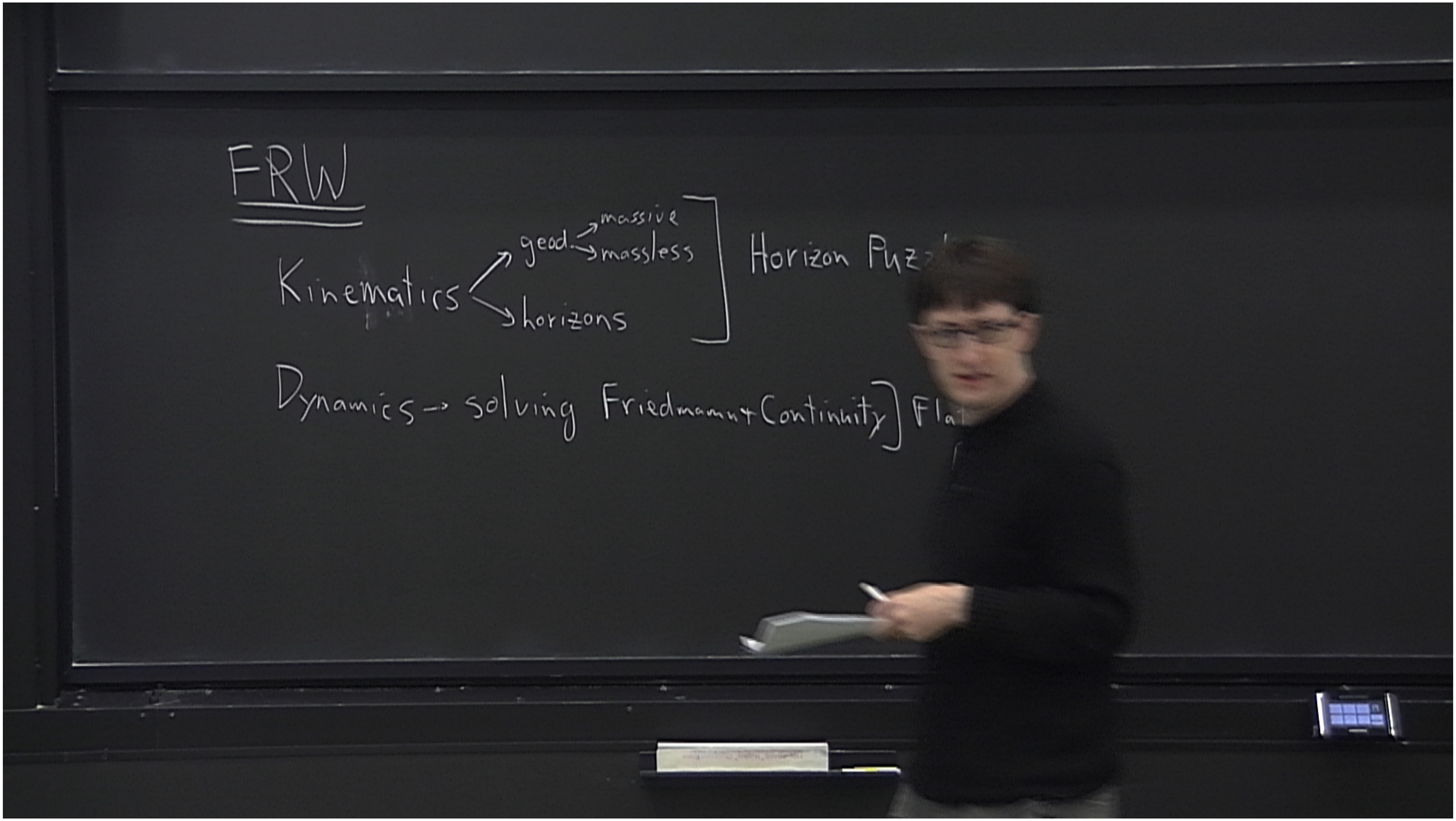
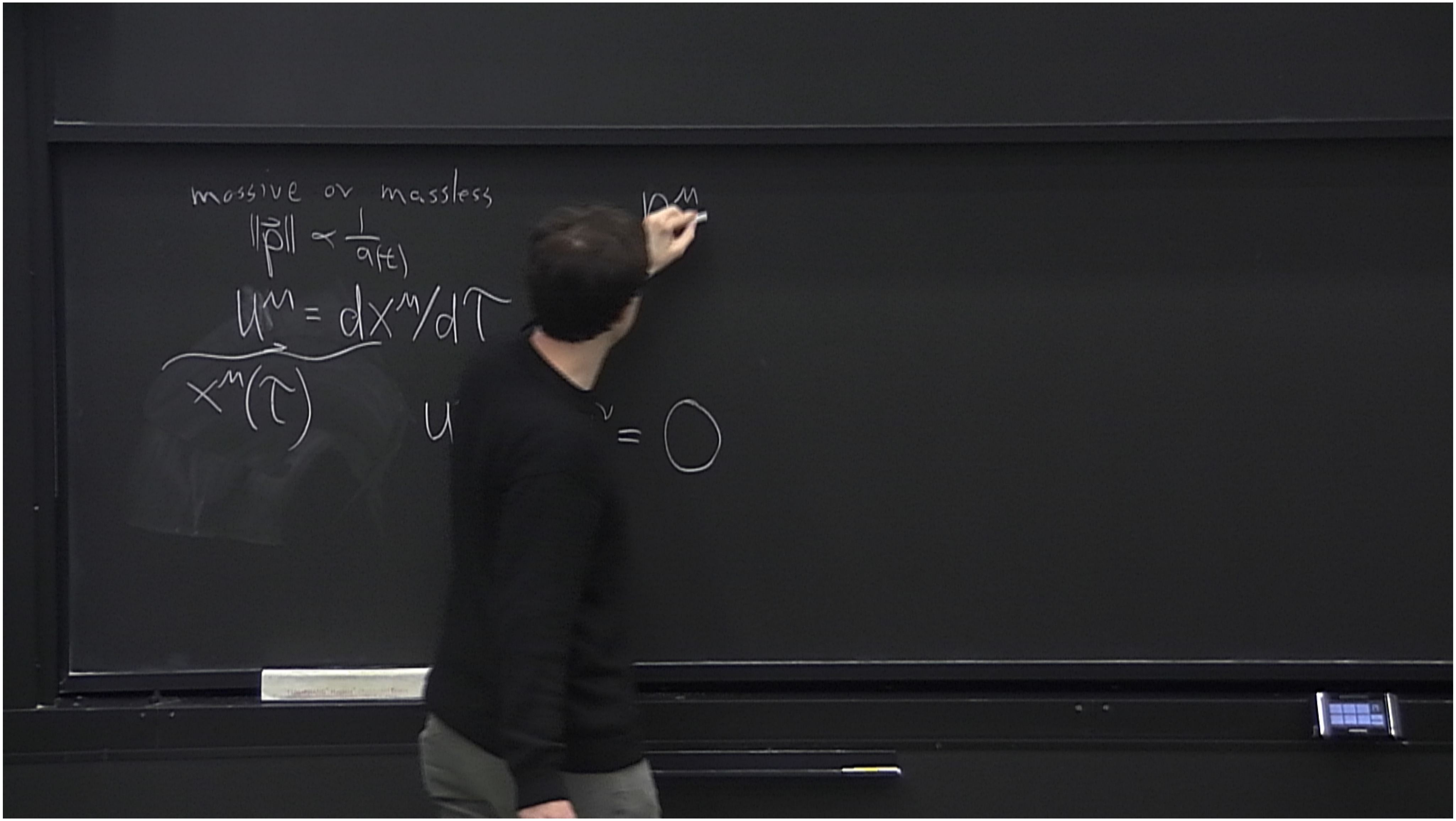
Title: 12/13 PSI - Cosmology Review Lecture 6

Date: Feb 26, 2013 11:30 AM

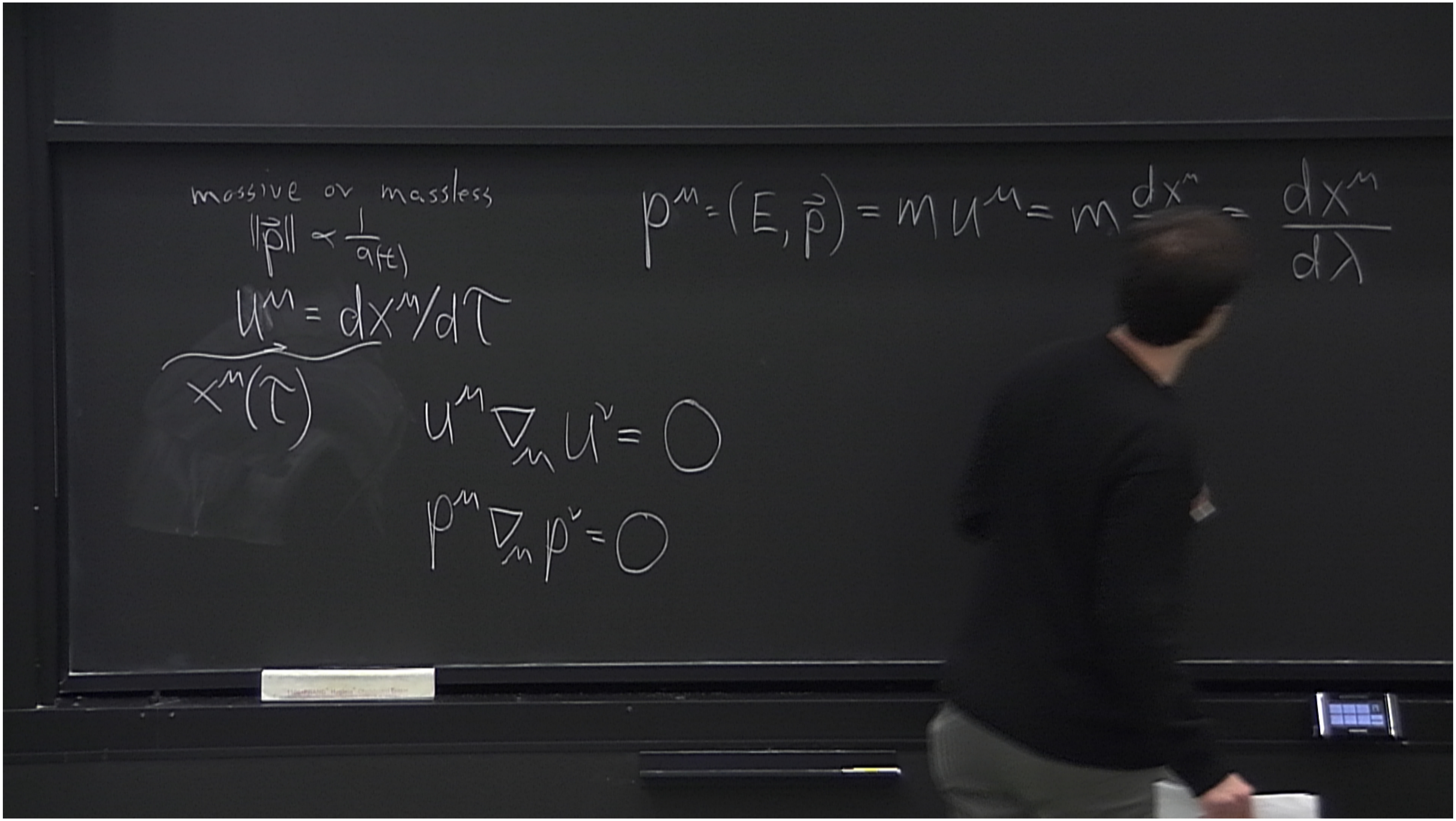
URL: http://www.pirsa.org/13020106

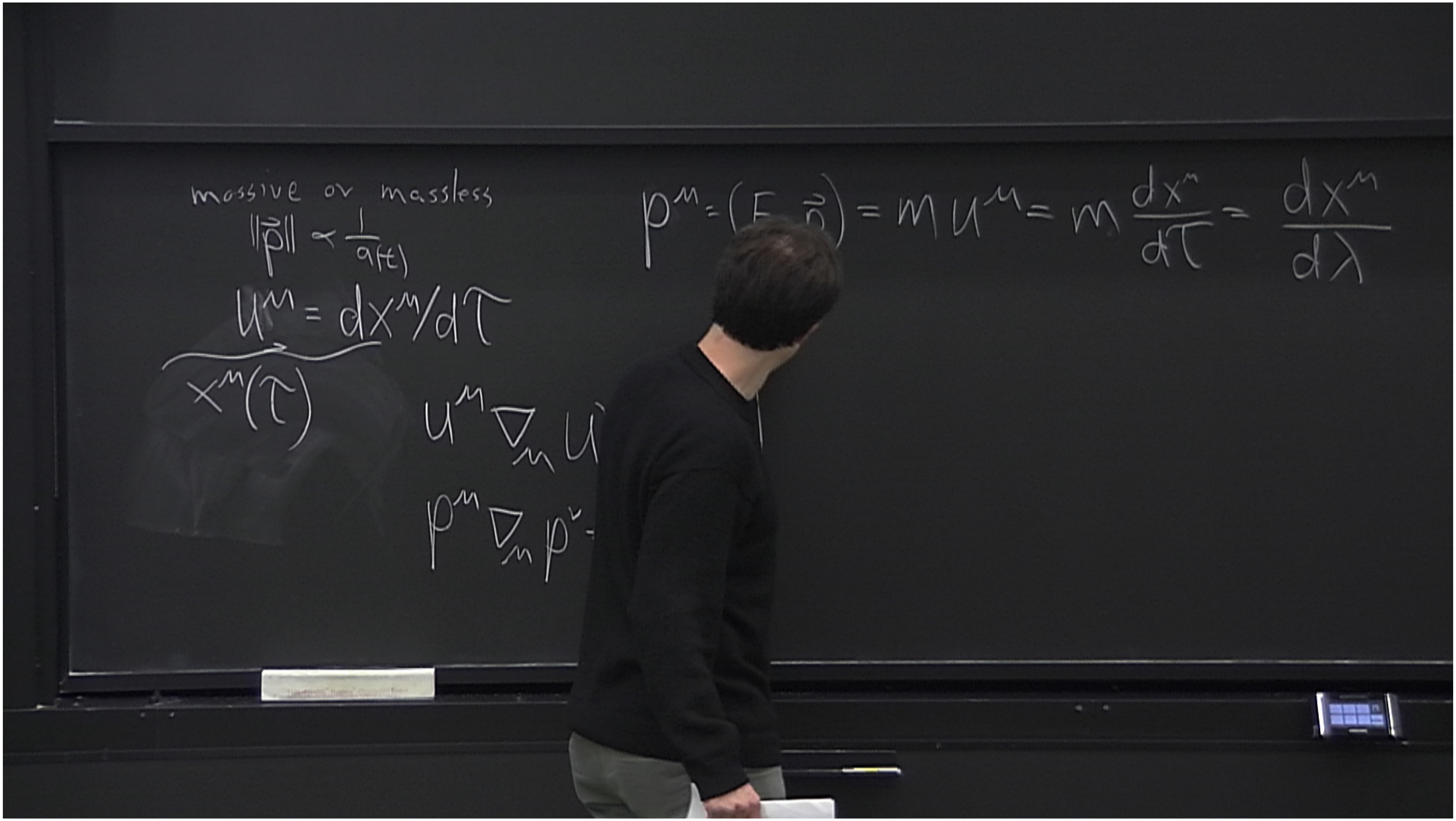
Abstract:

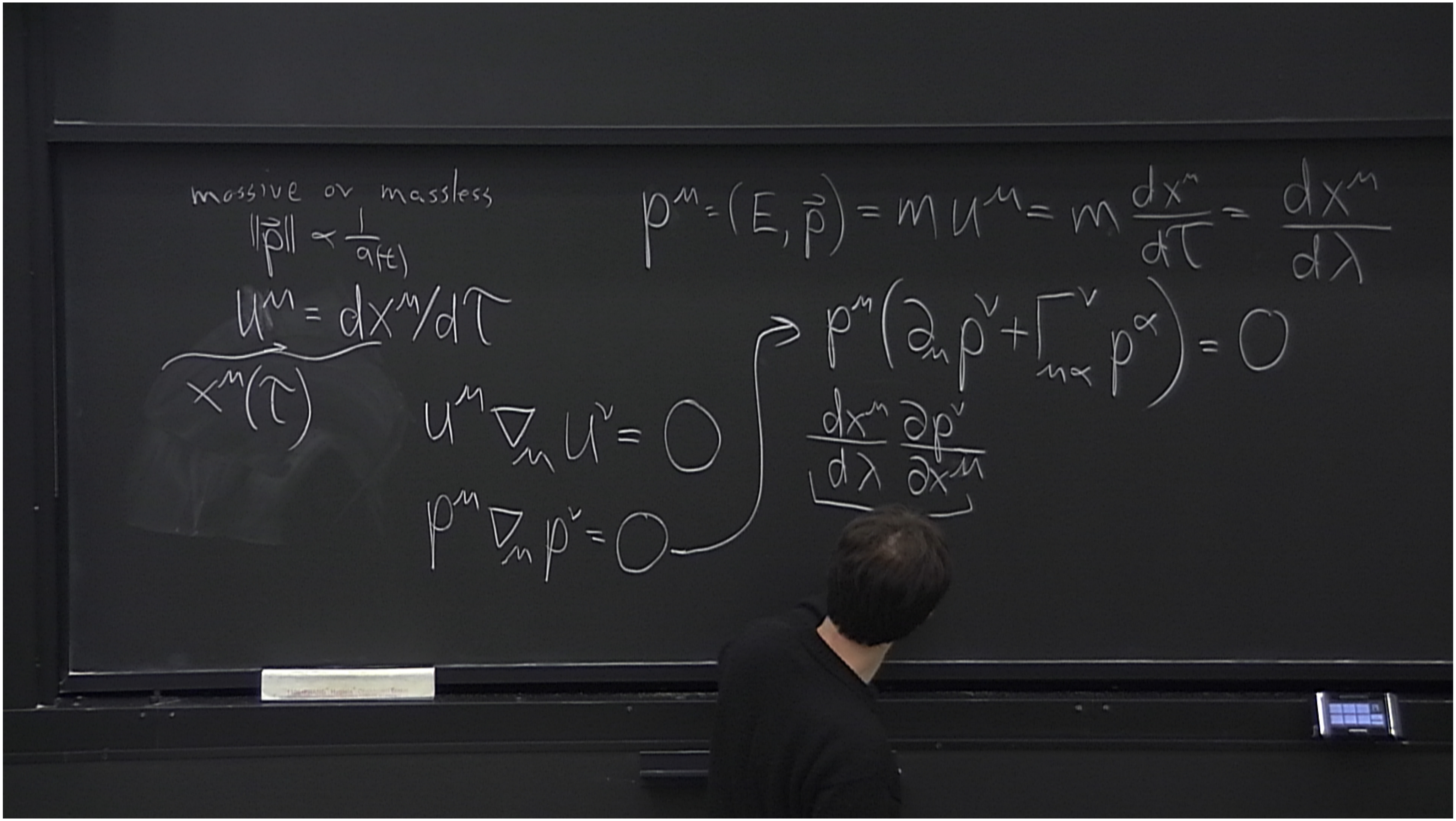

FRW
Kinematics
geod.
massive
massless
horizons
Horizon
Dynamics → solving Friedmann + Continuity



massive or massless
$\|\vec{p}\| \propto \frac{1}{a(t)}$
$u^\mu = dx^\mu/d\tau$
$x^\mu(\tau)$
$= 0$
p^μ


massive or massless
$\|\vec{p}\| \propto \frac{1}{a(t)}$
$u^\mu = dx^\mu / d\tau$
$x^\mu(\tau)$
$u^\mu \nabla_\mu u^\nu = 0$
$p^\mu \nabla_\mu p^\nu = 0$
$p^\mu = (E, \vec{p}) = m u^\mu = m \frac{dx^\mu}{\ } = \frac{dx^\mu}{d\lambda}$


massive or massless
$\|\vec{p}\| \propto \frac{1}{a(t)}$
$u^\mu = dx^\mu/d\tau$
$x^\mu(\tau)$
$u^\mu \nabla_\mu u$
$p^\mu \nabla_\mu p^\nu =$
$p^\mu = (E, \vec{p}) = m u^\mu = m \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\lambda}$


massive or massless
$\|\vec{p}\| \propto \frac{1}{a(t)}$
$u^\mu = dx^\mu/d\tau$
$x^\mu(\tau)$
$p^\mu = (E, \vec{p}) = m u^\mu = m \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\lambda}$
$p^\mu(\partial_\mu p^\nu + \Gamma^\nu_{\mu\alpha} p^\alpha) = 0$
$\frac{dx^\mu}{d\lambda} \frac{\partial p^\nu}{\partial x^\mu}$
$u^\mu \nabla_\mu u^\nu = 0$
$p^\mu \nabla_\mu p^\nu = 0$

massive or massless

$\|\vec{p}\| \propto \frac{1}{a(t)}$

$u^\mu = dx^\mu / d\tau$

$x^\mu(\tau)$

$$p^\mu = (E, \vec{p}) = m u^\mu = m \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\lambda}$$

$$u^\mu \nabla_\mu u^\nu = 0$$

$$p^\mu \nabla_\mu p^\nu = 0 \rightarrow p^\mu \left(\partial_\mu p^\nu + \Gamma^\nu_{\mu\alpha} p^\alpha \right) = 0 \longrightarrow$$

$$\underbrace{\frac{dx^\mu}{d\lambda} \frac{\partial p^\nu}{\partial x^\mu}}_{\frac{dp^\nu}{d\lambda}}$$

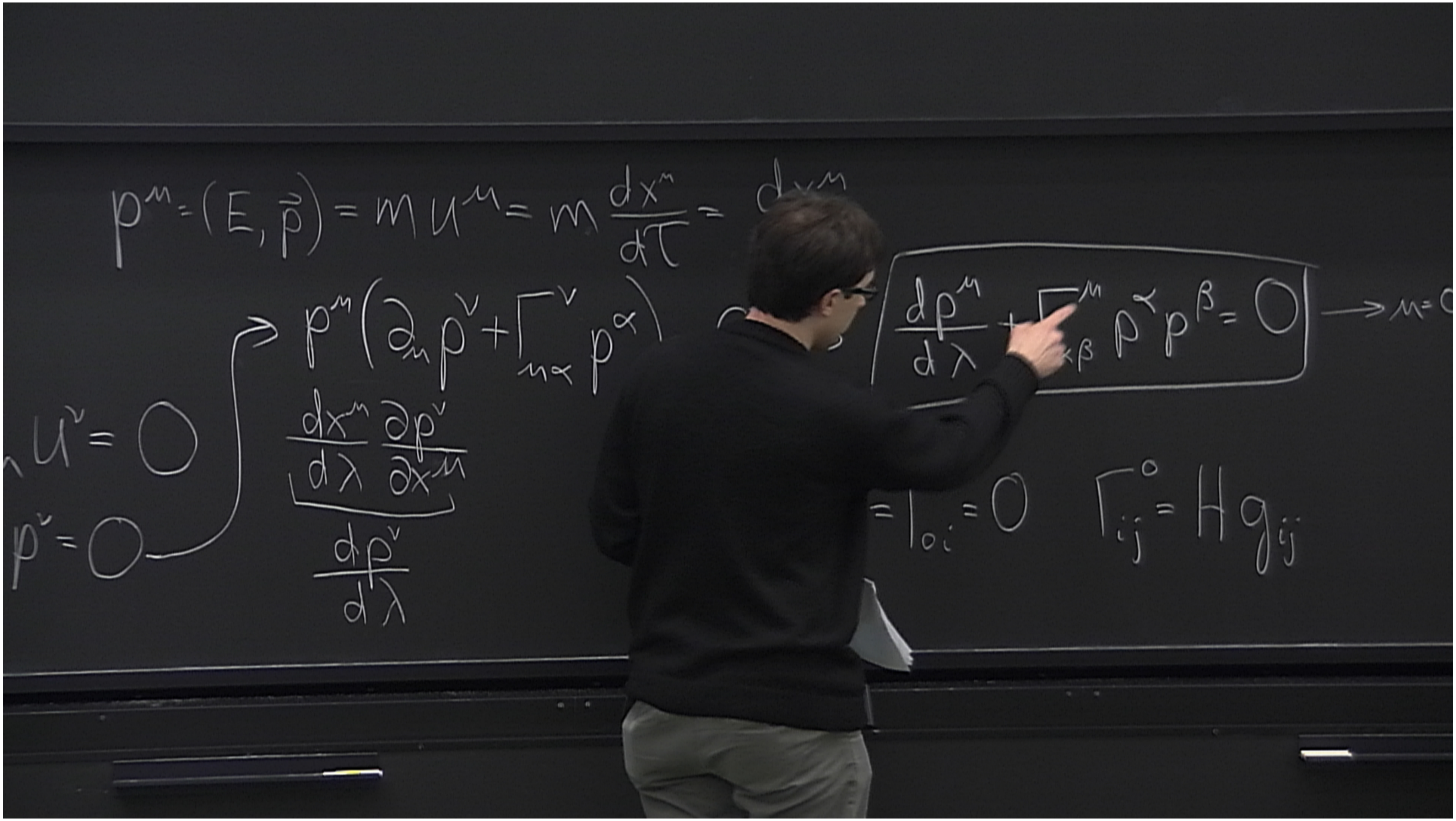
$$p^\mu = (E, \vec{p}) = m u^\mu = m \frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\lambda}$$

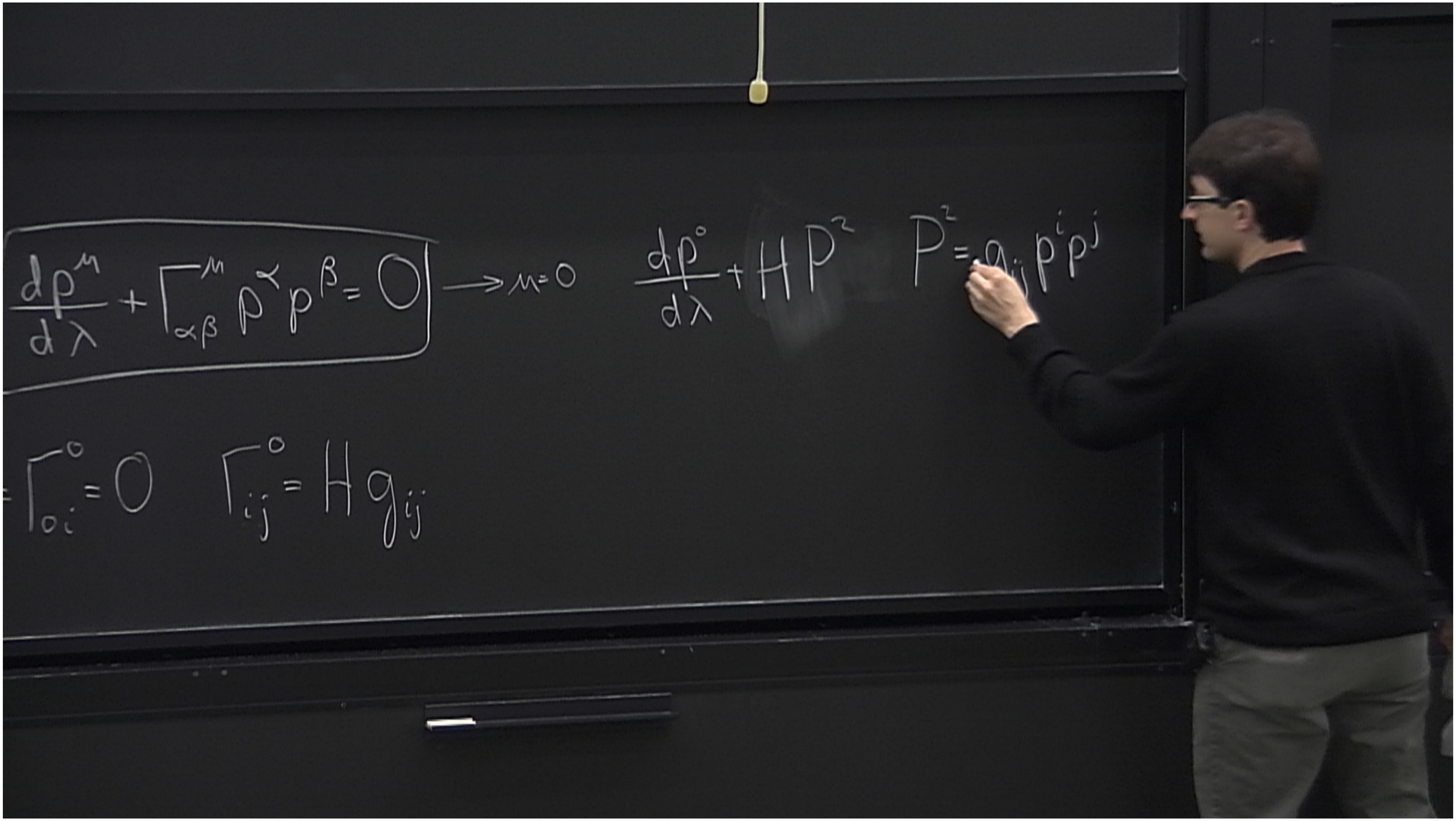
$$\ldots{}_\mu u^\nu = 0$$

$$\ldots p^\nu = 0$$

$$p^\mu\left(\partial_\mu p^\nu + \Gamma^\nu_{\mu\alpha} p^\alpha\right) = 0 \longrightarrow \boxed{\frac{dp^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = 0} \longrightarrow \mu = \ldots$$

$$\underbrace{\frac{dx^\mu}{d\lambda}\frac{\partial p^\nu}{\partial x^\mu}}_{\displaystyle \frac{dp^\nu}{d\lambda}}$$


$p^\mu = (E, \vec{p}) = m u^\mu = m \frac{dx^\mu}{d\tau} =$
$p^\mu(\partial_\mu p^\nu + \Gamma^\nu_{\mu\alpha} p^\alpha)$
$\frac{dp^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = 0 \rightarrow \mu = 0$
$u^\nu = 0$
$p^\nu = 0$
$\frac{dx^\mu}{d\lambda} \frac{\partial p^\nu}{\partial x^\mu}$
$\frac{dp^\nu}{d\lambda}$
$= \Gamma_{0i} = 0$
$\Gamma^0_{ij} = H g_{ij}$


$\boxed{\frac{dp^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} p^\alpha p^\beta = 0} \longrightarrow \mu = 0$
$\frac{dp^0}{d\lambda} + H P^2$
$P^2 = g_{ij} p^i p^j$
$\Gamma^0_{0i} = 0$
$\Gamma^0_{ij} = H g_{ij}$

$d\lambda$

$$\frac{1}{P}\frac{dP}{\underbrace{p^0\,d\lambda}_{=\,p^0 d\lambda\,=\,dt}} + \underbrace{H}_{=\,\frac{\dot a}{a}} = 0$$

$$\frac{1}{P}\,dP = -\frac{1}{a}\,da$$

$$\ln P = -\ln a + C$$

$$\boxed{P \propto \frac{1}{a}}$$

$d\lambda$

$$\frac{1}{P}\frac{dP}{\underbrace{p^0\, d\lambda}_{=\,p^0 d\lambda\,=\,dt}} + \underbrace{H}_{=\,\frac{\dot a}{a}} = 0$$

$$\frac{1}{P}\,dP = -\frac{1}{a}\,da$$

$$\ln P = -\ln a + C$$

$$\boxed{P \propto \frac{1}{a}}$$

$$P = mv$$

$$P \approx m v \gamma \qquad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$v \propto \frac{1}{a}$$

$d\lambda$

$$\frac{1}{P}\frac{dP}{\underbrace{p^0\, d\lambda}_{\substack{= p^0 d\lambda \\ = dt}}} + \underbrace{H}_{= \frac{\dot a}{a}} = 0$$

$$\frac{dx^\mu}{d\lambda}$$

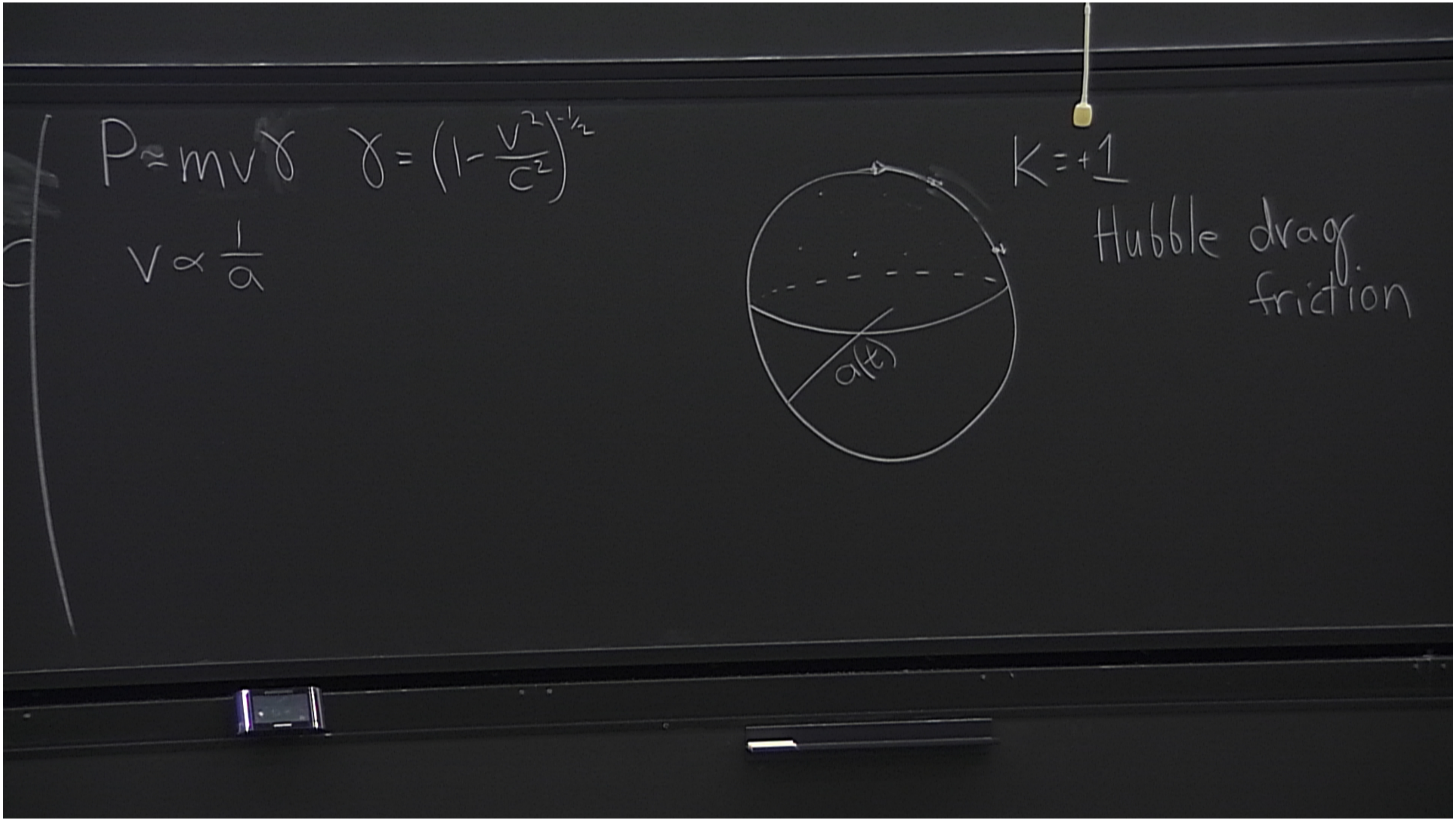
$$\frac{1}{P}\frac{dP}{} = -\frac{1}{a}\frac{da}{}$$

$$\ln P = -\ln a + C$$

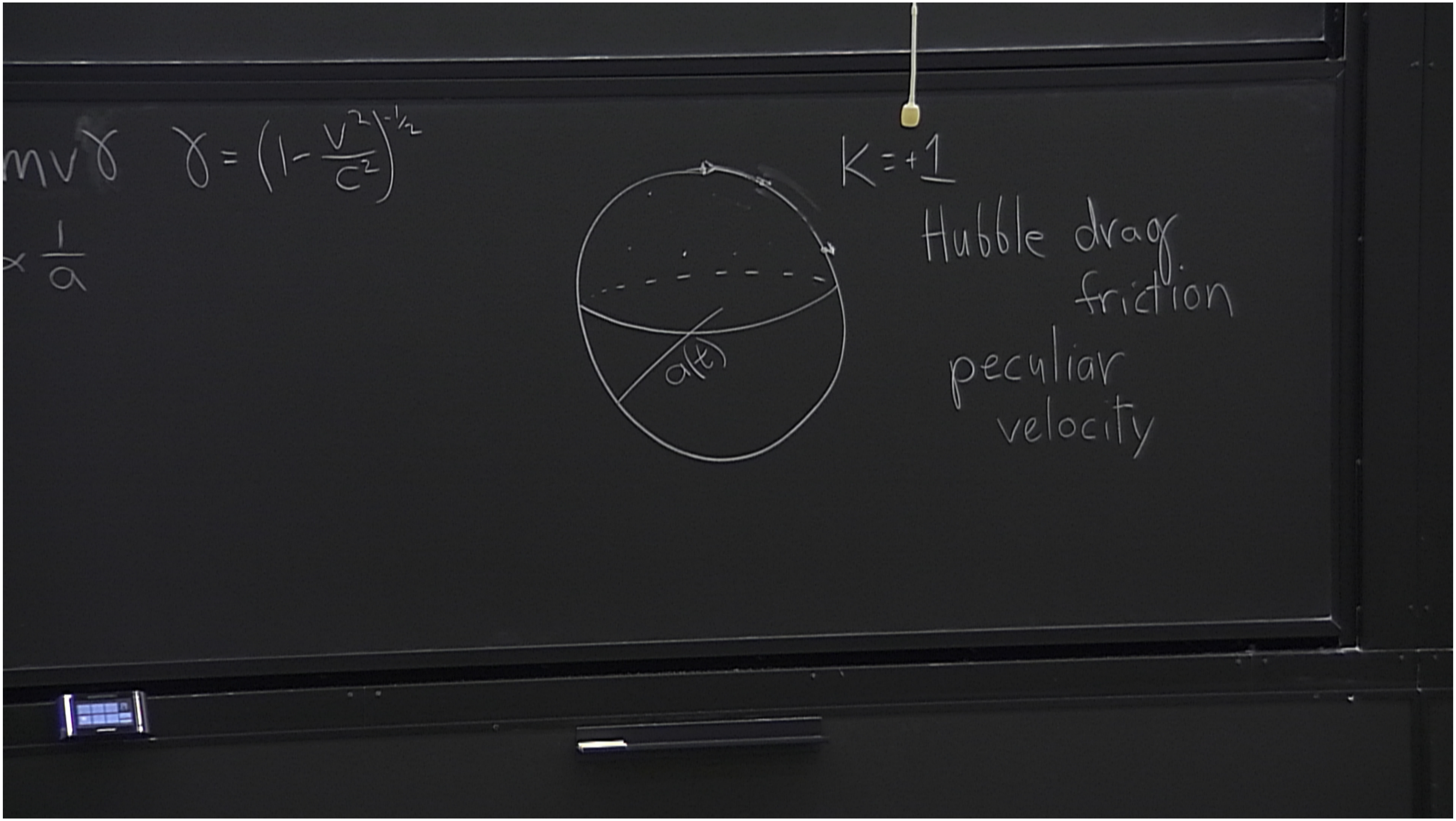
$$\boxed{P \propto \frac{1}{a}}$$

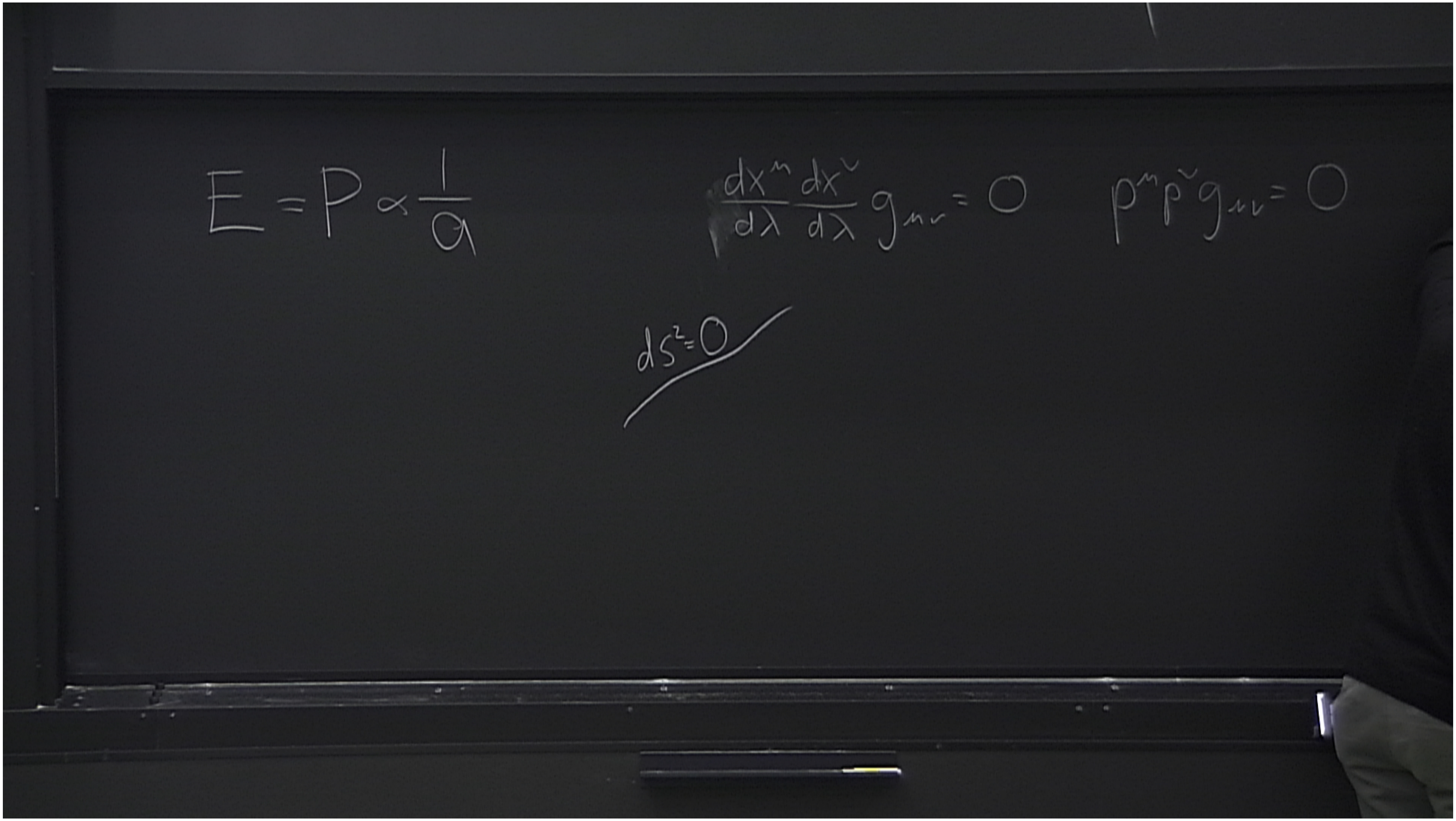
$$P \approx m v \gamma$$

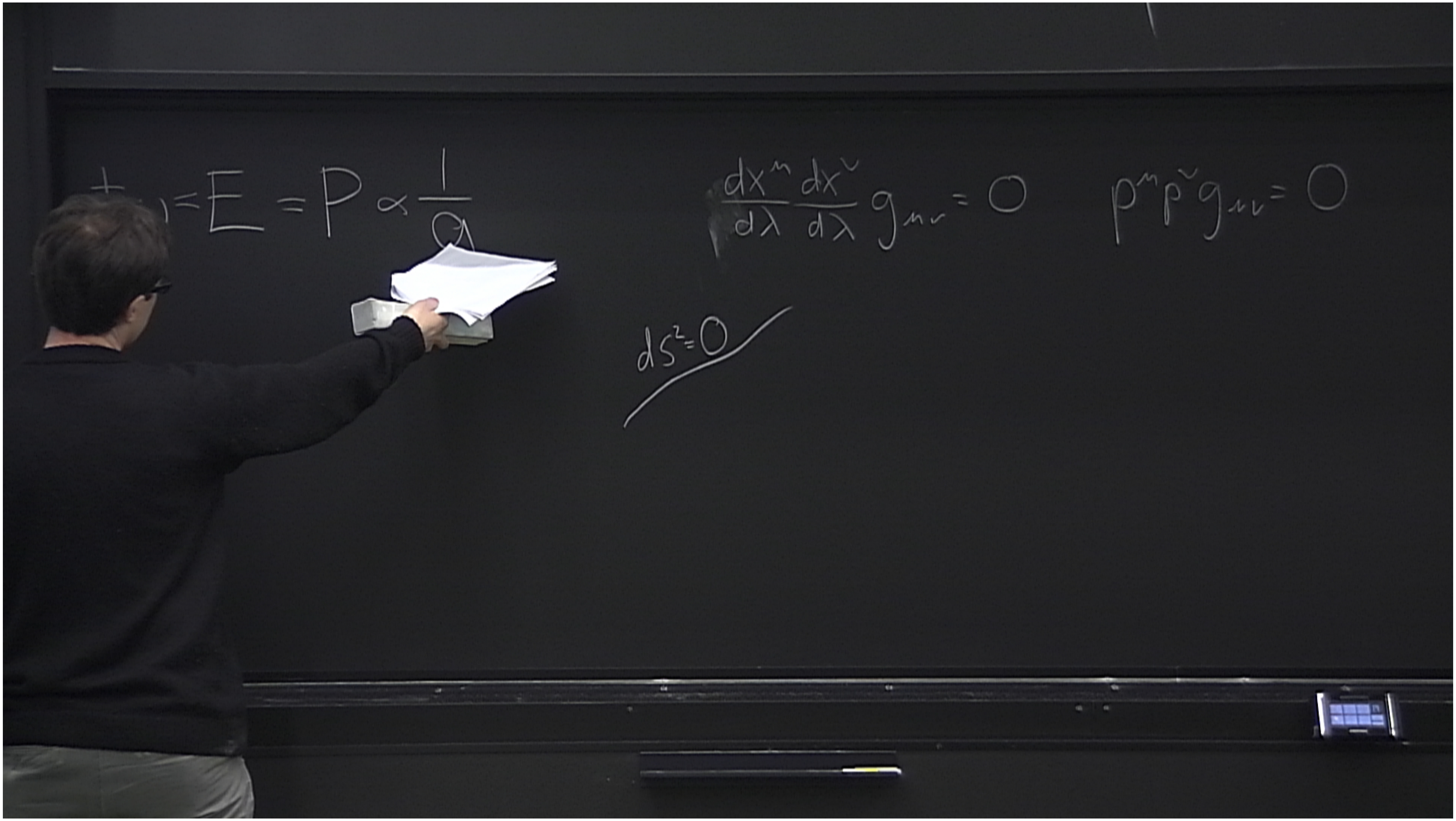
$$v \propto \frac{1}{a}$$

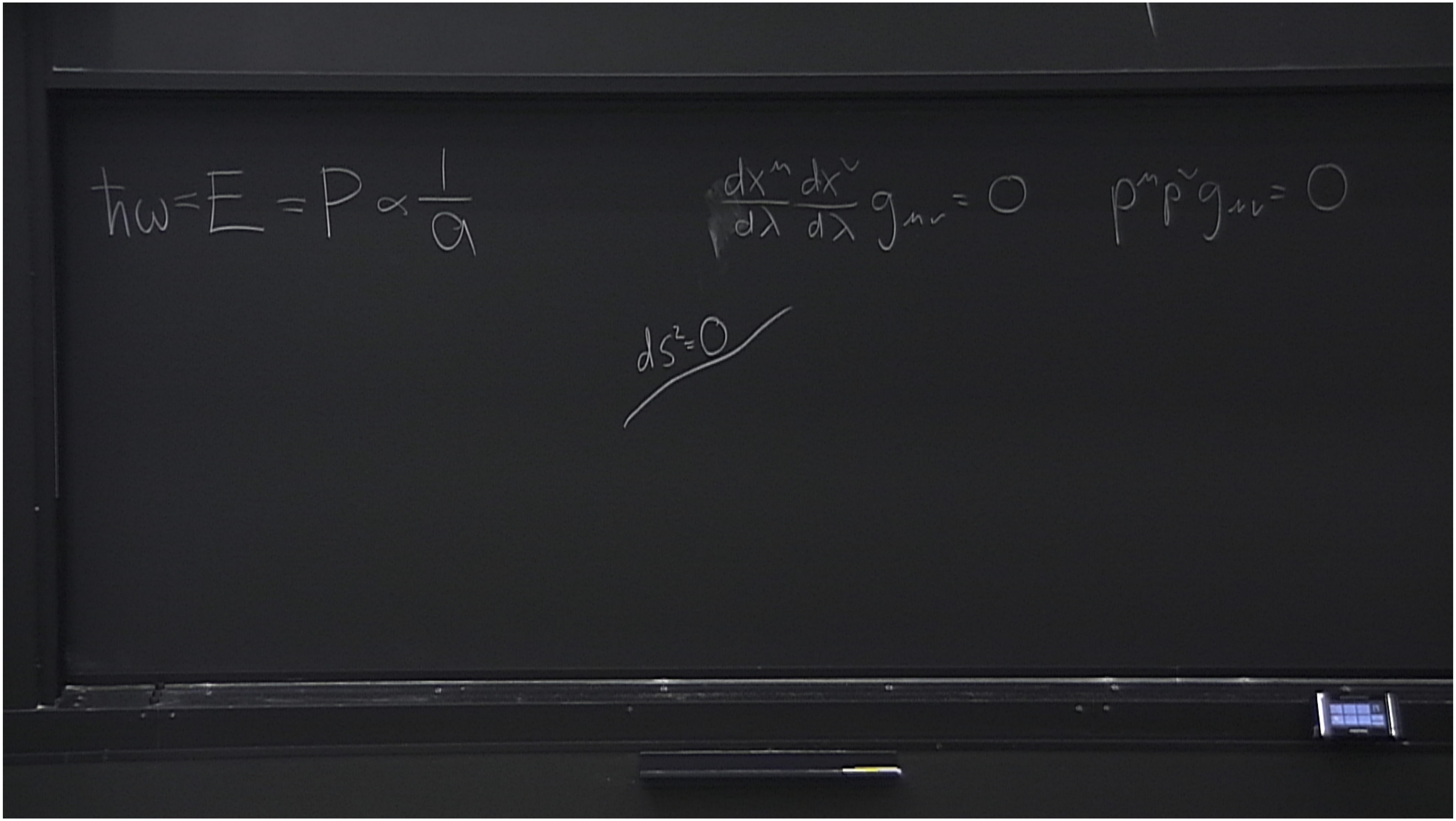


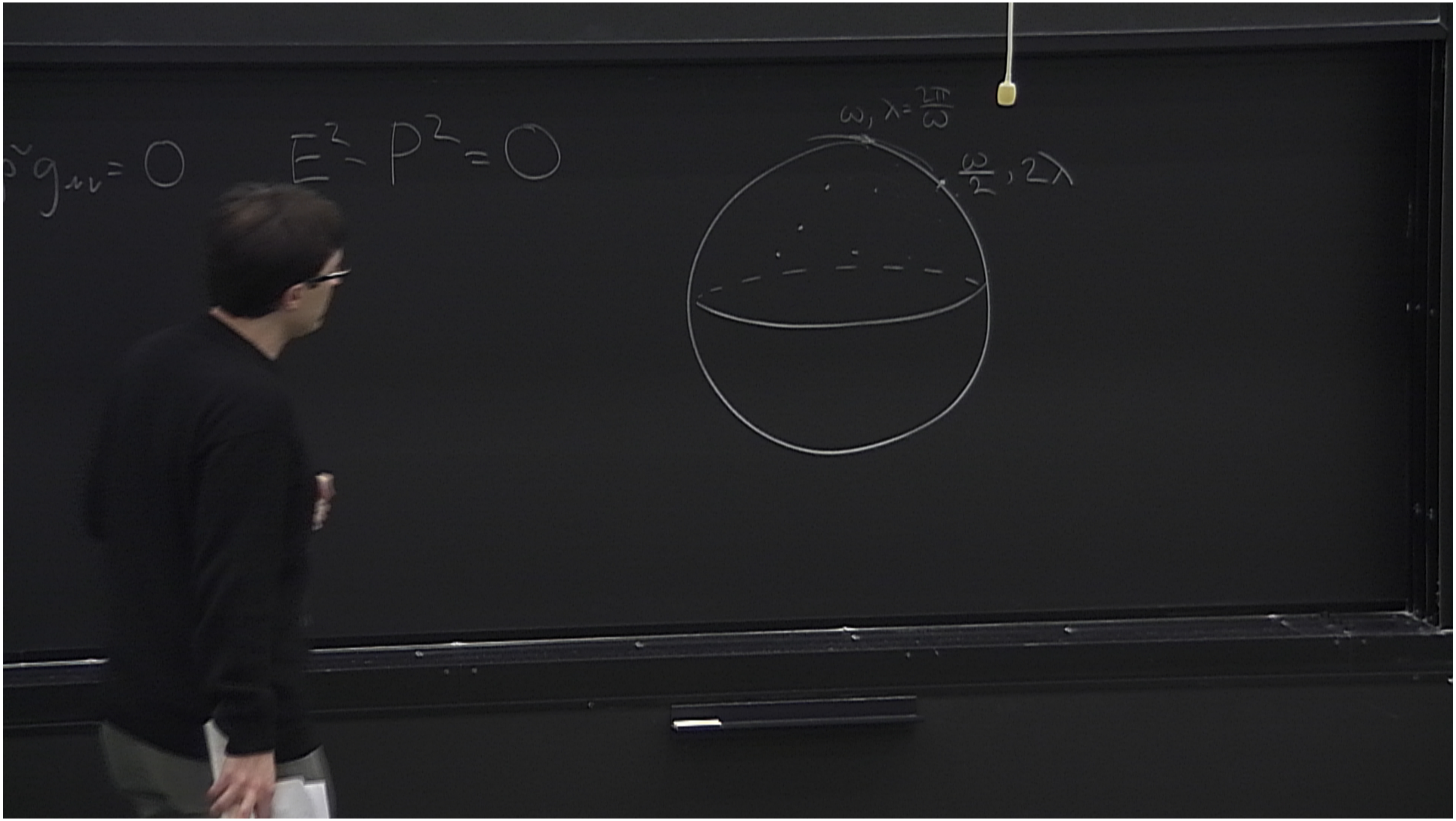
$P \simeq m v \gamma$ $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$
$v \propto \frac{1}{a}$
$K = +1$
$a(t)$
Hubble drag
friction

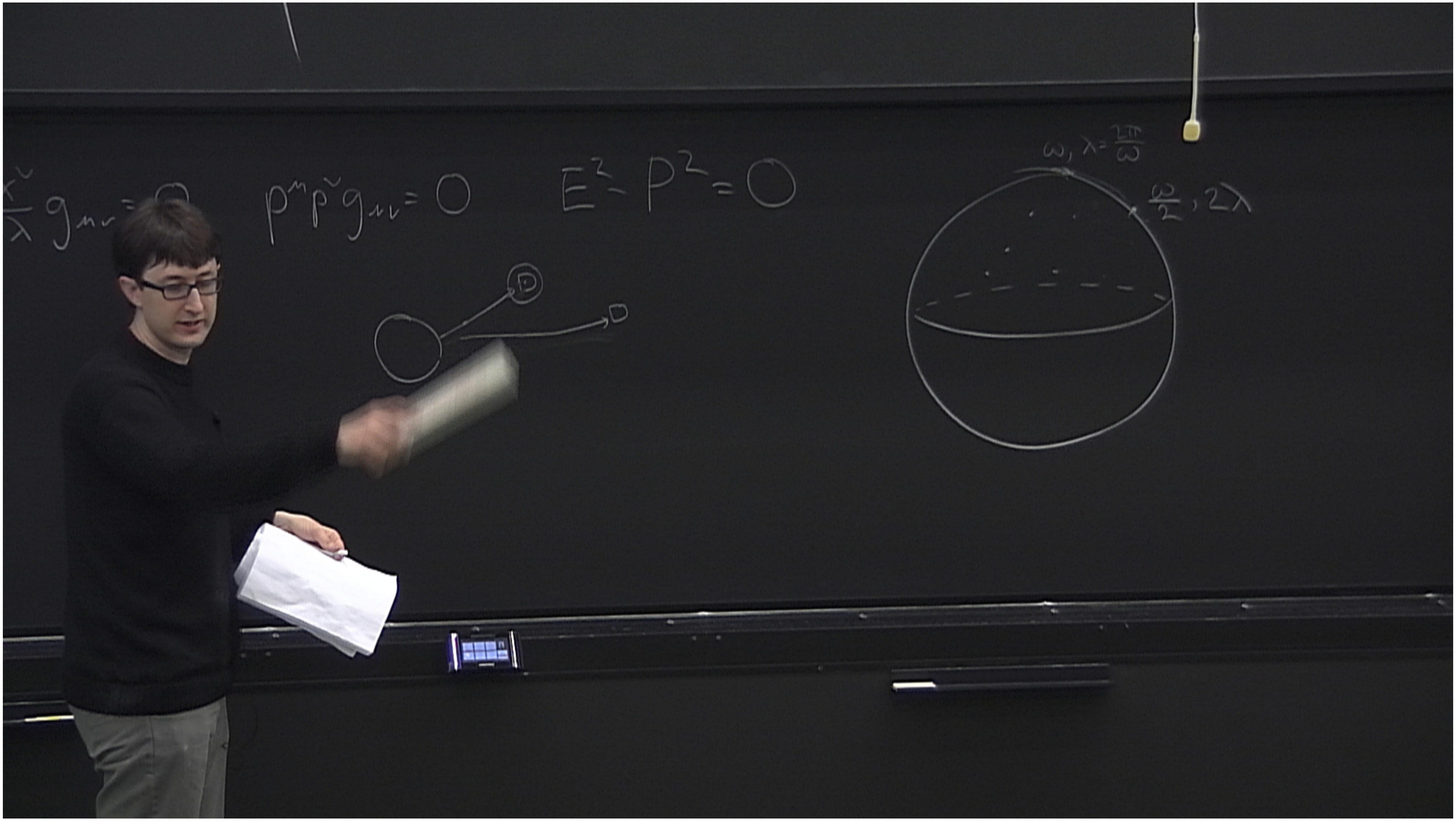

mvγ
$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$
$\propto \frac{1}{a}$
a(t)
K = +1
Hubble drag
friction
peculiar
velocity

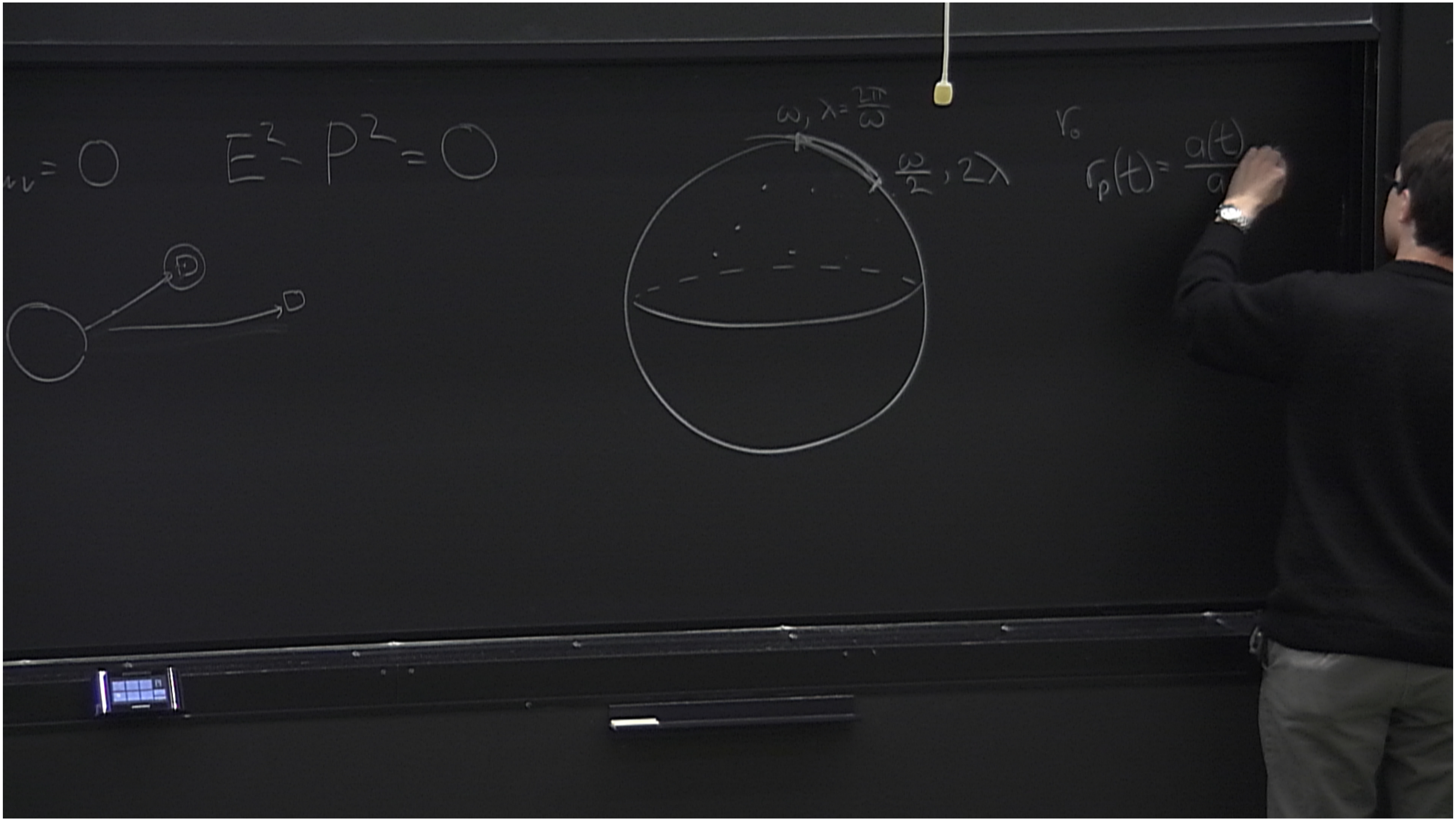

$E = P \propto \frac{1}{a}$
$\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} g_{\mu\nu} = 0$
$p^\mu p^\nu g_{\mu\nu} = 0$
$ds^2 = 0$

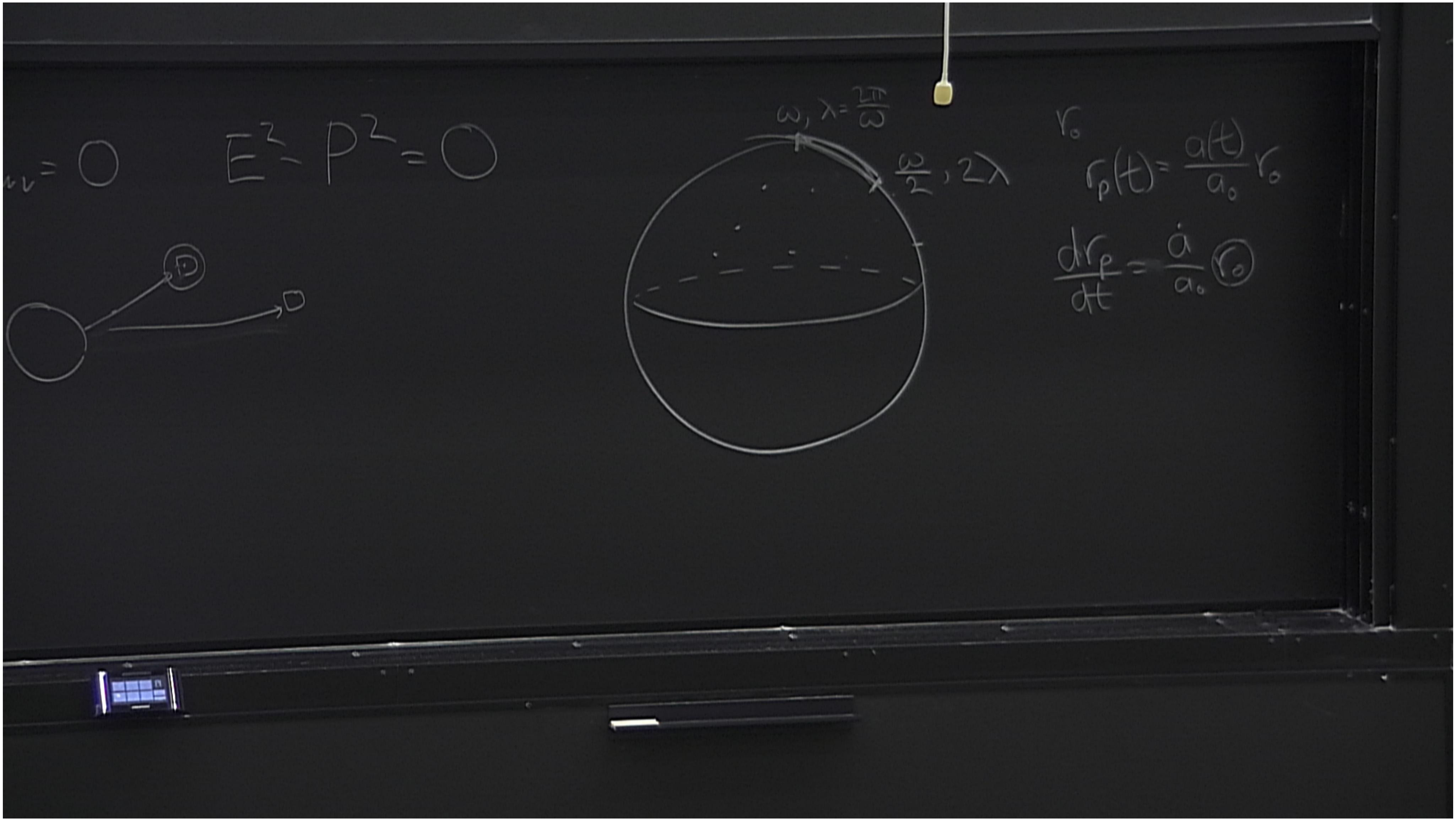

$\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}g_{\mu\nu}=0$
$p^\mu p^\nu g_{\mu\nu}=0$
$ds^2=0$

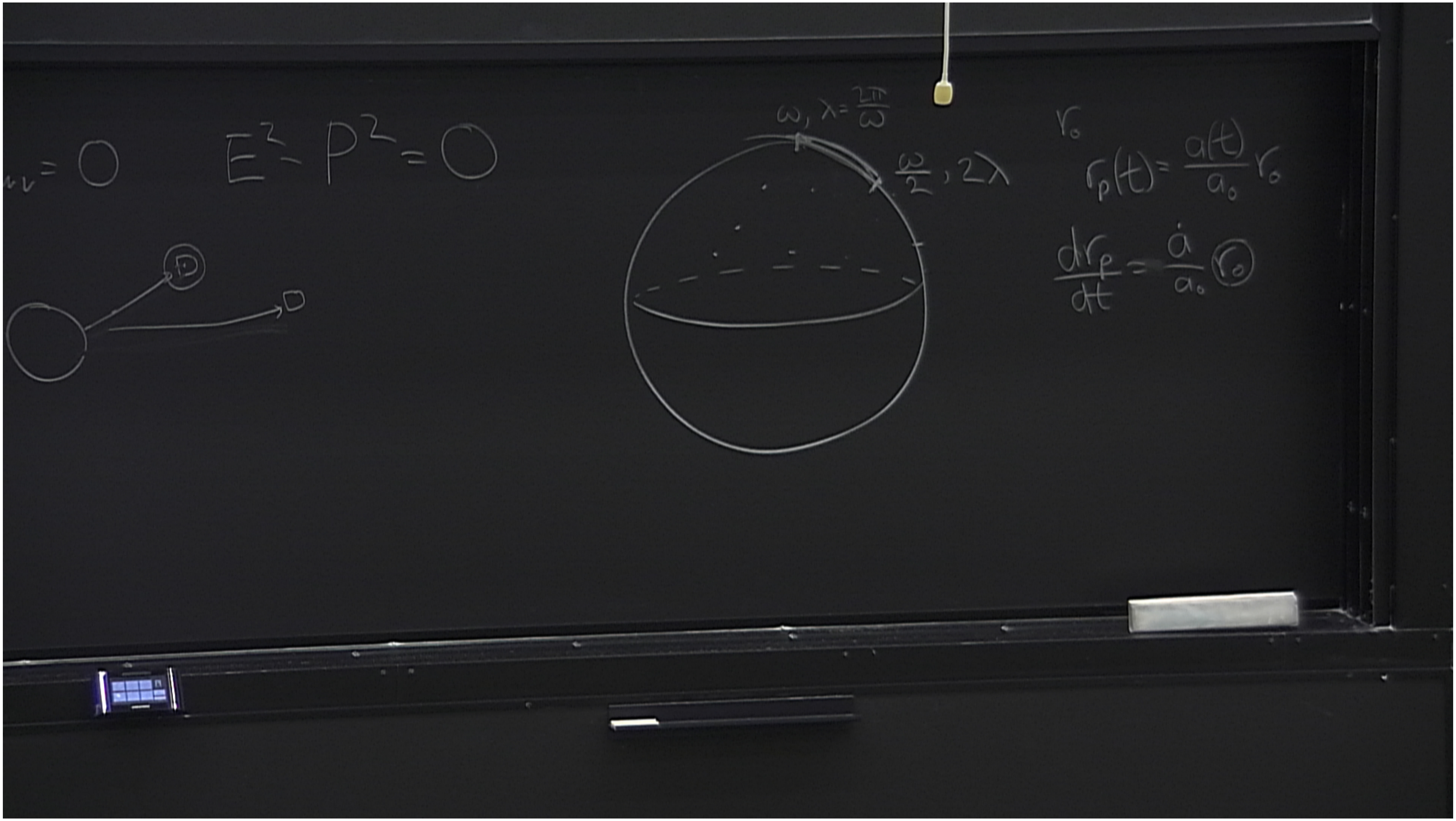

$\hbar\omega = E = P \propto \frac{1}{a}$
$\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} g_{\mu\nu} = 0$
$p^\mu p^\nu g_{\mu\nu} = 0$
$ds^2 = 0$

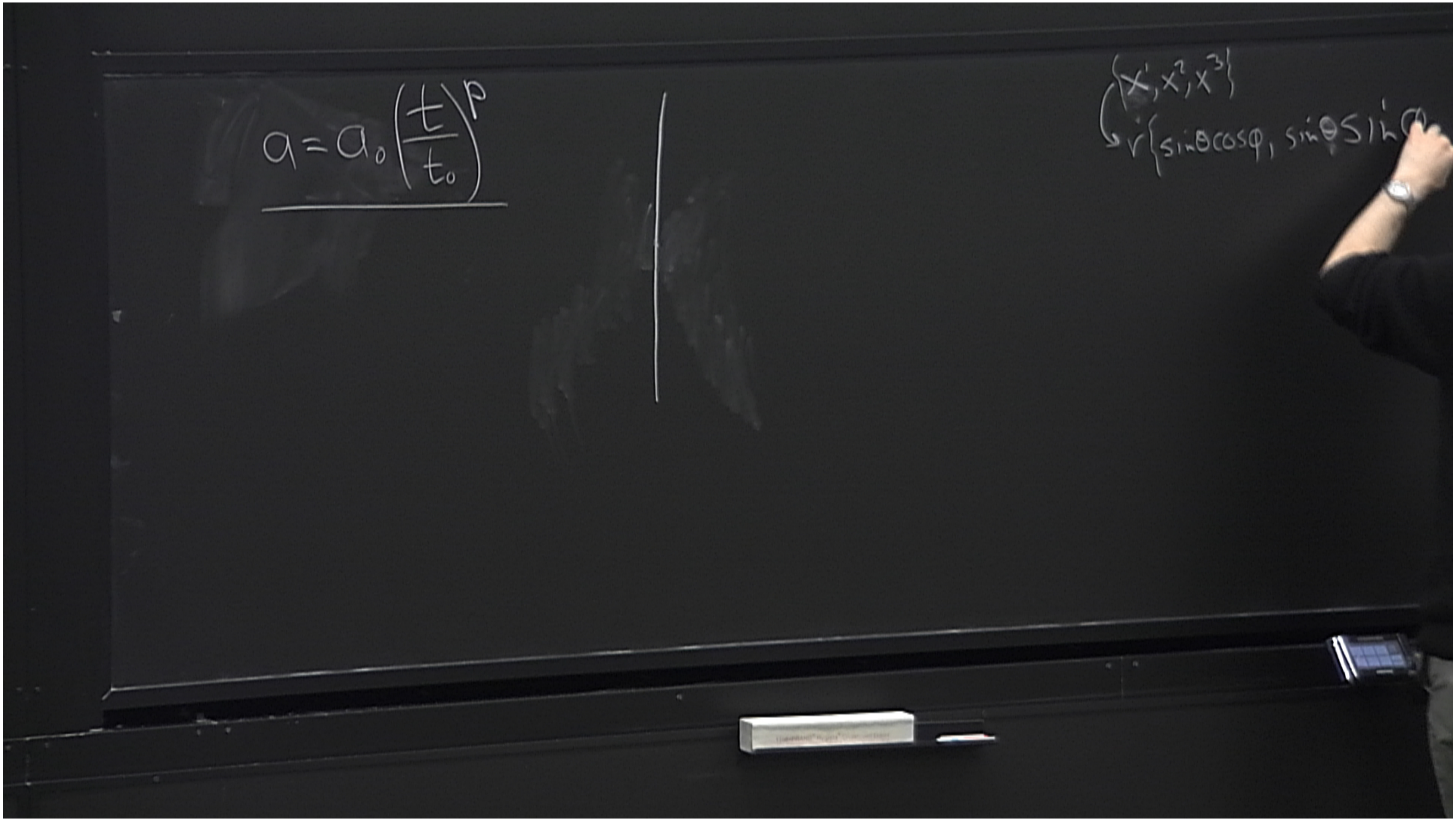

E² - P² = 0
ω, λ = 2π/ω
ω/2, 2λ

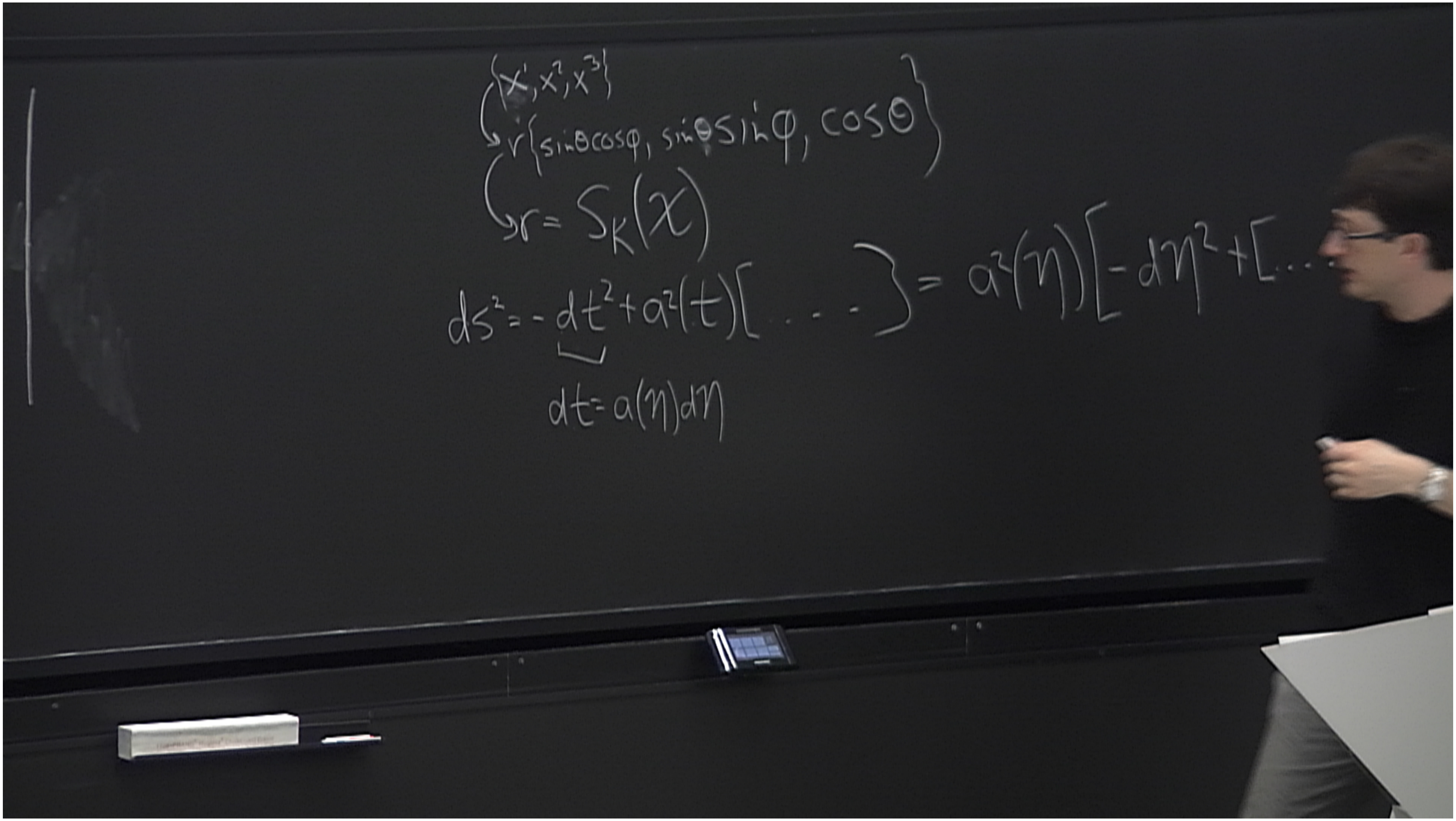

$p^\mu p^\nu g_{\mu\nu} = 0$
$E^2 - P^2 = 0$
$\omega, \lambda = \frac{2\pi}{\omega}$
$\frac{\omega}{2}, 2\lambda$


$E^2 - P^2 = 0$
$\omega, \lambda = \frac{2\pi}{\omega}$
$\frac{\omega}{2}, 2\lambda$
r_0
$r_p(t) = \frac{a(t)}{a}$

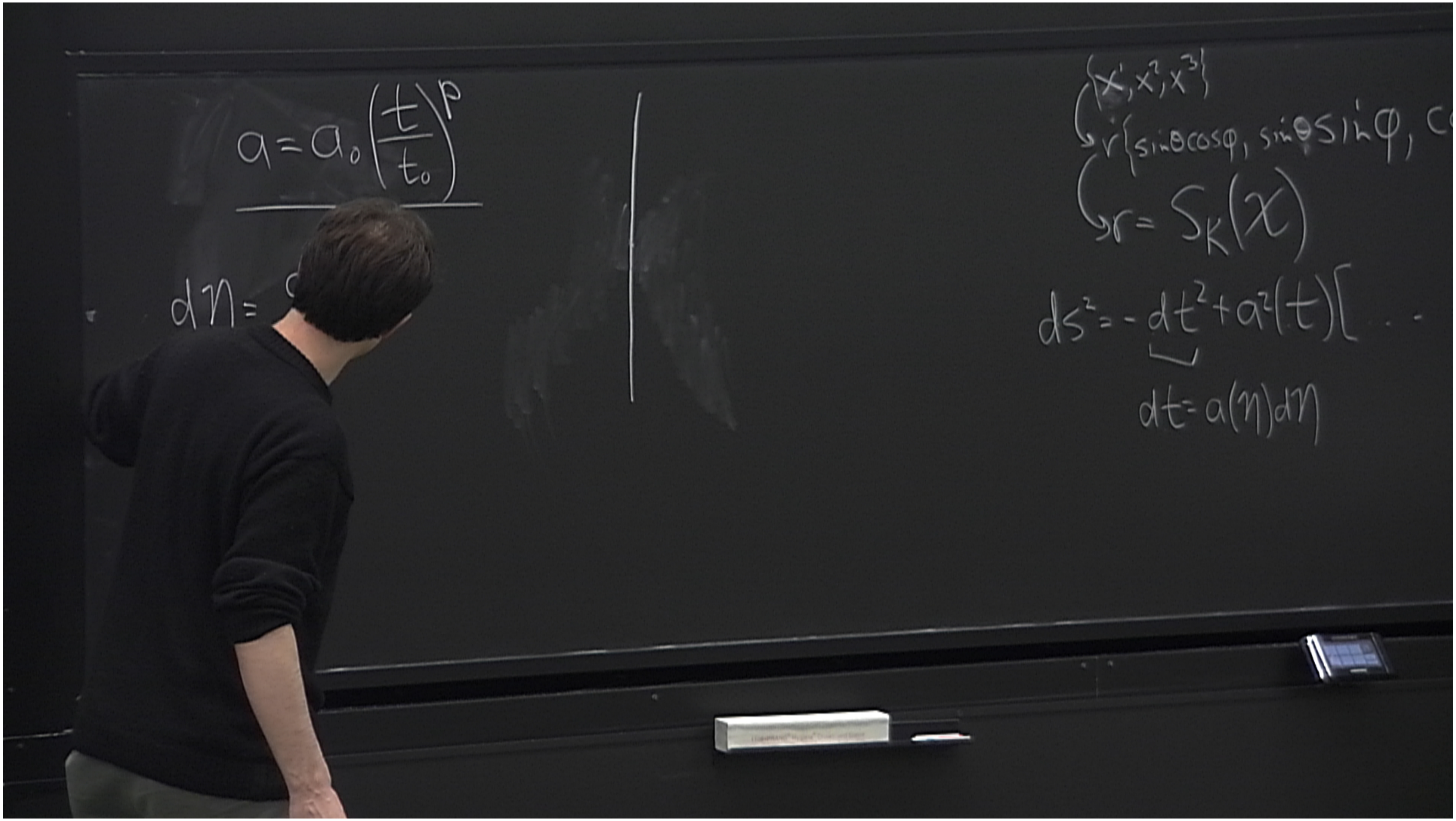

E² - P² = 0
ω, λ = 2π/ω
ω/2, 2λ
r₀
r_p(t) = a(t)/a₀ r₀
dr_p/dt = ȧ/a₀ r₀

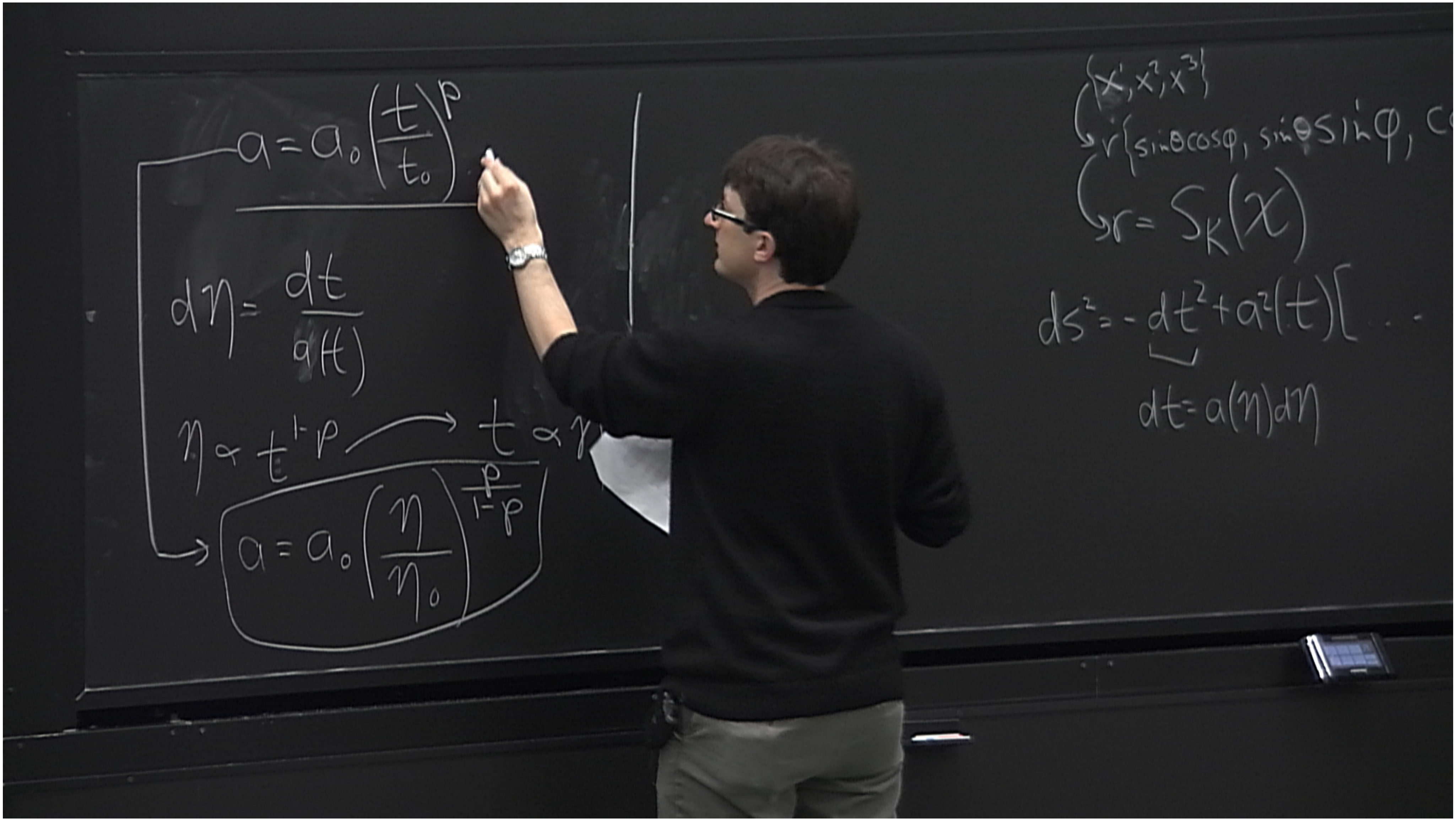

E² - P² = 0
ω, λ = 2π/ω
ω/2, 2λ
r₀
r_p(t) = a(t)/a₀ r₀
dr_p/dt = ȧ/a₀ r₀

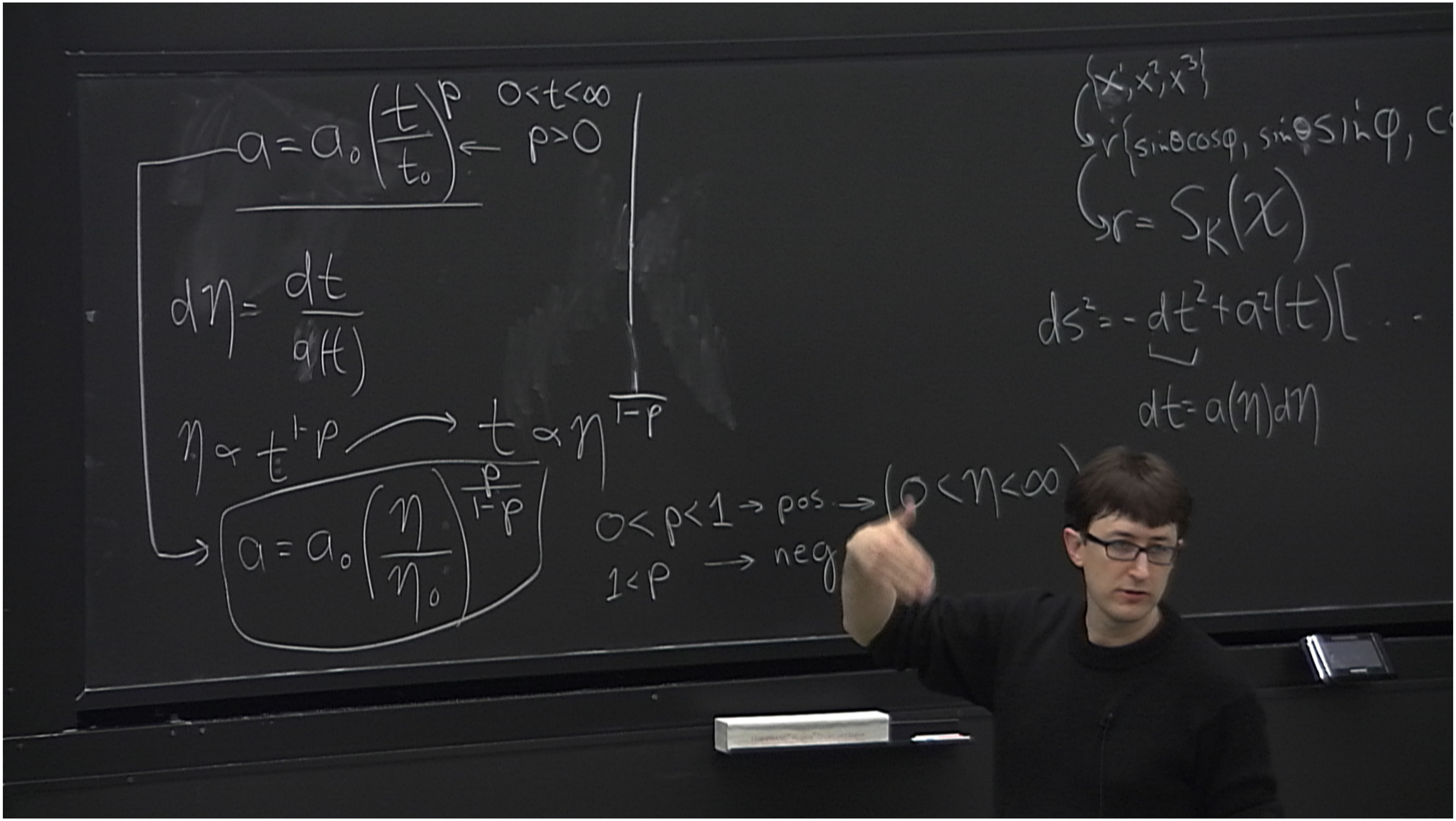

$a = a_0\left(\frac{t}{t_0}\right)^p$
$\{x^1, x^2, x^3\}$
$r\{\sin\theta\cos\varphi, \sin\theta\sin$



$\{x^1, x^2, x^3\}$
$r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$
$r = S_K(\chi)$
$ds^2 = -\underbrace{dt^2} + a^2(t)[\ldots] = a^2(\eta)[-d\eta^2 + [\ldots$
$dt = a(\eta)d\eta$


a = a₀ (t / t₀)^p
dη =
{x¹, x², x³}
r{sinθcosφ, sinθsinφ, c
r = S_K(χ)
ds² = - dt² + a²(t)[. . .
dt = a(η)dη


$a = a_0 \left(\frac{t}{t_0}\right)^p$
$d\eta = \frac{dt}{a(t)}$
$\eta \propto t^{1-p} \longrightarrow t \propto \eta^{\frac{1}{1-p}}$
$a = a_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}$
$\{x^1, x^2, x^3\}$
$r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, c$
$r = S_K(\chi)$
$ds^2 = -dt^2 + a^2(t)[\ \ldots$
$dt = a(\eta)d\eta$


a = a_0 (t/t_0)^p
0<t<∞
p>0
dη = dt/a(t)
η ∝ t^(1-p) → t ∝ η^(1/(1-p))
a = a_0 (η/η_0)^(p/(1-p))
0<p<1 → pos. → (0<η<∞)
1<p → neg
{x^1, x^2, x^3}
r{sinθcosφ, sinθsinφ, c
r = S_K(χ)
ds^2 = -dt^2 + a^2(t)[. . .
dt = a(η)dη

$\eta \propto t^{1-p} \longrightarrow t \propto \eta^{1-p}$

$dt = a(\eta)\,d\eta$

$$\boxed{a = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}}$$

$0 < p < 1 \to$ pos $\to (0 < \eta < \infty)$

$1 < p \longrightarrow$ neg $(-\infty < \eta < 0)$

$$a = a_0\left(\frac{t}{t_0}\right)^p \qquad 0<t<\infty \qquad p>0$$

$$d\eta = \frac{dt}{a(t)}$$

$$\eta \propto t^{1-p} \quad\to\quad t \propto \eta^{\frac{1}{1-p}}$$

$$\boxed{a = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}}$$

$$0<p<1 \to \text{pos} \to (0<\eta<\infty)$$

$$1<p \to \text{neg} \quad (-\infty<\eta<0)$$

$$\{x^1, x^2, x^3\} \to r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \text{c}\ldots \to r = S_K(\chi)$$

$$ds^2 = -\underbrace{dt^2}_{} + a^2(t)[\ \ldots$$

$$dt = a(\eta)\,d\eta$$

$$\{x^1, x^2, x^3\}$$

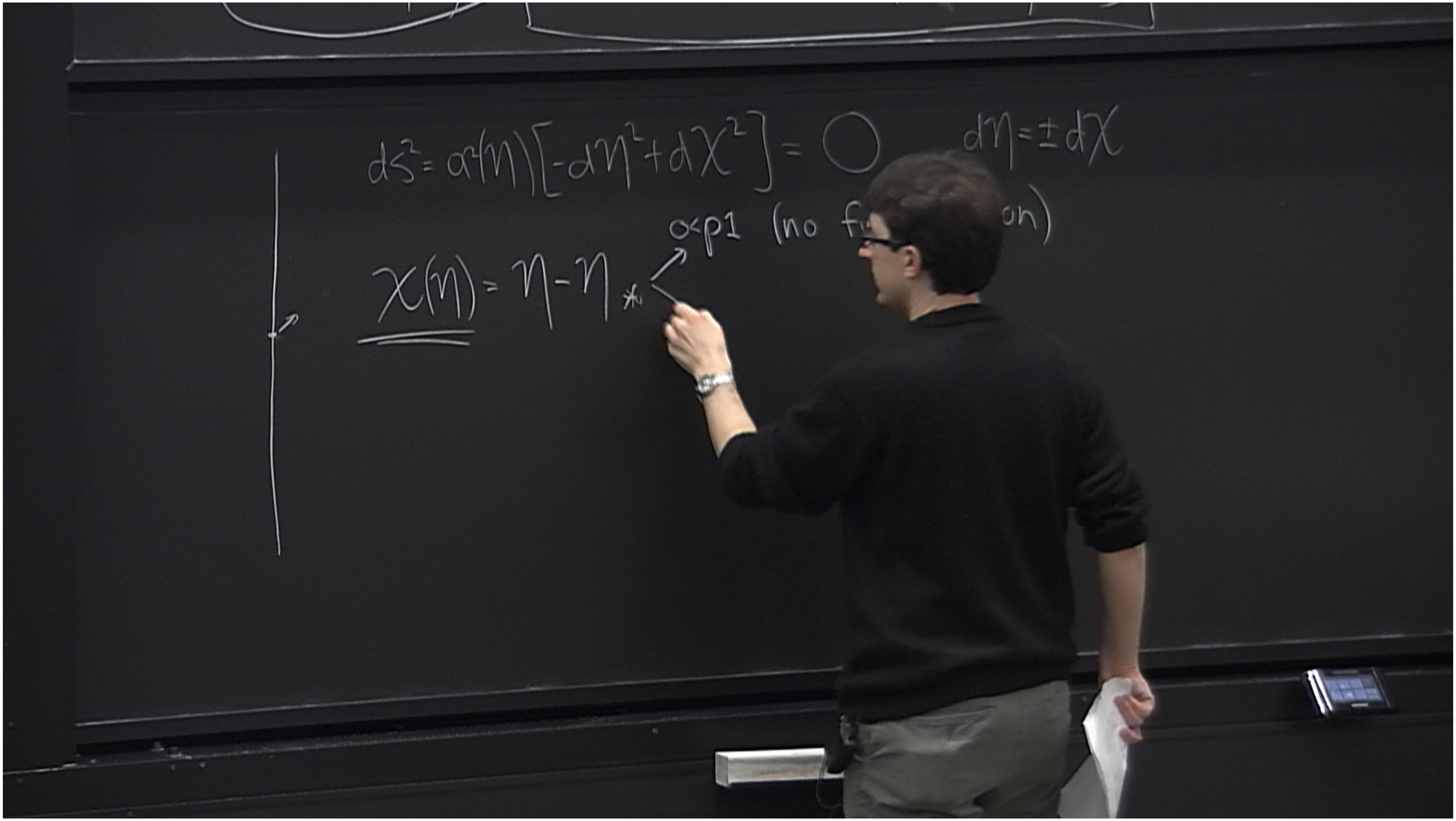
$$r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$$

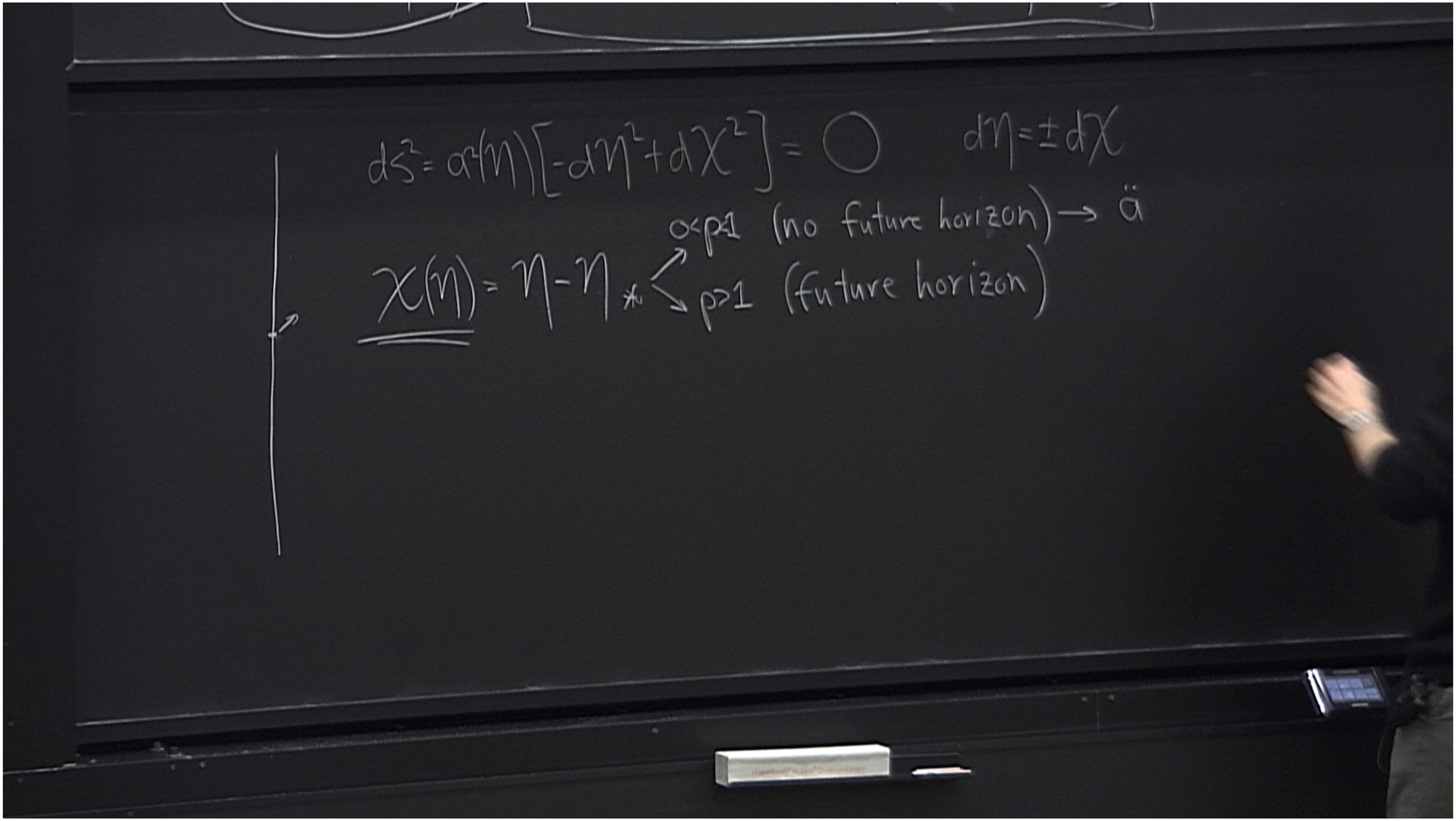
$$r = S_K(\chi)$$

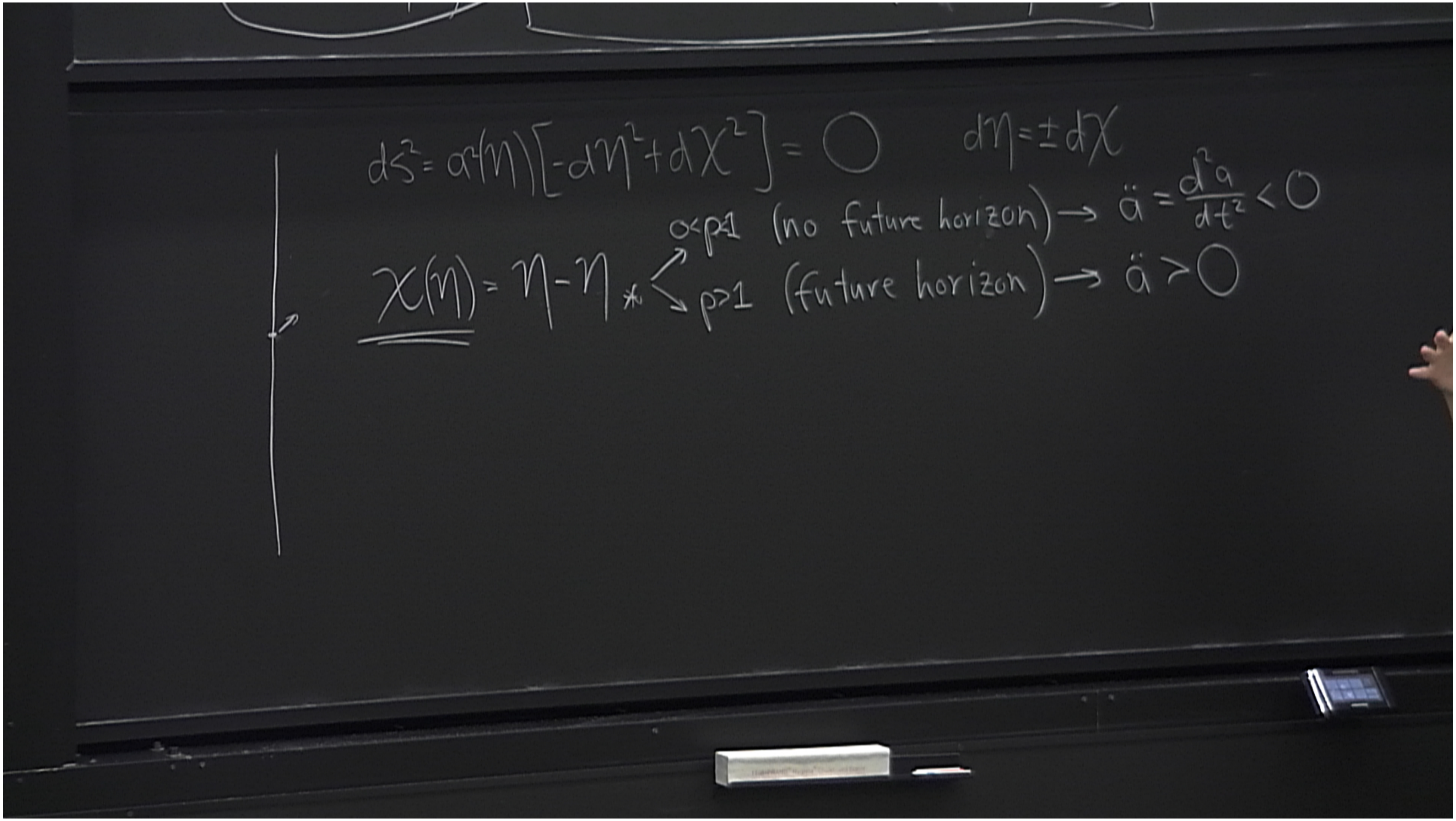
$$ds^2 = -\underbrace{dt^2}_{dt = a(\eta)d\eta} + a^2(t)[\;\ldots\;] = a^2(\eta)[-d\eta^2 + [\ldots]]$$

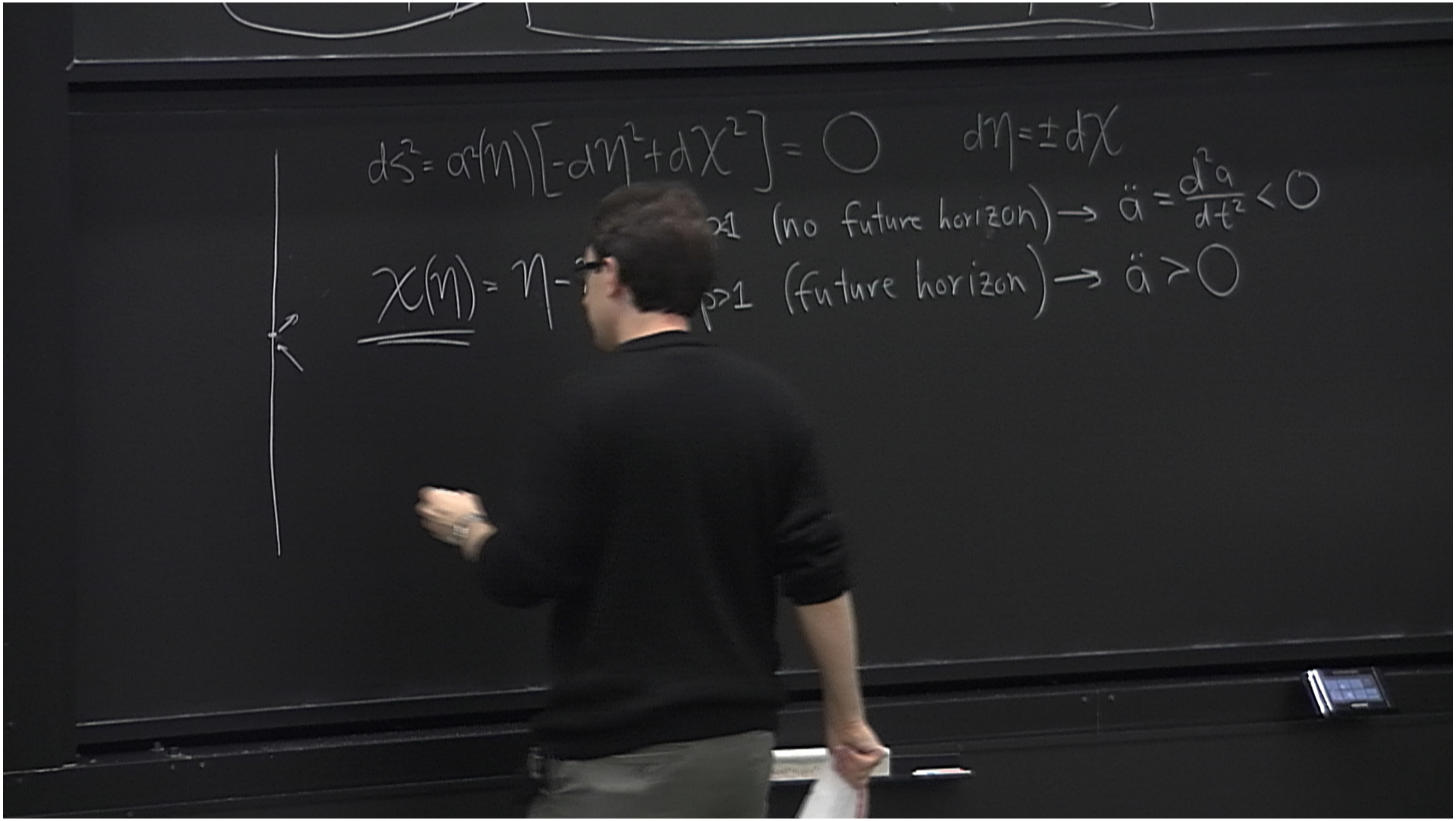
$$d\chi^2 + S_K^2(\chi)\,d\Omega^2$$

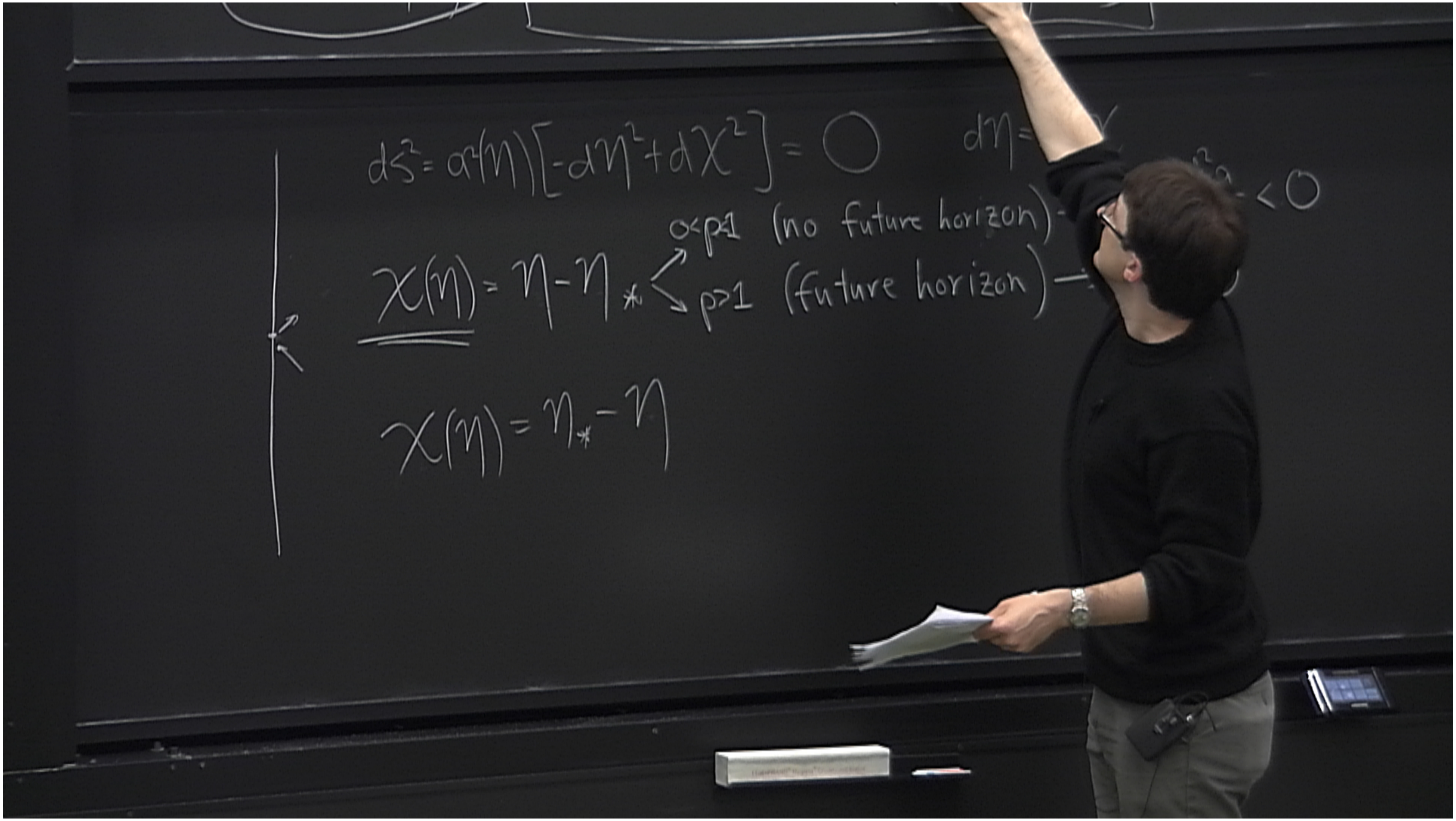
$$\boxed{\begin{array}{c}(0 < \eta < \infty) \\ (-\infty < \eta < 0)\end{array}}$$


ds² = a²(η)[-dη² + dχ²] = 0
dη = ±dχ
αp1 (no f
on)
χ(η) = η - η*


$ds^2 = a^2(\eta)[-d\eta^2 + d\chi^2] = 0$
$d\eta = \pm d\chi$
$0<p<1$ (no future horizon) $\rightarrow \ddot{a}$
$\chi(\eta) = \eta - \eta_*$
$p>1$ (future horizon)


$ds^2 = a^2(\eta)\left[-d\eta^2 + d\chi^2\right] = 0$
$d\eta = \pm d\chi$
$0<p<1$ (no future horizon) $\rightarrow \ddot{a} = \frac{d^2a}{dt^2} < 0$
$\chi(\eta) = \eta - \eta_*$
$p>1$ (future horizon) $\rightarrow \ddot{a} > 0$


ds² = a²(η)[-dη² + dχ²] = 0
dη = ± dχ
(no future horizon) → ä = d²a/dt² < 0
χ(η) = η −
p>1 (future horizon) → ä > 0



$ds^2 = a^2(\eta)[-d\eta^2 + d\chi^2] = 0$
$d\eta =$
< 0
$0<p<1$ (no future horizon)
$\chi(\eta) = \eta - \eta_*$
$p>1$ (future horizon)
$\chi(\eta) = \eta_* - \eta$

$$a = a_0\left(\frac{t}{t_0}\right)^p \quad 0<t<\infty \quad p>0$$

$$d\eta = \frac{dt}{a(t)}$$

$$\eta \propto t^{1-p} \longrightarrow t \propto \eta^{\frac{1}{1-p}}$$

$$\boxed{a = a_0\left(\frac{\eta}{\eta_0}\right)^{\frac{p}{1-p}}}$$

$0<p<1 \to \text{pos} \to (0<\eta<\infty)$

$1<p \longrightarrow \text{neg} \quad (-\infty<\eta<0)$

$\{x^1, x^2, x^3\}$

$r\{\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta\}$

$r = S_K(\chi)$

$$ds^2 = -\underbrace{dt^2}_{dt = a(\eta)d\eta} + a^2(t)[\ldots] = a^2(\eta)\left[-d\eta^2 + [\ldots]\right]$$

$$d\chi^2 + \cancel{S_K^2(\chi)\,d\Omega^2}$$

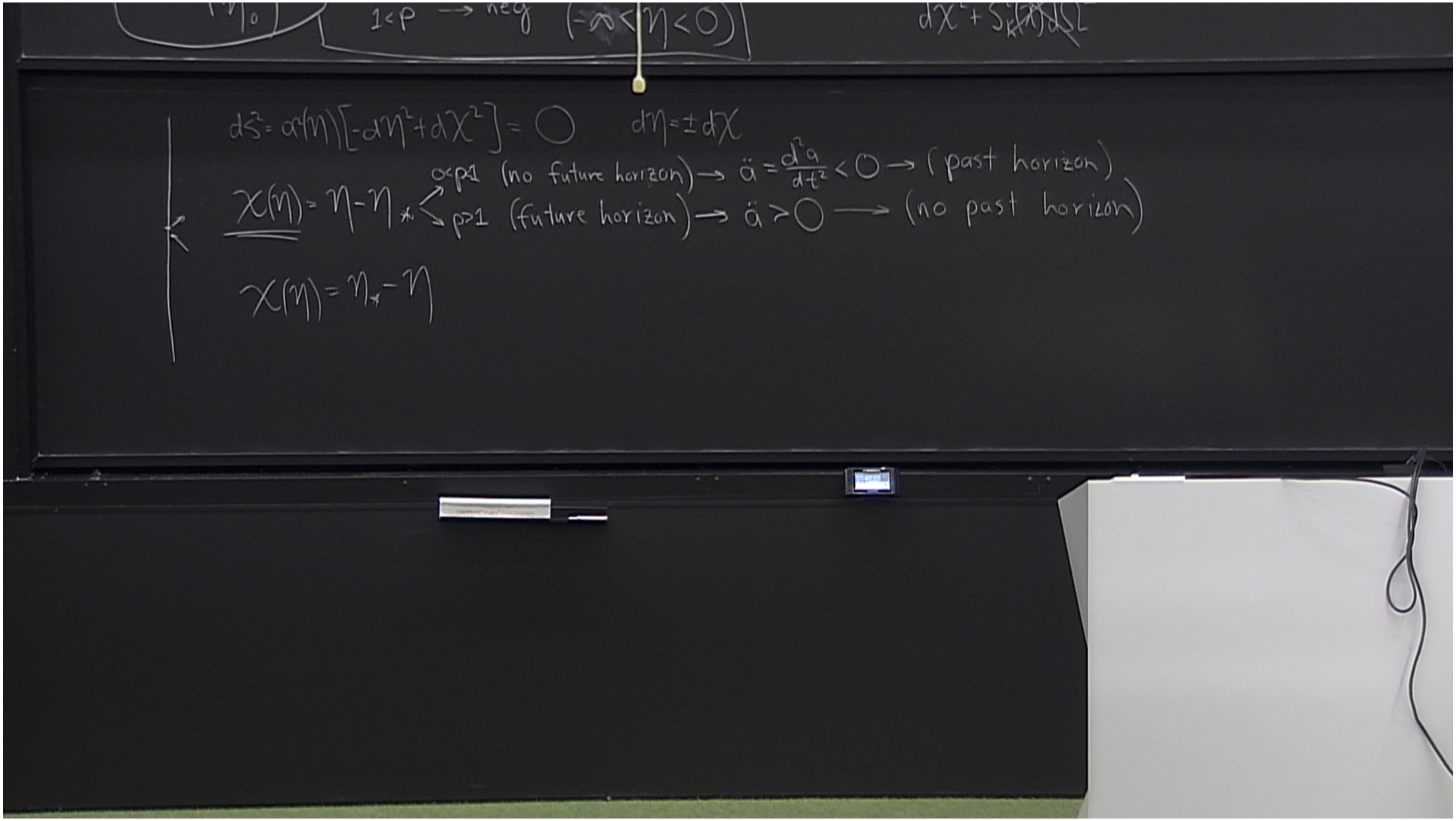
$$ds^2 = a^2(\eta)[-d\eta^2 + d\chi^2] = 0 \qquad d\eta = \pm d\chi$$

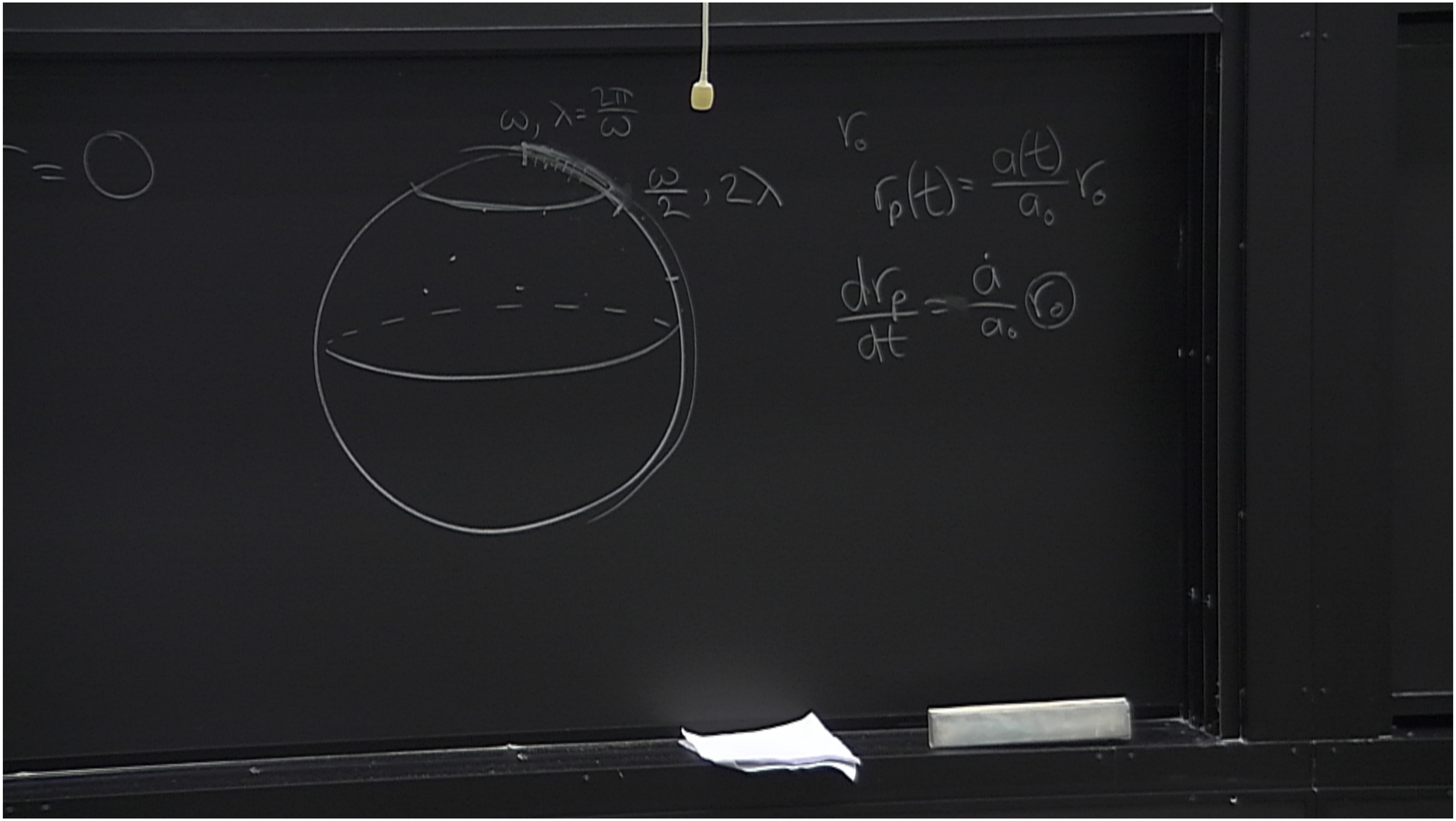
$0<p<1$ (no future horizon) $\to \ddot{a} = \frac{d^2a}{dt^2} < 0$

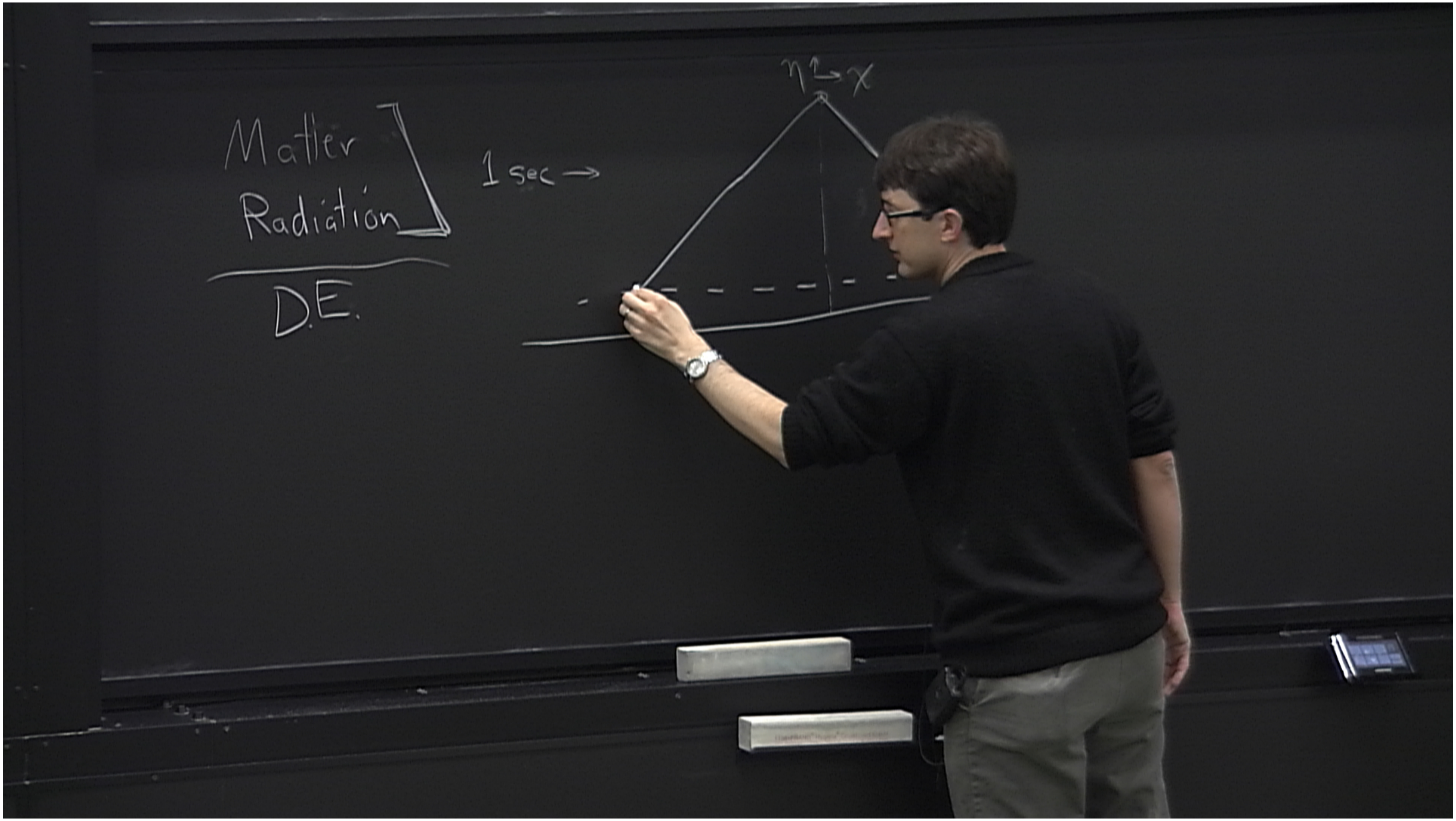
$\underline{\chi(\eta) = \eta - \eta_*}$

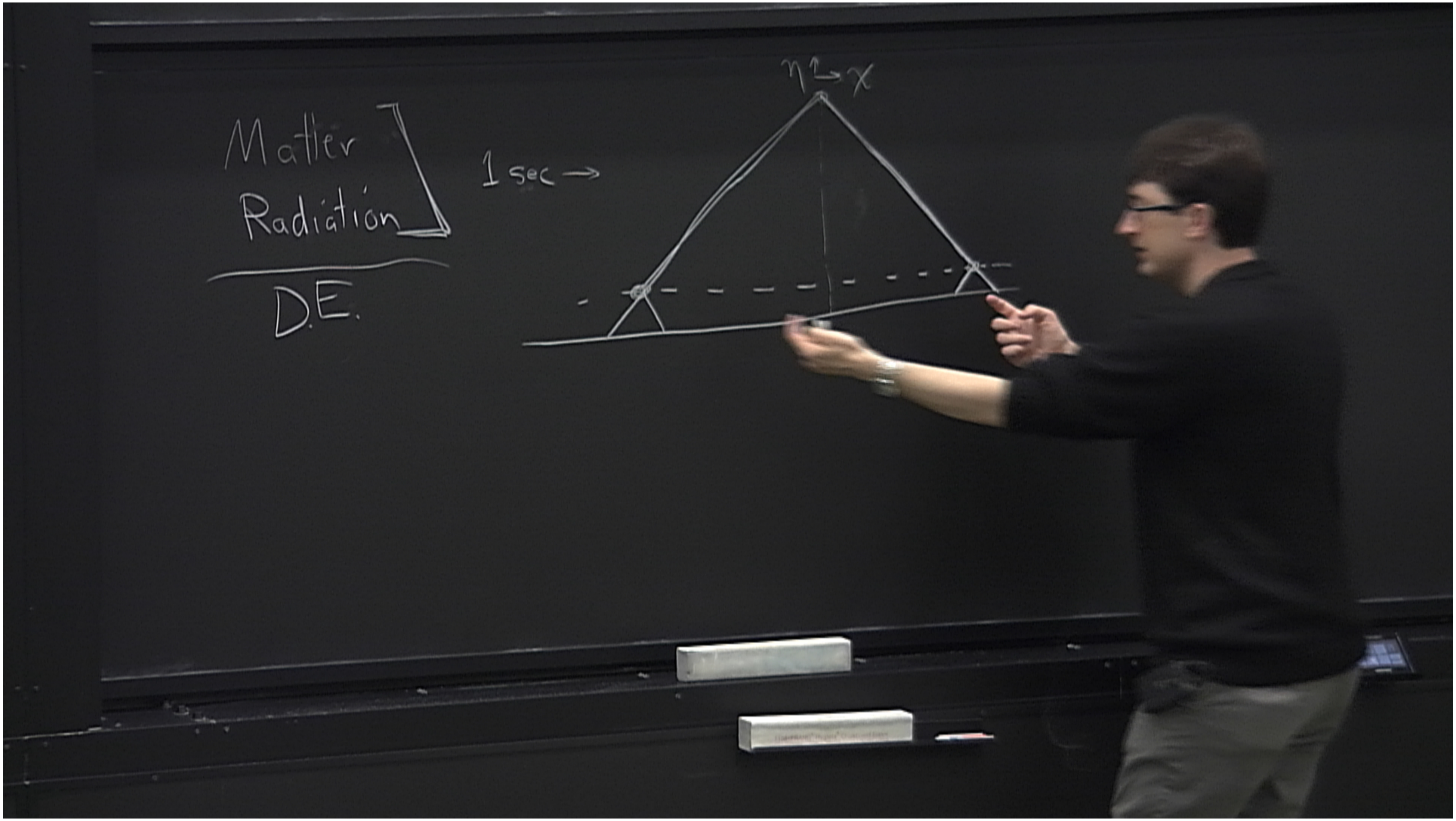
$p>1$ (future horizon) $\to \ddot{a} > 0$

$$\chi(\eta) = \eta_* - \eta$$

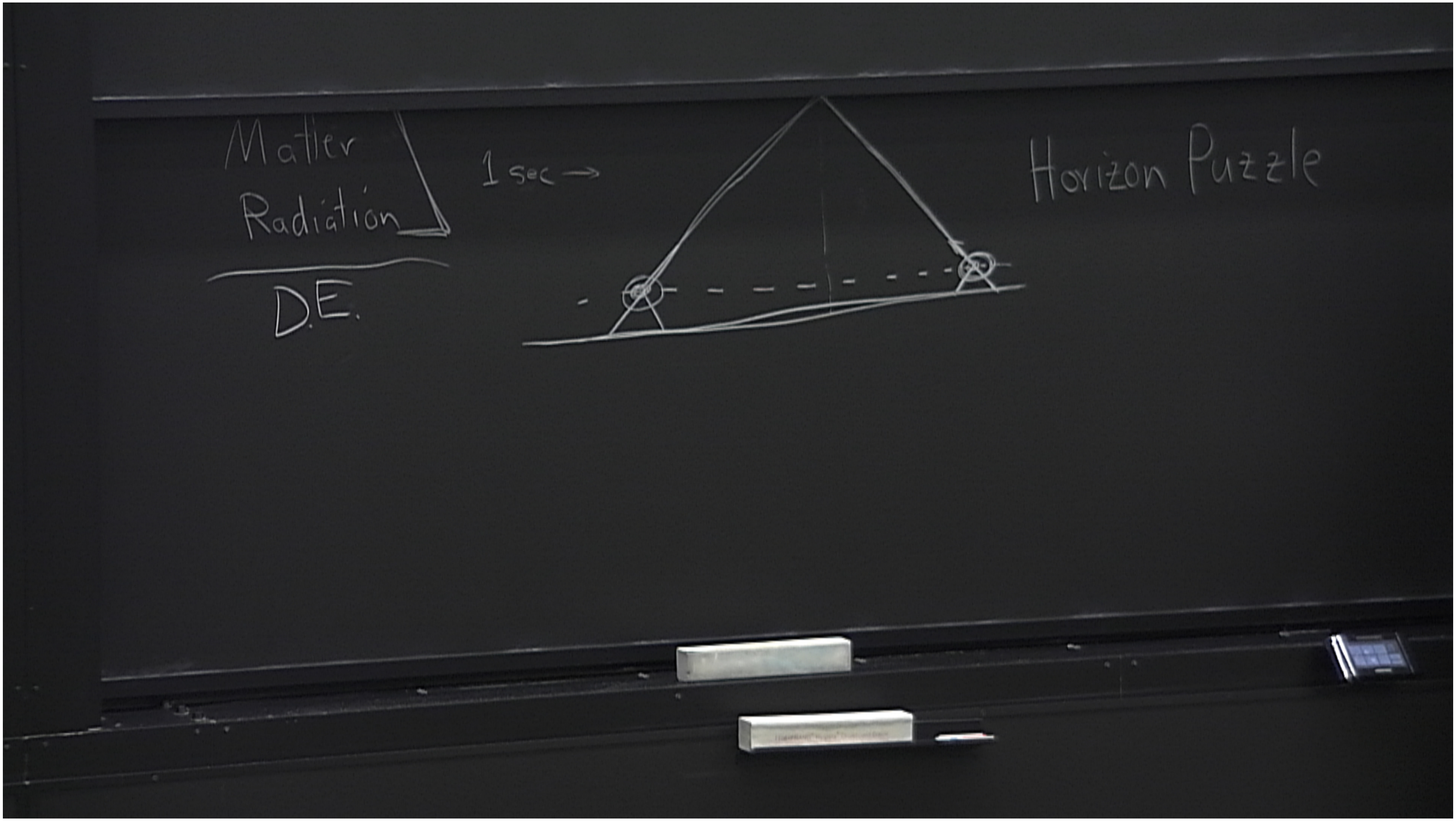

1<p → neg (-∞<η<0)
ds² = a²(η)[-dη² + dχ²] = 0
dη = ±dχ
0<p<1 (no future horizon) → ä = d²a/dt² < 0 → (past horizon)
χ(η) = η - η*
p>1 (future horizon) → ä > 0 → (no past horizon)
χ(η) = η* - η

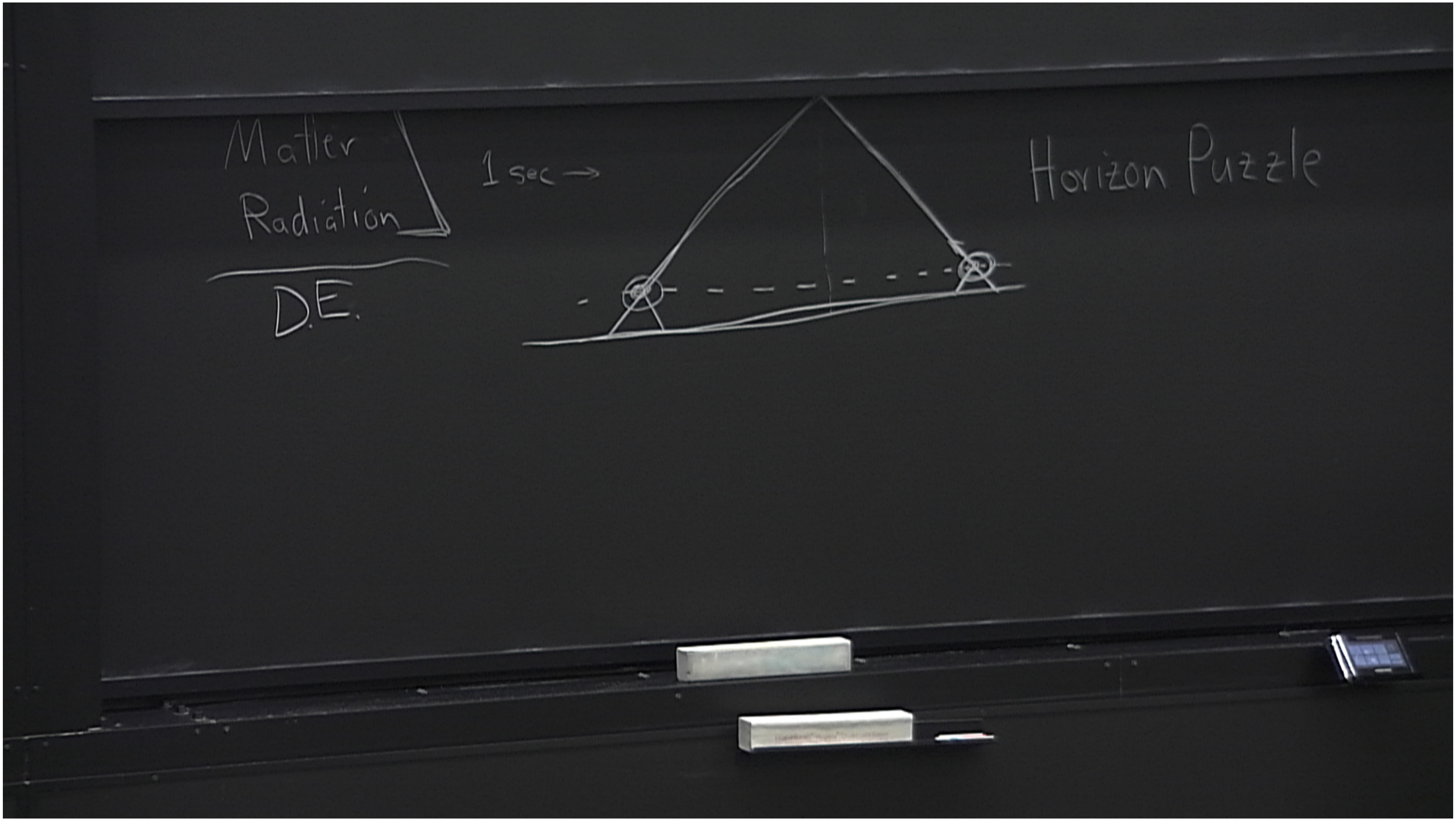

$\omega, \lambda = \frac{2\pi}{\omega}$
$\frac{\omega}{2}, 2\lambda$
r_0
$r_p(t) = \frac{a(t)}{a_0} r_0$
$\frac{dr_p}{dt} = \frac{\dot{a}}{a_0} r_0$


Matter
Radiation
D.E.
1 sec →
η → χ



Matter
Radiation
D.E.
1 sec →
η ↳ χ


Matter
Radiation
D.E.
1 sec →
Horizon Puzzle



Matter
Radiation
D.E.
1 sec →
Horizon Puzzle