Title: A positive and local formalism for quantum theory

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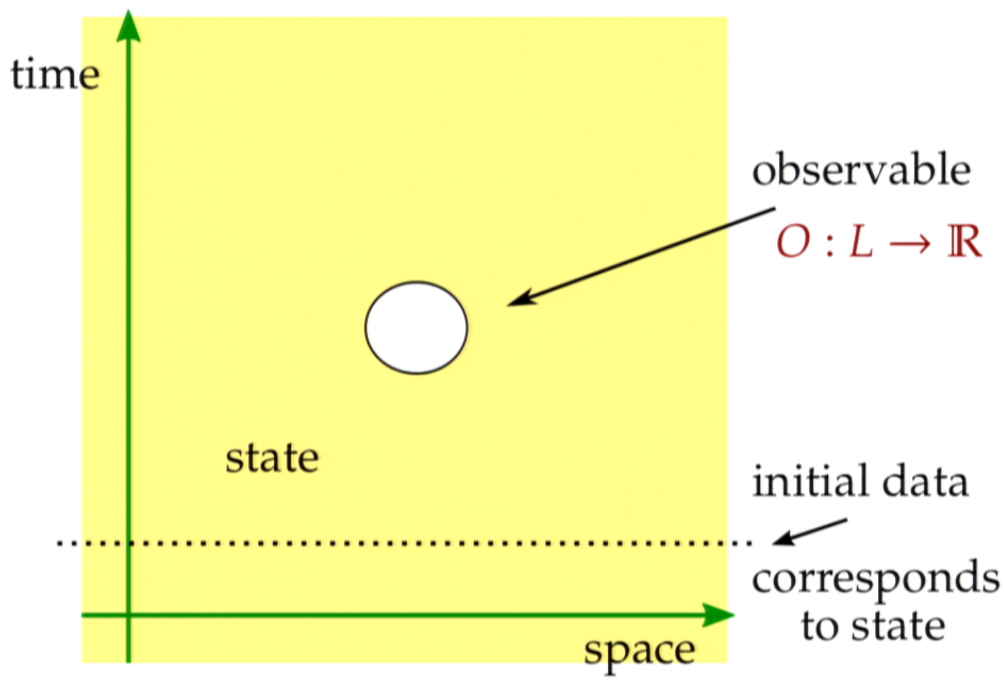
Abstract: The general boundary formulation (GBF) is an atemporal, but spacetime local formulation of quantum theory. Usually it is presented in terms of the amplitude formalism, which, in the presence of a background time, recovers the pure state formalism of the standard formulation of quantum theory. After reviewing the essentials of the amplitude formalism I will introduce a new "positive formalism", which recovers instead a mixed state formalism. This allows to define general quantum operations within the GBF and opens it to quantum information theory. Moreover, the transition to the positive formalism eliminates operationally irrelevant structure, making the extraction of measurement probabilities more direct. As a consequence, the probability interpretation takes on a remarkably simple and compelling form. I shall describe implications of the positive formalism, both for our understanding of quantum theory and for the practical formulation of quantum theories. I also observe a certain convergence with Lucien Hardy's operator tensor formulation of quantum theory, on which I hope to comment

Measurement in classical physics

The system is determined by a dynamical law and a state.



State space L.

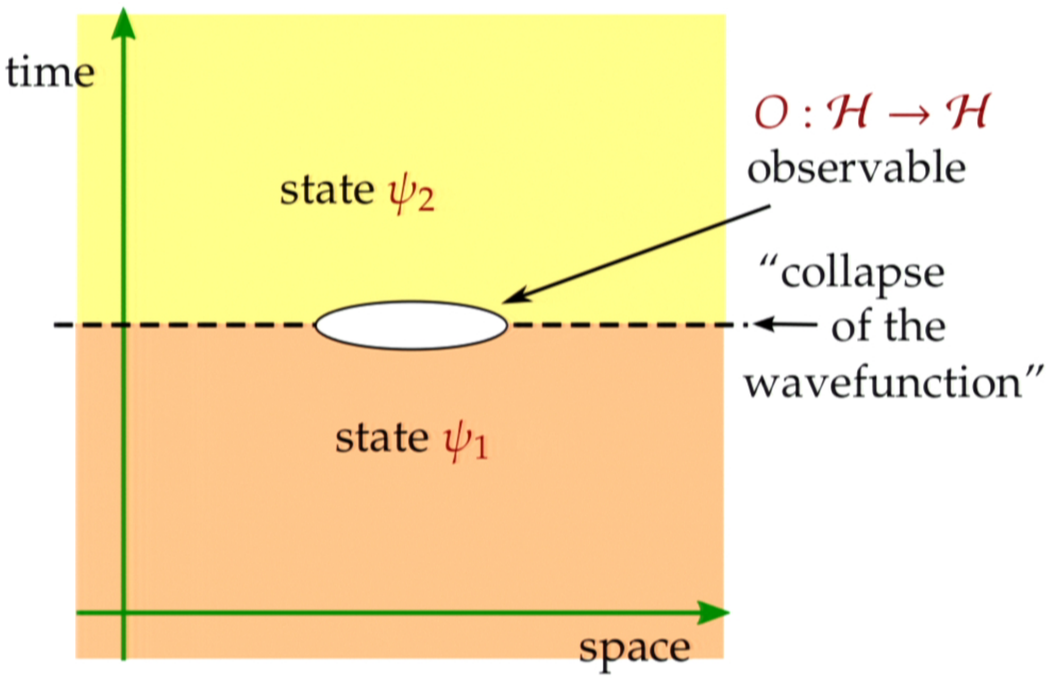
Measurements yield objective information about the state

States are global in spacetime

Measurement in quantum physics I

Standard formulation

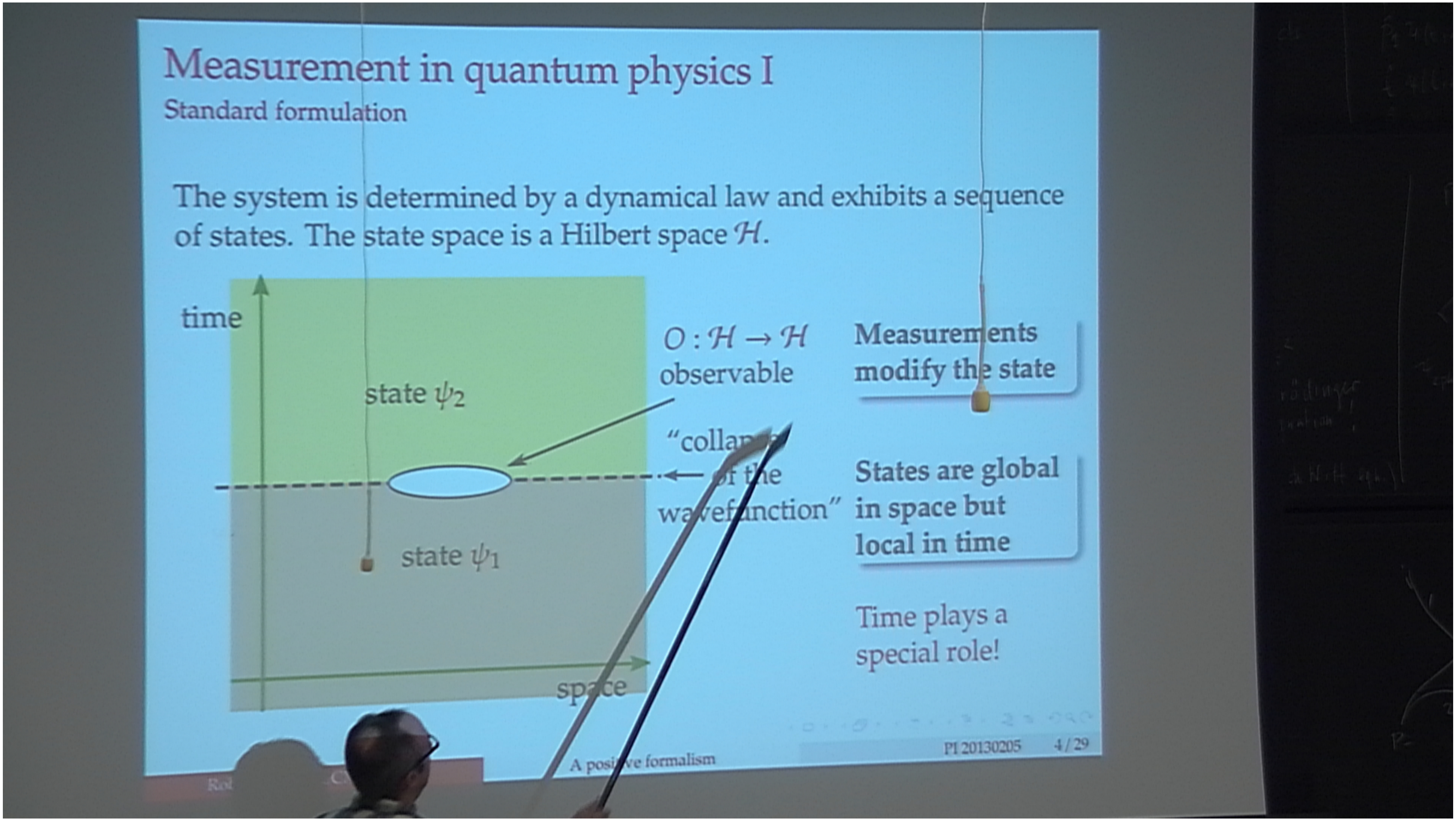
The system is determined by a dynamical law and exhibits a sequence of states. The state space is a Hilbert space $\mathcal{H}$.



Measurements modify the state

States are global in space but local in time

Time plays a special role!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$O : \mathcal{H} \to \mathcal{H}$
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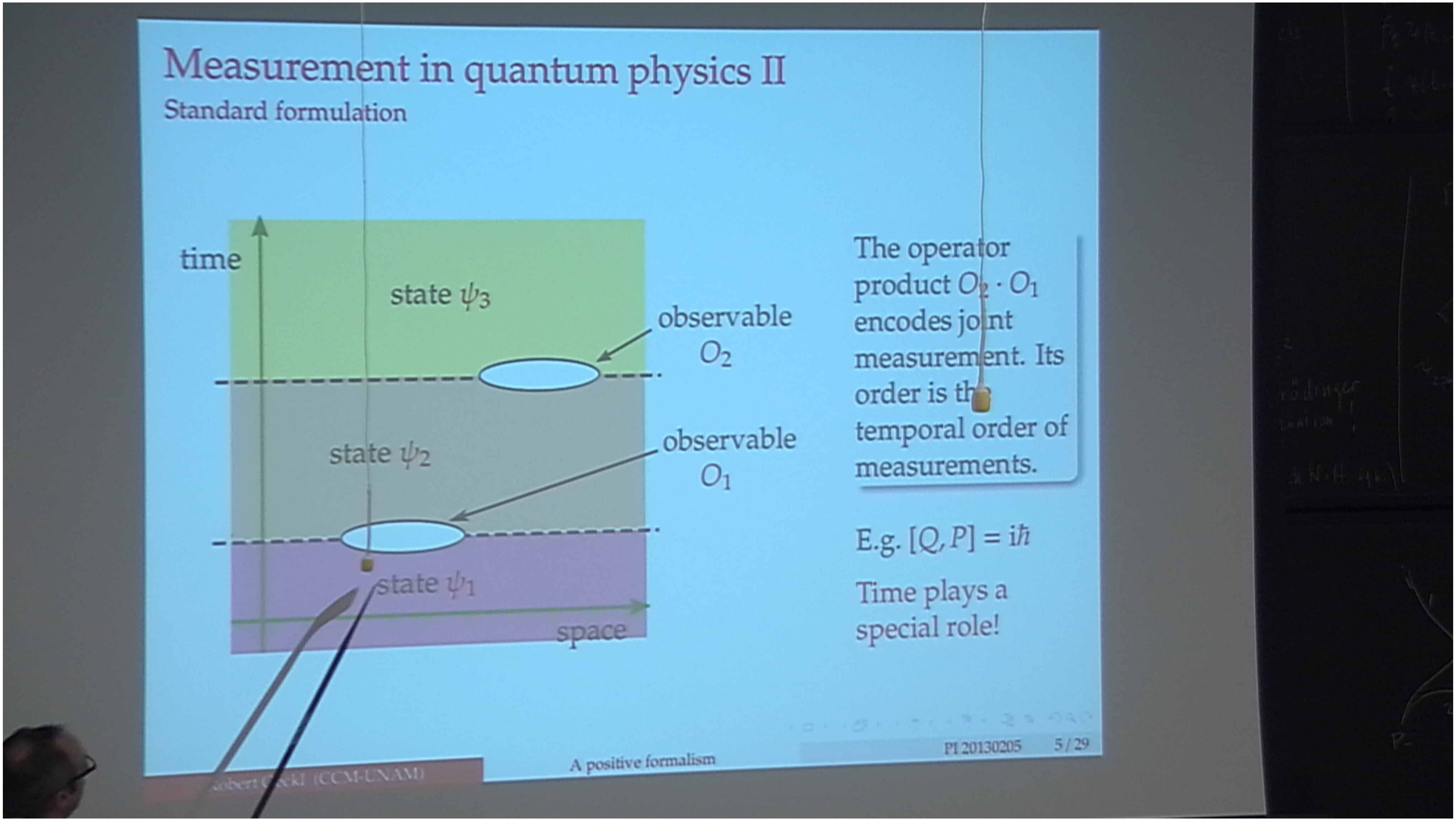
Measurement in quantum physics II
Standard formulation
time
state ψ_3
observable O_2
state ψ_2
observable O_1
state ψ_1
space
The operator product $O_2 \cdot O_1$ encodes joint measurement. Its order is the temporal order of measurements.
E.g. $[Q, P] = i\hbar$
Time plays a special role!
A positive formalism
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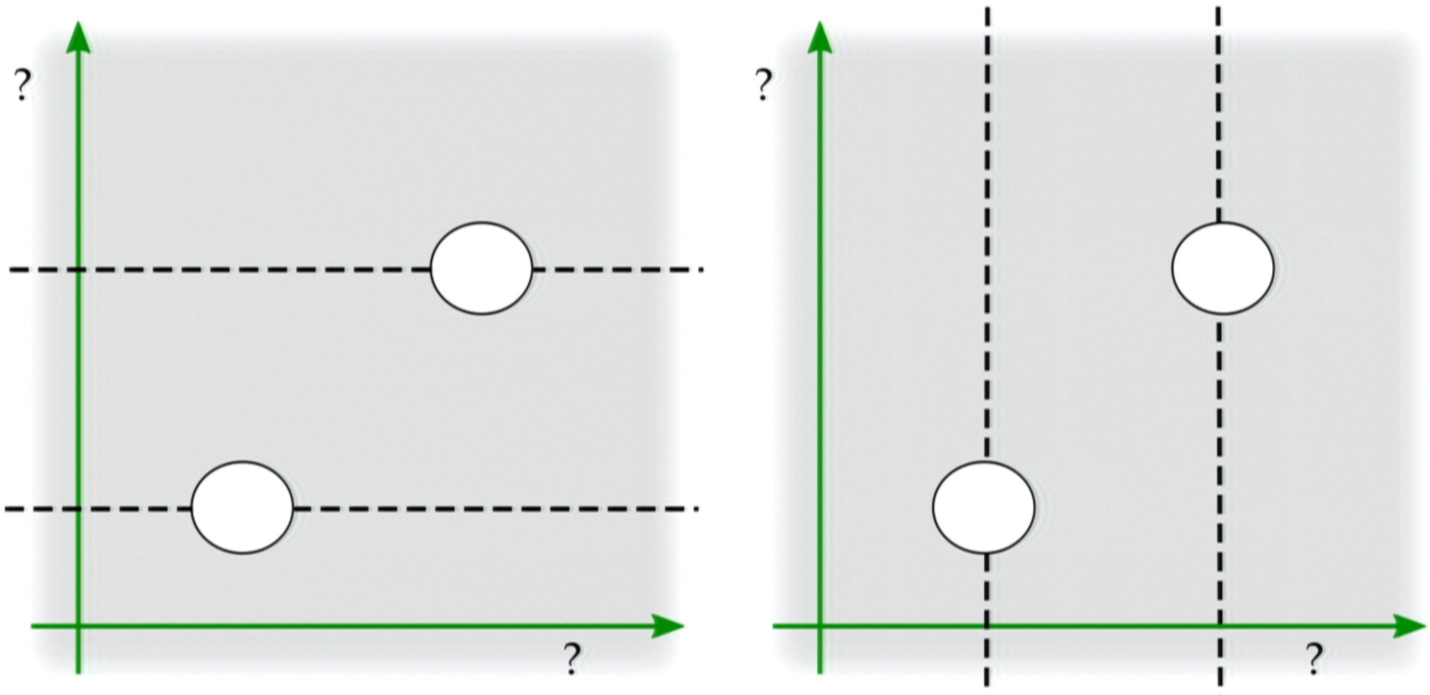
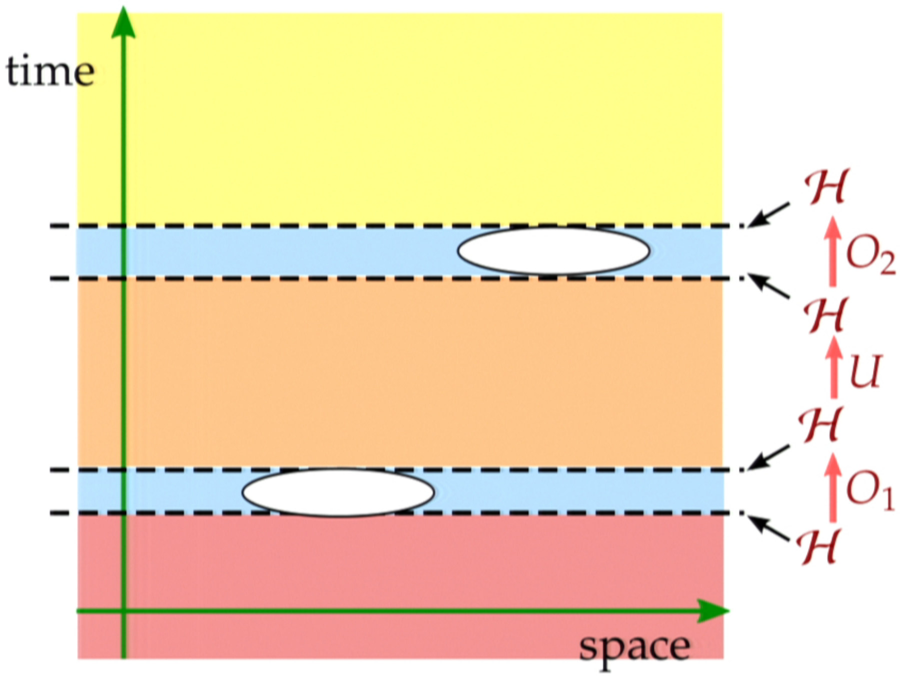
Quantum measurement without spacetime metric?

If spacetime is dynamical, there is no a priori metric "separating" space and time. What do we do?



A closer look at the formalism

Standard formulation

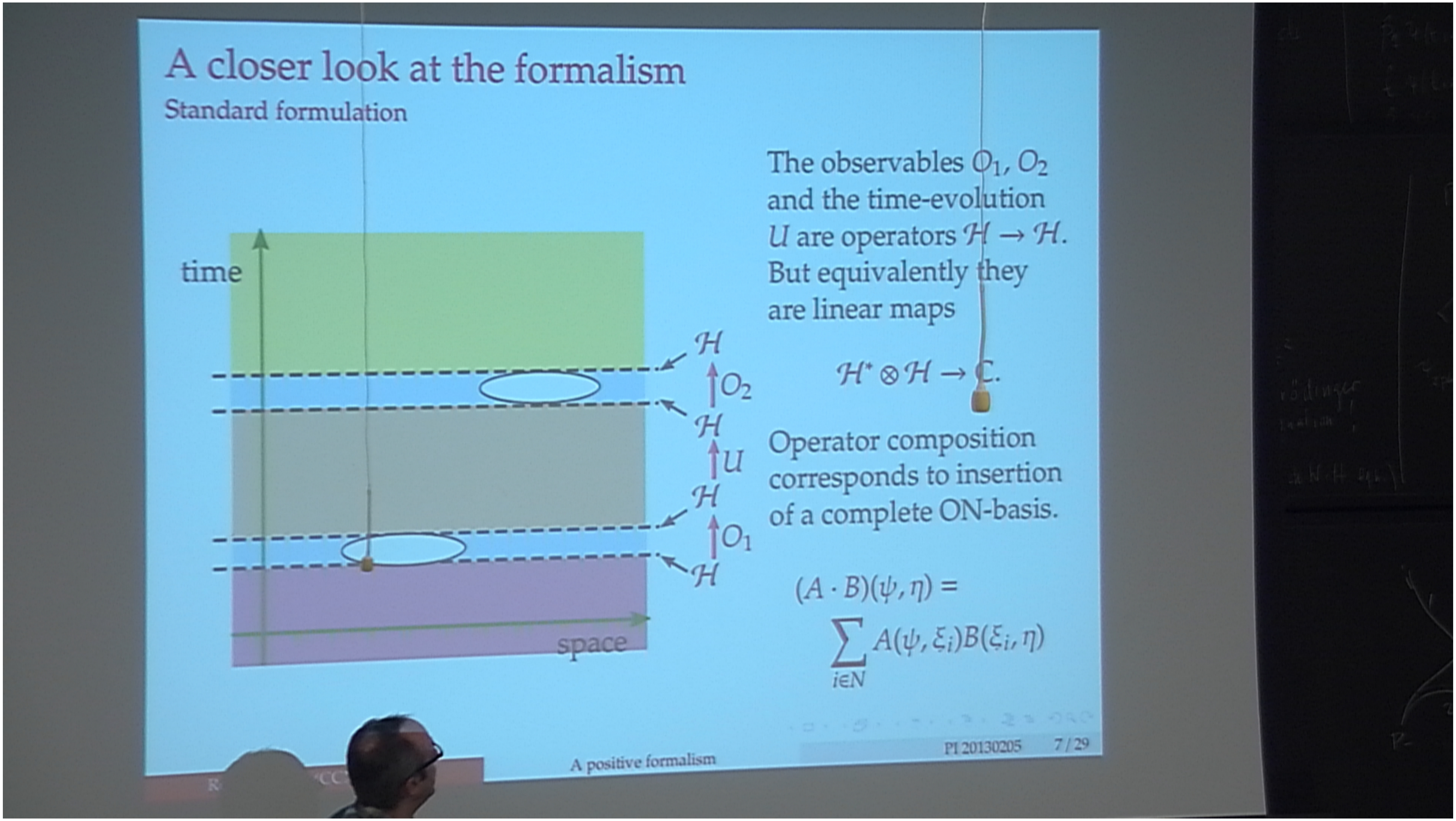


The observables O_1, O_2 and the time-evolution U are operators $\mathcal{H} \to \mathcal{H}$. But equivalently they are linear maps

$$\mathcal{H}^* \otimes \mathcal{H} \to \mathbb{C}.$$

Operator composition corresponds to insertion of a complete ON-basis.

$$(A \cdot B)(\psi, \eta) = \sum_{i \in N} A(\psi, \xi_i) B(\xi_i, \eta)$$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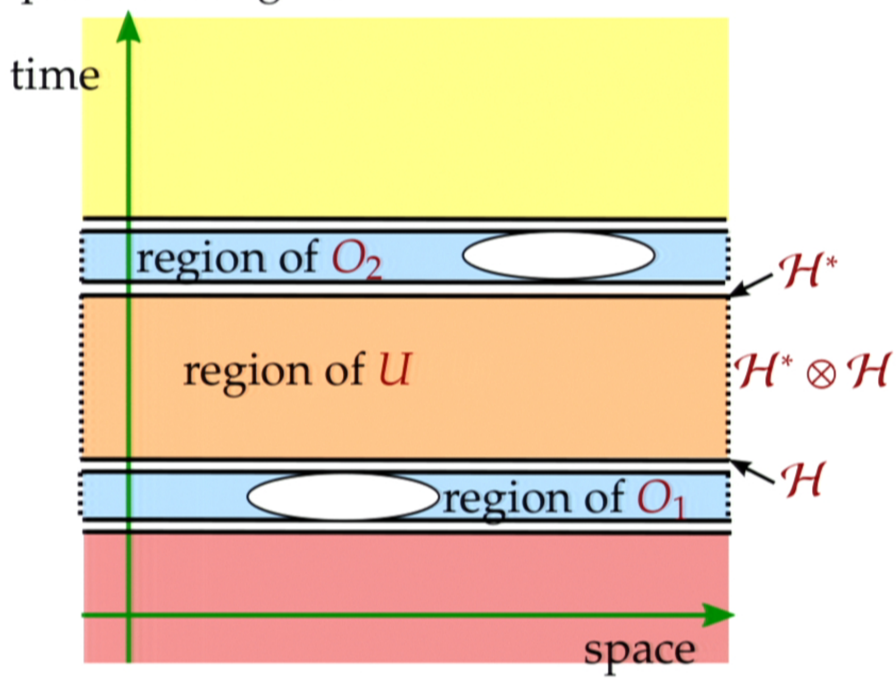
Rewriting the formalism

Standard formulation

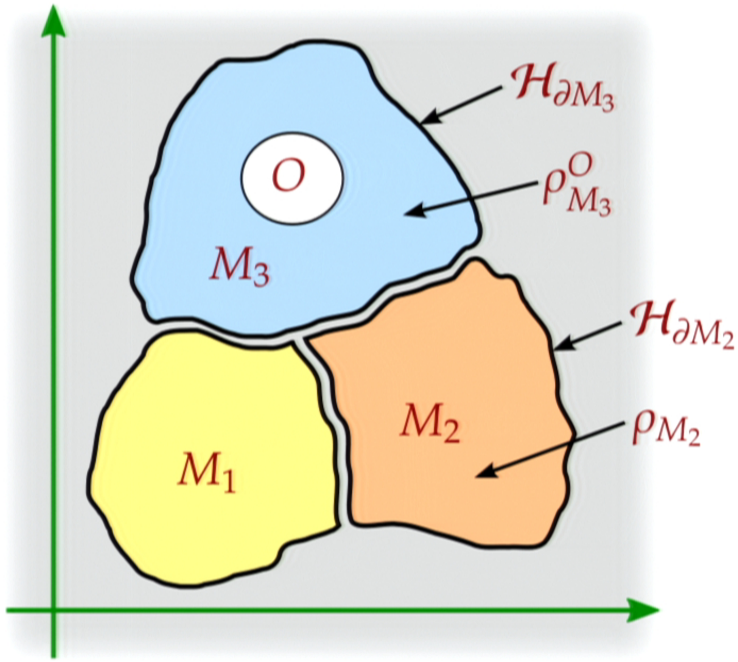
We can think of time-evolution and observables as localized in spacetime regions.



Associate $\mathcal{H}$ or $\mathcal{H}^*$ to each equal-time hypersurface depending on orientation. Associate the tensor product to unions. Then each region's boundary carries $\mathcal{H}^* \otimes \mathcal{H}$ and O_1, O_2, U are maps from this boundary Hilbert space to the complex numbers.

General boundary formulation (GBF)

Amplitude formalism



Generalizing, we associate

- to each **hypersurface** Σ a Hilbert space $\mathcal{H}_\Sigma$
- to each **region** M an **amplitude map** $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$
- to each region M that contains an observable O an **observable map** $\rho^O_M : \mathcal{H}_{\partial M} \to \mathbb{C}$

This is a version of **Topological Quantum Field Theory** [E. Witten, G. Segal, M. Atiyah etc. 1980's].

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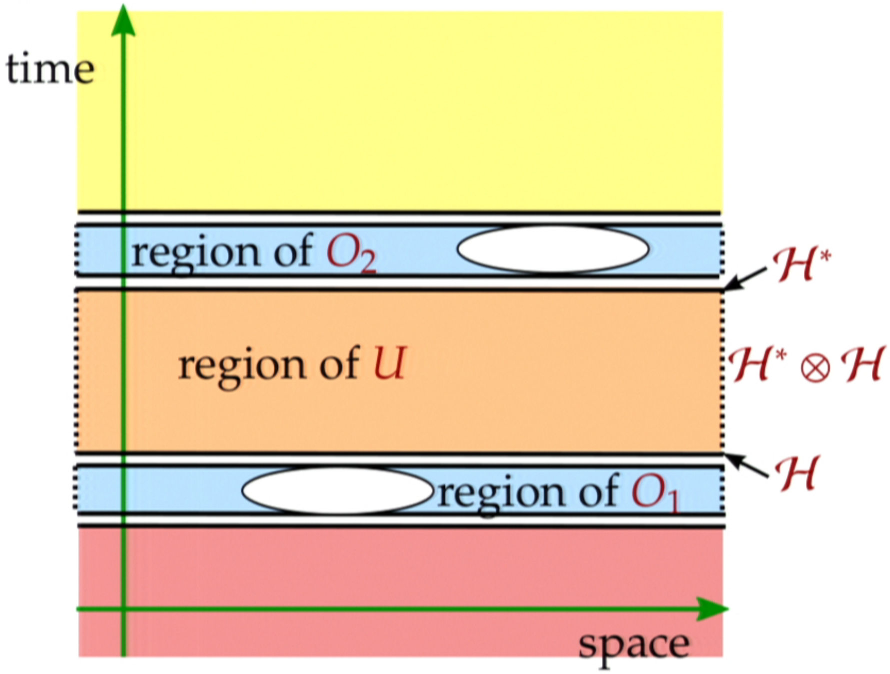

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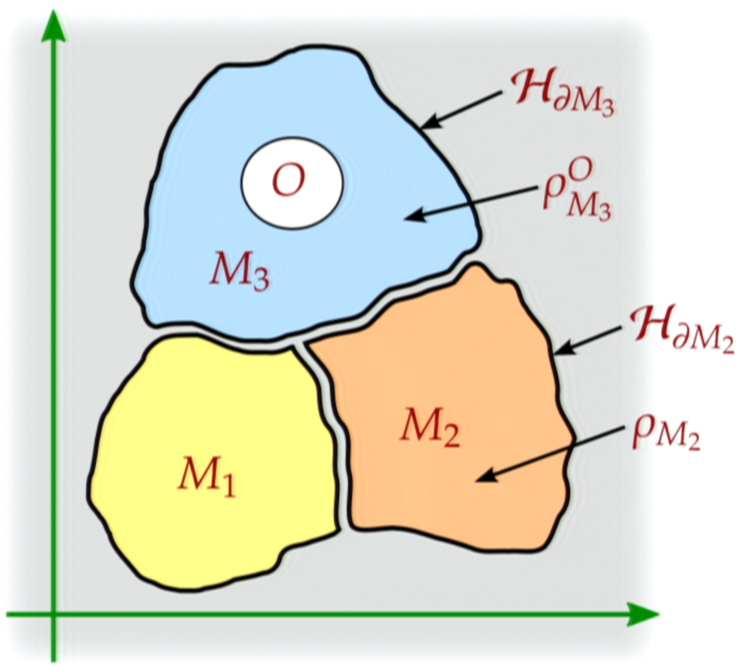

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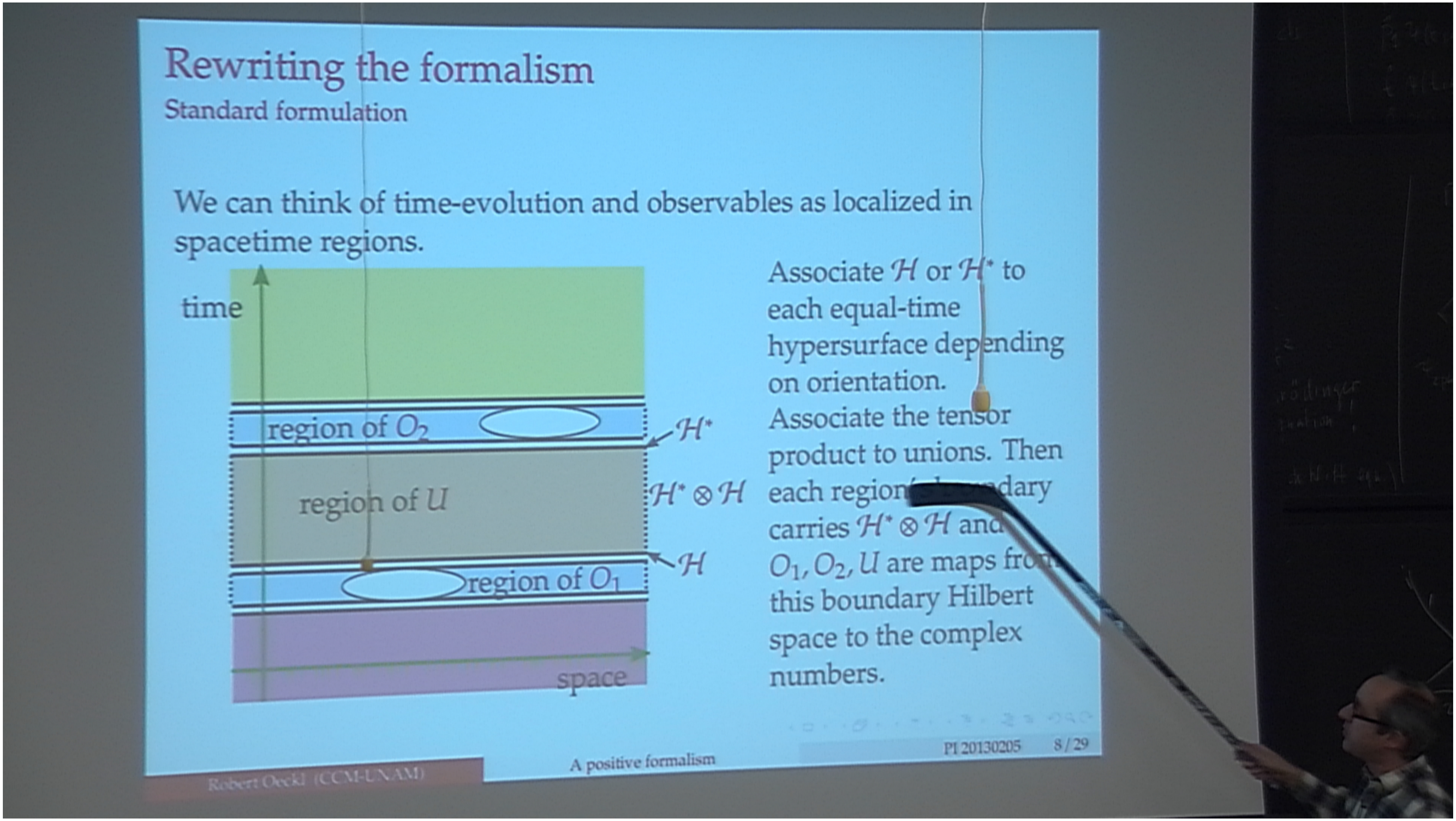
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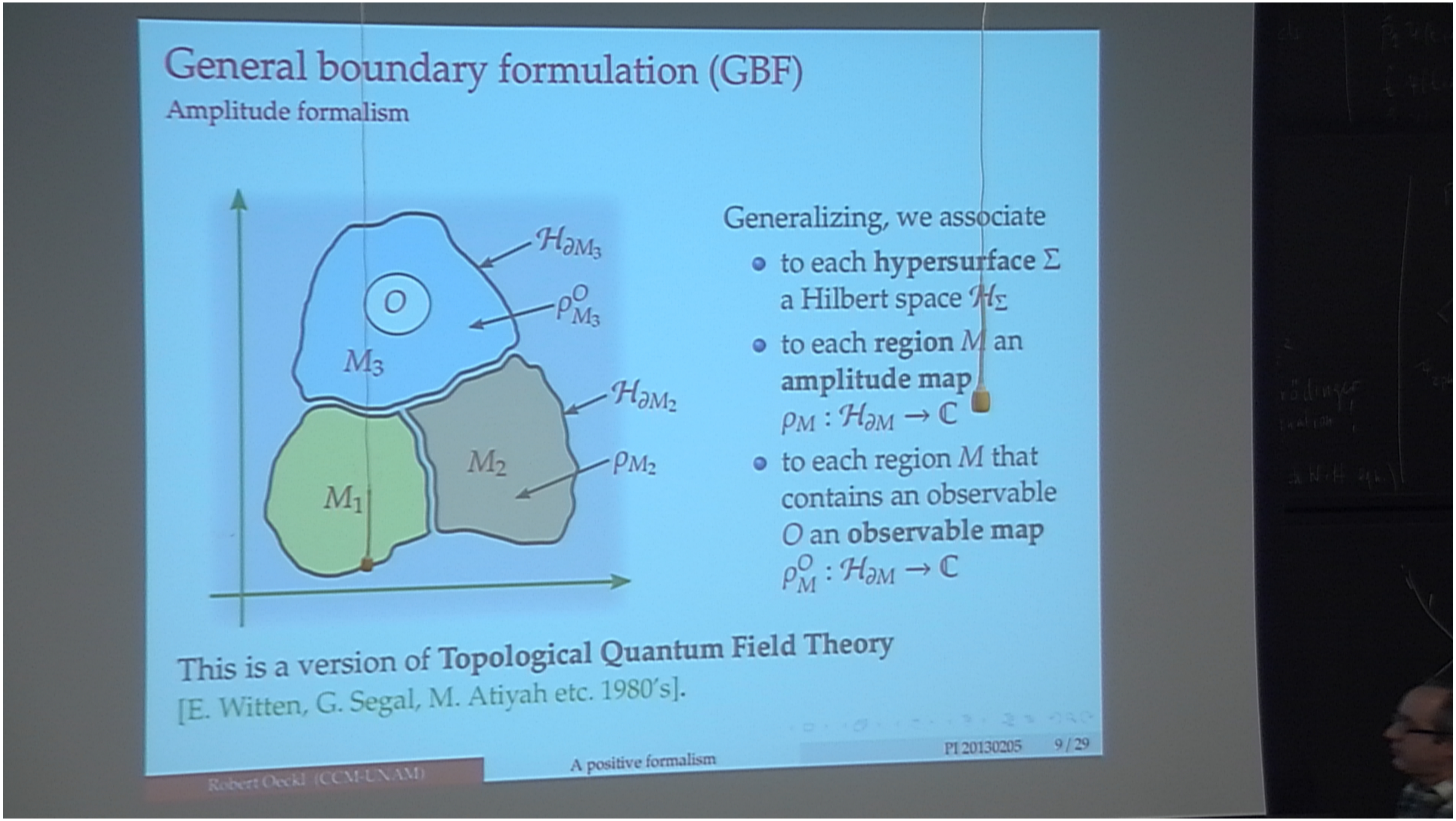
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Rewriting the formalism

Standard formulation

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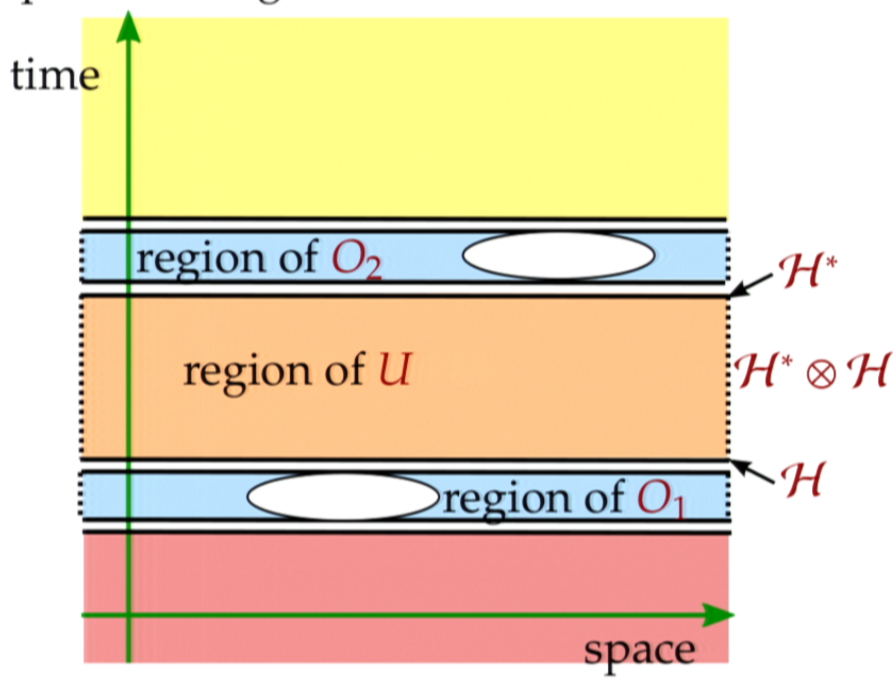

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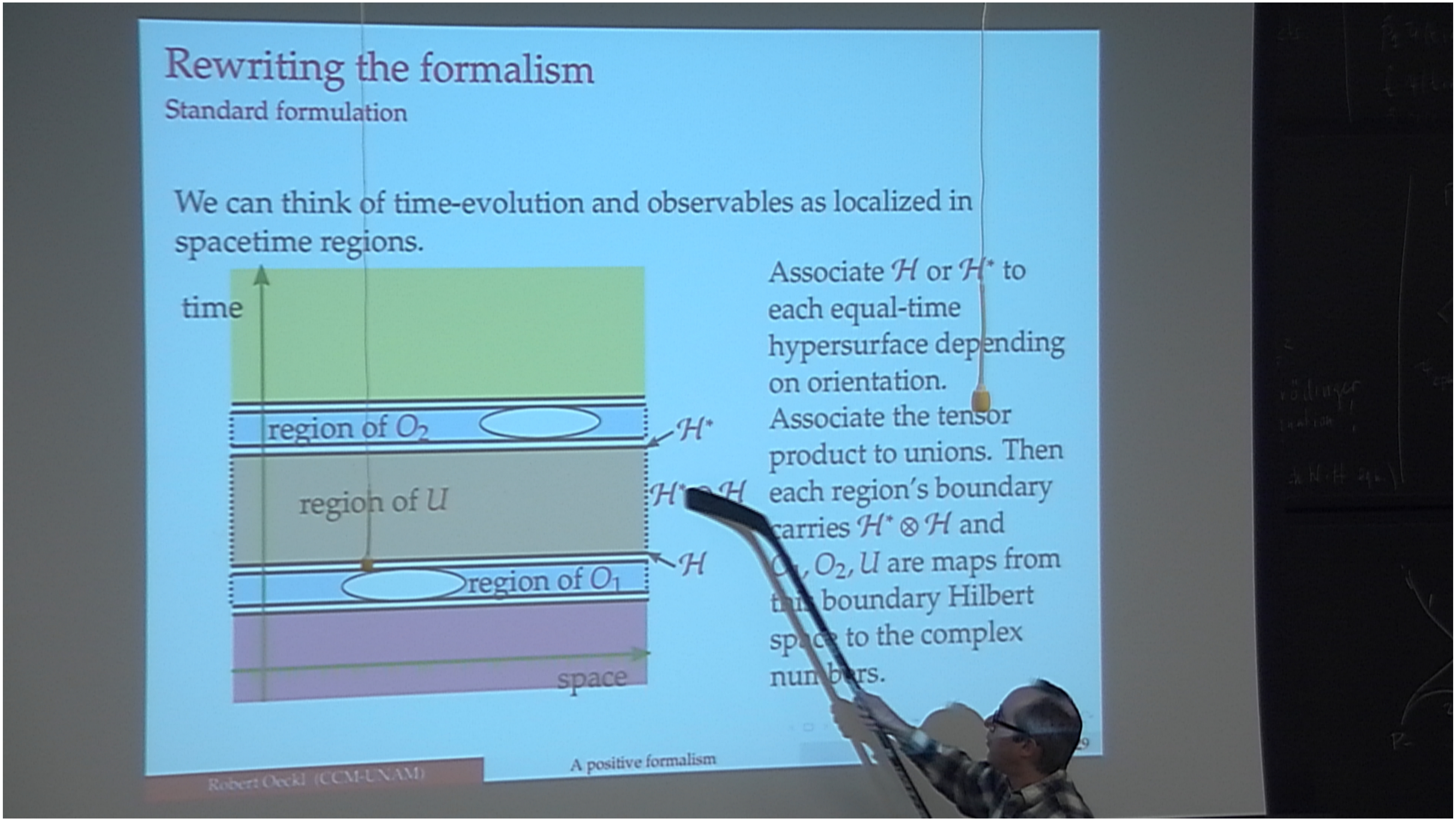
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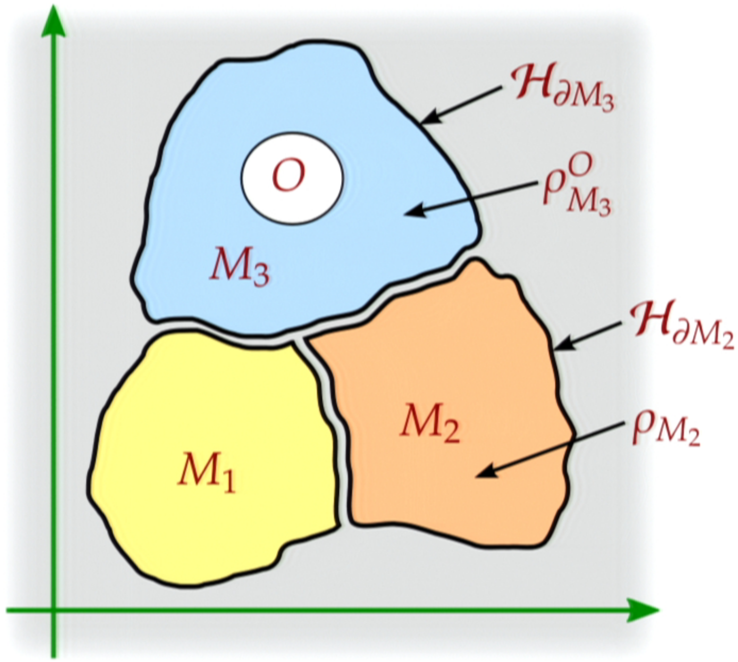

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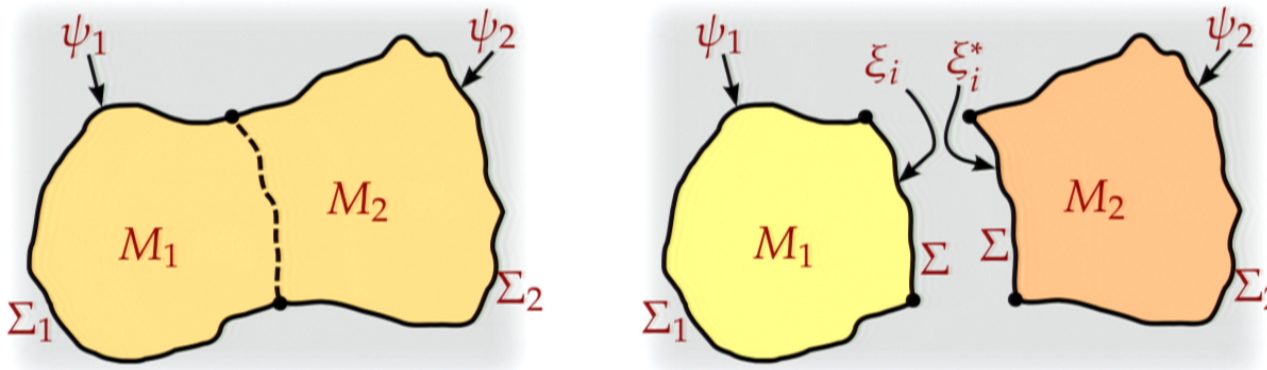
Core axioms

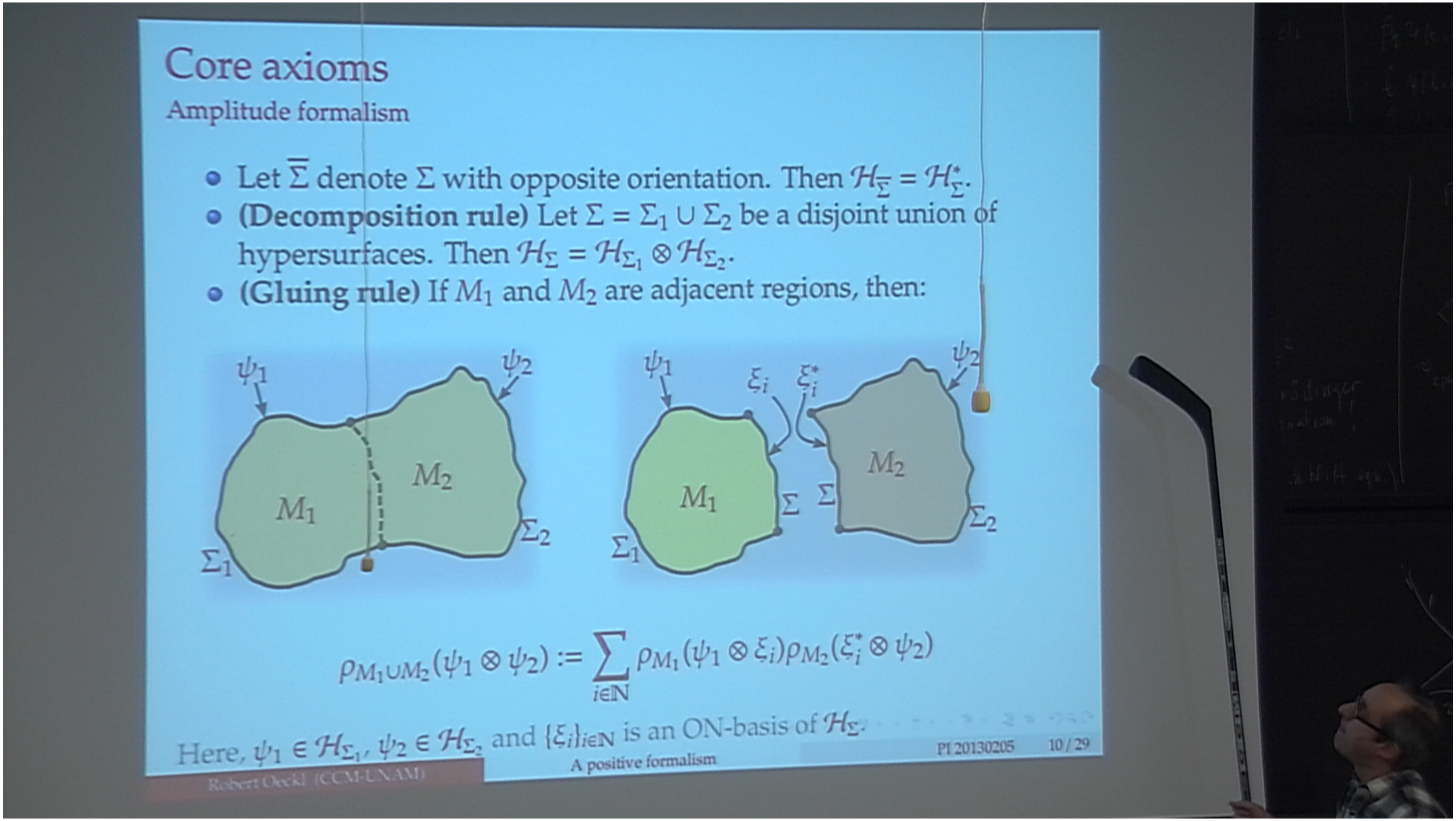
Amplitude formalism

- Let $\overline{\Sigma}$ denote Σ with opposite orientation. Then $\mathcal{H}_{\overline{\Sigma}} = \mathcal{H}_{\Sigma}^*$.
- **(Decomposition rule)** Let $\Sigma = \Sigma_1 \cup \Sigma_2$ be a disjoint union of hypersurfaces. Then $\mathcal{H}_{\Sigma} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\Sigma_2}$.
- **(Gluing rule)** If M_1 and M_2 are adjacent regions, then:



$$\rho_{M_1 \cup M_2}(\psi_1 \otimes \psi_2) := \sum_{i \in \mathbb{N}} \rho_{M_1}(\psi_1 \otimes \xi_i)\rho_{M_2}(\xi_i^* \otimes \psi_2)$$

Here, $\psi_1 \in \mathcal{H}_{\Sigma_1}$, $\psi_2 \in \mathcal{H}_{\Sigma_2}$ and $\{\xi_i\}_{i \in \mathbb{N}}$ is an ON-basis of $\mathcal{H}_{\Sigma}$.



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Probabilities

Amplitude formalism

Consider a spacetime region M. The associated amplitude ρ_M allows to extract probabilities for measurements in M.

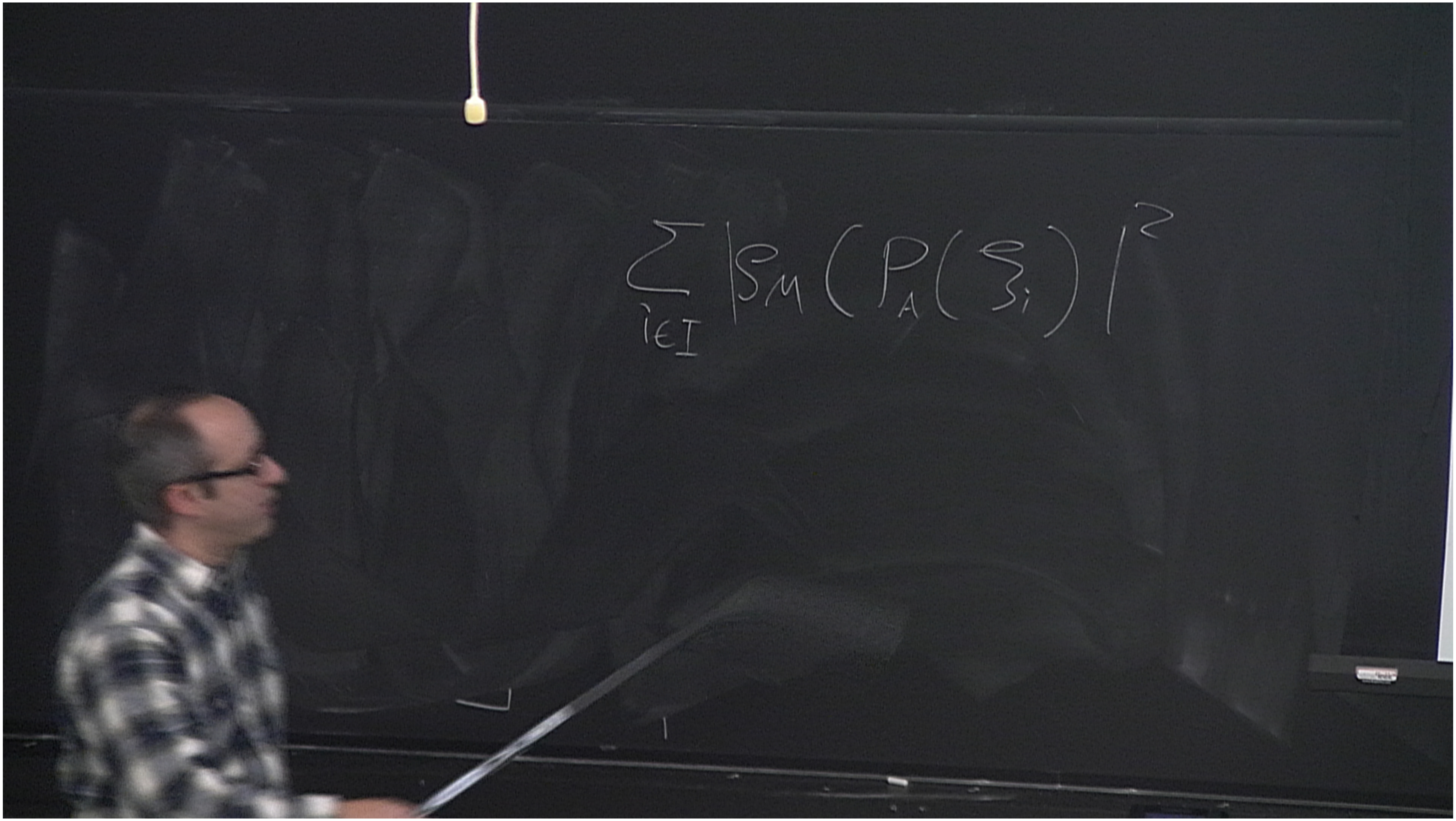
Probabilities in quantum theory are generally conditional probabilities. They depend on two pieces of information. Here these are:

- $\mathcal{S} \subseteq \mathcal{H}_{\partial M}$ representing **preparation** or **knowledge**
- $\mathcal{A} \subseteq \mathcal{H}_{\partial M}$ representing **observation** or the **question**

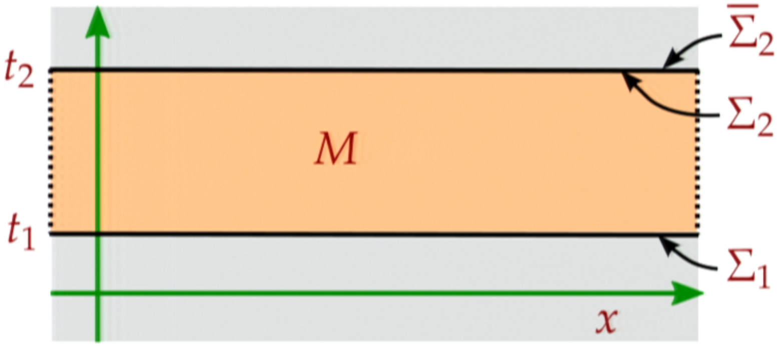
The probability that the physics in M is described by $\mathcal{A}$ given that it is described by $\mathcal{S}$ is: (here $\mathcal{A} \subseteq \mathcal{S}$) [RO 2005]

$$P(\mathcal{A}|\mathcal{S}) = \frac{\sum_{i\in I} \overline{\rho_M(\xi_i)}\, \rho_M(\mathsf{P}_{\mathcal{A}}(\xi_i))}{\sum_{i\in I} \overline{\rho_M(\xi_i)}\, \rho_M(\mathsf{P}_{\mathcal{S}}(\xi_i))}$$

$\mathsf{P}_{\mathcal{S}}$ and $\mathsf{P}_{\mathcal{A}}$ are the orthogonal projectors onto the subspaces $\mathcal{S}$ and $\mathcal{A}$; $\{\xi_i\}_{i\in I}$ an ON-basis of $\mathcal{H}_{\partial M}$.


$\sum_{i \in I} |\rho_M(P_A(\xi_i))|^2$

Recovering transition amplitudes and probabilities



- region: $M = [t_1, t_2] \times \mathbb{R}^3$
- boundary: $\partial M = \Sigma_1 \cup \overline{\Sigma}_2$
- state space: $\mathcal{H}_{\partial M} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}_{\overline{\Sigma}_2} = \mathcal{H}_{\Sigma_1} \otimes \mathcal{H}^*_{\Sigma_2}$

As before, we identify $\mathcal{H}_{\Sigma_1} \cong \mathcal{H}_{\Sigma_2} \cong \mathcal{H}$. Then,

$$\rho_{[t_1,t_2]}(\psi_1 \otimes \psi_2^*) = \langle \psi_2, U(t_1, t_2)\psi_1 \rangle.$$

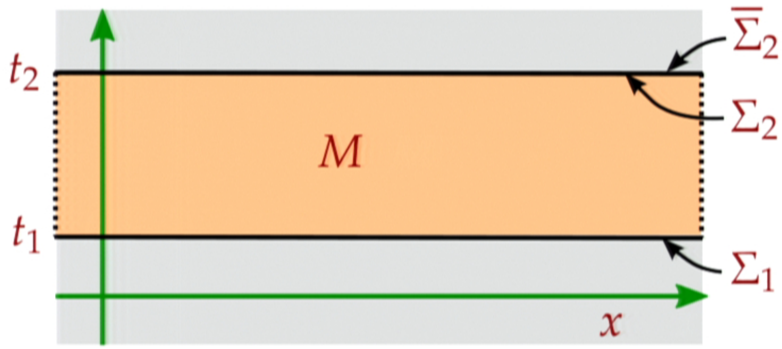
To compute the probability of measuring ψ_2 at t_2 given that we prepared ψ_1 at t_1 we set

$$\mathcal{S} = \psi_1 \otimes \mathcal{H}^*, \quad \mathcal{A} = \mathcal{H} \otimes \psi_2^*.$$

The resulting expression recovers precisely the transition probability

$$P(\mathcal{A}|\mathcal{S}) = |\langle \psi_2, U(t_1, t_2)\psi_1 \rangle|^2.$$

Recovering transition amplitudes and probabilities



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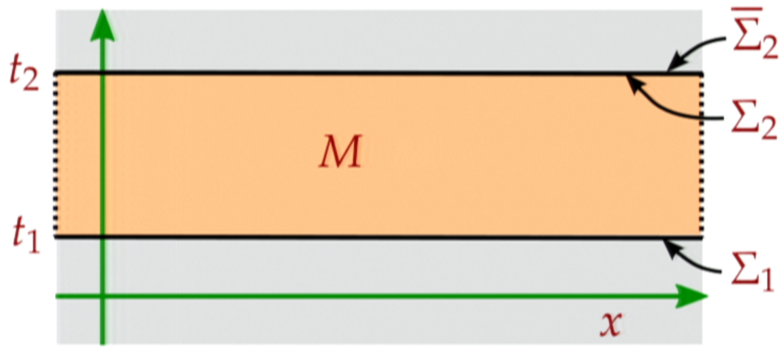

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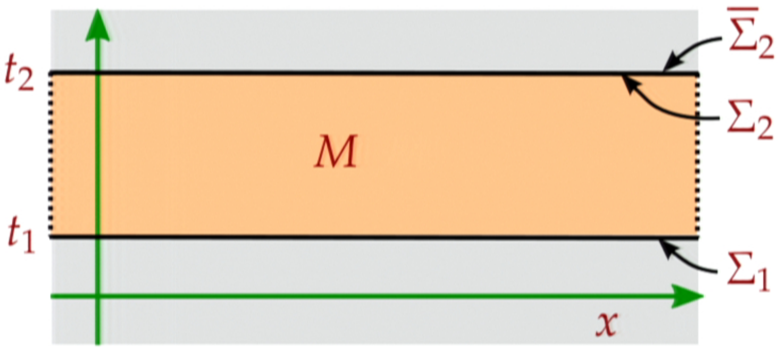

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Observables and expectation values

Amplitude formalism

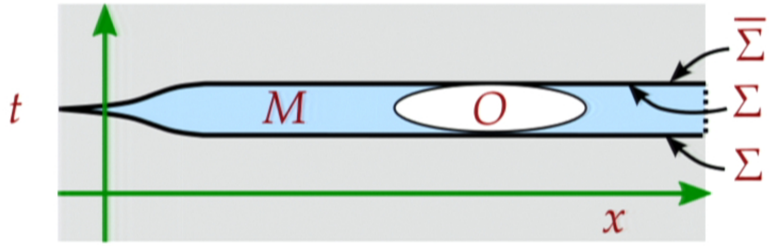
Consider a spacetime region M carrying an observable O. The associated observable map ρ_M^O allows to extract expectation values for measurements in M.

The **expectation value** of the observable O conditional on the system being prepared in the subspace $\mathcal{S} \subseteq \mathcal{H}_\Sigma$ can be represented as follows: [RO 2010]

$$\langle O \rangle_{\mathcal{S}} = \frac{\sum_{i \in I} \overline{\rho_M(\xi_i)}\, \rho_M^O(\mathsf{P}_{\mathcal{S}}(\xi_i))}{\sum_{i \in I} \overline{\rho_M(\xi_i)}\, \rho_M(\mathsf{P}_{\mathcal{S}}(\xi_i))}$$

$\mathsf{P}_{\mathcal{S}}$ is the orthogonal projector onto the subspace $\mathcal{S}$; $\{\xi_i\}_{i \in I}$ an ON-basis of $\mathcal{H}_{\partial M}$.

Recovering standard expectation values



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To compute the expectation value of observable O at time t given by

$$\rho^O_{[t,t]}(\psi_1 \otimes \psi_2^*) = \langle \psi_2, \hat{O}\psi_1 \rangle$$

in the state ψ we set

$$\mathcal{S} = \psi \otimes \mathcal{H}_\Sigma^*.$$

The standard expectation value is then correctly recovered as

$$\langle O \rangle_\mathcal{S} = \langle \psi, \hat{O}\psi \rangle.$$

A remark on fermions

The amplitude formalism in the form presented so far only applies to bosonic theories. In the presence of fermionic degrees of freedom certain modifications apply [RO 2012]:

- All structures are equipped with a $\mathbb{Z}_2$-grading that distinguishes even and odd fermion number.
- Hilbert spaces are replaced by **Krein spaces**. These are indefinite inner product spaces decomposing into a positive definite and negative definite part.

$$\mathcal{H}_\Sigma = \mathcal{H}_\Sigma^+ \oplus \mathcal{H}_\Sigma^-$$

The reason that these Krein spaces are "invisible" in ordinary QFT has to do with the restriction to spacelike hypersurfaces and to a global choice of time orientation.

For simplicity I will continue to restrict my attention to the purely bosonic case.

Applications of the amplitude formalism (AF)

- By restricting to spacetimes with spacelike foliations the standard formulation **is reproduced exactly**. [RO 2005; 2010]
- **Three dimensional quantum gravity** is already formulated as a TQFT and fits thus "automatically" into the AF.
- (Part of) the AF is extensively used in **spin foam quantum gravity**. [C. Rovelli et al.]
- A natural testing ground for the GBF is **quantum field theory**.
  - State spaces on timelike hypersurfaces and "evolution" in spacelike directions. [RO 2005]
  - New **S-matrix** type asymptotic amplitudes in Minkowski space, deSitter space, Anti-deSitter space. [D. Colosi, RO 2008; D. Colosi 2009; M. Dohse 2011; 2012]
  - **Quantum Yang-Mills theory** in 2 dimensions for arbitrary regions and hypersurfaces with corners. [RO 2006]
  - Rigorous and functorial quantization of linear and affine field theories without metric background. [RO 2010; 2011; 2012]
  - **Unruh effect**. [D. Colosi, D. Rätzel 2012]

Probabilities and expectation values

Positive formalism

Given a region M and subspaces $\mathcal{A} \subseteq \mathcal{S} \subseteq \mathcal{H}_{\partial M}$ we have $\mathsf{P}_{\mathcal{A}}, \mathsf{P}_{\mathcal{S}} \in \mathcal{D}_{\partial M}$. The probability for measuring $\mathcal{A}$ given $\mathcal{S}$ is,

$$\boxed{P(\mathcal{A}|\mathcal{S}) = \frac{A_M(\mathsf{P}_{\mathcal{A}})}{A_M(\mathsf{P}_{\mathcal{S}})}}$$

Given a region M carrying an observable O and given a subspace $\mathcal{S} \subseteq \mathcal{H}_{\partial M}$, the corresponding expectation value is,

$$\boxed{\langle O \rangle_{\mathcal{S}} = \frac{A_M^O(\mathsf{P}_{\mathcal{S}})}{A_M(\mathsf{P}_{\mathcal{S}})}}$$

This looks much simpler than in the amplitude formalism. . .

Realness and positivity

Positive formalism

. . . but it is also more natural!

- Consider the subset $\mathcal{D}^{\mathbb{R}}_{\Sigma} \subseteq \mathcal{D}_{\Sigma}$ of **self-adjoint** operators. This is a real vector space and $\mathcal{D}_{\Sigma}$ is its complexification.
- Consider the subset $\mathcal{D}^{+}_{\Sigma} \subseteq \mathcal{D}^{\mathbb{R}}_{\Sigma}$ of **positive** operators. This forms a generating proper cone in the real vector space $\mathcal{D}^{\mathbb{R}}_{\Sigma}$ making it into an **ordered vector space**.
- The orthogonal projection operators form a **lattice** in $\mathcal{D}^{\mathbb{R}}_{\Sigma}$. This is equivalent the lattice of closed subspaces of $\mathcal{H}_{\Sigma}$. That is,

$$\mathsf{P}_{\mathcal{A}_1} \le \mathsf{P}_{\mathcal{A}_2} \iff \mathcal{A}_1 \subseteq \mathcal{A}_2$$

- The probability map is **positive**, i.e.,

$$A_M(\sigma) \in \mathbb{R} \text{ if } \sigma \in \mathcal{D}^{\mathbb{R}}_{\Sigma} \quad \text{and} \quad A_M(\sigma) \ge 0 \text{ if } \sigma \in \mathcal{D}^{+}_{\Sigma}$$

  This implies $0 \le P(\mathcal{A}|\mathcal{S}) \le 1$.

Realness and positivity

Positive formalism

…but it is also more natural!

- Consider the subset $\mathcal{D}_\Sigma^\mathbb{R} \subseteq \mathcal{D}_\Sigma$ of **self-adjoint** operators. This is a real vector space and $\mathcal{D}_\Sigma$ is its complexification.
- Consider the subset $\mathcal{D}_\Sigma^+ \subseteq \mathcal{D}_\Sigma^\mathbb{R}$ of **positive** operators. This forms a generating proper cone in the real vector space $\mathcal{D}_\Sigma^\mathbb{R}$ making it into an **ordered vector space**.
- The orthogonal projection operators form a **lattice** in $\mathcal{D}_\Sigma^\mathbb{R}$. This is equivalent the lattice of closed subspaces of $\mathcal{H}_\Sigma$. That is,

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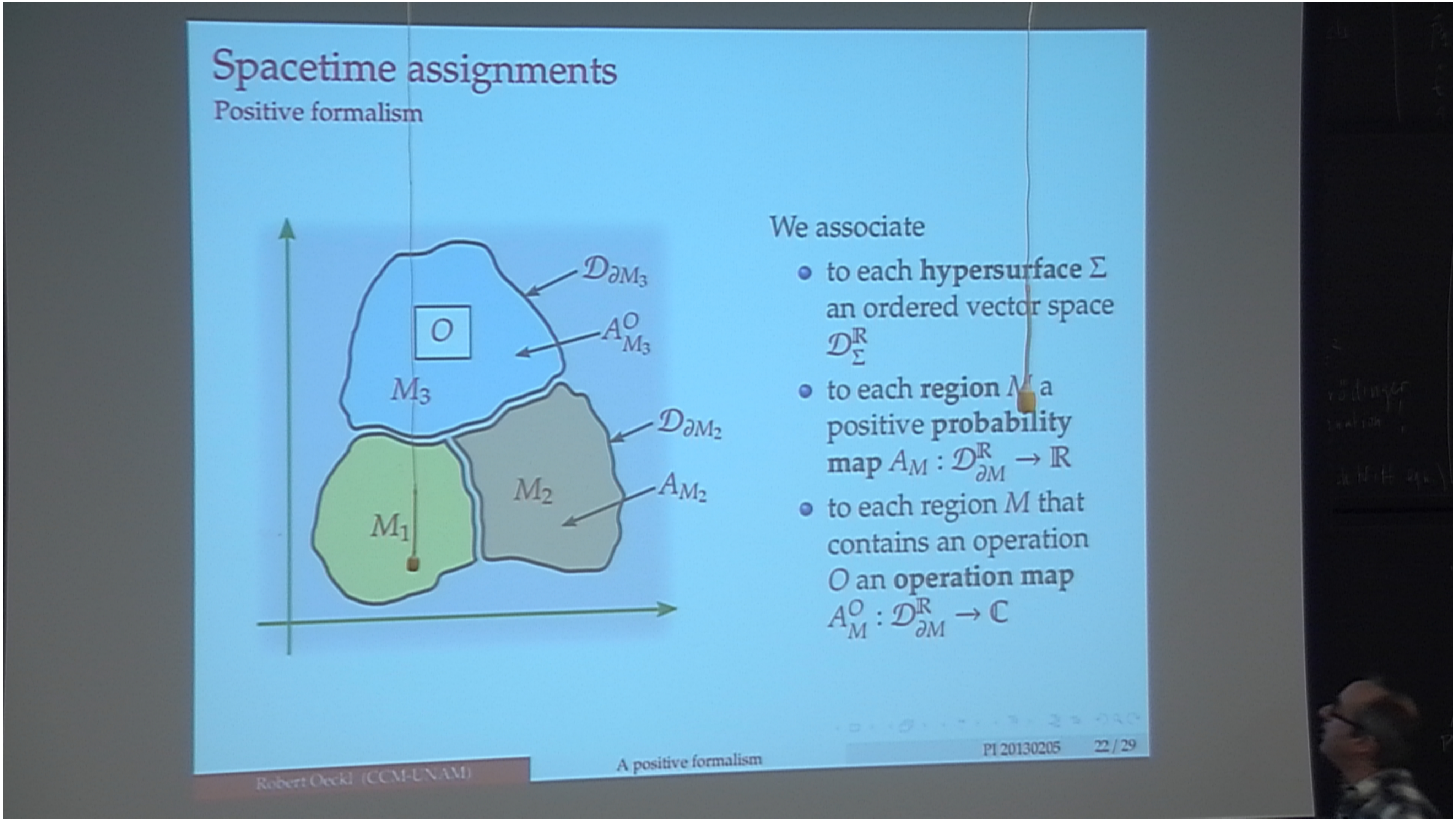
A formalism in its own right

Positive formalism

- Remarkably, the new structures $\mathcal{D}_\Sigma$, A_M and A_M^O satisfy axioms quite similar to those satisfied by $\mathcal{H}_\Sigma$, ρ_M and ρ_M^O.
- This suggests to postulate the new structures as objects in their own right, rather than to derive them from the amplitude formalism. This gives rise to the **positive formalism** [RO 2012].
- Positivity and normalization of probabilities now derive directly from the positivity of the probability map.
- We may restrict to the real vector spaces $\mathcal{D}_\Sigma^\mathbb{R}$, even forgetting $\mathcal{D}_\Sigma$.
- The latter step provokes a transition from an **oriented** to an **unoriented** formalism.
- We can generalize the expectation maps to not only represent observables, but more general **quantum operations**. We call these then **operation maps**.


Spacetime assignments
Positive formalism
We associate
to each **hypersurface** Σ an ordered vector space $\mathcal{D}_\Sigma^\mathbb{R}$
to each **region** M a positive **probability map** $A_M : \mathcal{D}_{\partial M}^\mathbb{R} \to \mathbb{R}$
to each region M that contains an operation O an **operation map** $A_M^O : \mathcal{D}_{\partial M}^\mathbb{R} \to \mathbb{C}$
$\mathcal{D}_{\partial M_3}$
$A_{M_3}^O$
O
M_3
$\mathcal{D}_{\partial M_2}$
M_2
A_{M_2}
M_1
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First summary

Positive formalism

The positive formalism is intriguing for a number of reasons:

- its spacetime locality and metric background independence (as an incarnation of the GBF)
- its wide applicability inherited from the amplitude formalism
- its potential applicability beyond the amplitude formalism
- its operationalism with a simple and elegant way to predict probabilities and expectation values
- its amenability to quantum information theory

At the same time it immediately invites many further questions. . .

No states, no collapse, but. . . ?

Question 1

- As becomes particularly clear in the positive formalism, the traditional concept of "state" as a specification of the reality of a system is untenable in the GBF. This also kills "collapse" interpretations and any model of the "collapse" as a physical event.
- Instead, the relevant mathematical objects entering the probability interpretation are the elements of the spaces $\mathcal{D}^+_{\partial M}$. We tentatively call them **quantum boundary conditions**. Only the "atomic" elements (one-dimensional projectors) correspond to elements in a Hilbert space. In turn, these coincide only in special circumstances with the traditional quantum states.
- But can we say anything more about the physical interpretation of the elements of $\mathcal{D}^+_\Sigma$? Do only special elements of $\mathcal{D}^+_\Sigma$ have a physical interpretation (e.g. the projectors)?

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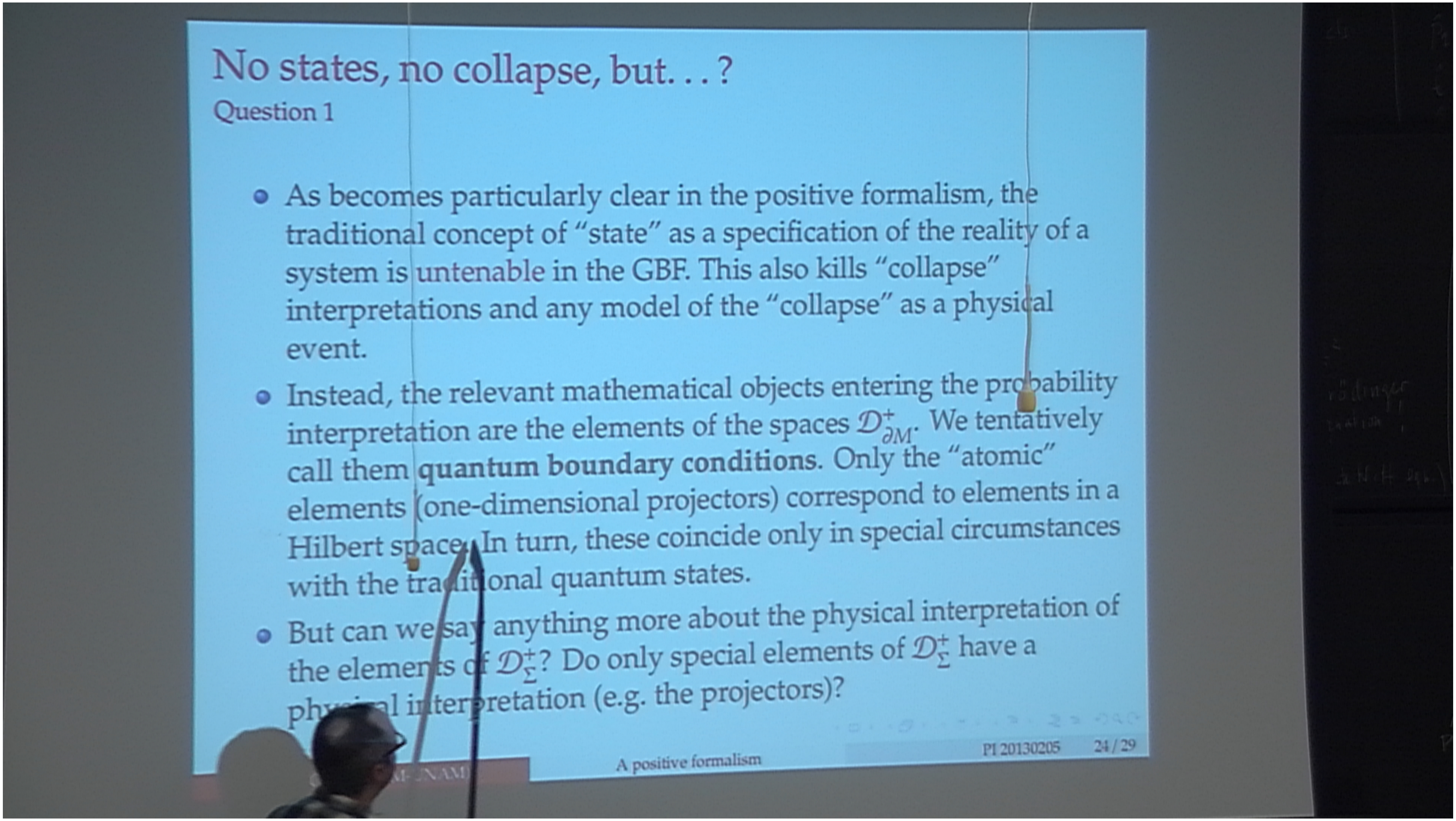
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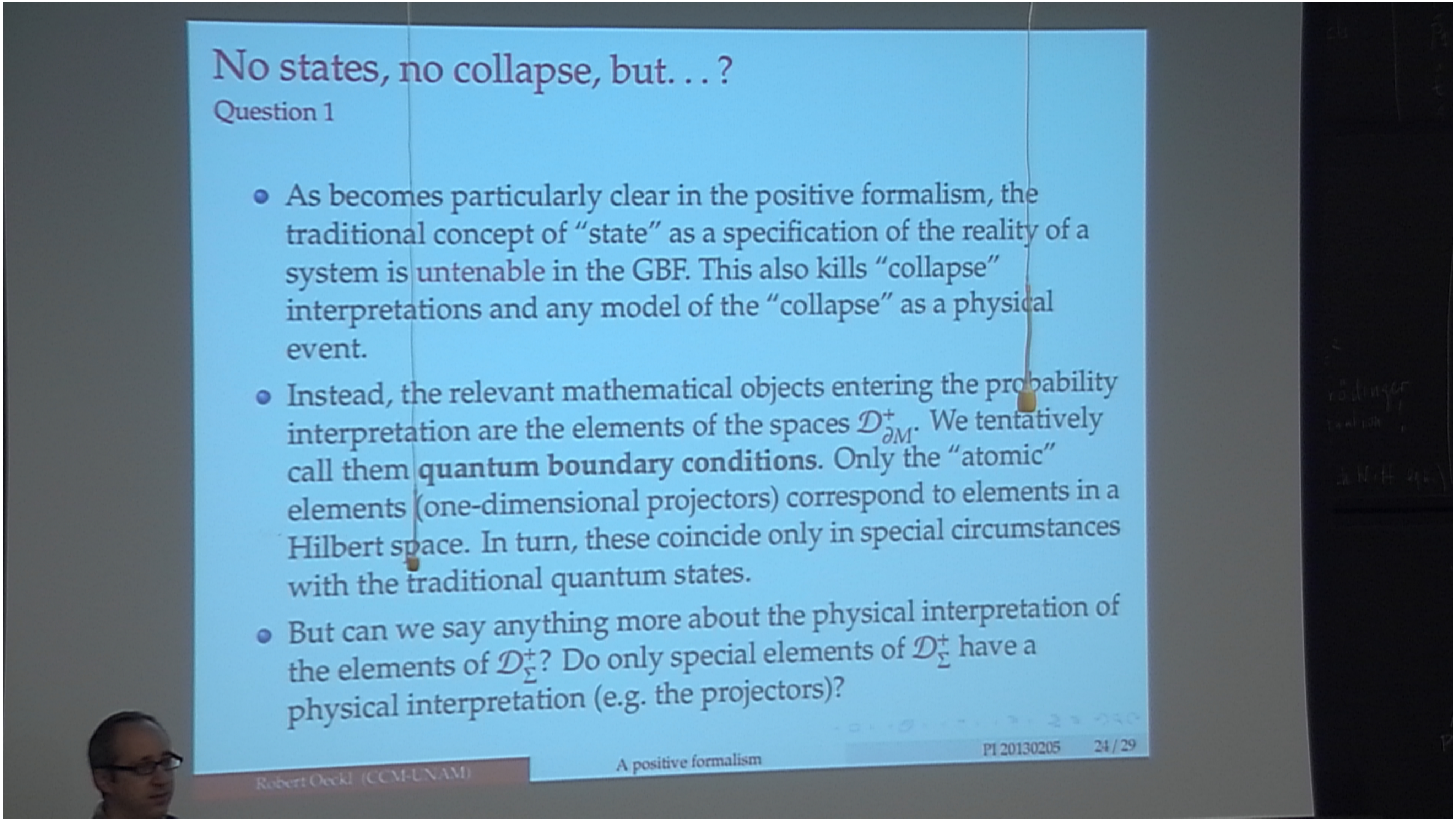
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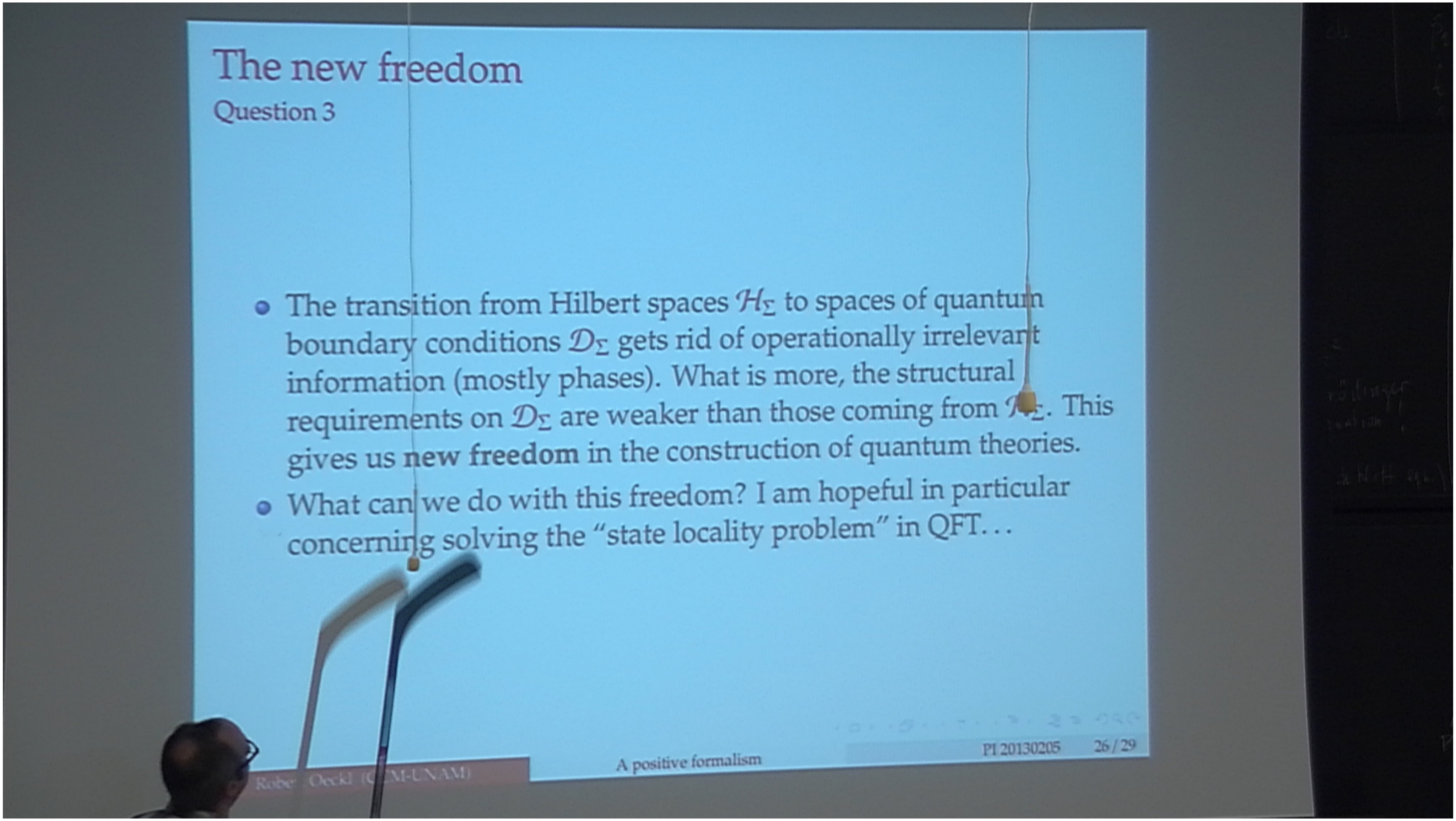
Spaces of quantum boundary conditions

Question 2

There are also mathematical questions about the spaces $\mathcal{D}_\Sigma$.

- Is the structure of ordered vector spaces sufficient? Do we need e.g., a Jordan product or even the "full" operator product? (In [RO 2012] I have also given them a Hilbert space structure.)
- What is the right "size" and topology for these spaces? In this talk I have assumed that these contain all bounded operators. In [RO 2012] I have assumed that these are only the Hilbert-Schmidt operators.

A related remark: The probability map A_M is actually not defined on $\mathcal{D}_{\partial M}$, but on a "dense" subspace $\mathcal{D}^\circ_{\partial M}$. Positivity suggests a solution to this problem. First, restrict A_M to $\mathcal{D}^{+\circ}_{\partial M}$. Second, extend the range of A_M from $[0, \infty]$ to $[0, \infty)$ to obtain a map $A_M : \mathcal{D}^+_{\partial M} \to [0, \infty)$.


The new freedom
Question 3
The transition from Hilbert spaces $\mathcal{H}_\Sigma$ to spaces of quantum boundary conditions $\mathcal{D}_\Sigma$ gets rid of operationally irrelevant information (mostly phases). What is more, the structural requirements on $\mathcal{D}_\Sigma$ are weaker than those coming from $\mathcal{H}_\Sigma$. This gives us **new freedom** in the construction of quantum theories.
What can we do with this freedom? I am hopeful in particular concerning solving the "state locality problem" in QFT. . .
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Quantum information theory

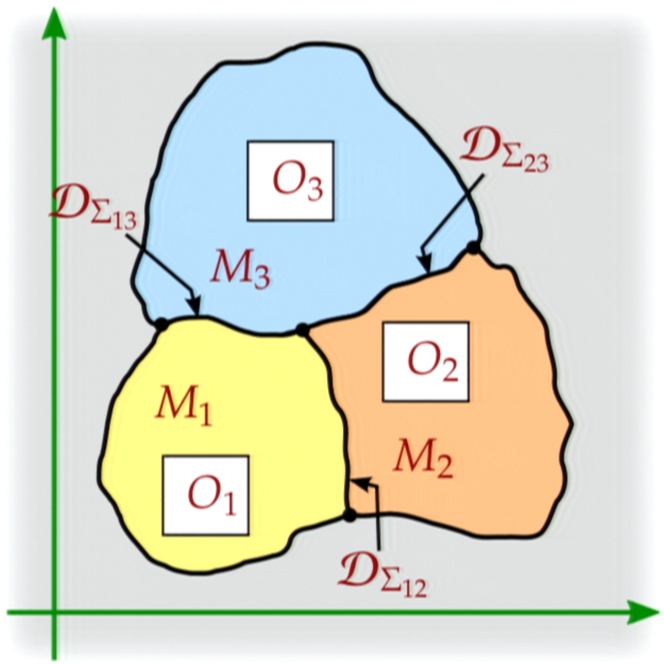
Question 4

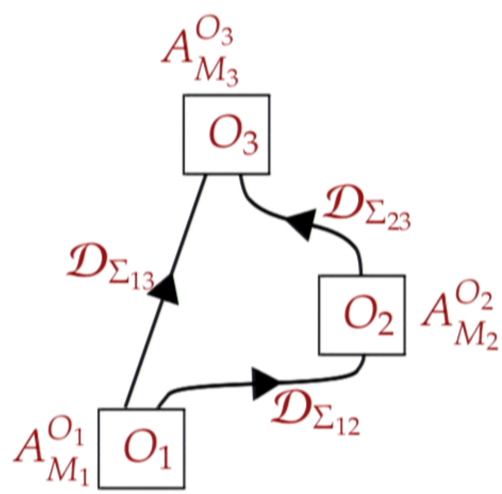
- The positive formalism enables us in particular to do within the GBF everything that can be done with the mixed state formalism of the standard formulation. We can implement arbitrary quantum operations, compose them, define notions of entropy, etc.
- [wild speculation] Can this help us to work towards a general relativistic (and quantum) framework for statistical physics, thermodynamics etc.?

Extracting operator tensors

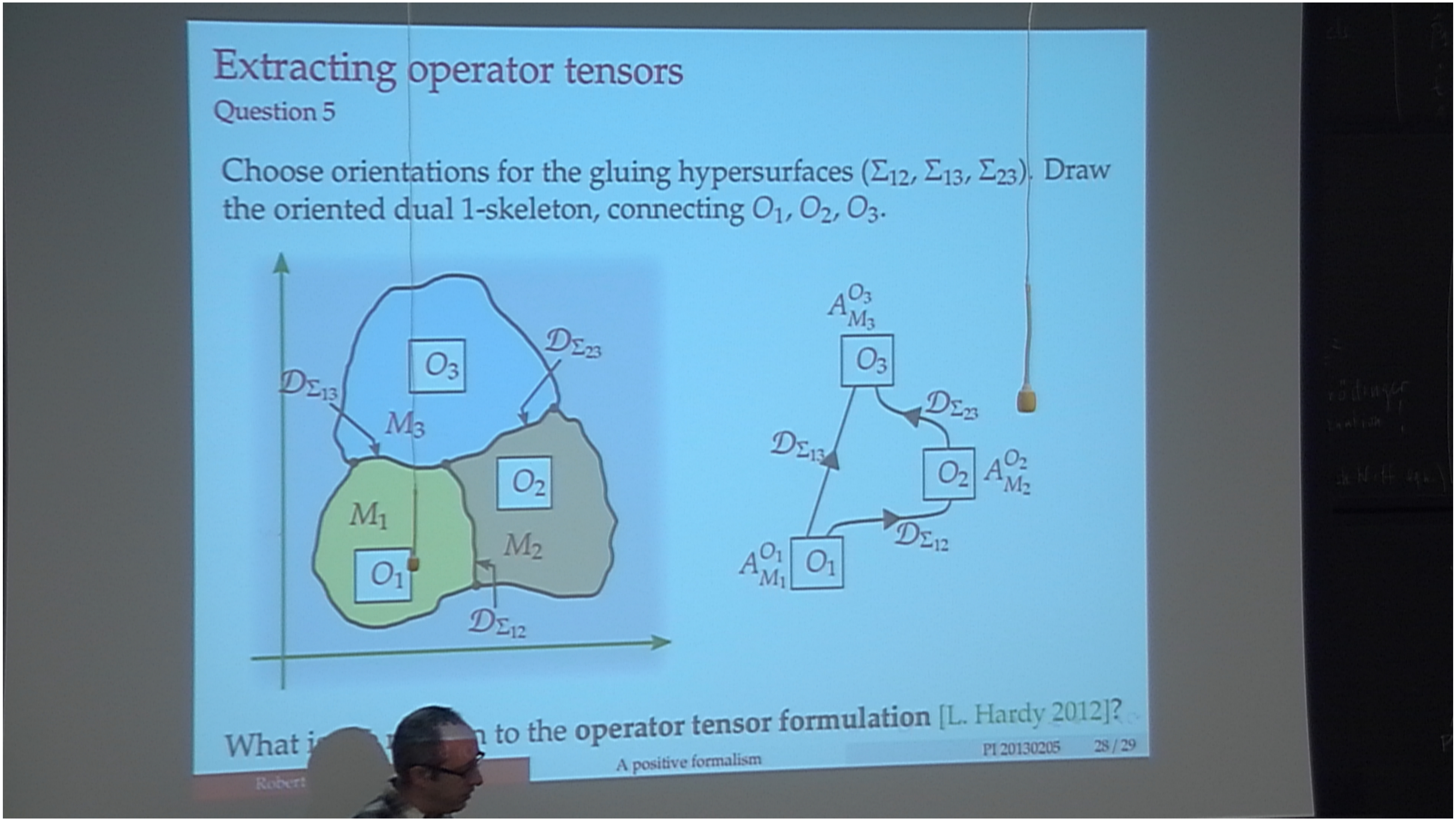
Question 5

Choose orientations for the gluing hypersurfaces (Σ_{12}, Σ_{13}, Σ_{23}). Draw the oriented dual 1-skeleton, connecting O_1, O_2, O_3.





What is the relation to the **operator tensor formulation** [L. Hardy 2012]?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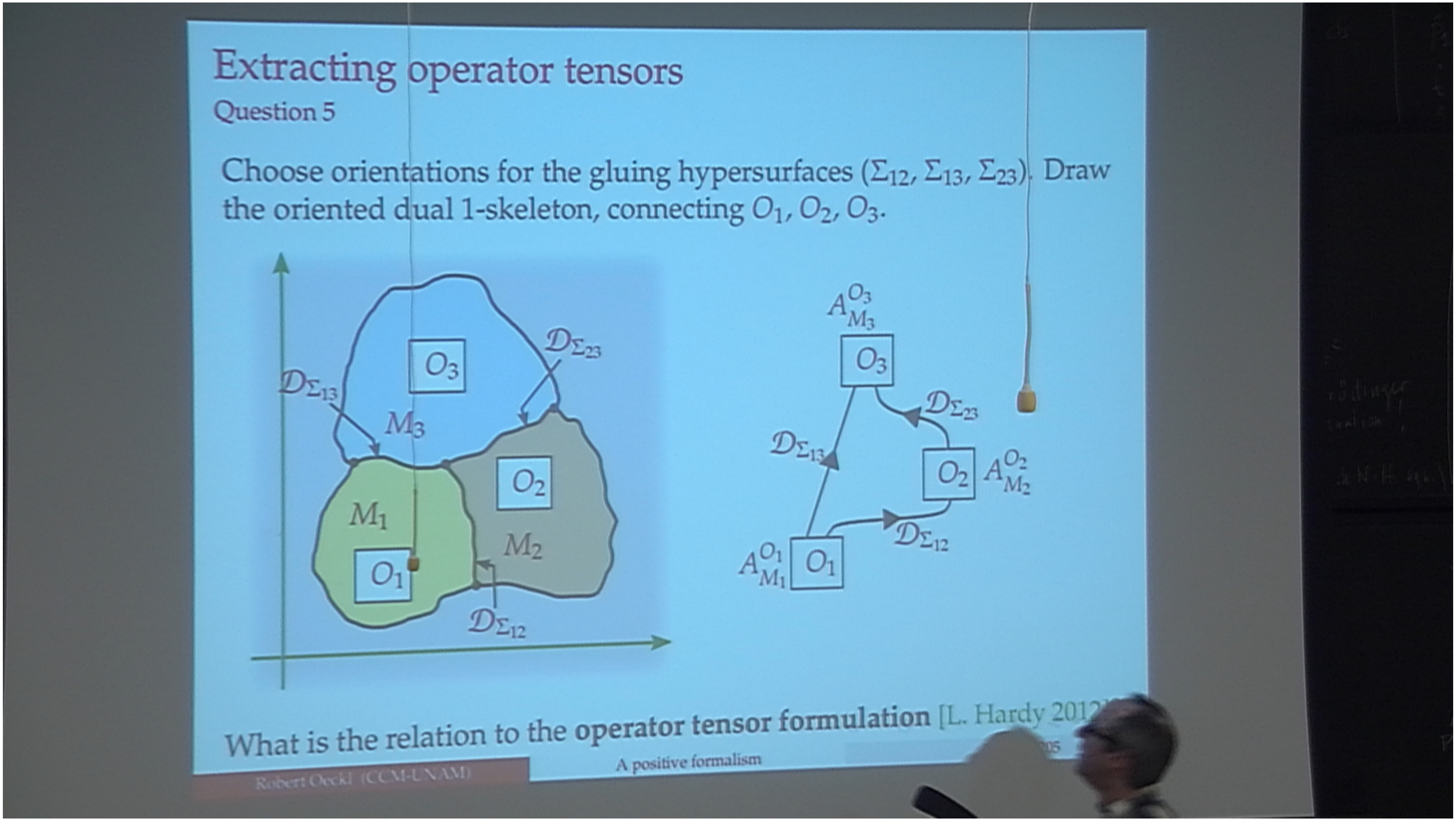
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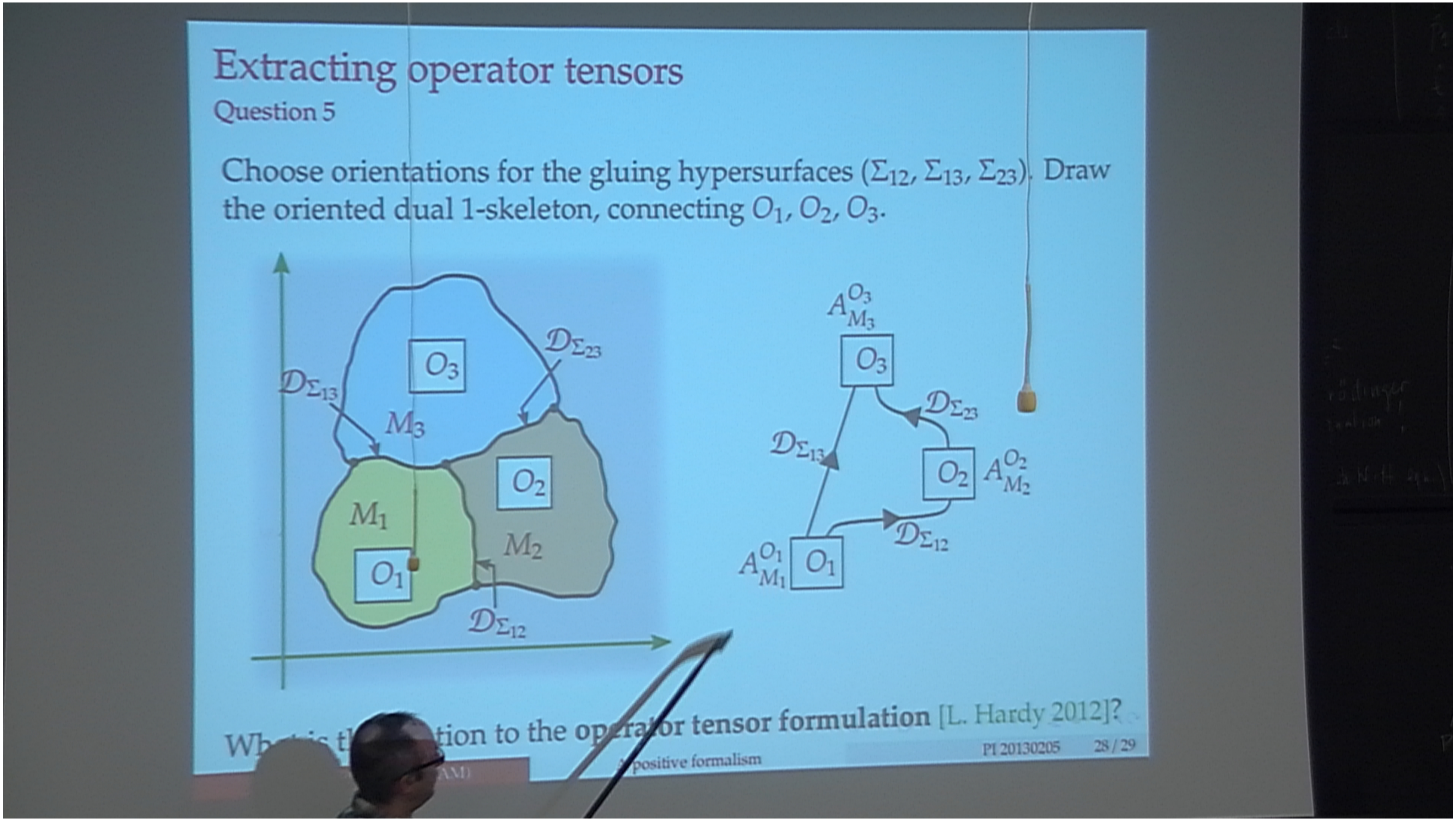
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