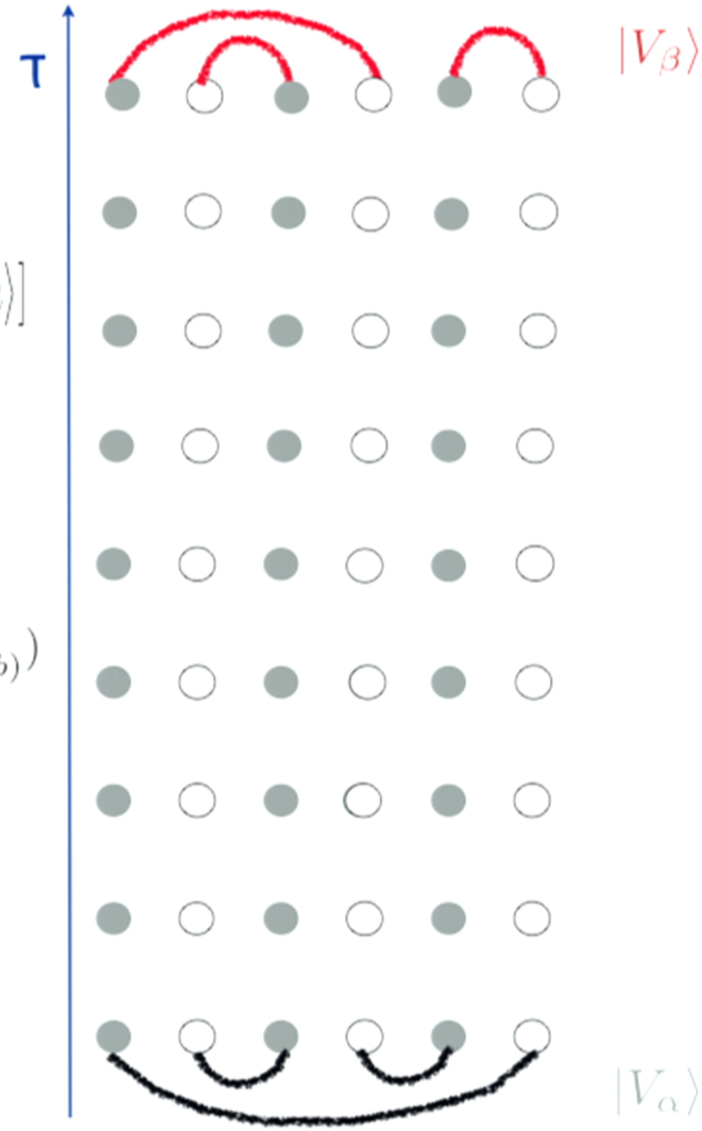
Projector MC

$$(-H)^m|\psi\rangle = c_0(-E_0)^m[|0\rangle + \sum_{m=1}^{\Lambda-1}\frac{c_n}{c_0}(\frac{E_n}{E_0})^m|n\rangle]$$

$C_{i,j} = 1/4 - \mathbf{S_i}\cdot\mathbf{S_j}$

$C_{1,b} = \frac{1}{4} - S^z_{i(b)}S^z_{j(b)}$

$C_{2,b} = \frac{1}{2}(S^+_{i(b)}S^-_{j(b)} + S^-_{i(b)}S^+_{j(b)})$



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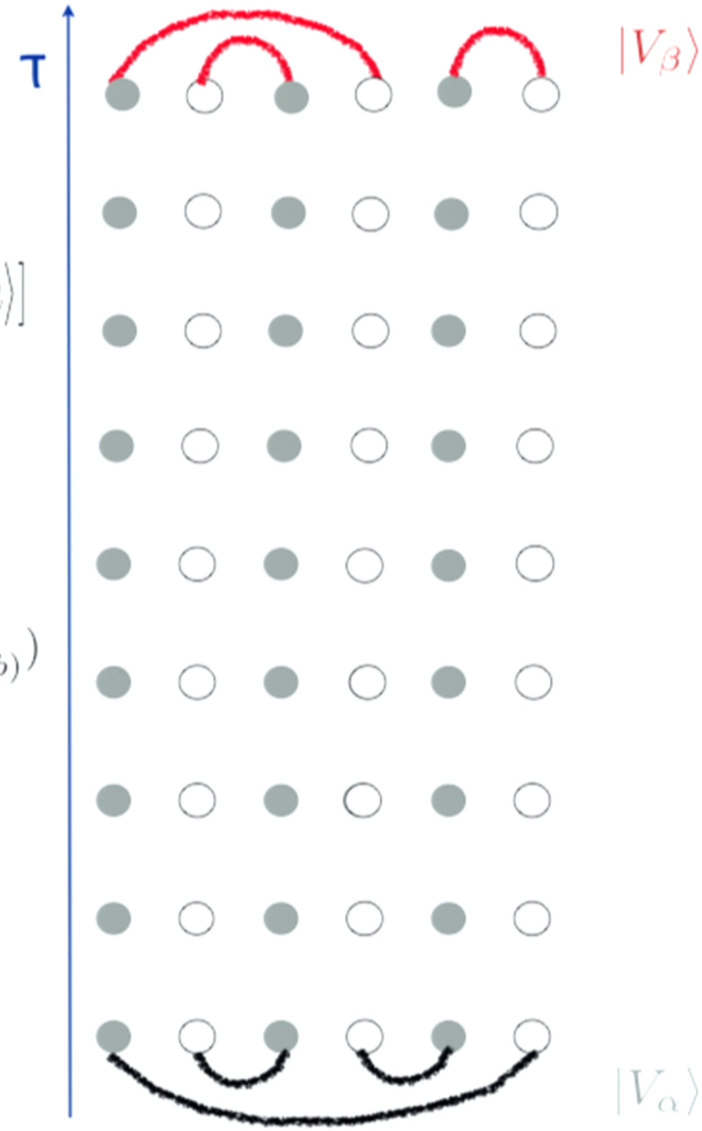

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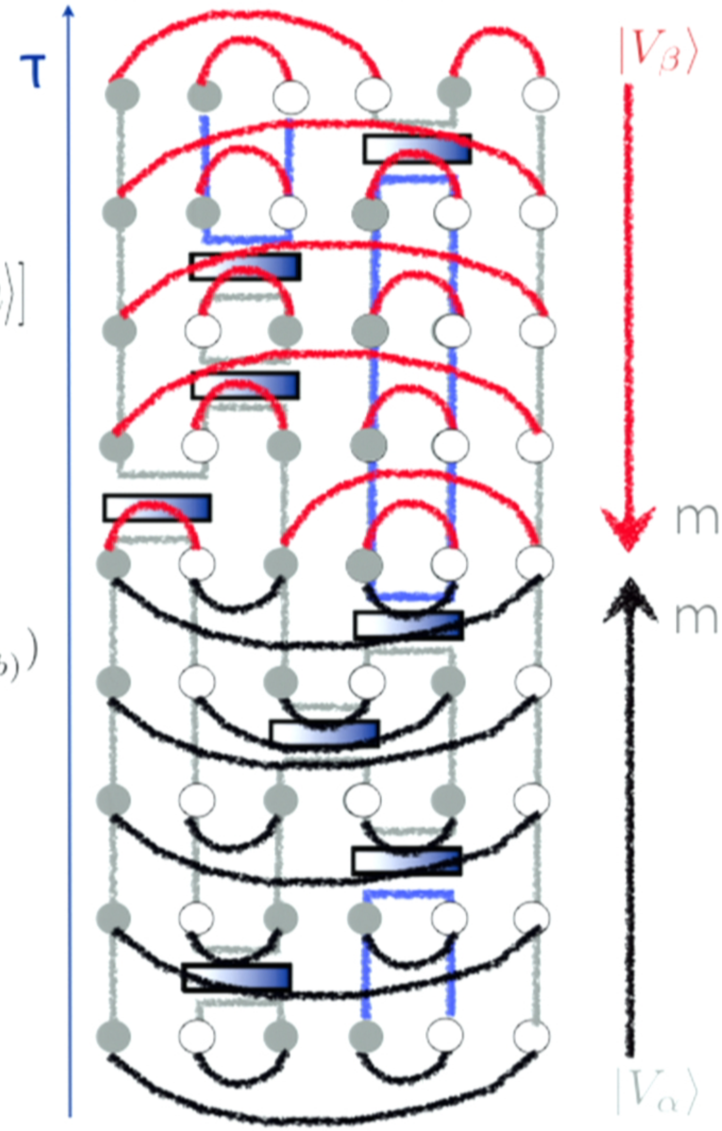
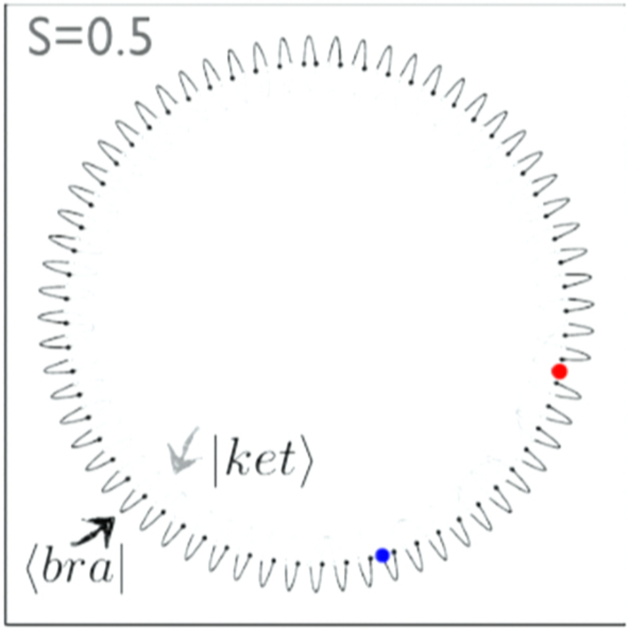
$\langle V'_\beta|V'_\alpha\rangle$



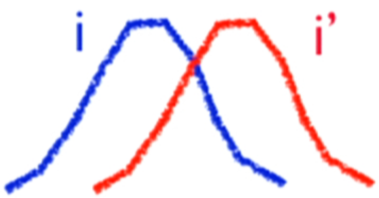
Spinon Size

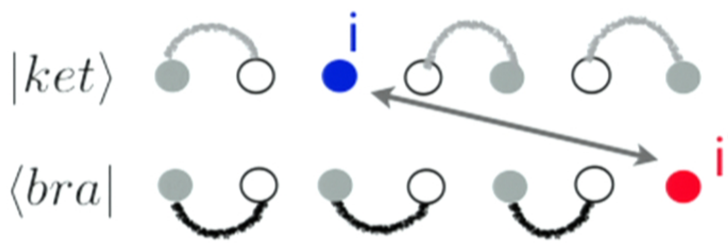


$$|\psi_{1/2}\rangle = \sum_i |\phi(i)\rangle$$
$$= \sum_i |singlet(N-1)\rangle \otimes |\uparrow_i\rangle$$

$$P(r_{i'i}) = \langle\phi(i')|\phi(i)\rangle \propto e^{-|r_i - r_i'|/\lambda}$$

$$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$$





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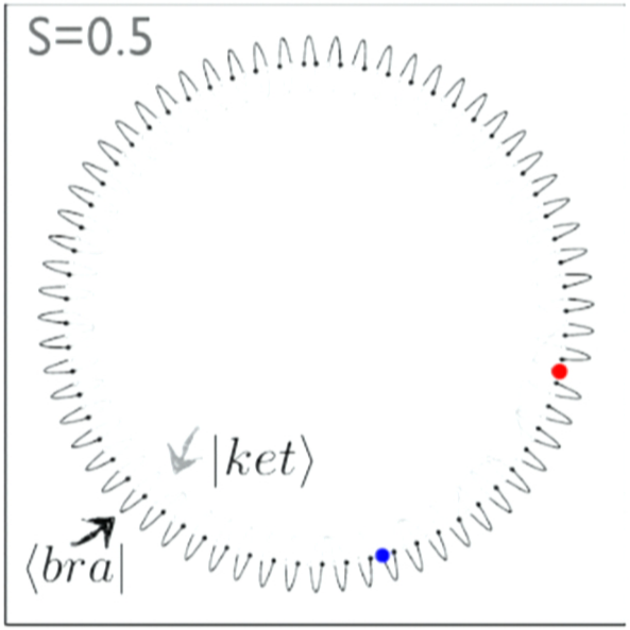

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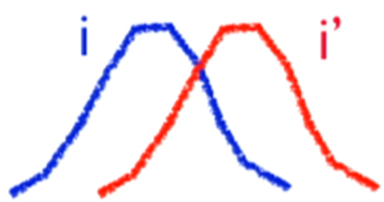

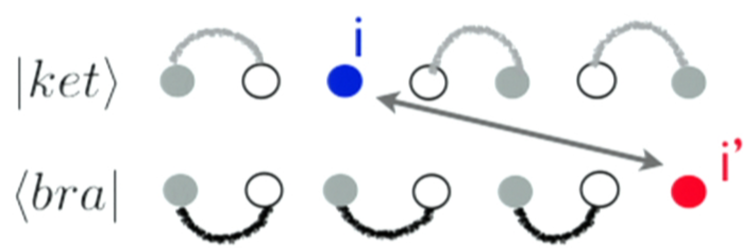

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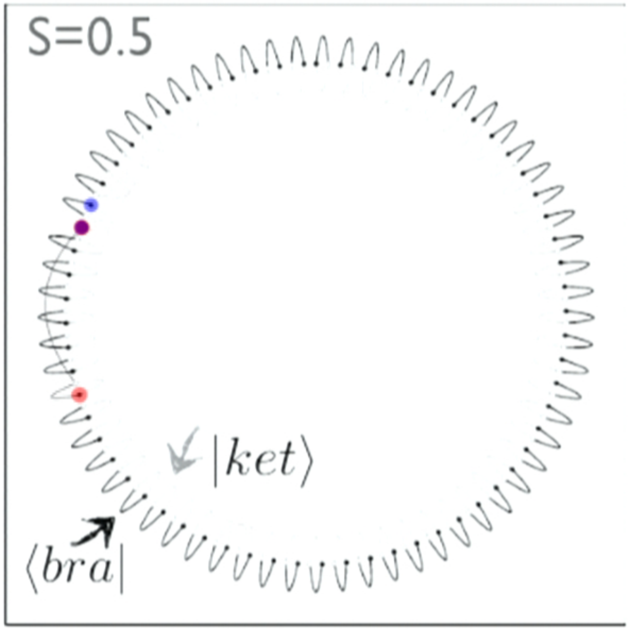

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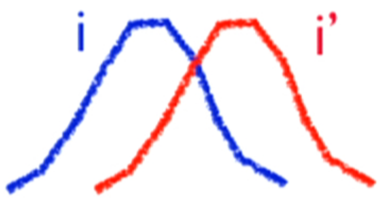
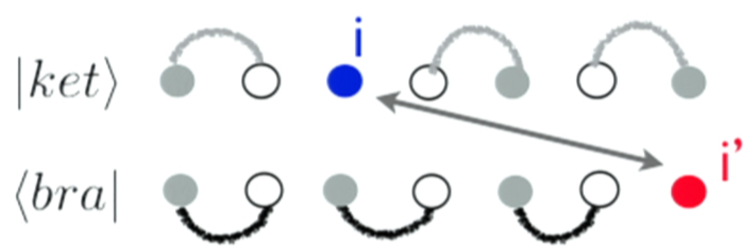
$$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$$

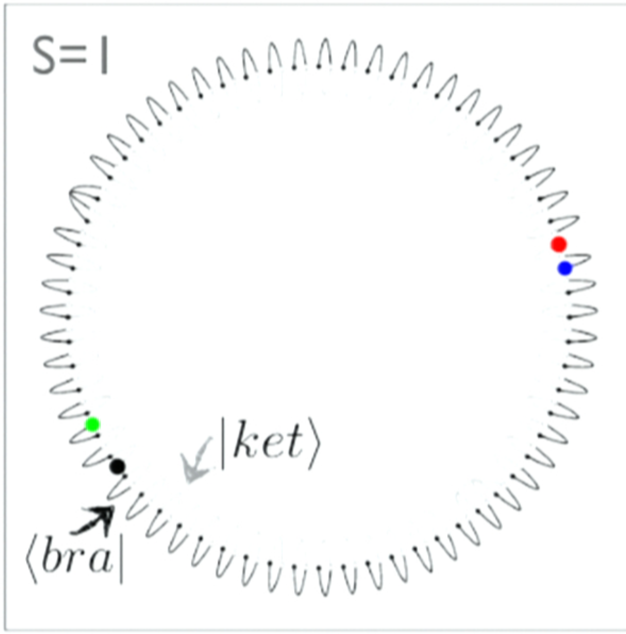
P_{AA}: distance distribution between two S=1/2 sites on the same sublattice

λ: the size of spinon





Spinon Interactions



Shastry and Sutherland, PRL (1981)

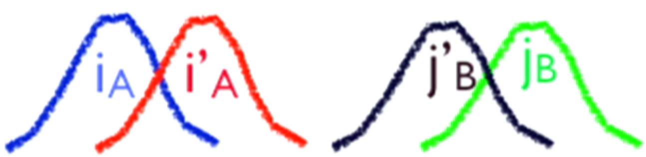
$$|\psi_1\rangle = \sum_{(i,j)} |\phi(i,j)\rangle$$
$$= \sum_{(i,j)} |singlet(N-2)\rangle \otimes |\uparrow_{i_A}\rangle \otimes |\uparrow_{j_B}\rangle$$

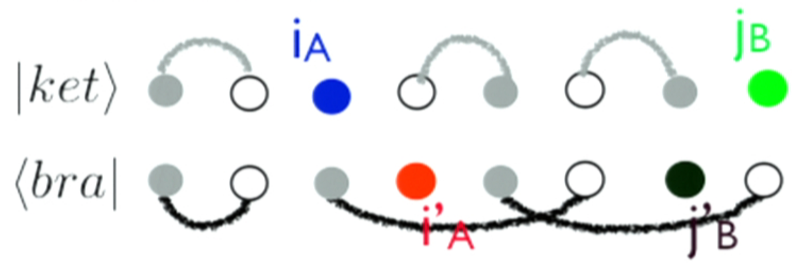
$$P(r) = \langle \phi(r_{i'_A}, r_{j'_B}) | \phi(r_{i_A}, r_{j_B}) \rangle$$

$$P_{AB}(r) \propto e^{-|r_{i_A} - r_{i'_B}|/\Lambda}$$

P_{AB}: distance distribution between two S=1/2 sites on different sublattice

Λ: confinement length





1D JQ$_3$ Chain
A test of the method

$$C_{i,j} = 1/4 - \mathbf{S_i} \cdot \mathbf{S_j}$$

$$H = -\sum_{i=1}^{N} (J C_{i,i+1} + Q_3 C_{i,i+1} C_{i+2,i+3} C_{i+4,i+5})$$

Critical $g_c = 0.1645$ VBS

+ Dimerization

$$H = -\sum_{i=2n+1}^{N/2-1} (J_1 C_{i,i+1} + J_2 C_{i+1,i+2}) - \sum_{i=1}^{N} Q_3 (C_{i,i+1} C_{i+2,i+3} C_{i+4,i+5})$$

1D JQ$_3$ Chain A test of the method

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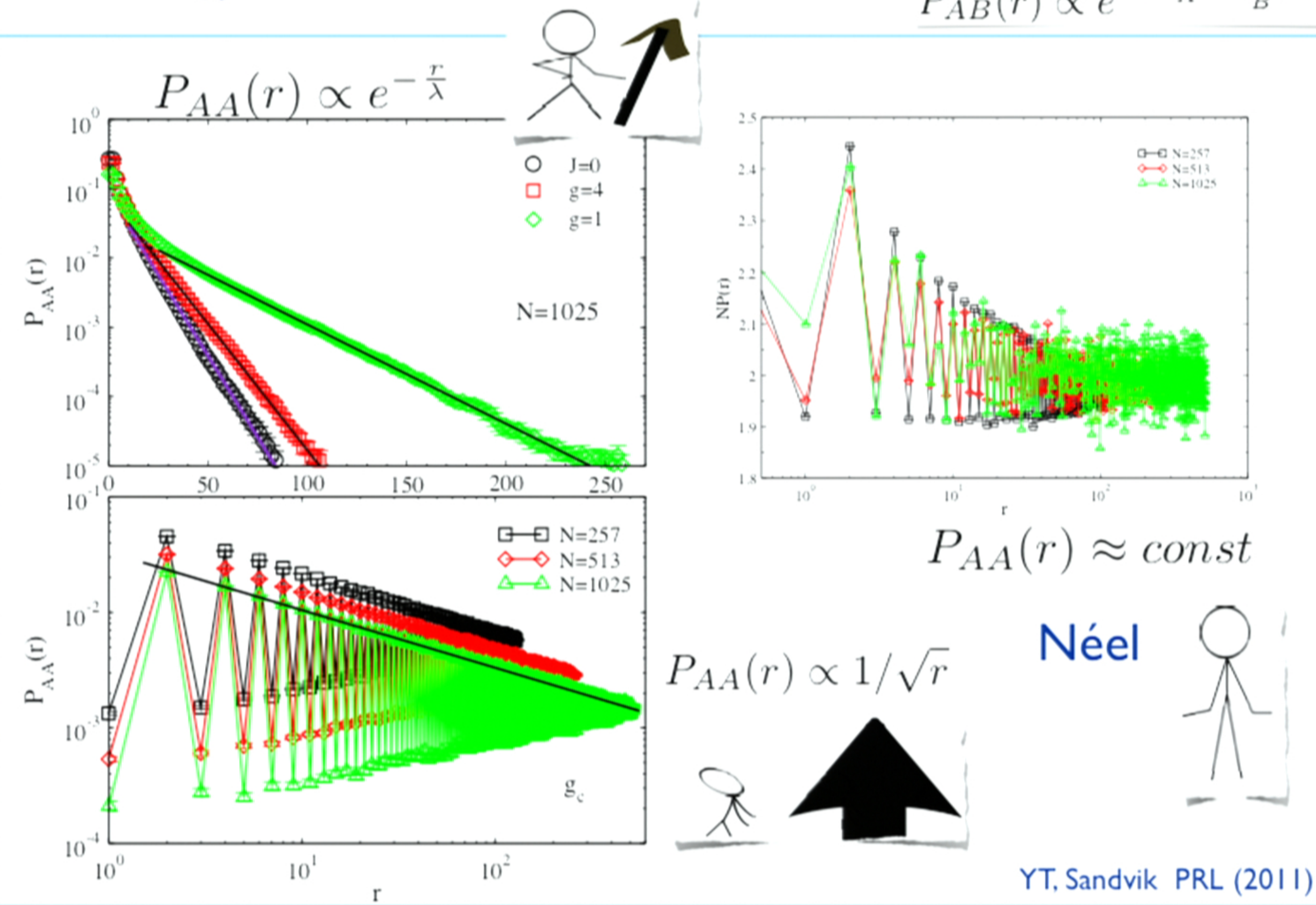
+ Dimerization

$$H = -\sum_{i=2n+1}^{N/2-1} (J_1 C_{i,i+1} + J_2 C_{i+1,i+2}) - \sum_{i=1}^{N} Q_3 (C_{i,i+1} C_{i+2,i+3} C_{i+4,i+5})$$

1D JQ$_3$ Chain S=1/2

$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$

$P_{AB}(r) \propto e^{-|r_{i_A} - r_{i'_B}|/\Lambda}$

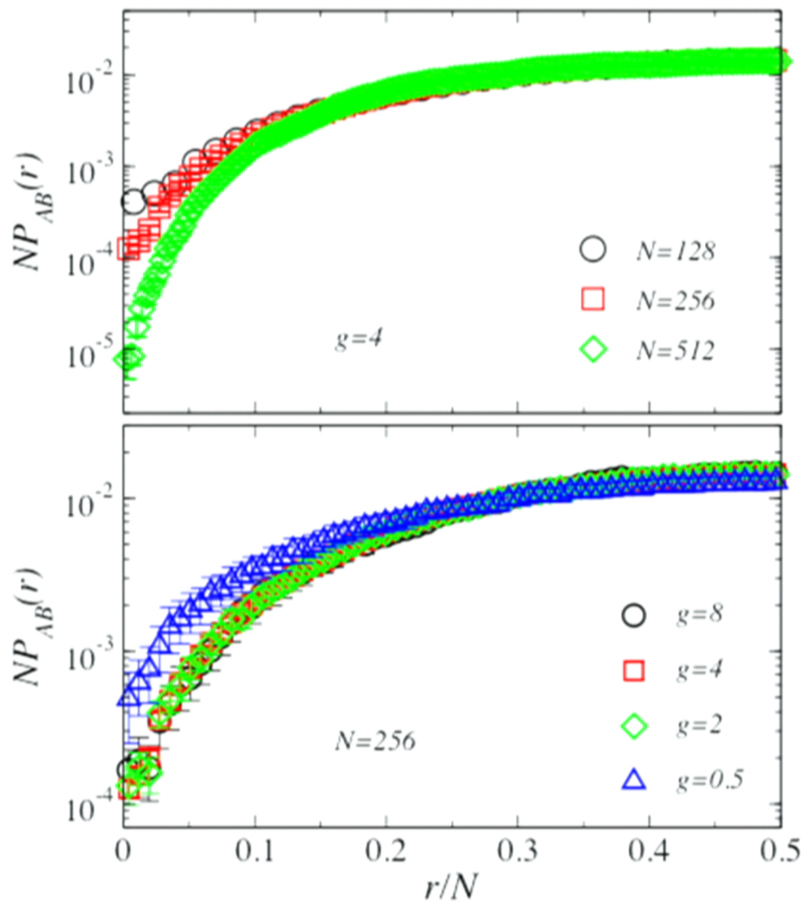


YT, Sandvik PRL (2011)

1D JQ$_3$ Chain S=1

$$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$$

$$P_{AB}(r) \propto e^{-|r_{i_A} - r_{i'_B}|/\Lambda}$$

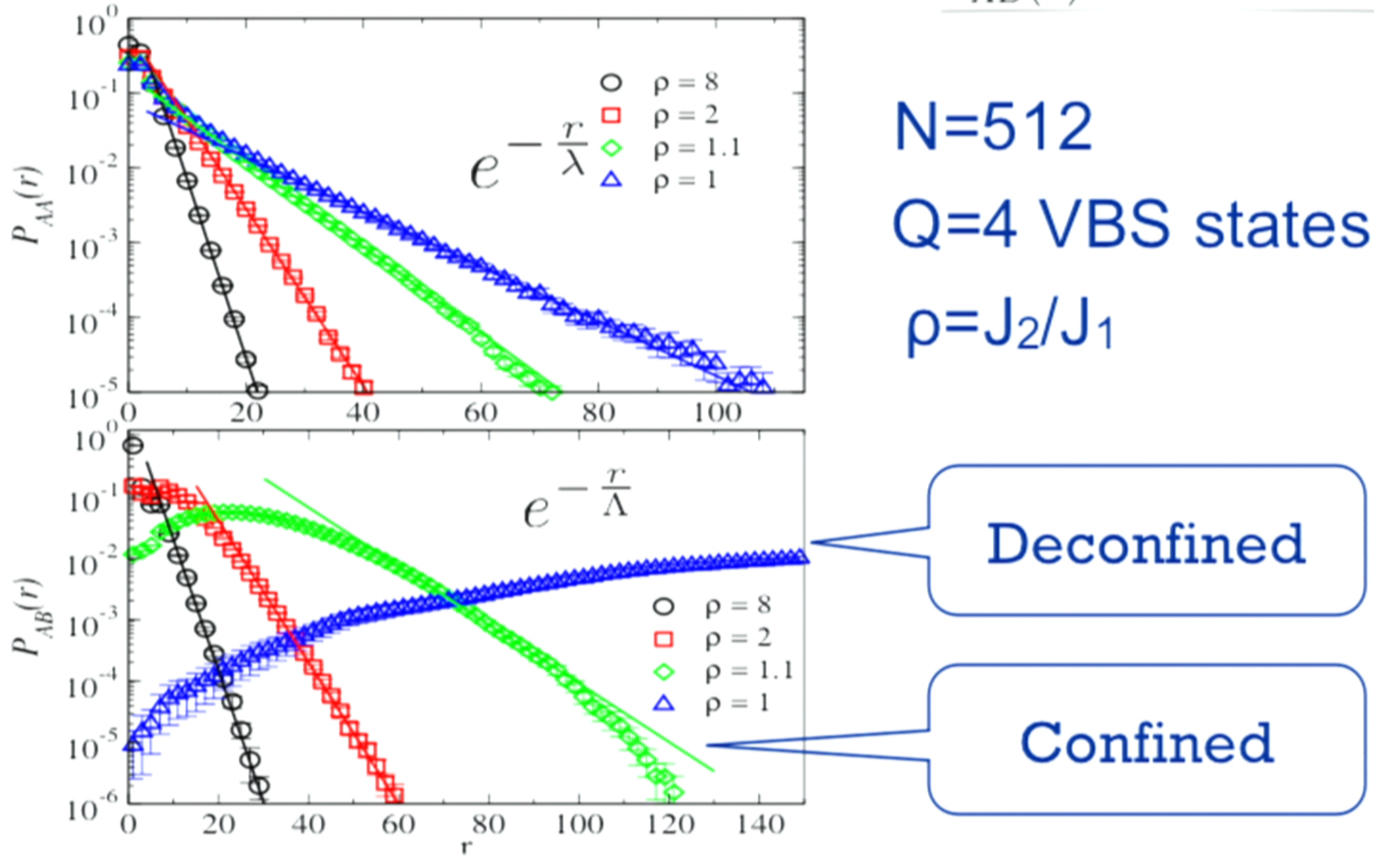


Deconfined Spinons

1D $J_1J_2Q_3$ Chain + Dimerization

$$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$$

$$P_{AB}(r) \propto e^{-|r_{i_A} - r_{i'_B}|/\Lambda}$$



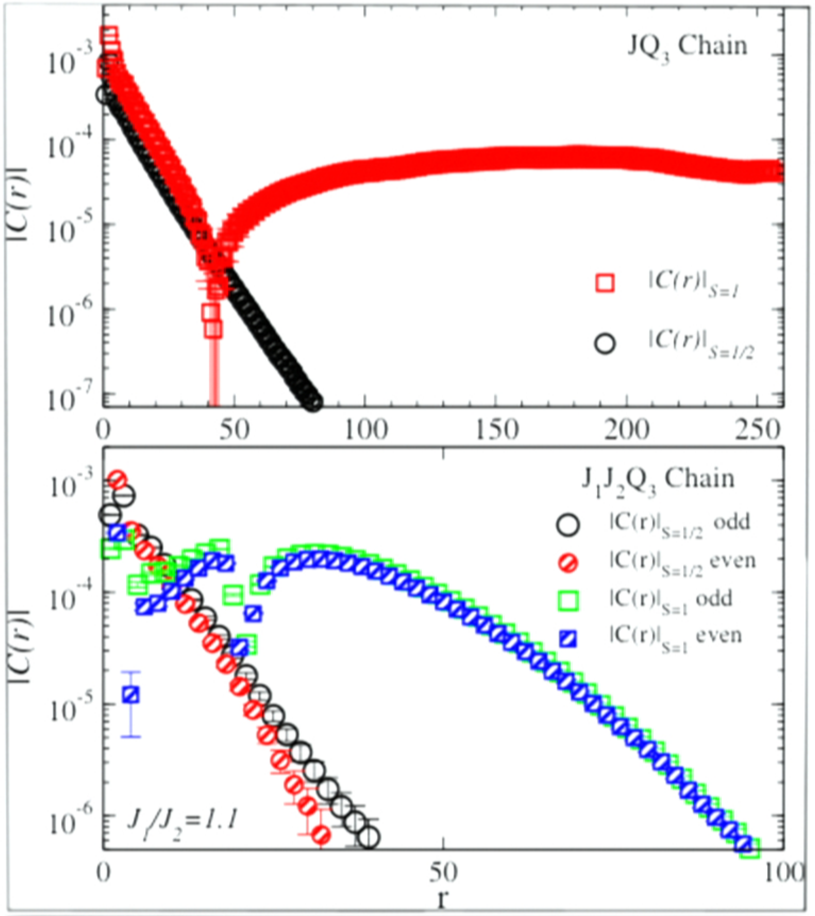
N=512

Q=4 VBS states

ρ=J_2/J_1

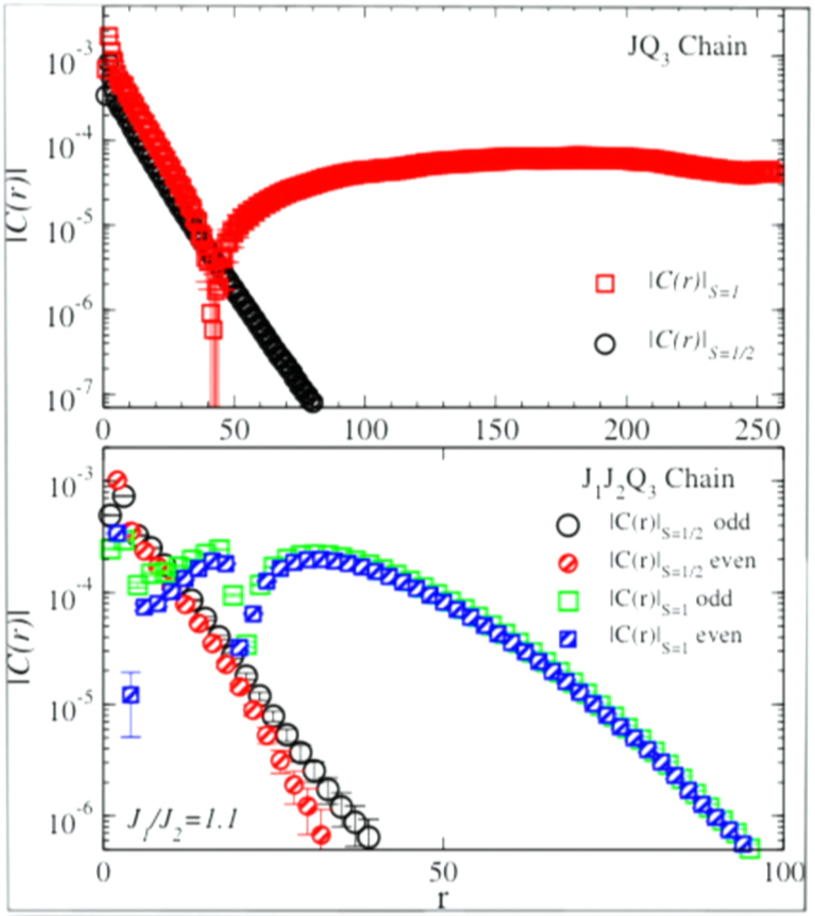
$$H = -\sum_{i=2n+1}^{N/2-1} (J_1 C_{i,i+1} + J_2 C_{i+1,i+2}) - \sum_{i=1}^{N} Q_3(C_{i,i+1}C_{i+2,i+3}C_{i+4,i+5})$$

Differences of Spin-Spin Correlation (z component)



$$|C(r)| = < S_i^z \cdot S_{i+r}^z >_{S=1/2,1} - < S_i^z \cdot S_{i+r}^z >_{S=0}$$

Differences of Spin-Spin Correlation (z component)



$$|C(r)| = \langle S_i^z \cdot S_{i+r}^z \rangle_{S=1/2,1} - \langle S_i^z \cdot S_{i+r}^z \rangle_{S=0}$$

Λ, λ are exhibited in spin correlation functions as well.

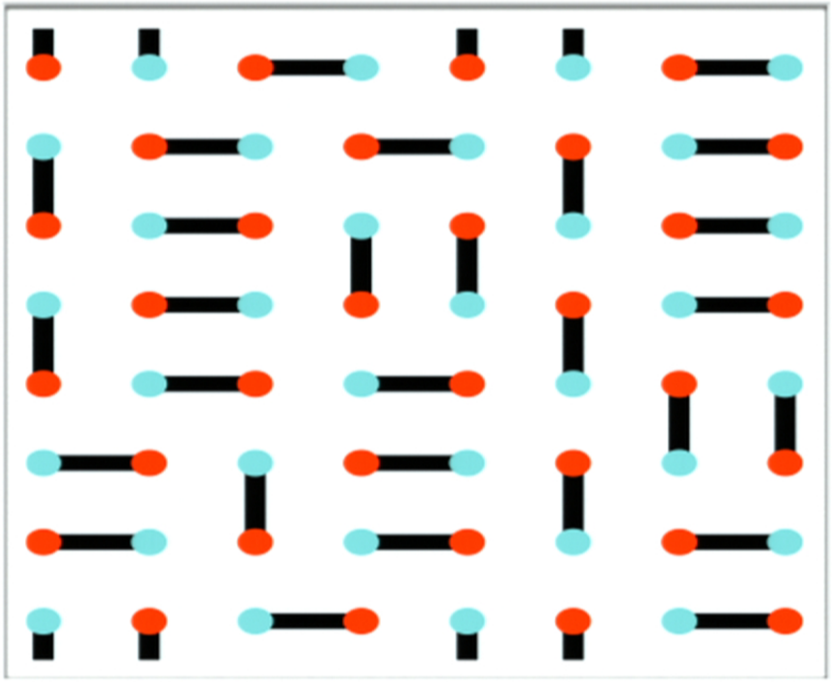
Conclusions of 1D results

- We develop a technique to define spinon size λ and confinement length Λ quantitatively.

- In 1D JQ model, we could observe finite spinon size and deconfinement.

Spinons in 2D

2D Resonating Valence Bond (RVB) Spin Liquid



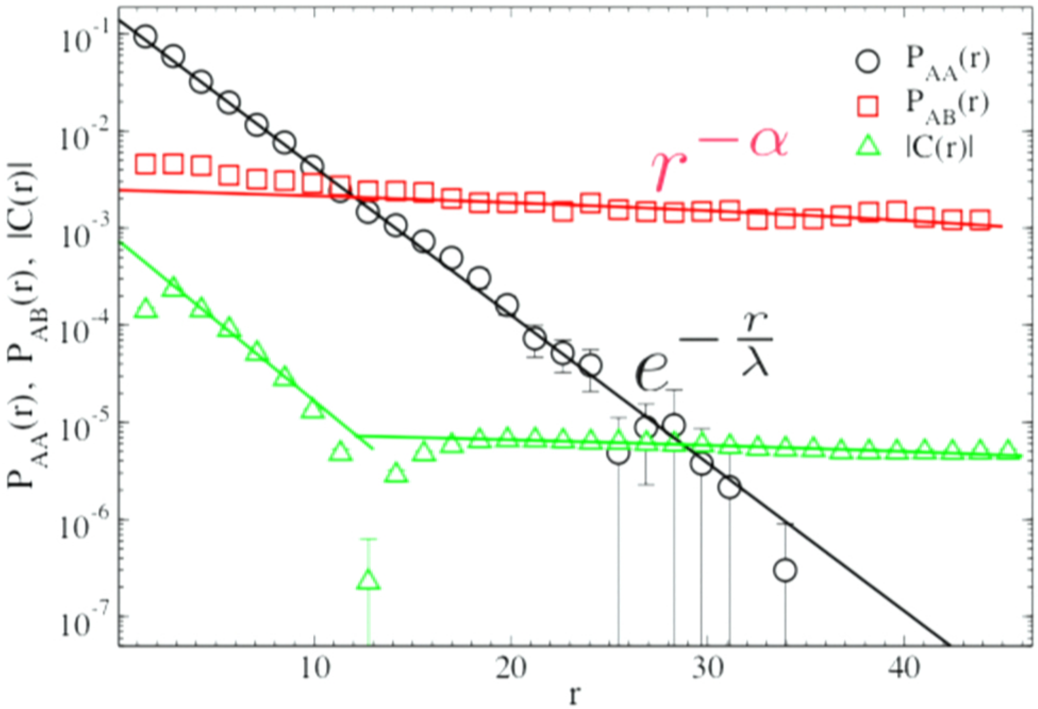
$$|RVB\rangle = \sum_{\alpha} |C_\alpha\rangle$$

Anderson Mater. Res. Bull (1973)
Anderson, Science (1987)
Read and Sachdev, PRL (1991)
⋮
Poilblanc, Lauchli, Mambrini and Mila,PRB(R) (2006)
Cano and Fendley, PRL (2010)
Albuquerque and Alet, PRB (2010)
YT, Sandvik, and Henley, PRB (2011)
Damle, Dhar and Ramola, PRL (2012)
Hu, Kallin,Fendley,Hasting and Melko, PRB (2012)
⋮

2D RVB Spin Liquid

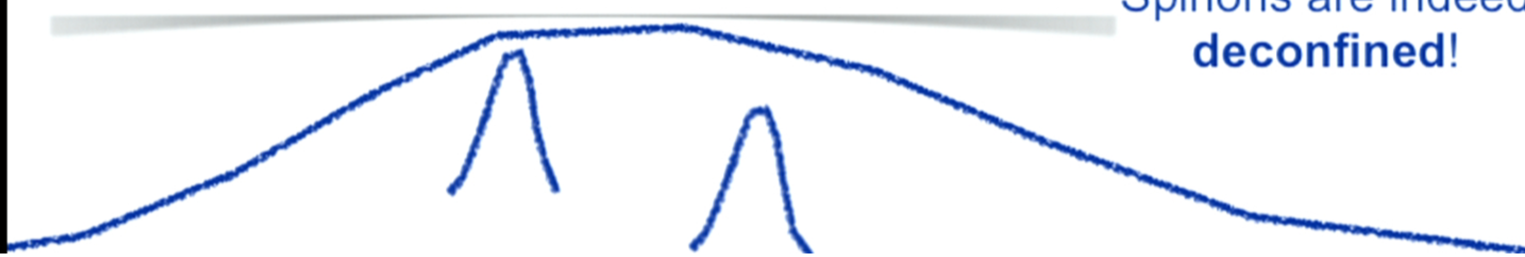
$$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$$

$$P_{AB}(r) \propto e^{-|r_{i_A} - r_{i'_B}|/\Lambda}$$



Λ >> λ

Spinons are indeed **deconfined**!



2D JQ₃ Models

$$H = -\sum_{<i,j>} J C_{i,j} - \sum_{<ijklmn>} Q_3 C_{i,j} C_{k,l} C_{m,n}$$

$$C_{i,j} = 1/4 - \mathbf{S_i} \cdot \mathbf{S_j}$$

2D RVB Spin Liquid

$$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$$

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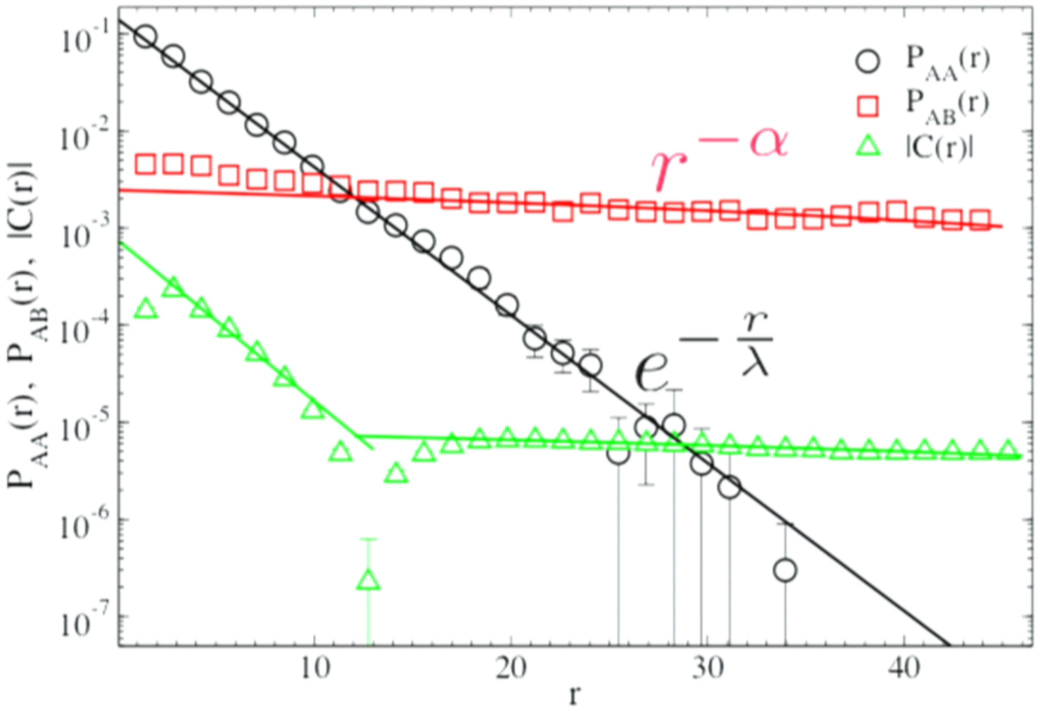


Λ >> λ

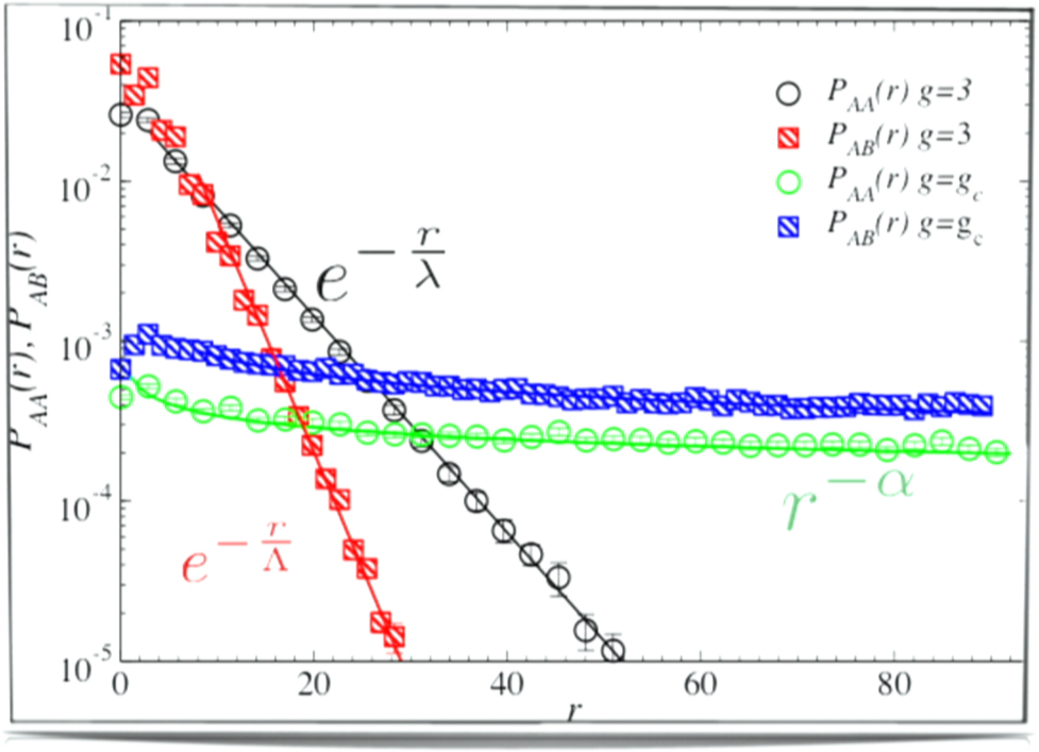
Spinons are indeed **deconfined**!

2D JQ3 Models

YT and Sandvik, arXiv:1301.3207

$$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$$

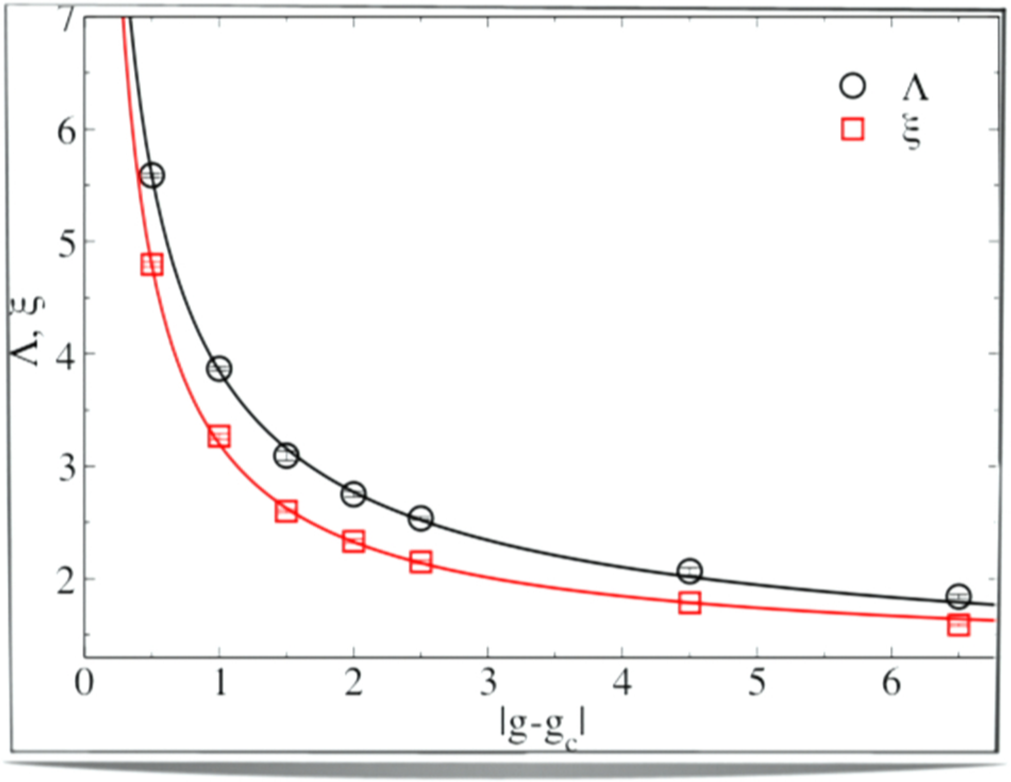
$$P_{AB}(r) \propto e^{-|r_{i_A} - r_{i'_B}|/\Lambda}$$



SPINON SHRINKS IN BOUND STATE !

Also seen in Superconductors: Chaves *et al* PRB (2011) Babaev *et al*, PRL (2009)

2D JQ$_3$ Model



$$\Lambda \propto \xi^{1+a}$$

$$\xi = a+b(g-g_c)^{-\nu}$$

$$\Lambda = a+b(g-g_c)^{-\mu}$$

μ	0.7(1)
ν	0.8(1)

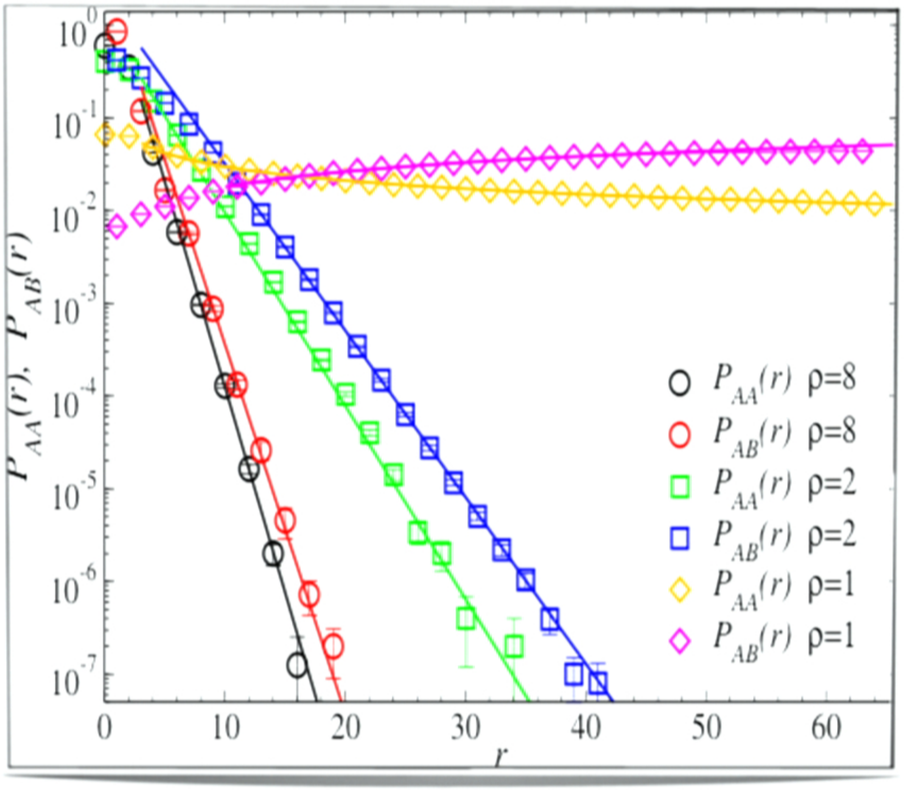
Confined ?

Deconfined ?

1D $J_1J_2Q_3$ Chain + Dimerization

$$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$$

$$P_{AB}(r) \propto e^{-|r_{i_A} - r_{i'_B}|/\Lambda}$$



$Q_3 = Q_c$
at Critical Point

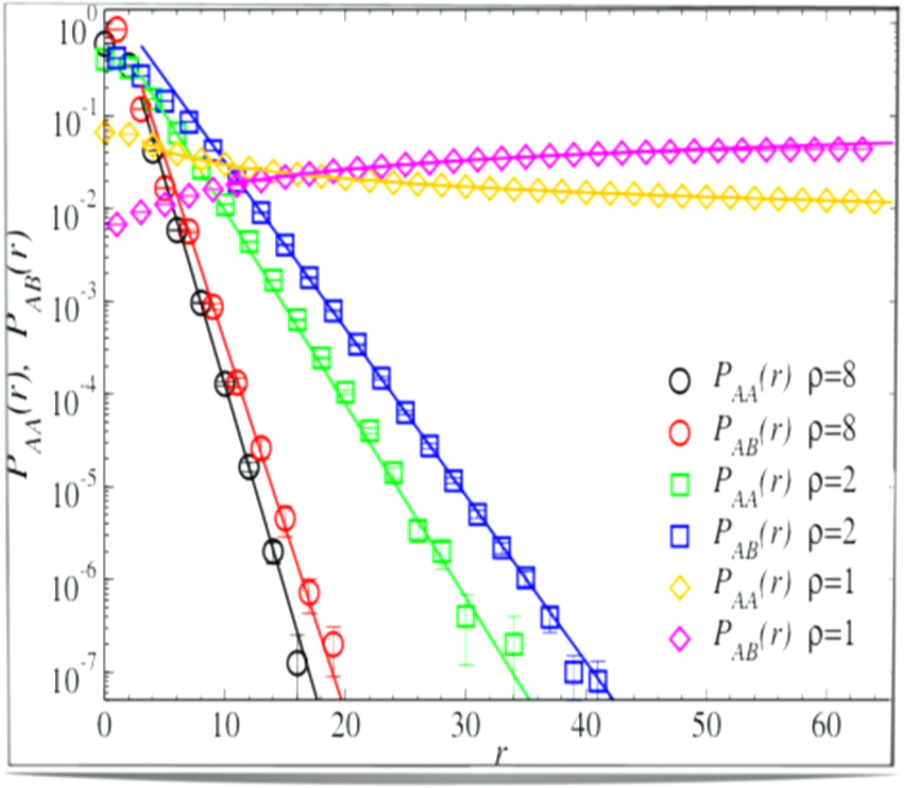
$\rho = J_2/J_1$

$$H = -\sum_{i=2n+1}^{N/2-1} (J_1 C_{i,i+1} + J_2 C_{i+1,i+2}) - \sum_{i=1}^{N} Q_3 (C_{i,i+1} C_{i+2,i+3} C_{i+4,i+5})$$

1D $J_1J_2Q_3$ Chain + Dimerization

$$P_{AA}(r) \propto e^{-|r_{i_A} - r_{i'_A}|/\lambda}$$

$$P_{AB}(r) \propto e^{-|r_{i_A} - r_{i'_B}|/\Lambda}$$



$Q_3=Q_c$
at Critical Point

$\rho=J_2/J_1$

Λ, λ are both diverging in this deconfined case.

Spinons have:

Repulsive interaction in 1D

$$H = -\sum_{i=2n+1}^{N/2-1} (J_1 C_{i,i+1} + J_2 C_{i+1,i+2}) - \sum_{i=1}^{N} Q_3 (C_{i,i+1} C_{i+2,i+3} C_{i+4,i+5})$$

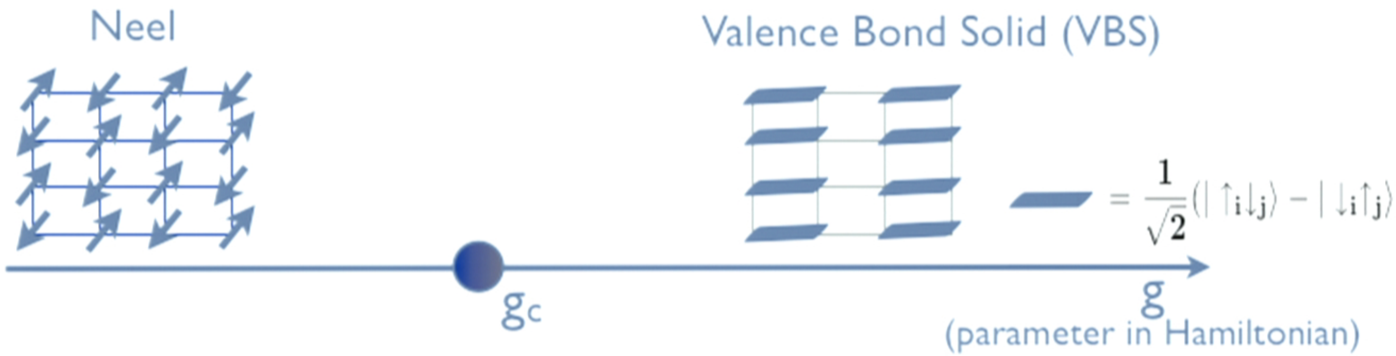
Conclusion

- We develop a technique to define spinon size and confinement length quantitatively.

Motivations

Antiferromagnetic Order ➡ Valence Bond Solid Order

T=0



- **Landau-Ginzburg-Wilson paradigm predicts first order phase transition**
- Numerical results suggest that it is a direct Continuous phase transition.
  - Sandvik, PRL (2007)
  - Lou, Sandvik and Kawashima, PRB (2009)
  - R. Kaul, R. G. Melko, PRB (2008)