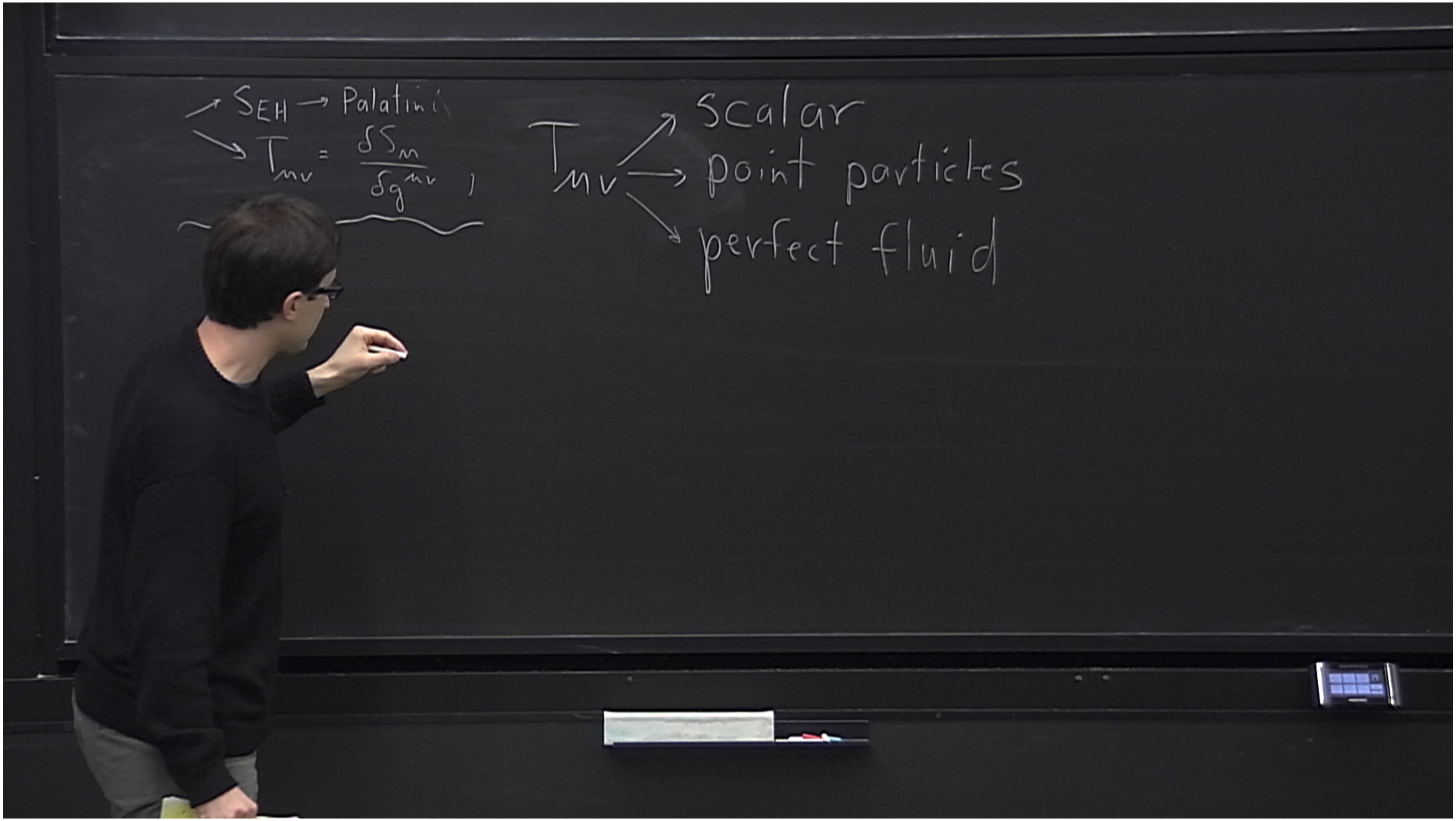
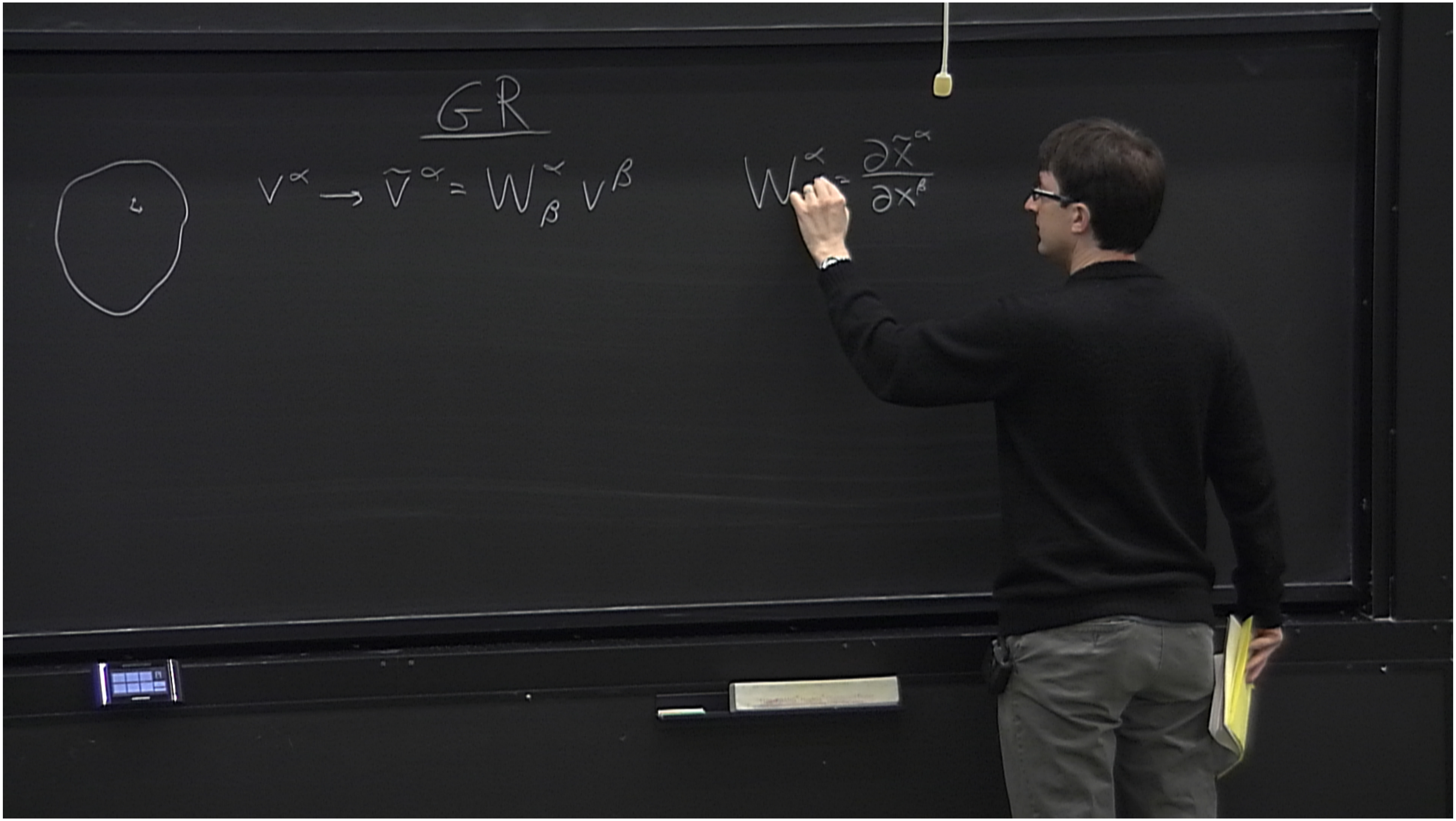
Title: 12/13 PSI - Cosmology Review Lecture 3

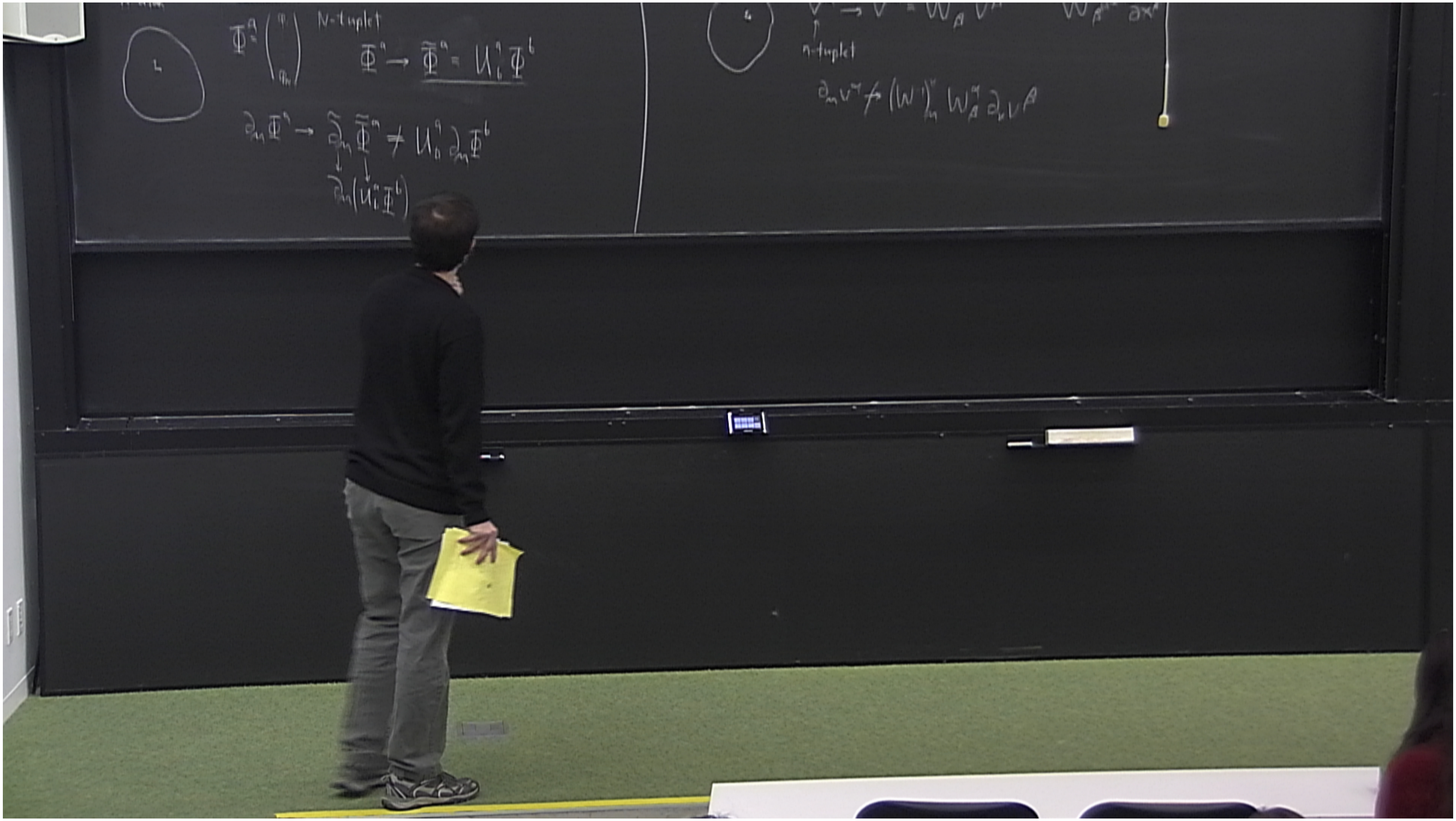
Date: Feb 21, 2013 11:30 AM

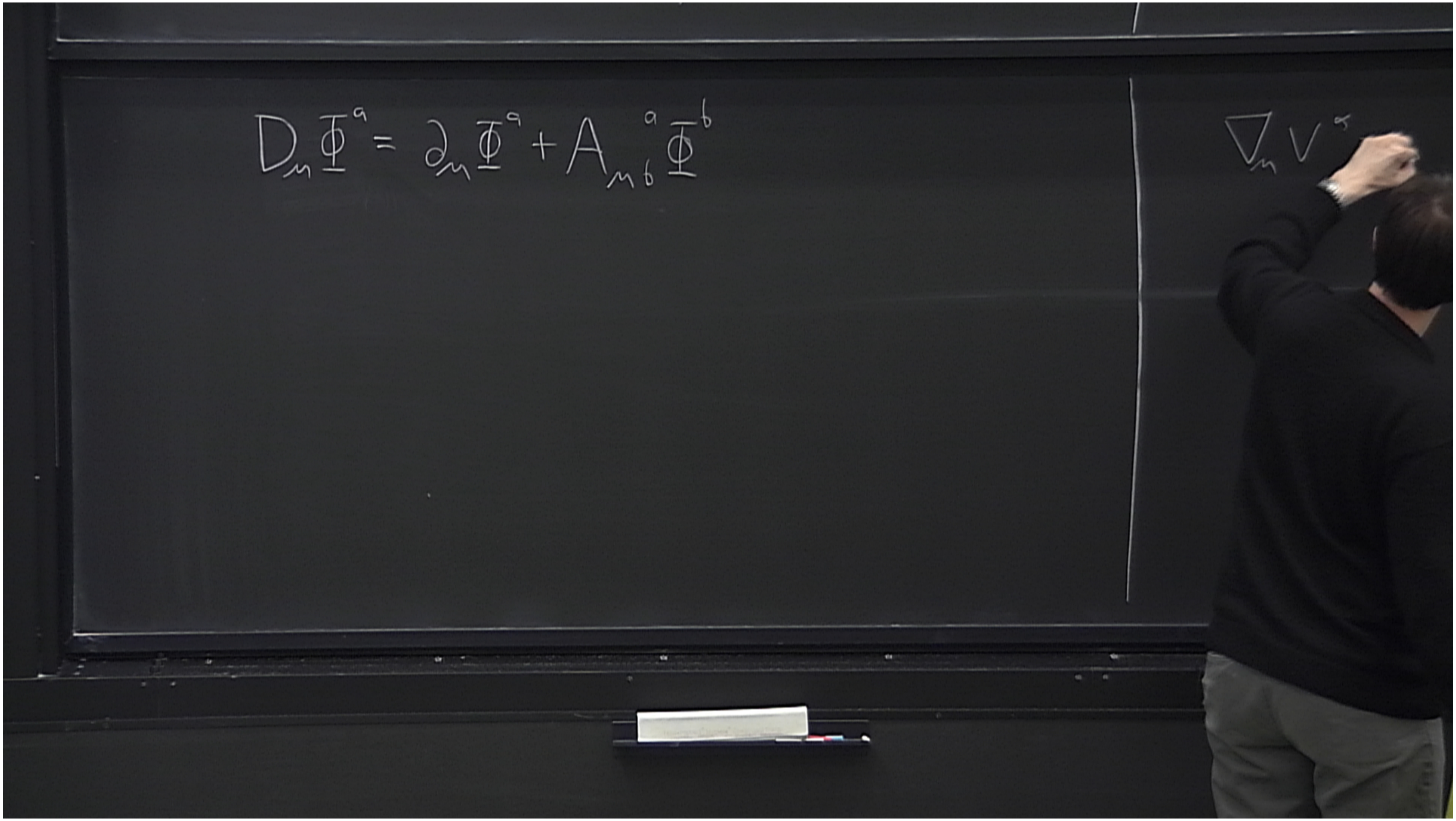
URL: http://www.pirsa.org/13020101

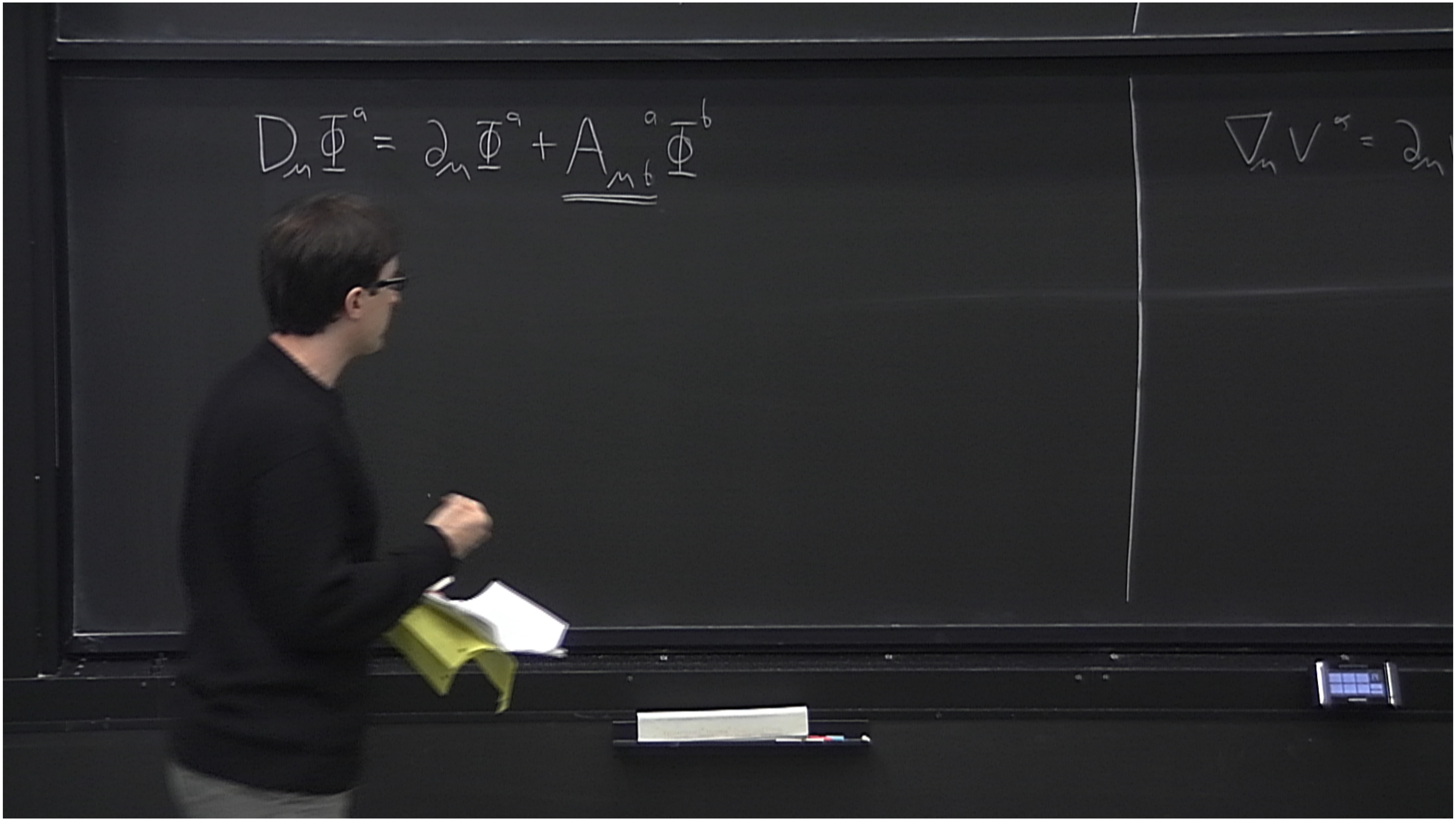
Abstract:

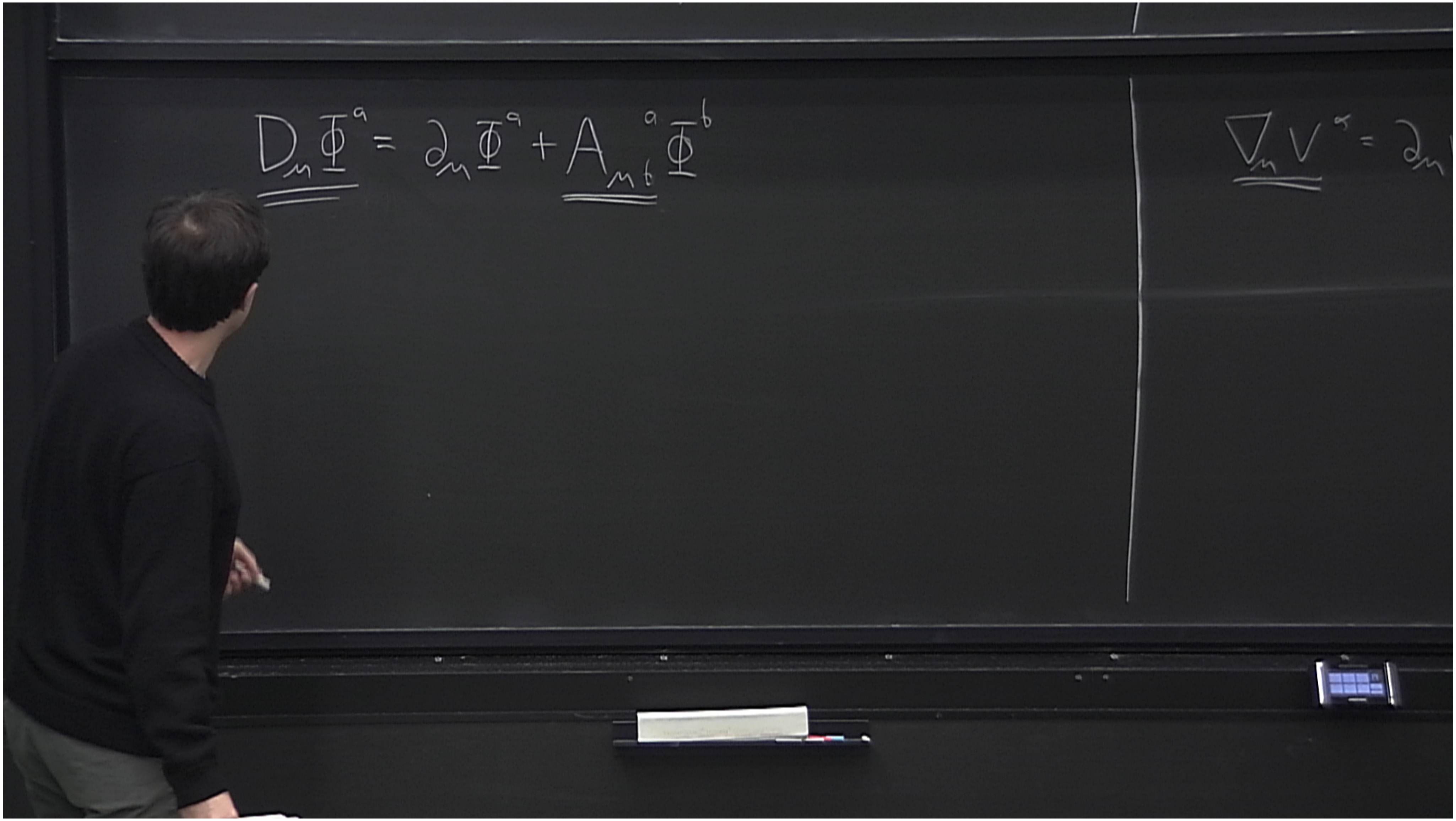

S_EH → Palatini
T_μν = δS_m / δg^μν ,
T_μν
scalar
point particles
perfect fluid

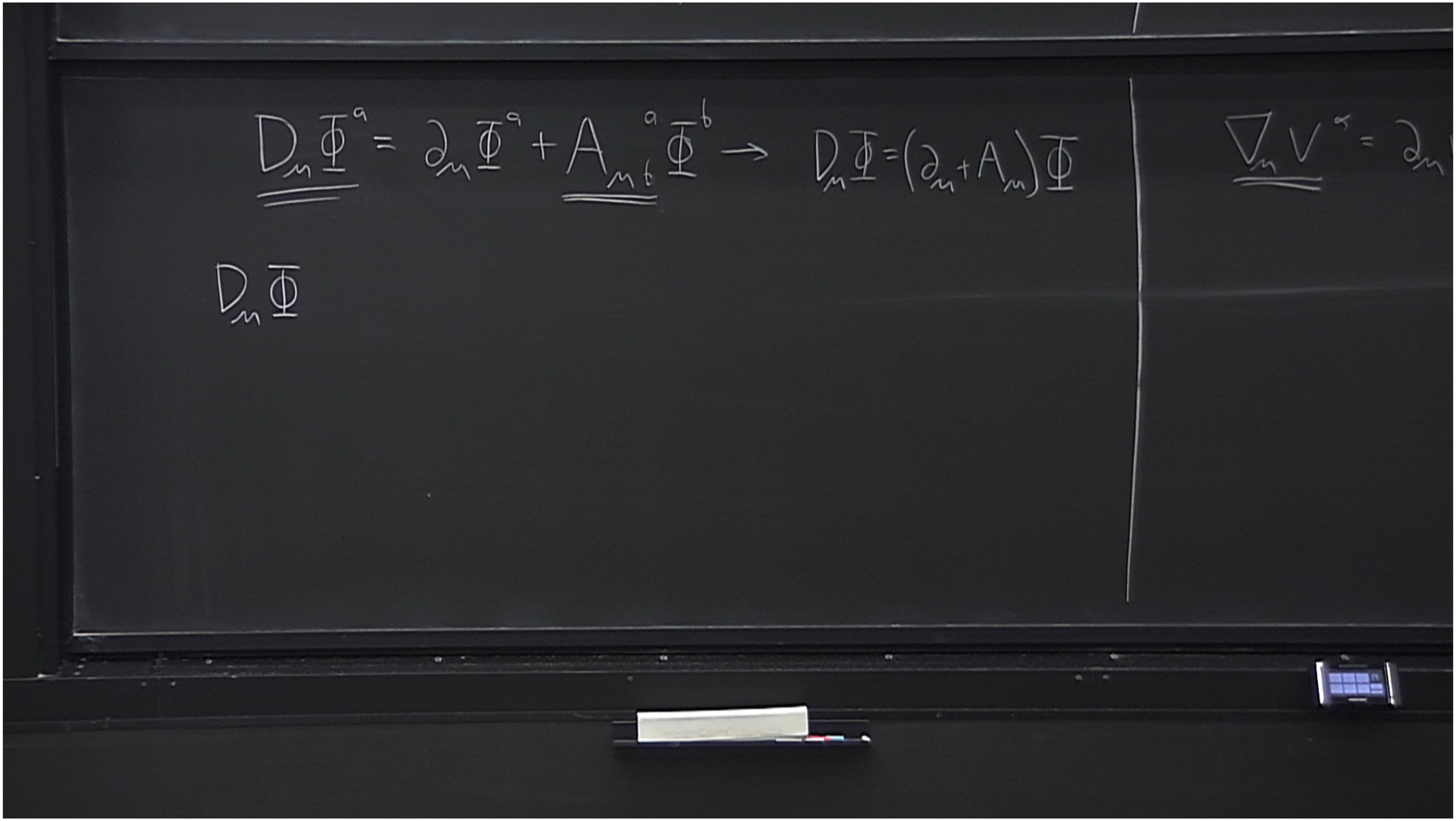

GR
$V^\alpha \longrightarrow \tilde{V}^\alpha = W^\alpha_\beta V^\beta$
$W^\alpha_\beta = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta}$

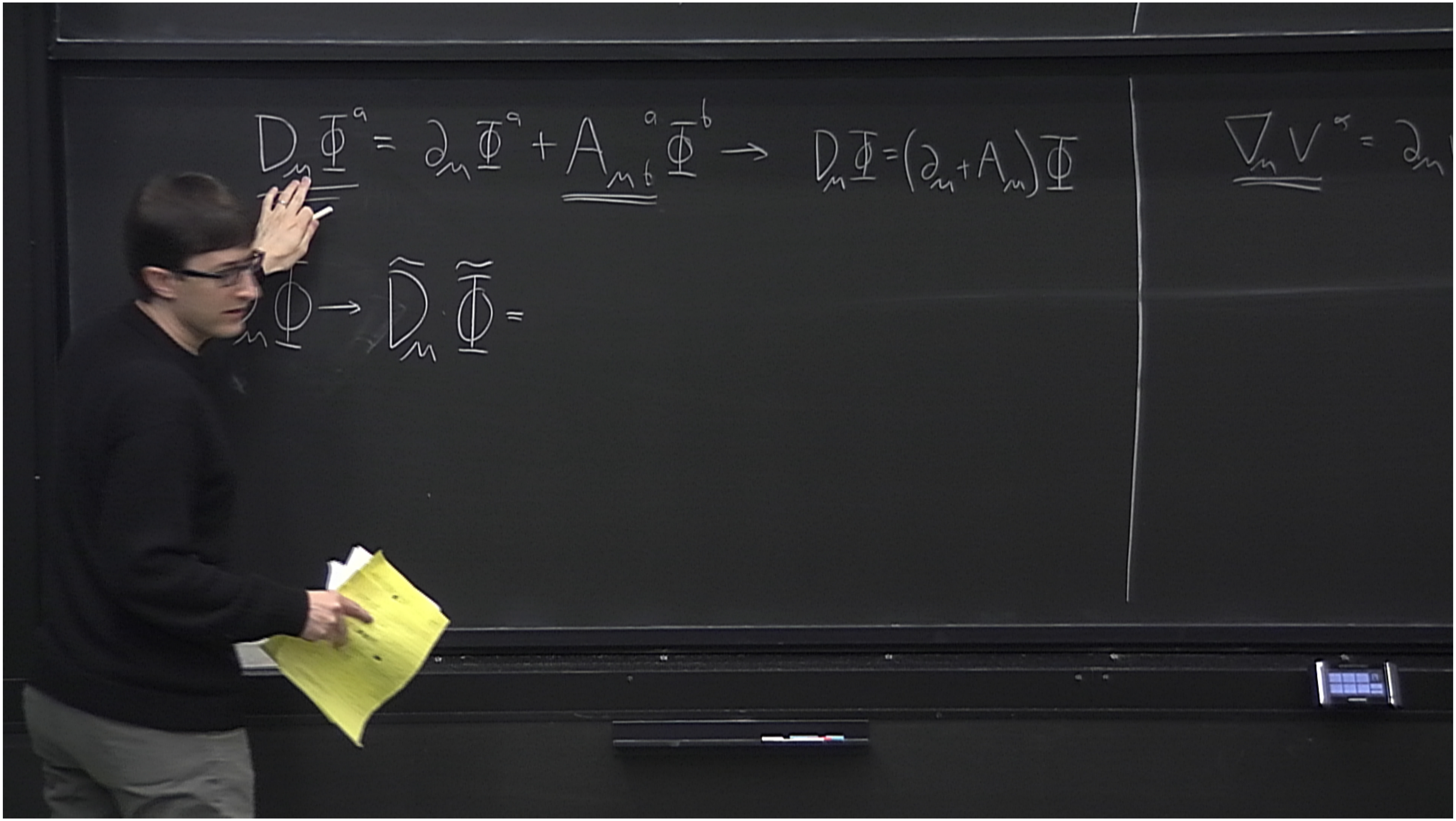

N-tuplet
n-tuplet

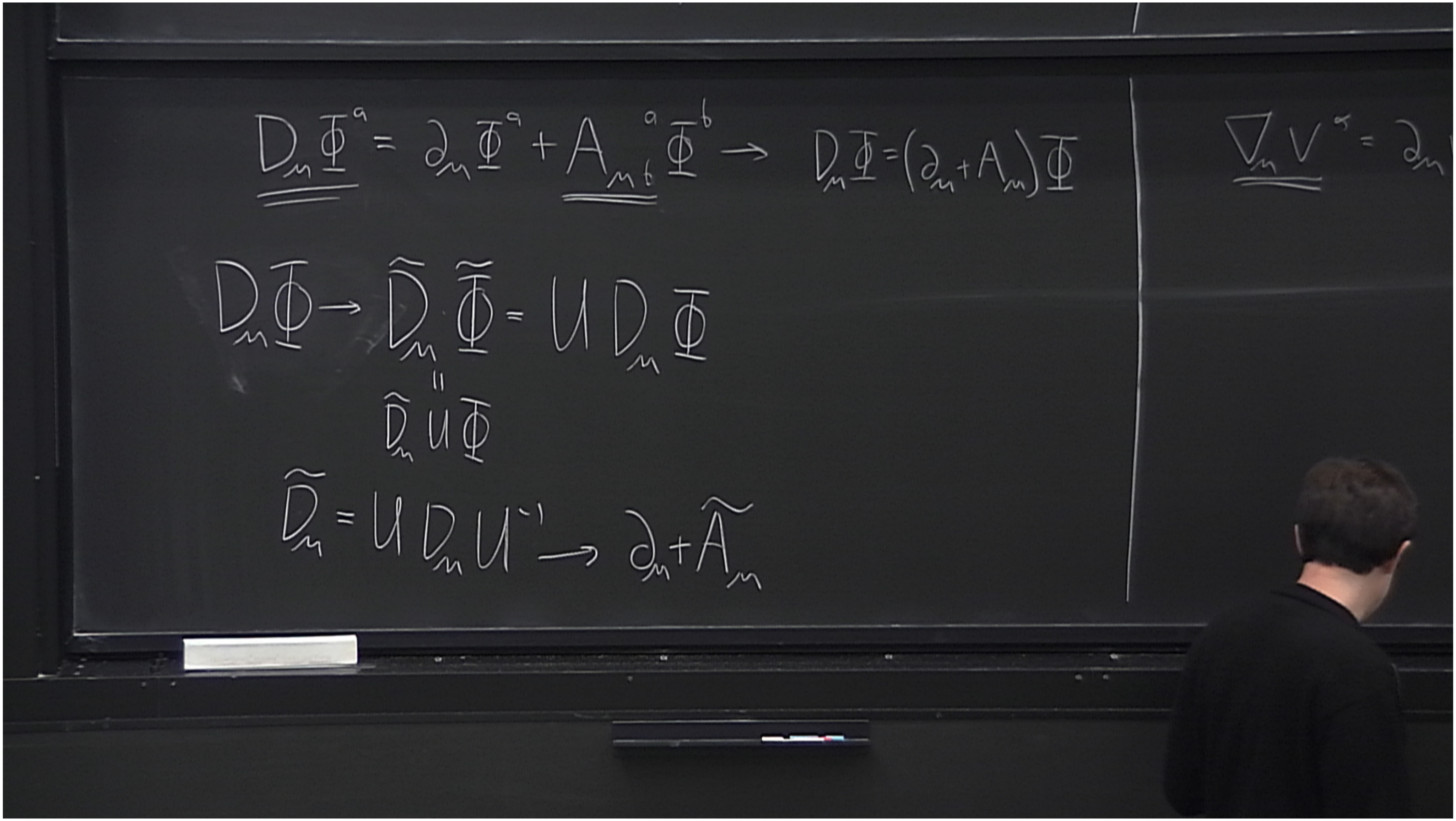

$D_\mu \Phi^a = \partial_\mu \Phi^a + A_{\mu}{}^{a}{}_{b} \Phi^b$
$\nabla_\mu V^a$

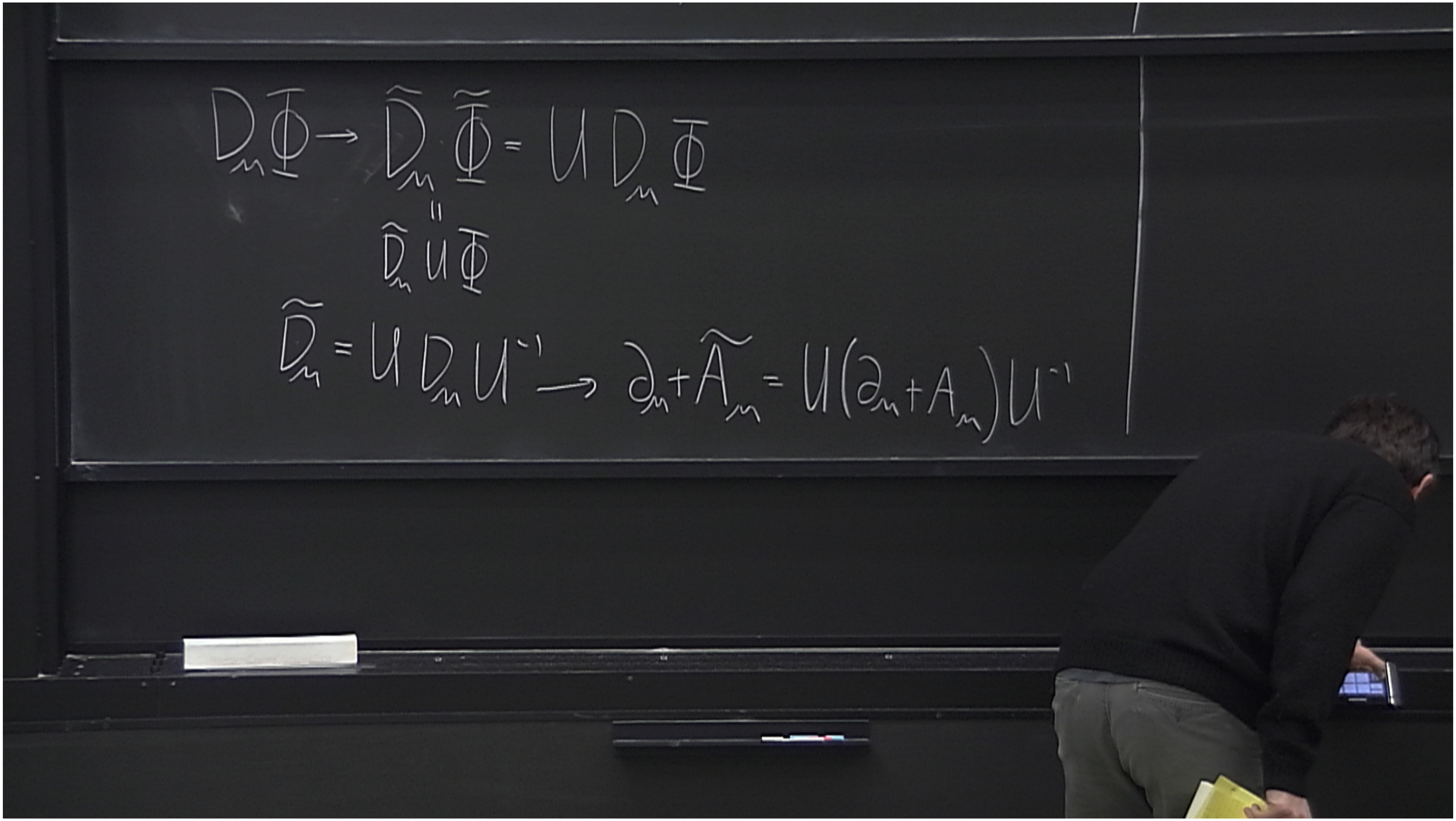

$D_\mu \Phi^a = \partial_\mu \Phi^a + A_{\mu\ b}^{\ a} \Phi^b$
$\nabla_\mu V^\sigma = \partial_\mu$


$D_\mu \Phi^a = \partial_\mu \Phi^a + A_{\mu\ b}^{\ a} \Phi^b$
$\nabla_\mu V^\alpha = \partial_\mu$

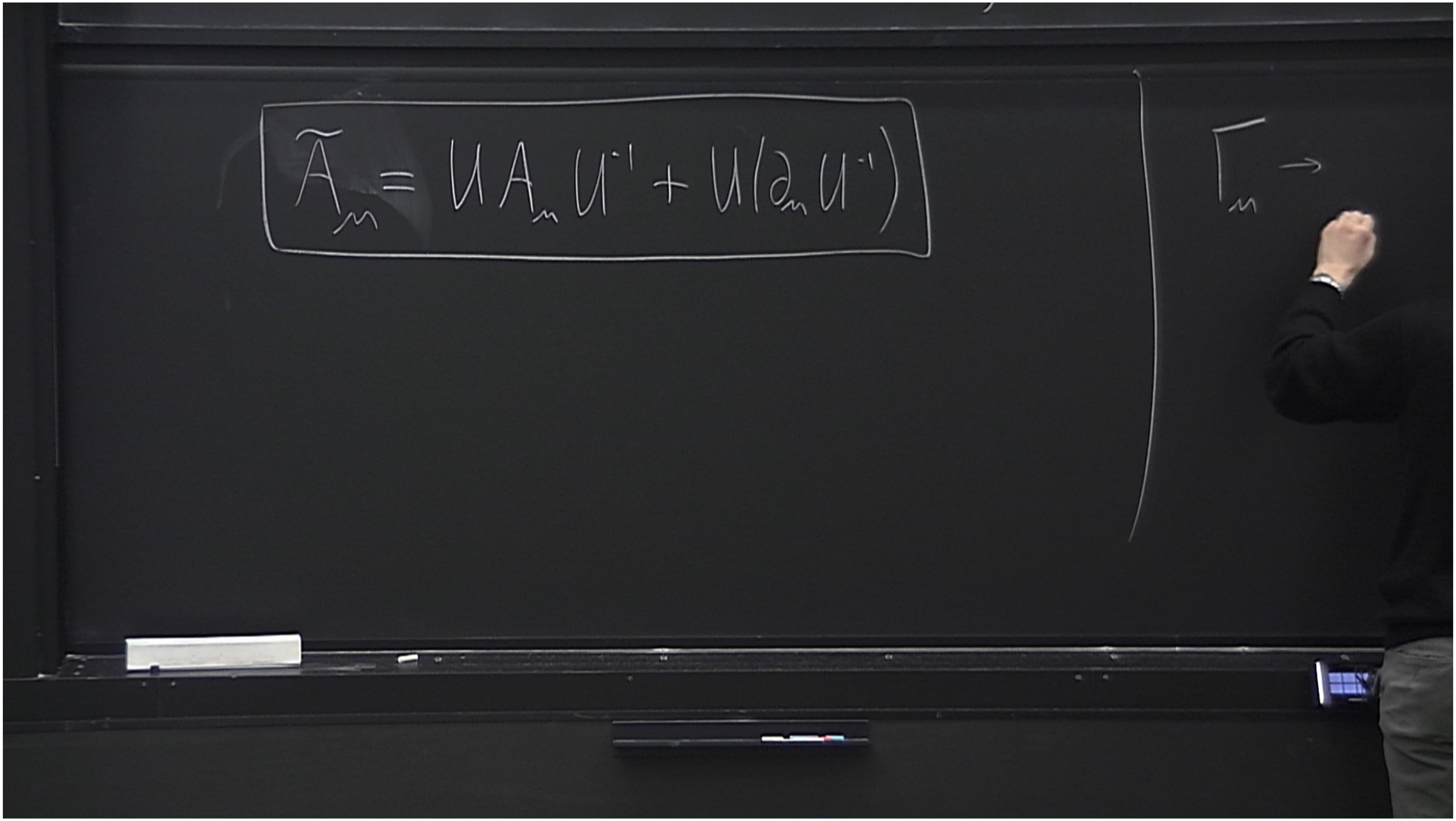

$D_\mu \Phi^a = \partial_\mu \Phi^a + A_{\mu\ b}^{\ a} \Phi^b \rightarrow D_\mu \Phi = (\partial_\mu + A_\mu) \Phi$
$\nabla_\mu V^\alpha = \partial_\mu$
$D_\mu \Phi$


$D_\mu \Phi^a = \partial_\mu \Phi^a + A_{\mu\, b}^{\ a} \Phi^b \rightarrow D_\mu \Phi = (\partial_\mu + A_\mu)\Phi$
$\Phi \rightarrow \tilde{D}_\mu \tilde{\Phi} =$
$\nabla_\mu V^\sigma = \partial_\mu$

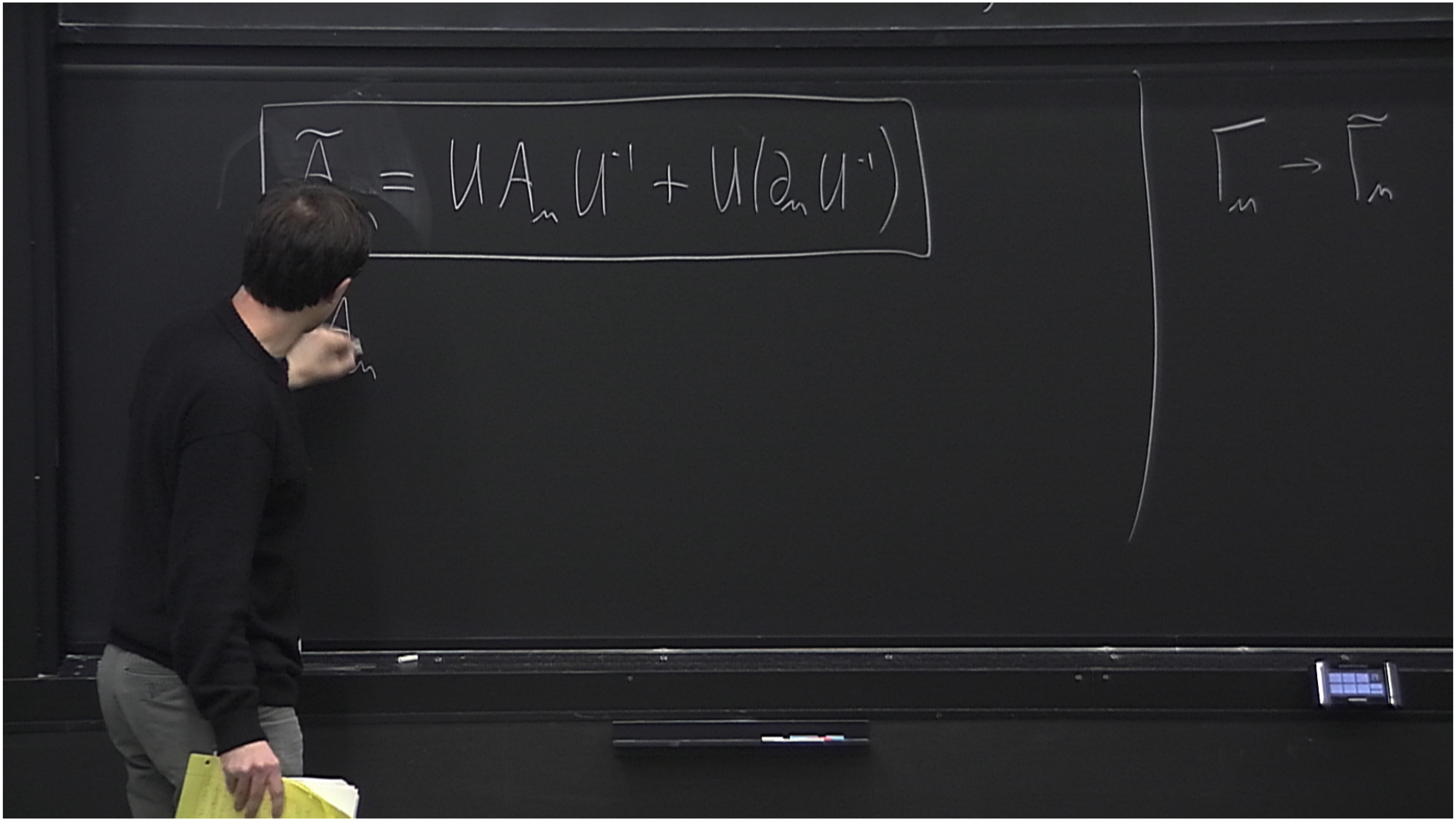

$D_\mu \Phi^a = \partial_\mu \Phi^a + A_\mu{}^a{}_b \Phi^b \rightarrow D_\mu \Phi = (\partial_\mu + A_\mu)\Phi$
$\nabla_\mu V^\sigma = \partial_\mu$
$D_\mu \Phi \rightarrow \tilde{D}_\mu \tilde{\Phi} = U D_\mu \Phi$
$= \tilde{D}_\mu U \Phi$
$\tilde{D}_\mu = U D_\mu U^{-1} \rightarrow \partial_\mu + \tilde{A}_\mu$

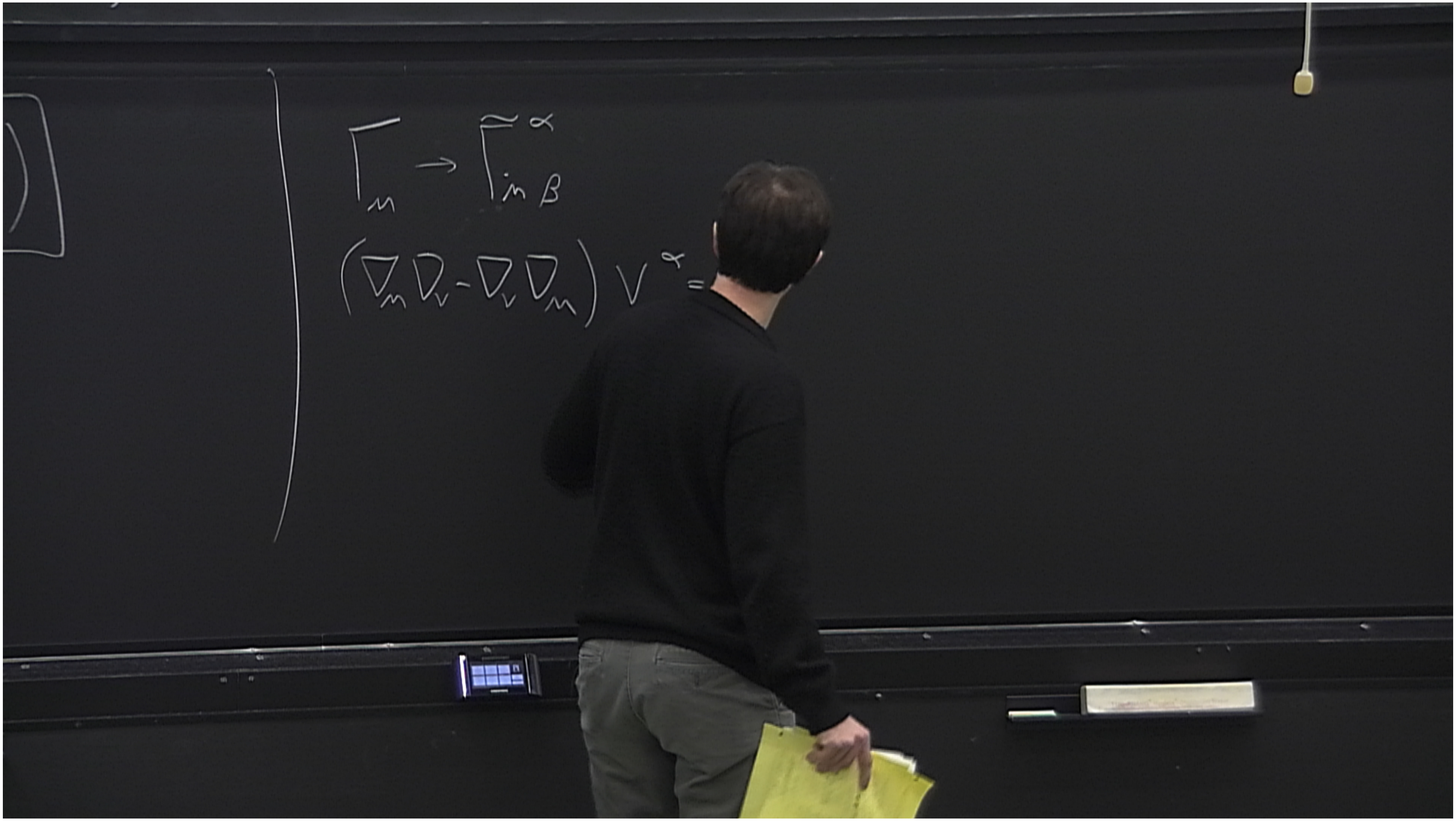


$D_\mu \Phi \to \tilde{D}_\mu \tilde{\Phi} = U D_\mu \Phi$
$\tilde{D}_\mu U \Phi$
$\tilde{D}_\mu = U D_\mu U^{-1} \to \partial_\mu + \tilde{A}_\mu = U(\partial_\mu + A_\mu)U^{-1}$



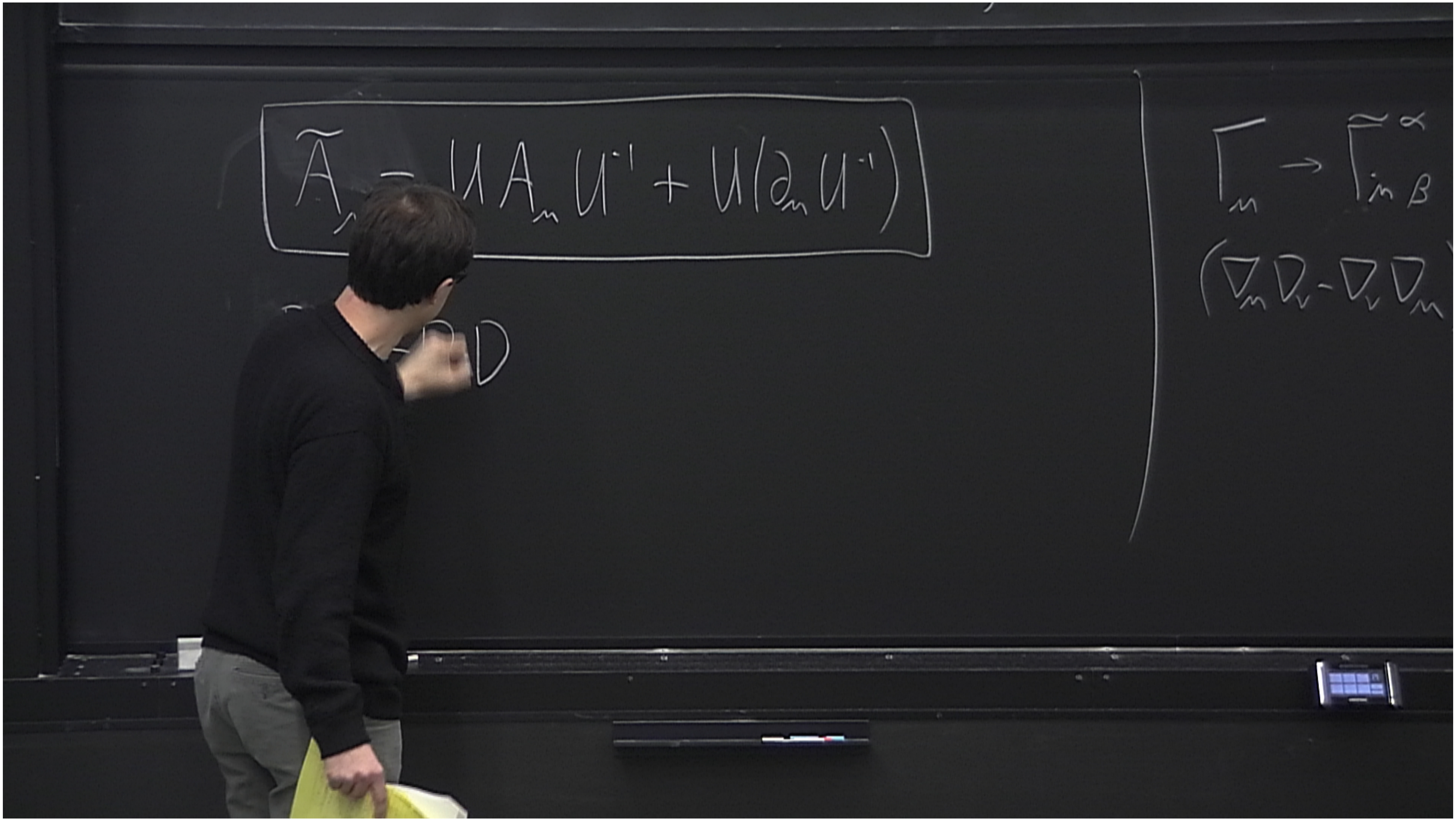
$\tilde{A}_\mu = U A_\mu U^{-1} + U(\partial_\mu U^{-1})$
$\Gamma_\mu \to$

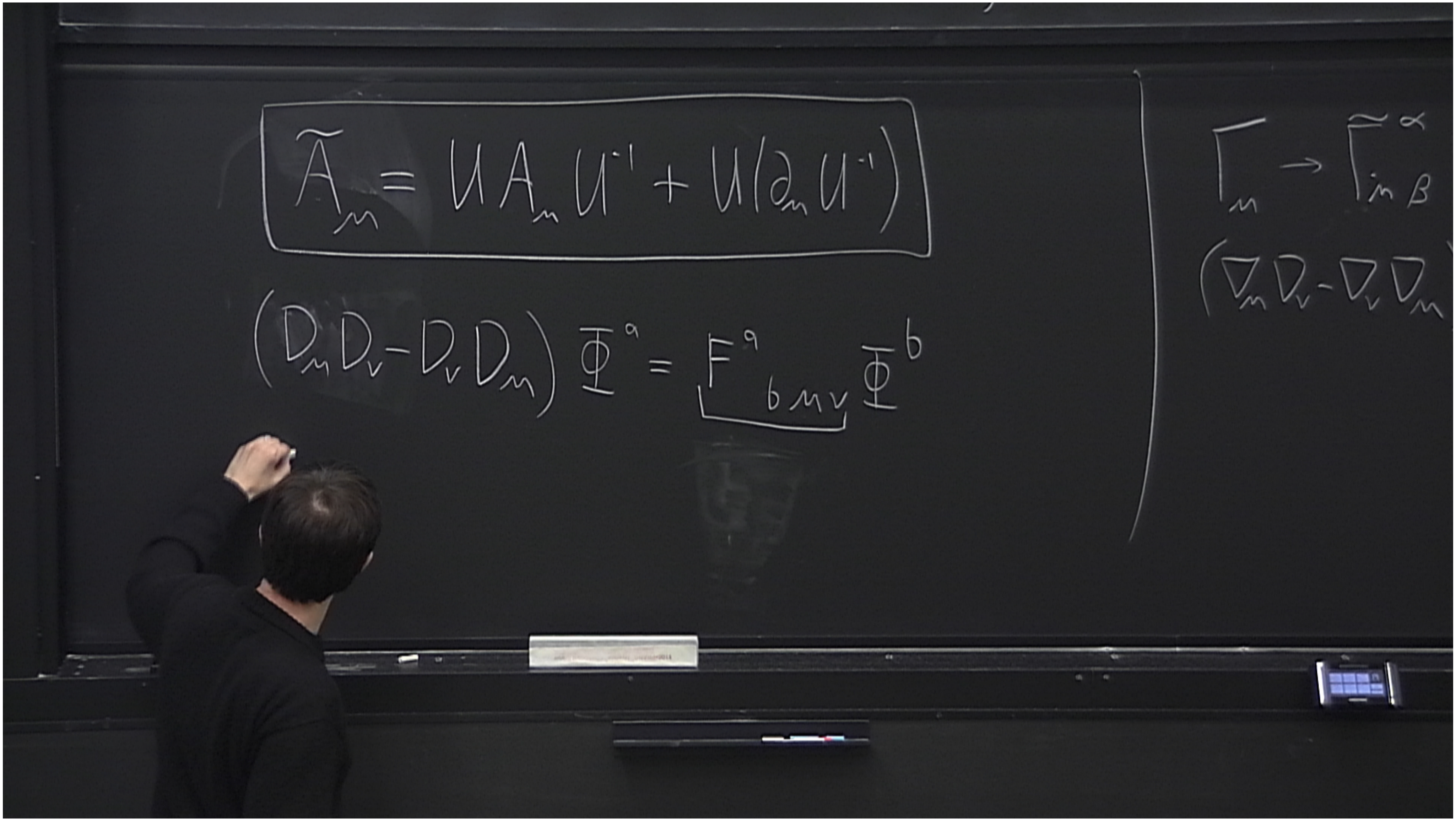

$\tilde{A}_\mu = U A_\mu U^{-1} + U(\partial_\mu U^{-1})$
$\Gamma_\mu \to \tilde{\Gamma}_\mu$
A_μ

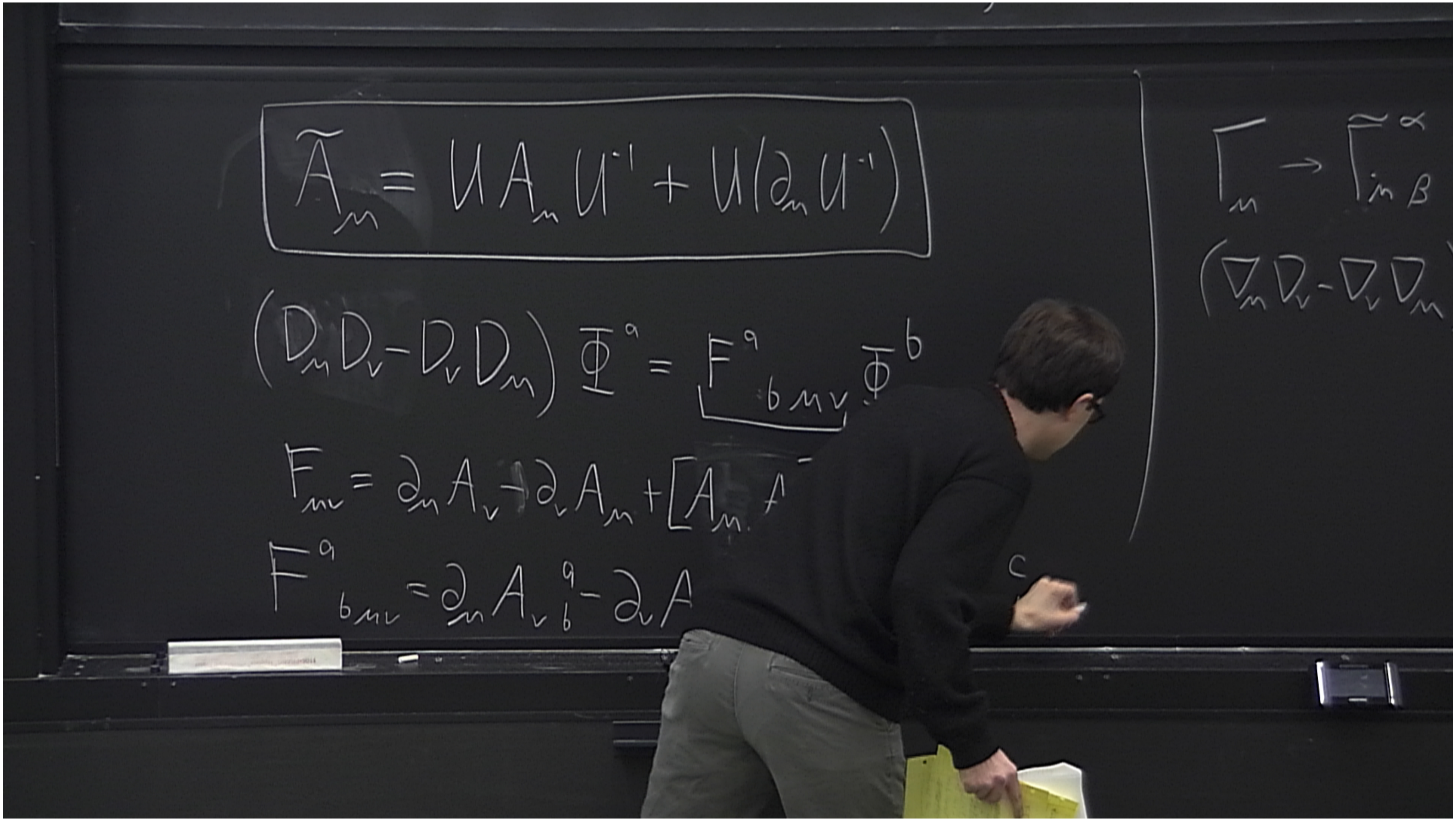

$\Gamma_\mu \rightarrow \tilde{\Gamma}^{\alpha}_{\mu\beta}$
$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) V^\alpha =$

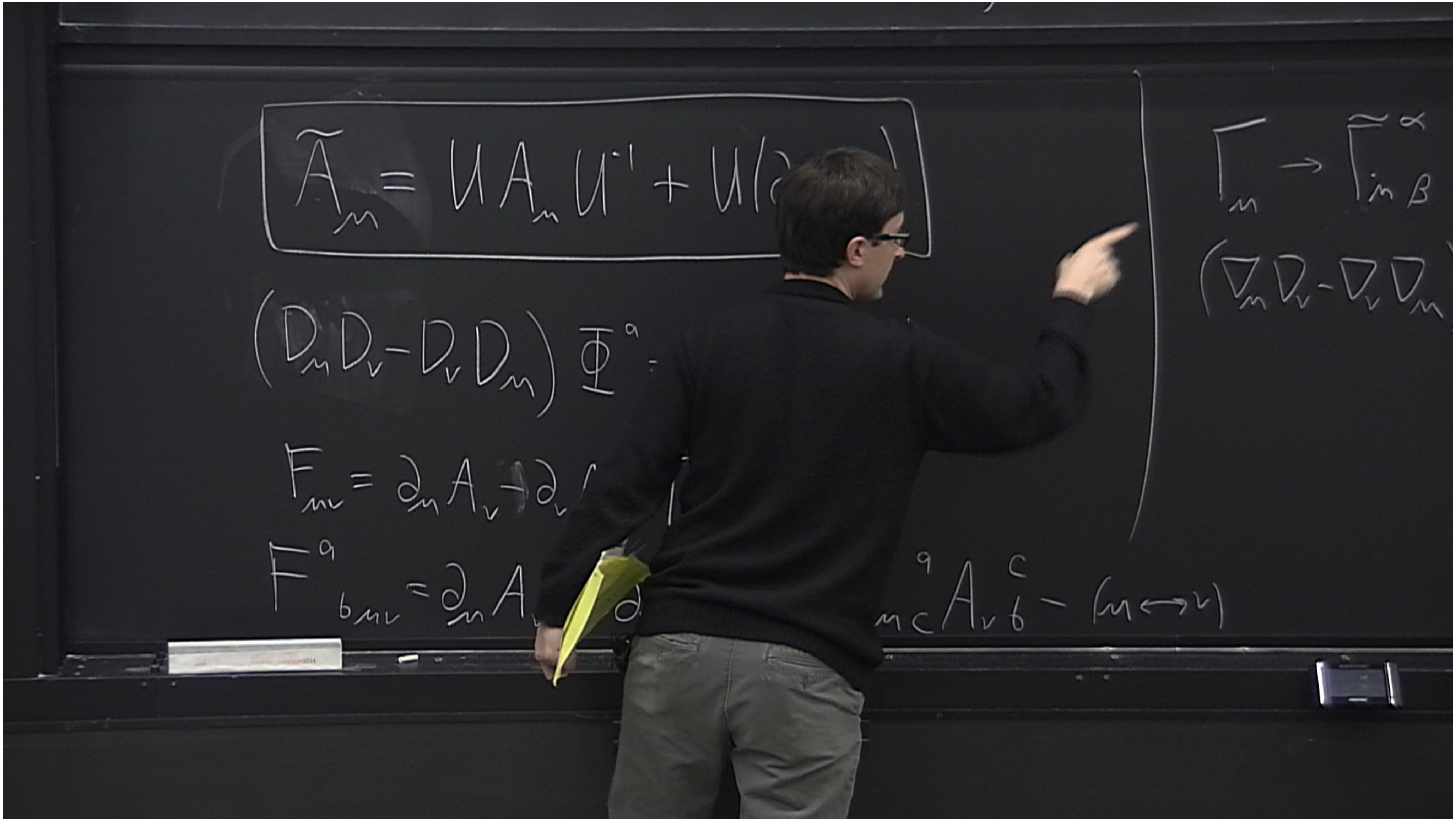
$$\Gamma_{\mu} \rightarrow \tilde{\Gamma}^{\alpha}_{\mu\beta}$$

$$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) V^\alpha = \underline{R^\alpha{}_{\beta\mu\nu}} V^\beta \qquad R^\alpha{}_{\beta\mu\nu} = \Gamma_{,} \; \Gamma_{,}$$

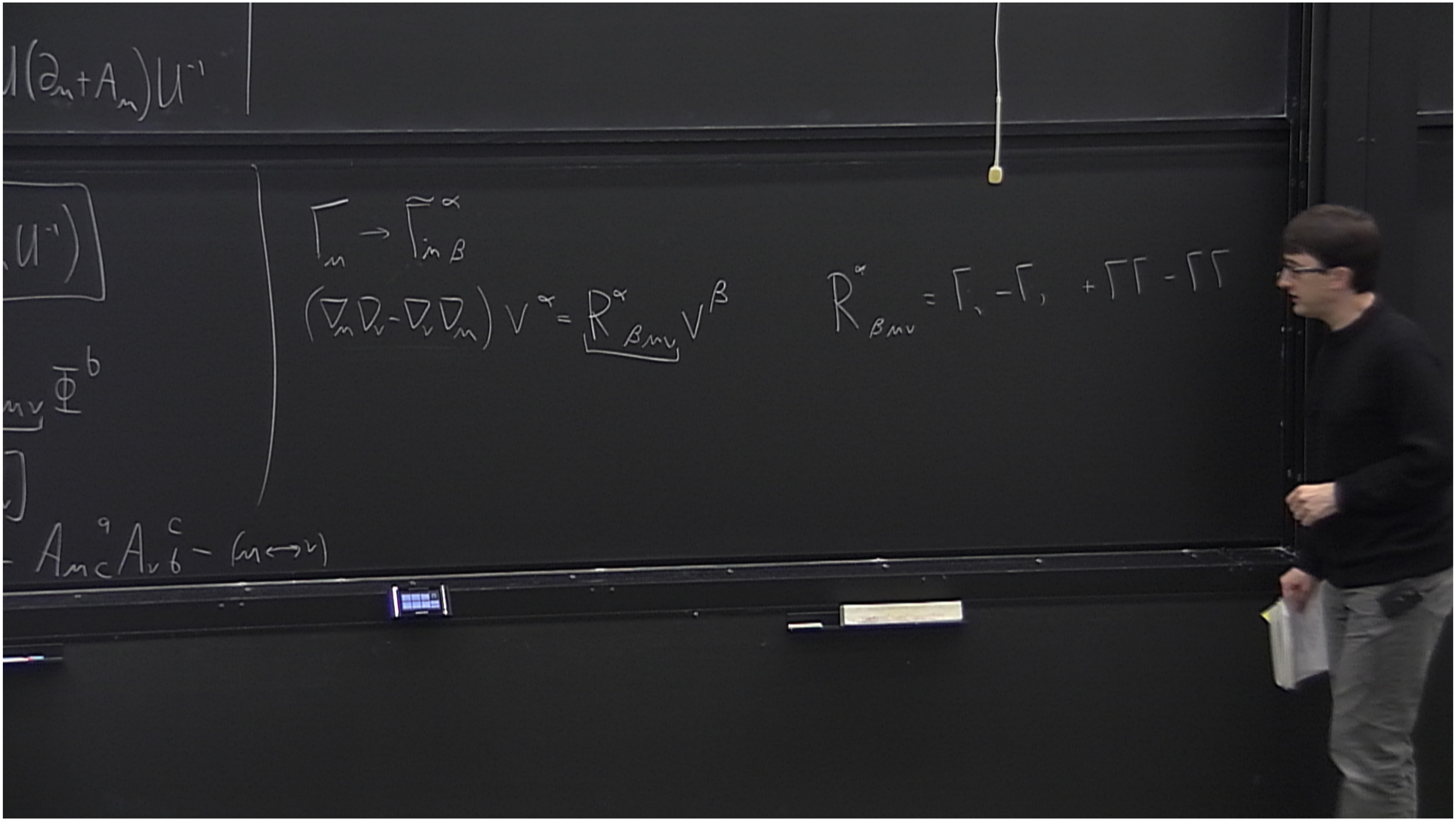


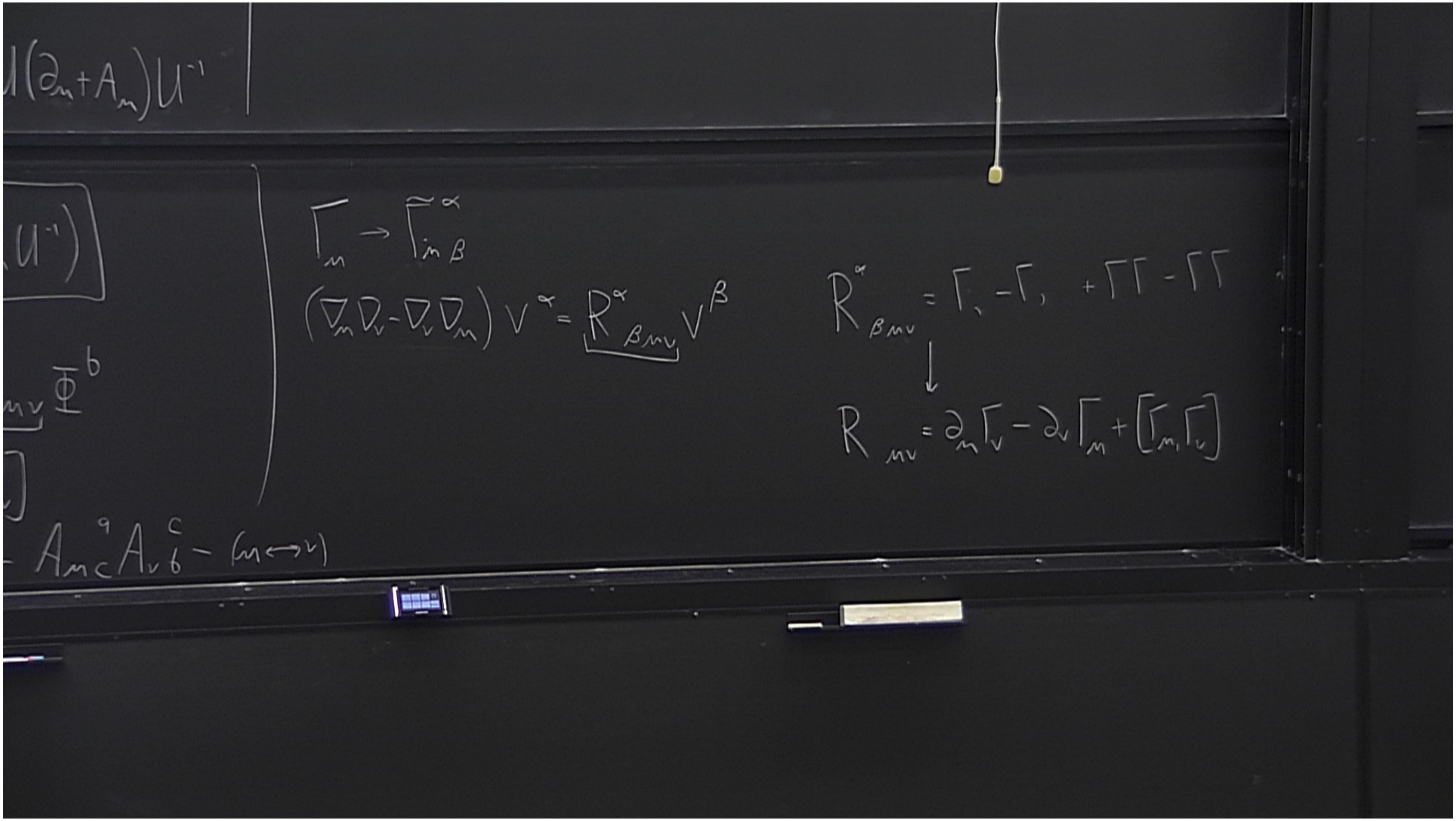

$\tilde{A}_\mu = U A_\mu U^{-1} + U(\partial_\mu U^{-1})$
$(D_\mu D_\nu - D_\nu D_\mu)\Phi^a = F^a{}_{b\mu\nu}\Phi^b$
$\Gamma_\mu \to \Gamma^\alpha_{\mu\beta}$
$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu$

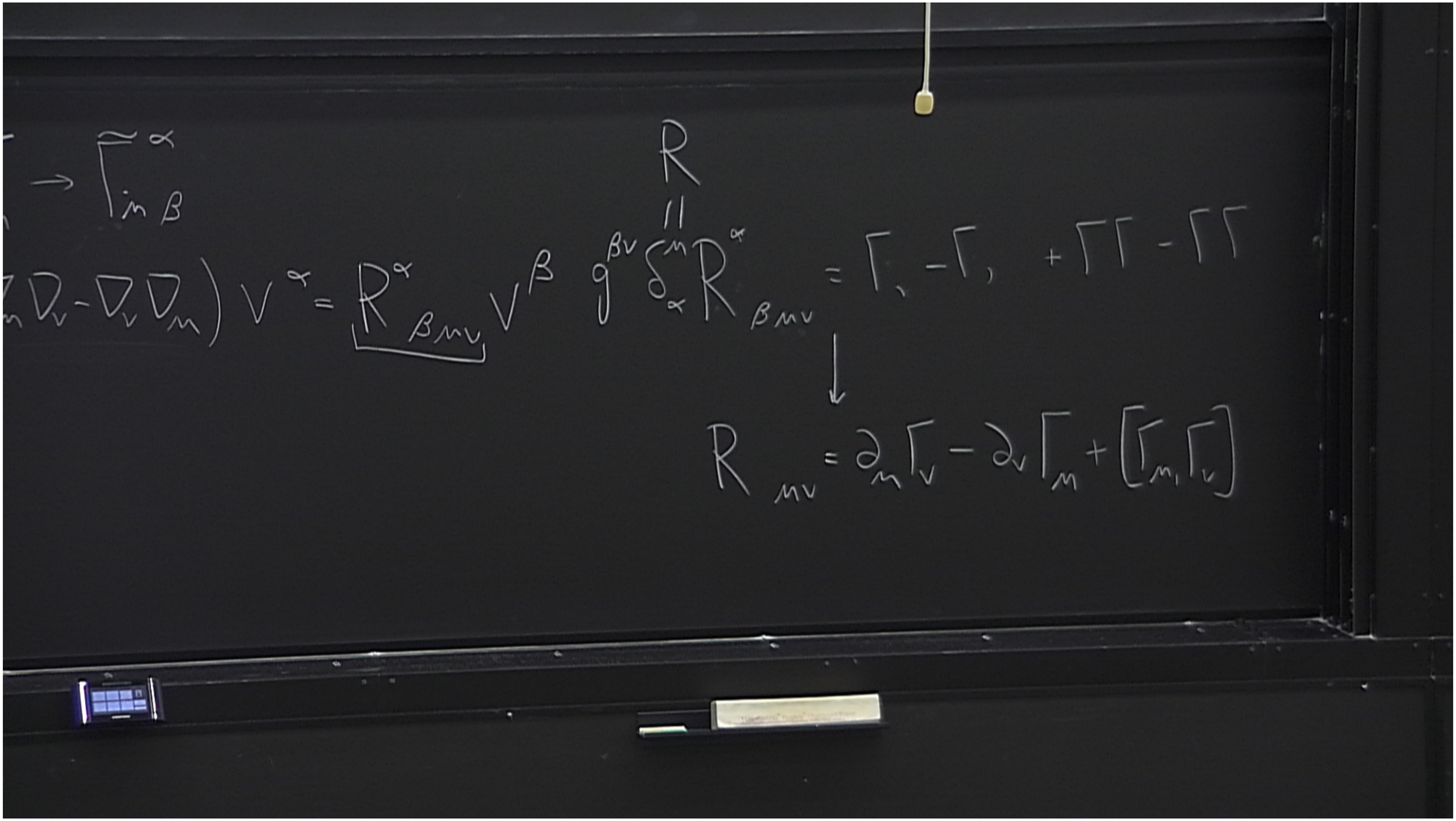

$\tilde{A}_\mu = U A_\mu U^{-1} + U(\partial_\mu U^{-1})$
$(D_\mu D_\nu - D_\nu D_\mu)\Phi^a = F^a{}_{b\mu\nu}\Phi^b$
$\Gamma_\mu \to \tilde{\Gamma}^\alpha_{\mu\beta}$

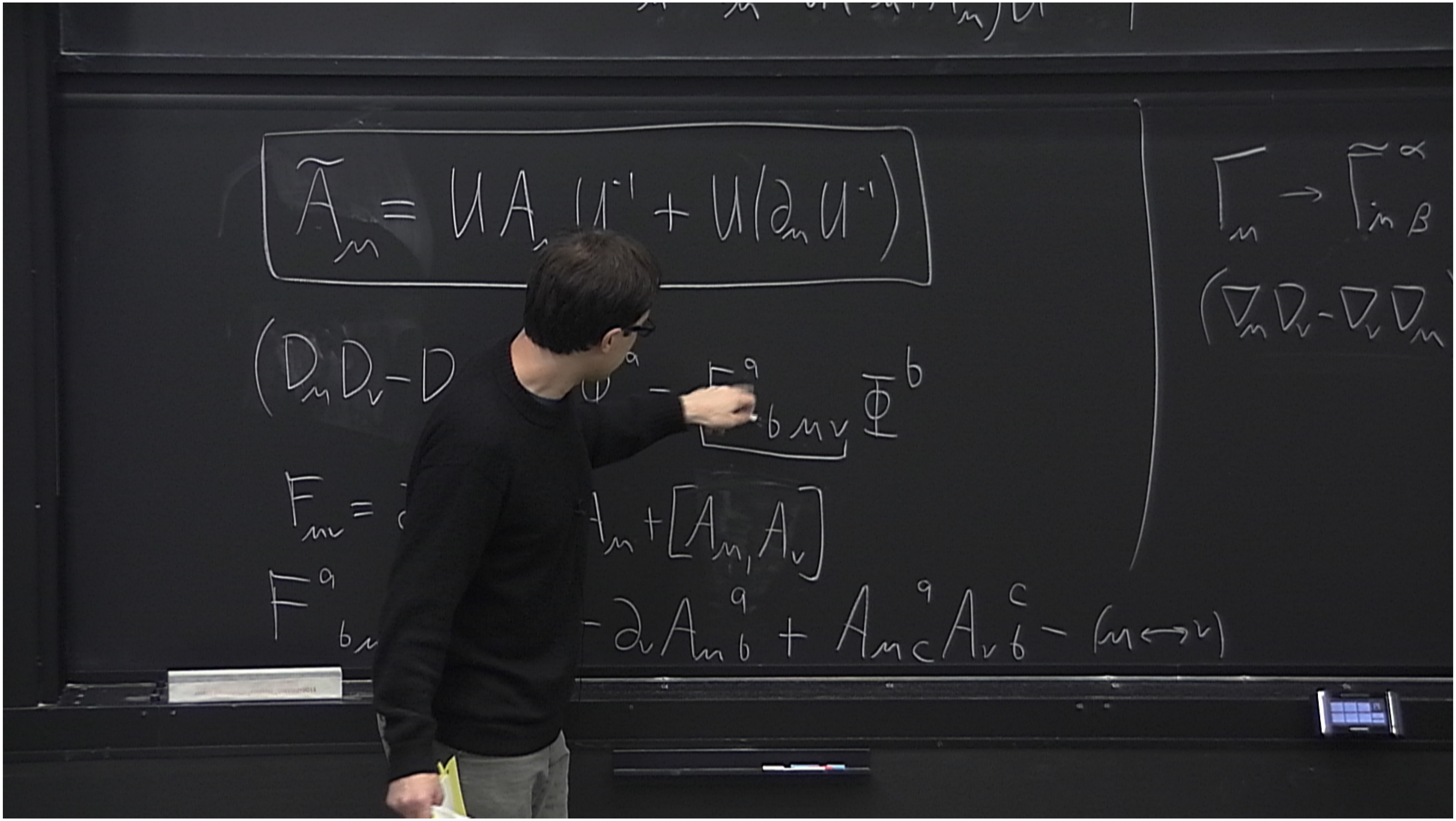


Ã_μ = U A_μ U⁻¹ + U(∂
(D_μ D_ν − D_ν D_μ) Φ^a =
F_μν = ∂_μ A_ν − ∂_ν
F^a_bμν = ∂_μ A
A_ν b − (μ ↔ ν)
Γ_μ → Γ̃^α_μβ
(∇_μ ∇_ν − ∇_ν ∇_μ


$\Gamma_\mu \to \tilde{\Gamma}^{\alpha}_{\mu\beta}$
$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) V^\alpha = R^\alpha{}_{\beta\mu\nu} V^\beta$
$R^\alpha{}_{\beta\mu\nu} = \Gamma_{,} - \Gamma_{,} + \Gamma\Gamma - \Gamma\Gamma$


U(∂_μ + A_μ)U^{-1}
Γ_μ → Γ̃^α_{μβ}
(∇_μ∇_ν − ∇_ν∇_μ) V^α = R^α_{βμν} V^β
R^α_{βμν} = Γ, − Γ, + ΓΓ − ΓΓ
R_{μν} = ∂_μΓ_ν − ∂_νΓ_μ + [Γ_μ, Γ_ν]
A_μ^a_c A_ν^c_b − (μ↔ν)


$\to \tilde{\Gamma}^{\alpha}_{\mu\beta}$
R
$\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) V^\alpha = R^\alpha{}_{\beta\mu\nu} V^\beta$
$g^{\beta\nu} \delta^\mu_\alpha R^\alpha{}_{\beta\mu\nu} = \Gamma_, - \Gamma_, + \Gamma\Gamma - \Gamma\Gamma$
$R_{\mu\nu} = \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu]$



$\tilde{A}_\mu = U A_\mu U^{-1} + U(\partial_\mu U^{-1})$
$\Gamma_\mu \to \Gamma^\alpha_{\mu\beta}$

$$\boxed{\tilde{A}_\mu = U A_\mu U^{-1} + U(\partial_\mu U^{-1})}$$

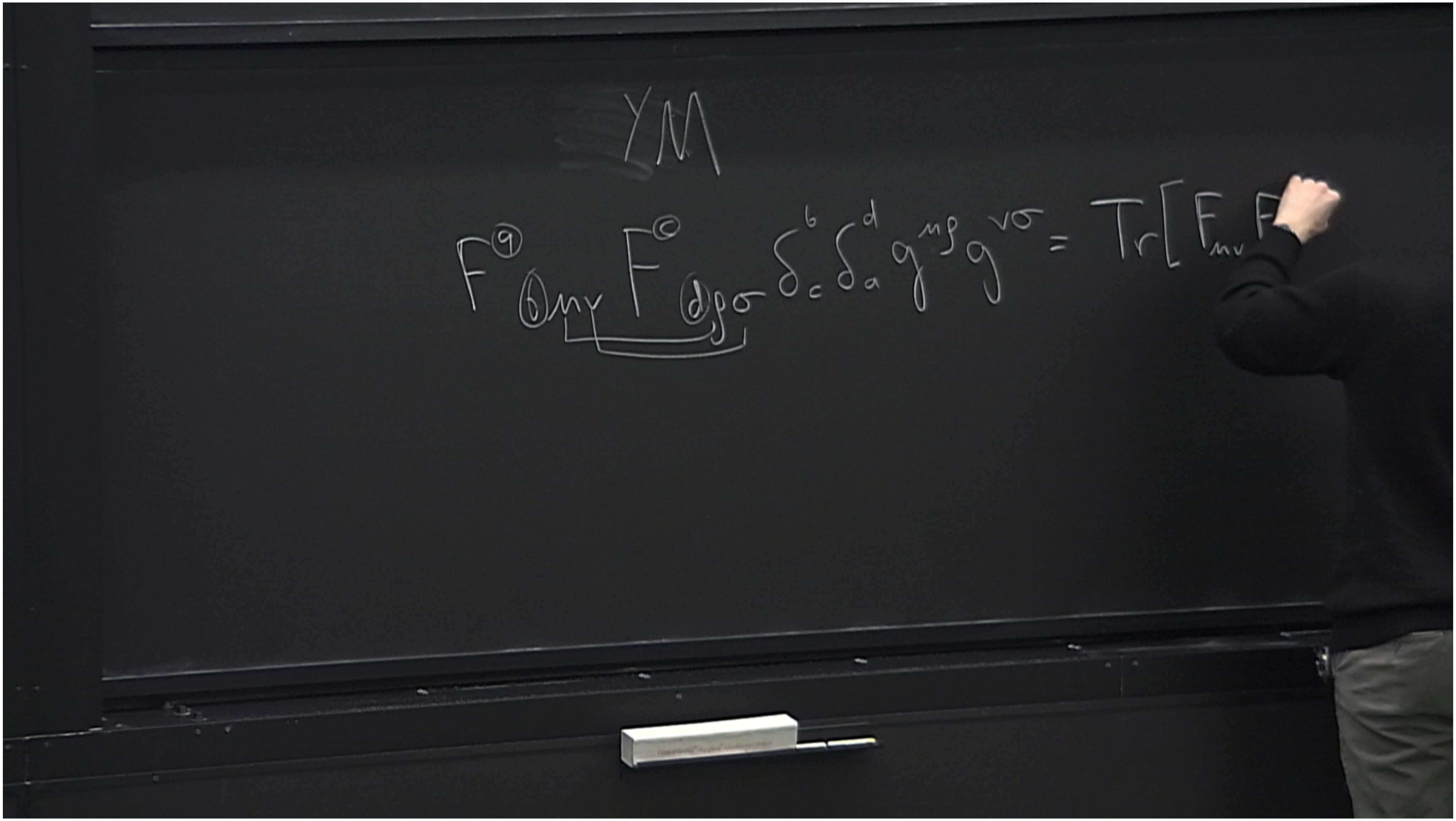
$$(D_\mu D_\nu - D_\nu D_\mu)\,\Phi^a = F^a{}_{b\mu\nu}\,\Phi^b$$

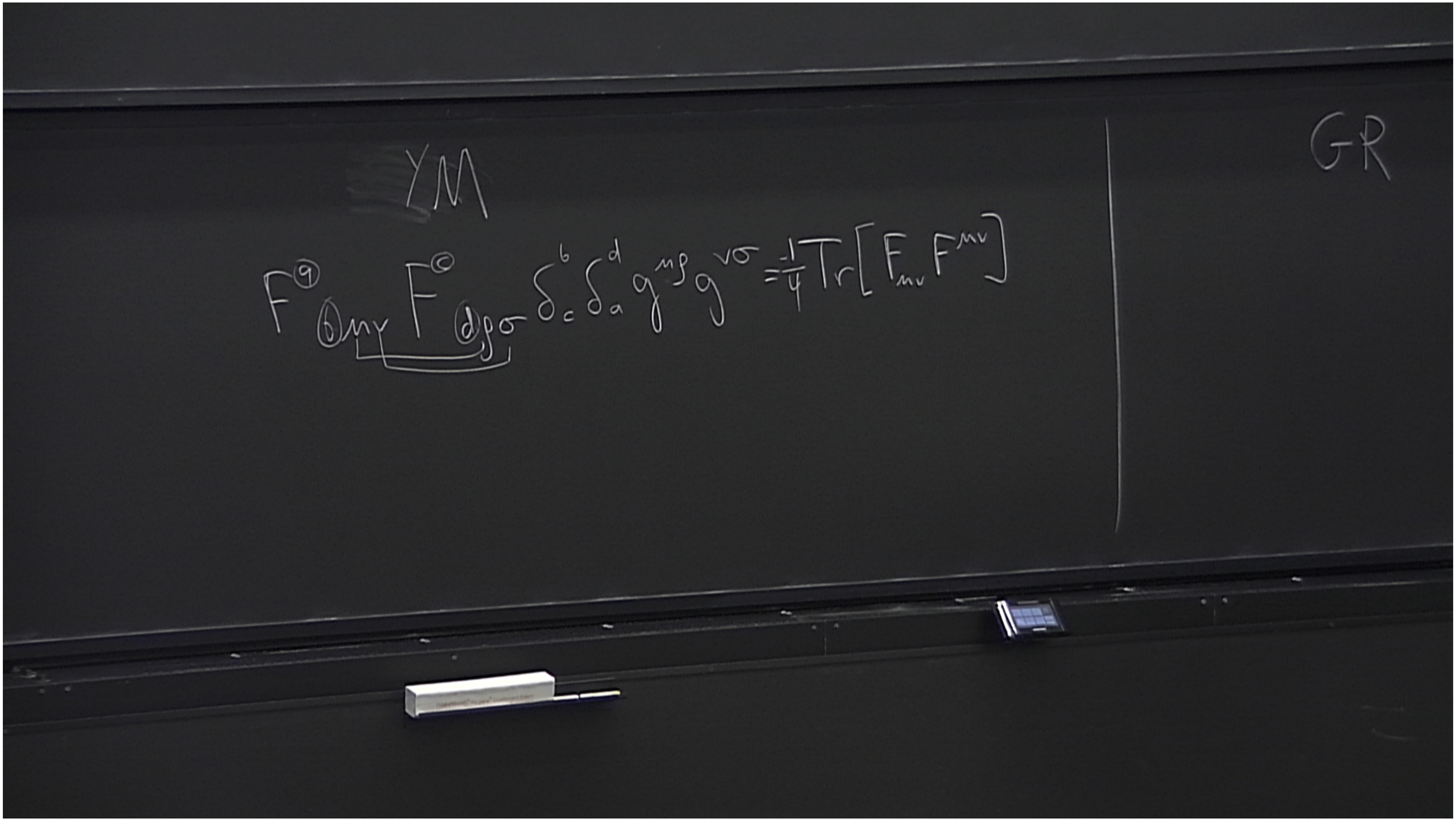
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

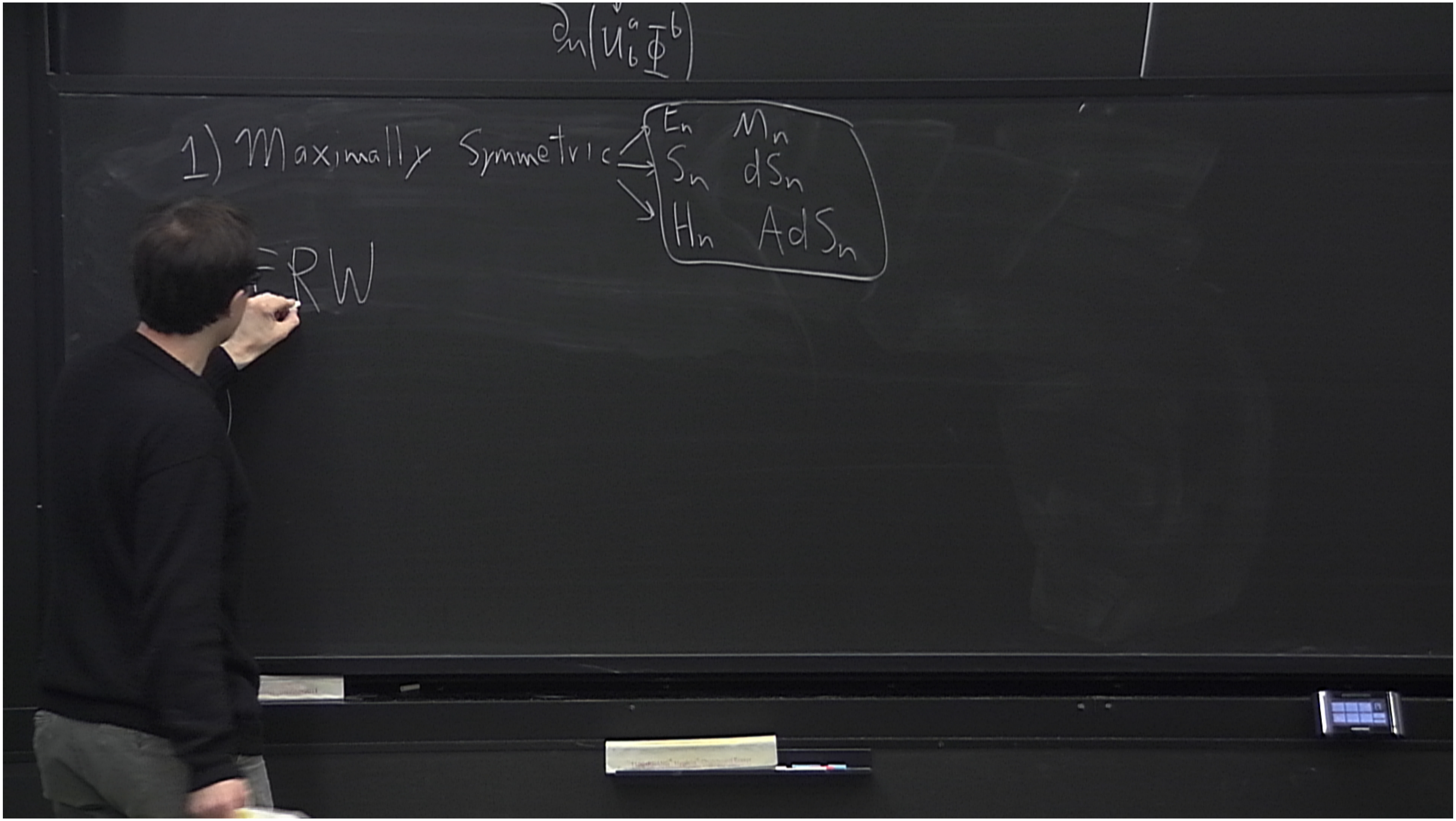
$$F^a{}_{b\mu\nu} = \partial_\mu A_\nu{}^a{}_b - \partial_\nu A_\mu{}^a{}_b + A_\mu{}^a{}_c A_\nu{}^c{}_b - (\mu \leftrightarrow \nu)$$

$$\Gamma_\mu \to \Gamma^\alpha_{\mu\beta}$$

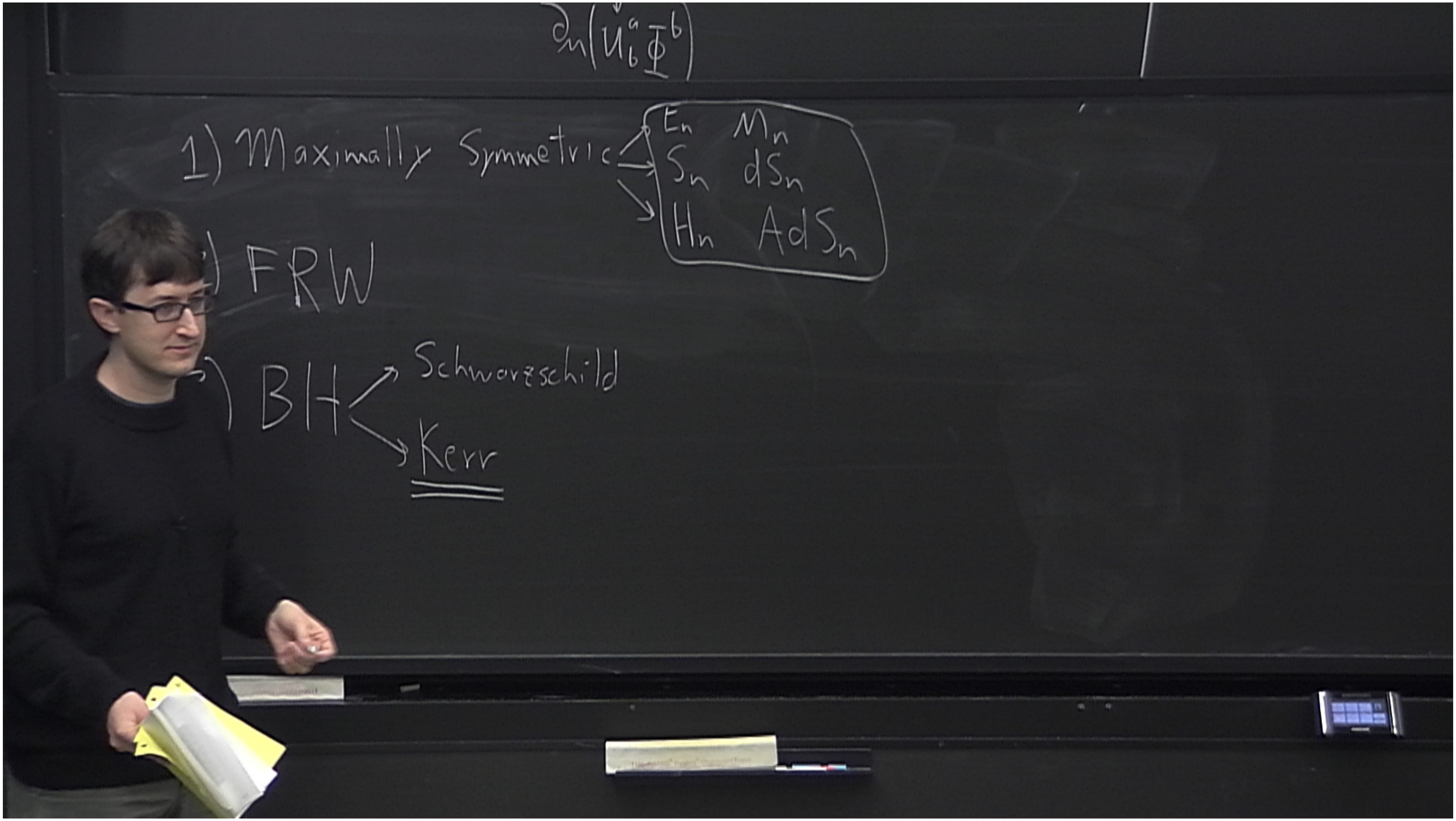
$$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu$$

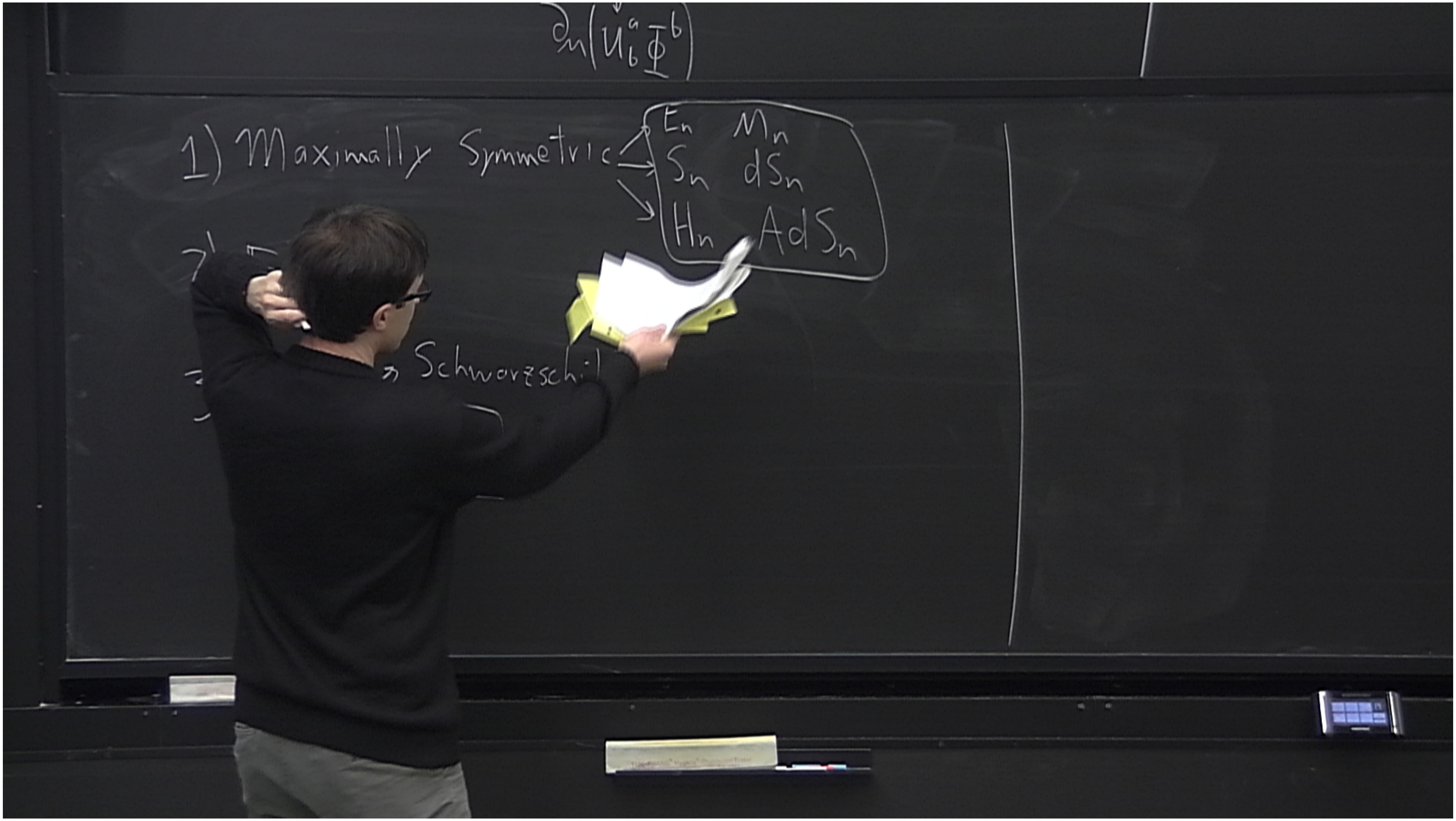

YM
F^a_{b\mu\nu} F^c_{d\rho\sigma} \delta^b_c \delta^d_a g^{\mu\rho} g^{\nu\sigma} = Tr[F_{\mu\nu} F


YM
$F^{a}_{b\mu\nu} F^{c}_{d\rho\sigma} \delta^{b}_{c} \delta^{d}_{a} g^{\mu\rho} g^{\nu\sigma} = -\frac{1}{4} Tr[F_{\mu\nu} F^{\mu\nu}]$
GR

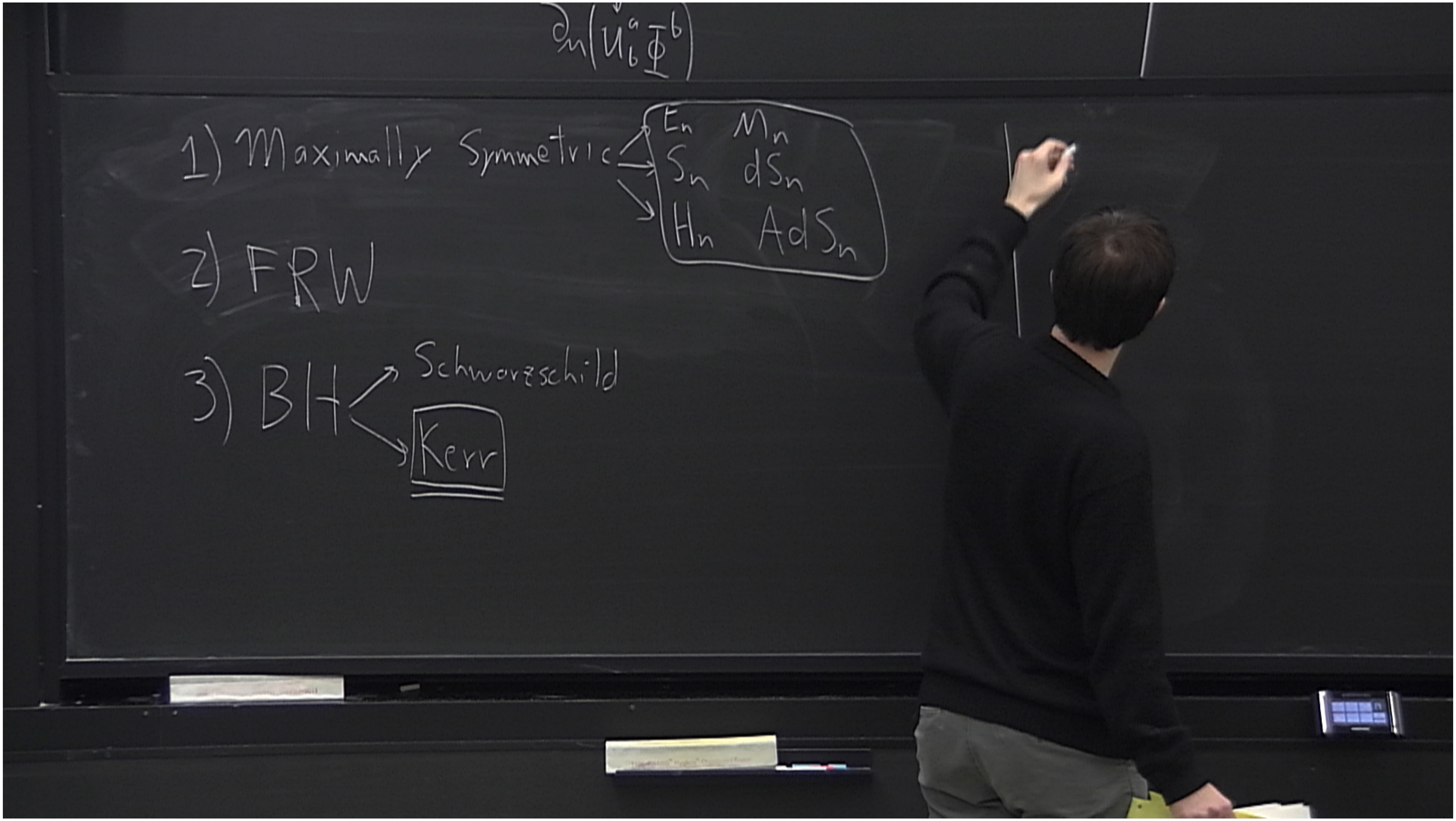


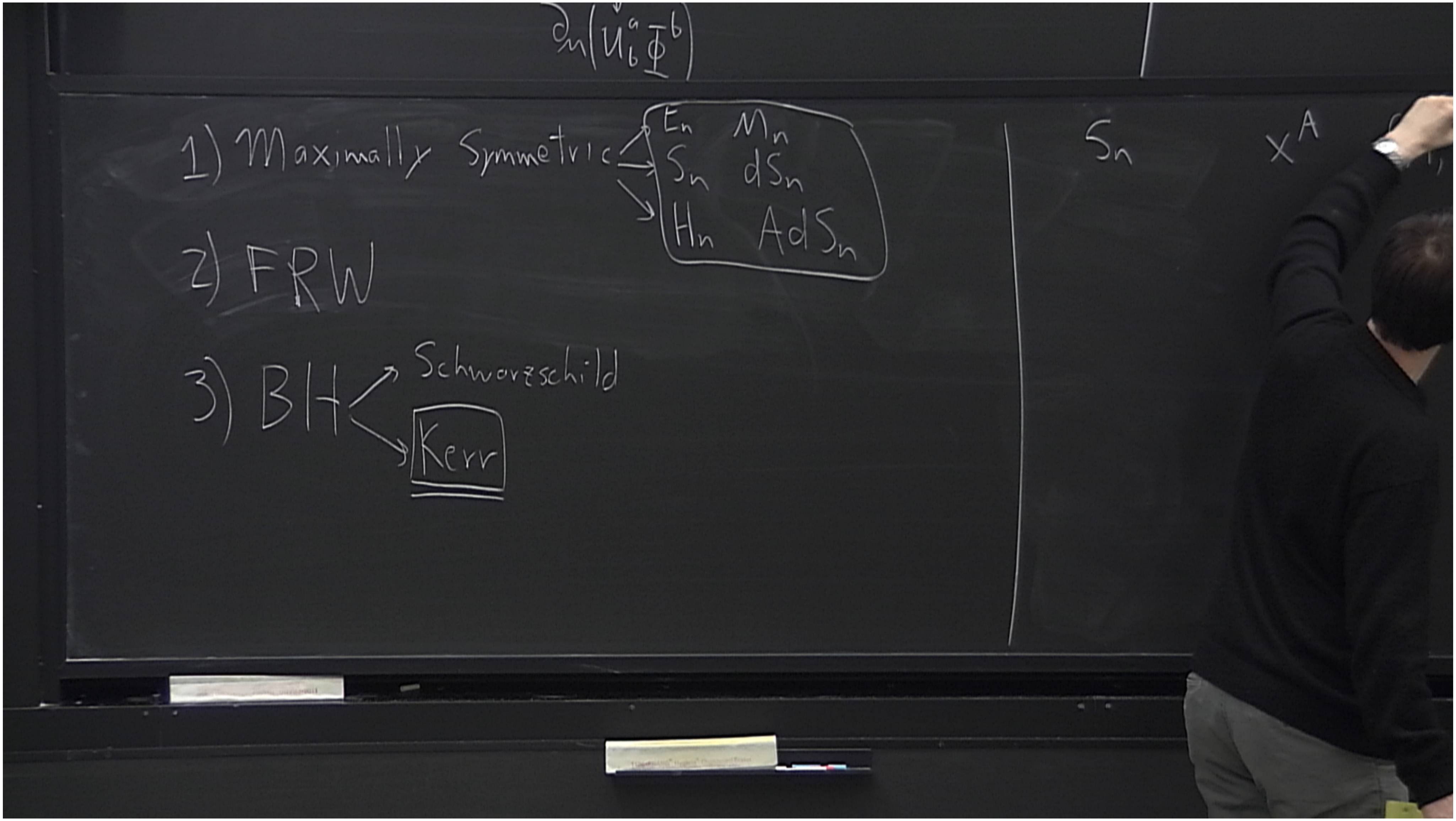
1) Maximally Symmetric
E_n M_n
S_n dS_n
H_n AdS_n
RW

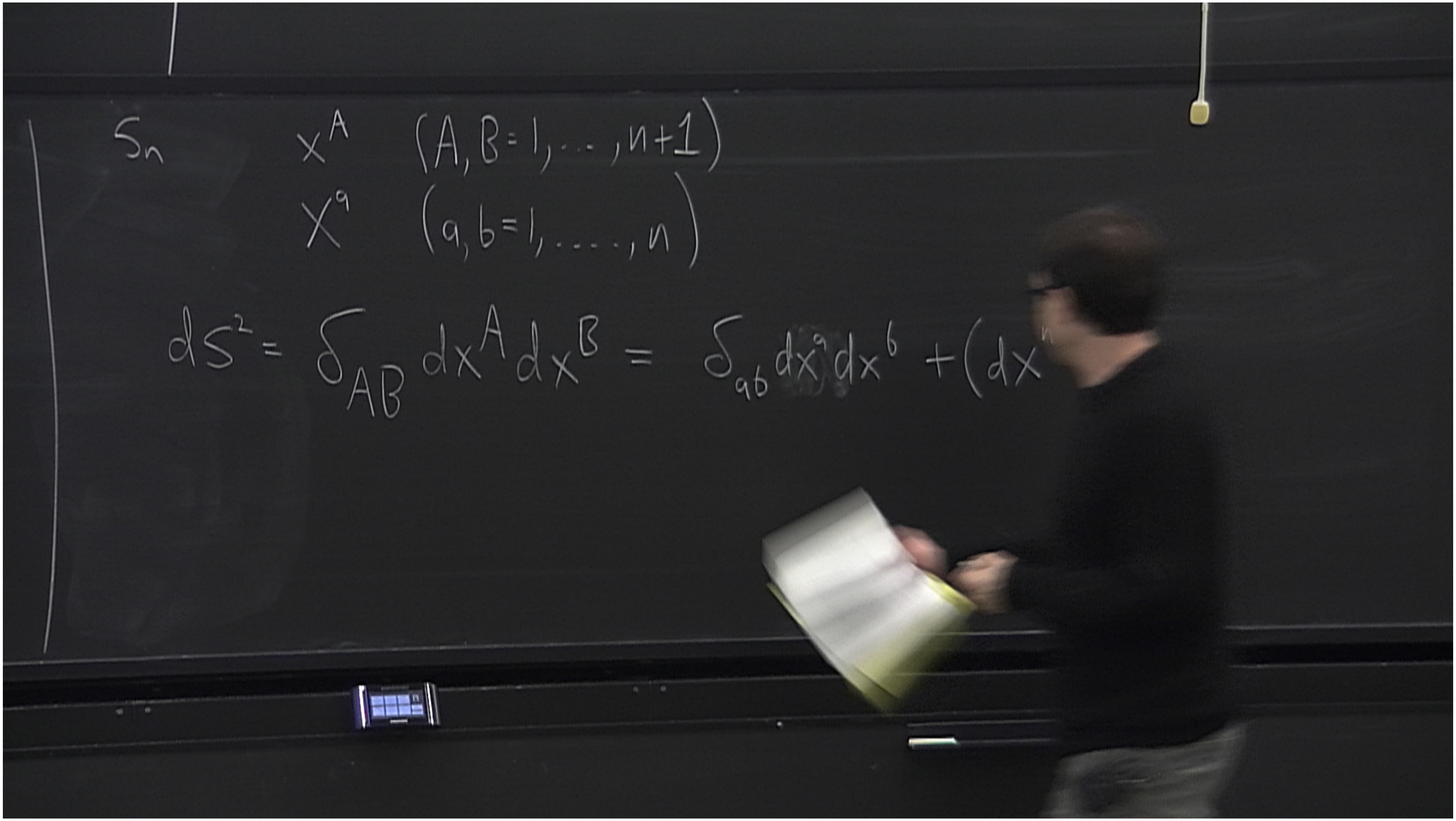

1) Maximally Symmetric
E_n M_n
S_n dS_n
H_n AdS_n
FRW
BH
Schwarzschild
Kerr



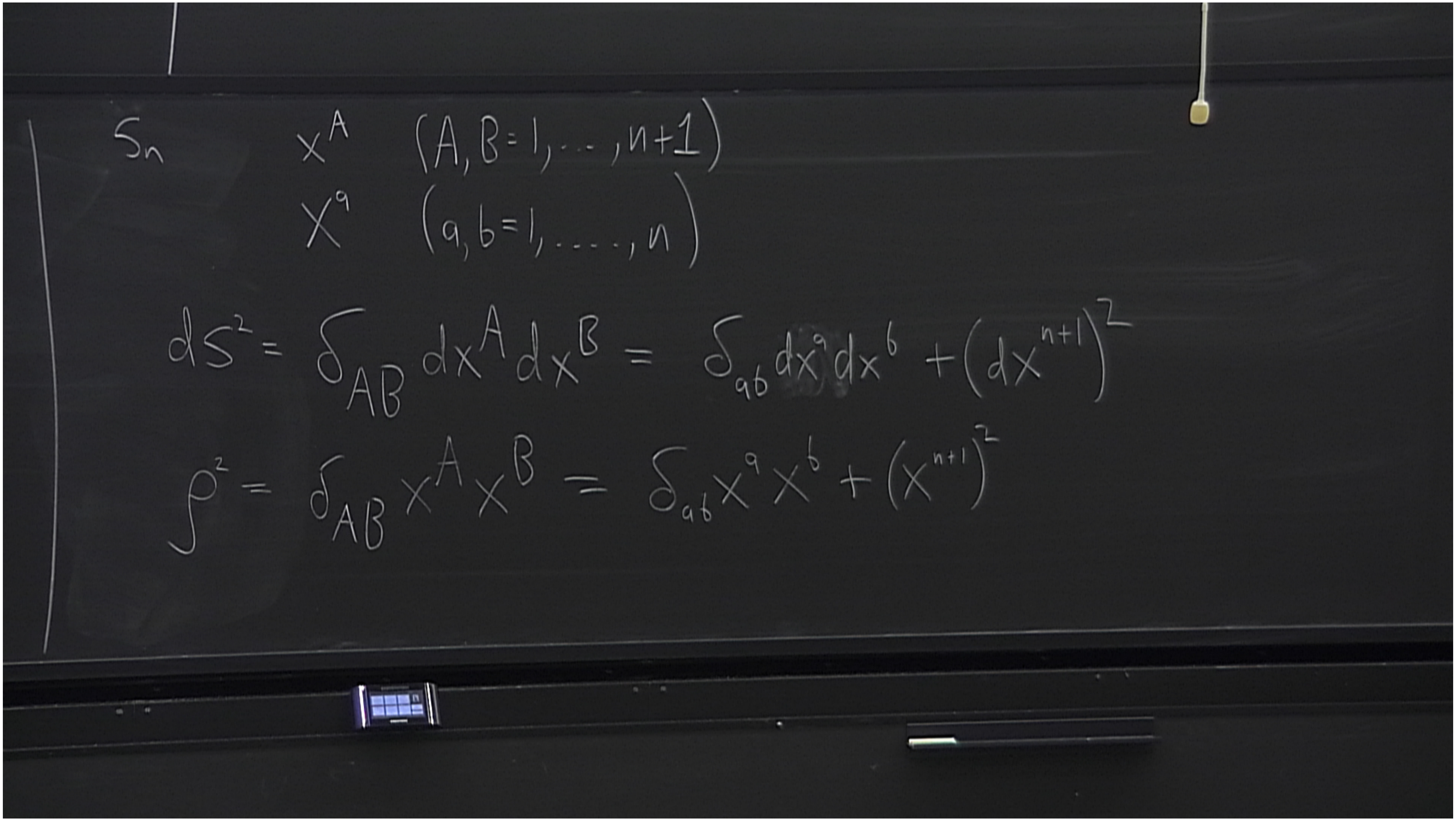
1) Maximally Symmetric
E_n M_n
S_n dS_n
H_n AdS_n


∂_n(U^a_b Φ^b)
1) Maximally Symmetric
E_n M_n
S_n dS_n
H_n AdS_n
2) FRW
3) BH
Schwarzschild
Kerr


$\partial_n(U^a_b \Phi^b)$
1) Maximally Symmetric
E_n M_n
S_n dS_n
H_n AdS_n
2) FRW
3) BH
Schwarzschild
Kerr
S_n
x^A



S_n x^A $(A, B = 1, \ldots, n+1)$
X^a $(a, b = 1, \ldots, n)$
$ds^2 = \delta_{AB}\, dx^A dx^B = \delta_{ab}\, dx^a dx^b + (dx$

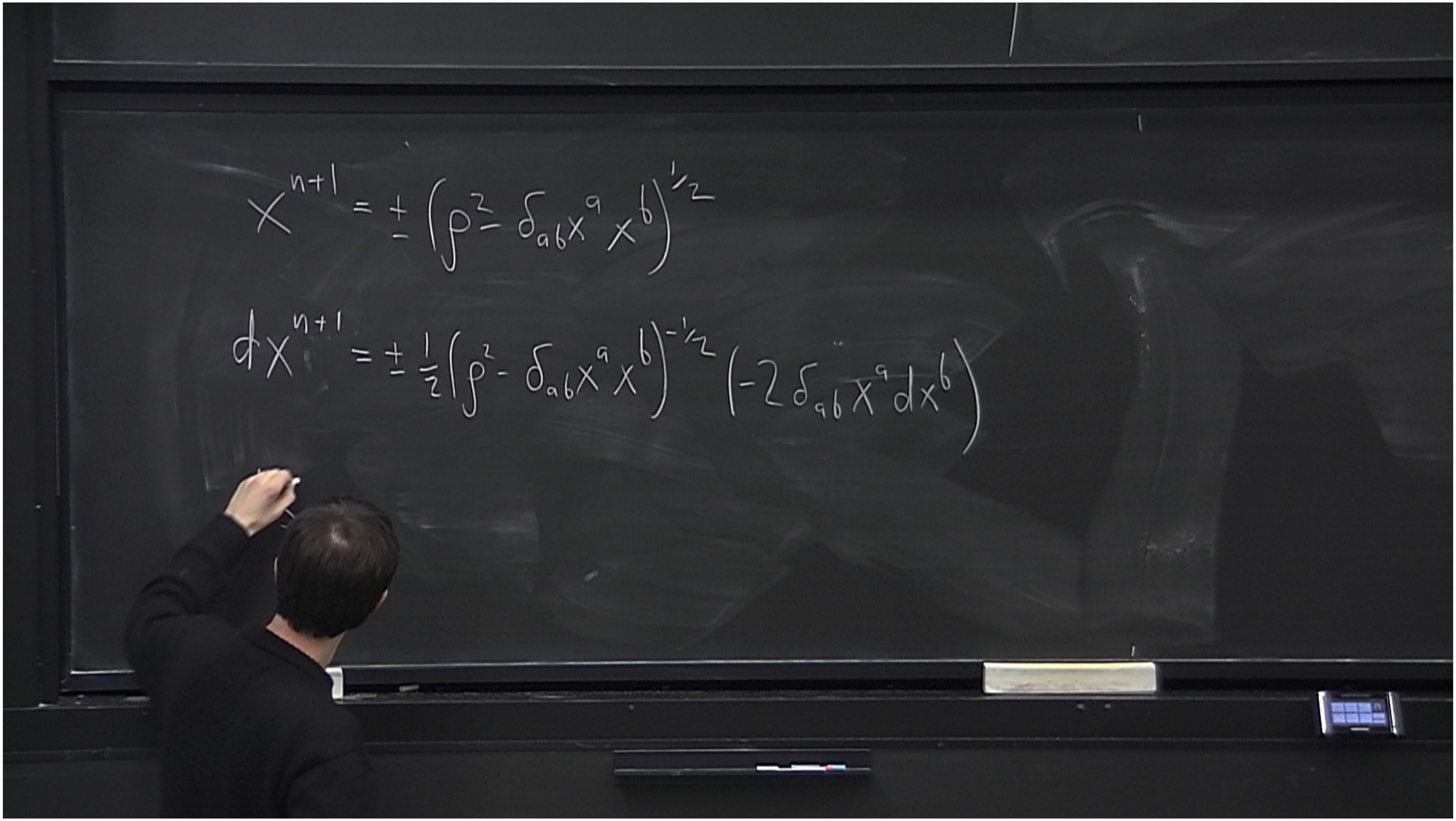

$S_n \quad x^A \quad (A, B = 1, \dots, n+1)$
$x^a \quad (a, b = 1, \dots, n)$
$ds^2 = \delta_{AB}\, dx^A dx^B = \delta_{ab}\, dx^a dx^b + (dx^{n+1})^2$
$\rho^2 = \delta_{AB}\, x^A x^B = \delta_{ab}\, x^a x^b + (x^{n+1})^2$

$S_n \qquad x^A \quad (A, B = 1, \ldots, n+1)$

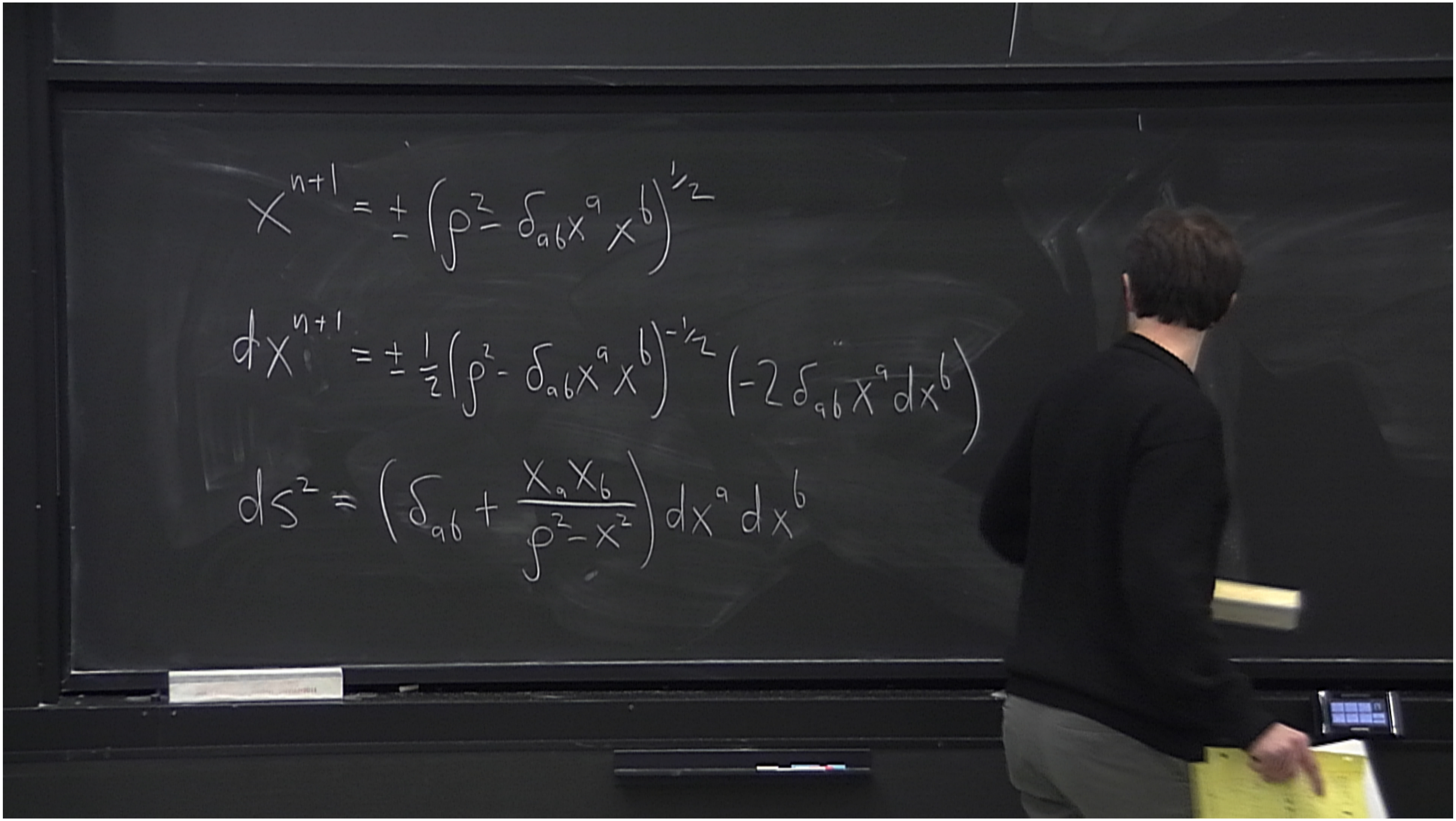
$\qquad\quad X^a \quad (a, b = 1, \ldots, n)$

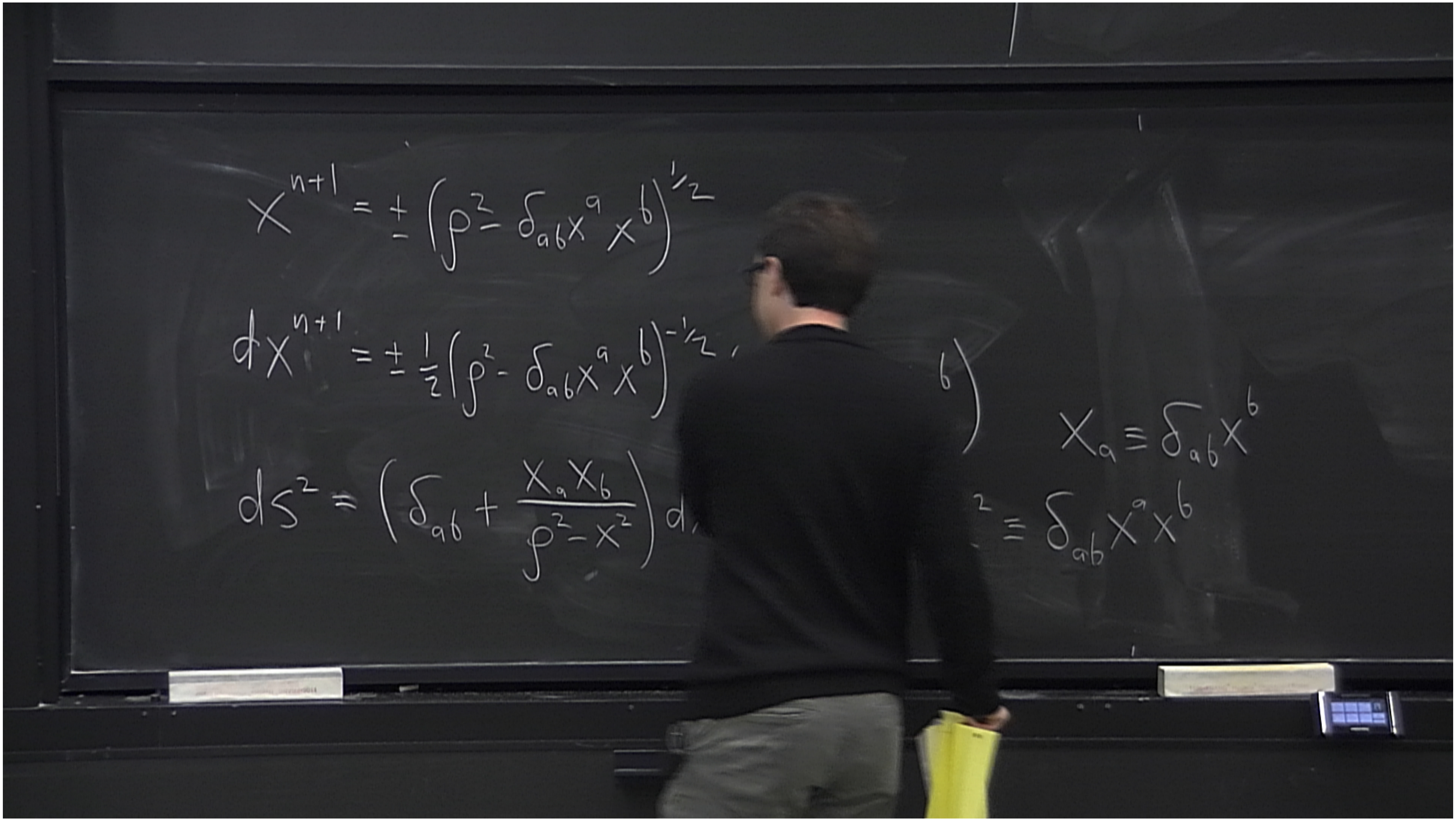
$$ds^2 = \delta_{AB}\, dx^A dx^B = \delta_{ab}\, dx^a dx^b + (dx^{n+1})^2$$

$$\rho^2 = \delta_{AB}\, x^A x^B = \delta_{ab}\, x^a x^b + (x^{n+1})^2$$

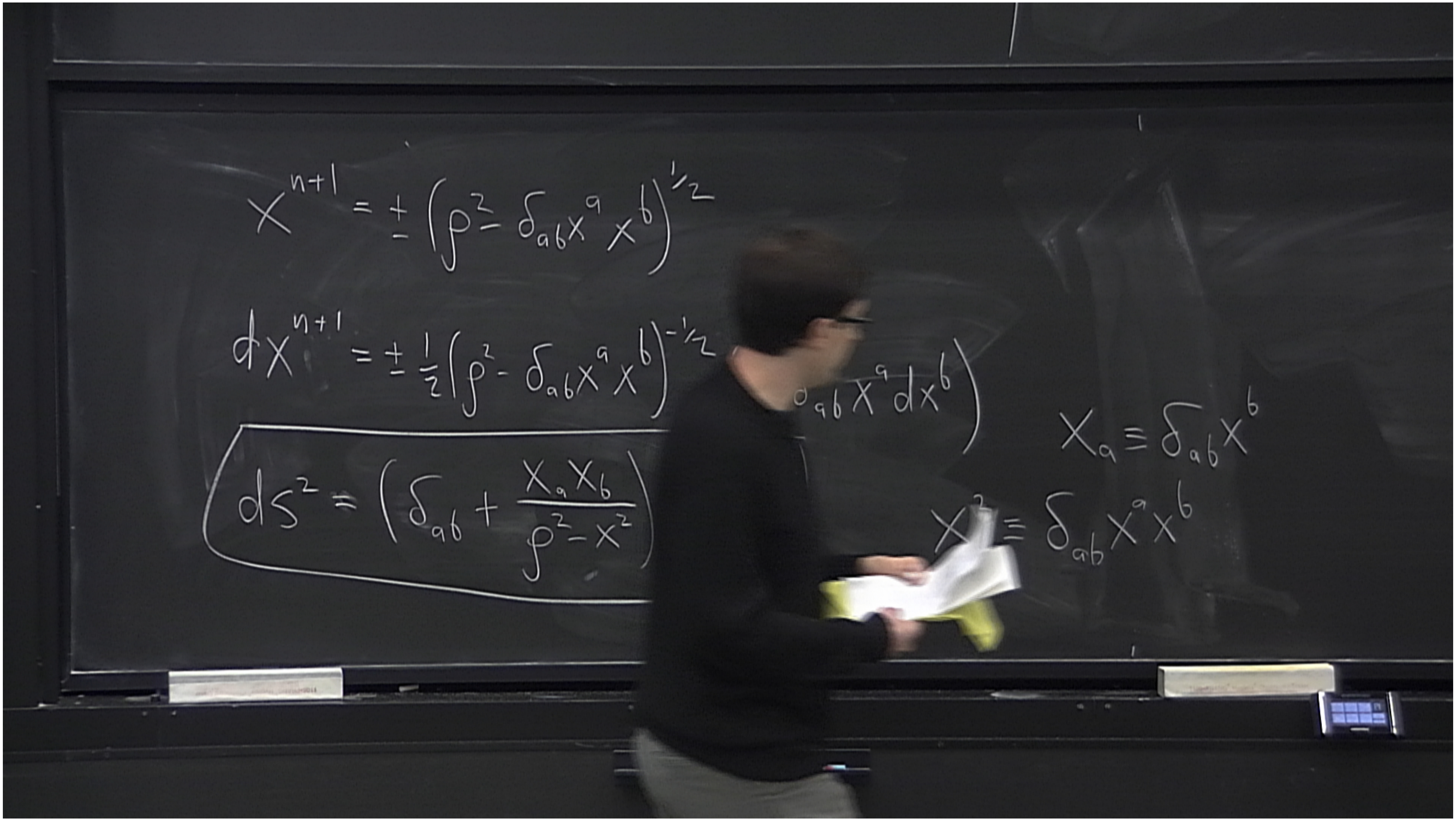


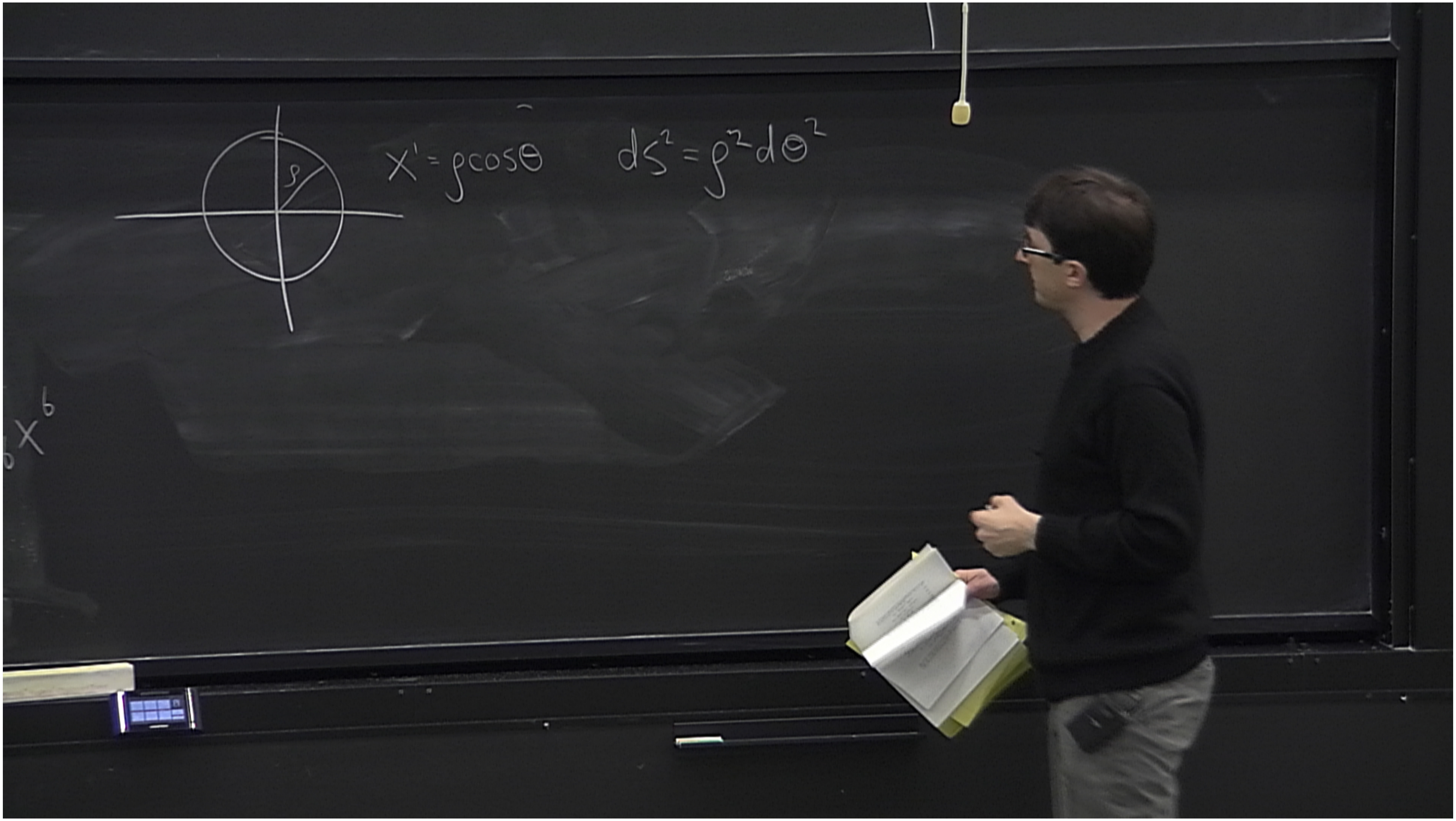
$x^{n+1} = \pm\left(\rho^2 - \delta_{ab}x^a x^b\right)^{1/2}$
$dx^{n+1} = \pm\frac{1}{2}\left(\rho^2 - \delta_{ab}x^a x^b\right)^{-1/2}\left(-2\delta_{ab}x^a dx^b\right)$

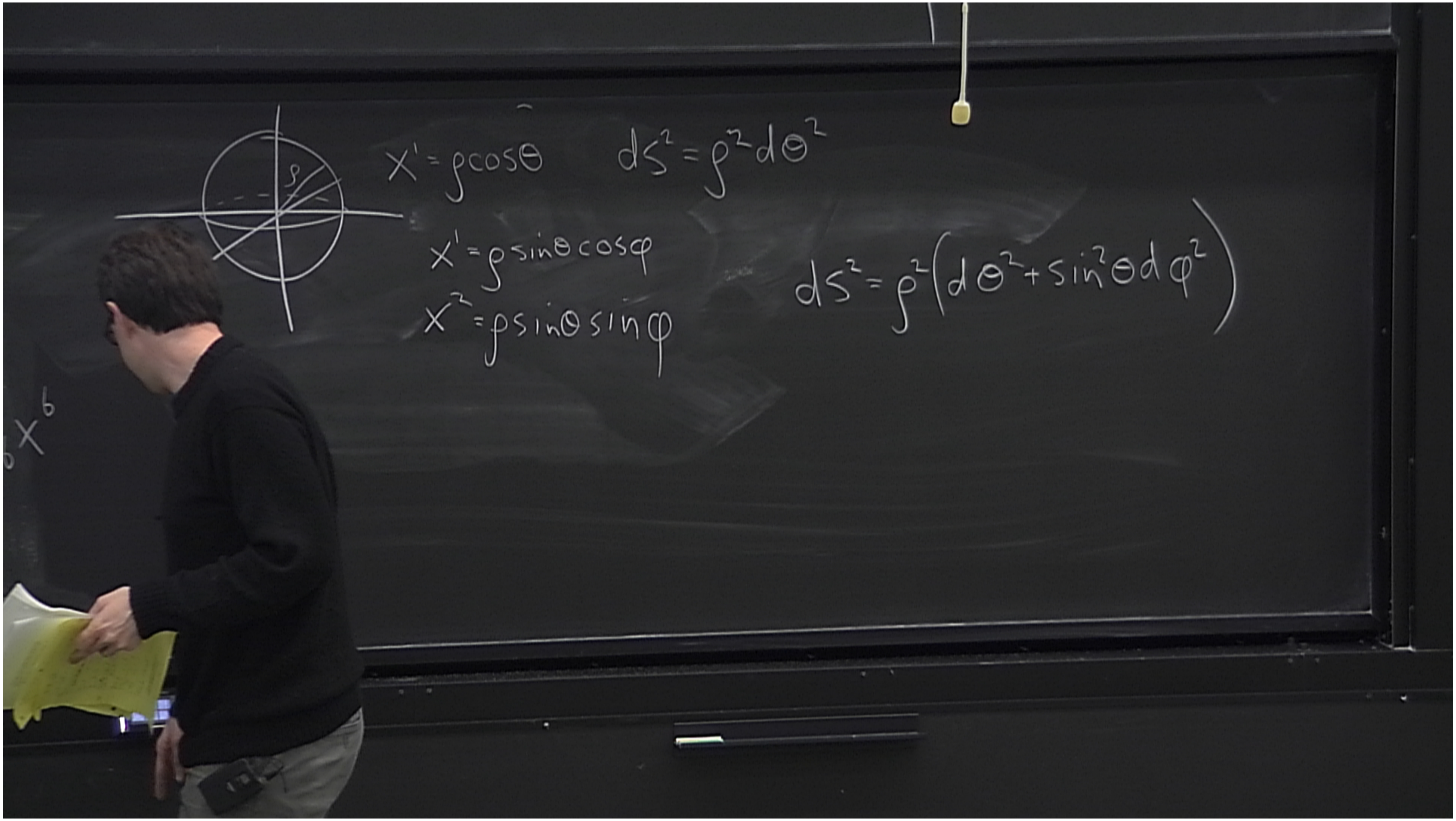

$x^{n+1} = \pm \left(\rho^2 - \delta_{ab} x^a x^b\right)^{1/2}$
$dx^{n+1} = \pm \frac{1}{2} \left(\rho^2 - \delta_{ab} x^a x^b\right)^{-1/2} \left(-2 \delta_{ab} x^a dx^b\right)$
$ds^2 = \left(\delta_{ab} + \frac{x_a x_b}{\rho^2 - x^2}\right) dx^a dx^b$



$X^{n+1} = \pm\left(\rho^2 - \delta_{ab}X^a X^b\right)^{1/2}$
$dX^{n+1} = \pm\frac{1}{2}\left(\rho^2 - \delta_{ab}X^a X^b\right)^{-1/2}$
$X_a \equiv \delta_{ab}X^b$
$ds^2 = \left(\delta_{ab} + \frac{X_a X_b}{\rho^2 - X^2}\right)$
$\equiv \delta_{ab}X^a X^b$

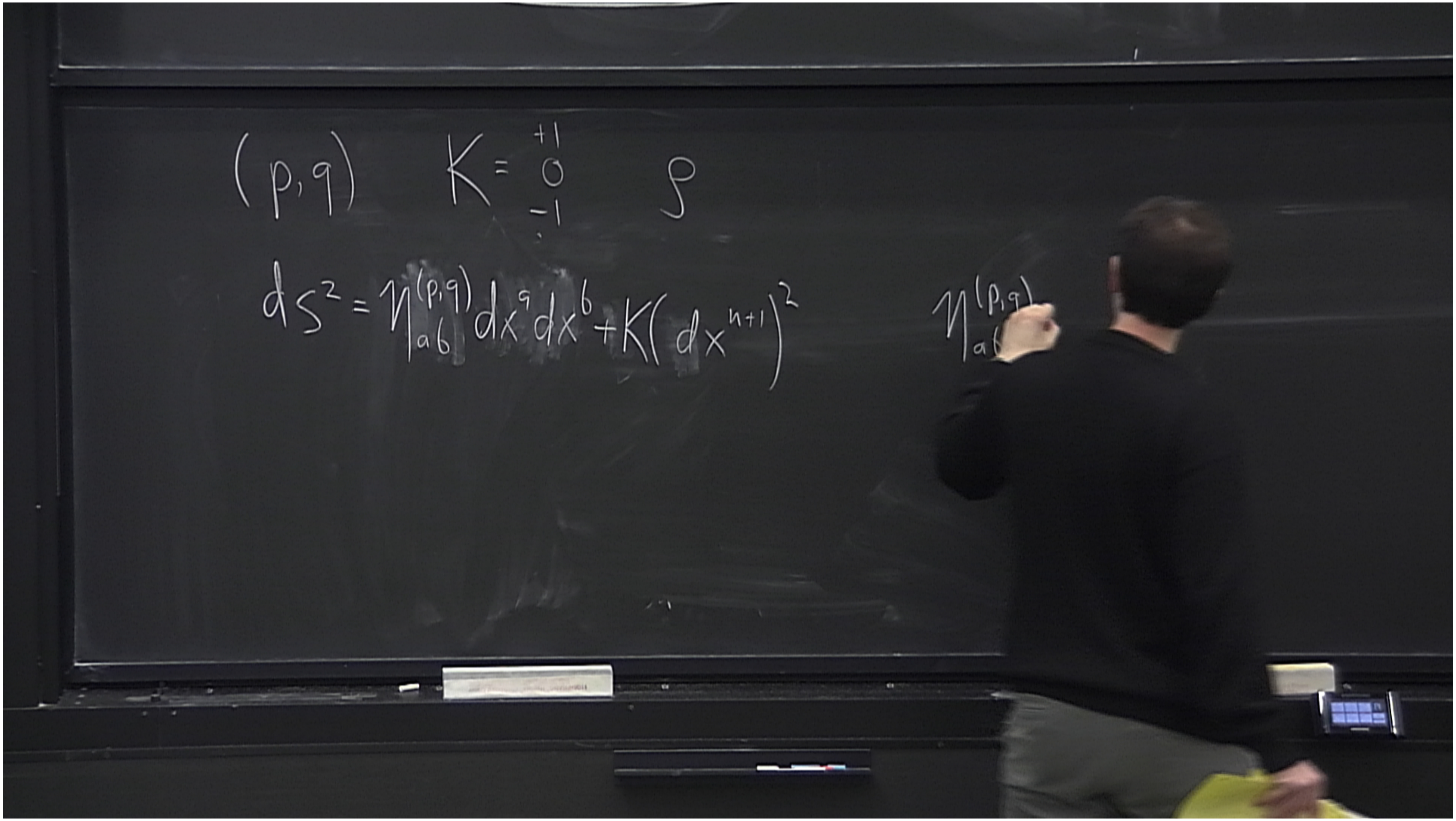

$X^{n+1} = \pm (\rho^2 - \delta_{ab} X^a X^b)^{1/2}$
$dX^{n+1} = \pm \frac{1}{2} (\rho^2 - \delta_{ab} X^a X^b)^{-1/2}$
$\delta_{ab} X^a dX^b)$
$ds^2 = \left(\delta_{ab} + \frac{X_a X_b}{\rho^2 - X^2} \right)$
$X_a \equiv \delta_{ab} X^b$
$X^2 \equiv \delta_{ab} X^a X^b$

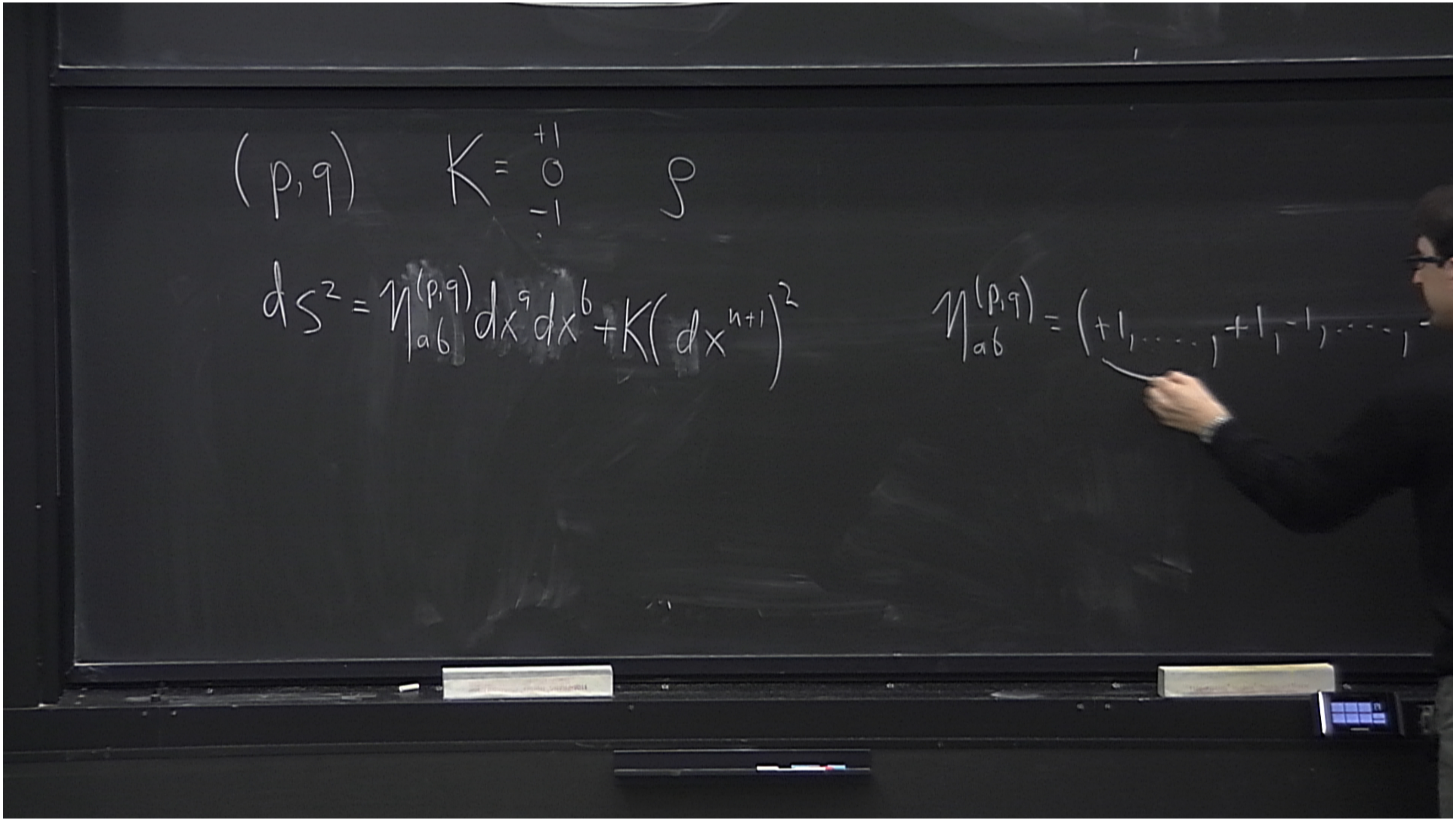

x¹ = ρ cos θ
ds² = ρ² dθ²
ρ
x⁶

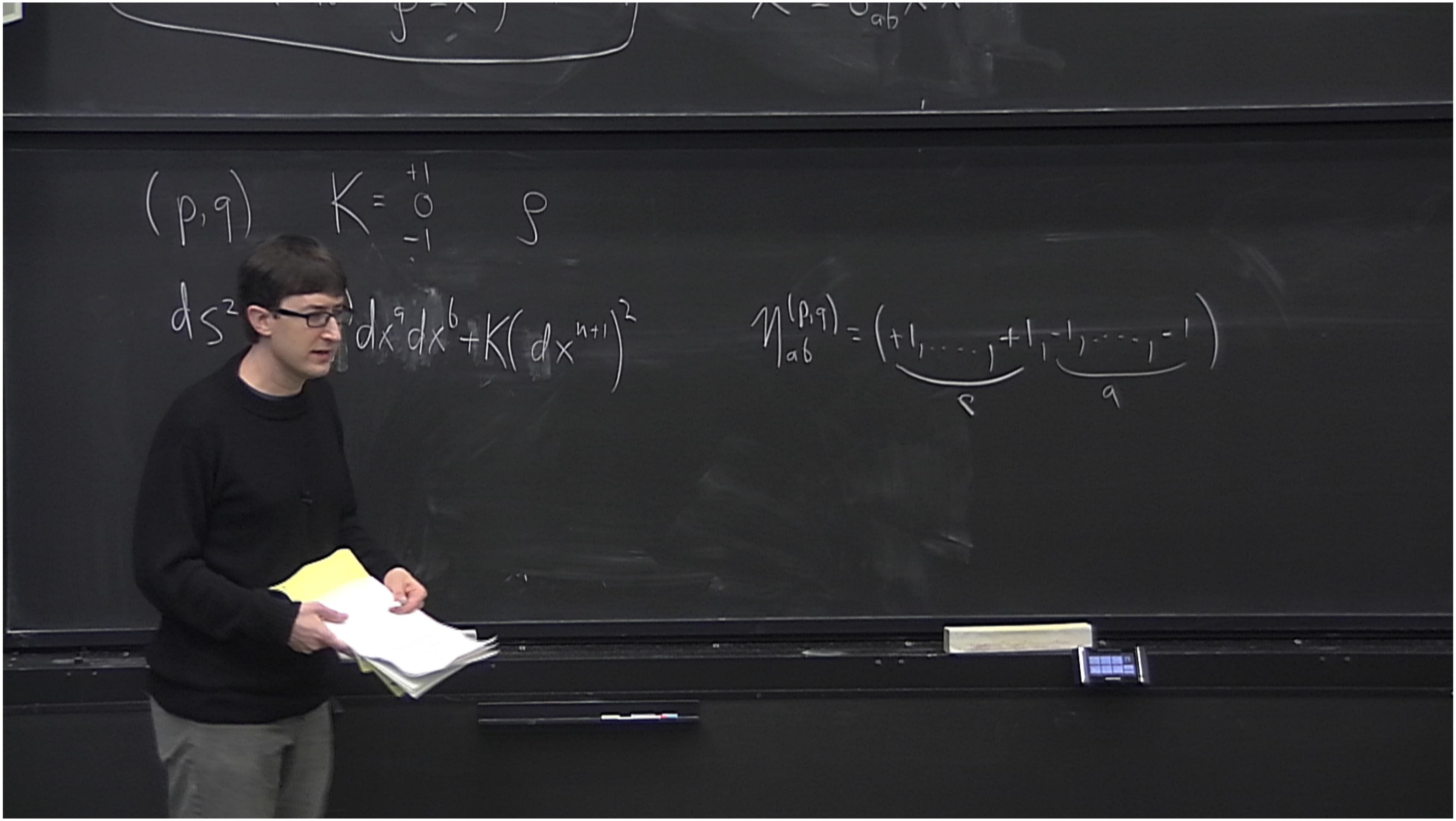

$x^1 = \rho\cos\theta$
$ds^2 = \rho^2 d\theta^2$
$x^1 = \rho\sin\theta\cos\varphi$
$x^2 = \rho\sin\theta\sin\varphi$
$ds^2 = \rho^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$

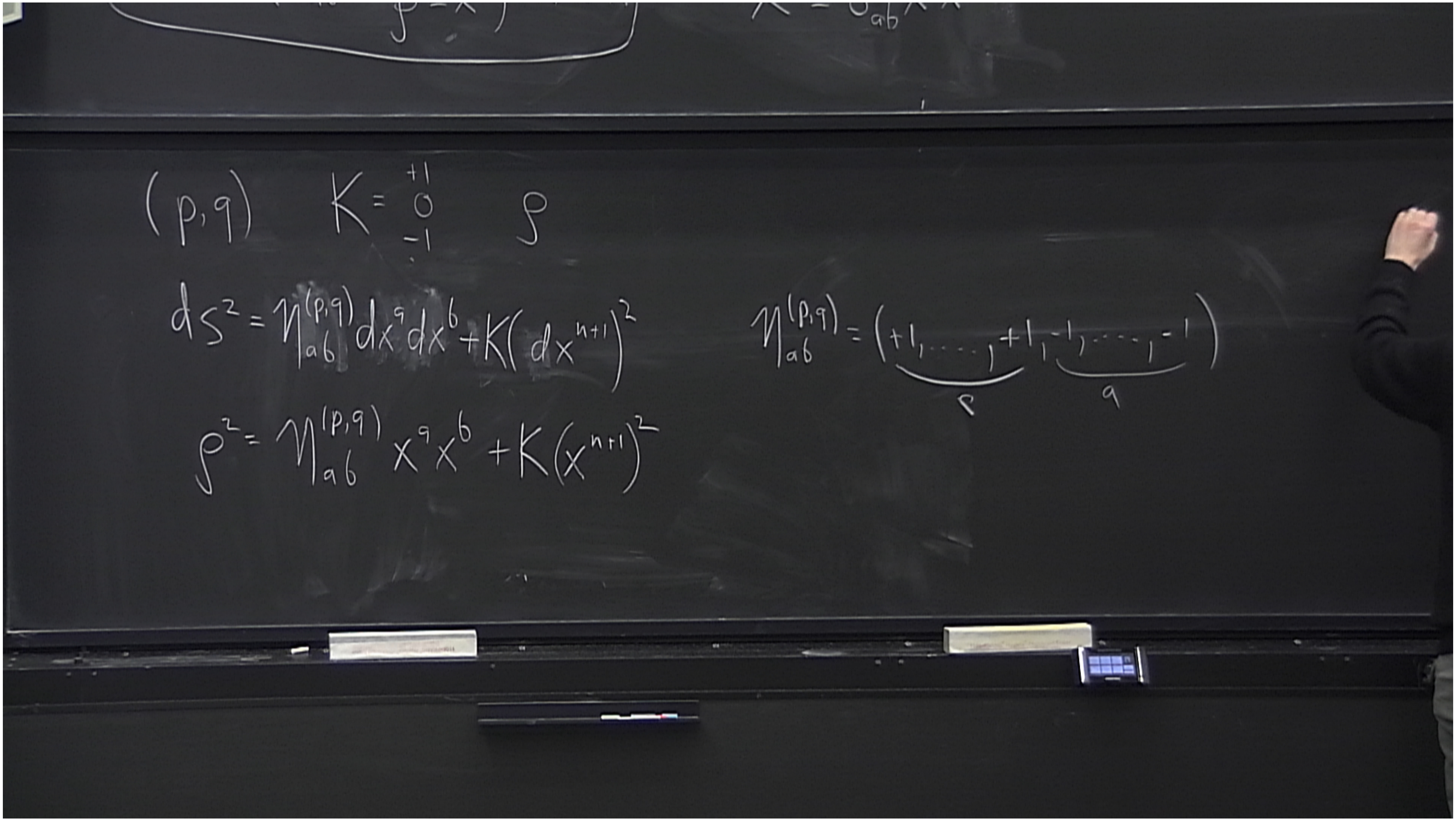
$(p,q) \qquad K = \begin{matrix} +1 \\ 0 \\ -1 \end{matrix} \qquad \rho$

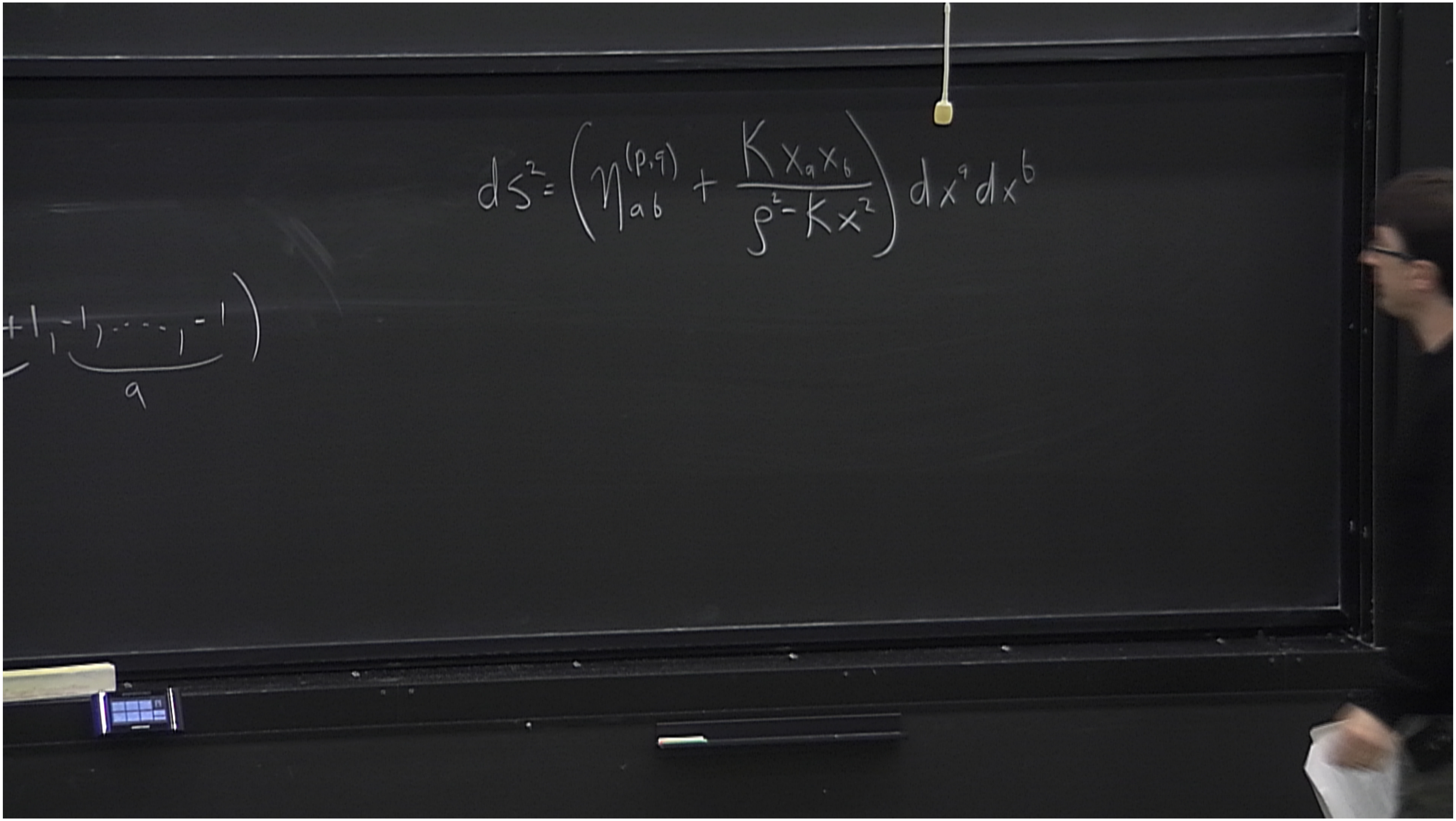
$$ds^2 = \delta_{ab}\, dx^a dx^b + (dx^{n+1})^2$$

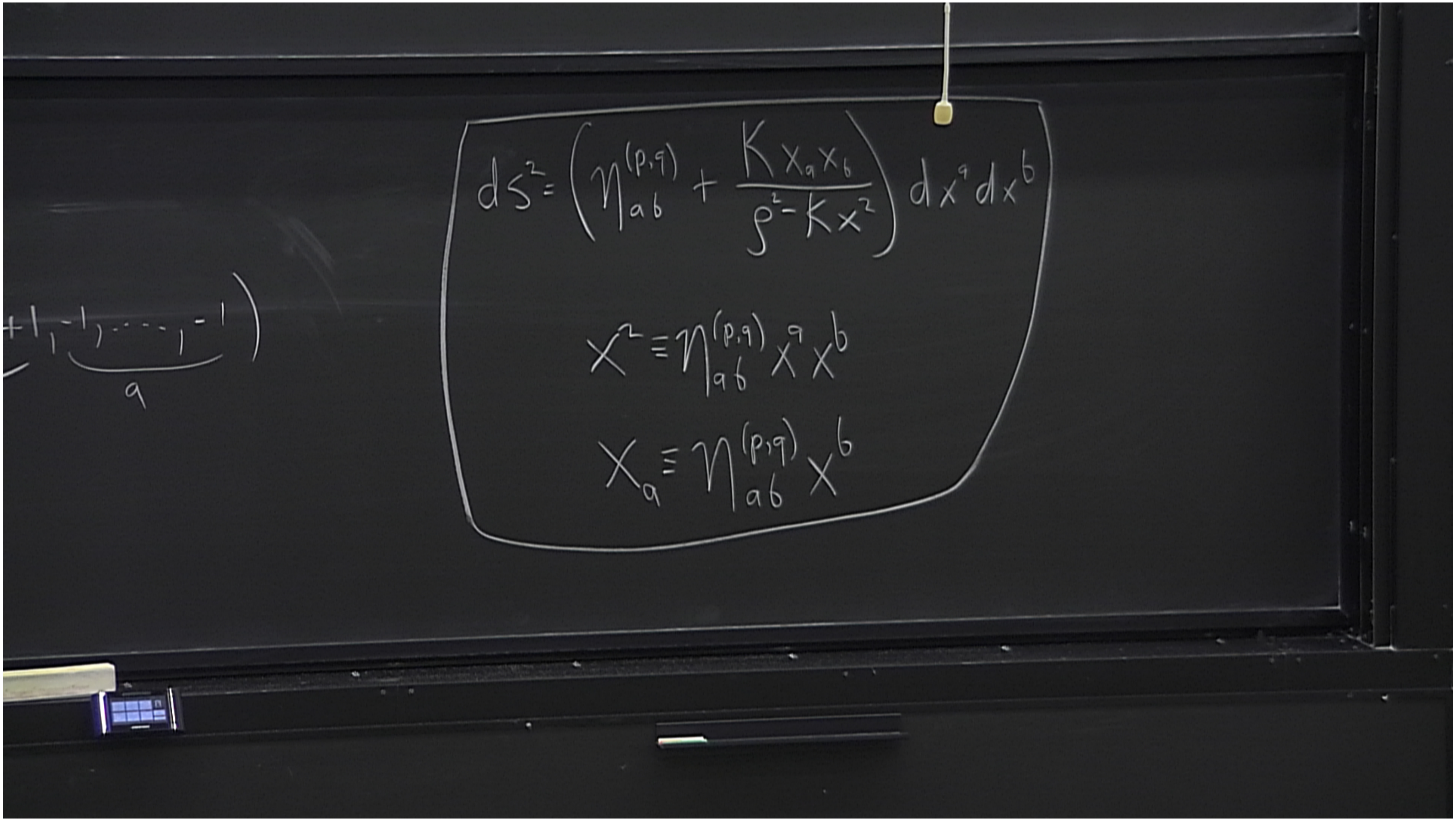

(p,q)
K = +1 0 −1
ρ
ds² = η^(p,q)_ab dx^a dx^b + K(dx^(n+1))²
η^(p,q)_ab


(p, q)
K = +1, 0, -1
ρ
ds² = η_ab^(p,q) dx^a dx^b + K(dx^(n+1))²
η_ab^(p,q) = (+1, ... , +1, -1, ... ,

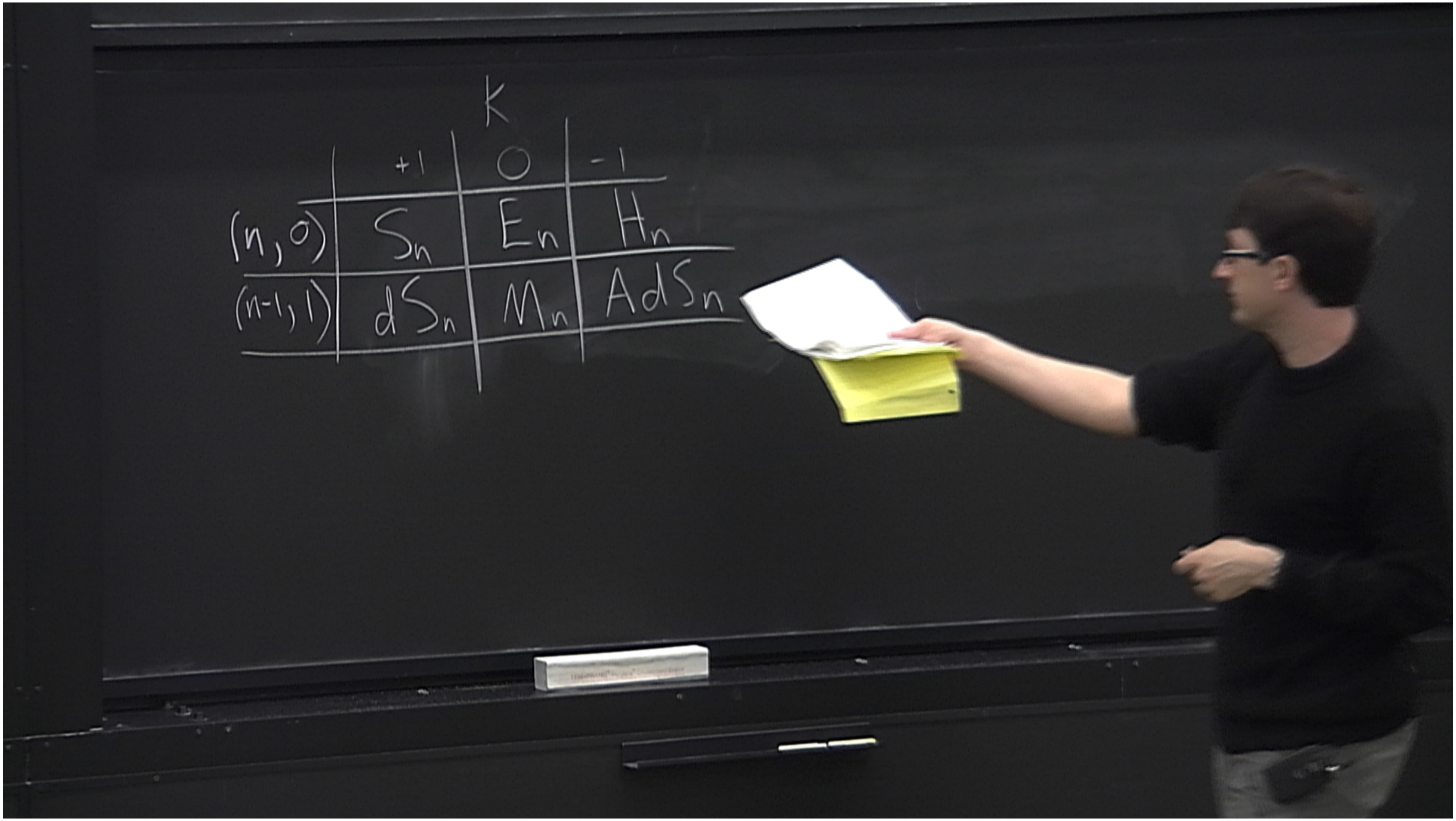

(p,q) K = +1, 0, -1 ρ
dx^a dx^b + K(dx^{n+1})^2
η_ab^(p,q) = (+1, . . . , +1, -1, . . . , -1)
p
q

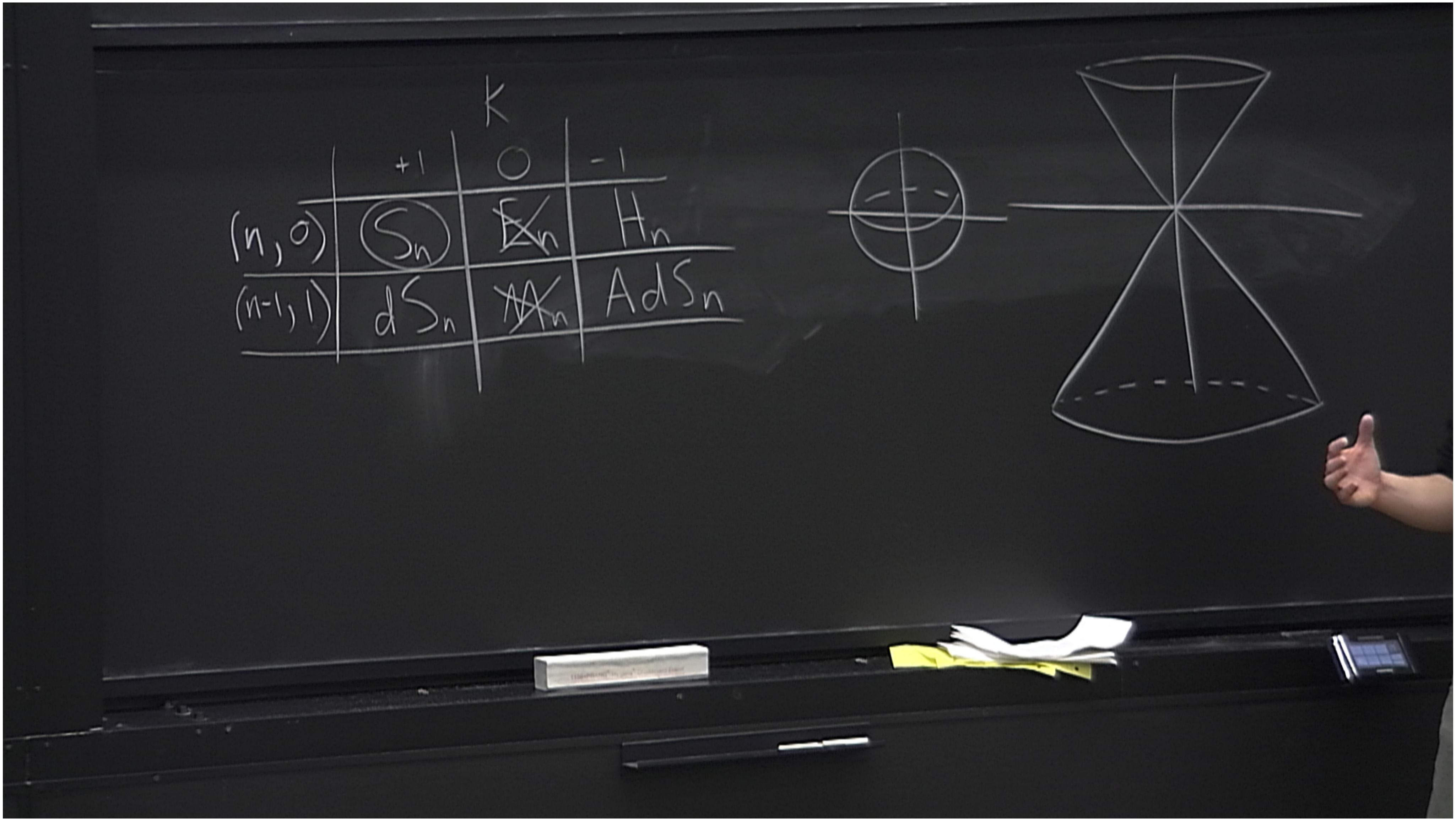

$(p,q) \quad K = \begin{matrix} +1 \\ 0 \\ -1 \end{matrix} \quad \rho$
$ds^2 = \eta^{(p,q)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$\rho^2 = \eta^{(p,q)}_{ab} x^a x^b + K(x^{n+1})^2$
$\eta^{(p,q)}_{ab} = (\underbrace{+1, \dots, +1}_{p}, \underbrace{-1, \dots, -1}_{q})$

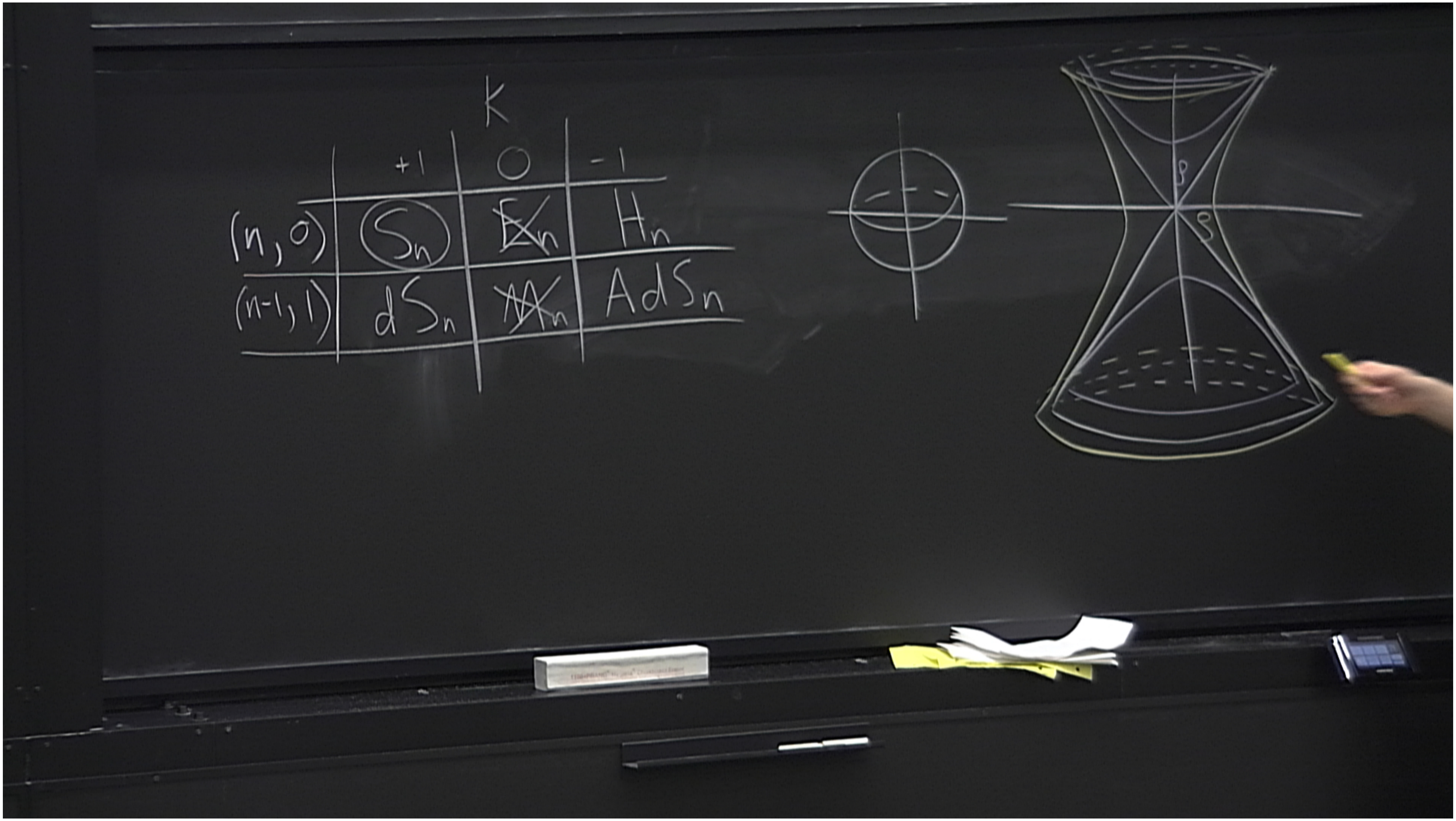

$ds^2 = \left(\eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2} \right) dx^a dx^b$
$+1, \underbrace{-1, \ldots, -1}_{q})$

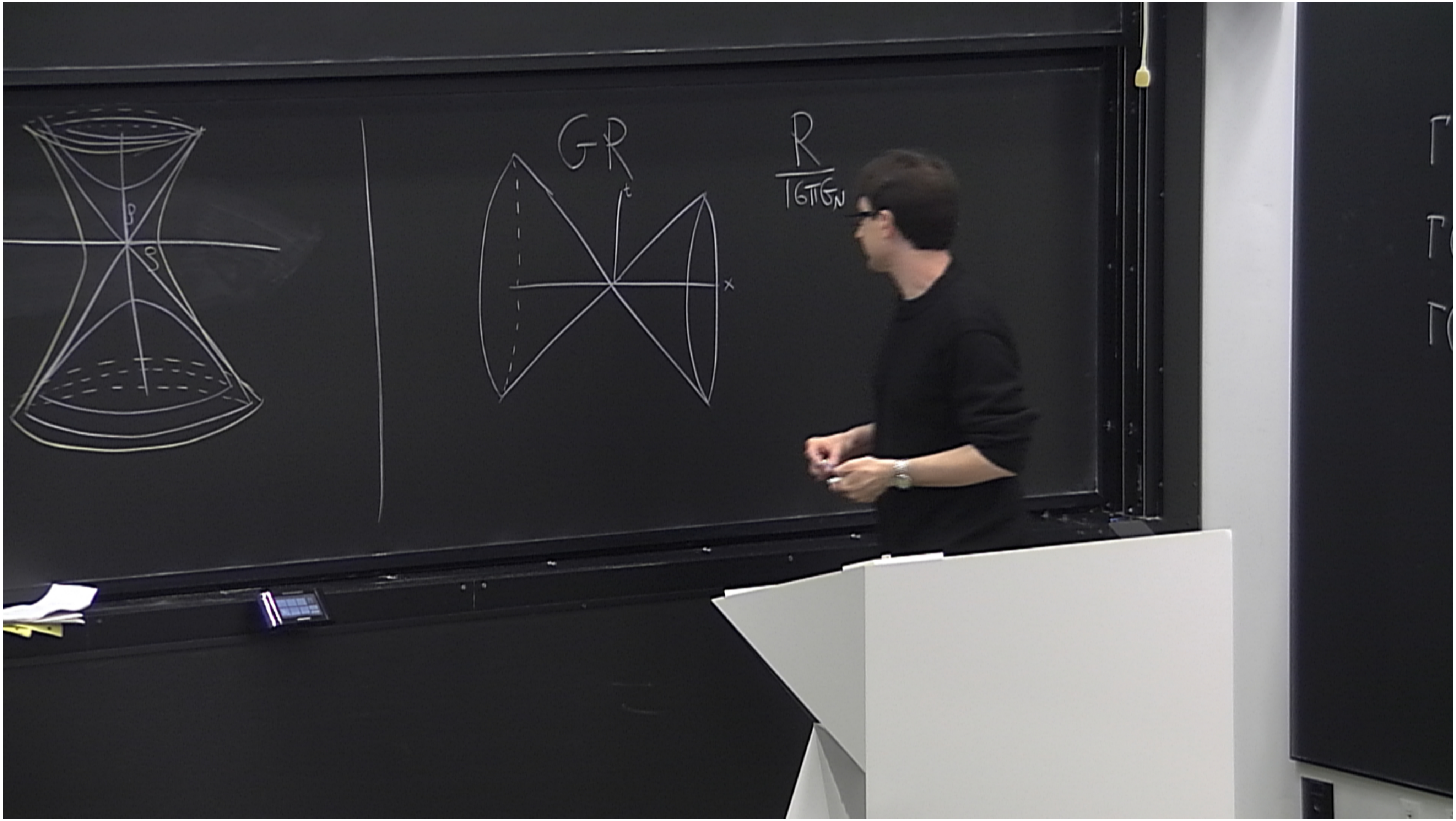


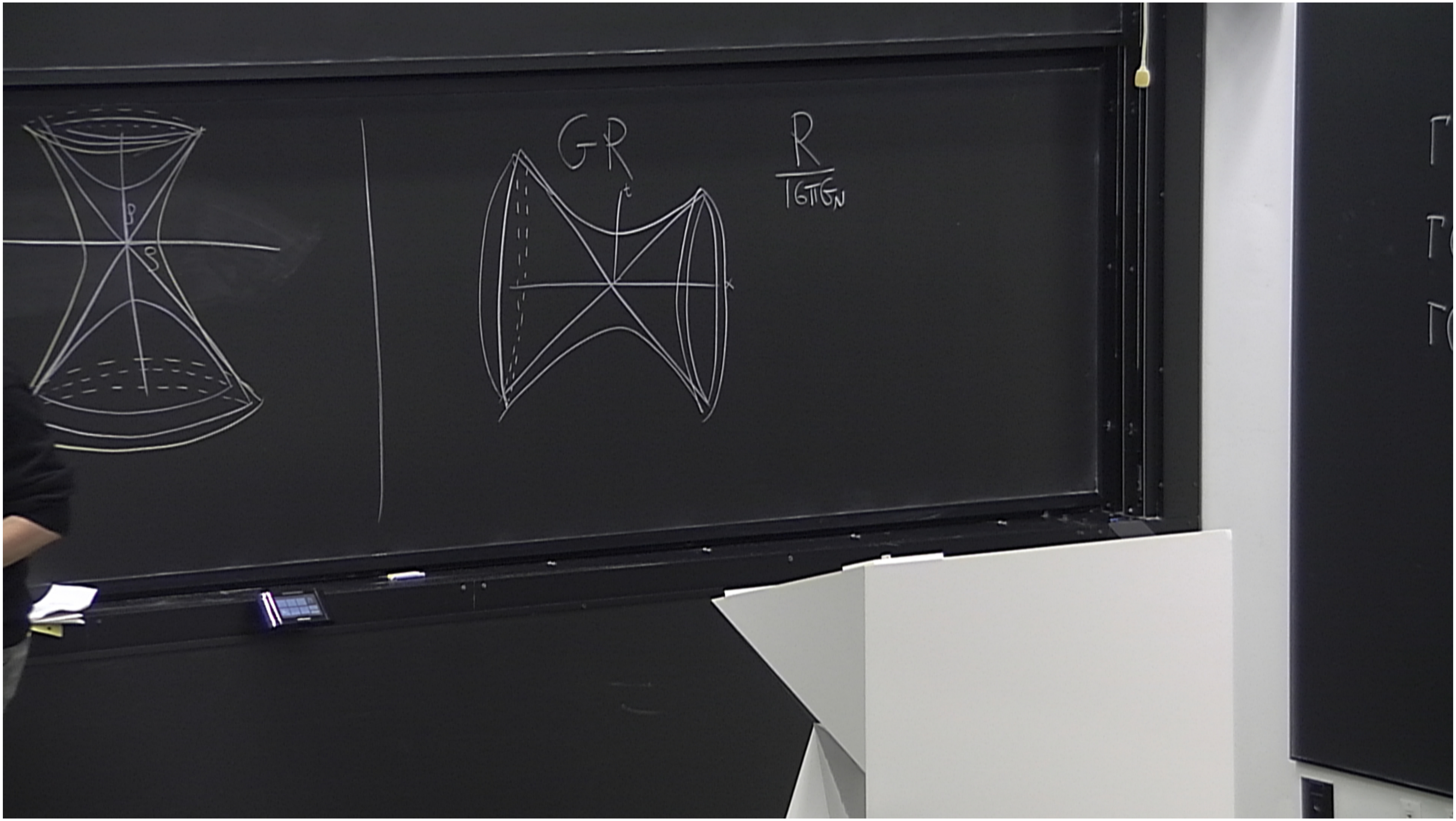
$ds^2 = \left(\eta^{(p,q)}_{ab} + \frac{K x_a x_b}{\rho^2 - K x^2} \right) dx^a dx^b$
$x^2 \equiv \eta^{(p,q)}_{ab} x^a x^b$
$x_a \equiv \eta^{(p,q)}_{ab} x^b$

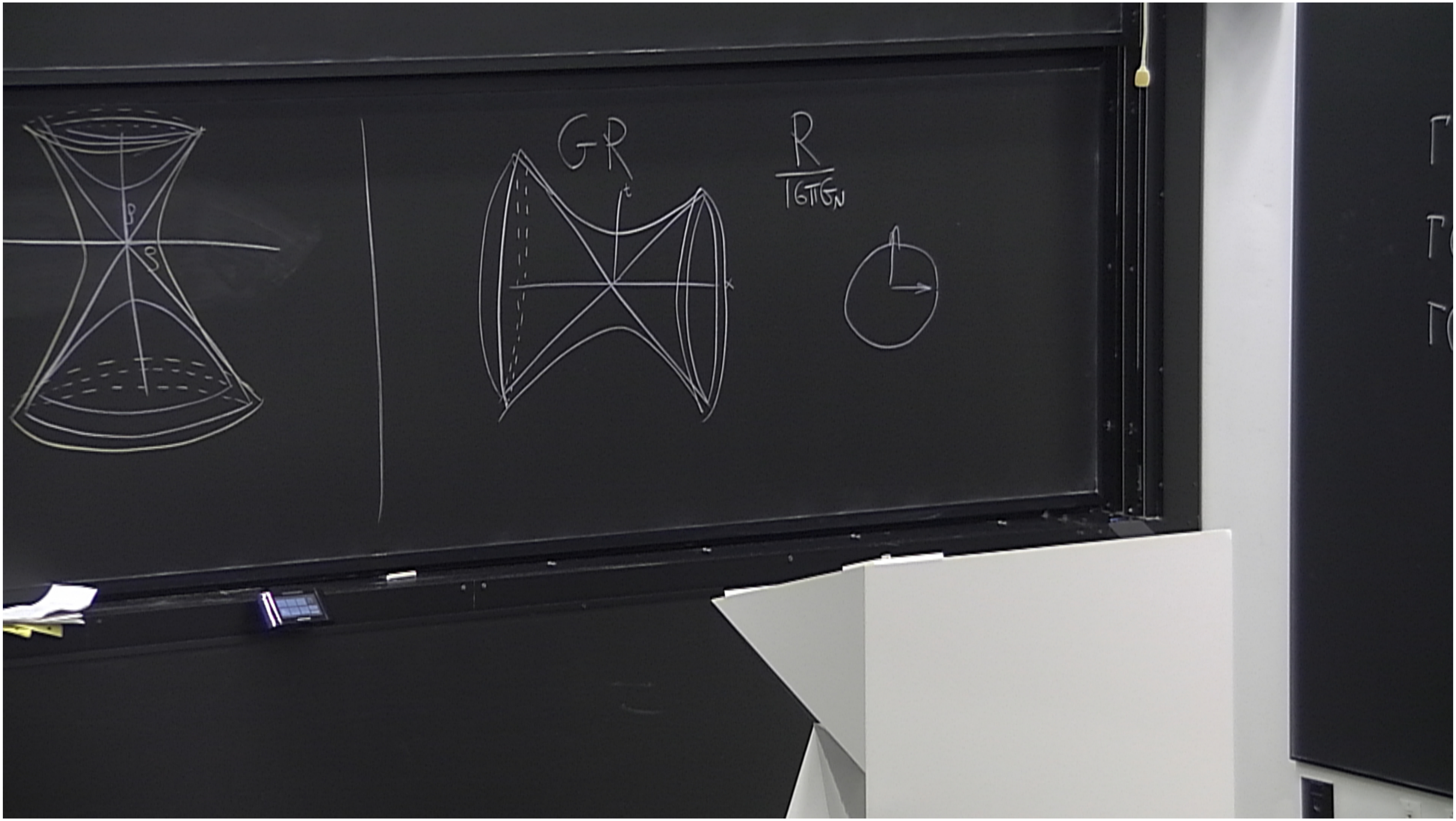

K
+1
0
-1
(n,0)
S_n
E_n
H_n
(n-1,1)
dS_n
M_n
AdS_n

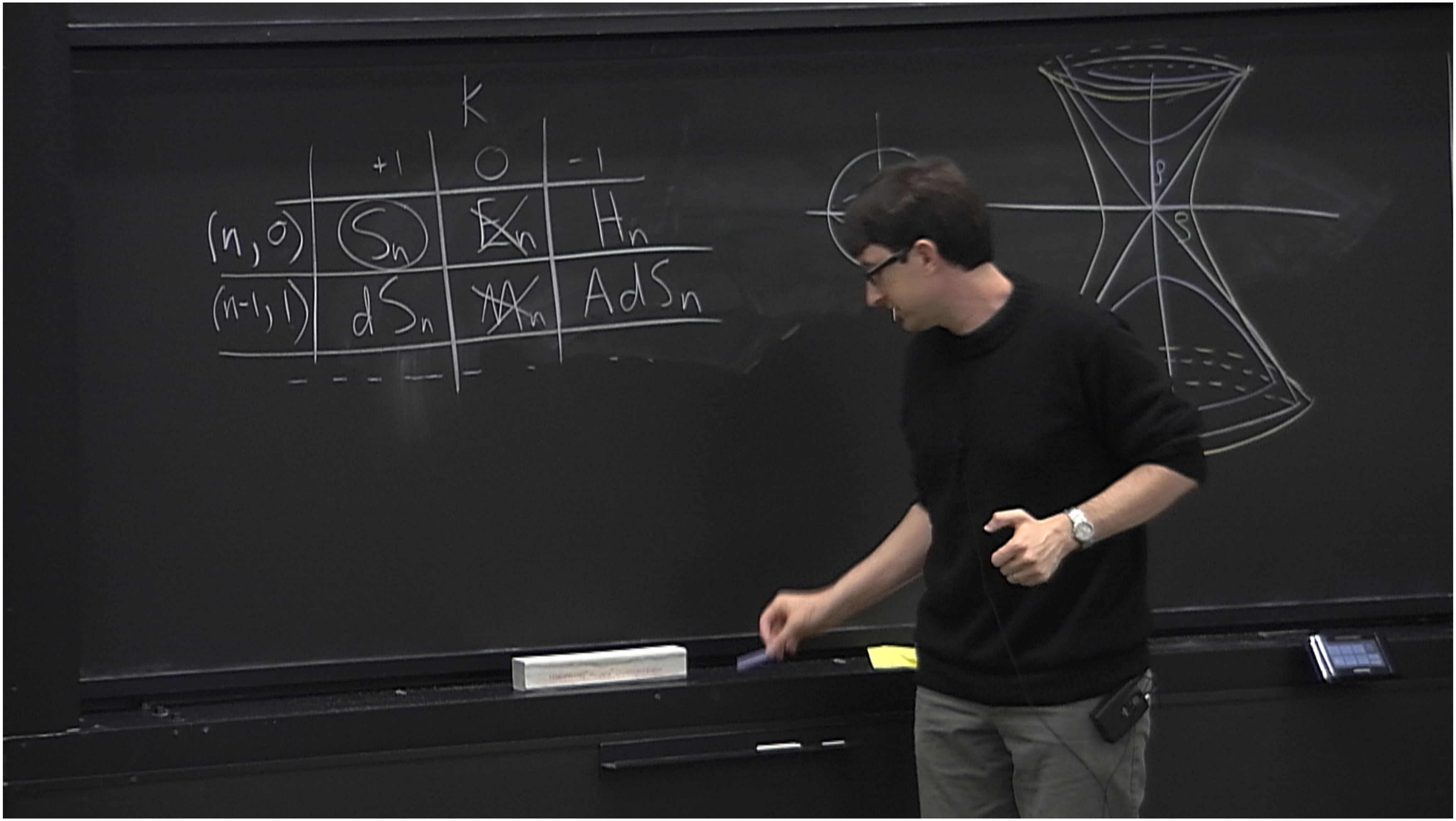

K
+1
0
-1
(n,0)
(n-1,1)
S_n
E_n
H_n
dS_n
AdS_n


K
+1
0
-1
(n, 0)
Sn
En
Hn
(n-1, 1)
dSn
Mn
AdSn


GR
t
x
R
16πG_N


GR
R
16πG_N


GR
$\frac{R}{16\pi G_N}$


K
+1
0
-1
(n, 0)
Sn
En
Hn
(n-1, 1)
dSn
Mn
AdSn