Title: Multifield Reheating and the Fate of the Primordial Observables

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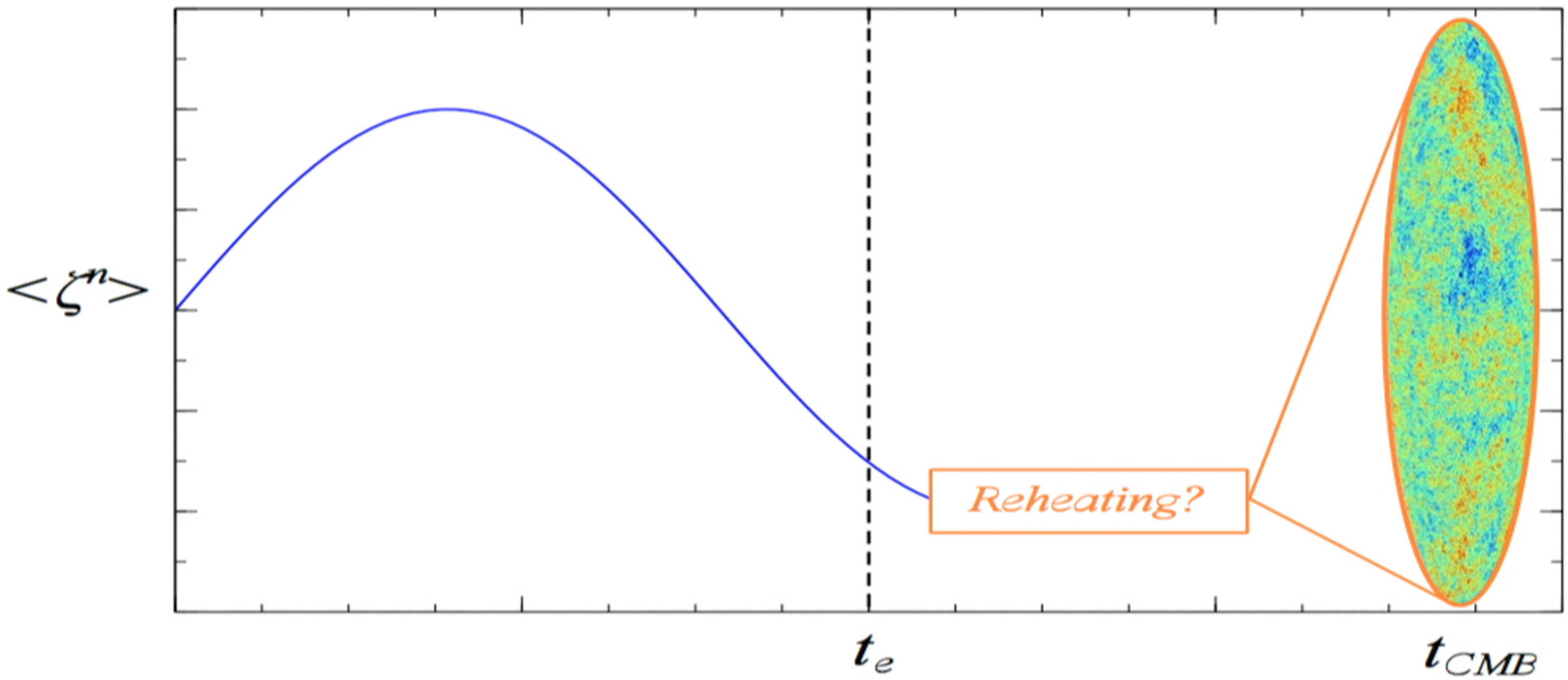
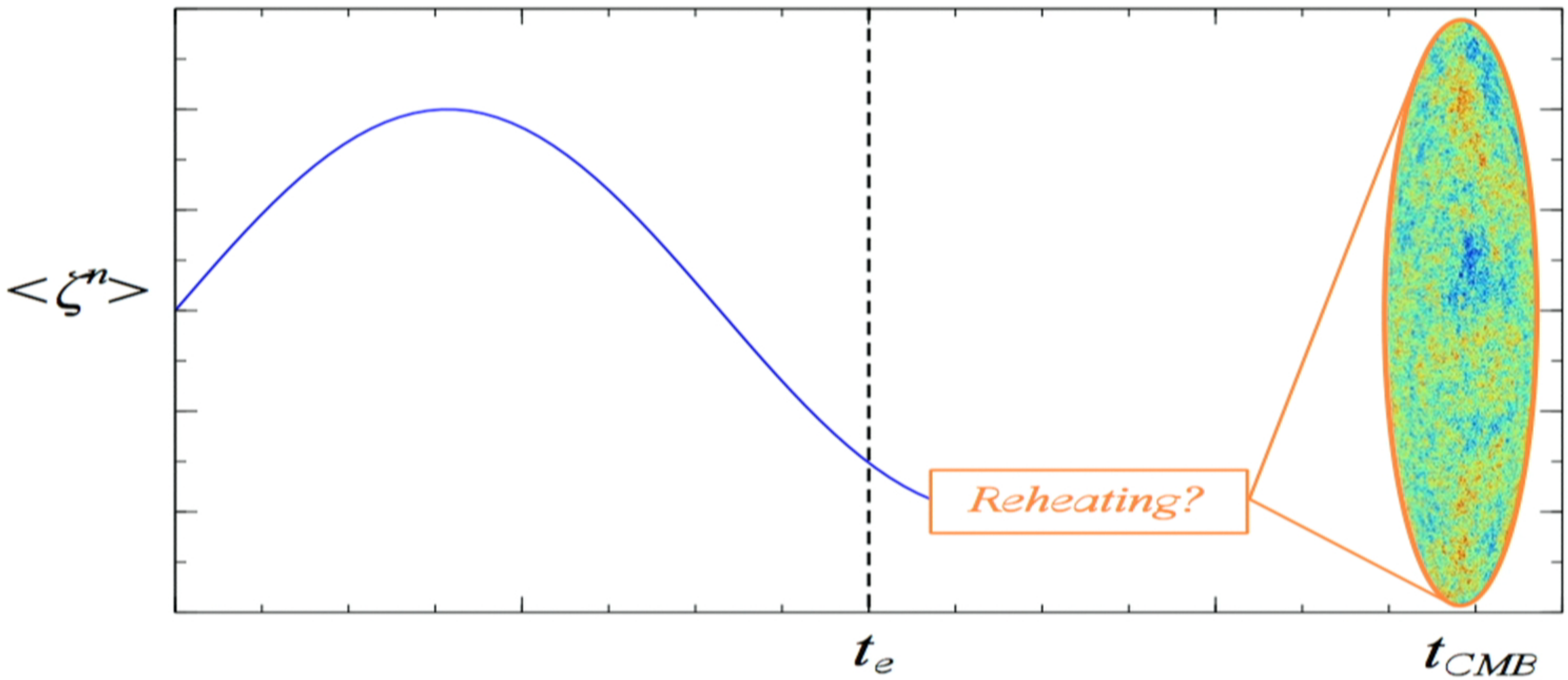
Abstract: The presence of additional light fields during inflation can source isocurvature fluctuations, which can cause the curvature perturbation ζ, and its statistics to evolve on superhorizon scales. I will demonstrate that if these fluctuations have not completely decayed before the onset of perturbative reheating, then primordial observables such as the level of non--Gaussianity can develop substantial reheating dependant corrections. I will argue that for inflationary models where an adiabatic condition is not reached before the relevant fields begin to decay, we must be careful in our interpretation of any observational constraints that place bounds on the statistics of ζ.



$<\zeta^n>$
Reheating?
t_e
t_{CMB}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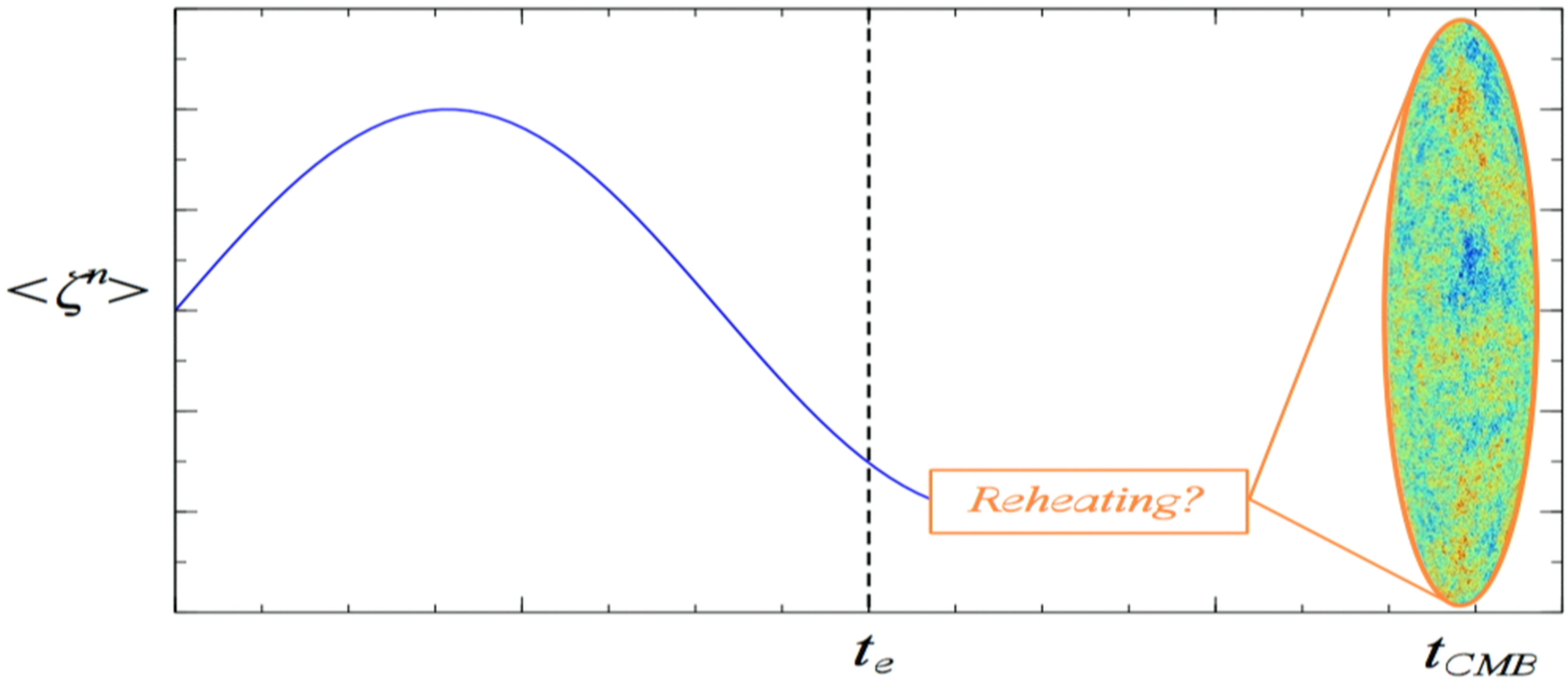
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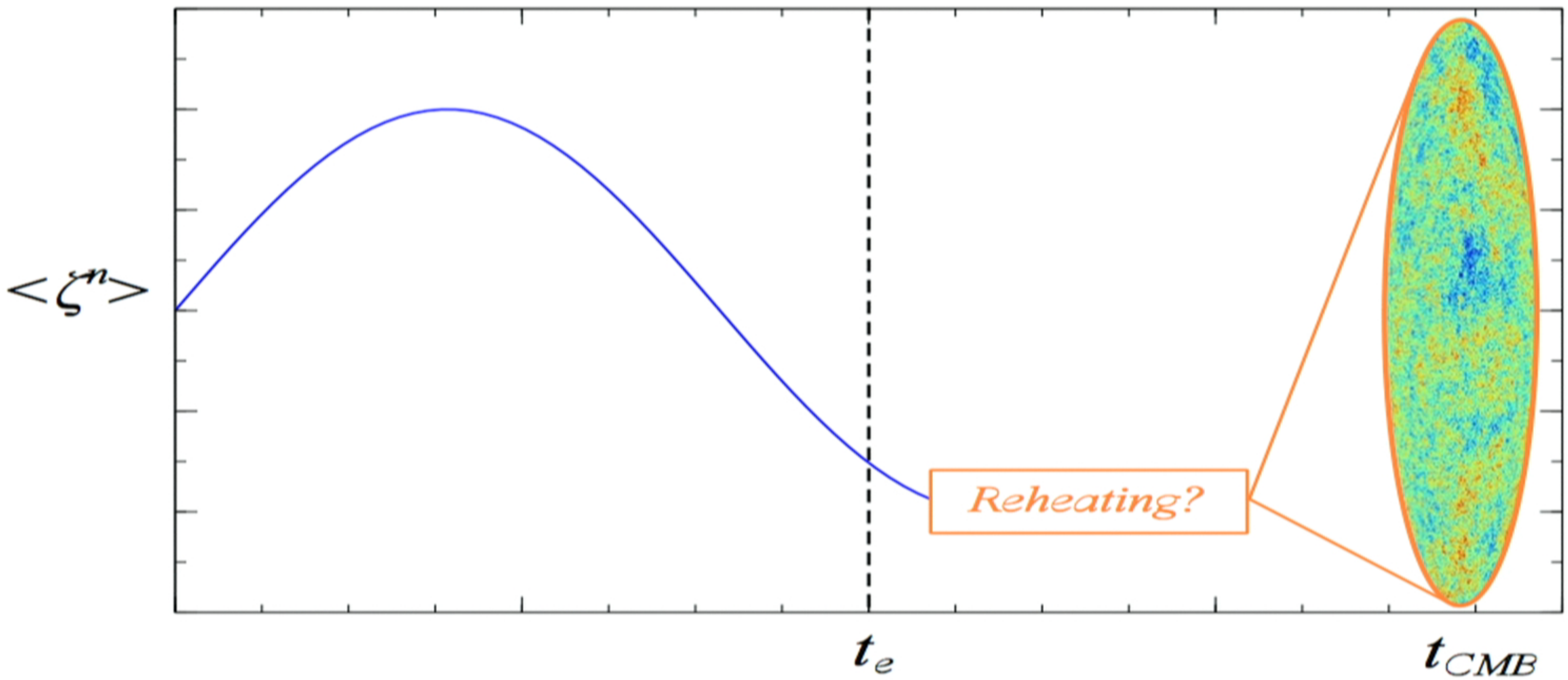
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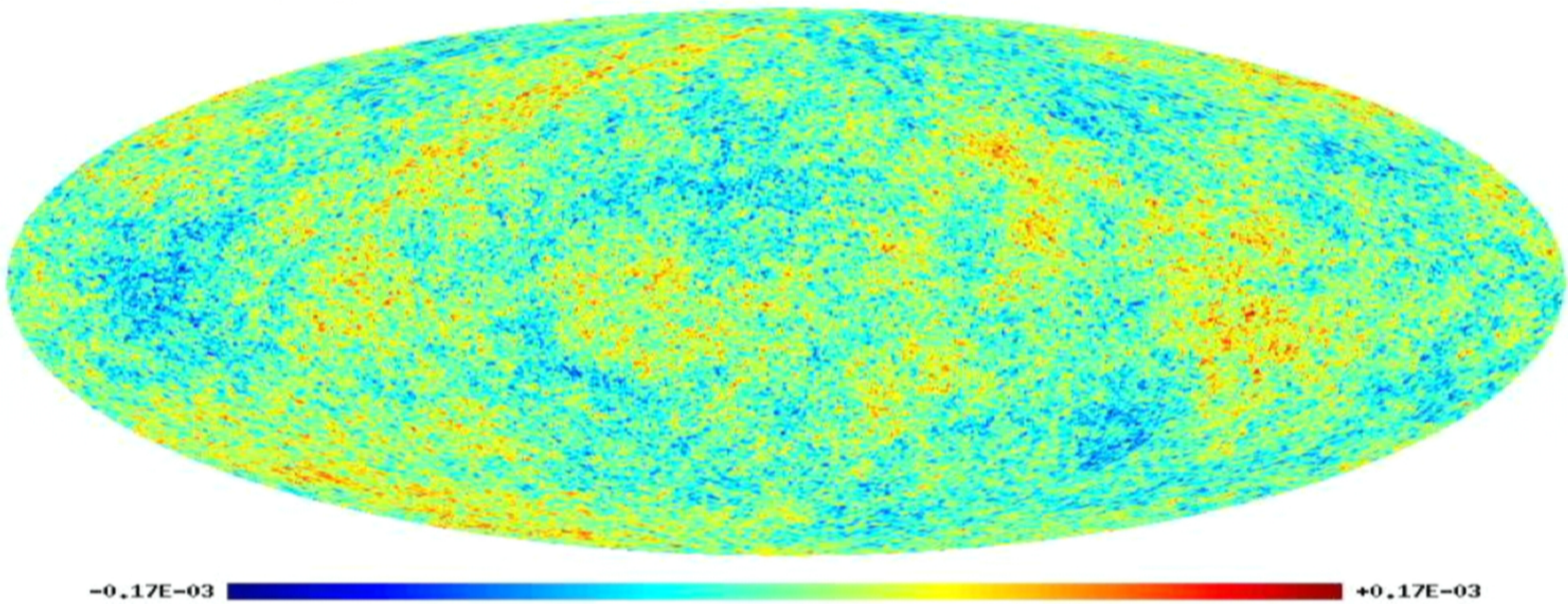
The Cosmic Microwave Background

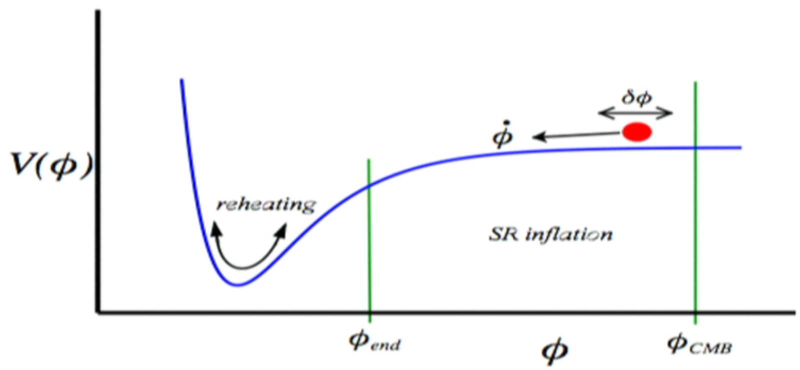
[Credit: Liguori et.al. (2003)]



Small temperature fluctuations $\Delta T/T \sim 10^{-5}$

Single Field Inflation

- Inflation: Quasi de–Sitter expansion, which causes the comoving Hubble radius to decrease
- Allows for amplification of quantum fluctuations of ϕ: $\delta\phi(t,\mathbf{x}) \leftrightarrow \delta\rho(t,\mathbf{x})$
- Solves horizon, flatness and relic problems: if $\epsilon \equiv -\dot{H}/H^2 \ll 1$ for at least 60 e–folds



$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{3M_{\rm p}^2}\left[\dot{\phi}^2 - V(\phi)\right]$$

- Produces robust, falsifiable predictions

Single field inflation: "*One direction in field space is solely responsible for the accelerated expansion AND the creation of curvature perturbations*"

Single Field Observables

- Nearly scale invariant spectrum of adiabatic curvature fluctuations:

$$n_\zeta - 1 \simeq -6\epsilon_* + 2\eta_* \,, \qquad \epsilon \equiv \frac{1}{2} M_{\rm p}^2 \left(\frac{V_\phi}{V}\right)^2 \,, \quad \eta \equiv M_{\rm p}^2 \left(\frac{V_{\phi\phi}}{V}\right)$$

- Small contribution from gravitational waves: $r \simeq 16\epsilon_*$
- Fluctuations have an almost Gaussian distribution:

$$f_{\rm NL}^{\rm (local)} = \frac{5}{12}(1 - n_\zeta)$$ [Maldacena (2002), Creminelli & Zaldarriaga (2004)]

- Consistent with current observational data: [*WMAP9*+eCMB+BAO+H_0]

$$\begin{aligned} n_\zeta &= 0.9636^{+0.0084}_{-0.0084} \qquad \text{(assuming } r = 0) \\ r &< 0.13 \quad (95\%\ \text{CL}) \\ -3 &< f_{\rm NL}^{\rm (local)} < 77 \quad (95\%\ \text{CL}) \end{aligned}$$ [*WMAP9*]

Aside: The primordial curvature perturbation $\zeta(t, \mathbf{x})$ and δN

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- Unperturbed universe: $ds^2 = -dt^2 + g_{ij} dx^i dx^j$, $\qquad g_{ij} = a^2(t)\delta_{ij}$
- Perturbed universe: $g_{ij} = a^2(t) e^{2\zeta(t,\mathbf{x})} \gamma_{ij}(t, \mathbf{x})$
- Define a local scale factor: $\tilde{a}(t, \mathbf{x}) = a(t) e^{\zeta(t,\mathbf{x})}$, volume: $\mathcal{V} \propto \tilde{a}^3(t, \mathbf{x})$

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- Start from a flat slice ($\zeta = 0$) at $t = t_*$. Define the amount of expansion to a uniform density hypersurface at $t = t_c$ as: $\mathcal{N}(t_*, t_c, \mathbf{x}) \equiv \ln \left| \frac{\tilde{a}(t_c, \mathbf{x})}{a(t_*)} \right|$, then:

$$\zeta(t_c, \mathbf{x}) \simeq \delta N \equiv \mathcal{N}(t_*, t_c, \mathbf{x}) - N_0(t_*, t_c) , \quad N_0(t_*, t_c) \equiv \ln \left| \frac{a(t_c)}{a(t_*)} \right|$$

- Assume that on super–Hubble scales ($aH \gg k$) the local evolution is well approximated by the evolution of some unperturbed universe. This is the separate universe approximation [Lyth & Rodriguez (2005)]
- In the absence of isocurvature (entropy) modes, ζ becomes a conserved quantity on super–Hubble scales [Lyth, Malik & Sasaki (2005)]

Non–Gaussianity of ζ

- The statistical properties of ζ are completely specified by the two–point correlation function if and only if the statistical distribution of ζ have a Gaussian distribution with random phases:

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2}) \frac{2\pi^2}{k_1^3} \mathcal{P}_\zeta(k_1)$$

- Any information contained in the departure from a perfect Gaussian, non–Gaussianity, is not encoded in the power spectrum, but has to be extracted from measurements of higher–order correlation functions

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k_1} + \mathbf{k_2} + \mathbf{k_3}) B_\zeta(k_1, k_2, k_3)$$

- Observations constrain the nonlinear parameter $f_{\rm NL}$:

$$\frac{6}{5} f_{\rm NL} \equiv \frac{\prod_i k_i^3}{\sum_i k_i^3} \frac{B_\zeta(k_1, k_2, k_3)}{4\pi^4 \mathcal{P}_\zeta^2}$$

[Maldacena (2002)]

- The bispectrum has an amplitude and a shape
- Violation of:
  - Single field $\longrightarrow$ squeezed configuration $k_3 \ll k_2 \simeq k_1$
  - Canonical kinetic terms $\longrightarrow$ equilateral configuration $k_1 = k_2 = k_3$
  - Bunch–Davies Vacuum $\longrightarrow$ flattened configuration $k_1 \simeq 2k_2 \simeq 2k_3$
- A plethora of different mechanisms for generating a large $f_{\rm NL}$ have been proposed:

- Features in the inflaton potential [Chen et.al. (2007)]
- Non–local inflation [Barnaby & Cline et.al. (2008)]
- Non–canonical kinetic terms (e.g.DBI) [Silverstein & Tong (2004), Huang & Shiu (2006)]
- Modulated reheating [Divali et.al. (2004), Suyama & Yamaguchi (2008), Kobayashi (2011)]
- Inhomogeneous end to inflation [Bernardeau & Uzan (2003), Lyth (2005)]
- Preheating [Chambers & Rajantie (2008), Bond et.al. (2009)]
- Curvaton scenario [Lyth & Wands (2002), Sasaki et.al. (2006), Kobayashi et.al. (2012)]
- Multiple fields [Enqvist et.al. (2005), Brynes et.al. (2008), Kim et.al. (2012)]

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$$S = \int d^4x\sqrt{-g}\left[M_{\rm p}^2\frac{R}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - W(\varphi,\chi)\right]$$

- 3–pt statistics obtained in the same fashion: [Maldacena (2002), Vernizzi & Wands (2006)]

$$\frac{6}{5}f_{\mathrm{NL}} = \frac{r}{16}\left[1 + f(k_1, k_2, k_3)\right] + \frac{\sum_{IJ} N_{,IJ} N_{,I} N_{,J}}{\left(\sum_I N_{,I}^2\right)^2}$$

Computing the Observables (II)

- Single field: SR solution forms a 1D phase space $\rightarrow$ once the inflationary attractor has been reached, unique trajectory
  - straight–forward to compute δN derivatives
- Multi field: infinite number of classical SR trajectories in phase space:
  - makes computation much more complicated

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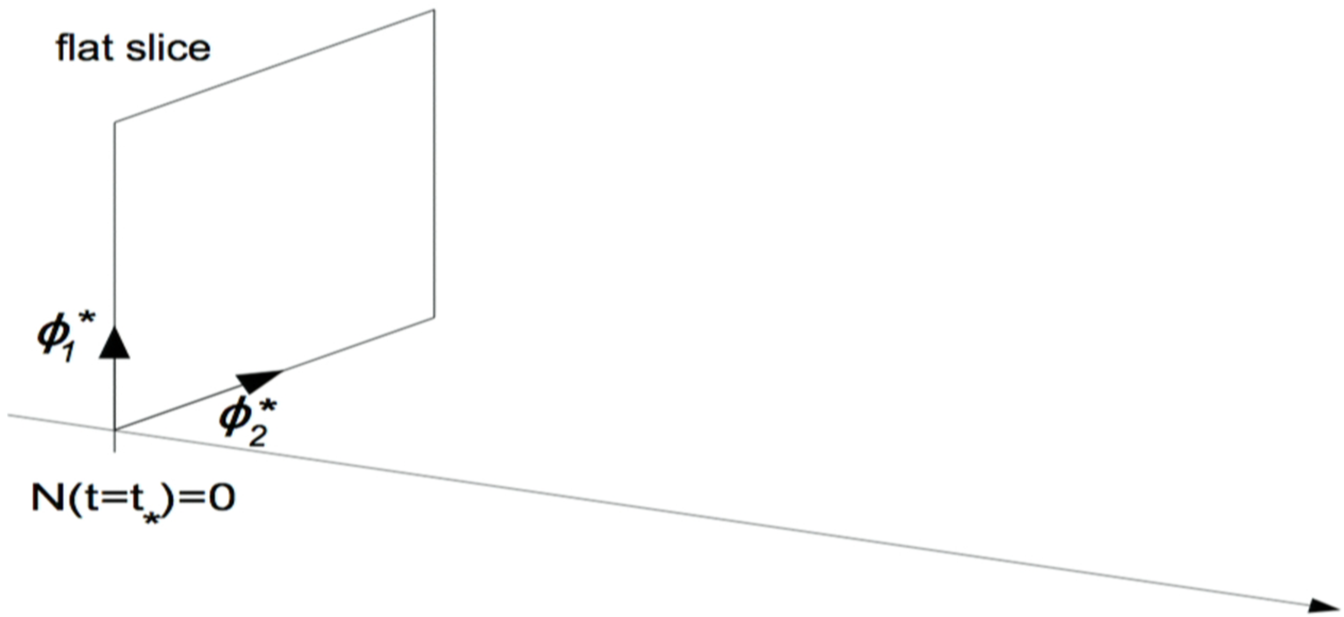
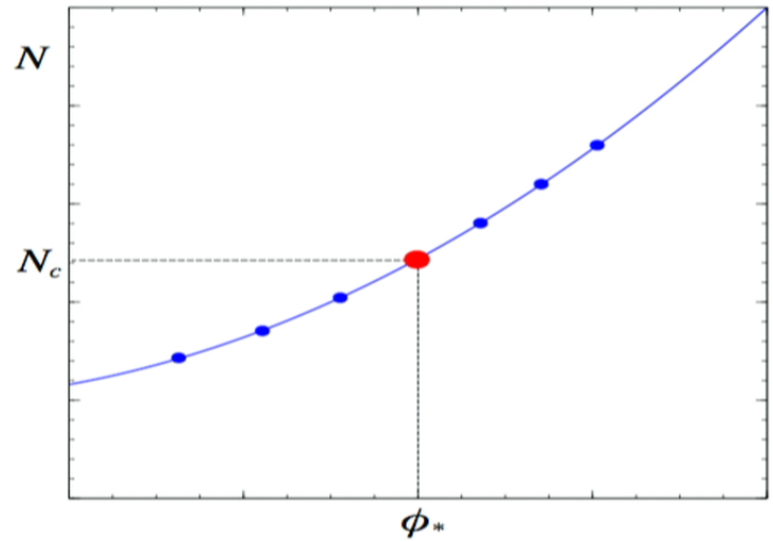
- Multi field: infinite number of classical SR trajectories in phase space:
  - makes computation much more complicated
- For example: $W(\varphi, \chi) = U(\varphi) + V(\chi)$

$$N(t_c, t_*) = \underbrace{-\frac{1}{M_{\rm p}^2}\int_*^c \frac{U}{U_\varphi}\, d\varphi}_{\text{depends on } \chi_*} \underbrace{-\frac{1}{M_{\rm p}^2}\int_*^c \frac{V}{V_\chi}\, d\chi}_{\text{depends on } \varphi_*}$$

- An alternative is to solve the background equations numerically



- δN derivatives computed with 7–point stencil finite difference method (5–pt for cross derivatives)
- Demands excellent error control:
  - long EoM integration time + oscillating fields during reheating
  - $N(\varphi_*, \chi_*)$ can be very flat: must deal with truncation and round–off error when taking derivatives
- Tested against only known relevant exact solution beyond SR [Brynes & Tasinato (2009)]



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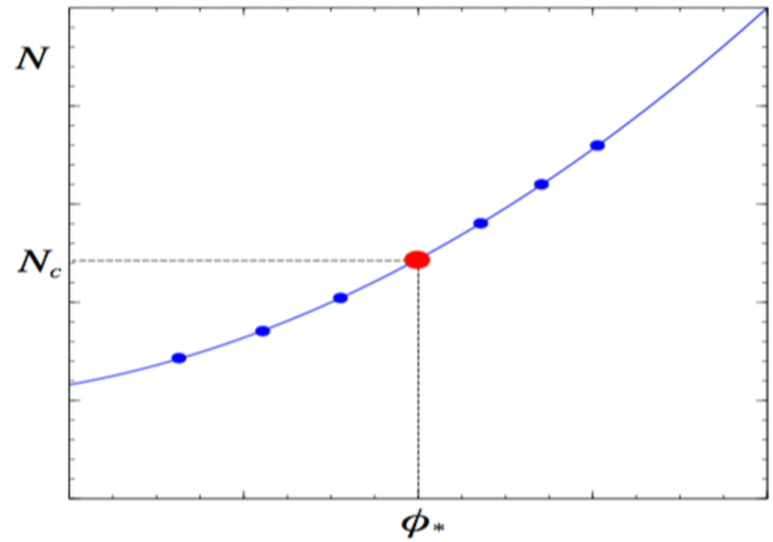
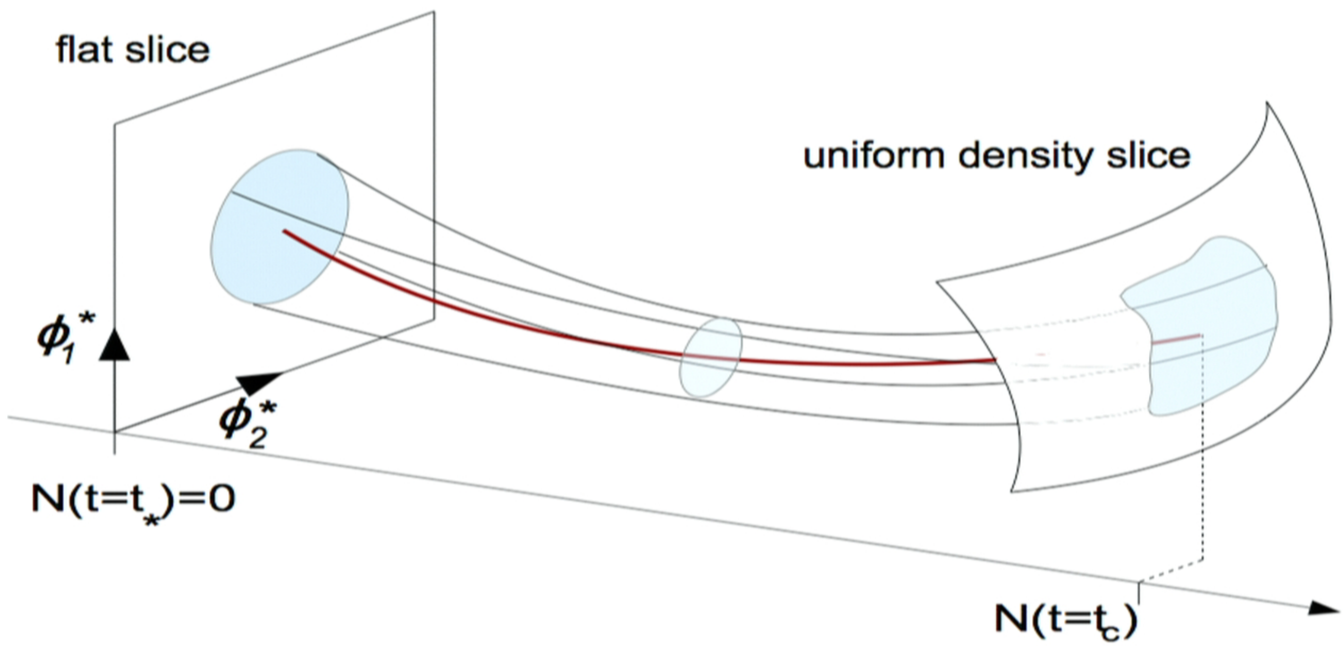



flat slice
uniform density slice
ϕ_1^*
ϕ_2^*
$N(t=t_*)=0$
$N(t=t_c)$

When can I "stop calculating"?

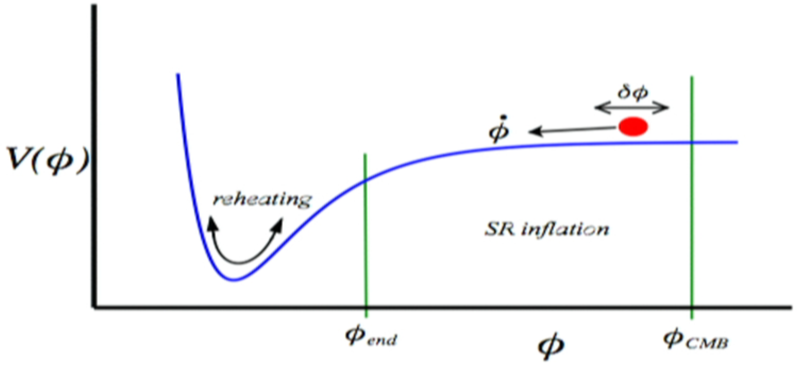
At least two ways in which multifield perturbations can become adiabatic:

- Pass through a sufficiently long phase of single field inflation
- $f_{\rm NL}$ is generically damped away [Meyers & Sivanadam (2011)] (except in special cases)

- Local thermal equilibrium [Weinberg (2004,2009), Meyers (2012)]
- We find that if $f_{\rm NL}$ is large at the end of inflation, it typically remains large at the completion of reheating

Currently no evidence for non–adiabatic fluctuations in CMB:
uncorrelated: $a_0 < 0.047$, correlated: $a_{-1} < 0.0039$ [*WMAP9*+eCMB+BAO+H_0]

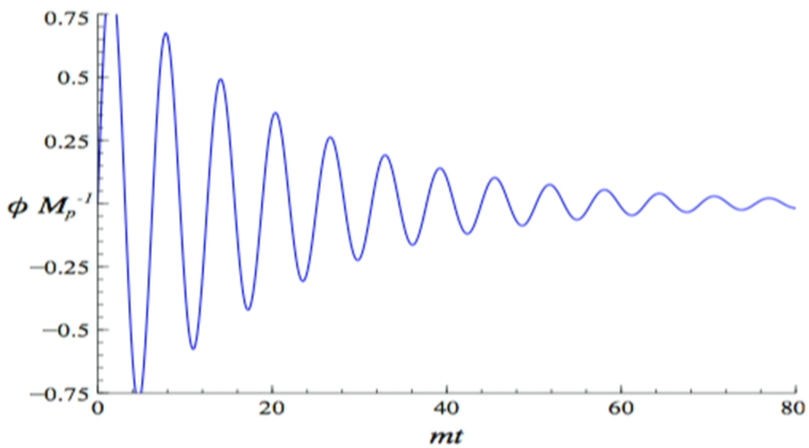
Perturbative Reheating



- During the oscillating phase:

$$\phi(t) \simeq \frac{\phi_0}{a^{3/2}(t)}\sin(mt)$$
$$a(t) \simeq t^{2/3}$$

- Near the minimum $V(\phi) \simeq \frac{1}{2}m^2\phi^2$



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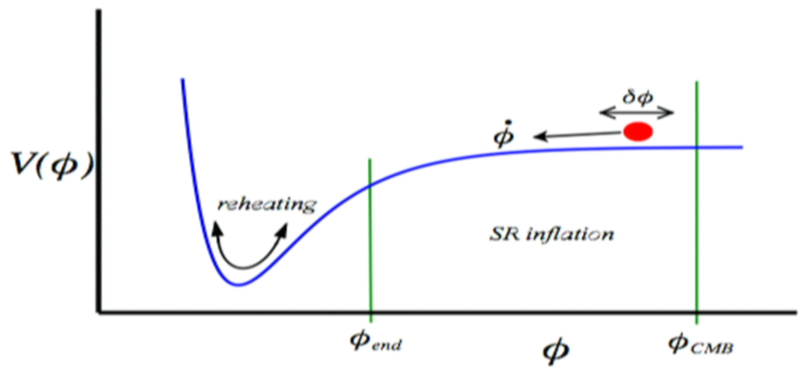

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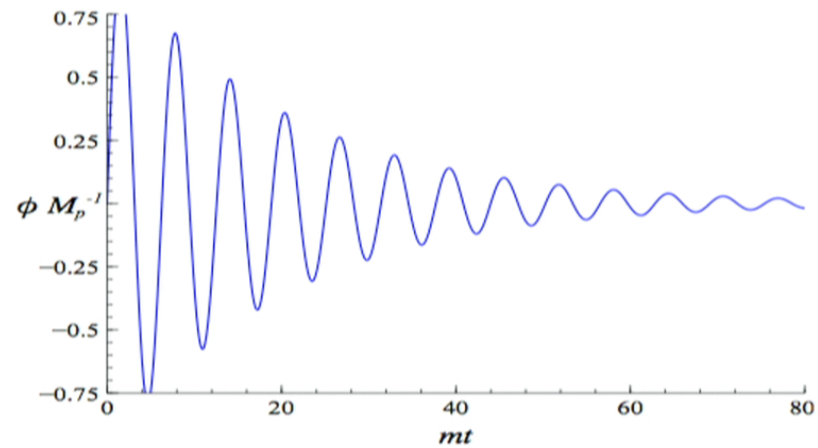

- oscillations behave like pressureless dust

- Want to see the oscillating inflaton condensate decay

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}(\partial\psi)^2 - \frac{g^2}{2}\phi^2\psi^2 - \frac{1}{2}\sigma\phi\psi^2 - \frac{\lambda}{4}\psi^4$$

- Bogoliubov calculation $\rightarrow$ $\alpha_k(\tau)$ and $\beta_k(\tau)$ to obtain frequency of ψ modes ω_k, and mode occupation numbers n_k
- Boltzmann equation to determine density in ψ field

$$a^4\rho_\psi \equiv \int \frac{\mathrm{d}^3k}{(2\pi)^3}\omega_k n_k \quad \rightarrow \quad a^{-4}\frac{\mathrm{d}}{\mathrm{d}t}(a^4\rho_\psi) \simeq \Gamma_{\phi\rightarrow\psi\psi}\rho_\phi$$

[see e.g. Braden, Kofman & Barnaby (2010)]

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- Energy conservation:

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- Subject to Hubble constraint: $3M_{\rm p}^2 H^2 = \sum_I \rho_{\phi_I} + \rho_\gamma$

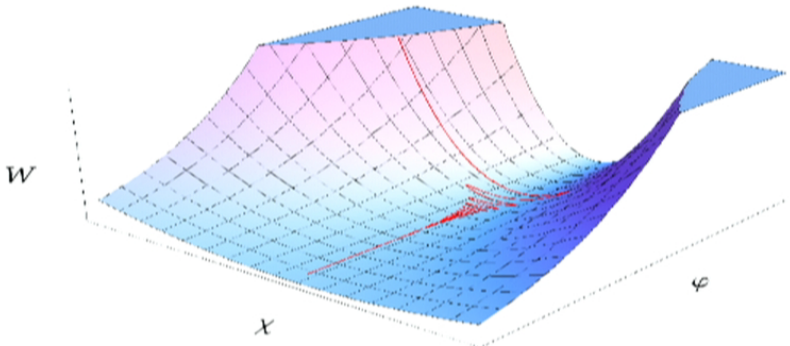
Case Study I: Single Minimum

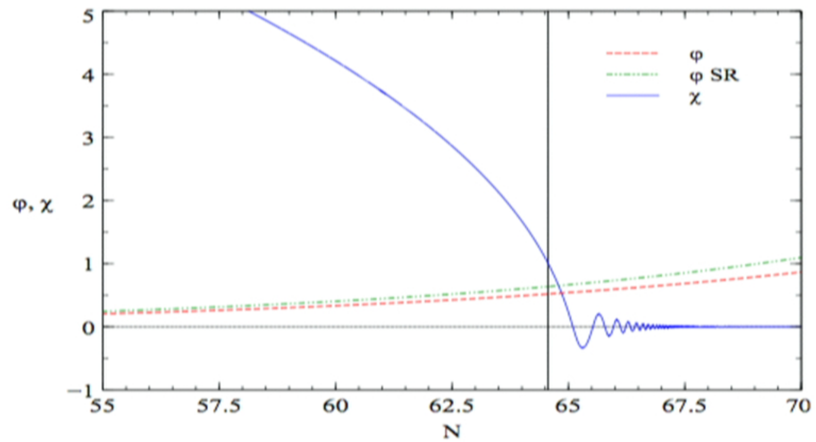
$$W(\varphi,\chi) = W_0 \chi^2 e^{-\lambda\varphi^2}$$

[Byrnes et.al., (2008), Elliston et.al. (2011), Watanabe (2012), Huston (2012), Kaiser et.al. (2012)]

χ drives inflation, φ remains SR and sources the isocurvature modes

$\varphi_* = 10^{-3} M_\mathrm{p}$, $\chi_* = 16.0 M_\mathrm{p}$, $\lambda = 0.05$





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- Subject to Hubble constraint: $3M_{\rm p}^2 H^2 = \sum_I \rho_{\phi_I} + \rho_\gamma$
- Only valid when ϕ_I rapidly oscillating – "particle creation" terms should not be present during inflation
- Require $m \gg \max(H, \Gamma)$
- Reheating temperature $T_R \sim 0.1\sqrt{\Gamma M_{\rm p}}$
- BBN: $T_R \geq 5\,{\rm MeV} \rightarrow \Gamma \geq 4 \times 10^{-40} M_{\rm p}$ [e.g. Kawasaki, Kohri & Moroi (2005)]
- Also constraints on T_R from CMB [Martin & Ringeval (2010)]

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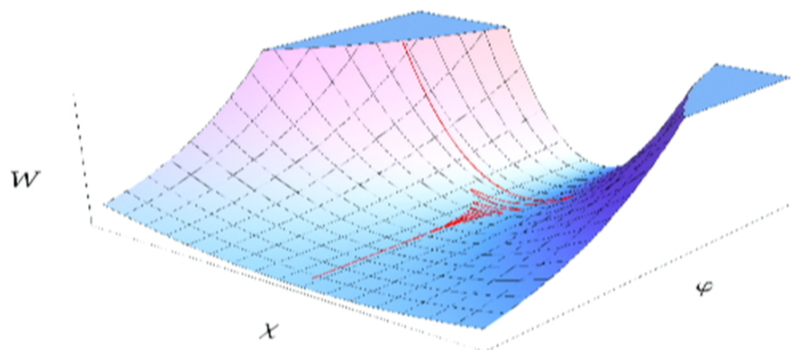

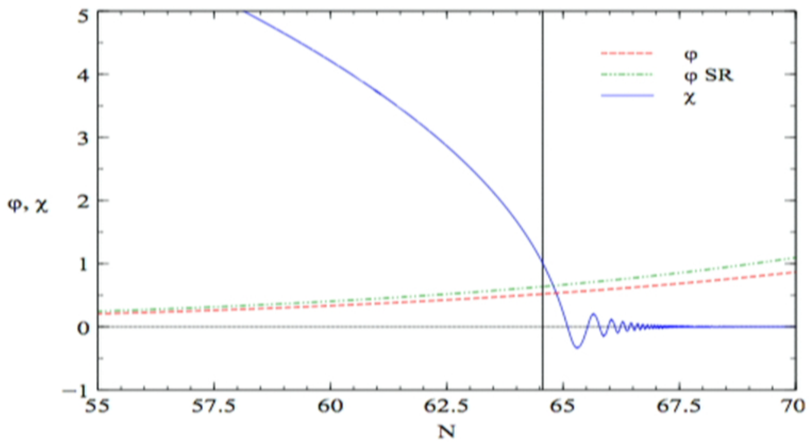

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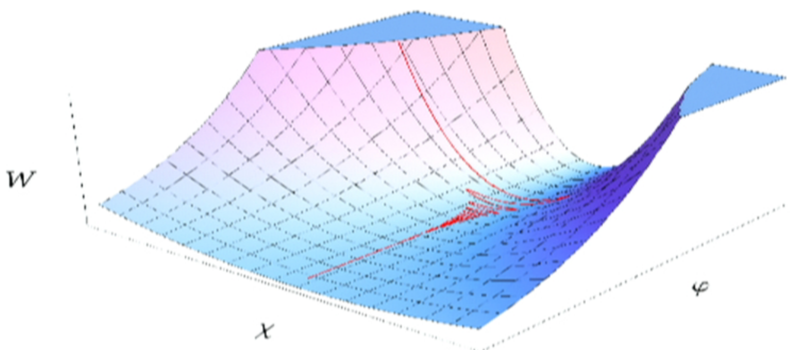

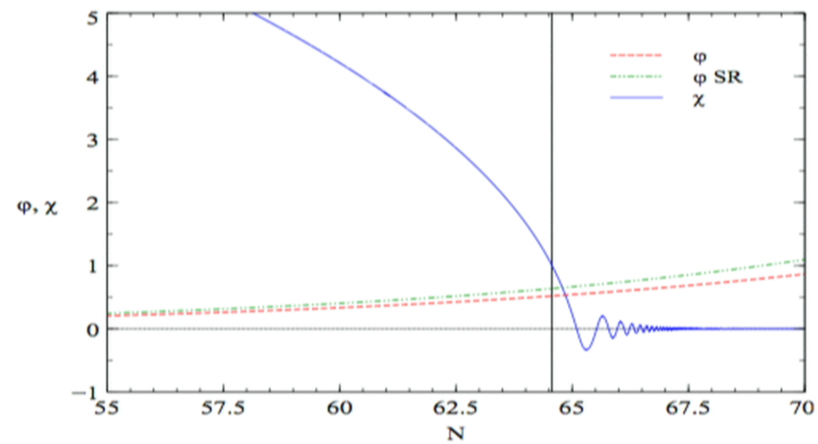

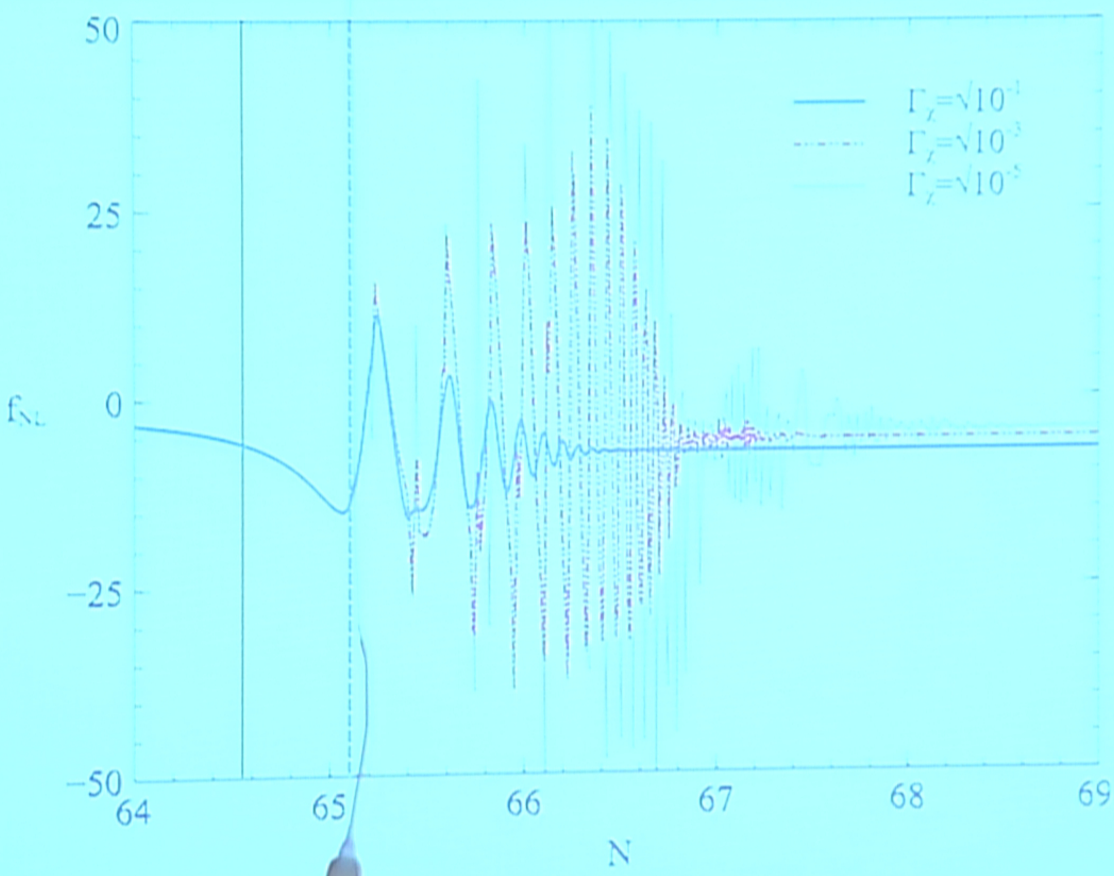

vertical **black** line: end of inflation, vertical **blue** line: start of reheating

$$\frac{6}{5}f_{\rm NL} = \frac{N_{\varphi\varphi}N_\varphi^2}{\left[N_\varphi^2 + N_\chi^2\right]^2} + 2\frac{N_{\varphi\chi}N_\chi N_\varphi}{\left[N_\varphi^2 + N_\chi^2\right]^2} + \frac{N_{\chi\chi}N_\chi^2}{\left[N_\varphi^2 + N_\chi^2\right]^2}$$

$$N(t_c, t_*) = \int_*^c H\mathrm{d}t = \int_*^c \frac{\mathrm{d}H^2}{2\dot{H}}$$

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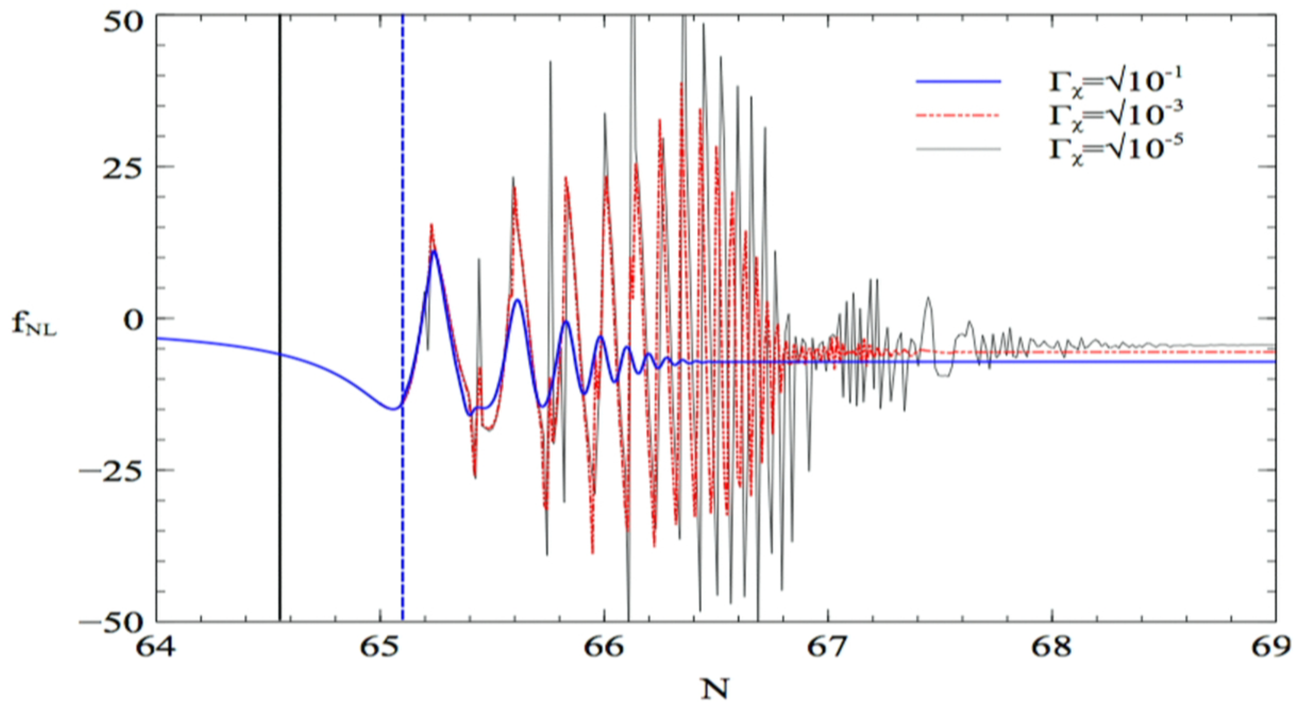
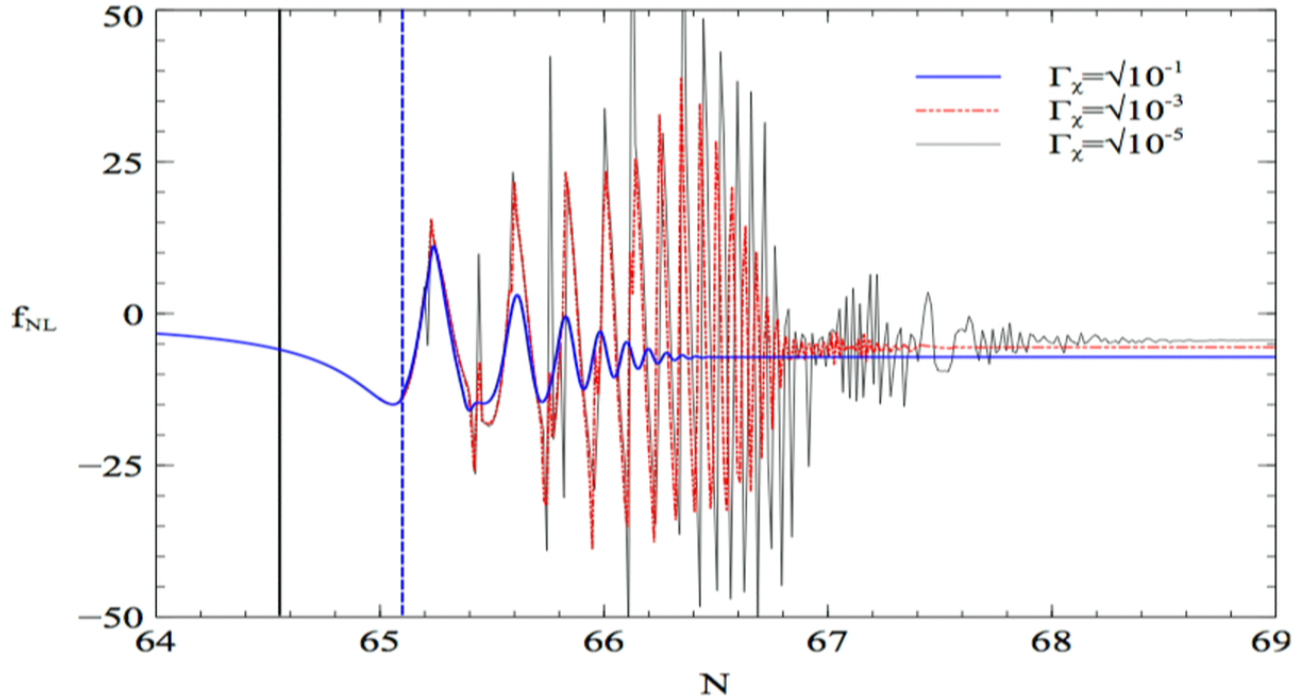


vertical **black** line: end of inflation, vertical **blue** line: start of reheating



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$$f_{\rm NL}(t_e) = -5.93,$$
$$n_\zeta(t_e) = 0.763,\ r(t_e) = 2.8 \times 10^{-4}$$

Γ_χ	$f_{\rm NL}^{\rm final}$	$n_\zeta^{\rm final}$	$r^{\rm final}$
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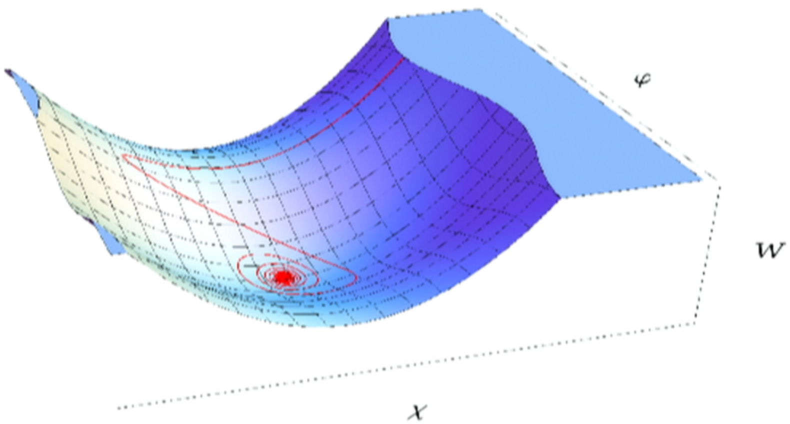
Case Study II: Two Minima

$$W_{\rm eff}(\varphi,\chi) = W_0\left[\frac{1}{2}m^2\chi^2 + \Lambda^4\left(1-\cos\left(\frac{2\pi}{f}\varphi\right)\right)\right]$$



- Motivated by axion N–flation: $W = \sum_i \Lambda_i^4\left[1-\cos\left(\frac{2\pi}{f_i}\varphi_i\right)\right]$ [Dimopoulos et.al. (2005)]
- De–phasing of field oscillations suppress parametric resonance $\Rightarrow$ reheating proceeds perturbatively [Braden, Kofman & Barnaby (2010)]
- Reheat from both fields

$\Lambda^4 = m^2 f^2/4\pi^2$, $\varphi_* = (\frac{f}{2} - 0.001)M_{\rm p}$, $\chi_* = 16M_{\rm p}$, $f = m = 1$

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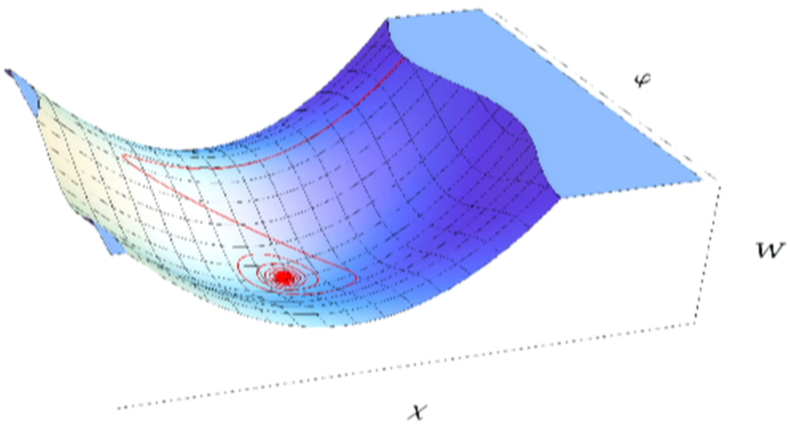

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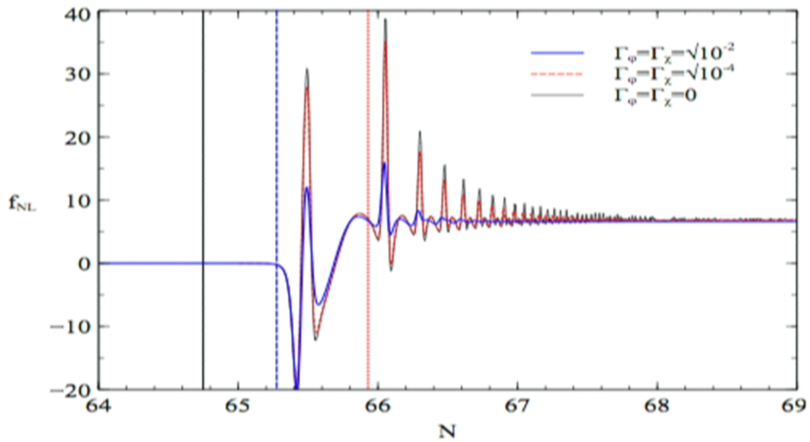

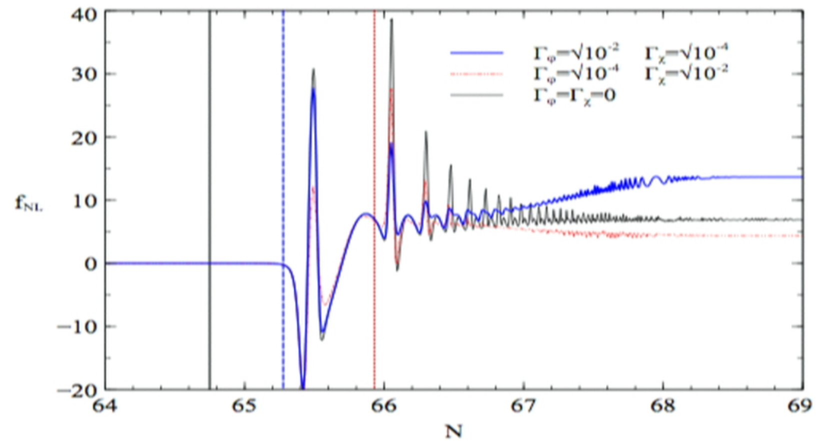

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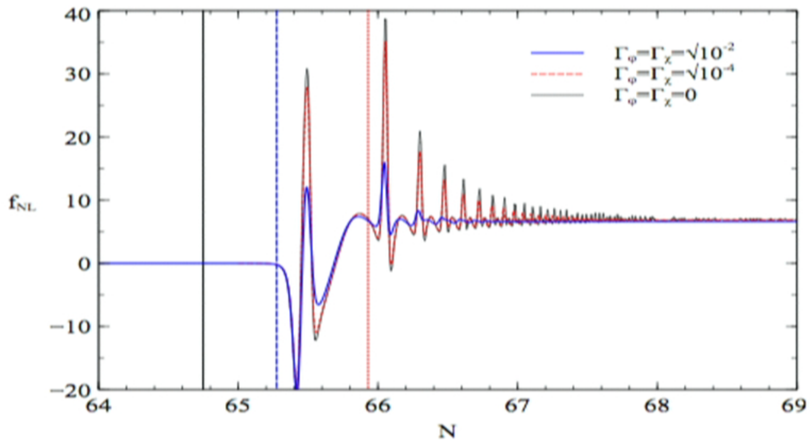
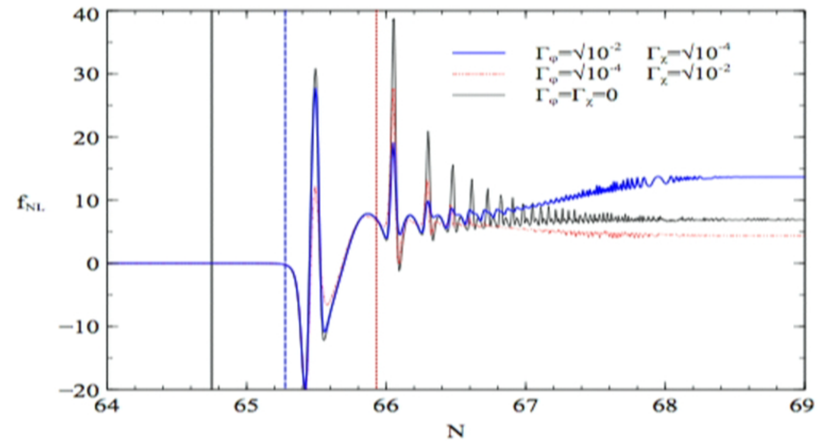
Similar behaviour to the Curvaton/Axionic–Curvaton scenario

[T.Kobayashi, M.Kawasaki, and F.Takahashi (2011,2012)]





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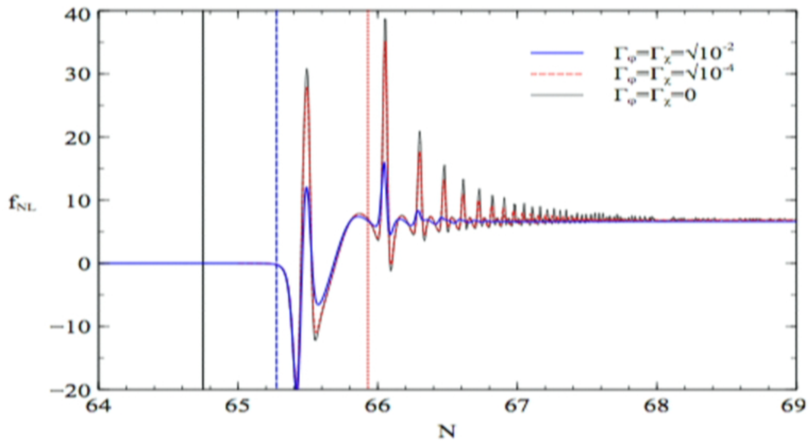
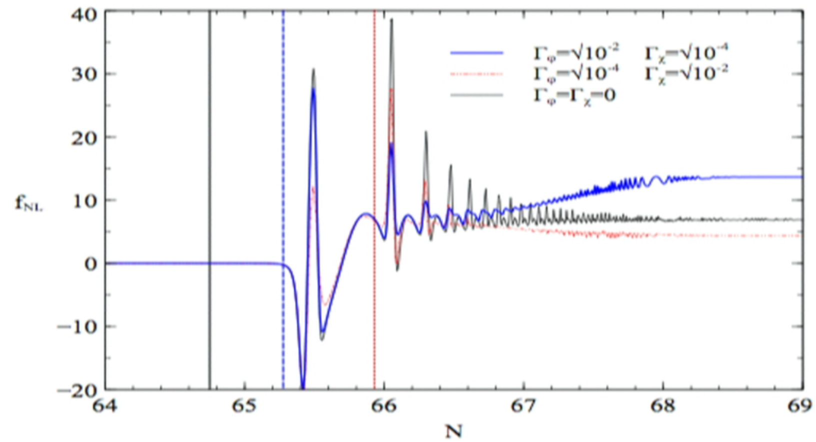
$$f_{\rm NL}(t_e) \approx 0,$$
$$n_s(t_e) = 0.969,\ r(t_e) = 0.124$$

Γ_φ	Γ_χ	$f_{\rm NL}^{\rm final}$	$n_s^{\rm final}$	$r^{\rm final}$
0	0	6.88	0.935	4.6×10^{-4}
$\sqrt{10^{-2}}$	$\sqrt{10^{-2}}$	6.59	0.969	4.3×10^{-4}
$\sqrt{10^{-4}}$	$\sqrt{10^{-4}}$	6.83	0.965	4.6×10^{-4}
$\sqrt{10^{-2}}$	$\sqrt{10^{-4}}$	13.66	0.963	1.0×10^{-3}
$\sqrt{10^{-4}}$	$\sqrt{10^{-2}}$	4.37	0.974	2.7×10^{-4}





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Summary and Outlook

- Amplitude of $f_{\mathrm{NL}}^{\mathrm{local}}$ has limited power to discriminate between models
- Except its sign which seems to be preserved
- Sensitivity to reheating is model (and parameter) dependent
- If $f_{\mathrm{NL}}^{\mathrm{local}}$ is large at the end of inflation, it typically remains large (reverse not always true)
- Spectral index is less sensitive to reheating
- Worth keeping in mind the degree of fine tuning required to generate large f_{NL} in the first place

- More models – how generic are our conclusions?
- Does reheating affect other bispectrum shapes: folded, equilateral?
- Sensitivity of "higher order" observables $\Rightarrow$ the trispectrum: τ_{NL}, g_{NL}
- Signatures of reheating in consistency relations between observables, such as $\tau_{\mathrm{NL}} \geq (\frac{6}{5} f_{\mathrm{NL}})^2$ [Suyama & Yamaguchi (2008)]
- Scale dependence of non–Gaussianity: $n_{f_{\mathrm{NL}}} = \frac{1}{f_{\mathrm{NL}}} \frac{\mathrm{d} \log f_{\mathrm{NL}}}{\mathrm{d} \log k}$
- Observables or combinations of observables that are insensitive to reheating
- Sensitivity of super–Hubble evolution to preheating [Chambers & Rajantie (2008,2009), Bond, Frolov, Huang & Kofman (2009)]