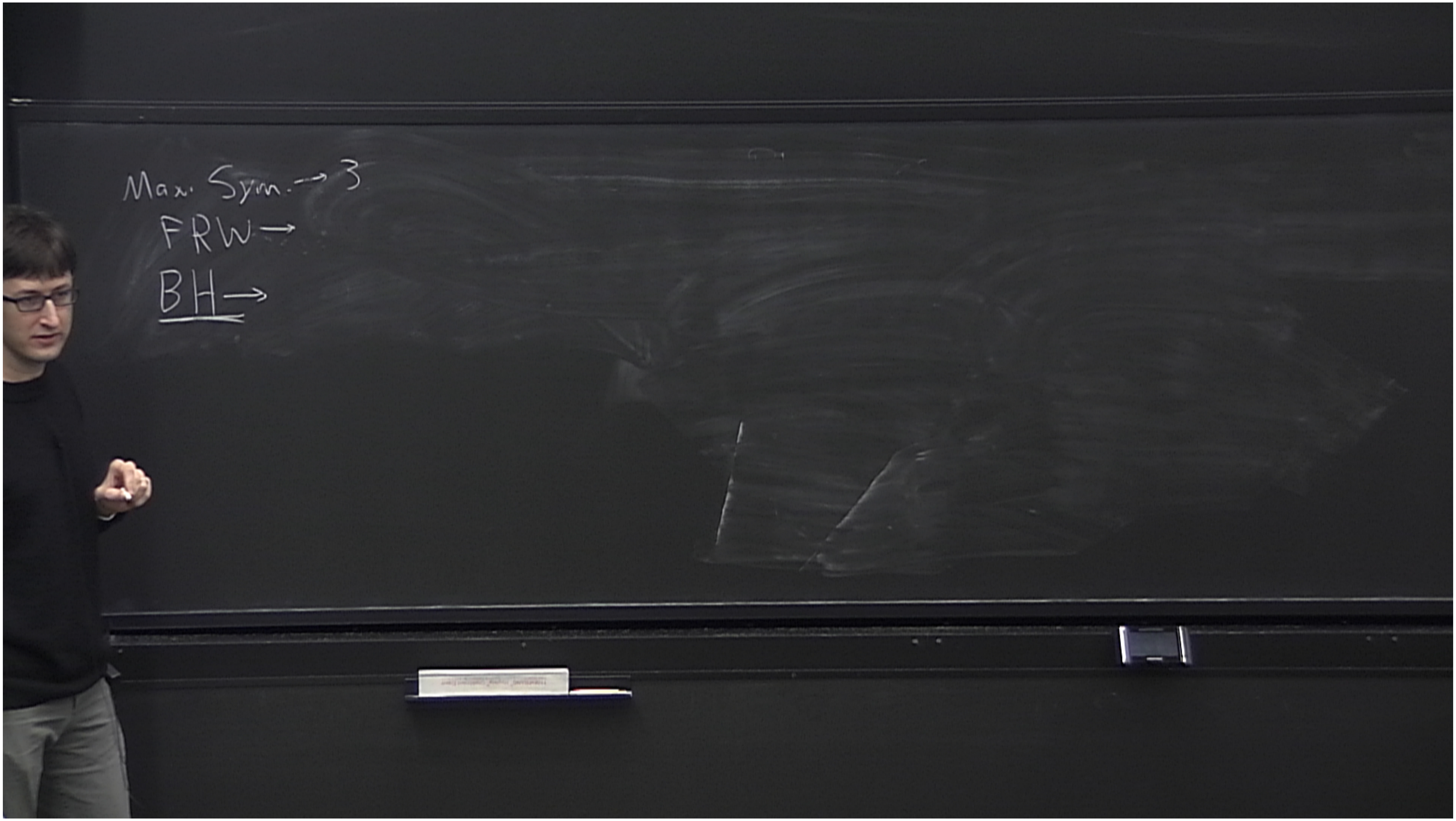
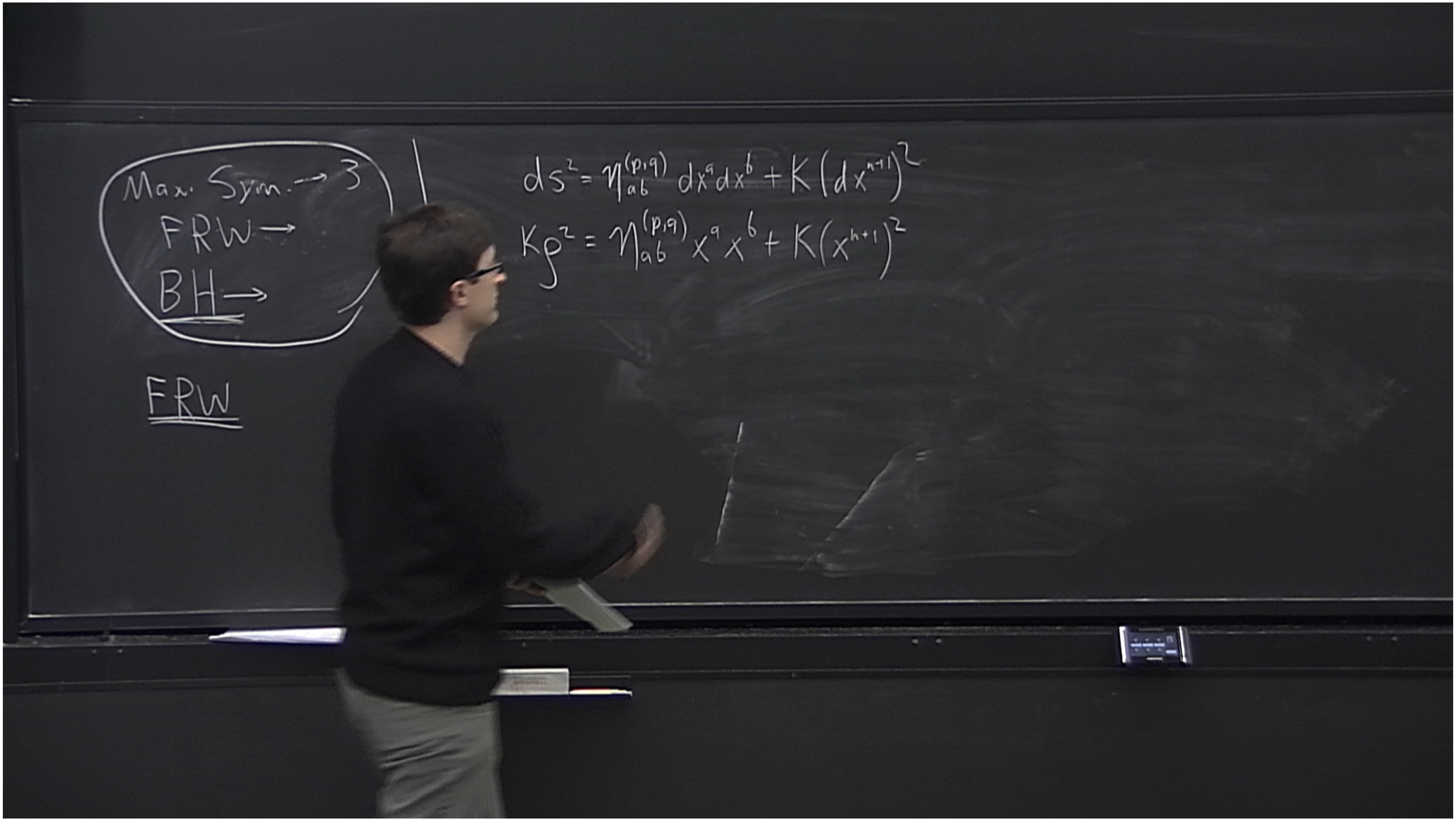
Title: 12/13 PSI - Cosmology Review Lecture 5

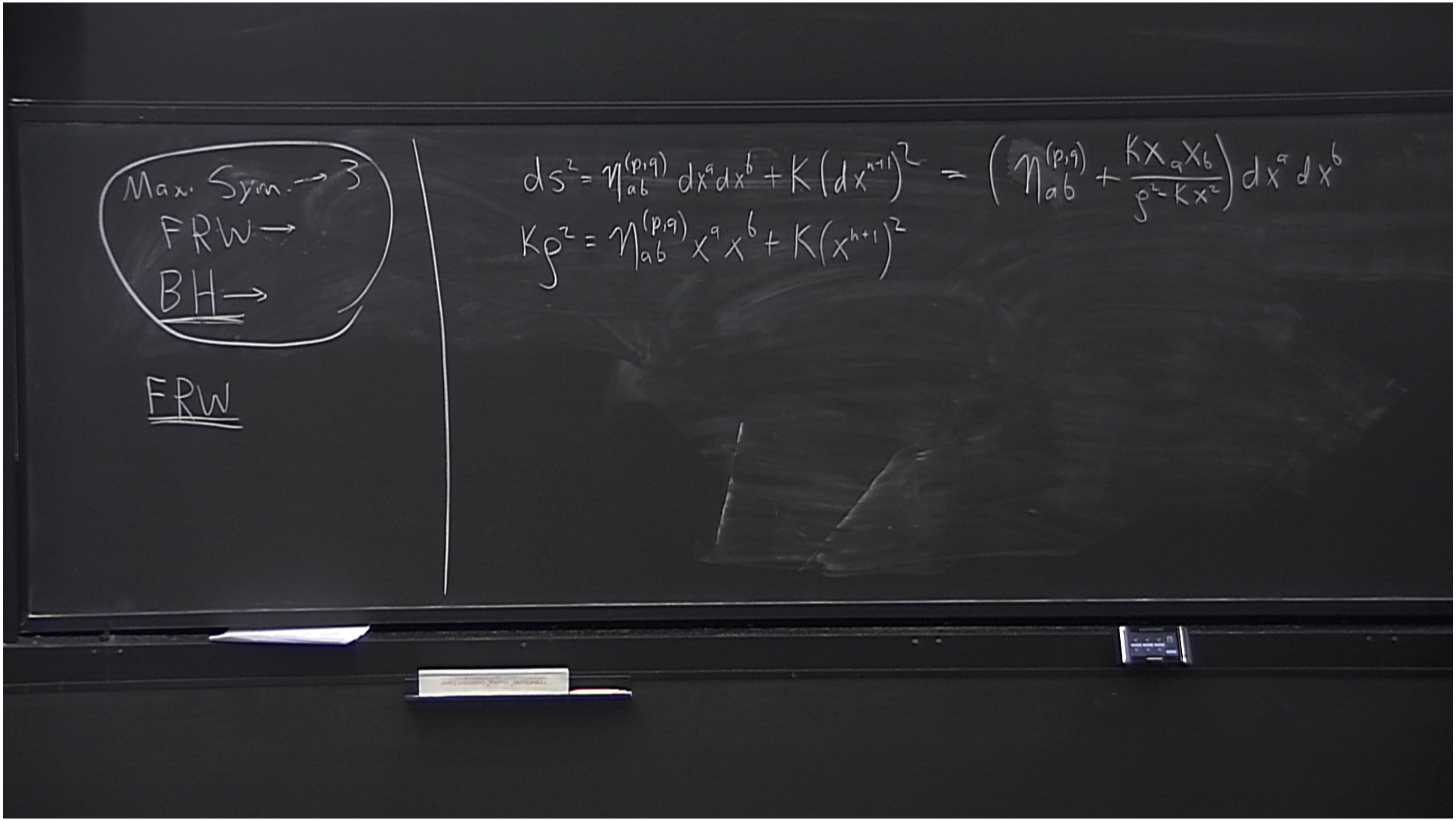
Date: Feb 25, 2013 11:30 AM

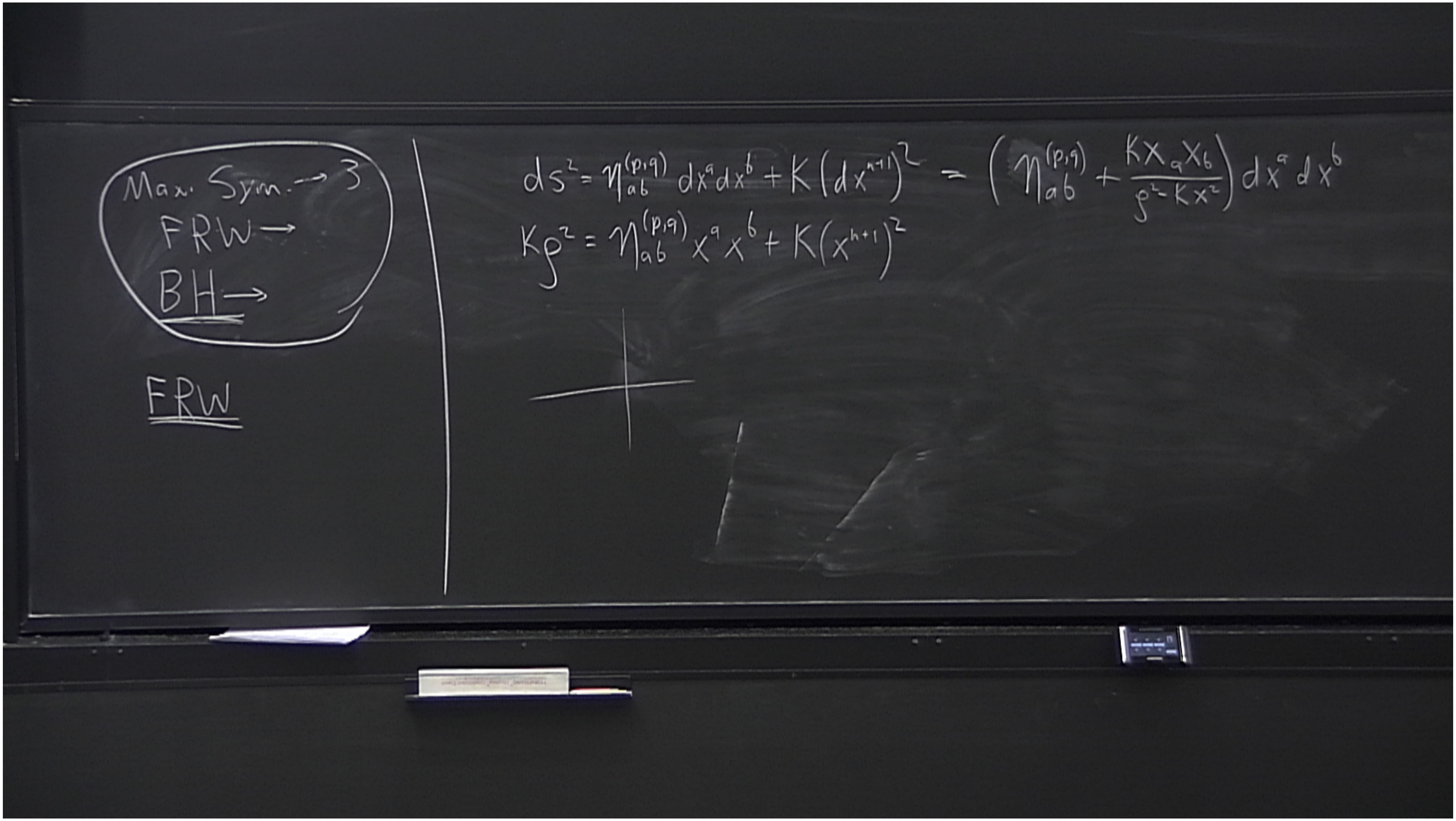
URL: http://pirsa.org/13020105

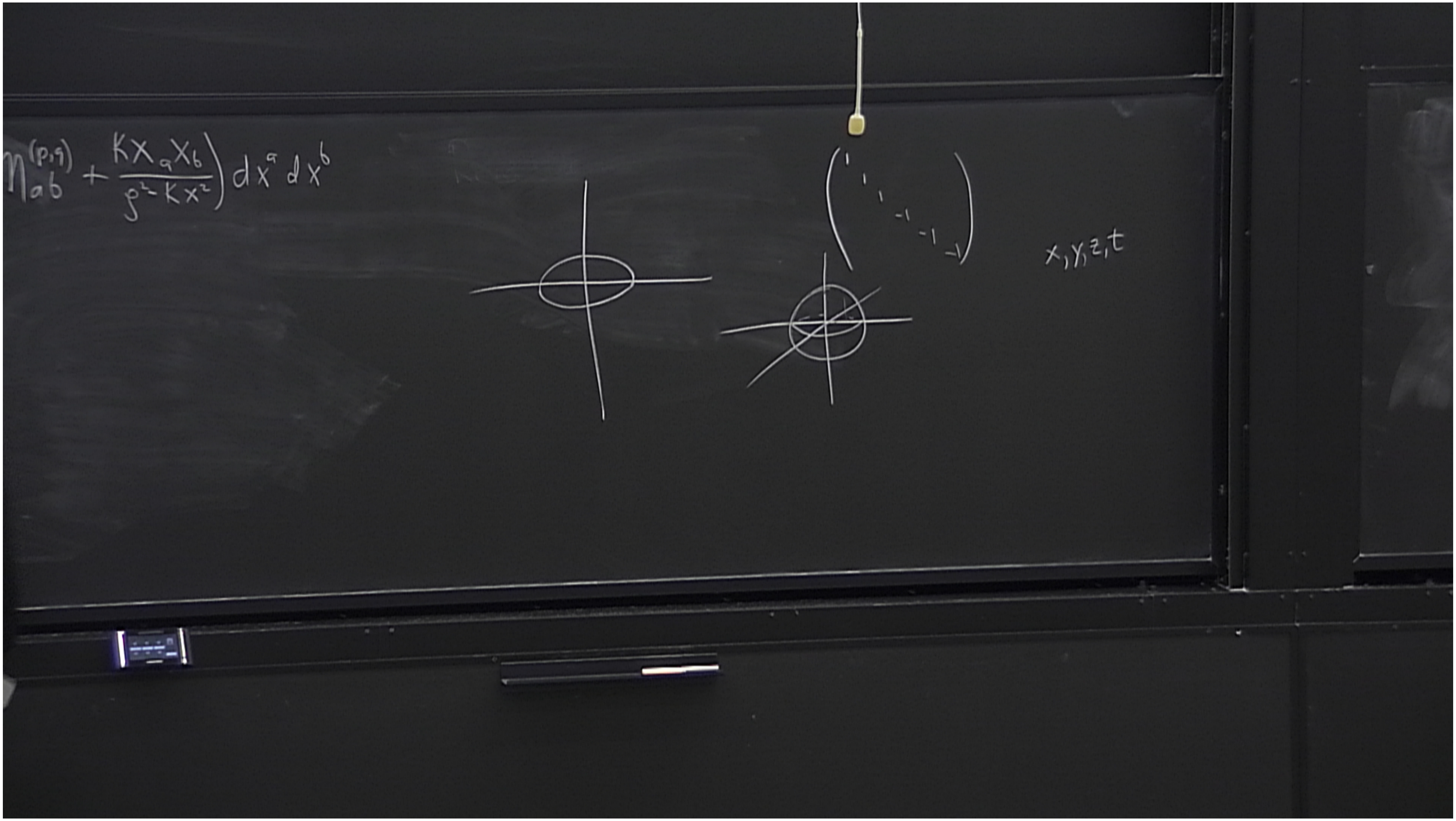
Abstract:

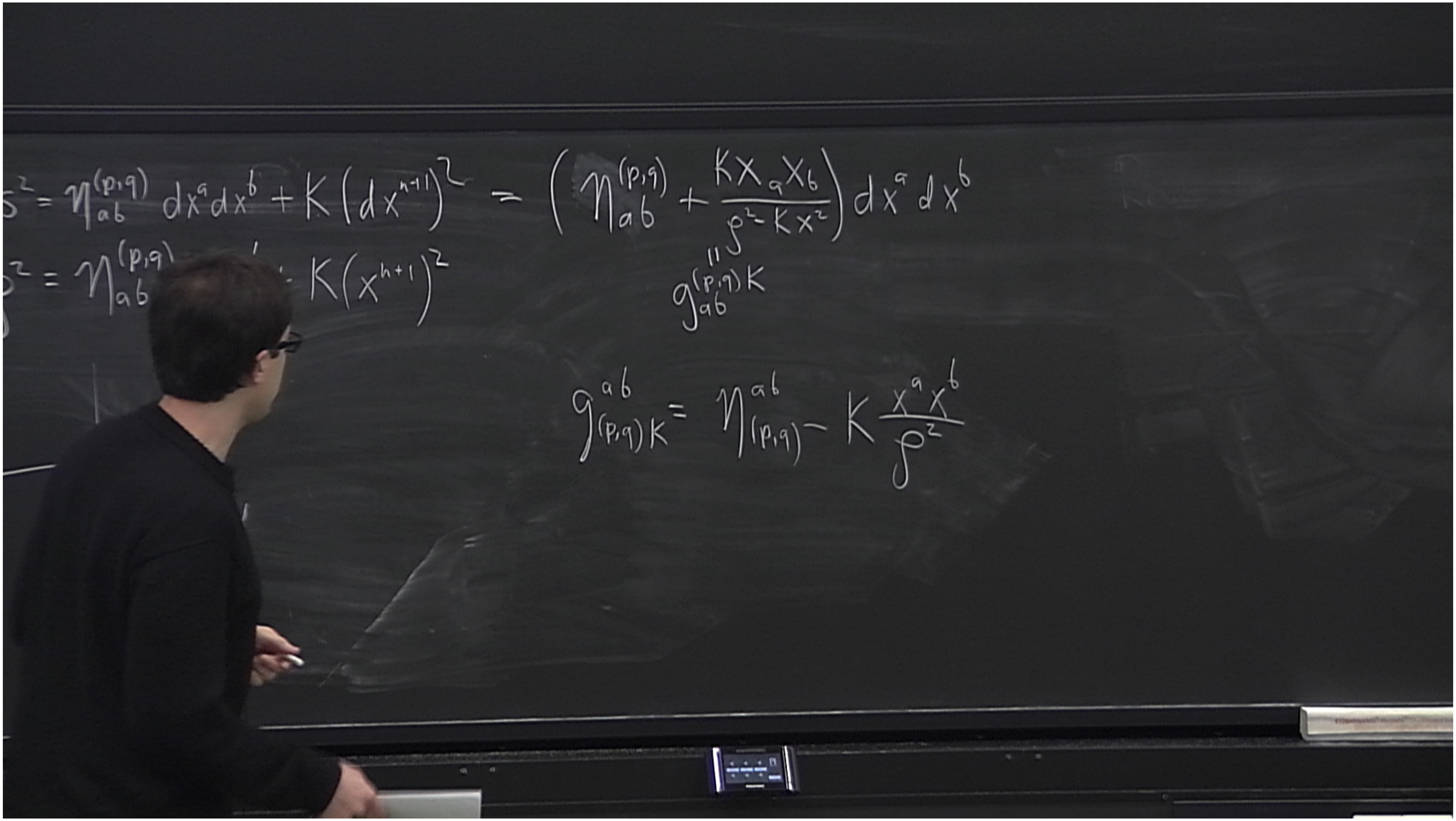

Max. Sym. → 3
FRW →
BH →


Max. Sym. → 3
FRW →
BH →
FRW
$ds^2 = \eta_{ab}^{(p,q)} dx^a dx^b + K (dx^{n+1})^2$
$K\rho^2 = \eta_{ab}^{(p,q)} x^a x^b + K (x^{n+1})^2$


Max. Sym. → 3
FRW →
BH →
FRW
$ds^2 = \eta_{ab}^{(p,q)} dx^a dx^b + K (dx^{n+1})^2 = \left(\eta_{ab}^{(p,q)} + \frac{K x_a x_b}{\rho^2 - K x^2}\right) dx^a dx^b$
$K\rho^2 = \eta_{ab}^{(p,q)} x^a x^b + K (x^{n+1})^2$


Max. Sym. → 3
FRW →
BH →
FRW
$ds^2 = \eta_{ab}^{(p,q)} dx^a dx^b + K(dx^{n+1})^2 = \left(\eta_{ab}^{(p,q)} + \frac{K x_a x_b}{\rho^2 - K x^2}\right) dx^a dx^b$
$K\rho^2 = \eta_{ab}^{(p,q)} x^a x^b + K(x^{n+1})^2$


ηab(p,q) + Kxaxb / ρ²-Kx²) dxa dxb
1 1 1 -1 -1 -1
x, y, z, t

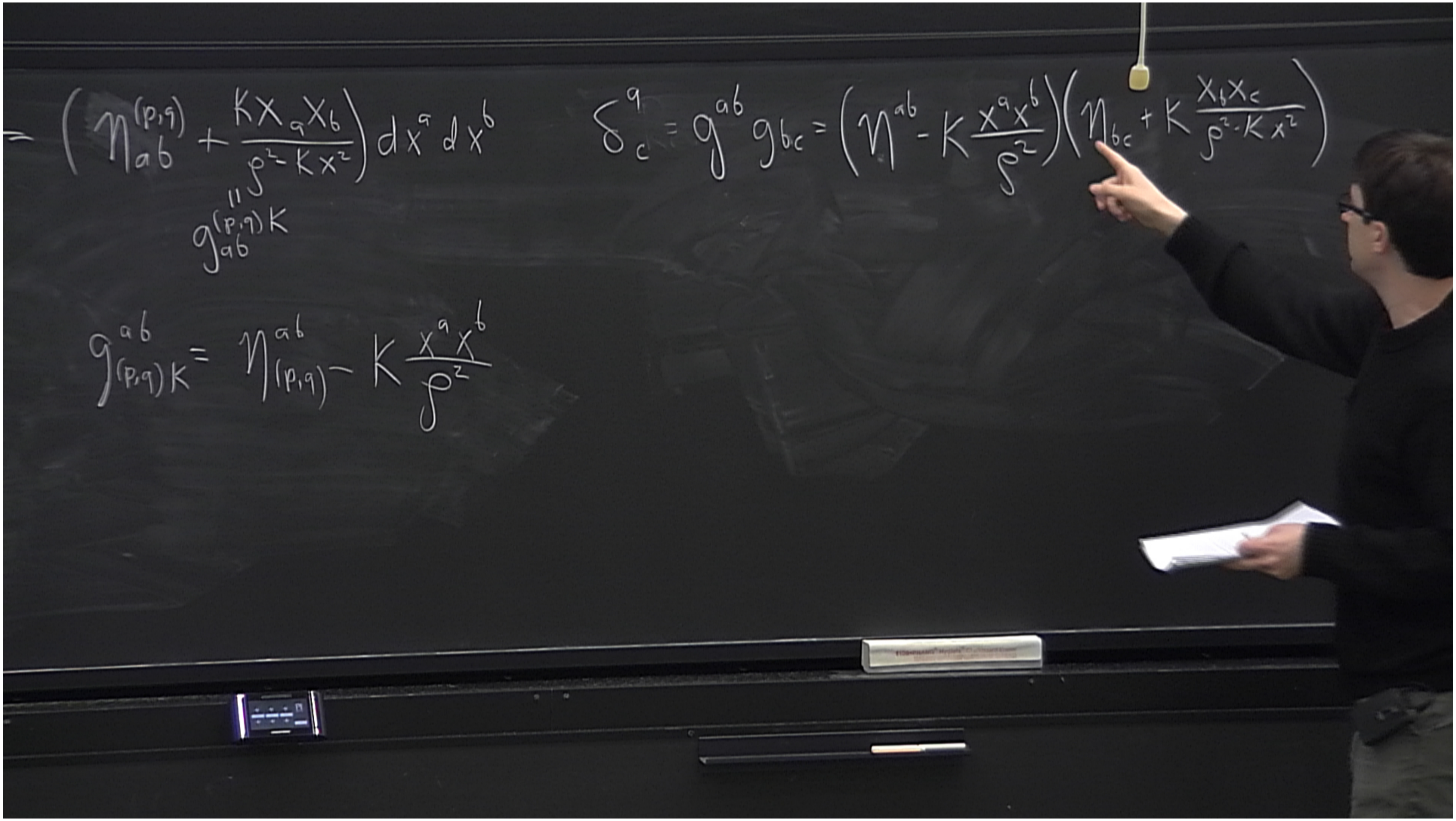

$s^2 = \eta_{ab}^{(p,q)} dx^a dx^b + K(dx^{n+1})^2 = \left(\eta_{ab}^{(p,q)} + \frac{K x_a x_b}{\rho^2 - K x^2}\right) dx^a dx^b$
$g_{ab}^{(p,q)K}$
$g^{ab}_{(p,q)K} = \eta^{ab}_{(p,q)} - K \frac{x^a x^b}{\rho^2}$

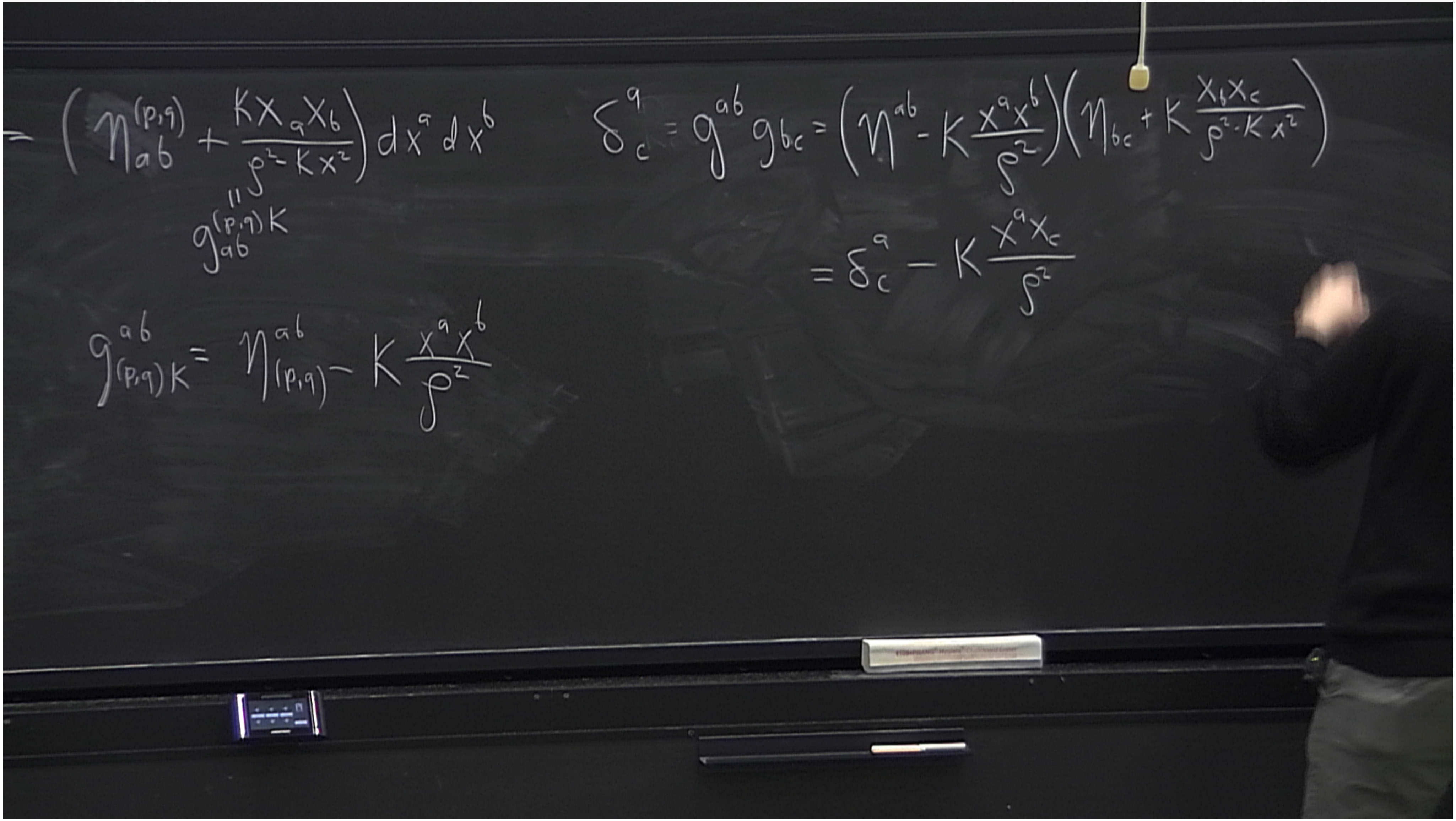
$$= \left(\eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2}\right) dx^a \, dx^b$$

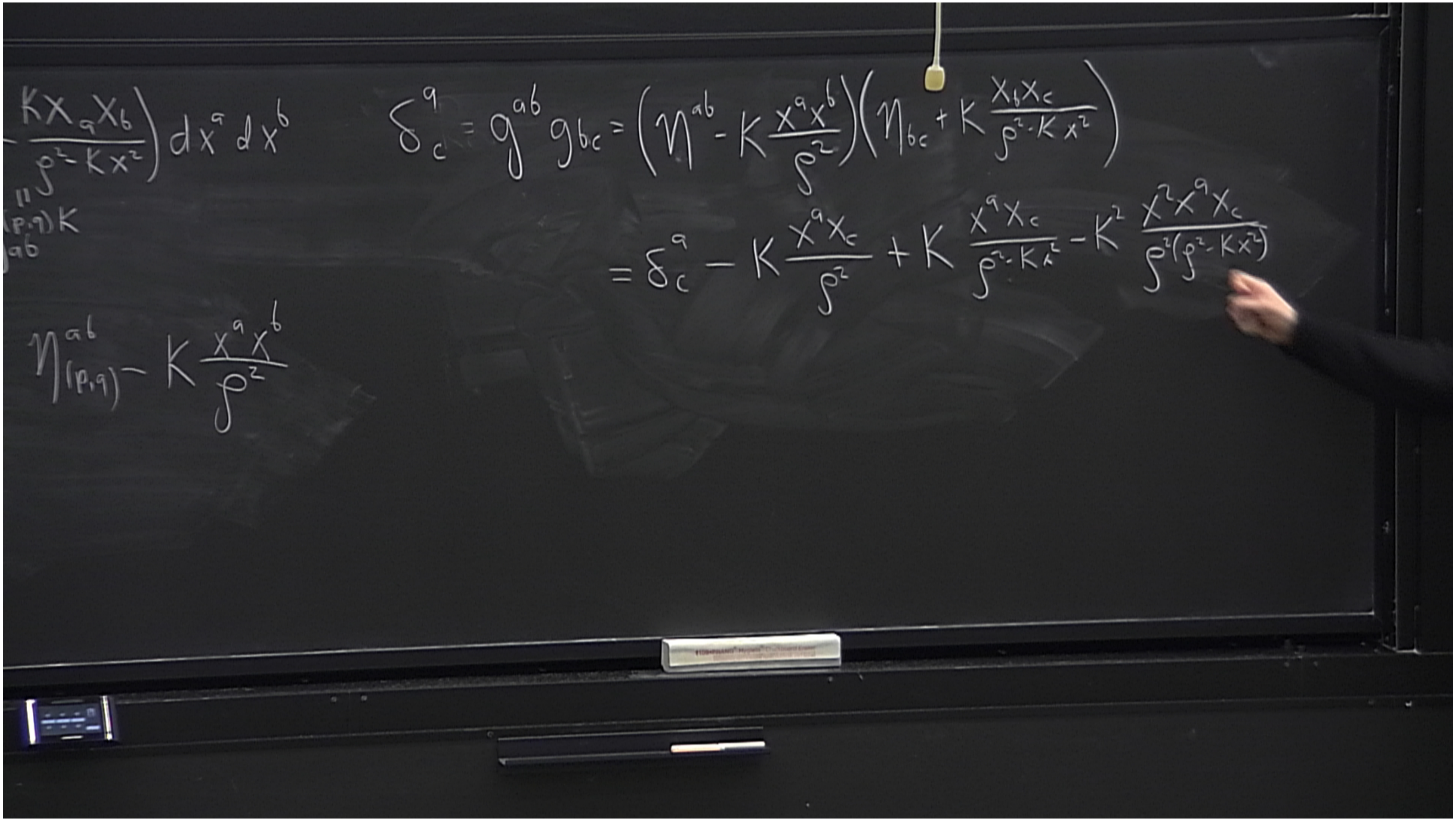
$$\left(\eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2}\right) = g^{(p,q)K}_{ab}$$

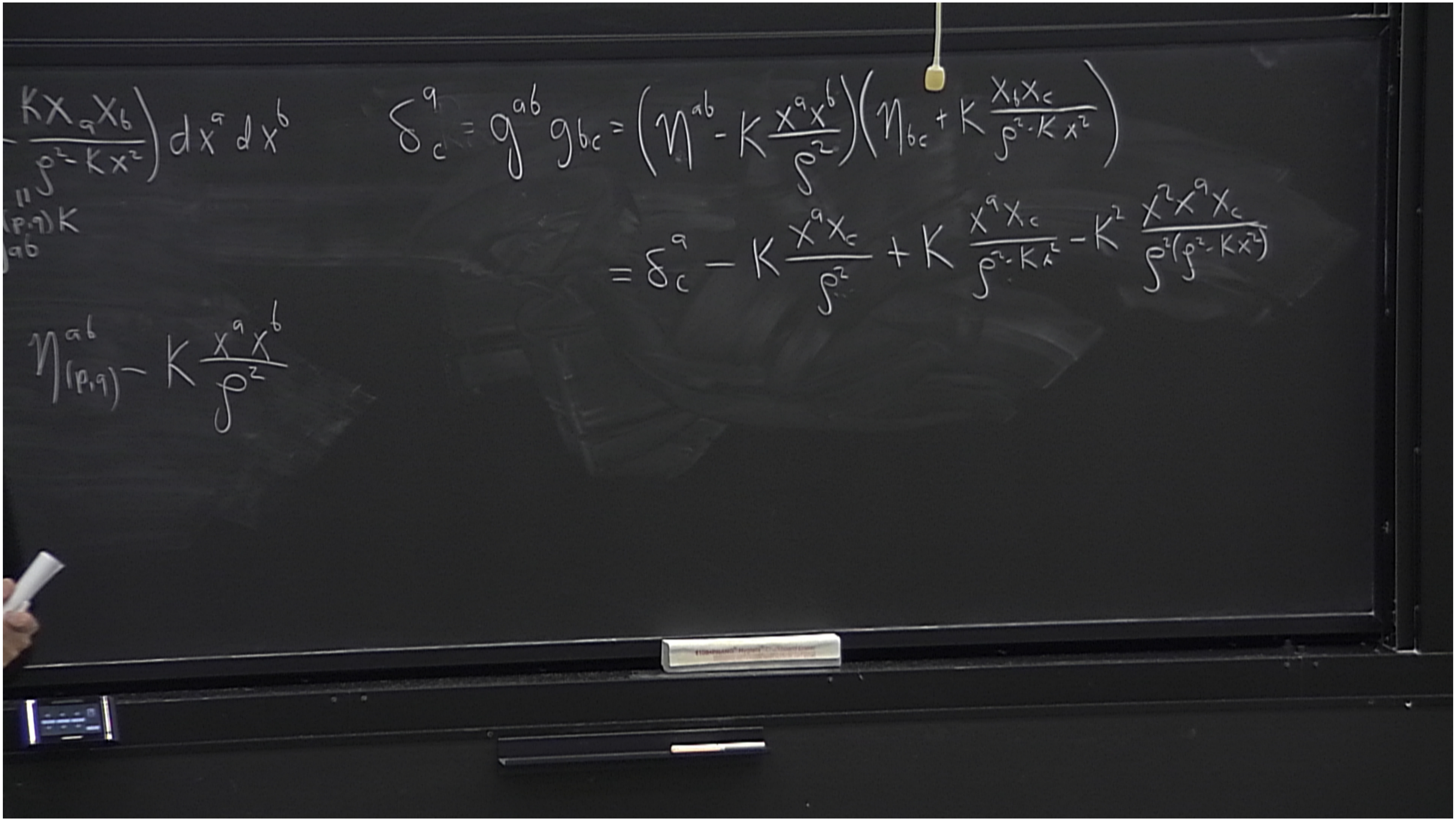
$$g^{ab} g_{bc} = \delta^a_c$$

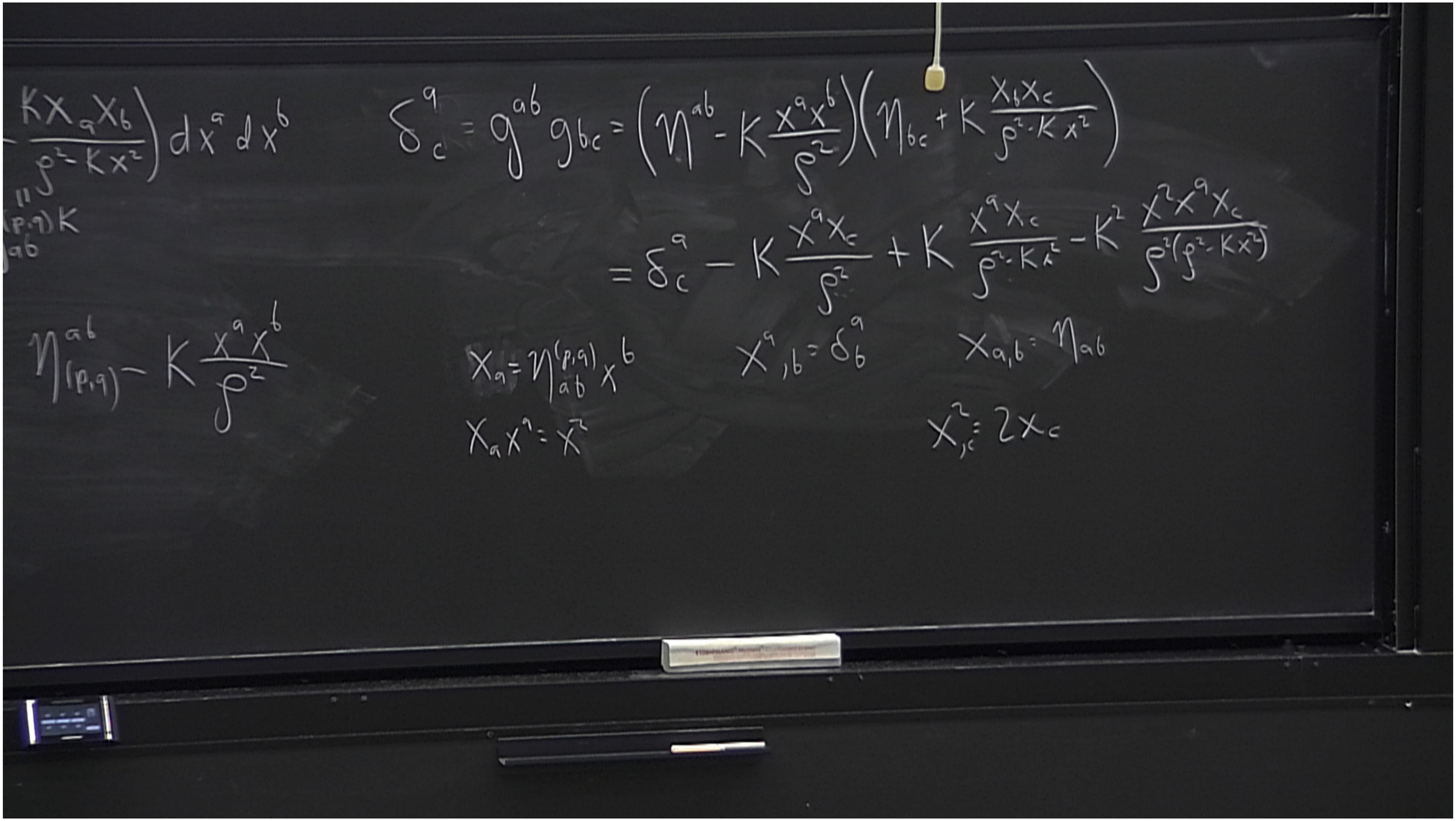
$$g^{ab}_{(p,q)K} = \eta^{ab}_{(p,q)} - K \frac{x^a x^b}{\rho^2}$$

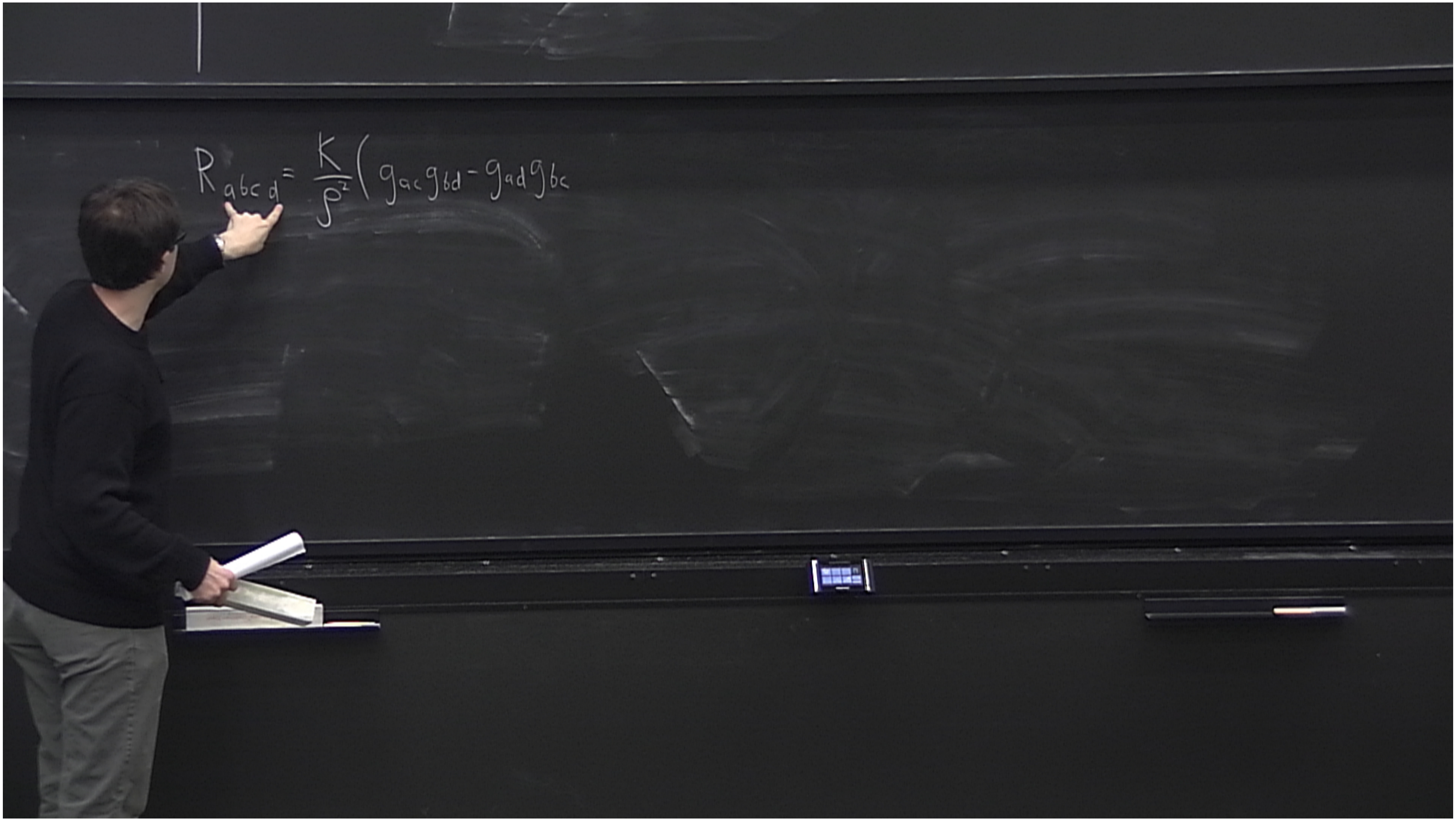

$= \left(\eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2} \right) dx^a dx^b$
$= g^{(p,q)K}_{ab}$
$\delta^a_c = g^{ab} g_{bc} = \left(\eta^{ab} - K \frac{x^a x^b}{\rho^2} \right) \left(\eta_{bc} + K \frac{x_b x_c}{\rho^2 - K x^2} \right)$
$g^{ab}_{(p,q)K} = \eta^{ab}_{(p,q)} - K \frac{x^a x^b}{\rho^2}$

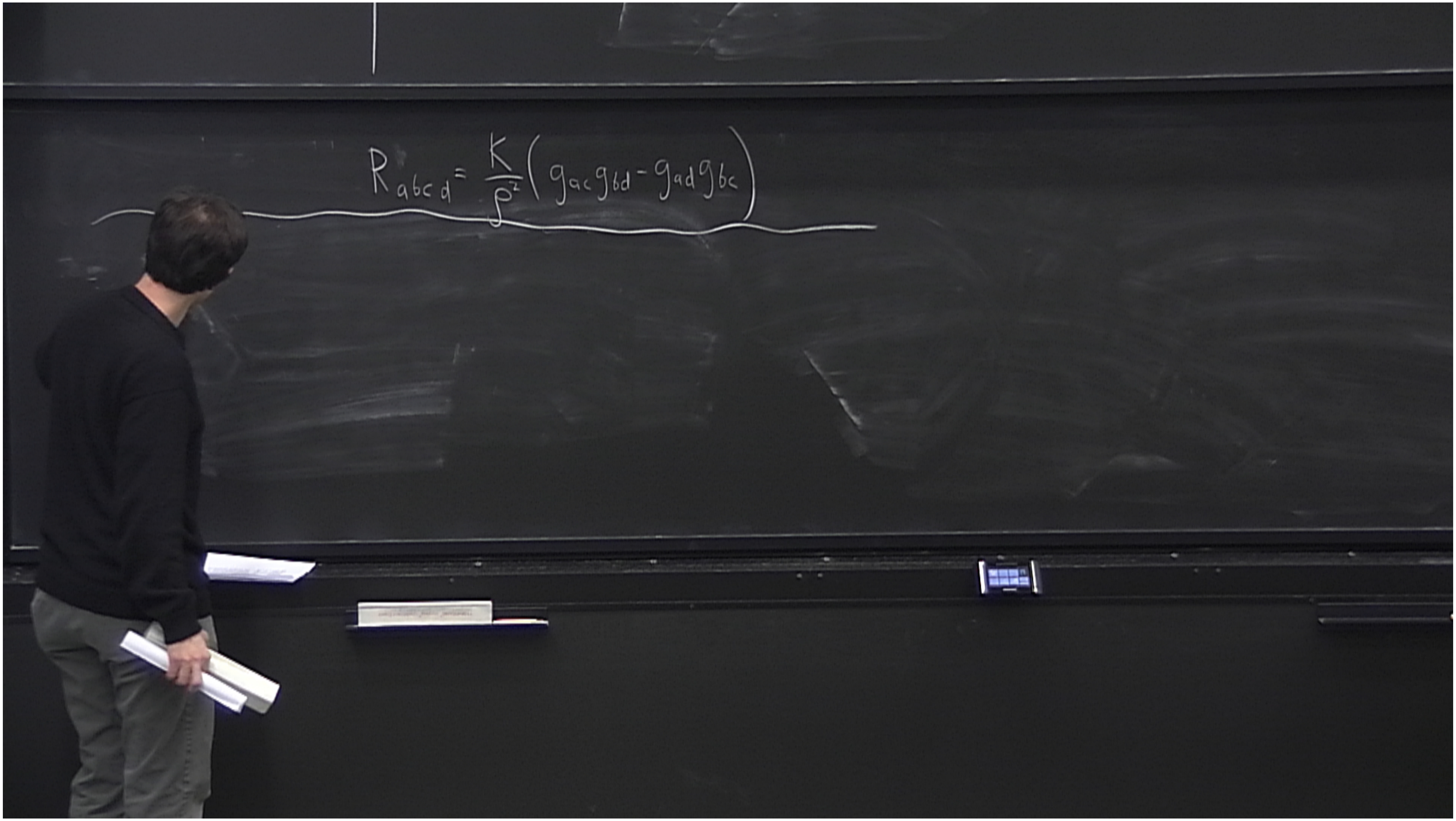

$= \left(\eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2} \right) dx^a dx^b$
$= g^{(p,q)K}_{ab}$
$g^{ab}_{(p,q)K} = \eta^{ab}_{(p,q)} - K \frac{x^a x^b}{\rho^2}$
$\delta^a_c = g^{ab} g_{bc} = \left(\eta^{ab} - K \frac{x^a x^b}{\rho^2} \right) \left(\eta_{bc} + K \frac{x_b x_c}{\rho^2 - K x^2} \right)$
$= \delta^a_c - K \frac{x^a x_c}{\rho^2}$

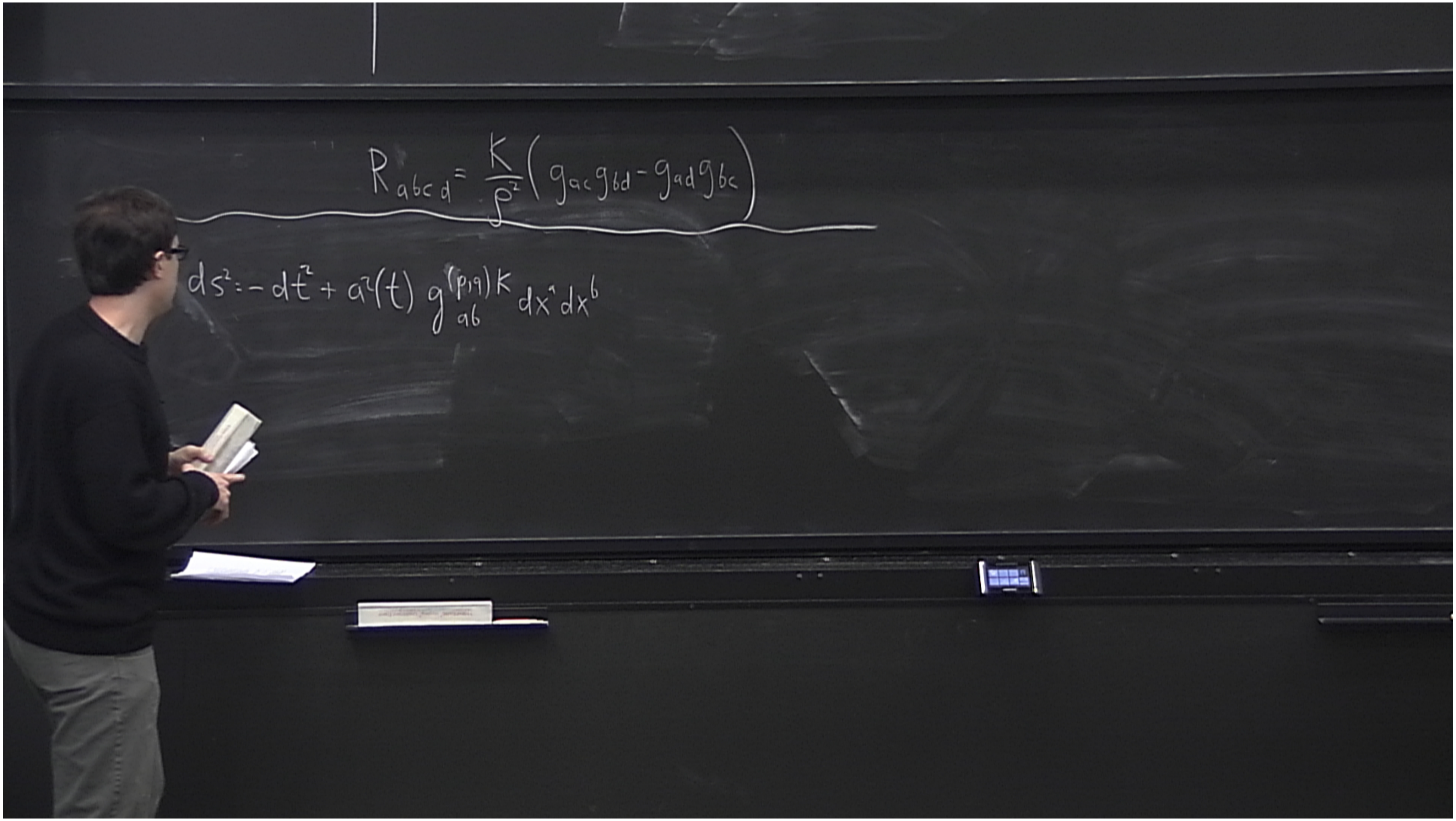

$\delta^a_c = g^{ab} g_{bc} = \left(\eta^{ab} - K\frac{x^a x^b}{\rho^2}\right)\left(\eta_{bc} + K\frac{x_b x_c}{\rho^2 - K x^2}\right)$
$= \delta^a_c - K\frac{x^a x_c}{\rho^2} + K\frac{x^a x_c}{\rho^2 - K x^2} - K^2\frac{x^2 x^a x_c}{\rho^2(\rho^2 - K x^2)}$
$\eta^{ab}_{(p,q)} - K\frac{x^a x^b}{\rho^2}$
$\frac{K x_a x_b}{\rho^2 - K x^2}\right) dx^a dx^b$

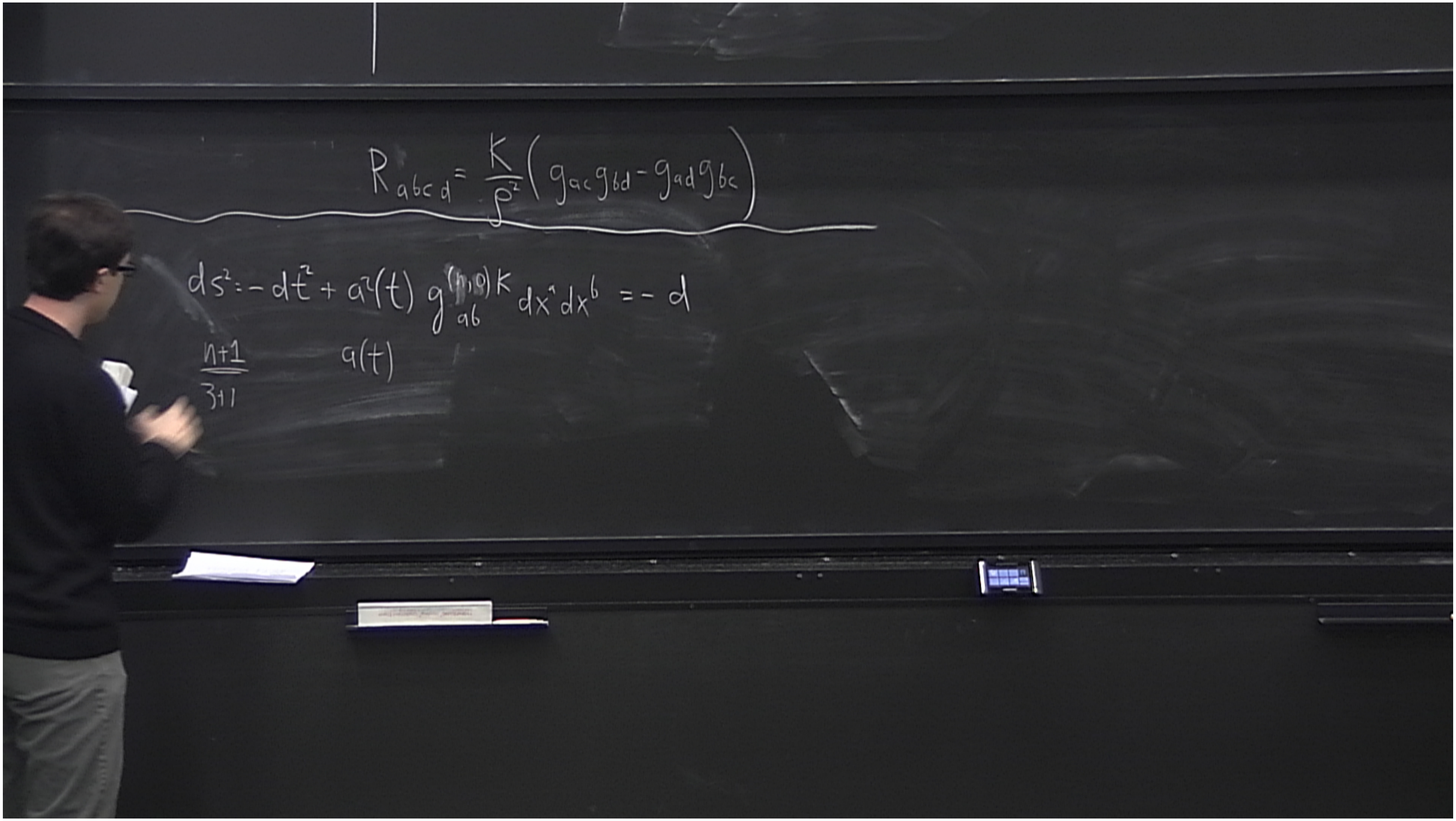

$\frac{K x_a x_b}{\rho^2 - K x^2}\Big) dx^a dx^b$
$(p,q)K$
ab
$\delta^a_c = g^{ab} g_{bc} = \left(\eta^{ab} - K\frac{x^a x^b}{\rho^2}\right)\left(\eta_{bc} + K\frac{x_b x_c}{\rho^2 - K x^2}\right)$
$= \delta^a_c - K\frac{x^a x_c}{\rho^2} + K\frac{x^a x_c}{\rho^2 - K x^2} - K^2\frac{x^2 x^a x_c}{\rho^2(\rho^2 - K x^2)}$
$\eta^{ab}_{(p,q)} - K\frac{x^a x^b}{\rho^2}$

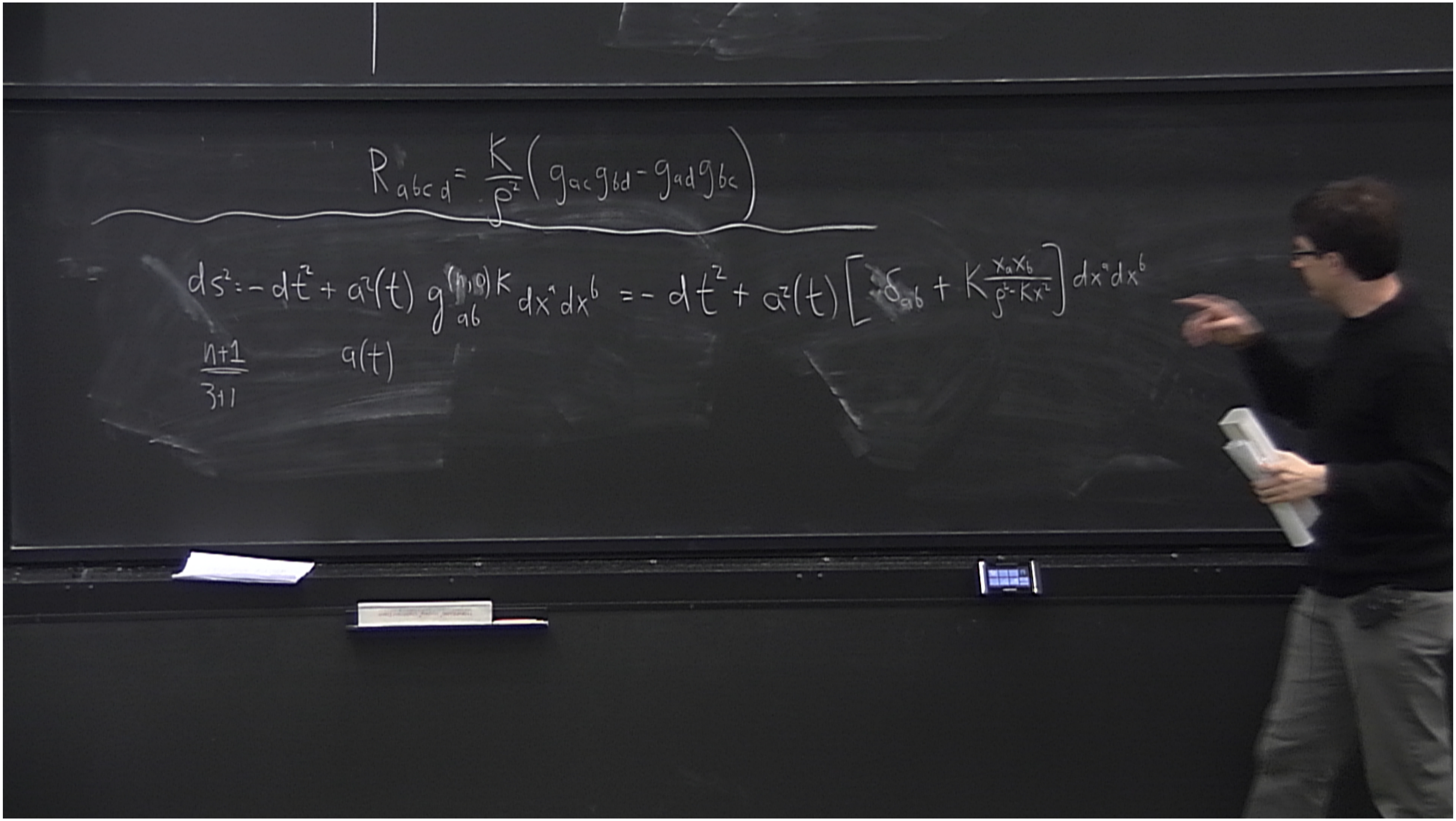

$\frac{K x_a x_b}{\rho^2 - K x^2}\Big) dx^a dx^b$
$(p,q) K$
ab
$\delta^a_c = g^{ab} g_{bc} = \left(\eta^{ab} - K \frac{x^a x^b}{\rho^2}\right)\left(\eta_{bc} + K \frac{x_b x_c}{\rho^2 - K x^2}\right)$
$= \delta^a_c - K \frac{x^a x_c}{\rho^2} + K \frac{x^a x_c}{\rho^2 - K x^2} - K^2 \frac{x^2 x^a x_c}{\rho^2(\rho^2 - K x^2)}$
$\eta^{ab}_{(p,q)} - K \frac{x^a x^b}{\rho^2}$
$x_a = \eta^{(p,q)}_{ab} x^b$
$x^a{}_{,b} = \delta^a_b$
$x_{a,b} = \eta_{ab}$
$x_a x^a = x^2$
$x^2{}_{,c} = 2 x_c$

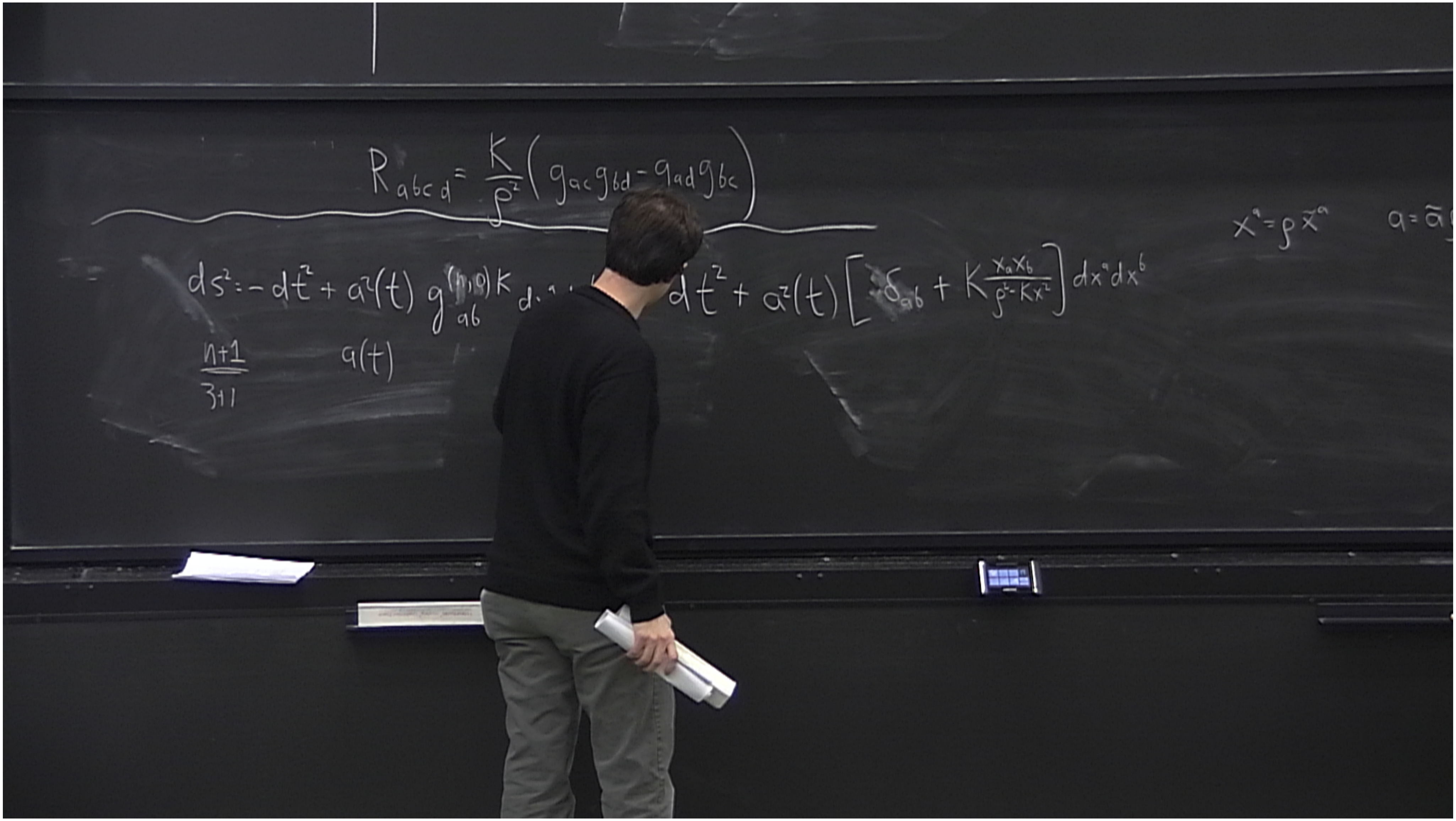

$R_{abcd} = \frac{K}{\rho^2}(g_{ac}g_{bd} - g_{ad}g_{bc}$

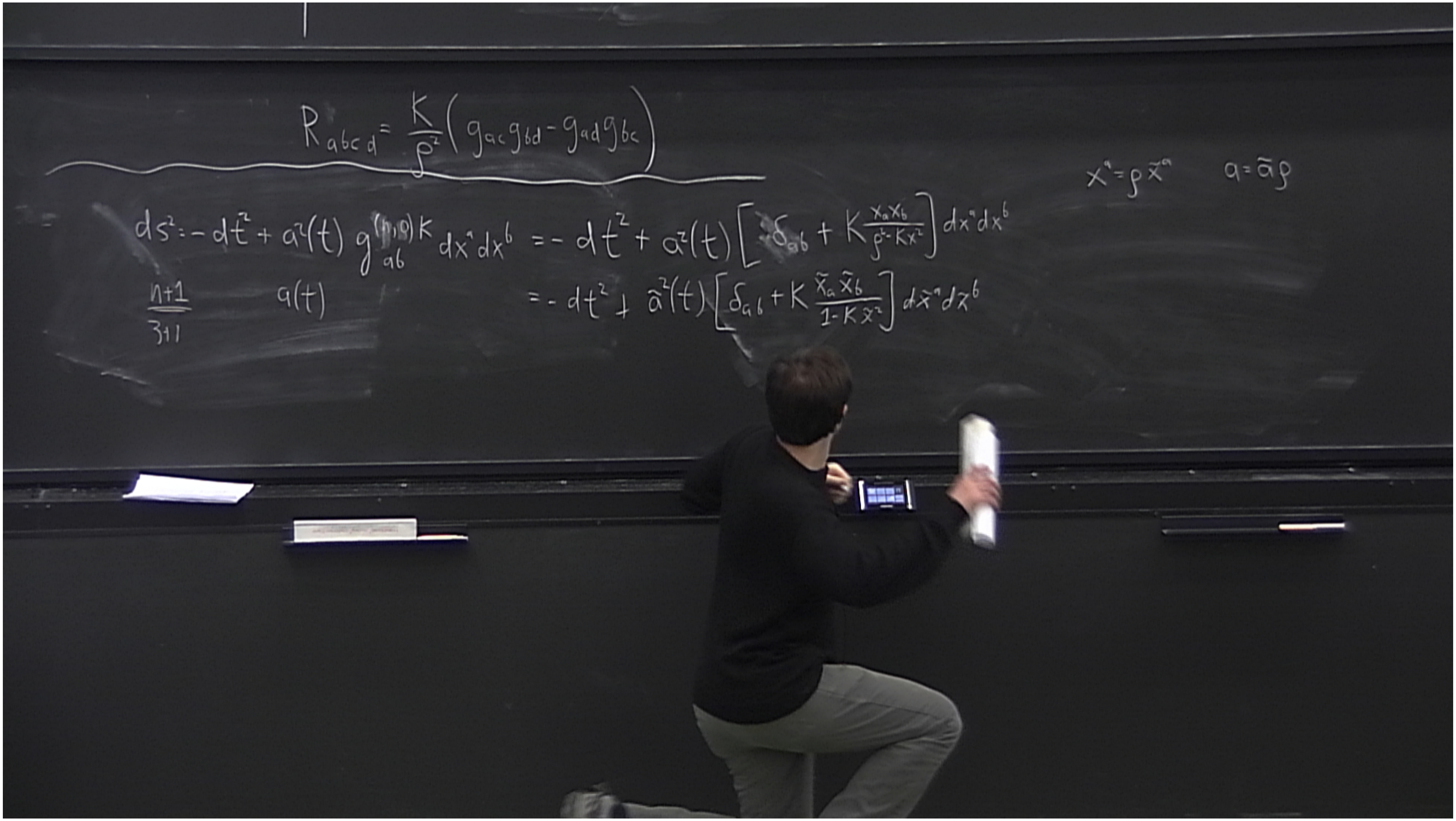

$R_{abcd} = \frac{K}{\rho^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$

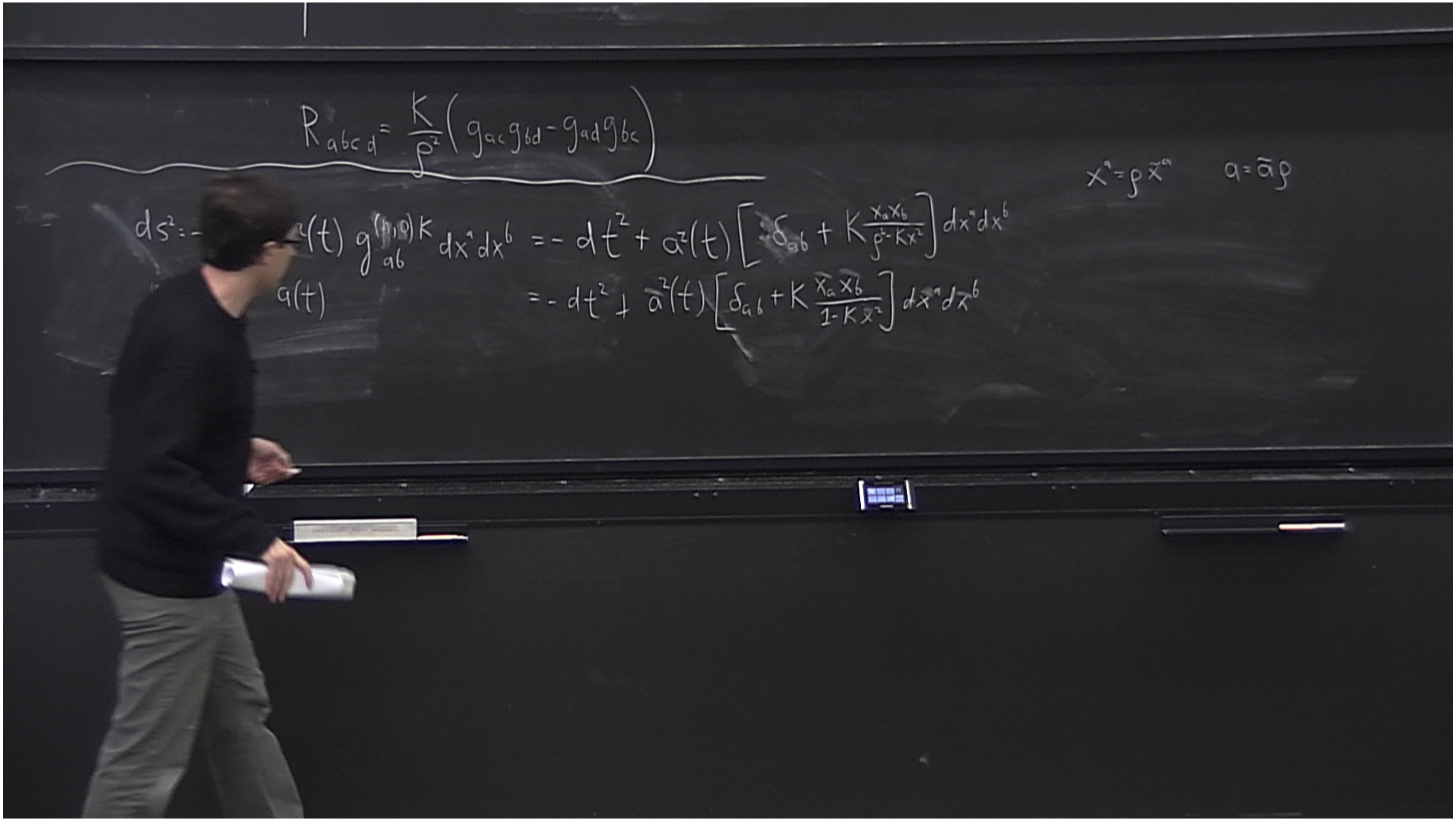

$R_{abcd} = \frac{K}{\rho^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$
$ds^2 = -dt^2 + a^2(t)\, g^{(p,q)K}_{ab}\, dx^a dx^b$

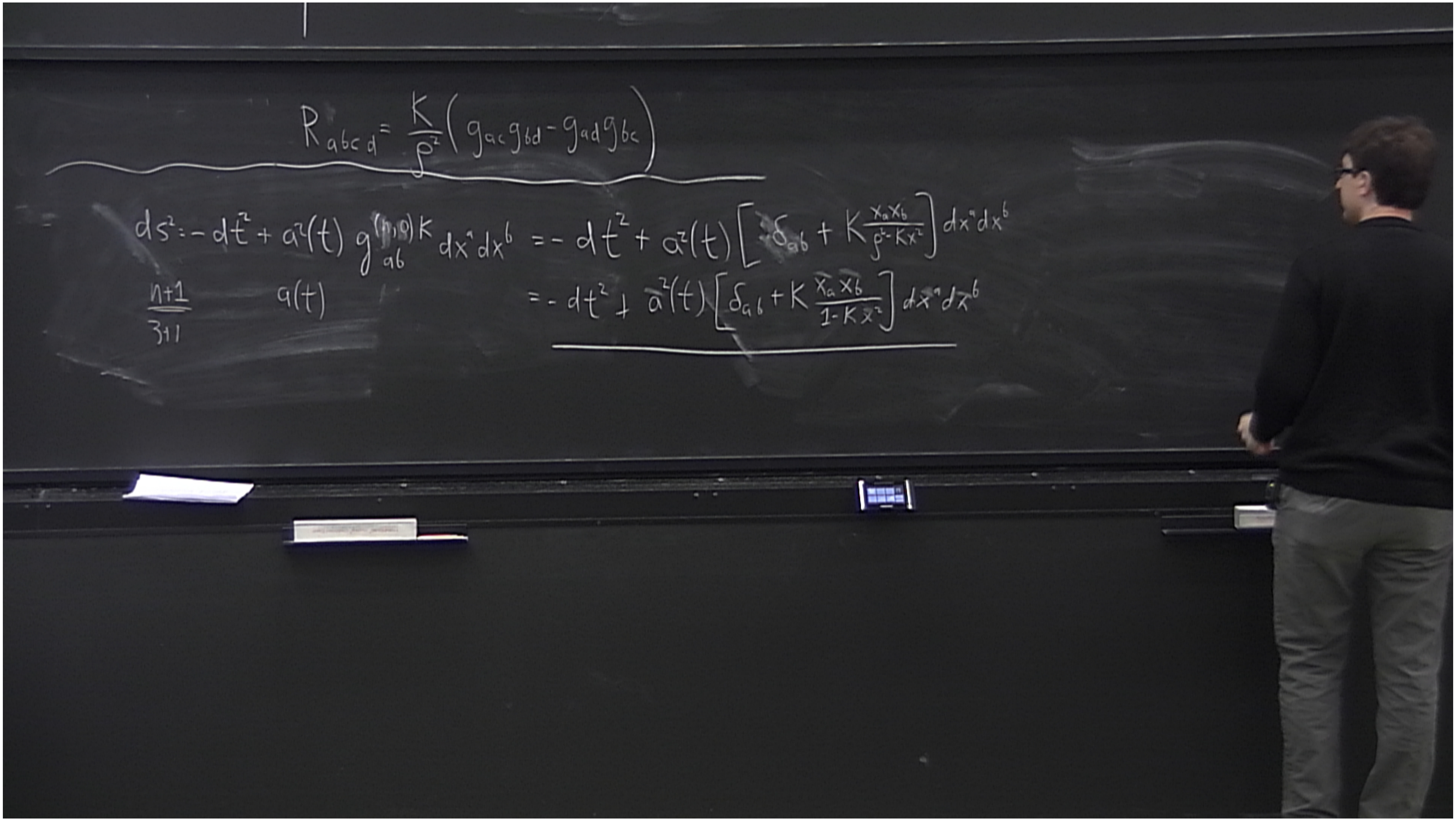

$R_{abcd} = \frac{K}{\rho^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$
$ds^2 = -dt^2 + a^2(t)\, g^{(n,0)K}_{ab}\, dx^a dx^b = -d$
n+1
3+1
a(t)

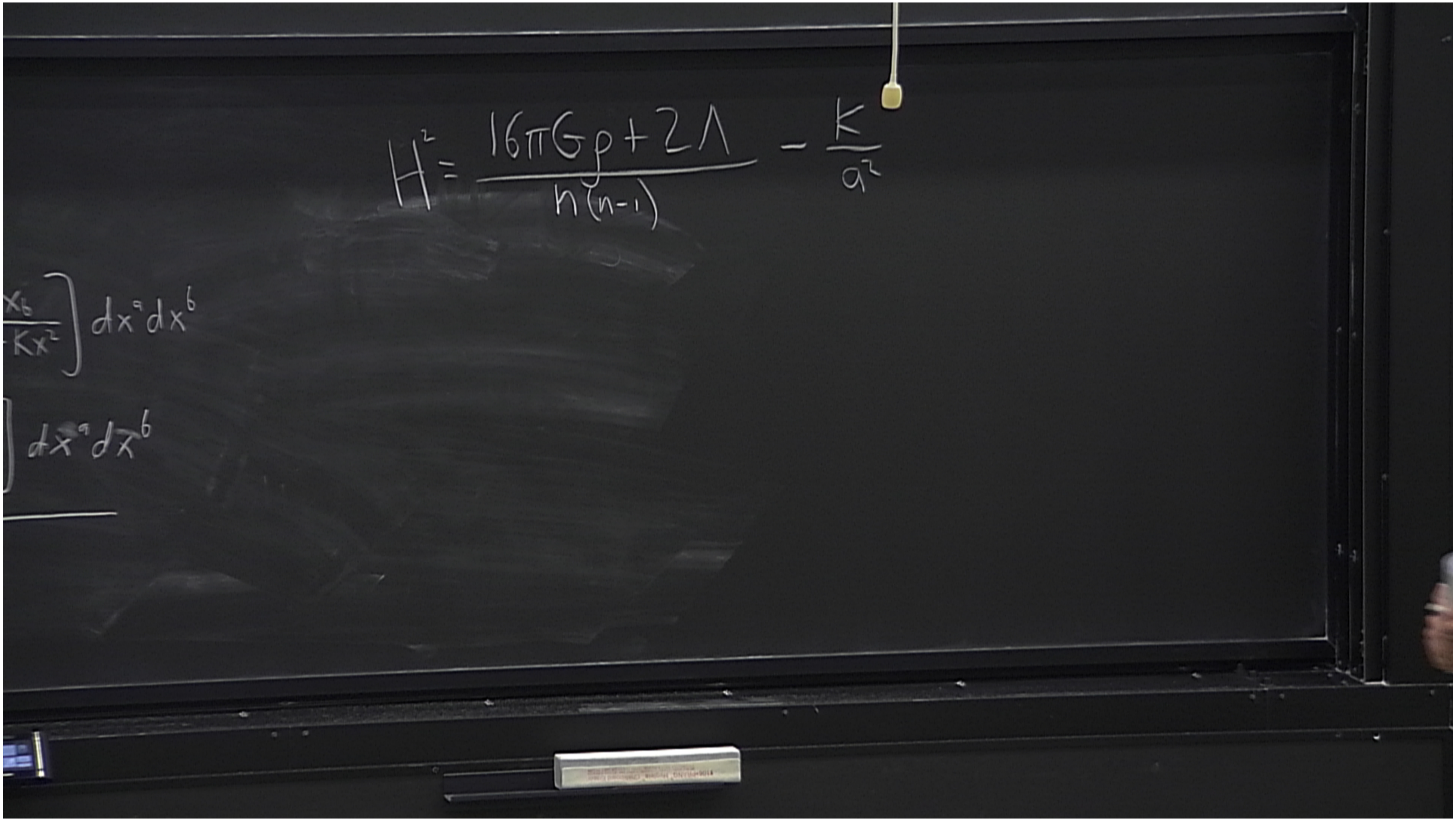

$R_{abcd} = \frac{K}{\rho^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$
$ds^2 = -dt^2 + a^2(t)\, g^{(n,0)K}_{ab}\, dx^a dx^b = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{\rho^2 - Kx^2}\right] dx^a dx^b$
$n+1$
$a(t)$
$3+1$

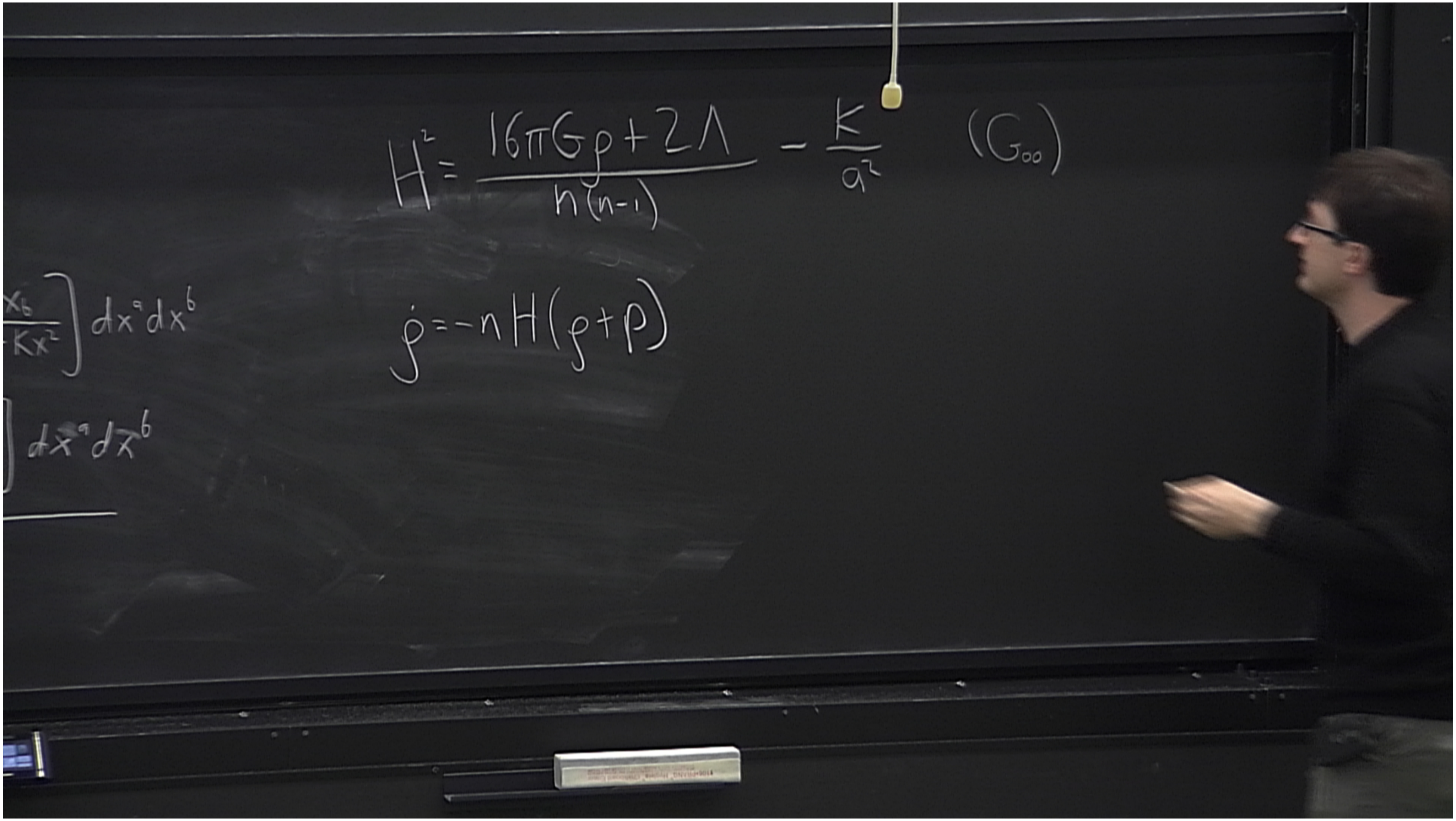

$R_{abcd} = \frac{K}{\rho^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$
$x^a = \rho \tilde{x}^a$
$ds^2 = -dt^2 + a^2(t)\, g^{(n,0)K}_{ab}$
$dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{\rho^2 - Kx^2}\right] dx^a dx^b$
n+1
3+1
a(t)

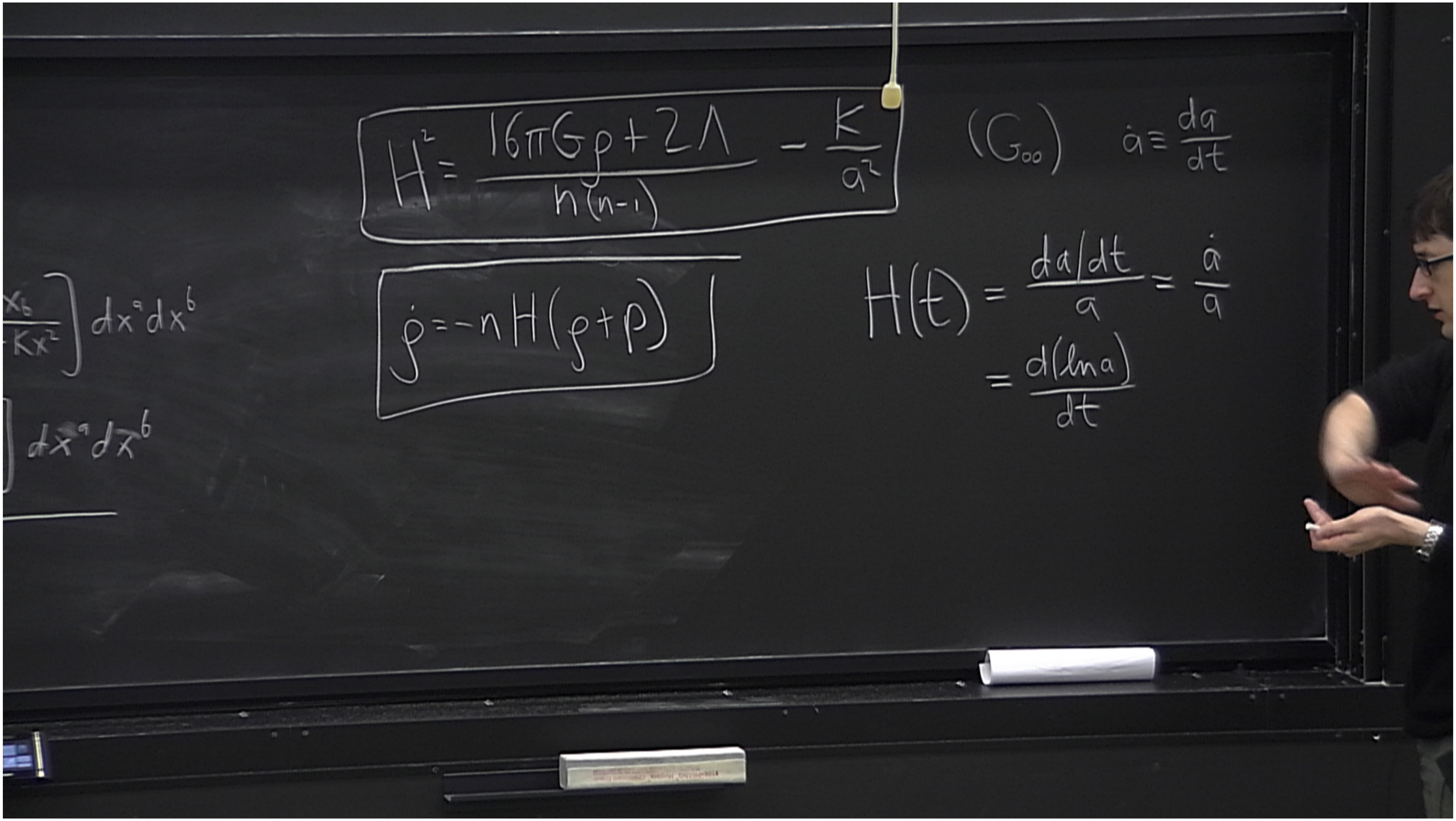

$R_{abcd} = \frac{K}{\rho^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$
$x^a = \rho\tilde{x}^a$ $a = \tilde{a}\rho$
$ds^2 = -dt^2 + a^2(t)\, g^{(n,0)K}_{ab}\, dx^a dx^b = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{\rho^2 - Kx^2}\right] dx^a dx^b$
$= -dt^2 + \tilde{a}^2(t)\left[\delta_{ab} + K\frac{\tilde{x}_a \tilde{x}_b}{1 - K\tilde{x}^2}\right] d\tilde{x}^a d\tilde{x}^b$
n+1
3+1
a(t)

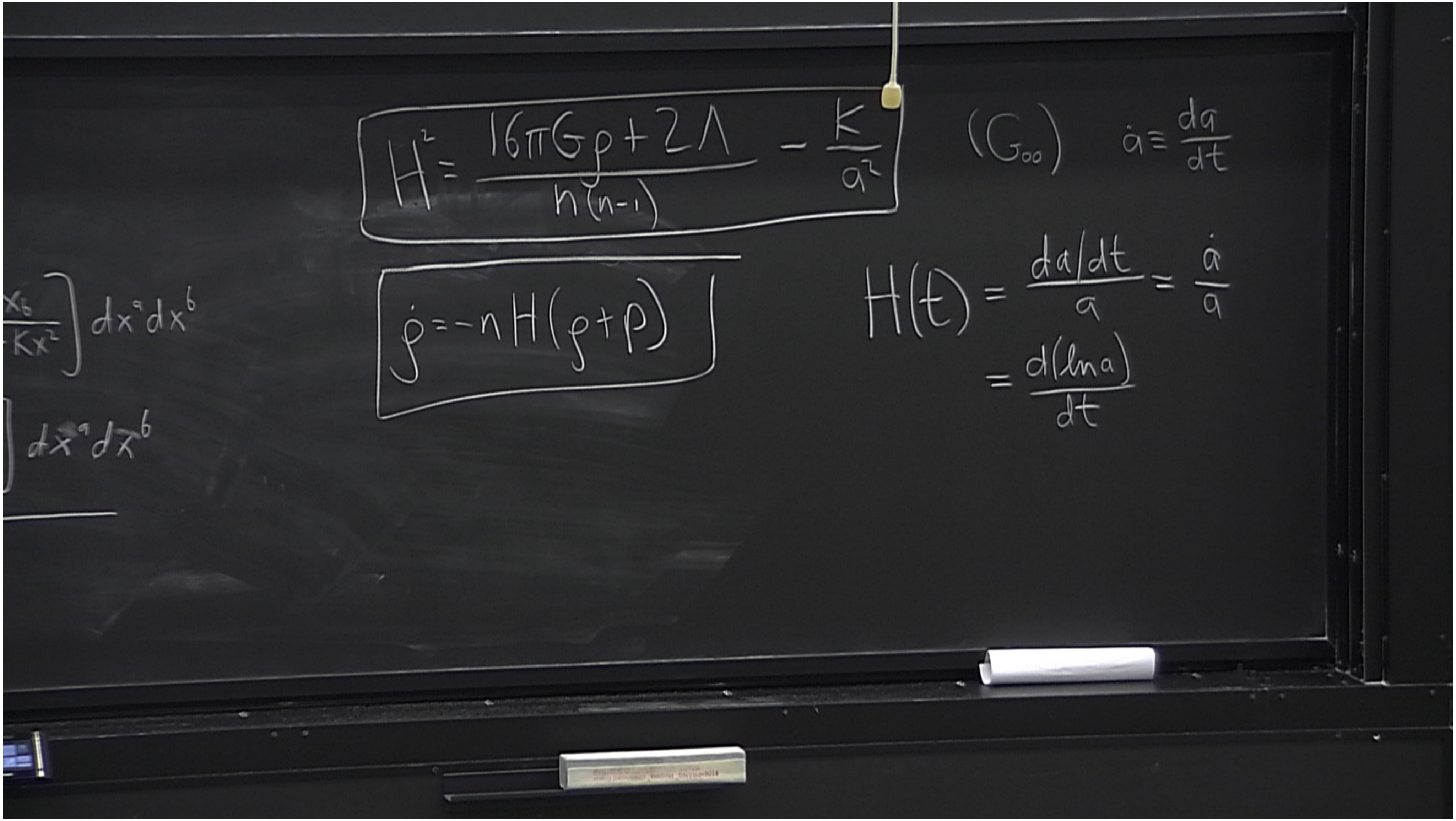

$R_{abcd} = \frac{K}{\rho^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$
$x^a = \rho \tilde{x}^a$
$a = \tilde{a}\rho$
$ds^2 = -$
$(t)\, g^{(n,0)K}_{ab}\, dx^a dx^b = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{\rho^2 - Kx^2}\right] dx^a dx^b$
$a(t)$
$= -dt^2 + \tilde{a}^2(t)\left[\delta_{ab} + K\frac{\tilde{x}_a \tilde{x}_b}{1 - K\tilde{x}^2}\right] d\tilde{x}^a d\tilde{x}^b$

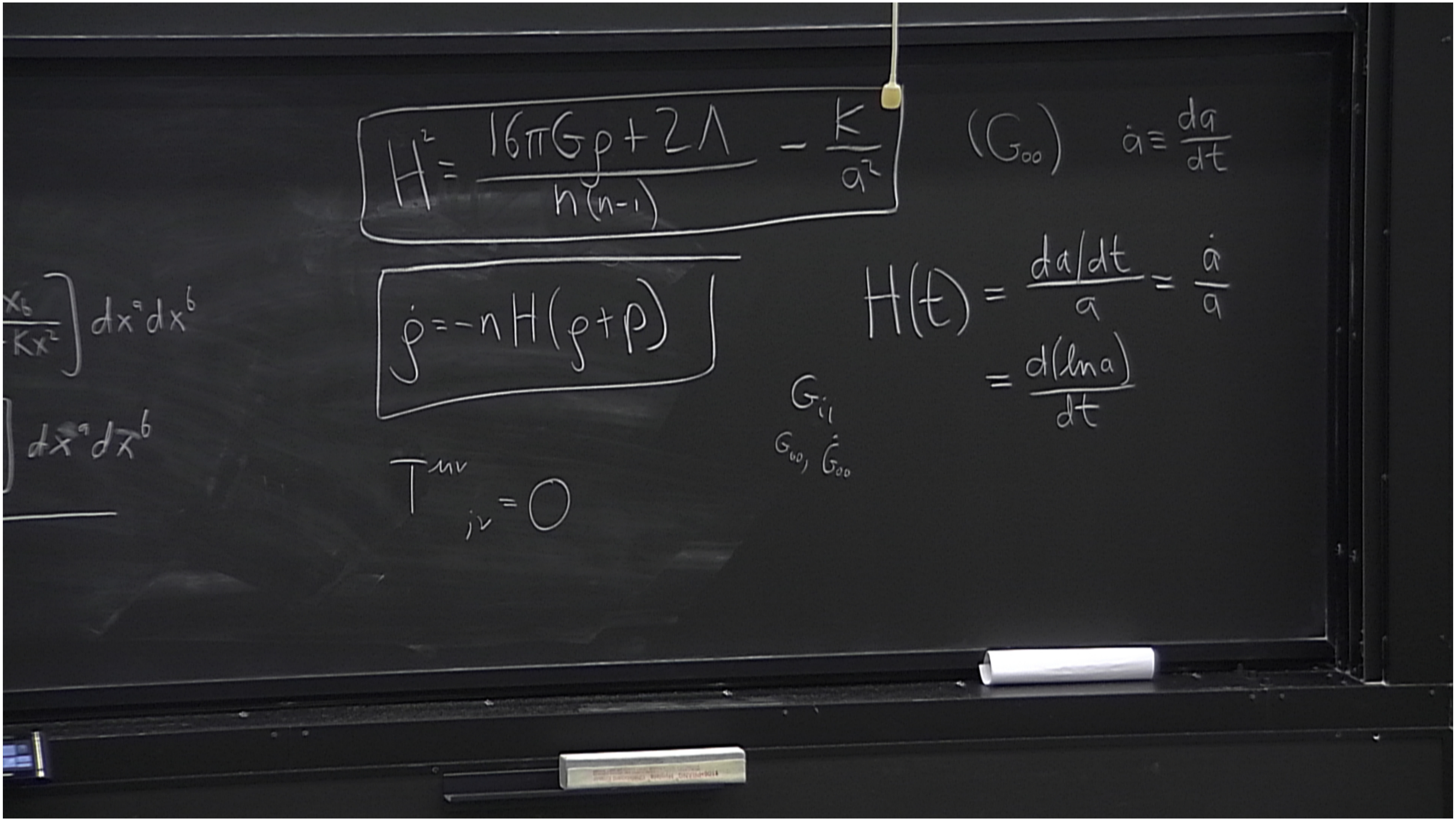

$R_{abcd} = \frac{K}{\rho^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$
$ds^2 = -dt^2 + a^2(t)\, g^{(n,0)K}_{ab}\, dx^a dx^b = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{\rho^2 - Kx^2}\right] dx^a dx^b$
$n+1$
$3+1$
$a(t)$
$= -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{1 - Kx^2}\right] dx^a dx^b$


$H^2 = \frac{16\pi G\rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$
$dx^a dx^b$
$d\bar{x}^a d\bar{x}^b$

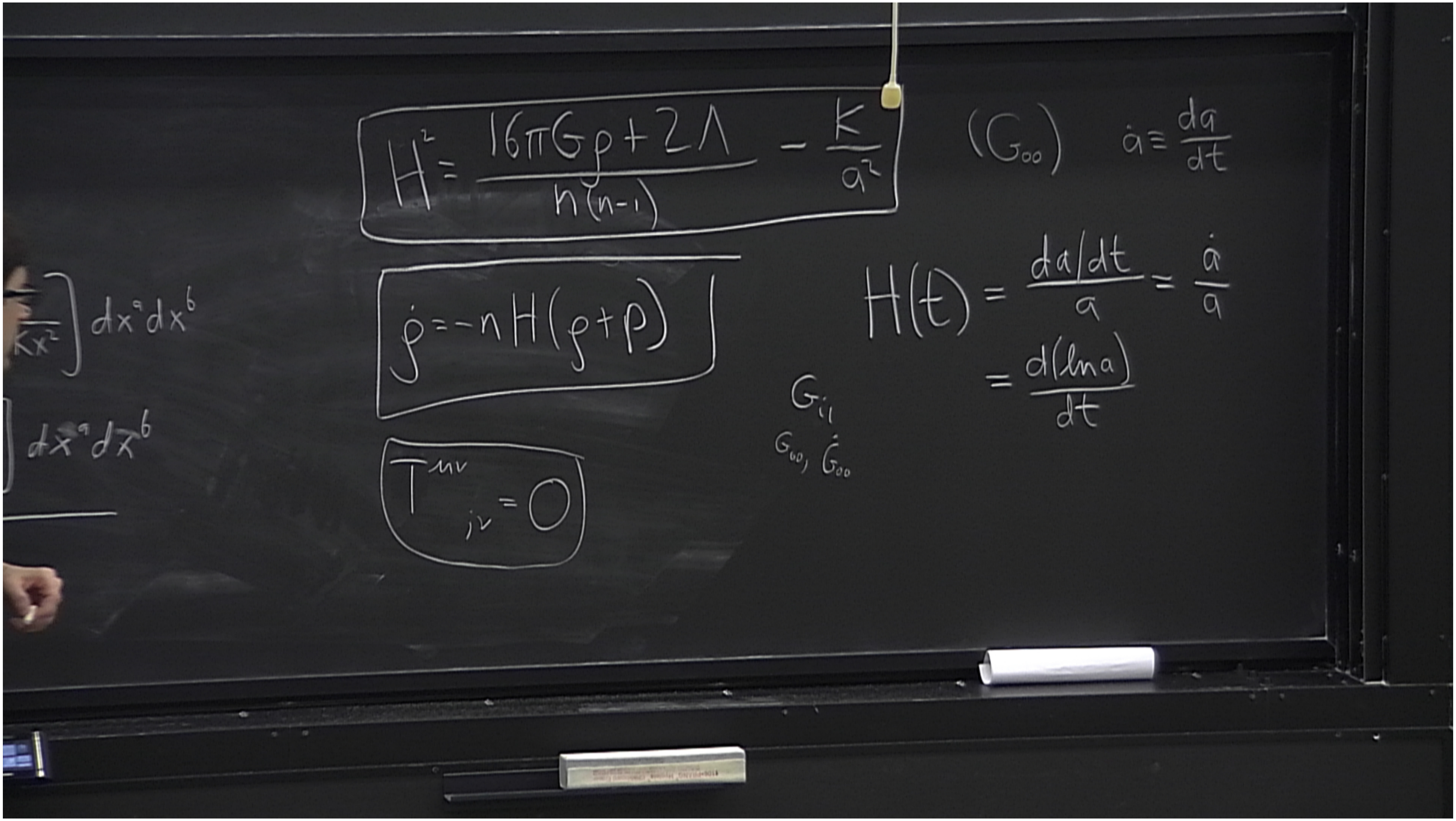

$H^2 = \frac{16\pi G\rho + 2\Lambda}{n(n-1)} - \frac{k}{a^2}$ (G_{00})
$\dot{\rho} = -nH(\rho + p)$

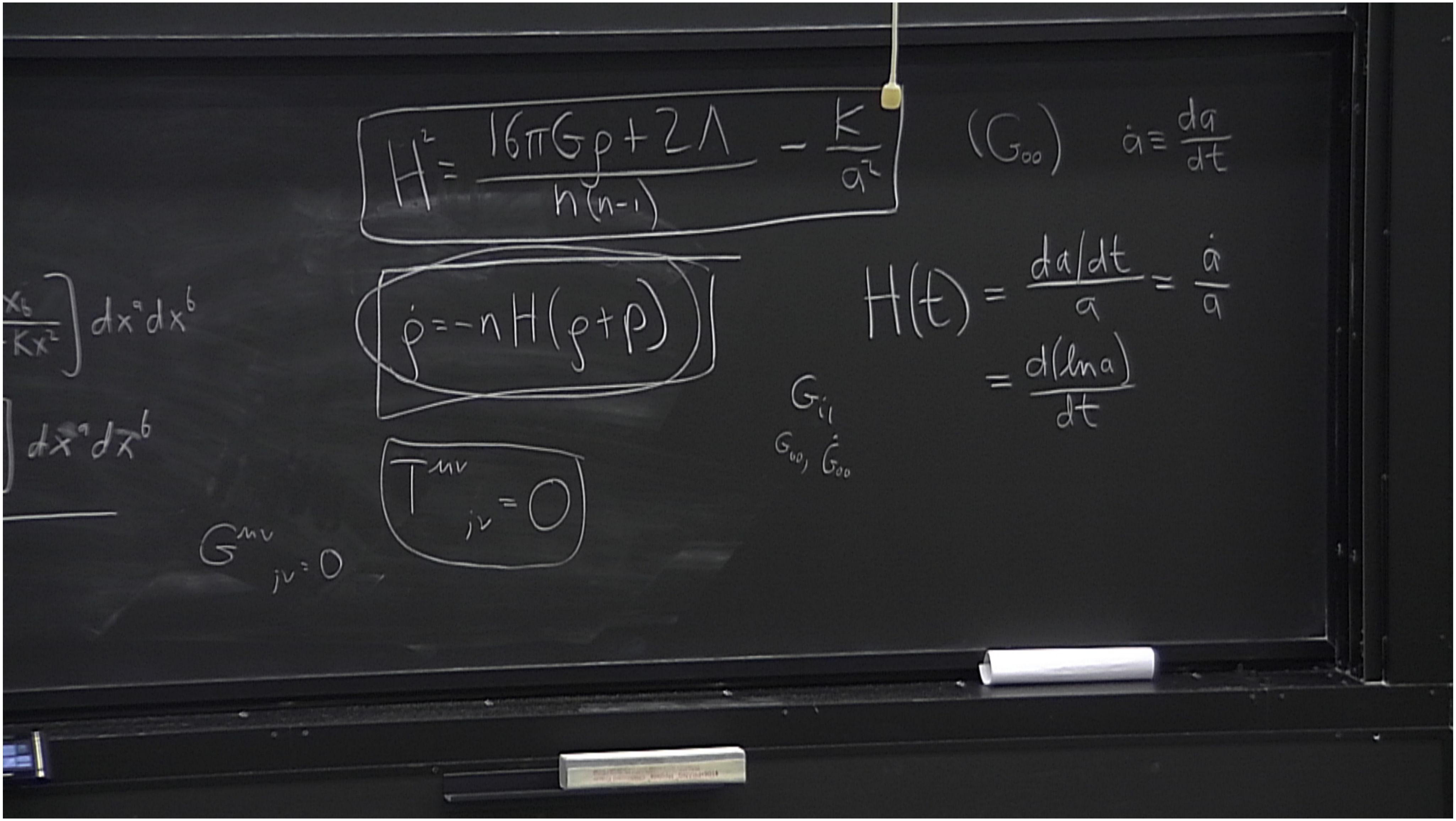

$H^2 = \frac{16\pi G\rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$
(G_{00})
$\dot{a} \equiv \frac{da}{dt}$
$\dot{\rho} = -nH(\rho + p)$
$H(t) = \frac{da/dt}{a} = \frac{\dot{a}}{a}$
$= \frac{d(\ln a)}{dt}$
$dx^a dx^b$

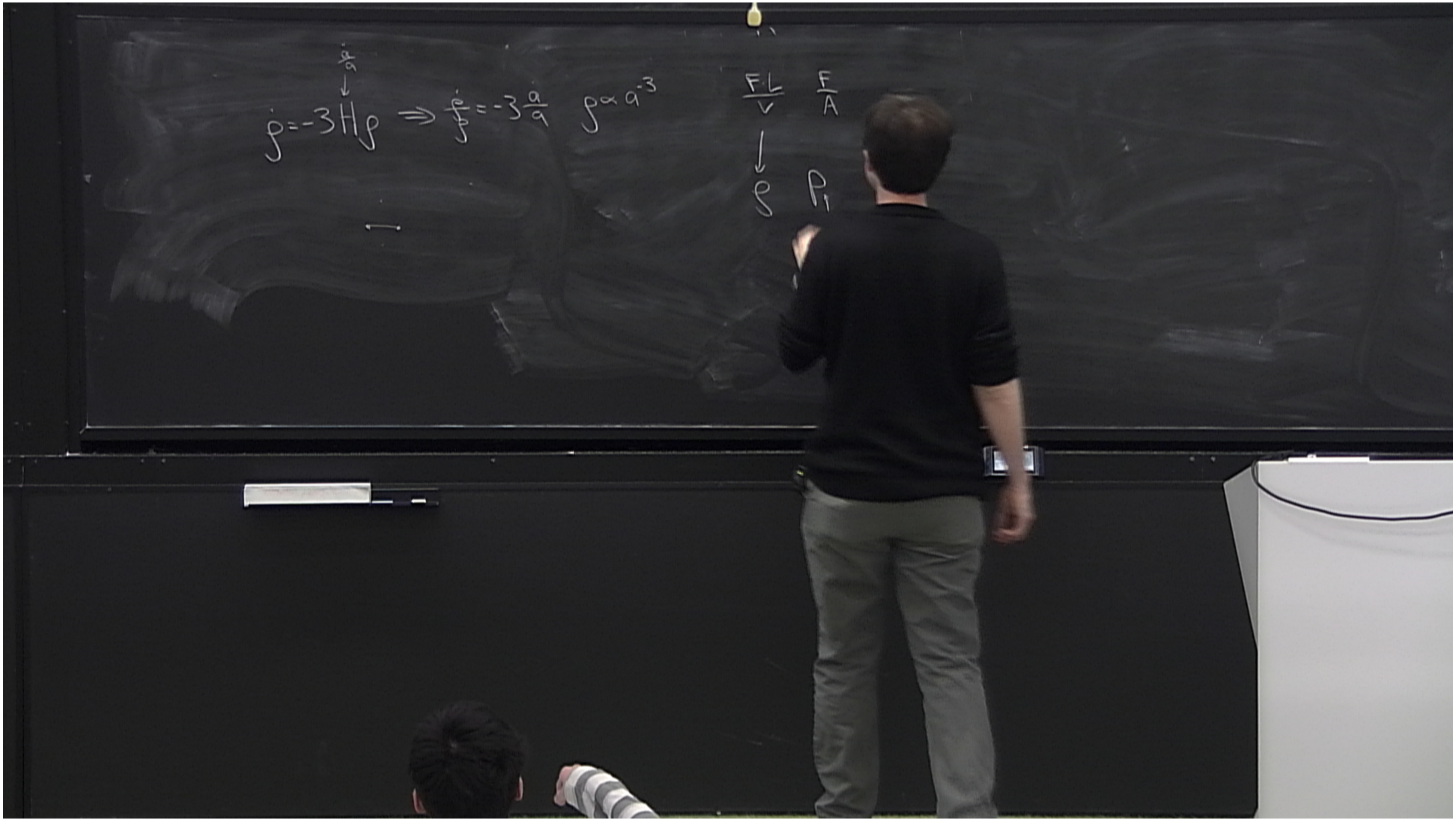

$H^2 = \frac{16\pi G\rho + 2\Lambda}{n(n-1)} - \frac{k}{a^2}$
(G_{00})
$\dot{a} \equiv \frac{da}{dt}$
$\dot{\rho} = -nH(\rho + p)$
$H(t) = \frac{da/dt}{a} = \frac{\dot{a}}{a}$
$= \frac{d(\ln a)}{dt}$
$dx^a dx^b$
$d\bar{x}^a d\bar{x}^b$

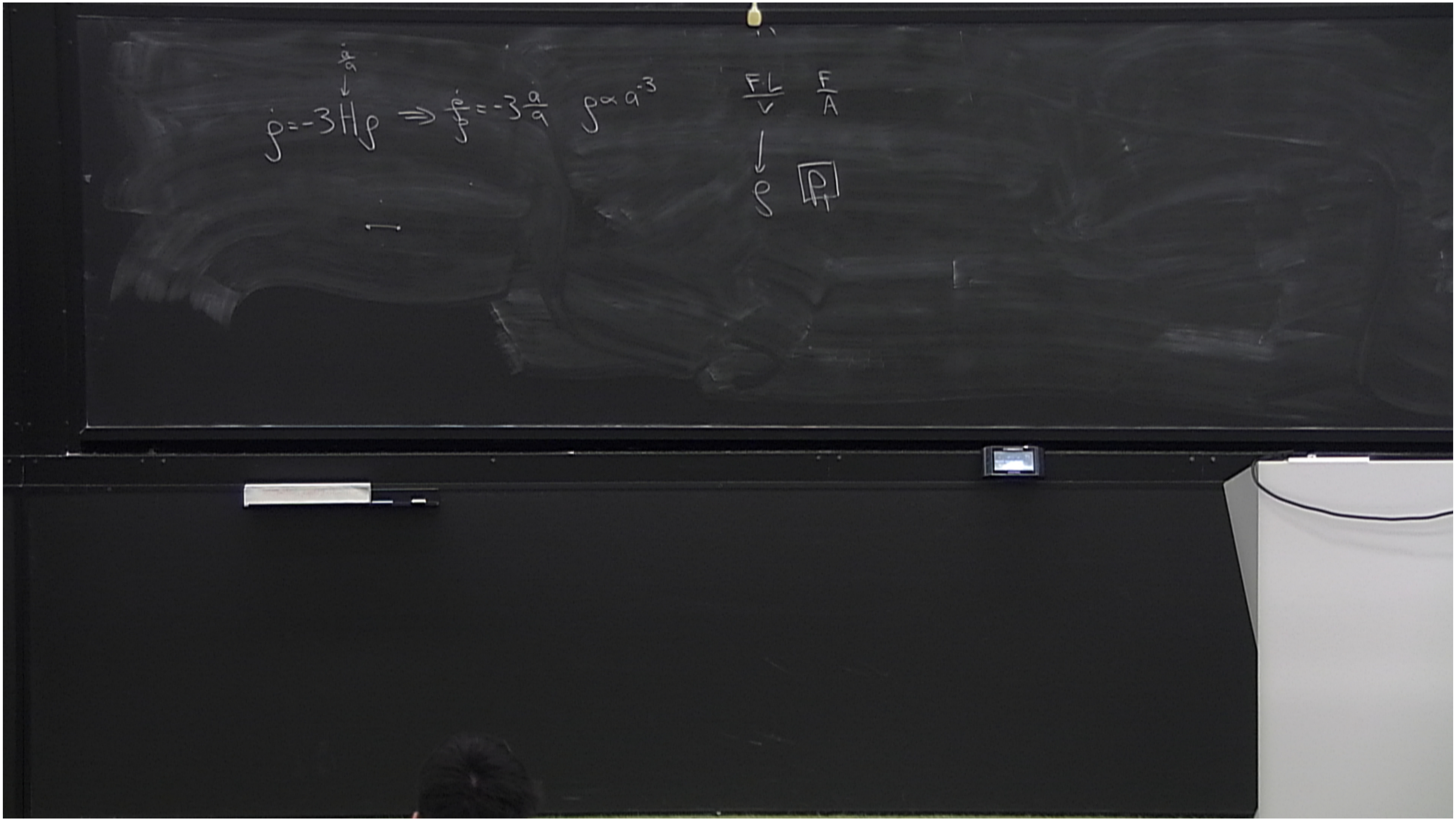


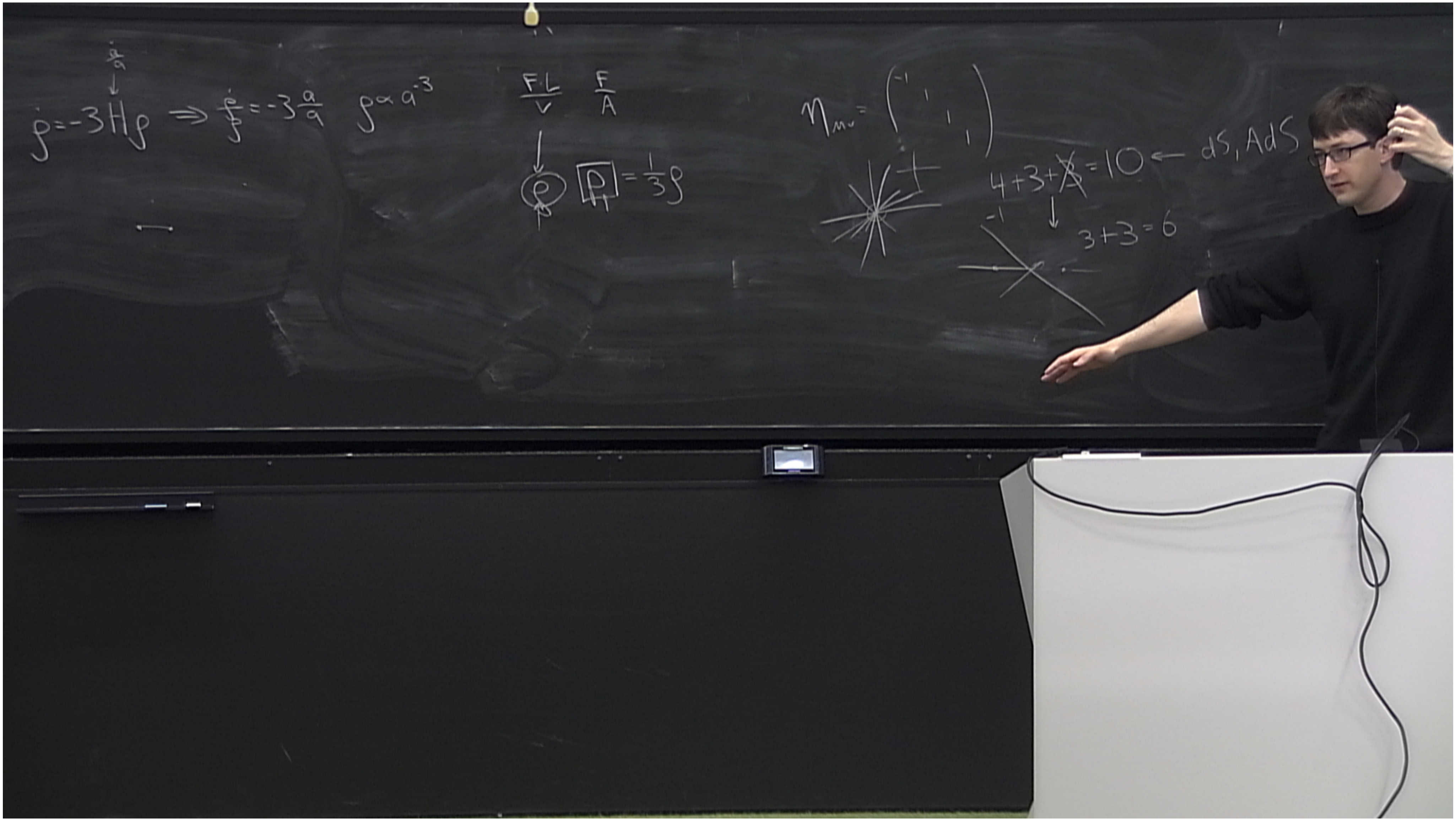
$H^2 = \frac{16\pi G\rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$
(G_{00})
$\dot{a} \equiv \frac{da}{dt}$
$\dot{\rho} = -nH(\rho + p)$
$H(t) = \frac{da/dt}{a} = \frac{\dot{a}}{a}$
$= \frac{d(\ln a)}{dt}$
G_{ii}
$G_{00}, \dot{G}_{00}$
$T^{\mu\nu}{}_{;\nu} = 0$
$dx^a dx^b$

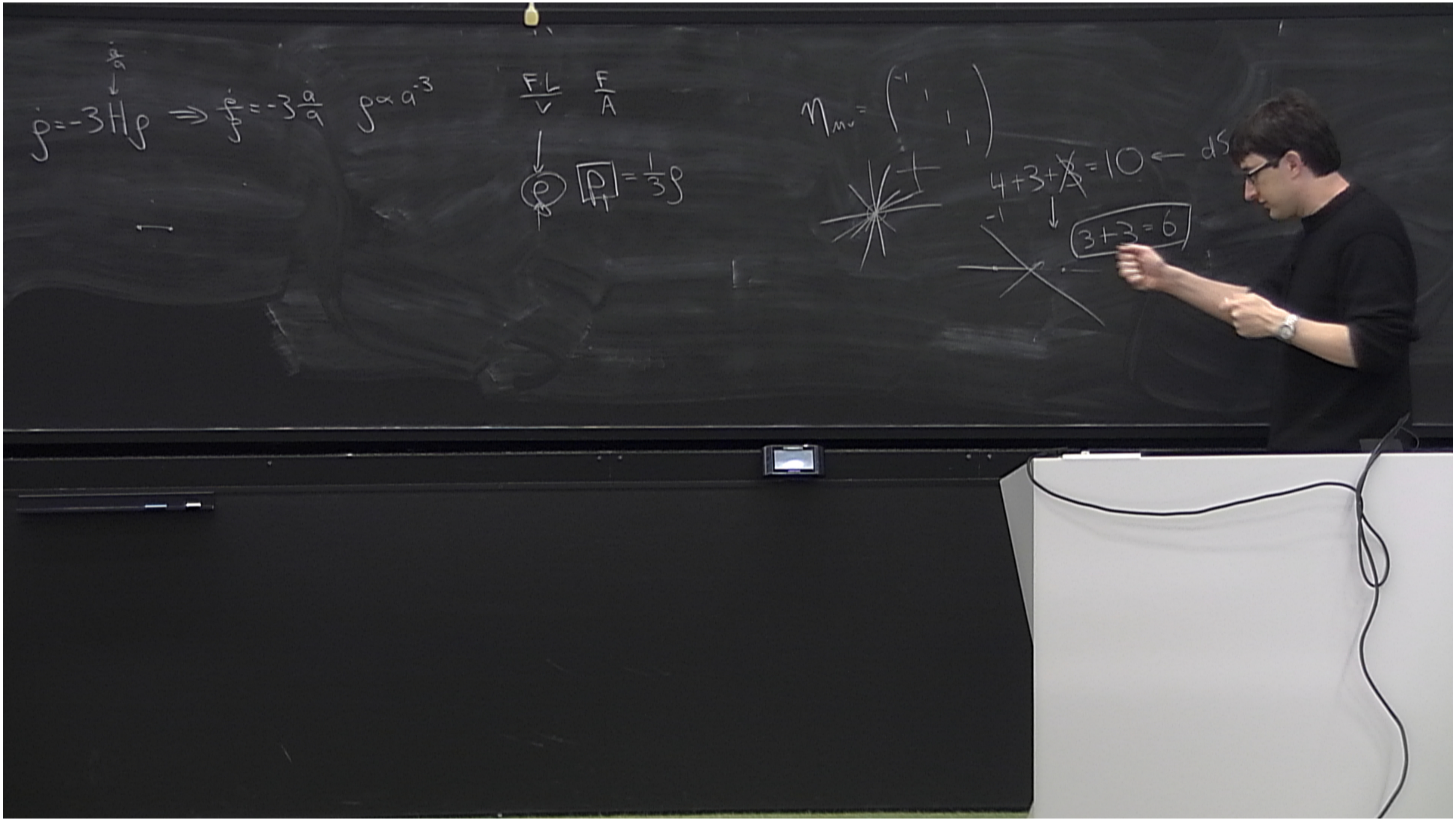

$H^2 = \frac{16\pi G\rho + 2\Lambda}{n(n-1)} - \frac{k}{a^2}$
(G_{00})
$\dot{a} \equiv \frac{da}{dt}$
$H(t) = \frac{da/dt}{a} = \frac{\dot{a}}{a}$
$= \frac{d(\ln a)}{dt}$
$\dot{\rho} = -nH(\rho + p)$
G_{ii}
$G_{00}, \dot{G}_{00}$
$T^{\mu\nu}{}_{;\nu} = 0$
$dx^a dx^b$
$d\tilde{x}^a d\tilde{x}^b$

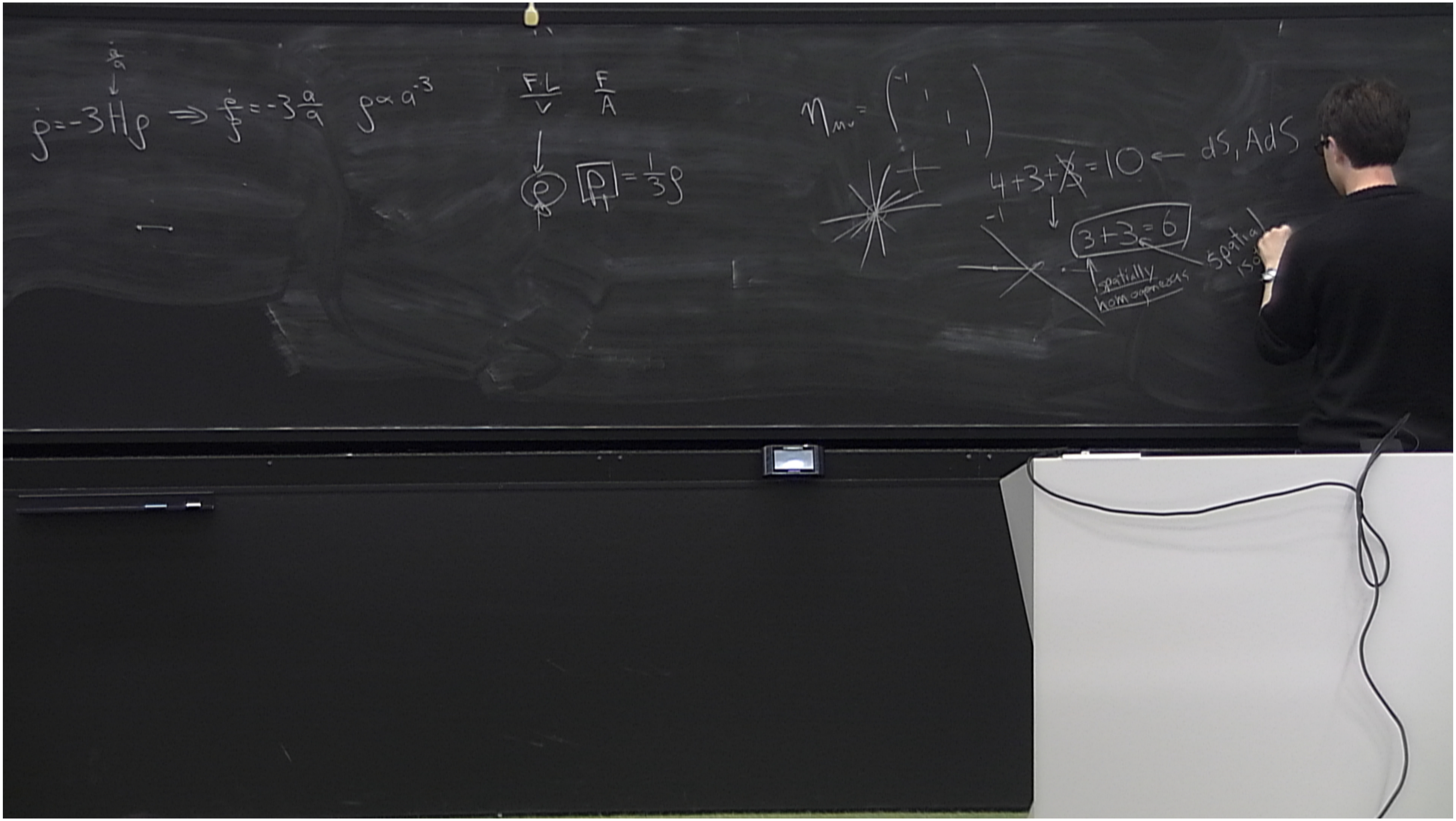

$H^2 = \frac{16\pi G\rho + 2\Lambda}{n(n-1)} - \frac{k}{a^2}$
(G_{00})
$\dot{a} \equiv \frac{da}{dt}$
$\dot{\rho} = -nH(\rho + p)$
$H(t) = \frac{da/dt}{a} = \frac{\dot{a}}{a}$
$= \frac{d(\ln a)}{dt}$
G_{ii}
$G_{00}, \dot{G}_{00}$
$T^{\mu\nu}{}_{;\nu} = 0$
$G^{\mu\nu}{}_{;\nu} = 0$
$dx^a dx^b$
$d\bar{x}^a d\bar{x}^b$

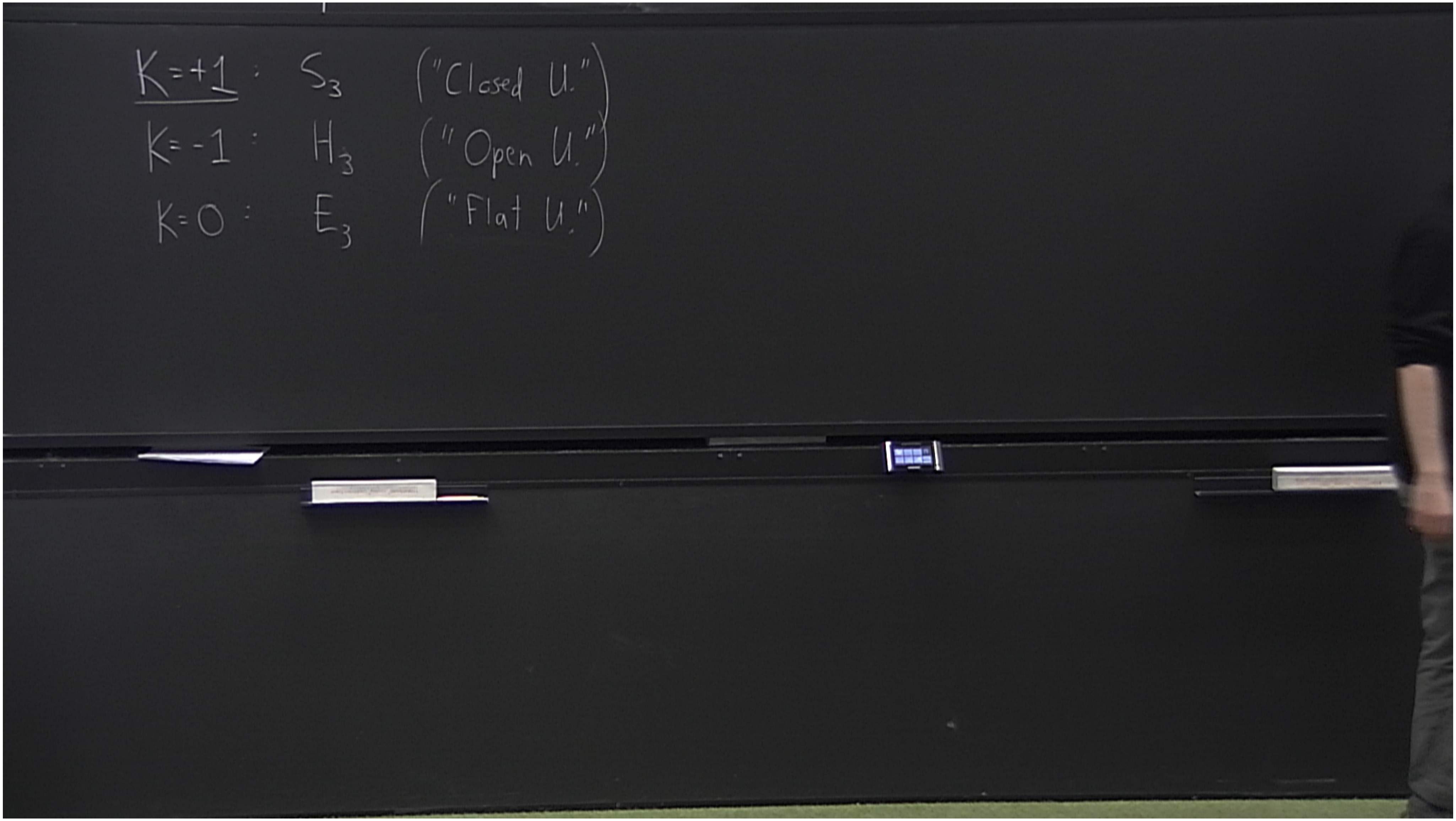

ρ̇ = -3Hρ ⇒ ρ̇/ρ = -3 ȧ/a   ρ ∝ a^-3
F·L / V
F / A
ρ
P

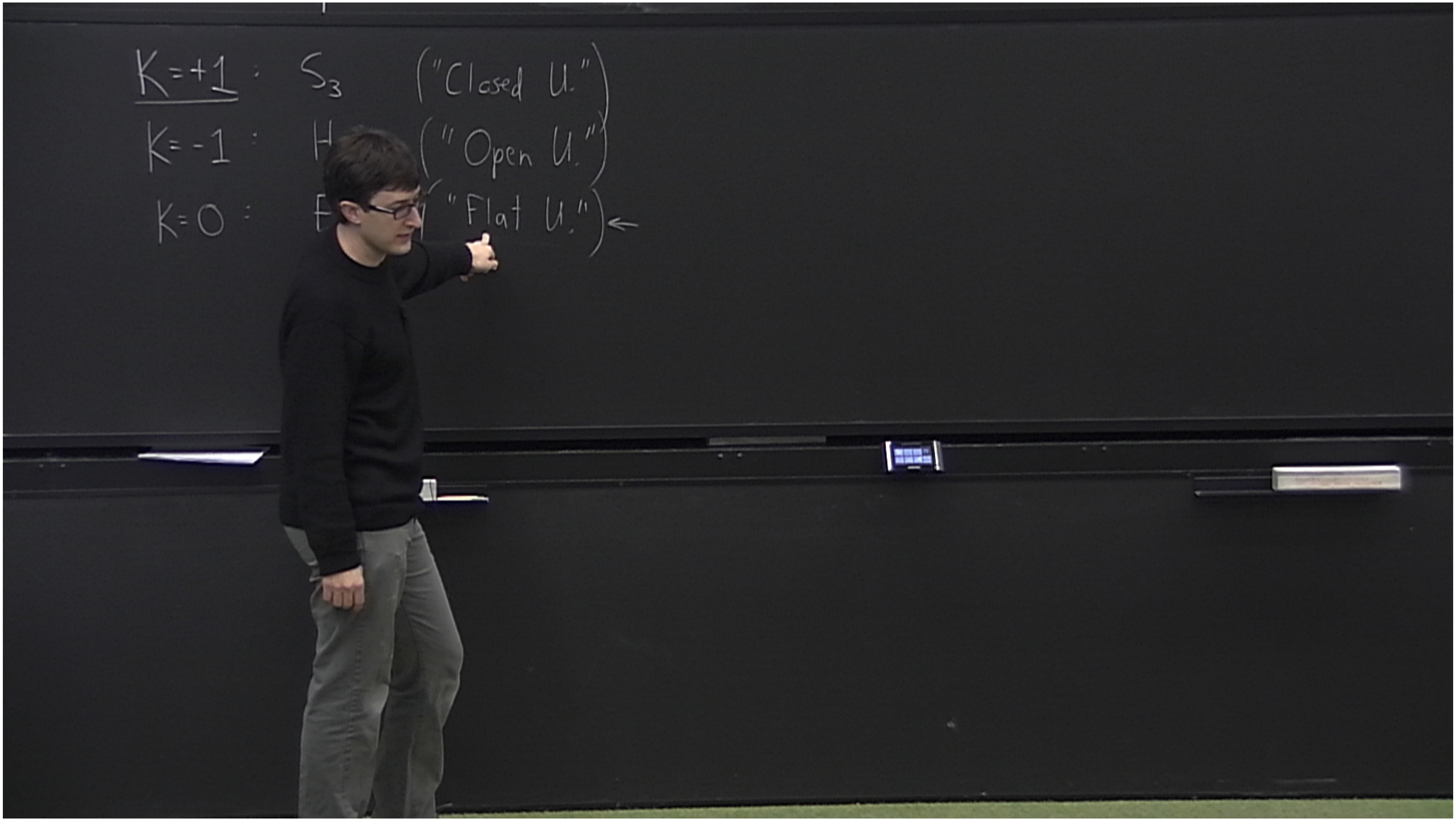

ρ̇ = -3Hρ ⇒ ρ̇/ρ = -3 ȧ/a   ρ ∝ a^-3
F·L/V   F/A
ρ   P

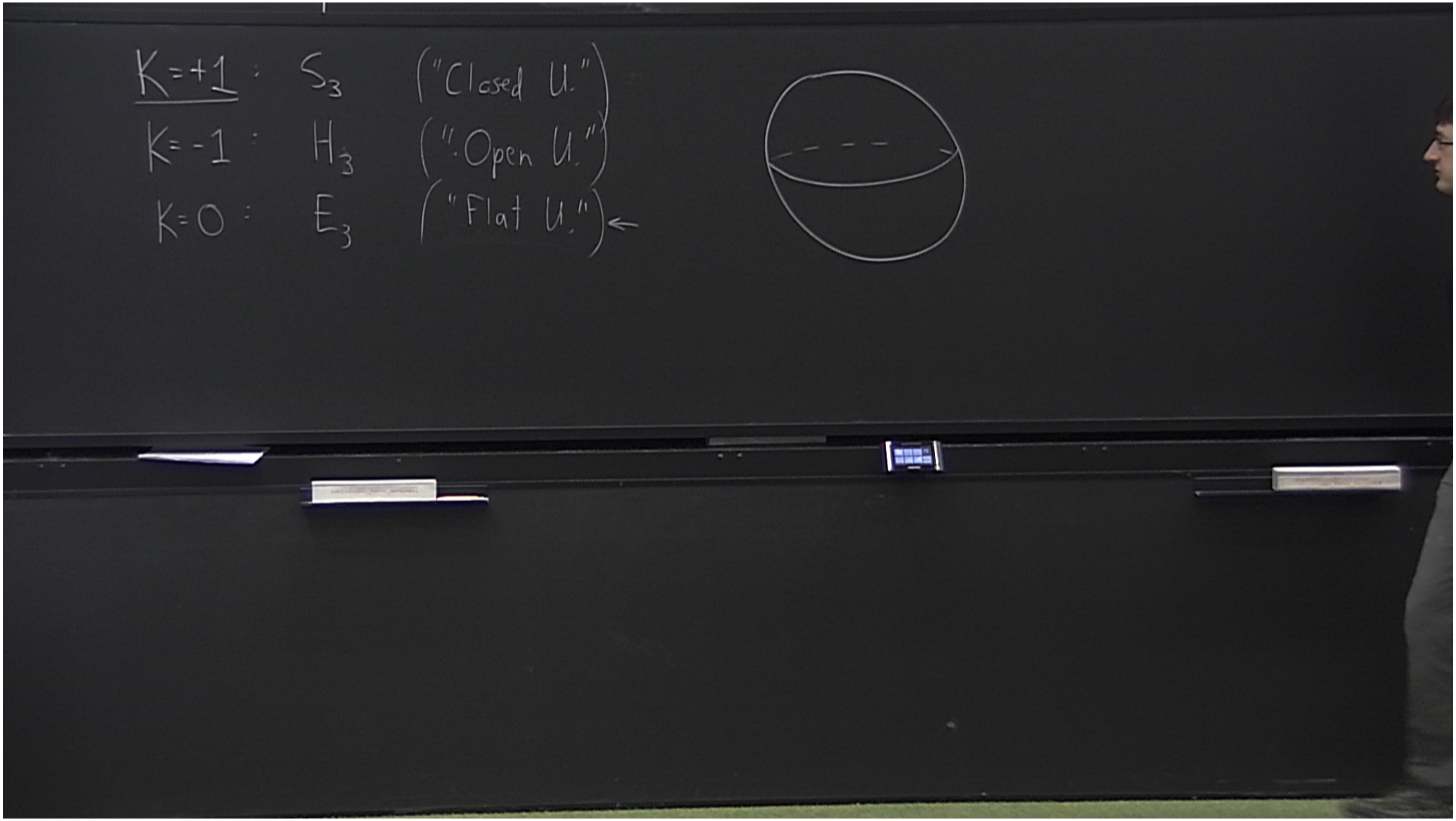

ρ̇ = -3Hρ ⇒ ρ̇/ρ = -3 ȧ/a
ρ ∝ a^-3
F·L/V
F/A
P = 1/3 ρ
η_μν =
4+3+3 = 10 ← dS, AdS
3+3 = 6

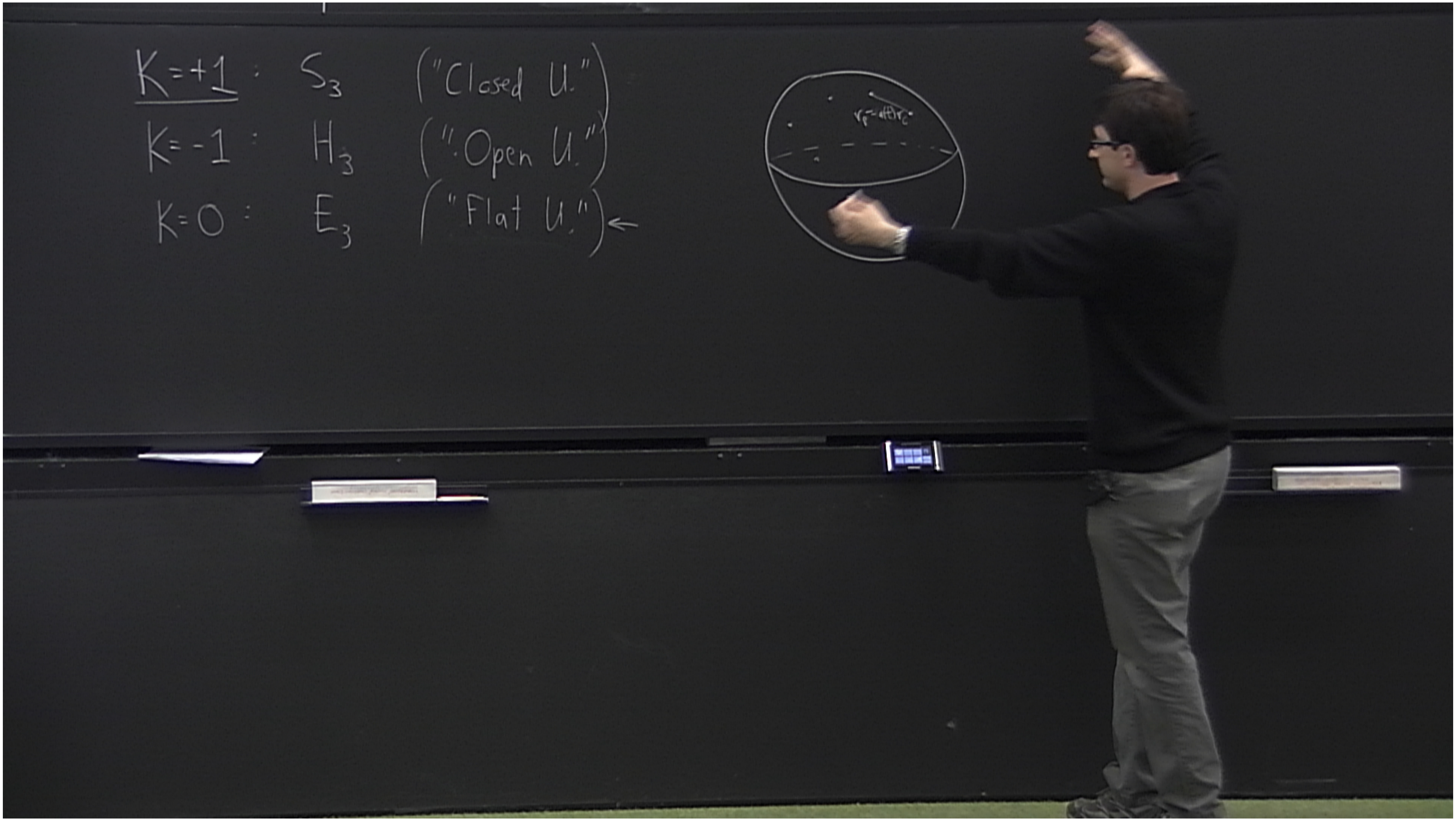


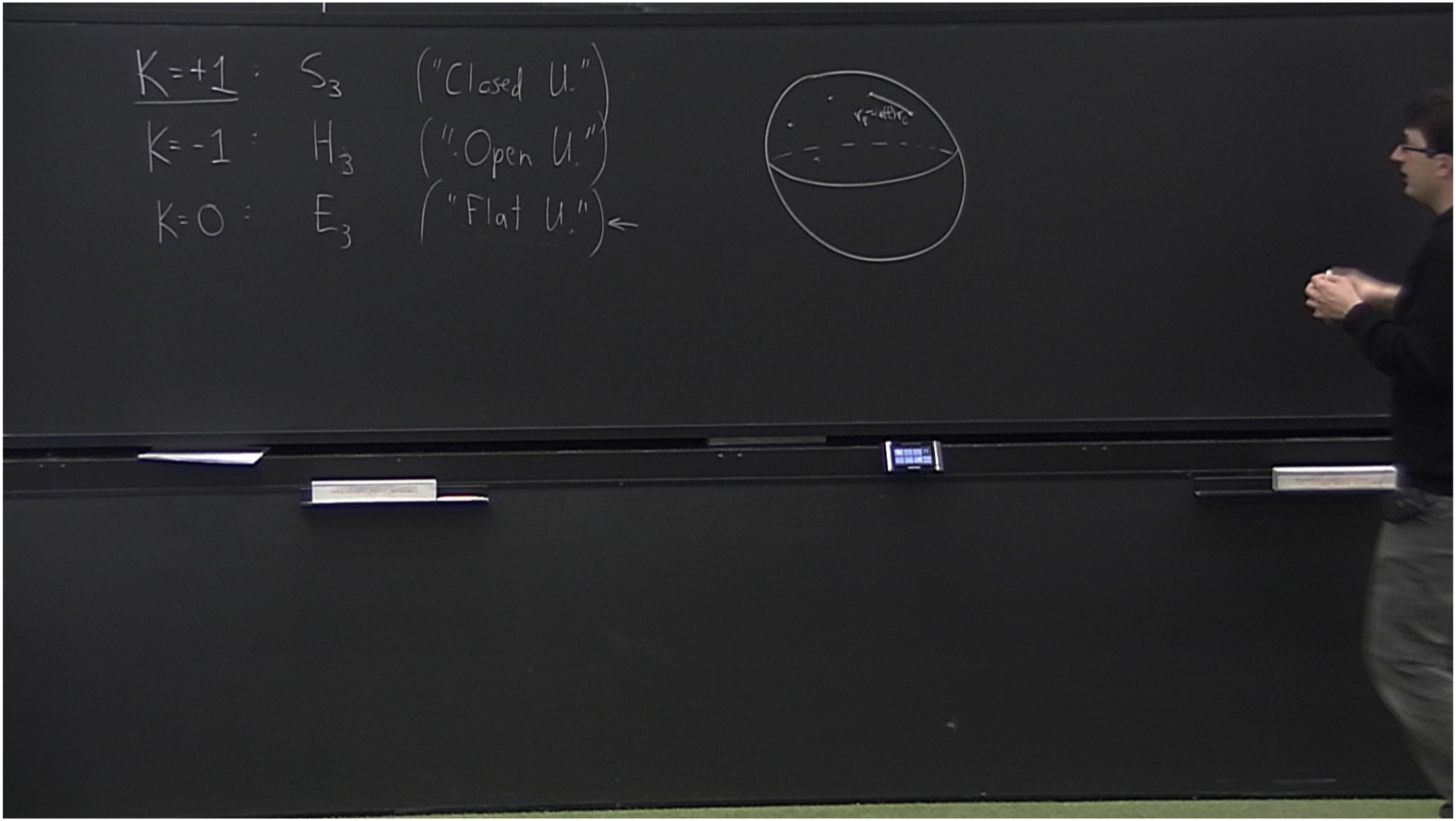

ρ̇ = -3Hρ ⇒ ρ̇/ρ = -3 ȧ/a
ρ ∝ a^-3
FL/V
F/A
p = 1/3 ρ
η_μν =
= 10 ← dS, AdS
3+3 = 6
spatially homogeneous

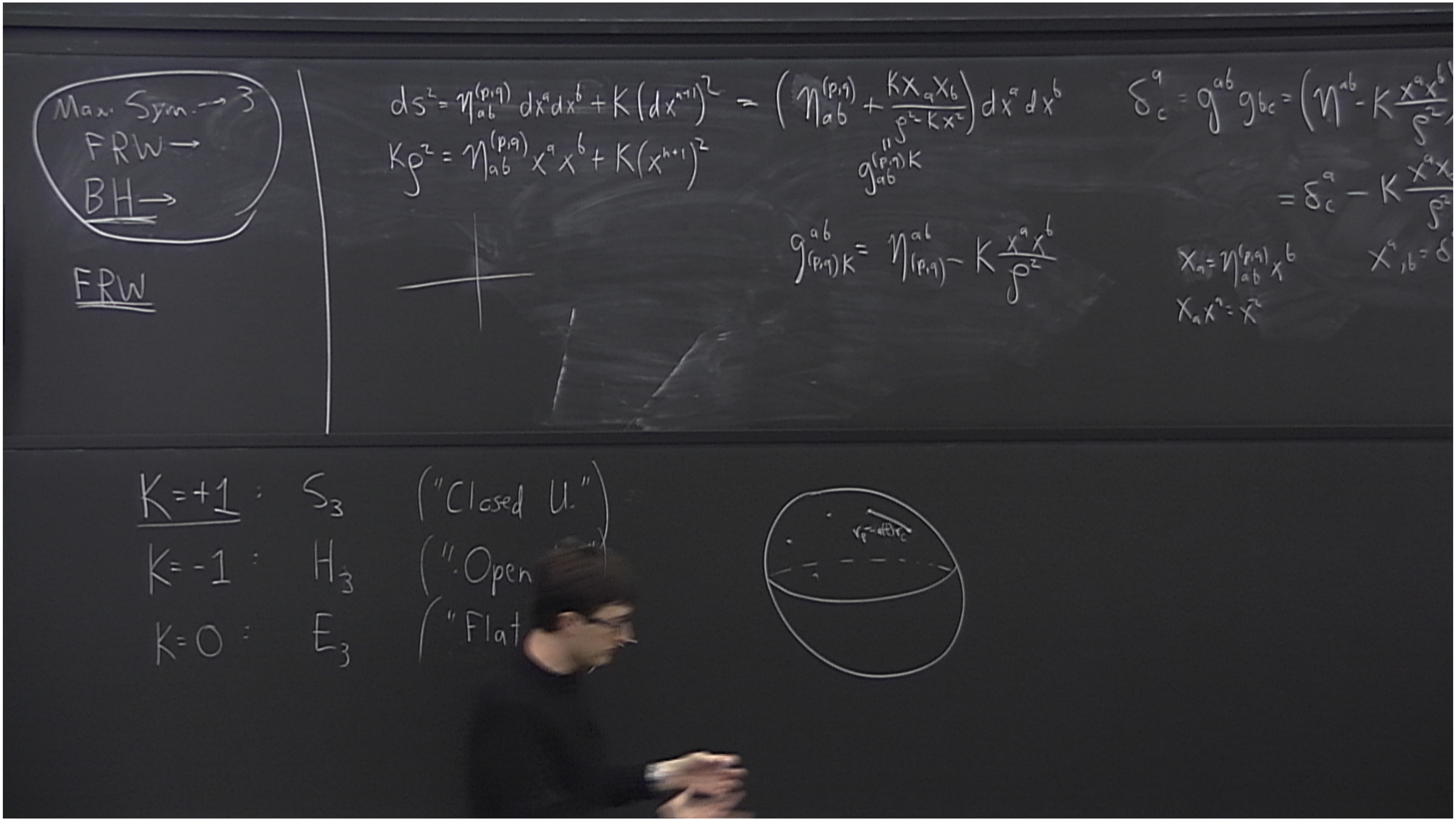

K=+1 : S3 ("Closed U.")
K=-1 : H3 ("Open U.")
K=0 : E3 ("Flat U.")

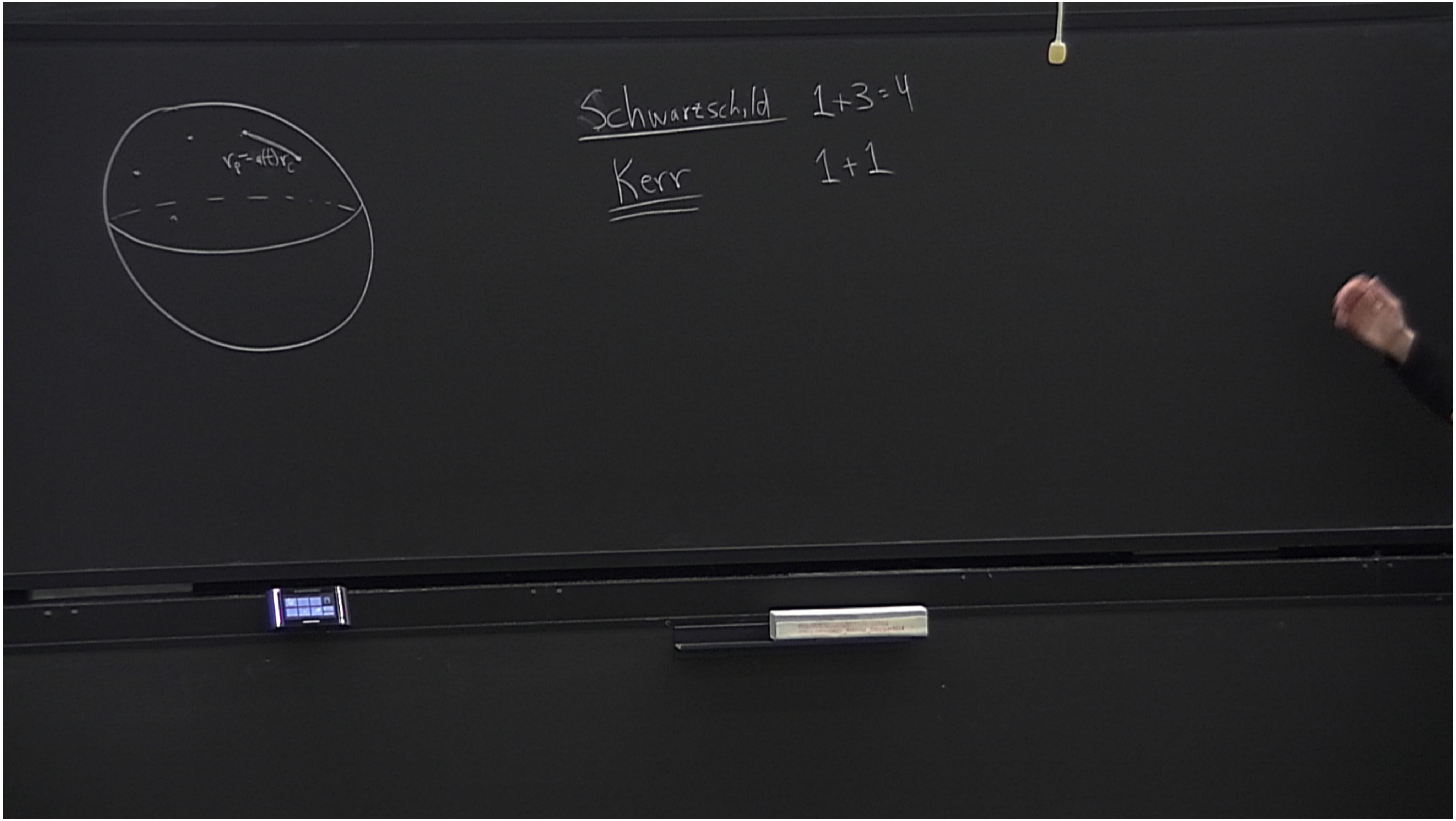

K=+1 : S3 ("Closed U.")
K=-1 : H ("Open U.")
K=0 : F ("Flat U.")

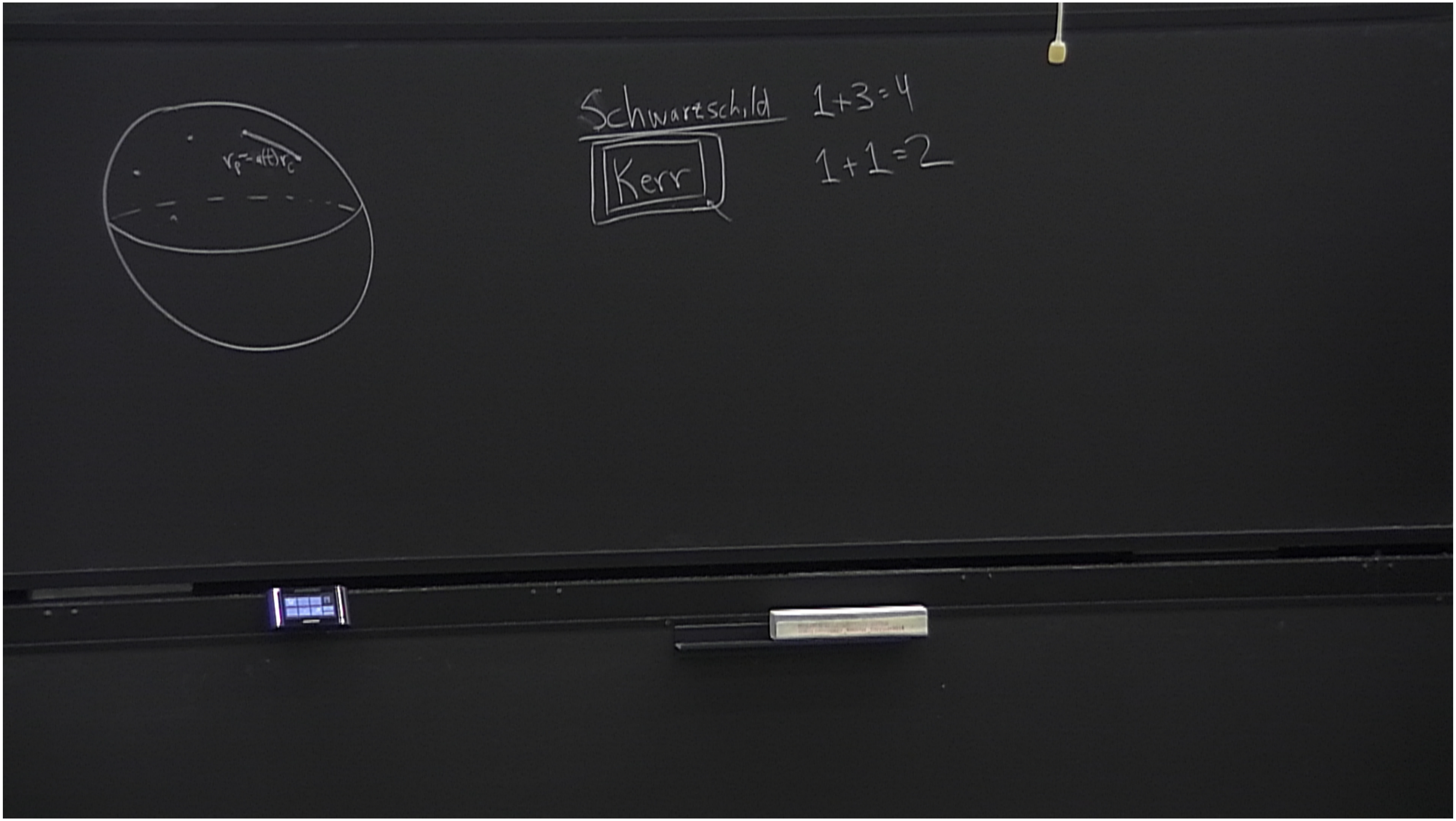

K=+1 : S3 ("Closed U.")
K=-1 : H3 ("Open U.")
K=0 : E3 ("Flat U.")

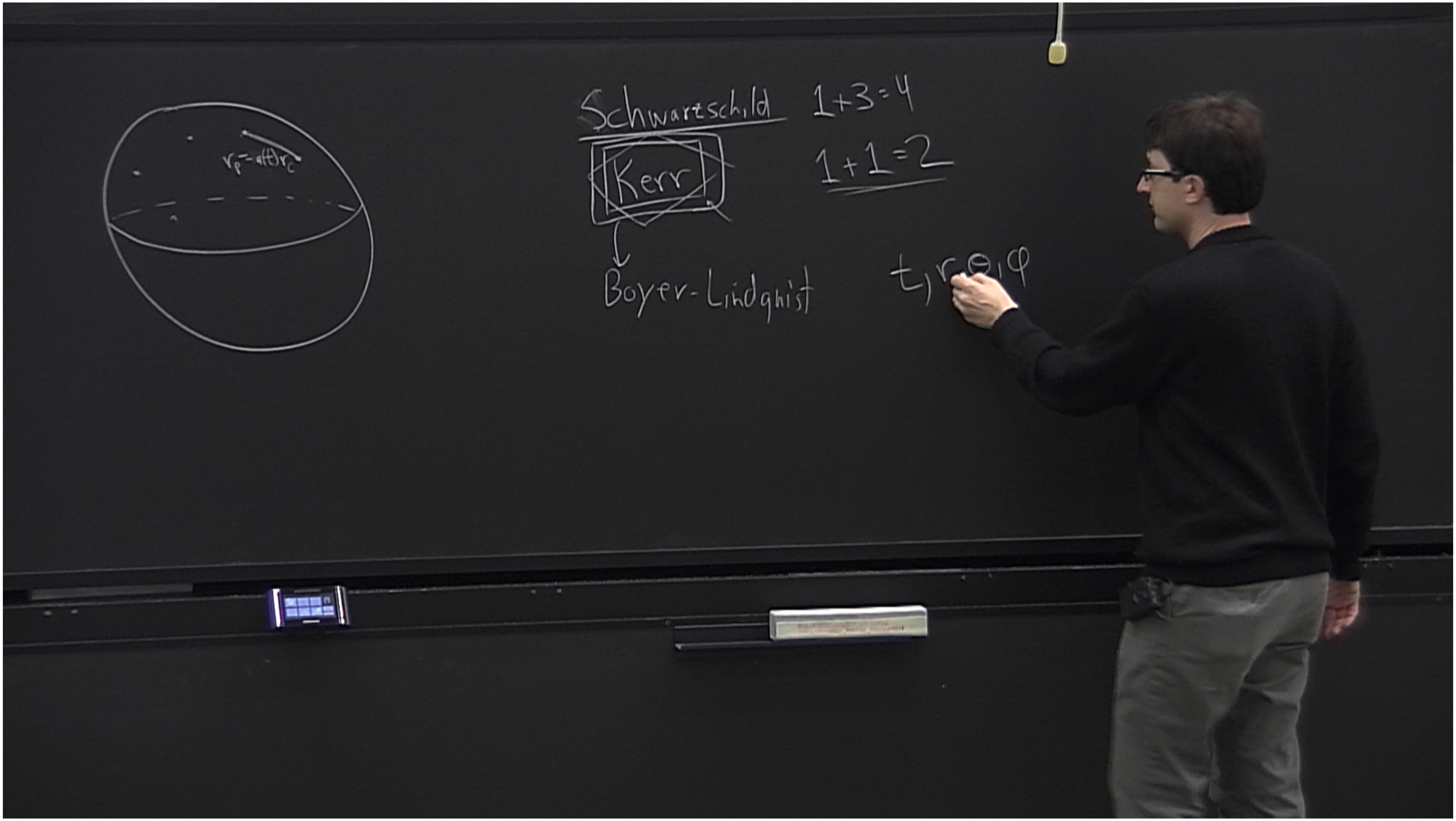

K=+1 : S3 ("Closed U.")
K=-1 : H3 ("Open U.")
K=0 : E3 ("Flat U.") ←
rp = a(t) rc


K=+1 : S3 ("Closed U.")
K=-1 : H3 ("Open U.")
K=0 : E3 ("Flat U.")

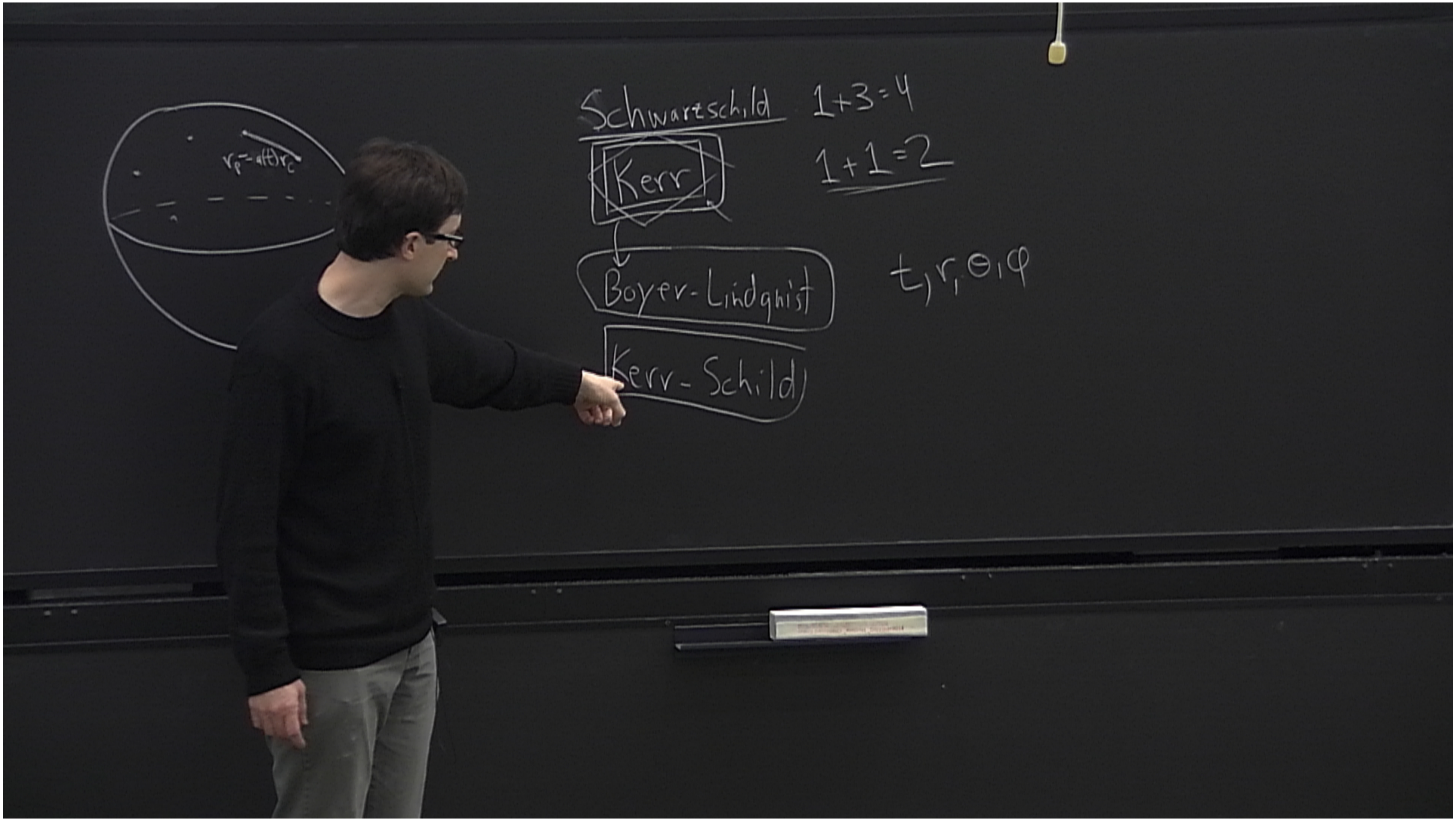

Max. Sym. → 3
FRW →
BH →
FRW
K=+1 : S3 ("Closed U.")
K=-1 : H3 ("Open
K=0 : E3 ("Flat

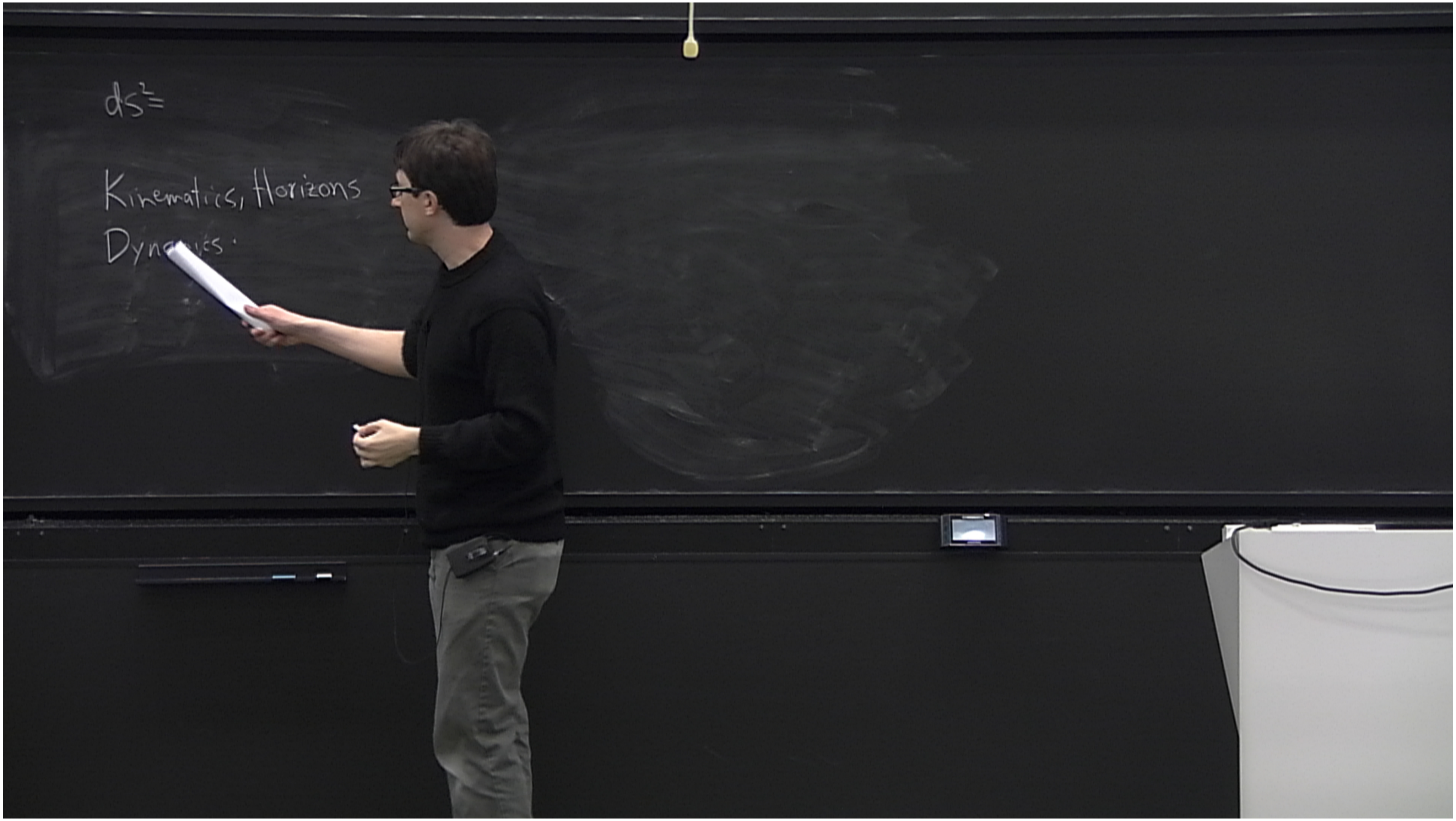

rp = a(t) rc
Schwartzschild 1+3=4
Kerr 1+1

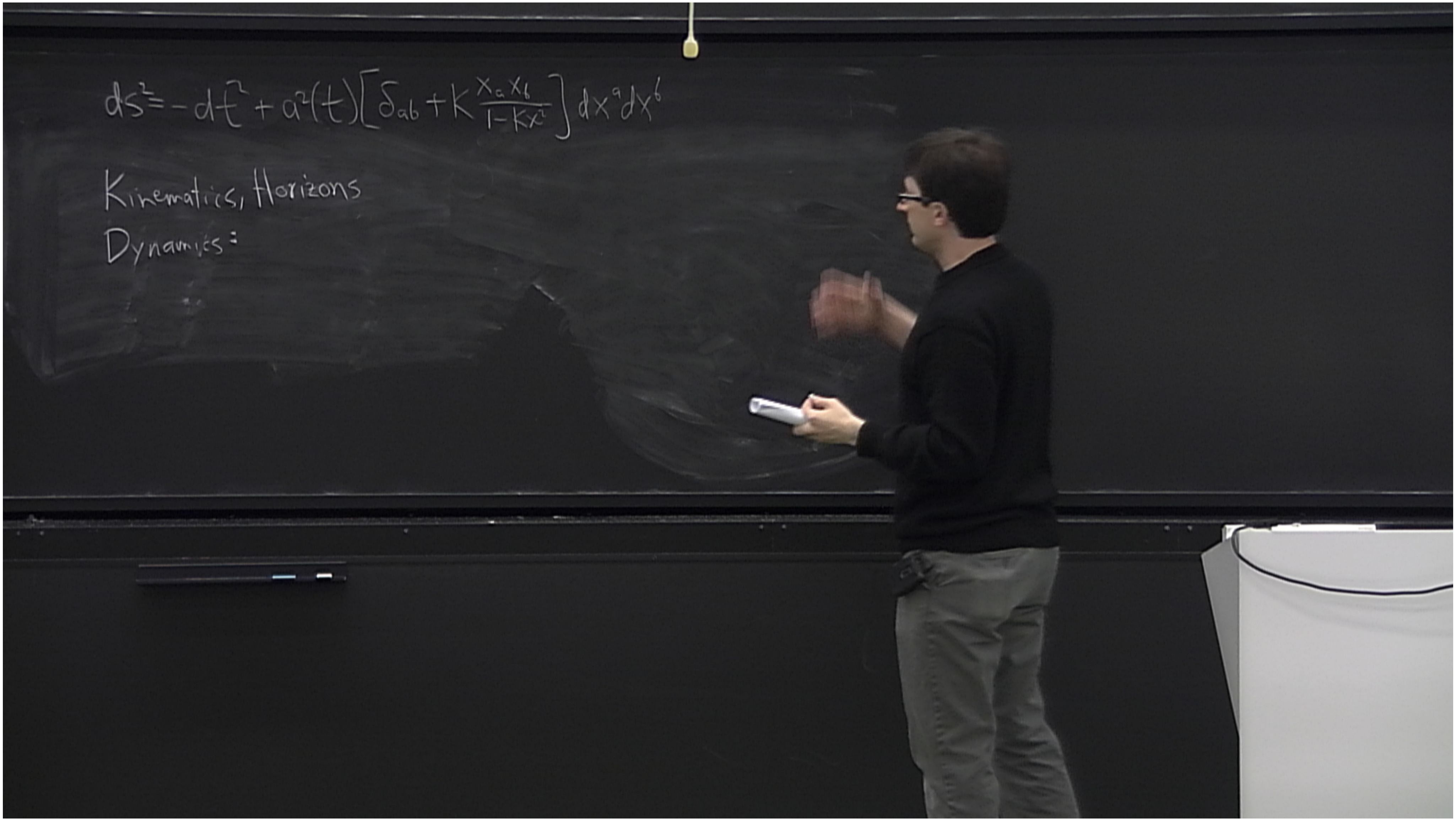

$r_p = a(t) r_c$
Schwartschild
Kerr
1+3=4
1+1=2

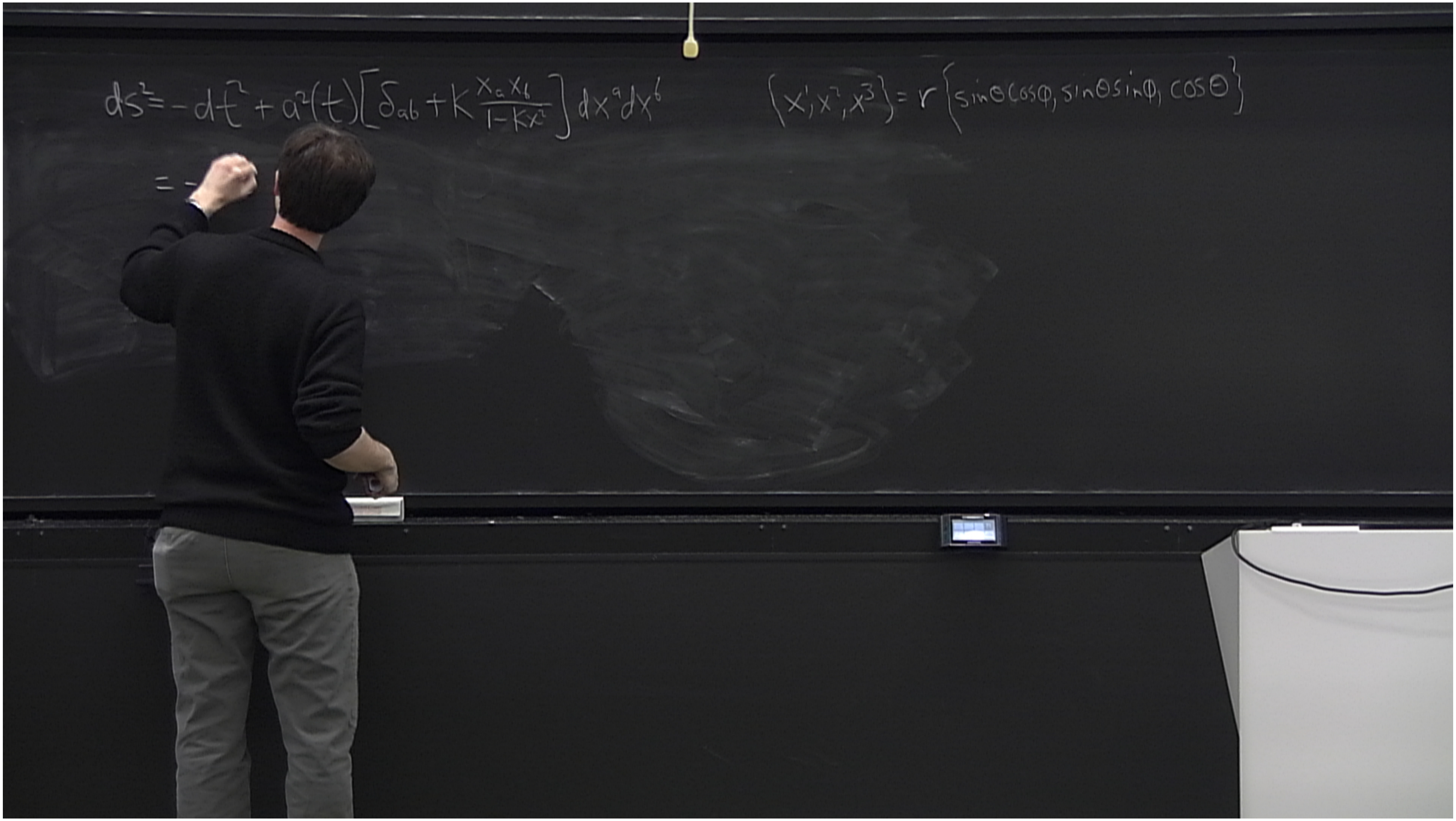


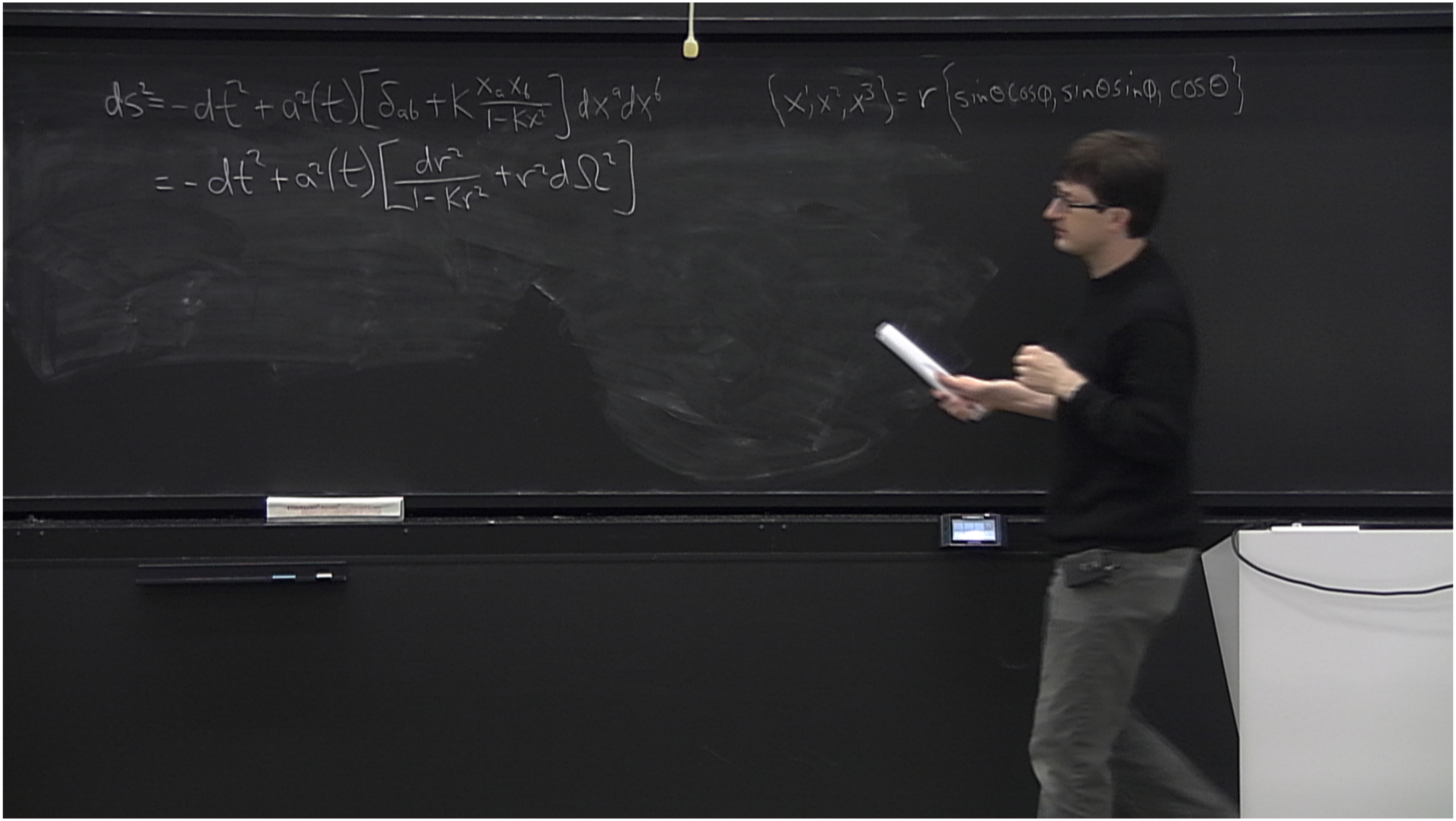
r_p = a(t) r_c
Schwarzschild
1+3=4
Kerr
1+1=2
Boyer-Lindquist
t, r, θ, φ

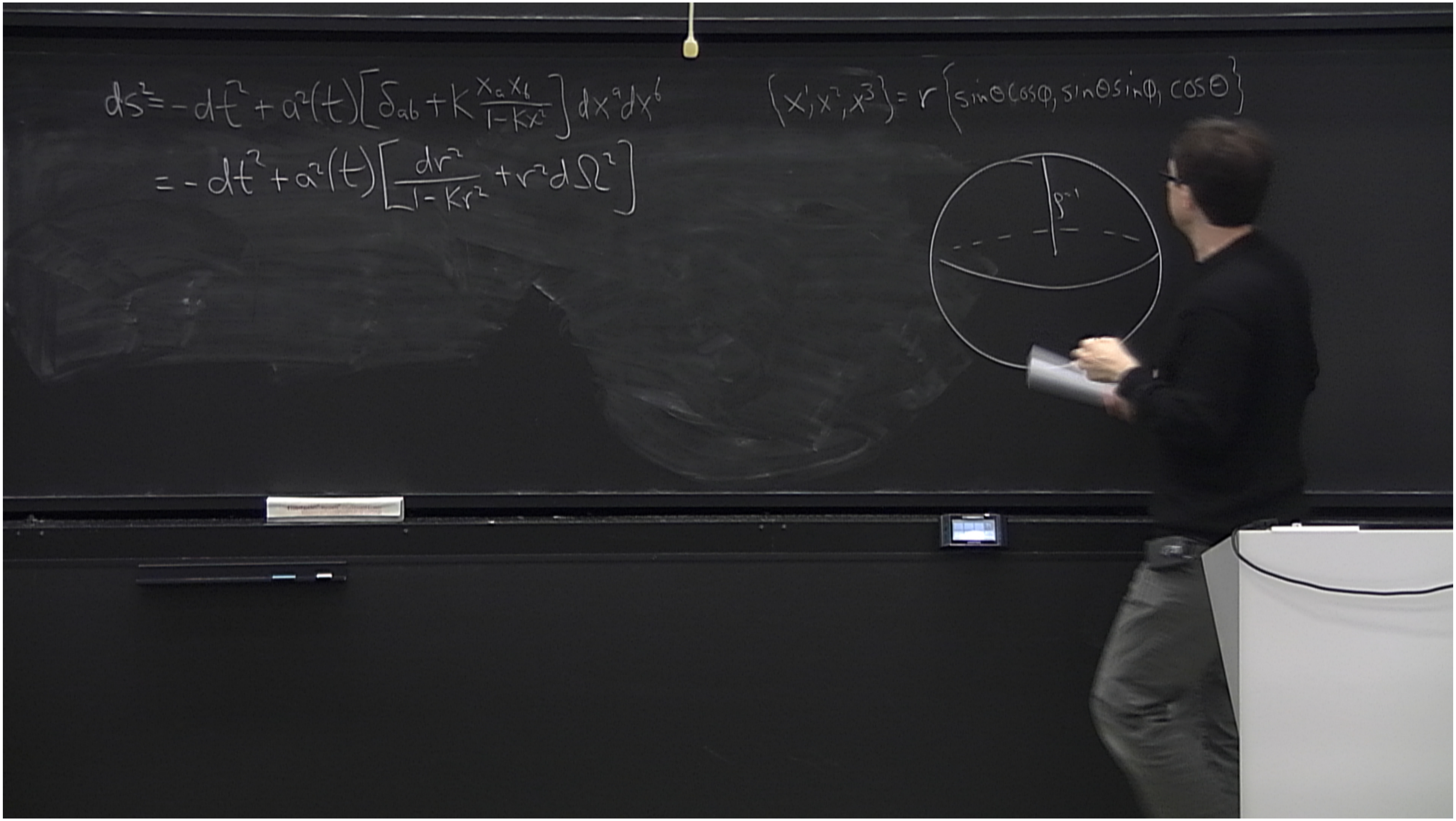

Schwartschild
1+3=4
Kerr
1+1=2
Boyer-Lindqnist
t, r, θ, φ
Kerr-Schild

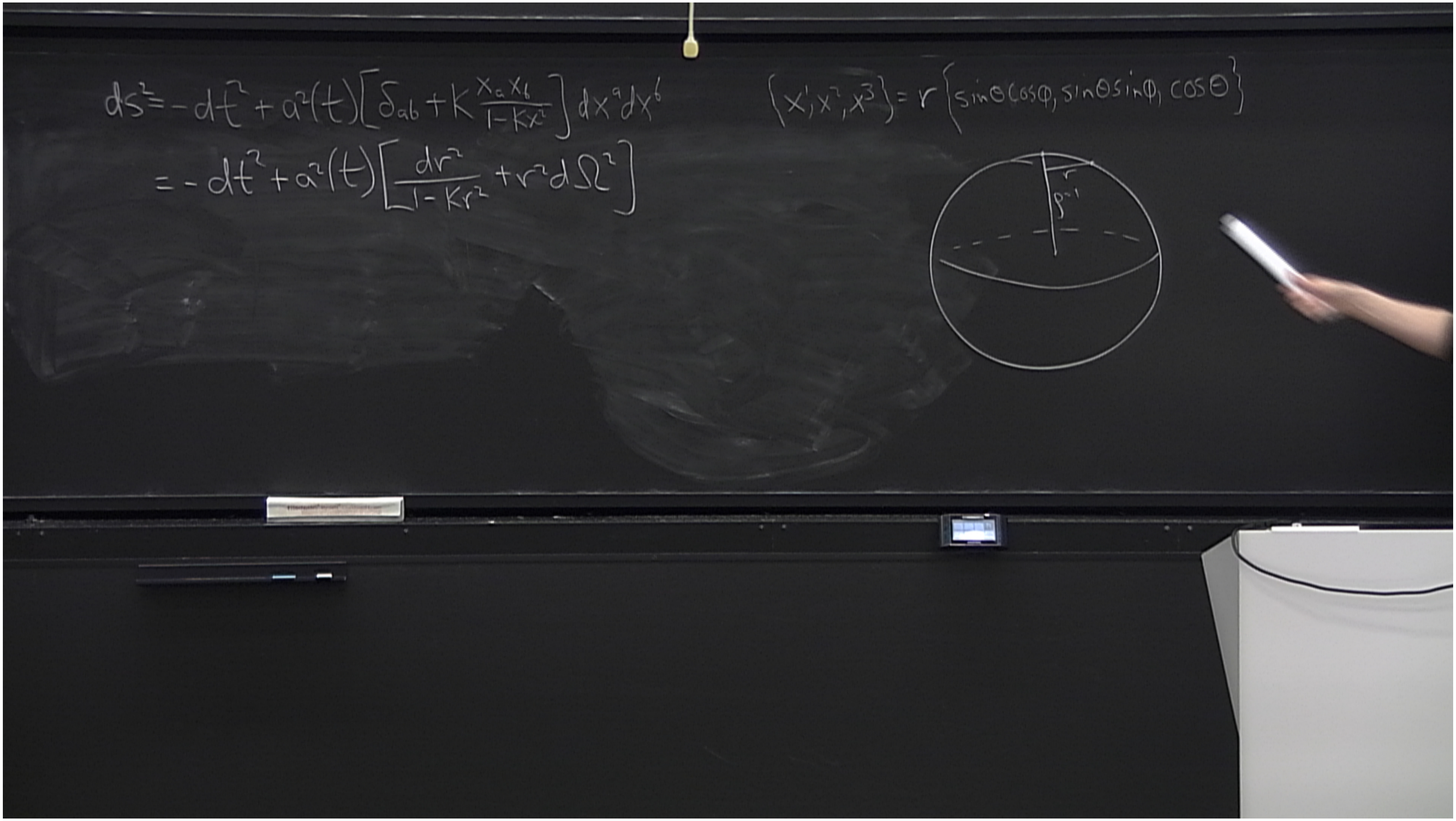

ds² =
Kinematics, Horizons

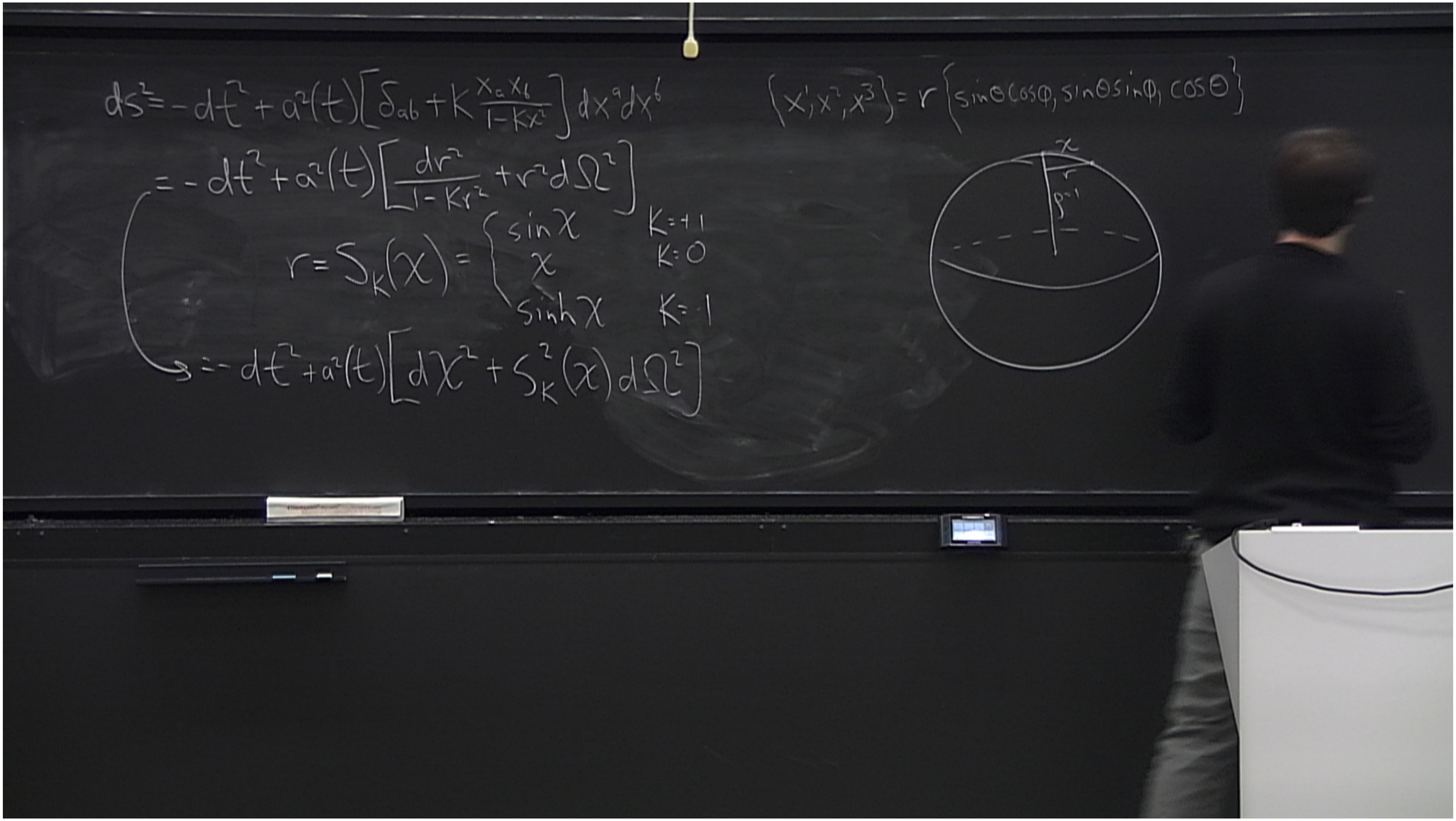

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{1-Kx^2}\right]dx^a dx^b$
Kinematics, Horizons
Dynamics:

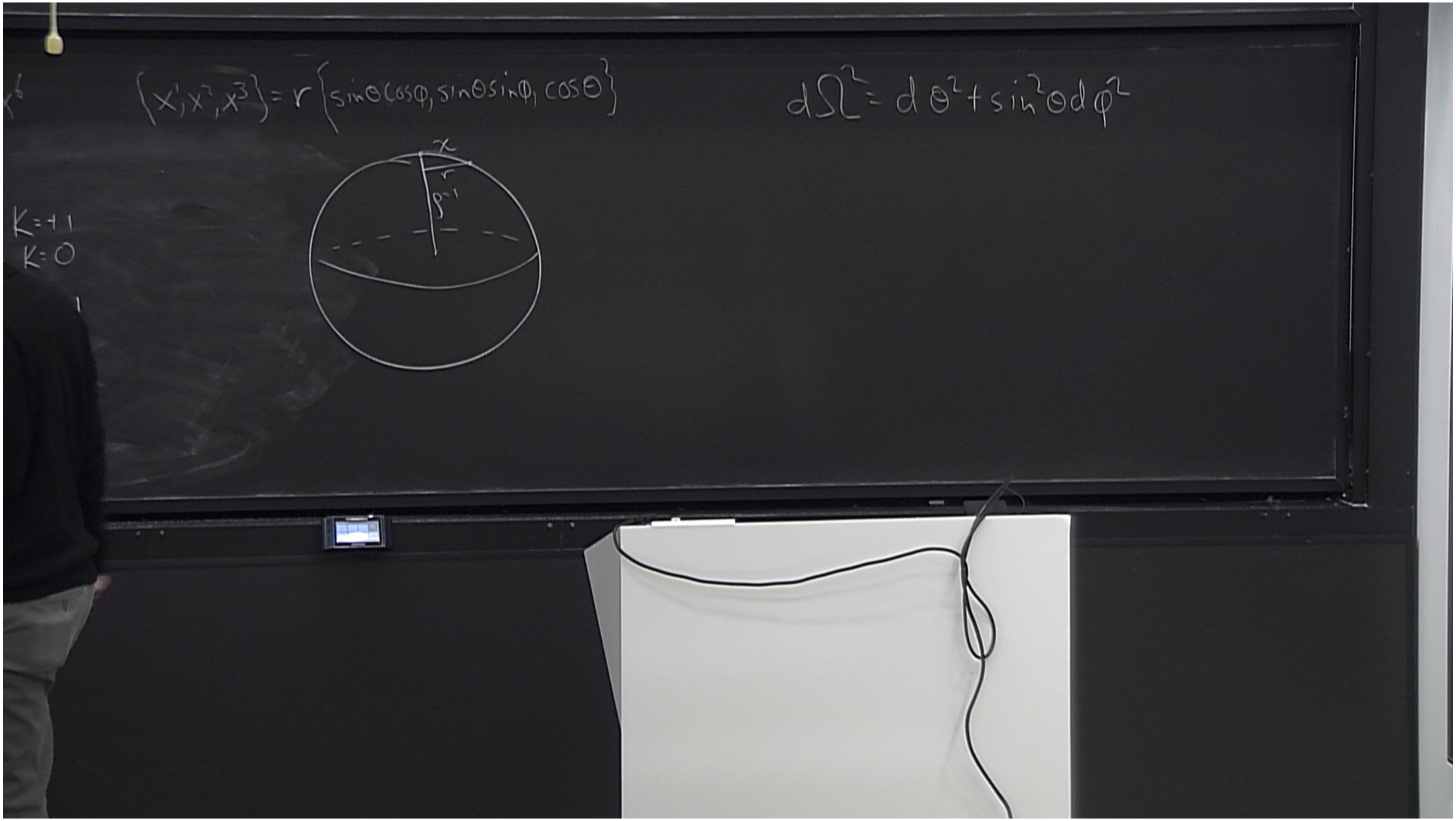

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ab} + k\frac{x_a x_b}{1-kx^2}\right]dx^a dx^b$
$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$

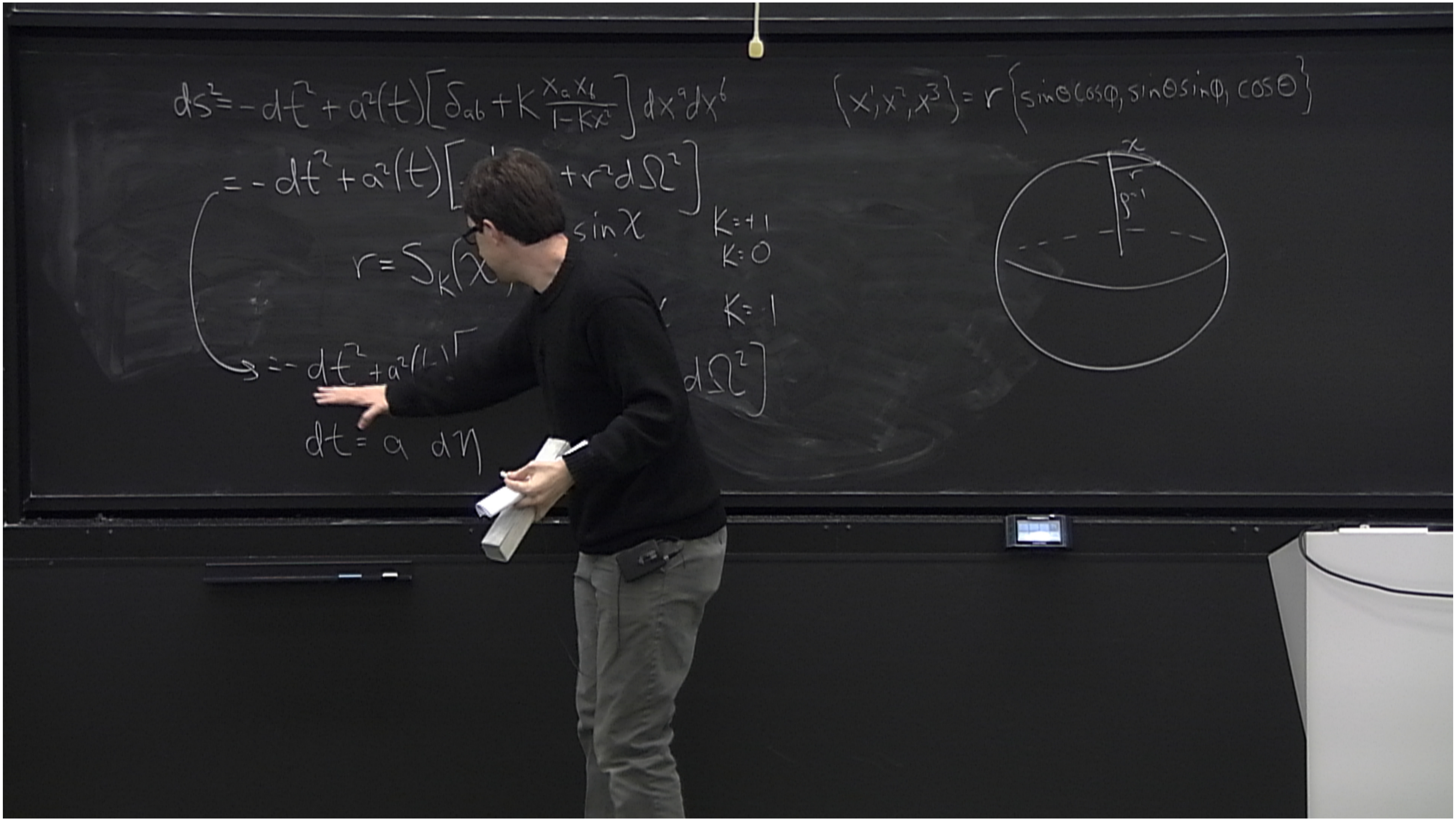

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{1-Kx^2}\right]dx^a dx^b$
$= -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$\{x^1, x^2, x^3\} = r\{\sin\Theta\cos\Phi, \sin\Theta\sin\Phi, \cos\Theta\}$

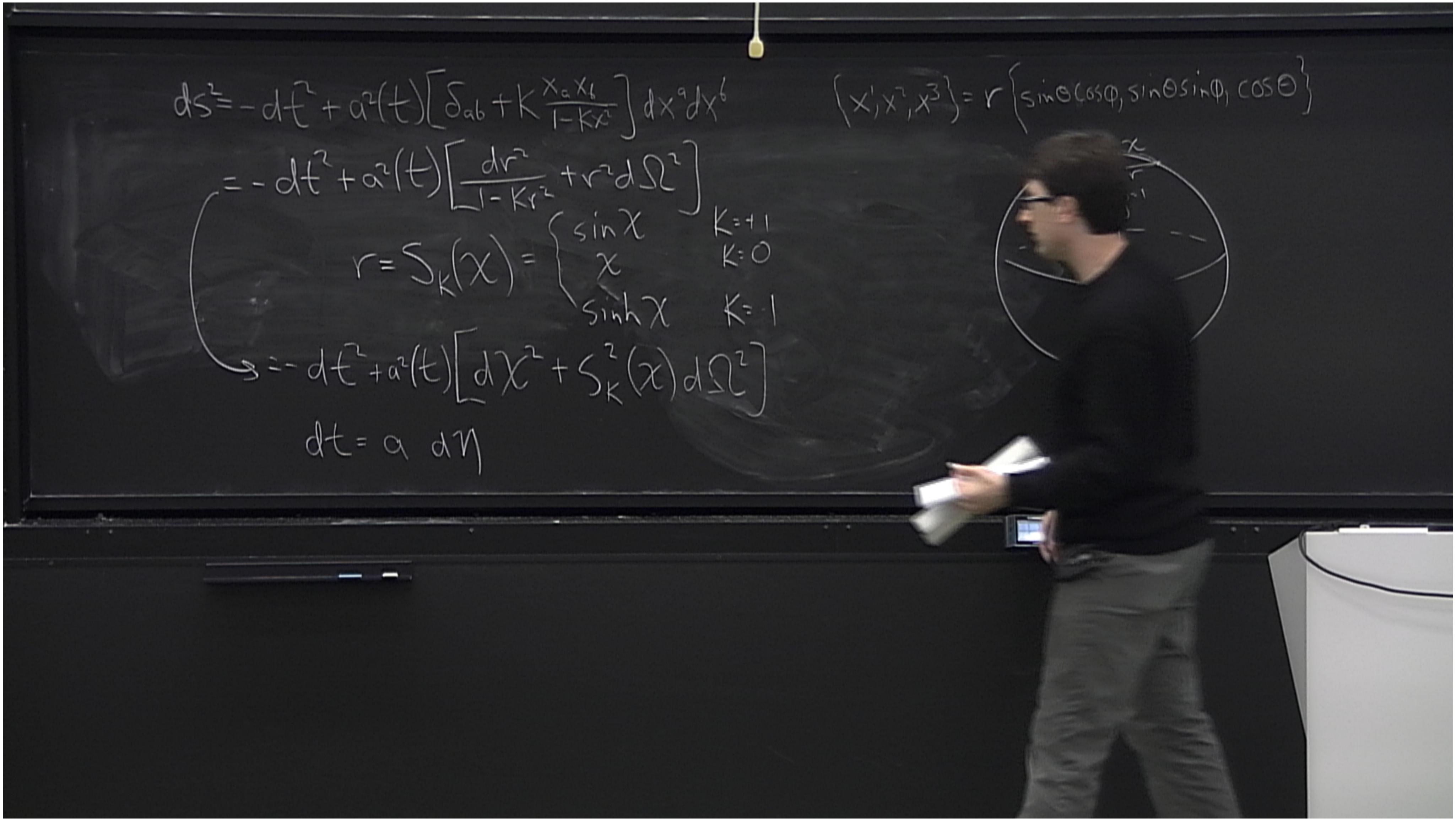

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{1-Kx^2}\right]dx^a dx^b$
$= -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$

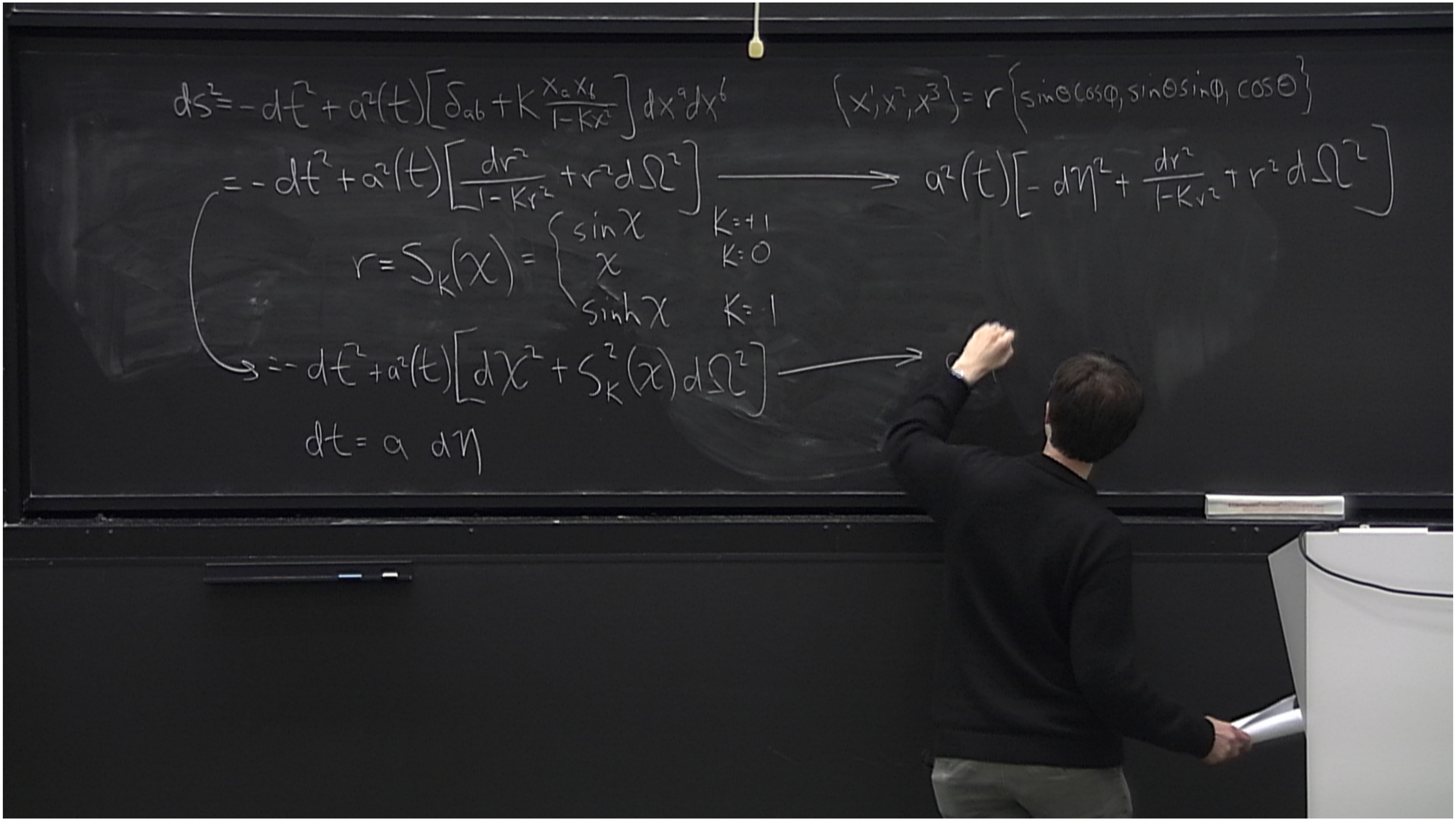

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{1-Kx^2}\right]dx^a dx^b$
$= -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$
r

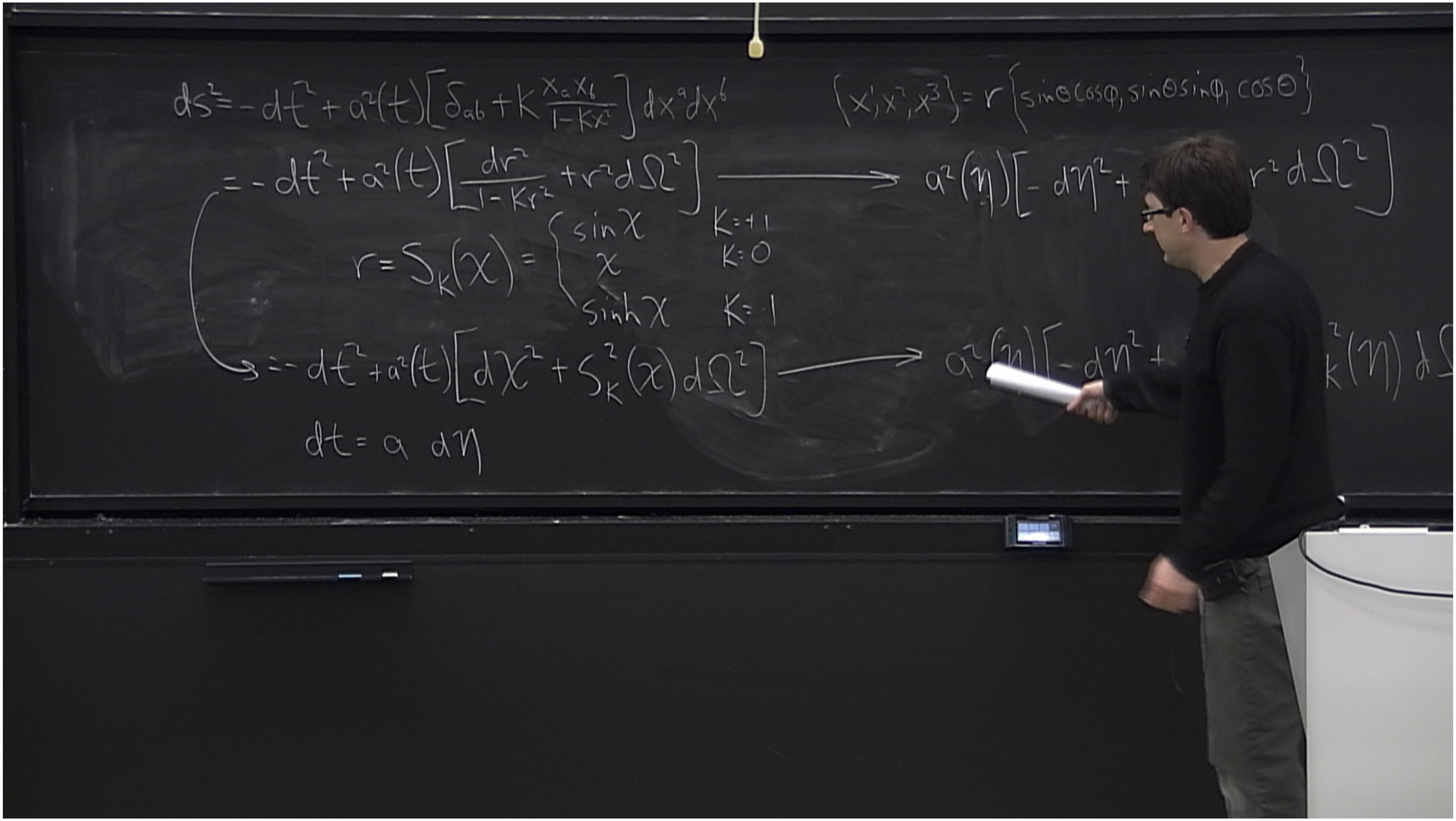

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{1-Kx^2}\right]dx^a dx^b$
$= -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$r = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
$= -dt^2 + a^2(t)\left[d\chi^2 + S_K^2(\chi)\, d\Omega^2\right]$
$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$
χ
r
$\rho = 1$

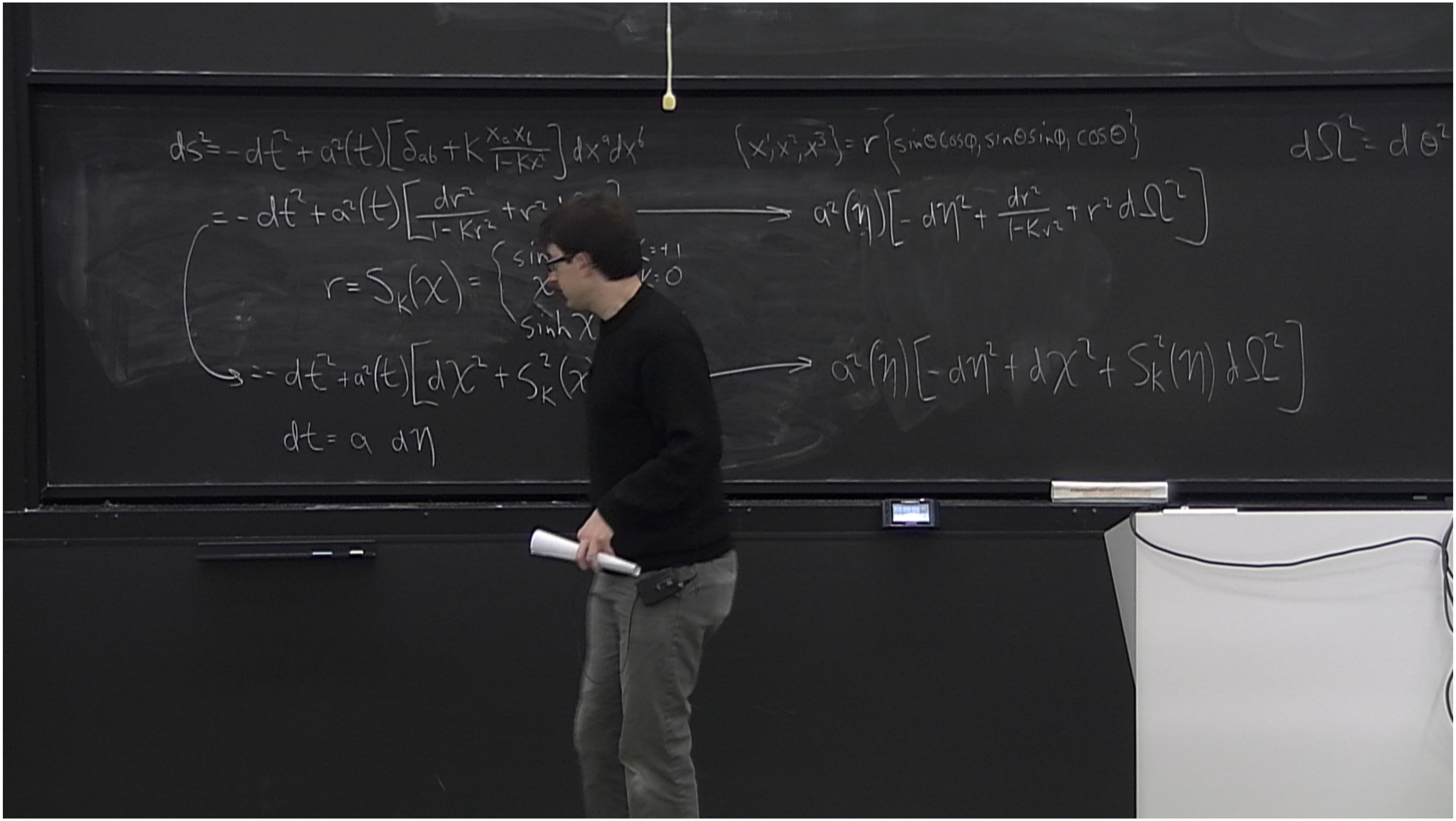

{x¹, x², x³} = r {sinθ cosφ, sinθ sinφ, cosθ}
dΩ² = dθ² + sin²θ dφ²
x
r
ρ=1
K=+1
K=0

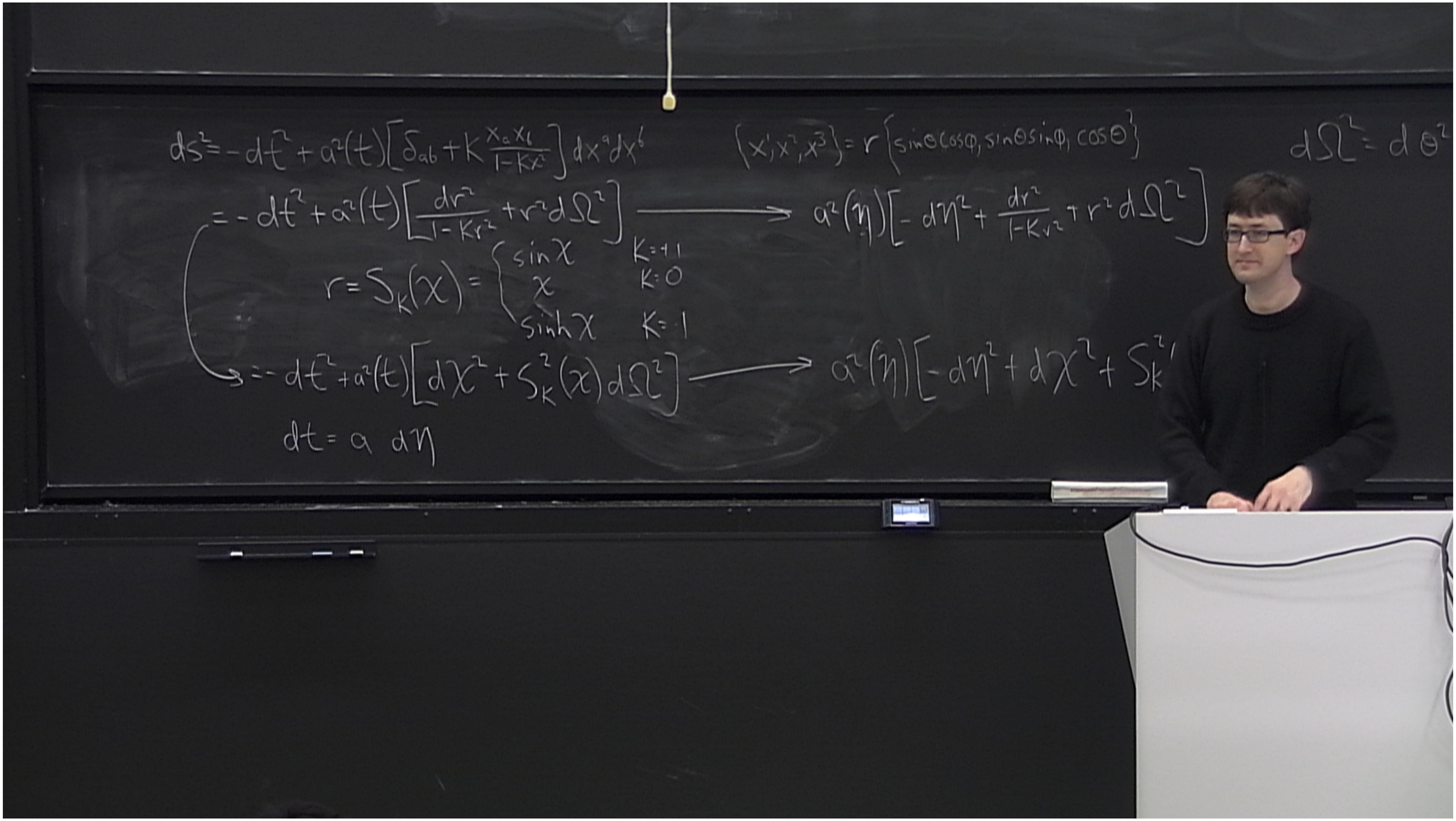

ds² = -dt² + a²(t) [δab + K xa xb / 1-Kx²] dxᵃ dxᵇ
{x¹, x², x³} = r {sinΘ cosΦ, sinΘ sinΦ, cosΘ}
K=+1
K=0
K=-1
dt = a dη


ds² = -dt² + a²(t)[δab + K xa xb / (1-Kx²)] dx^a dx^b
{x¹, x², x³} = r {sinθ cosφ, sinθ sinφ, cosθ}
= -dt² + a²(t)[dr²/(1-Kr²) + r² dΩ²]
r = S_K(χ) = { sin χ K=+1; χ K=0; sinh χ K=-1
= -dt² + a²(t)[dχ² + S_K²(χ) dΩ²]
dt = a dη


$ds^2 = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{1-Kx^2}\right]dx^a dx^b$
$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$
$= -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right] \longrightarrow a^2(t)\left[-d\eta^2 + \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$r = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
$= -dt^2 + a^2(t)\left[d\chi^2 + S_K^2(\chi)\, d\Omega^2\right] \longrightarrow$
$dt = a\, d\eta$


$ds^2 = -dt^2 + a^2(t)\left[\delta_{ab} + K\frac{x_a x_b}{1-Kx^2}\right]dx^a dx^b$
$\{x^1, x^2, x^3\} = r\{\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta\}$
$= -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$a^2(\eta)[-d\eta^2 +$
$r^2 d\Omega^2]$
$r = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
$= -dt^2 + a^2(t)\left[d\chi^2 + S_K^2(\chi)\, d\Omega^2\right]$
$a^2(\eta)[-d\eta^2 +$
$dt = a\, d\eta$


ds² = - dt² + a²(t)[δ_ab + K x_a x_b / (1-Kx²)] dx^a dx^b
{x¹, x², x³} = r {sinθ cosφ, sinθ sinφ, cosθ}
dΩ² = dθ²
= - dt² + a²(t)[dr²/(1-Kr²) + r²
→ a²(η)[- dη² + dr²/(1-Kr²) + r² dΩ²]
r = S_K(χ) =
sinh χ
= - dt² + a²(t)[dχ² + S_K²(χ
→ a²(η)[- dη² + dχ² + S_K²(η) dΩ²]
dt = a dη


ds² = -dt² + a²(t)[δab + K xa xb / (1-Kx²)] dx^a dx^b
{x¹, x², x³} = r {sinθ cosφ, sinθ sinφ, cosθ}
dΩ² = dθ²
= -dt² + a²(t)[dr²/(1-Kr²) + r² dΩ²]
a²(η)[-dη² + dr²/(1-Kr²) + r² dΩ²]
r = S_K(χ) = sin χ K=+1; χ K=0; sinh χ K=-1
= -dt² + a²(t)[dχ² + S_K²(χ) dΩ²]
a²(η)[-dη² + dχ² + S_K²
dt = a dη