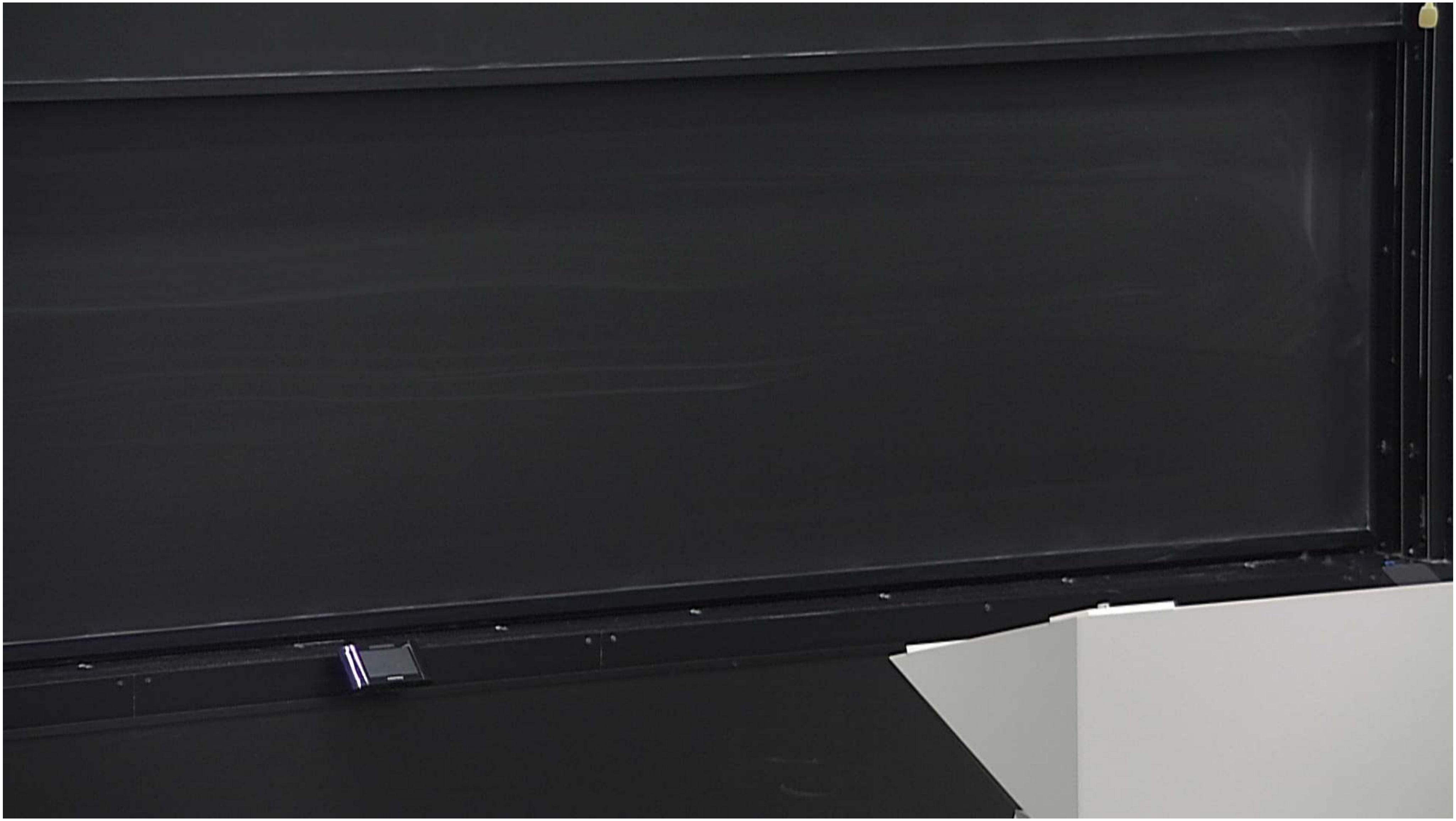
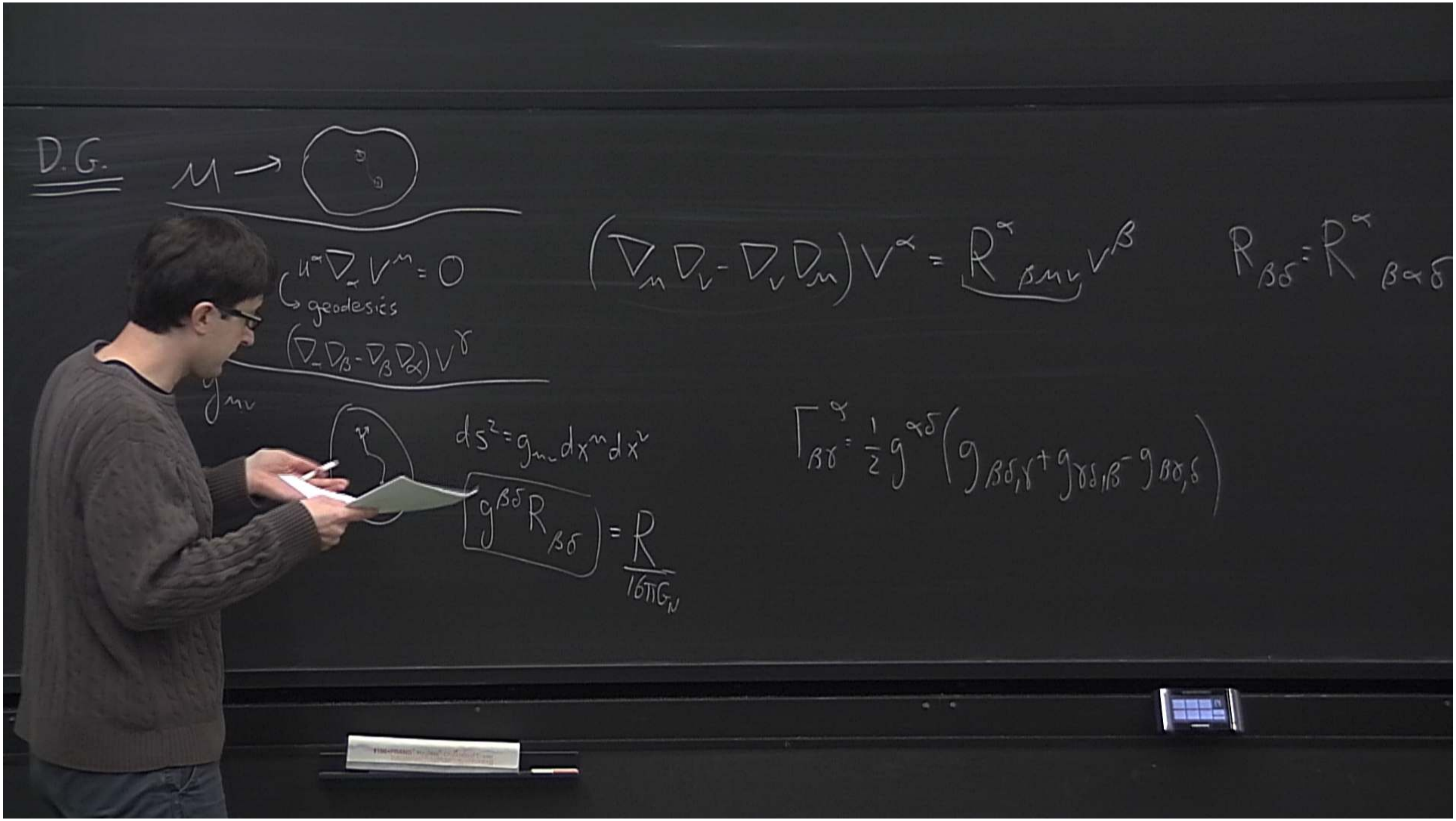
Title: 12/13 PSI - Cosmology Review Lecture 2

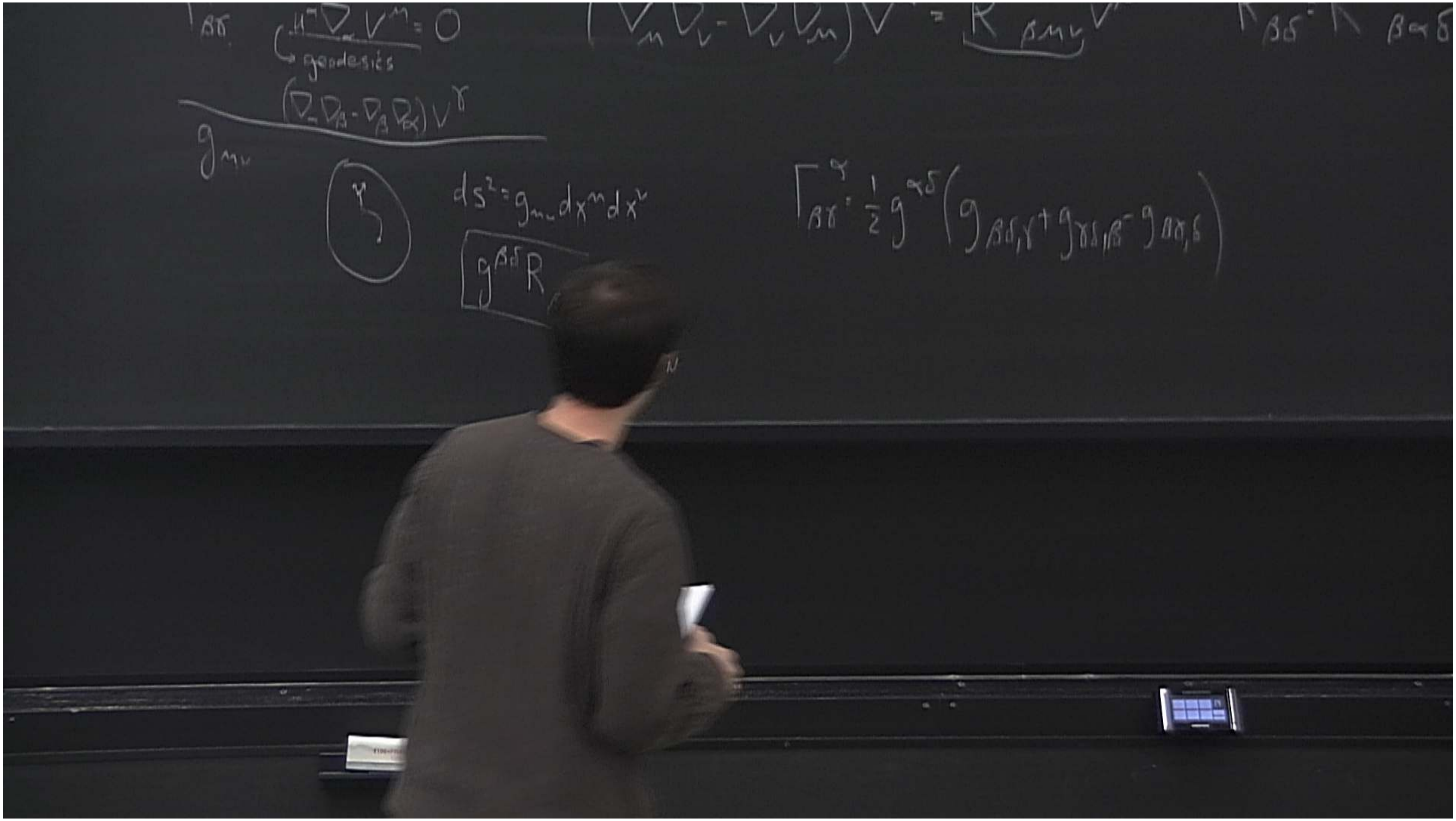
Date: Feb 20, 2013 11:30 AM

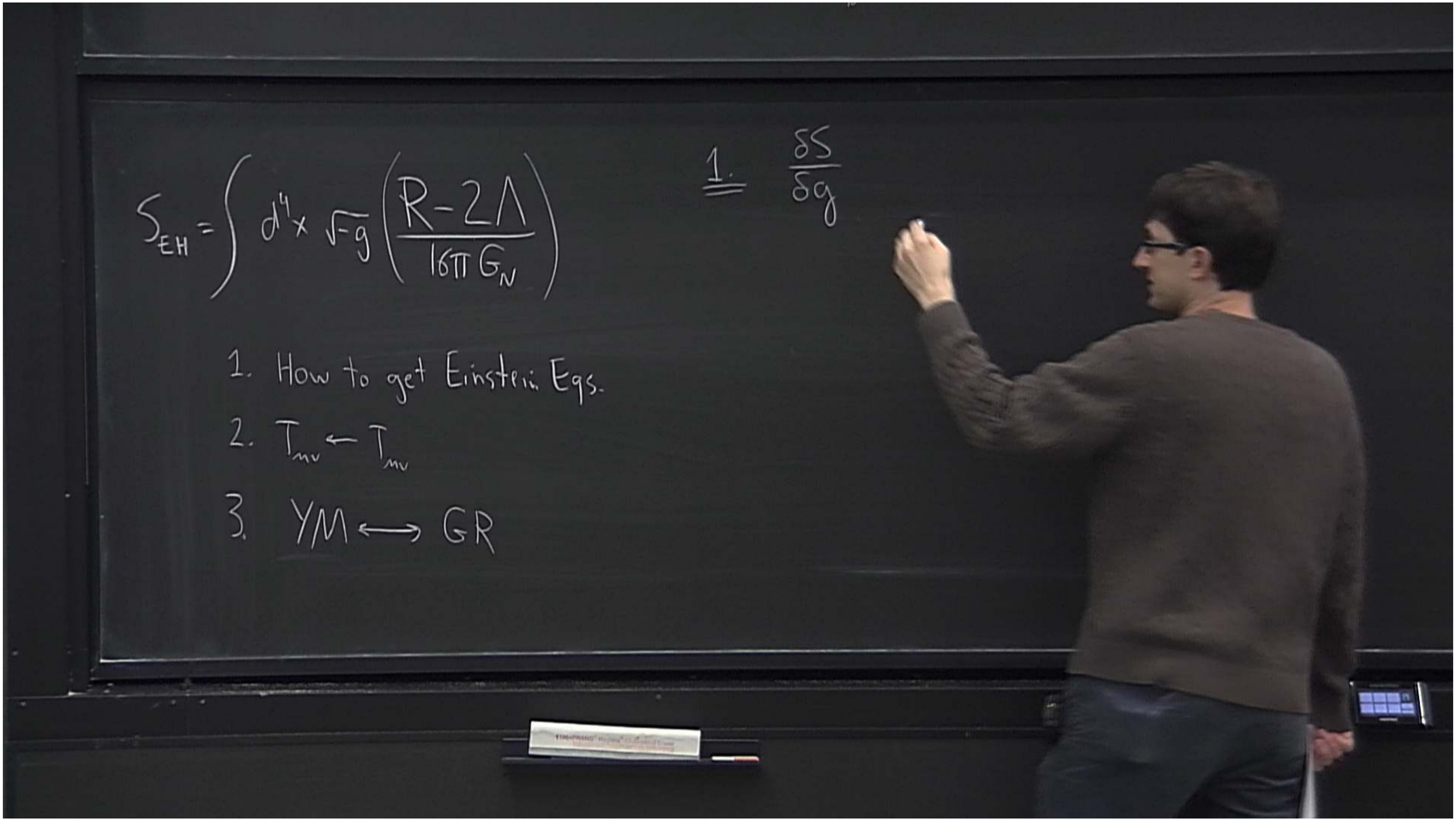
URL: http://pirsa.org/13020100

Abstract:




D.G.
M →
u^α ∇_α V^μ = 0
↳ geodesics
(∇_α ∇_β - ∇_β ∇_α) V^γ
g_μν
ds² = g_μν dx^μ dx^ν
g^βδ R_βδ = R/16πG_N
(∇_μ ∇_ν - ∇_ν ∇_μ) V^α = R^α_βμν V^β
R_βδ = R^α_βαδ
Γ^α_βγ = 1/2 g^αδ (g_βδ,γ + g_γδ,β - g_βγ,δ)


geodesics
ds² = g_μν dx^μ dx^ν
Γ^α_βγ = ½ g^αδ (g_βδ,γ + g_γδ,β − g_βγ,δ)


$S_{EH} = \int d^4x \sqrt{-g} \left(\frac{R - 2\Lambda}{16\pi G_N} \right)$
1. How to get Einstein Eqs.
2. $T_{\mu\nu} \leftarrow T_{\mu\nu}$
3. YM $\longleftrightarrow$ GR
1. $\frac{\delta S}{\delta g}$

$$S_{EH} = \int d^4x \sqrt{-g} \left(\frac{R - 2\Lambda}{16\pi G_N} \right)$$

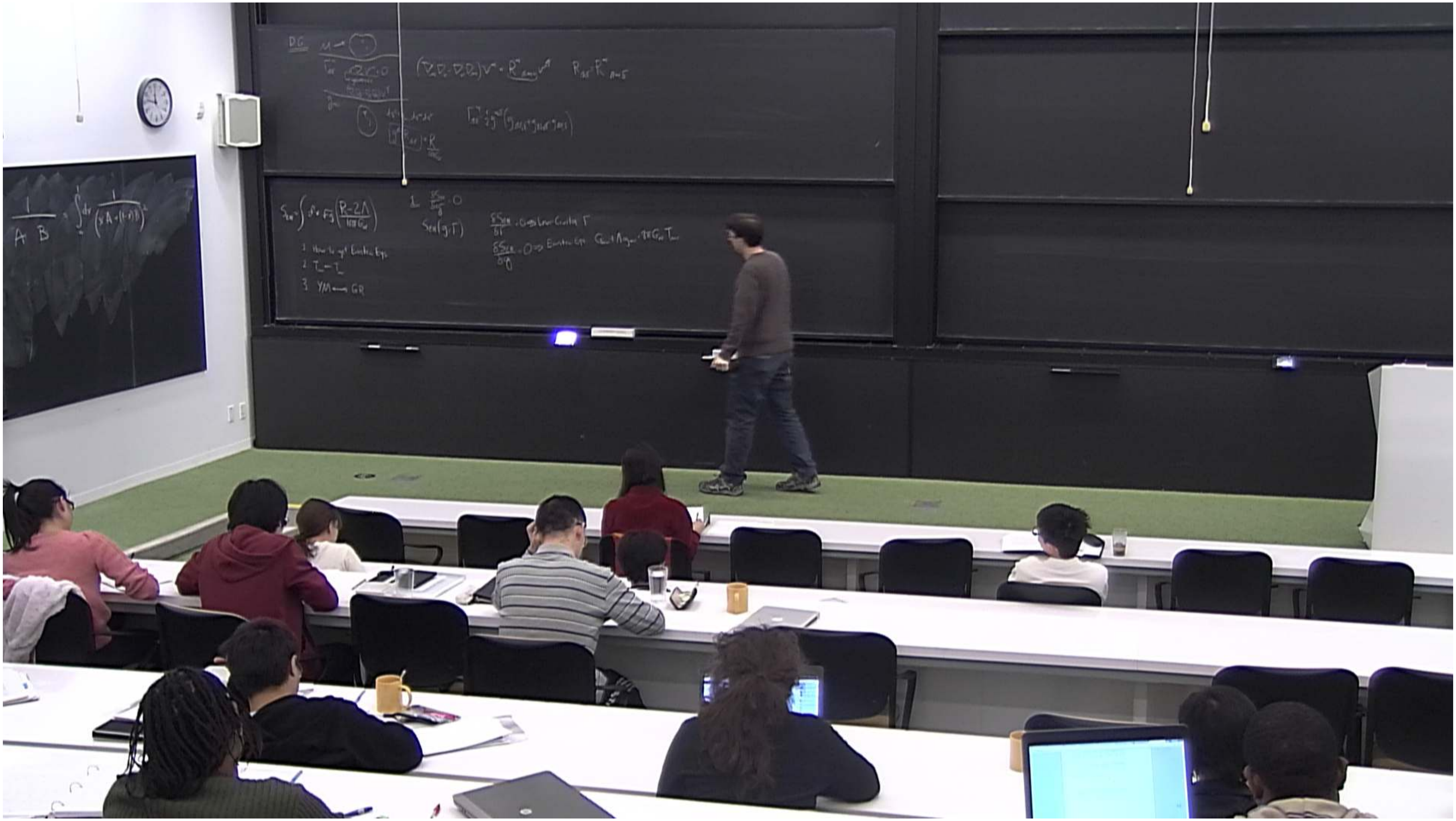
1. How to get Einstein Eqs.
2. $T_{\mu\nu} \leftarrow T_{\mu\nu}$
3. $YM \longleftrightarrow GR$

$\underline{\underline{1.}}$ $$\frac{\delta S_{EH}}{\delta g^{\mu\nu}} = 0$$

$$S_{EH} = \int d^4x \sqrt{-g} \left(\frac{R - 2\Lambda}{16\pi G_N} \right)$$

1. How to get Einstein Eqs.
2. $T_{\mu\nu} \leftarrow T_{\mu\nu}$
3. $YM \longleftrightarrow GR$

1. $\frac{\delta S_{EH}}{\delta g^{\mu\nu}} = 0$

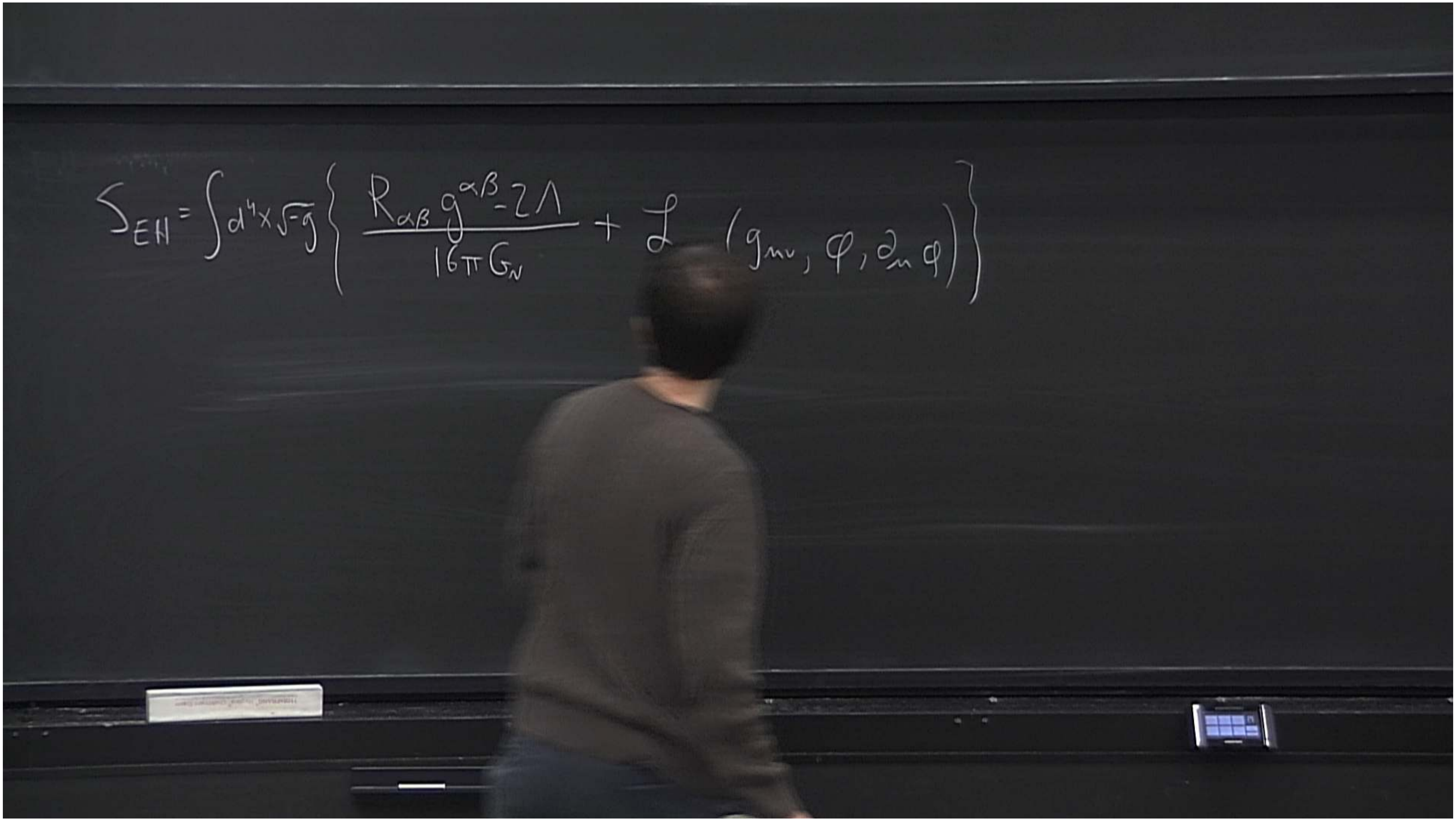


1. $\dfrac{\delta S_{EH}}{\delta g^{\mu\nu}} = 0$

$S_{EH}(g, \Gamma)$

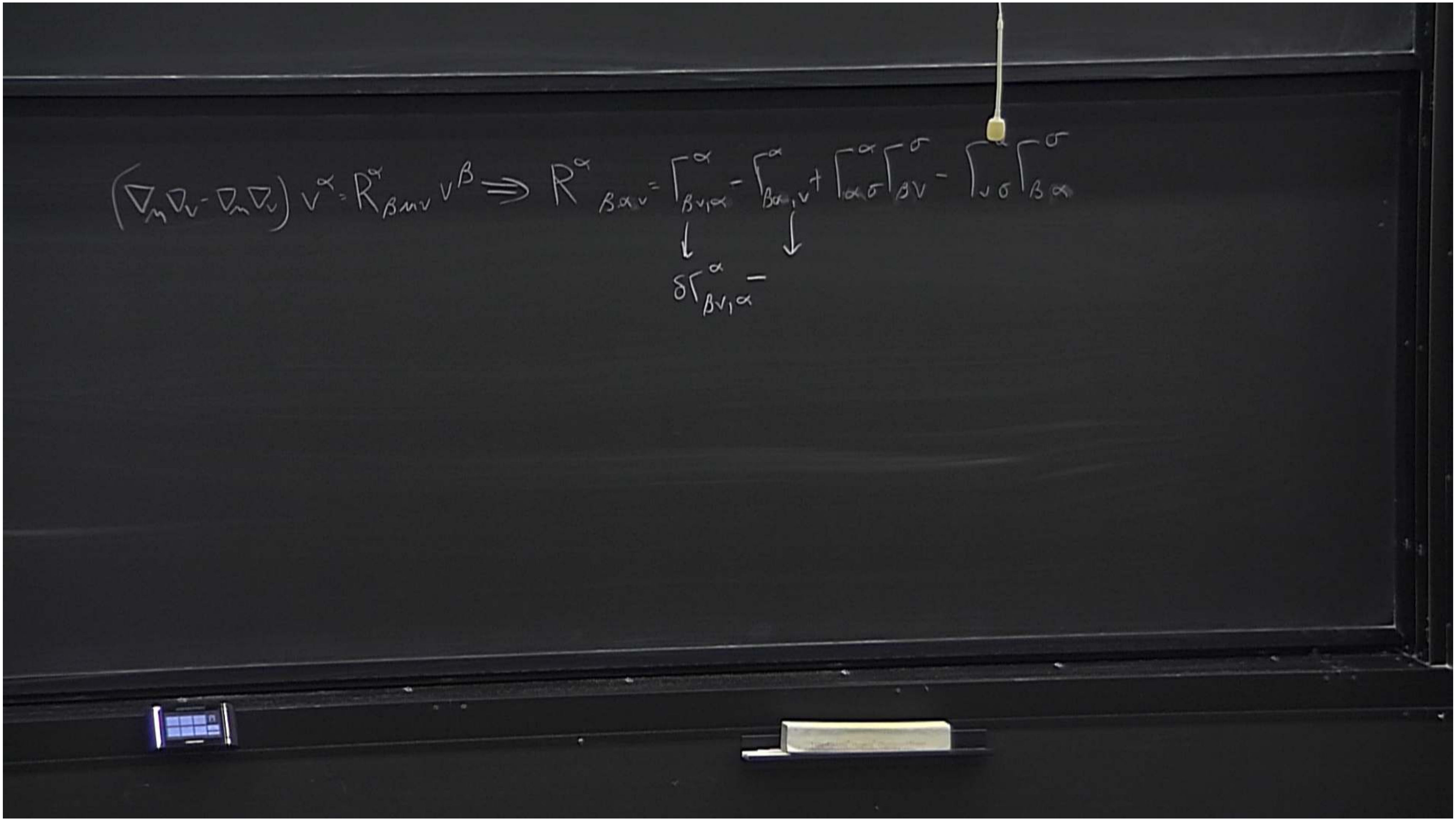
$\dfrac{\delta S_{EH}}{\delta \Gamma} = 0 \Rightarrow$ Levi-Civita Γ

$\dfrac{\delta S_{EH}}{\delta g} = 0 \Rightarrow$ Einstein Eqs $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$

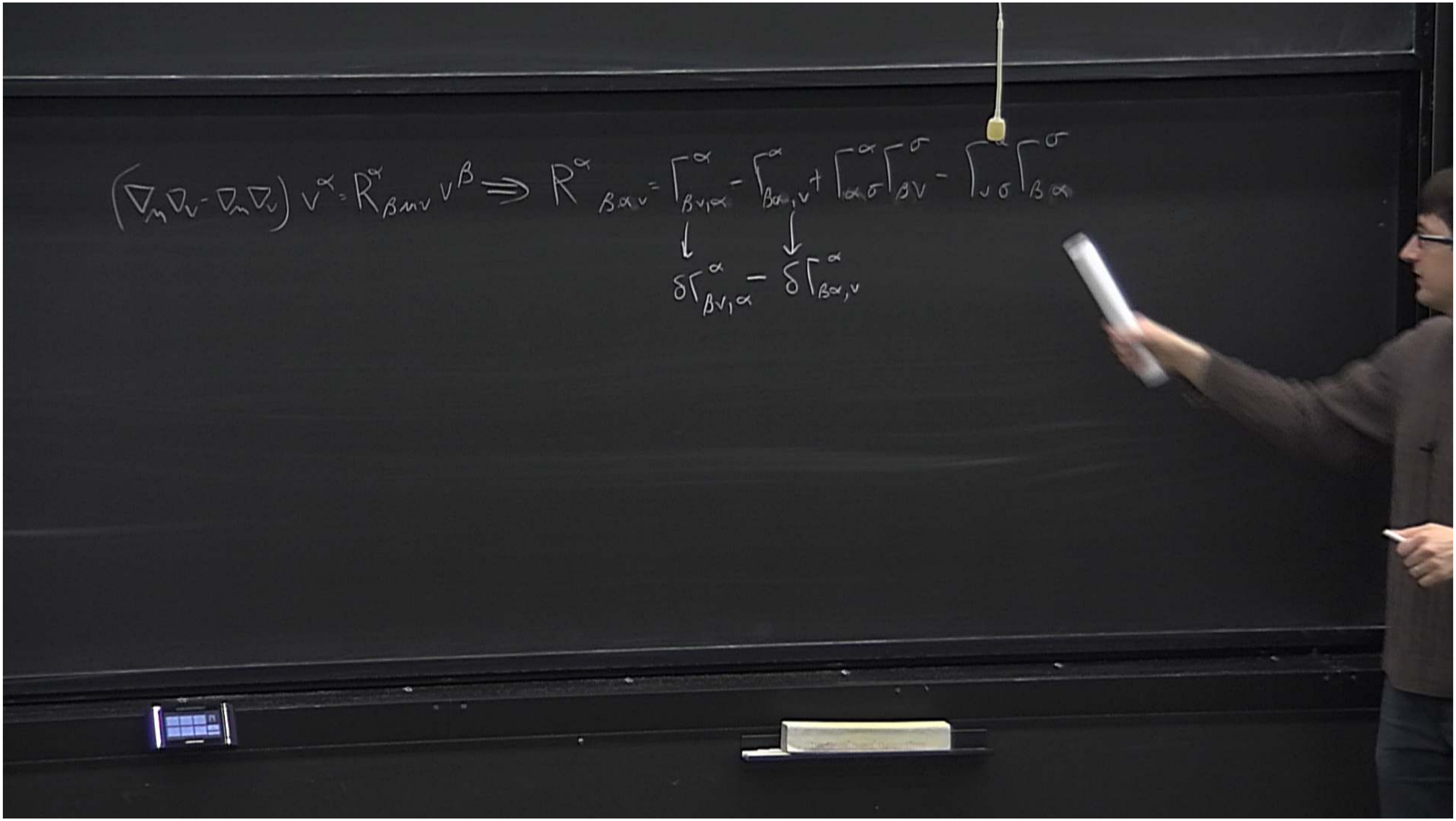

$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{R_{\alpha\beta} g^{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}(g_{\mu\nu}, \varphi, \partial_\mu \varphi) \right\}$

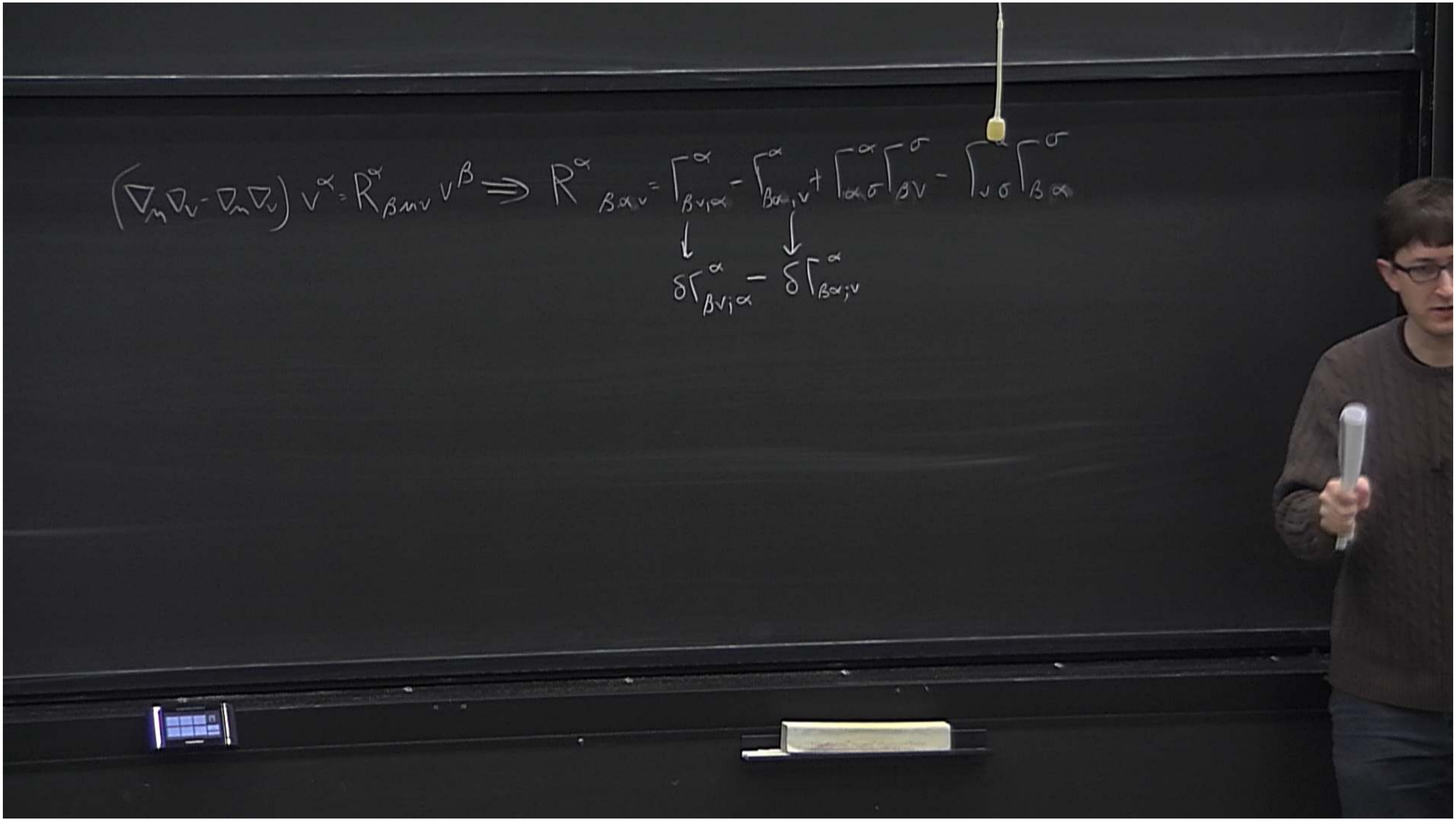
$$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{R_{\alpha\beta}\, g^{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m\left(g_{\mu\nu}, \varphi, \partial_\mu \varphi\right) \right\}$$

$$R_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$



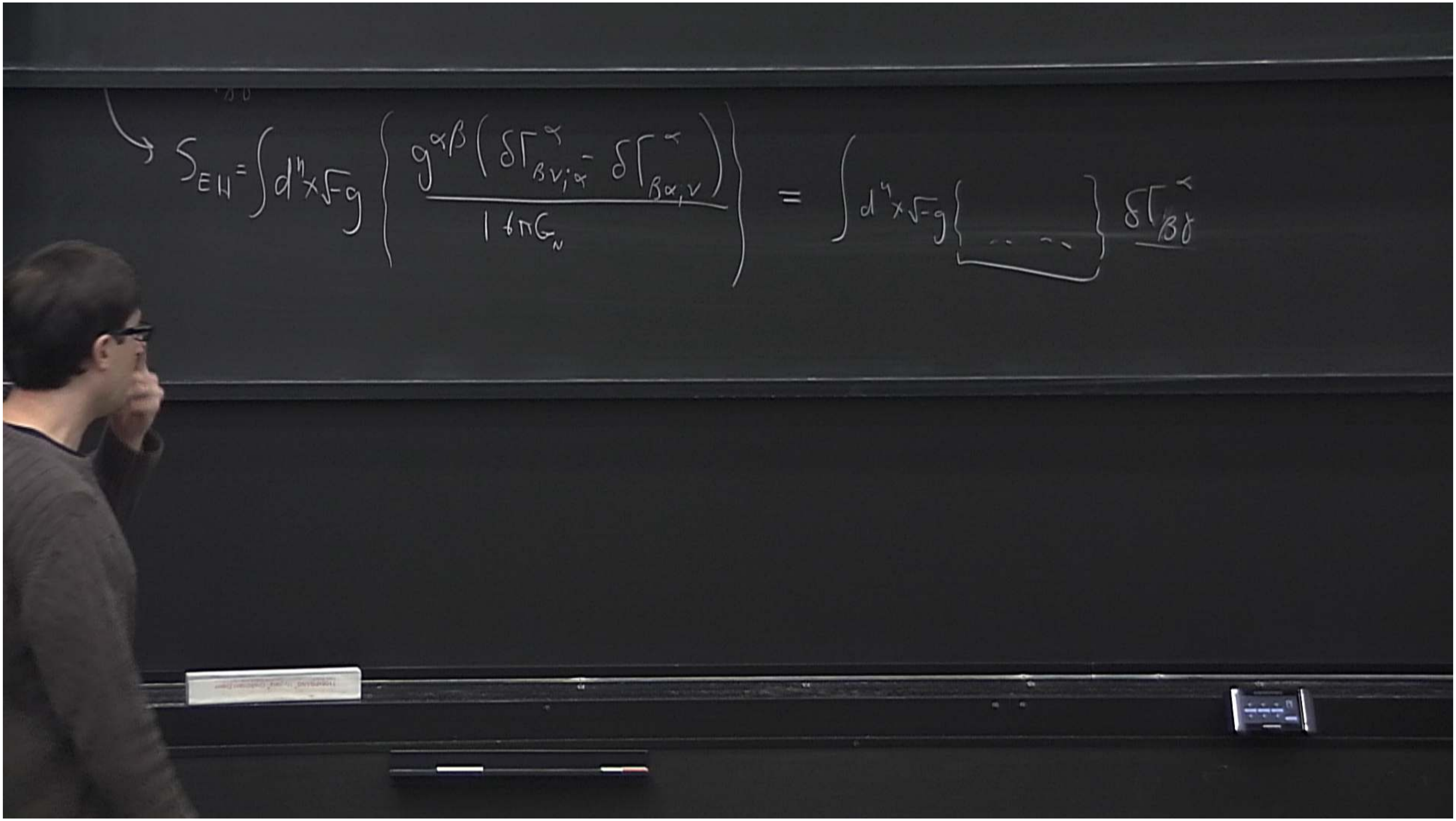
$(\nabla_\mu\nabla_\nu - \nabla_\mu\nabla_\nu) V^\alpha = R^\alpha{}_{\beta\mu\nu} V^\beta \Rightarrow R^\alpha{}_{\beta\alpha\nu} = \Gamma^\alpha_{\beta\nu,\alpha} - \Gamma^\alpha_{\beta\alpha,\nu} + \Gamma^\alpha_{\alpha\sigma}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\nu\sigma}\Gamma^\sigma_{\beta\alpha}$
$\delta\Gamma^\alpha_{\beta\nu,\alpha} -$

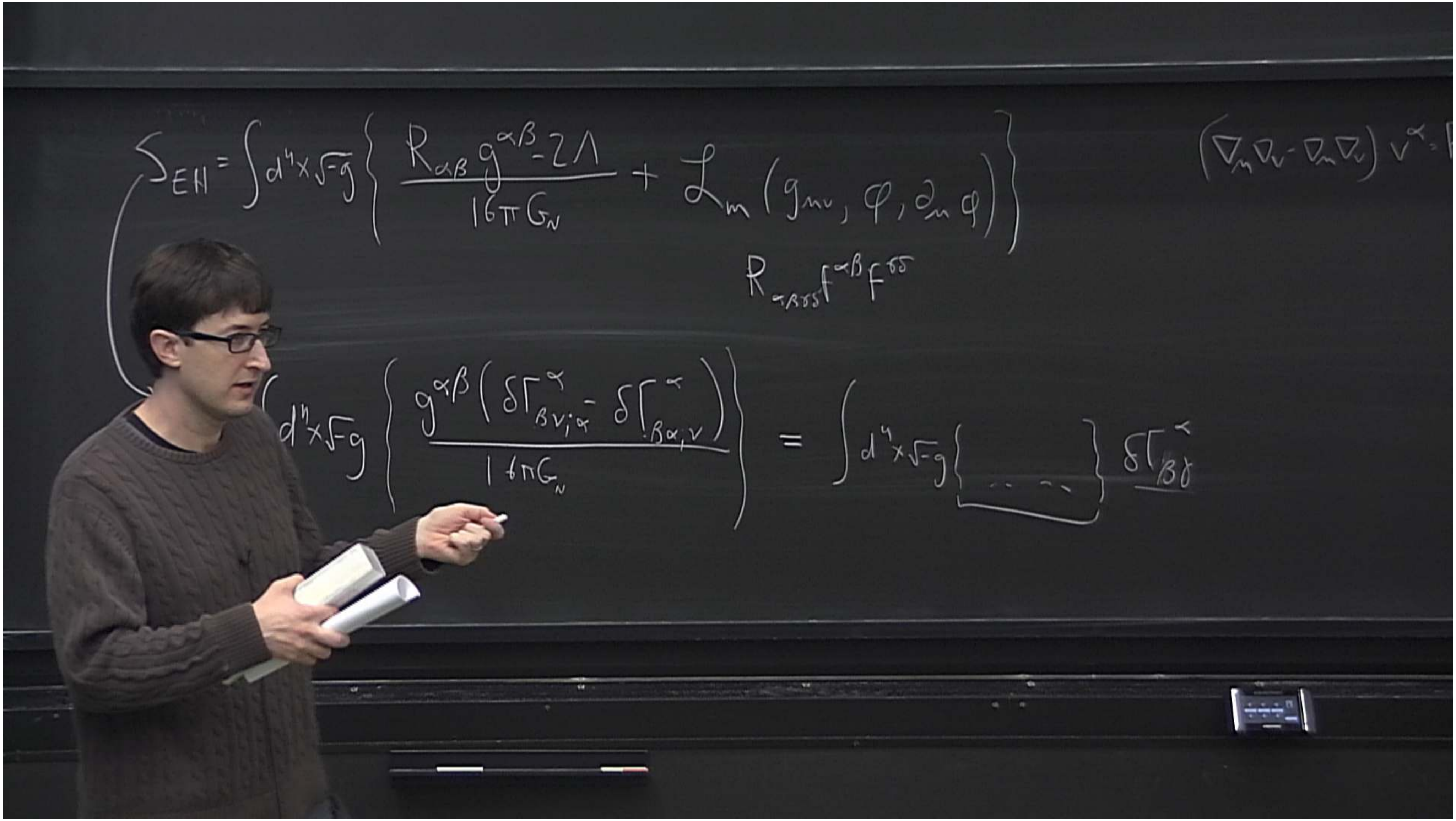

$(\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu) V^\alpha = R^\alpha{}_{\beta\mu\nu} V^\beta \Rightarrow R^\alpha{}_{\beta\alpha\nu} = \Gamma^\alpha_{\beta\nu,\alpha} - \Gamma^\alpha_{\beta\alpha,\nu} + \Gamma^\alpha_{\alpha\sigma}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\nu\sigma}\Gamma^\sigma_{\beta\alpha}$
$\delta\Gamma^\alpha_{\beta\nu,\alpha} - \delta\Gamma^\alpha_{\beta\alpha,\nu}$


$(\nabla_\mu \nabla_\nu - \nabla_\mu \nabla_\nu) V^\alpha = R^\alpha{}_{\beta\mu\nu} V^\beta \Rightarrow R^\alpha{}_{\beta\alpha\nu} = \Gamma^\alpha_{\beta\nu,\alpha} - \Gamma^\alpha_{\beta\alpha,\nu} + \Gamma^\alpha_{\alpha\sigma}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\nu\sigma}\Gamma^\sigma_{\beta\alpha}$
$\delta\Gamma^\alpha_{\beta\nu;\alpha} - \delta\Gamma^\alpha_{\beta\alpha;\nu}$

$$(\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu)\, V^\alpha = R^\alpha{}_{\beta\mu\nu} V^\beta \Rightarrow R^\alpha{}_{\beta\alpha\nu} = \Gamma^\alpha_{\beta\nu,\alpha} - \Gamma^\alpha_{\beta\alpha,\nu} + \Gamma^\alpha_{\alpha\sigma}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\nu\sigma}\Gamma^\sigma_{\beta\alpha}$$

$$\delta\Gamma^\alpha_{\beta\nu;\alpha} - \delta\Gamma^\alpha_{\beta\alpha;\nu}$$


$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta}\left(\delta\Gamma^{\alpha}_{\beta\nu;\alpha} - \delta\Gamma^{\alpha}_{\beta\alpha;\nu}\right)}{16\pi G_N} \right\} = \int d^4x \sqrt{-g} \{ \ldots \} \delta\Gamma^{\alpha}_{\beta\gamma}$


$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{R_{\alpha\beta} g^{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m (g_{\mu\nu}, \varphi, \partial_\mu \varphi) \right\}$
$R_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$
$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) V^\alpha =$
$\int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} (\delta\Gamma^\alpha_{\beta\nu;\alpha} - \delta\Gamma^\alpha_{\beta\alpha;\nu})}{16\pi G_N} \right\} = \int d^4x \sqrt{-g} \{ \dots \} \delta\Gamma^\alpha_{\beta\gamma}$

$$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{R_{\alpha\beta} g^{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \left(g_{\mu\nu}, \varphi, \partial_\mu \varphi \right) \right\}$$

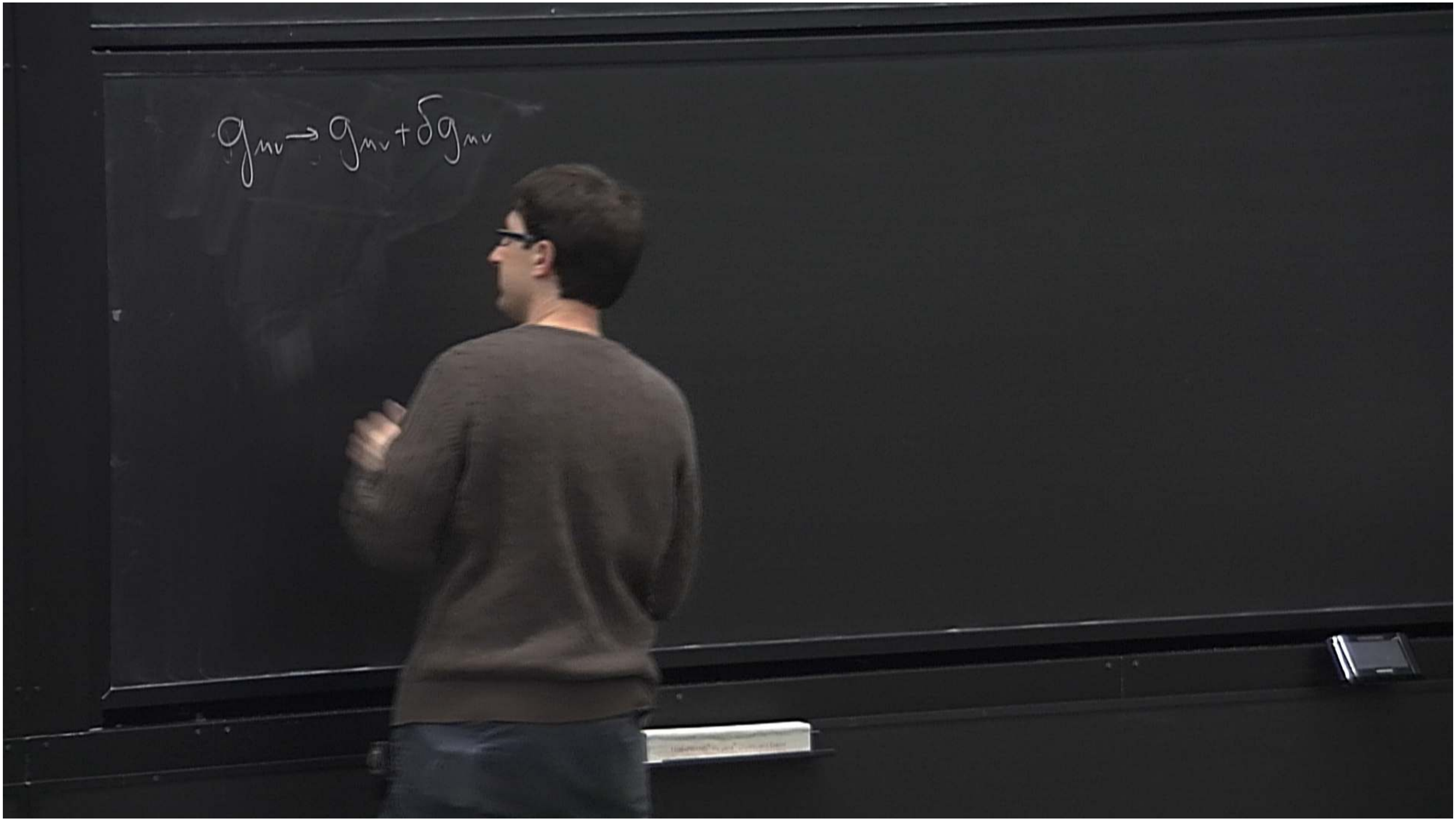
$$R_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$

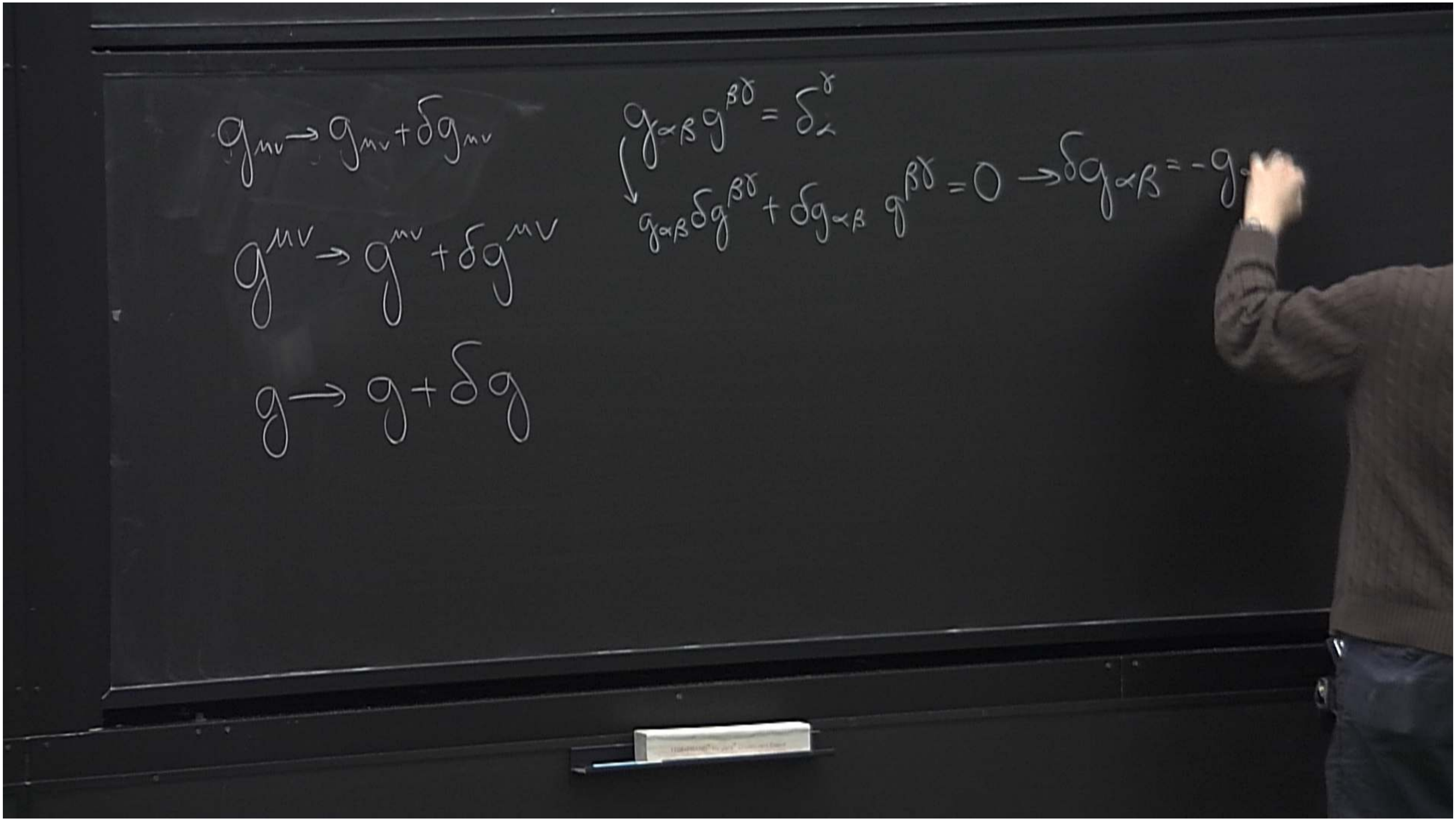
$$\delta\Gamma^{\alpha}_{\beta\gamma}$$

$$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} \left(\delta\Gamma^{\alpha}_{\beta\nu;\alpha} - \delta\Gamma^{\alpha}_{\beta\alpha;\nu} \right)}{16\pi G_N} \right\} = \int d^4x \sqrt{-g} \underbrace{\left\{ \ldots \right\}} \delta\Gamma^{\alpha}_{\beta\gamma}$$

$$\left(\nabla_\mu \nabla_\nu - \nabla_\mu \nabla_\nu \right) v^\alpha =$$

$$\left(\sqrt{-g} \right.$$


$g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$


$g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$
$g^{\mu\nu} \to g^{\mu\nu} + \delta g^{\mu\nu}$
$g \to g + \delta g$
$g_{\alpha\beta} g^{\beta\gamma} = \delta_\alpha^\gamma$
$g_{\alpha\beta} \delta g^{\beta\gamma} + \delta g_{\alpha\beta} \, g^{\beta\gamma} = 0 \to \delta g_{\alpha\beta} = -g$

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + \delta g_{\mu\nu}$$

$$g^{\mu\nu} \rightarrow g^{\mu\nu} + \delta g^{\mu\nu}$$

$$g \rightarrow g + \delta g$$

$$g_{\alpha\beta} g^{\beta\gamma} = \delta_\alpha^\gamma$$

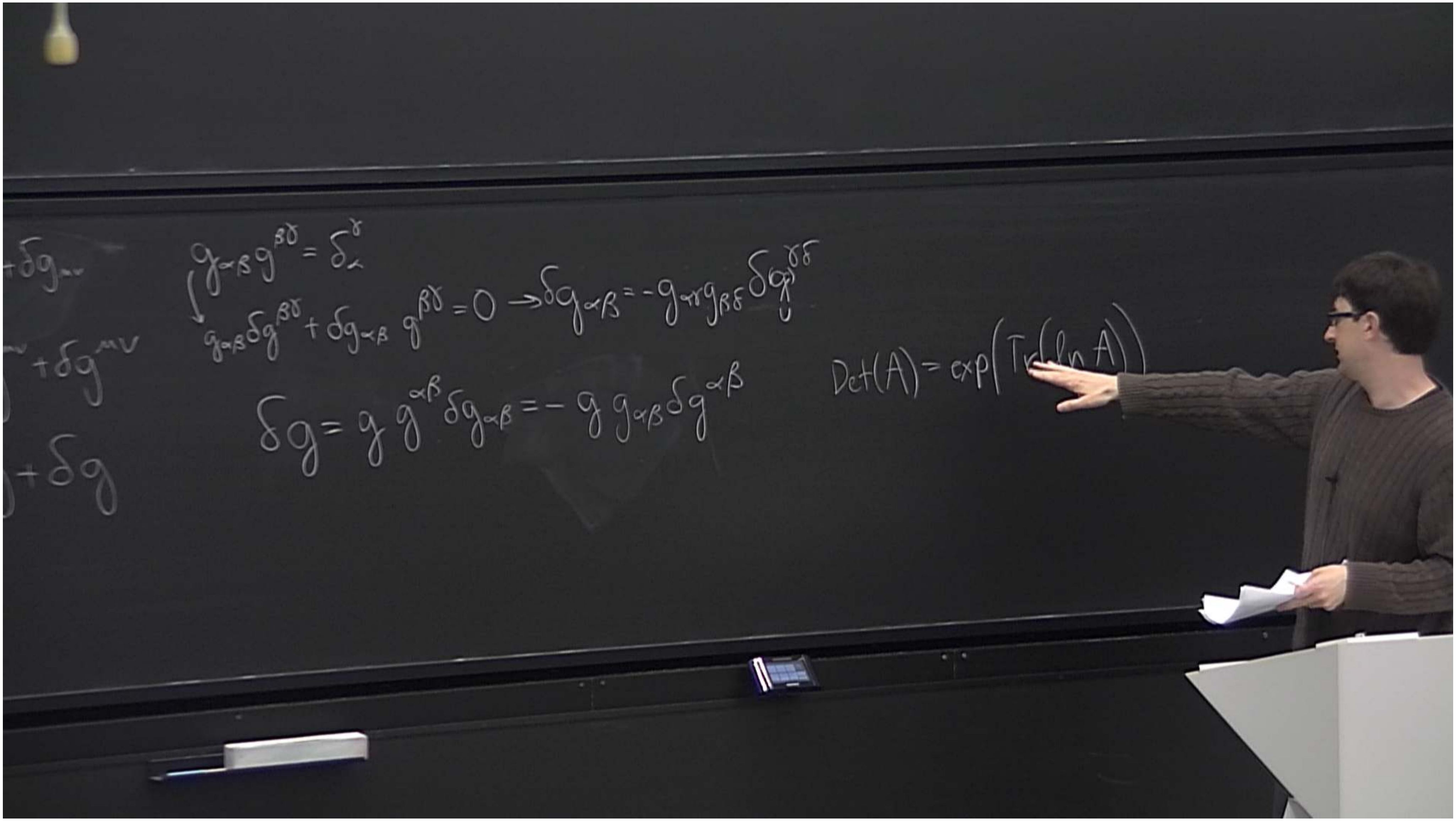
$$g_{\alpha\beta} \delta g^{\beta\gamma} + \delta g_{\alpha\beta} \, g^{\beta\gamma} = 0 \rightarrow \delta g_{\alpha\beta} = - g_{\alpha\gamma} g_{\beta\delta} \delta g^{\gamma\delta}$$

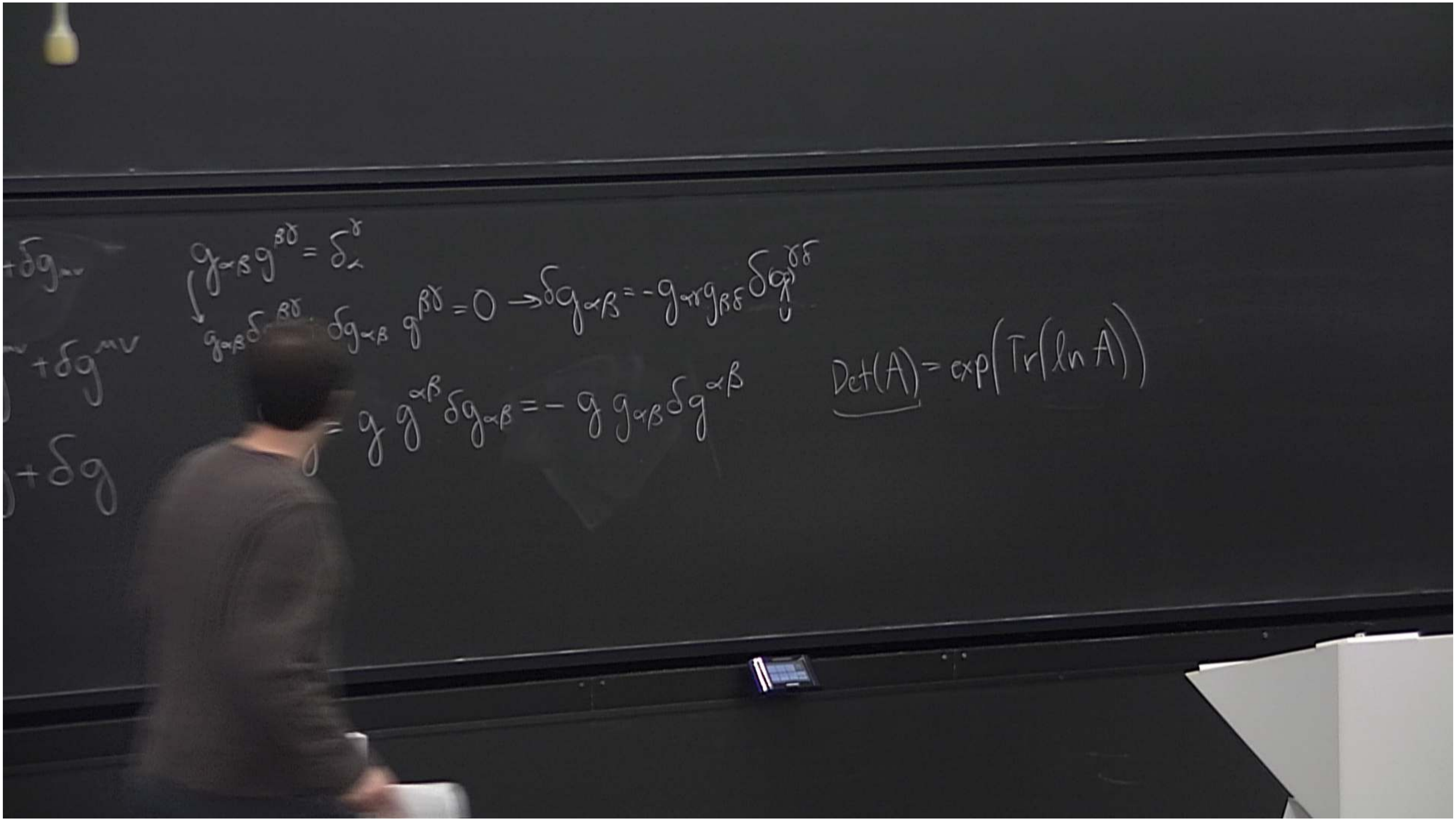
$g^{\mu\nu}$

g

$$g_{\alpha\beta}\, g^{\beta\gamma} = \delta_\alpha^\gamma$$

$$g_{\alpha\beta}\,\delta g^{\beta\gamma} + \delta g_{\alpha\beta}\, g^{\beta\gamma} = 0 \rightarrow \delta g_{\alpha\beta} = -g_{\alpha\gamma}\, g_{\beta\delta}\, \delta g^{\gamma\delta}$$


$+\delta g_{\mu\nu}$
$+\delta g^{\mu\nu}$
$+\delta g$
$g_{\alpha\beta} g^{\beta\gamma} = \delta_\alpha^\gamma$
$g_{\alpha\beta}\delta g^{\beta\gamma} + \delta g_{\alpha\beta}\, g^{\beta\gamma} = 0 \rightarrow \delta g_{\alpha\beta} = -g_{\alpha\gamma} g_{\beta\delta}\delta g^{\gamma\delta}$
$\delta g = g\, g^{\alpha\beta}\delta g_{\alpha\beta} = -g\, g_{\alpha\beta}\delta g^{\alpha\beta}$
$\mathrm{Det}(A) = \exp(\mathrm{Tr}(\ln A))$



$+\delta g_{\mu\nu}$
$+\delta g^{\mu\nu}$
$+\delta g$
$g_{\alpha\beta} g^{\beta\gamma} = \delta_\alpha^\gamma$
$\delta g_{\alpha\beta}\, g^{\beta\gamma} = 0 \rightarrow \delta g_{\alpha\beta} = -g_{\alpha\gamma} g_{\beta\delta}\, \delta g^{\gamma\delta}$
$= g\, g^{\alpha\beta}\delta g_{\alpha\beta} = -g\, g_{\alpha\beta}\delta g^{\alpha\beta}$
$\mathrm{Det}(A) = \exp(\mathrm{Tr}(\ln A))$

$+\delta g_{\mu\nu}$

$+\delta g^{\mu\nu}$

$+\delta g$

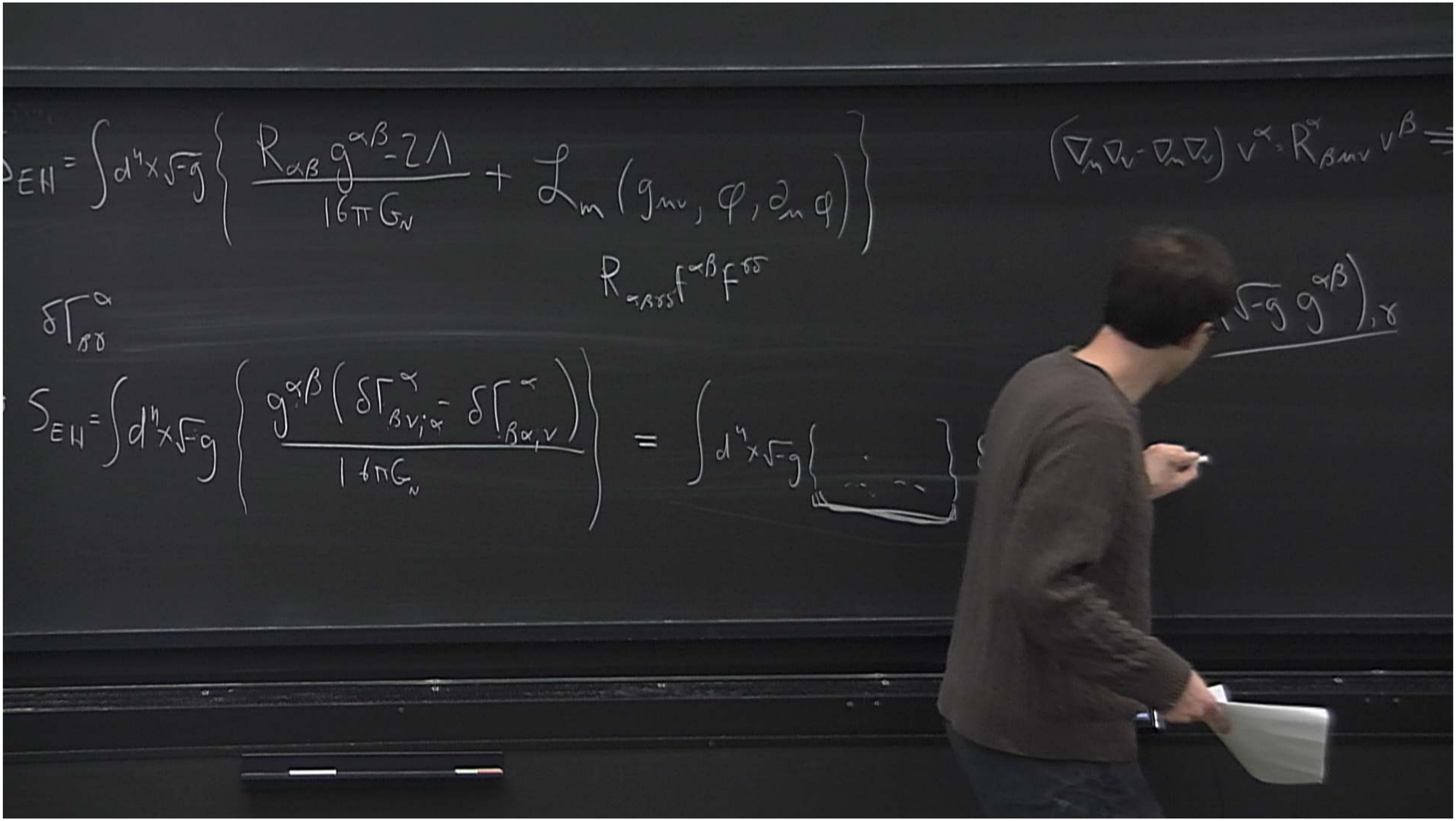
$$g_{\alpha\beta}\, g^{\beta\gamma} = \delta_\alpha^\gamma$$

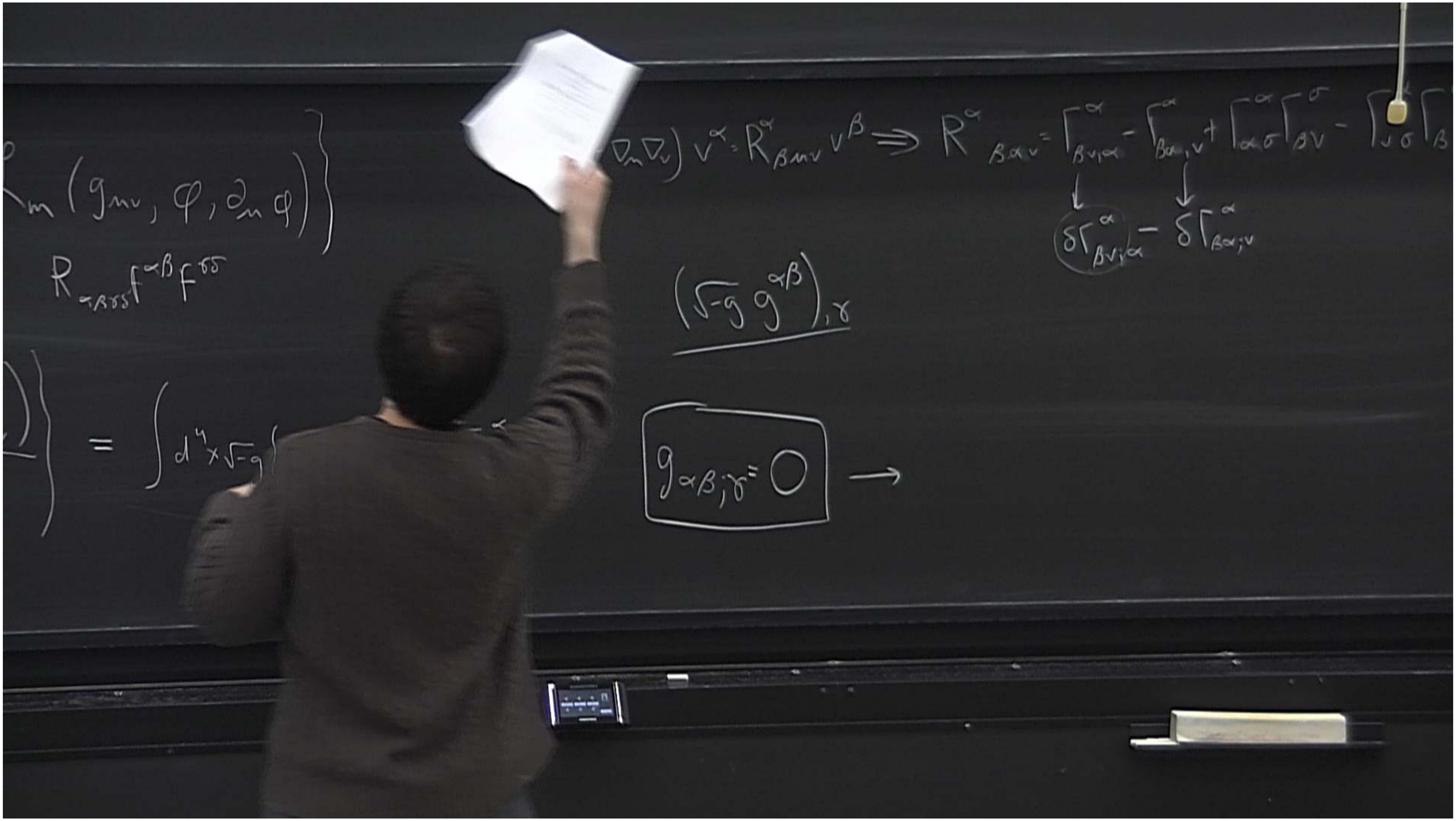
$$g_{\alpha\beta}\,\delta g^{\beta\gamma} + \delta g_{\alpha\beta}\, g^{\beta\gamma} = 0 \rightarrow \delta g_{\alpha\beta} = -g_{\alpha\gamma}\, g_{\beta\delta}\,\delta g^{\gamma\delta}$$

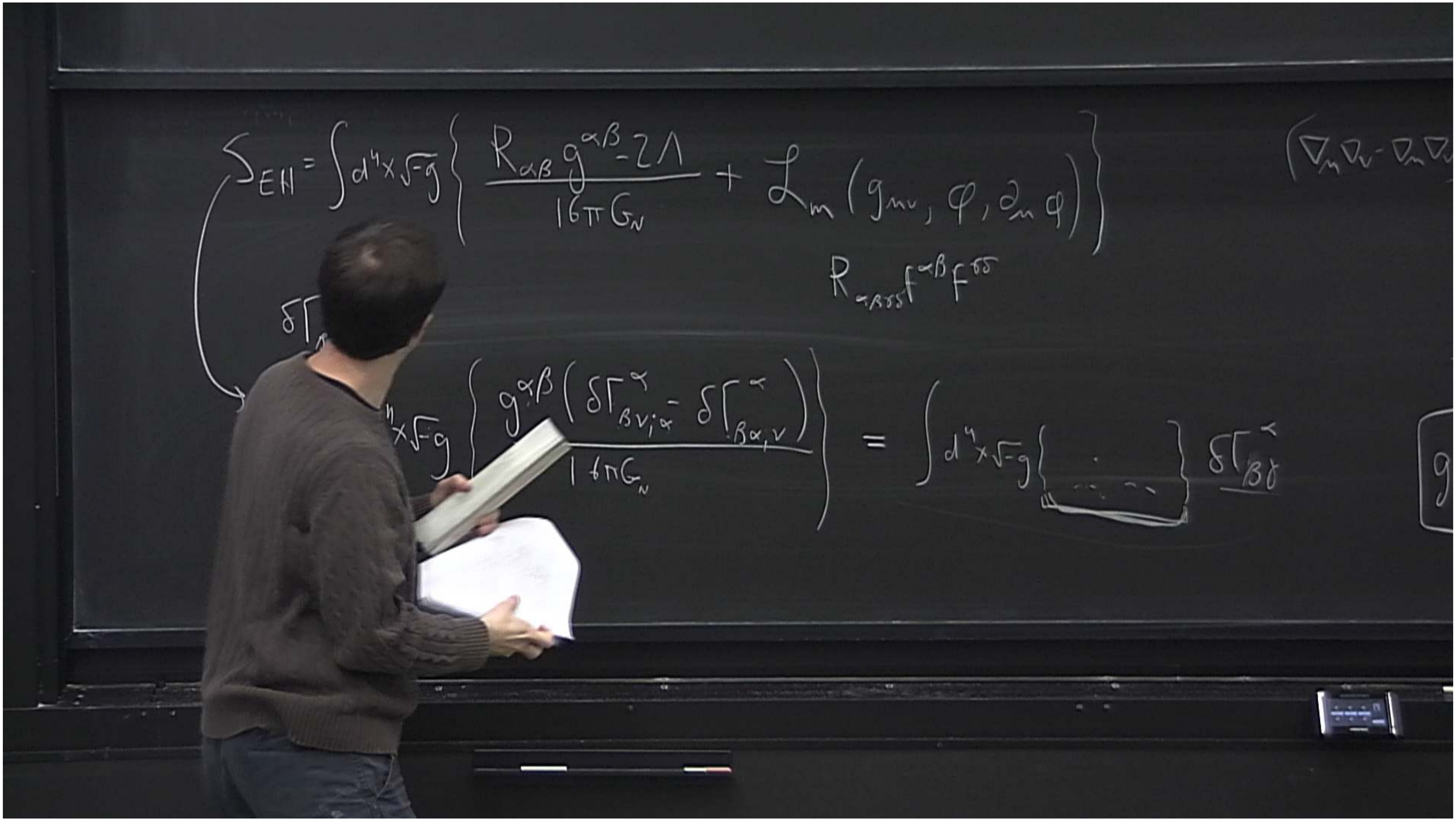
$$\delta g = g\, g^{\alpha\beta}\,\delta g_{\alpha\beta} = -g\, g_{\alpha\beta}\,\delta g^{\alpha\beta}$$

$$\boxed{g_{,\gamma} = g\, g^{\alpha\beta}\,\delta g_{\alpha\beta,\gamma} = -g\, g_{\alpha\beta}\,\delta g^{\alpha\beta}{}_{,\gamma}}$$

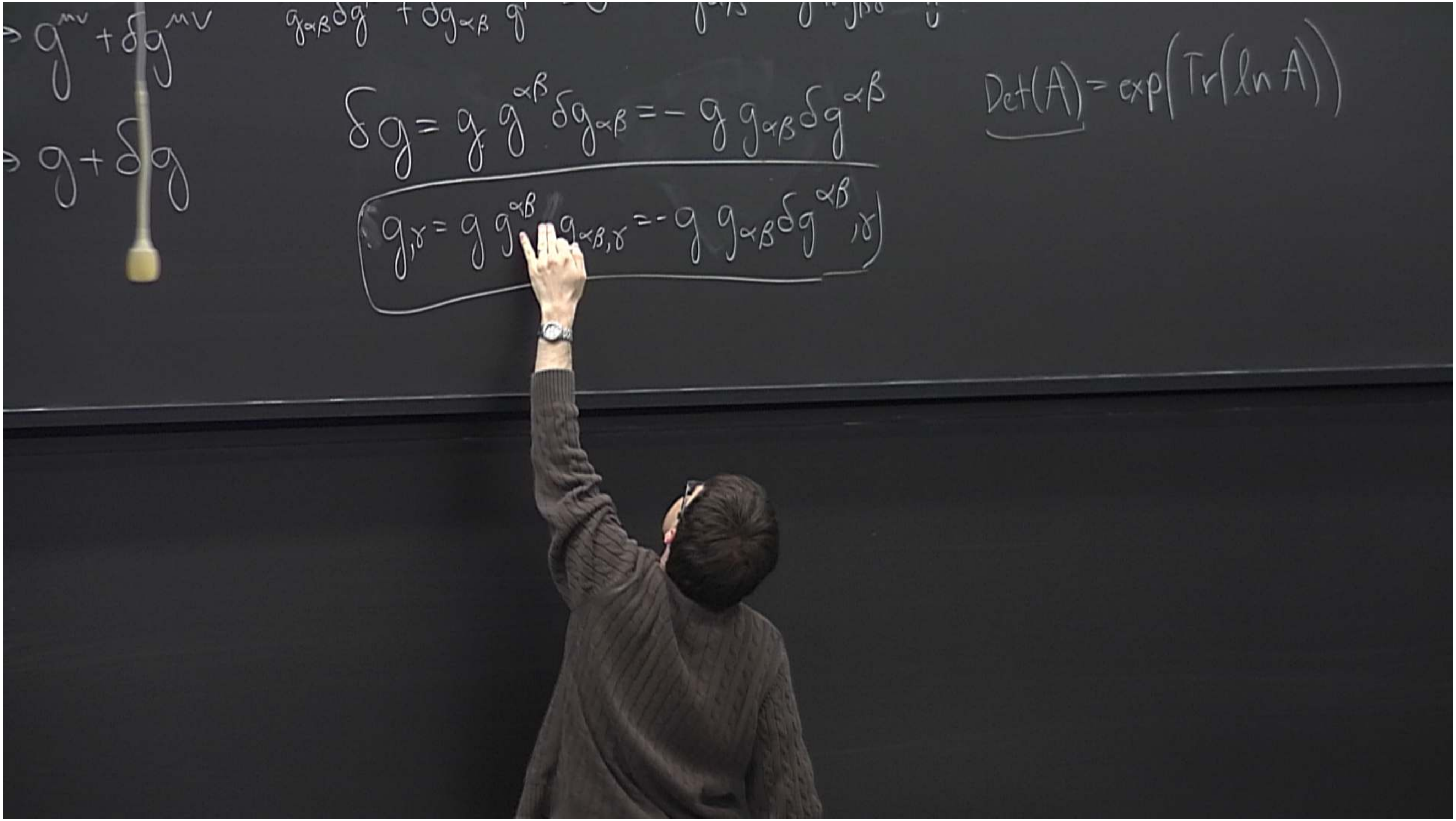
$$\mathrm{Det}(A) = \exp\left(\mathrm{Tr}\left(\ln A\right)\right)$$


$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{R_{\alpha\beta} g^{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m (g_{\mu\nu}, \varphi, \partial_\mu \varphi) \right\}$
$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) v^\alpha = R^\alpha_{\ \beta\mu\nu} v^\beta$
$R_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$
$\delta\Gamma^\alpha_{\beta\gamma}$
$\left(\sqrt{-g}\, g^{\alpha\beta}\right)_{,\gamma}$
$\delta S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} \left(\delta\Gamma^\alpha_{\beta\nu;\alpha} - \delta\Gamma^\alpha_{\beta\alpha;\nu} \right)}{16\pi G_N} \right\} = \int d^4x \sqrt{-g} \{ \quad \}$


$(\nabla_\mu \nabla_\nu) v^\alpha = R^\alpha{}_{\beta\mu\nu} v^\beta \Rightarrow R^\alpha{}_{\beta\alpha\nu} = \Gamma^\alpha_{\beta\nu,\alpha} - \Gamma^\alpha_{\beta\alpha,\nu} + \Gamma^\alpha_{\alpha\sigma}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\nu\sigma}\Gamma_{\beta}$
$\delta\Gamma^\alpha_{\beta\nu;\alpha} - \delta\Gamma^\alpha_{\beta\alpha;\nu}$
$\mathcal{L}_m(g_{\mu\nu}, \varphi, \partial_\mu\varphi)$
$R_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$
$(\sqrt{-g}\, g^{\alpha\beta})_{,\gamma}$
$= \int d^4x \sqrt{-g}$
$g_{\alpha\beta;\gamma} = 0 \longrightarrow$



$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{R_{\alpha\beta} g^{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m(g_{\mu\nu}, \varphi, \partial_\mu \varphi) \right\}$
$R_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$
$\delta\Gamma$
$x\sqrt{-g} \left\{ \frac{g^{\alpha\beta}\left(\delta\Gamma^{\alpha}_{\beta\nu;\alpha} - \delta\Gamma^{\alpha}_{\beta\alpha,\nu}\right)}{16\pi G_N} \right\} = \int d^4x \sqrt{-g} \{ \quad \} \delta\Gamma^{\alpha}_{\beta\gamma}$
$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla$


δg = g g^αβ δg_αβ = − g g_αβ δg^αβ
g,γ = g g^αβ g_αβ,γ = − g g_αβ δg^αβ,γ
Det(A) = exp(Tr(ln A))

$$\delta\left((-g)^{\frac{1}{2}}\right) = \frac{1}{2}(-g)^{-\frac{1}{2}}(-\delta g)$$

$$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{R_{\alpha\beta}\, g^{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m\left(g_{\mu\nu}, \varphi, \partial_\mu \varphi\right) \right\}$$

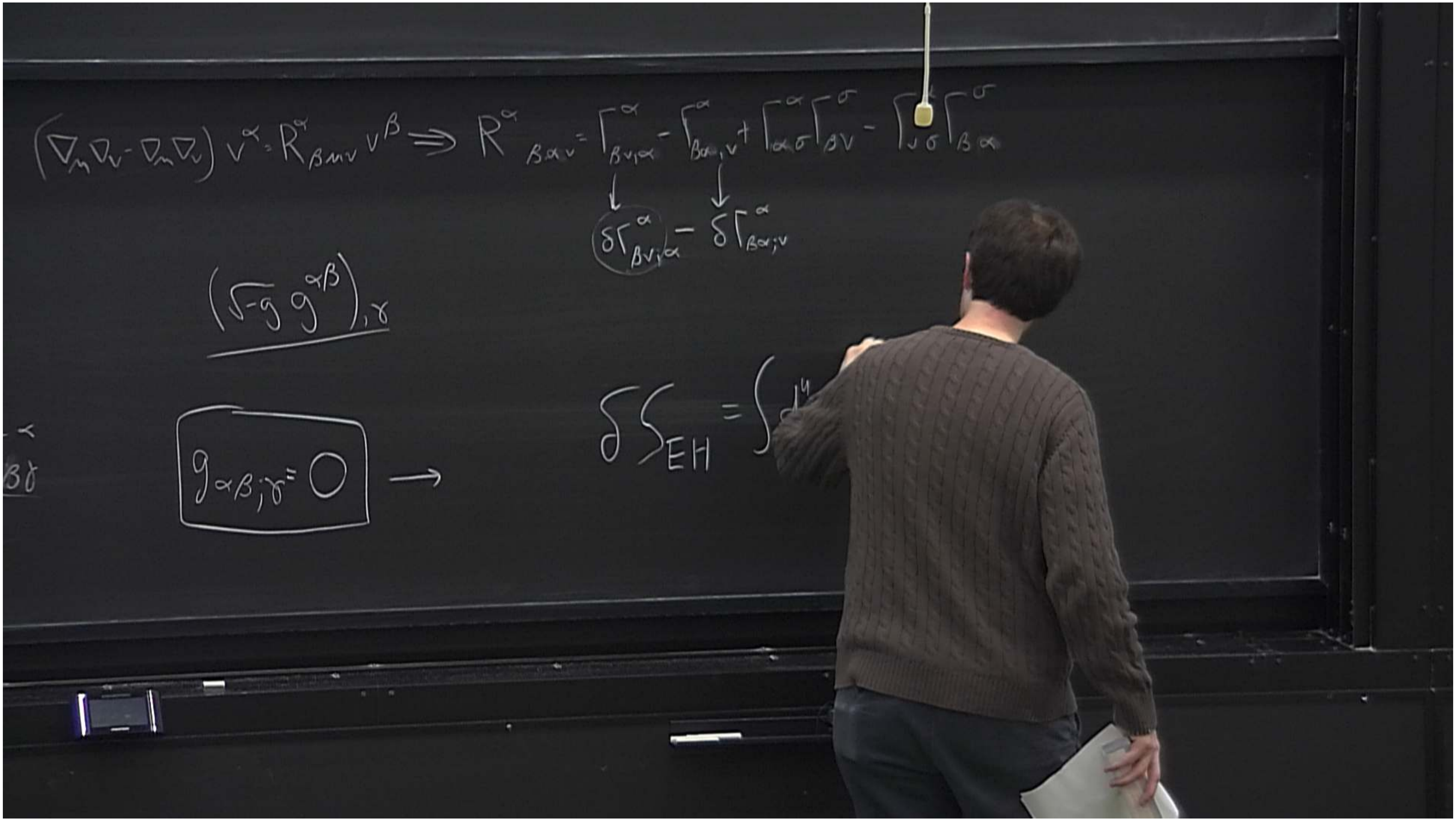
$$R_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$

$$\delta\Gamma^{\alpha}_{\beta\gamma}$$

$$S_{EH} = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta}\left(\delta\Gamma^{\alpha}_{\beta\nu;\alpha} - \delta\Gamma^{\alpha}_{\beta\alpha,\nu}\right)}{16\pi G_N} \right\} = \int d^4x \sqrt{-g} \{ \ldots \} \, \delta\Gamma^{\alpha}_{\beta\gamma}$$

$(\nabla_\mu \nabla_\nu - \nabla_\mu \nabla_\nu$

g


$(\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu) v^\alpha = R^\alpha{}_{\beta\mu\nu} v^\beta \Rightarrow R^\alpha{}_{\beta\alpha\nu} = \Gamma^\alpha_{\beta\nu,\alpha} - \Gamma^\alpha_{\beta\alpha,\nu} + \Gamma^\alpha_{\alpha\sigma}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\nu\sigma}\Gamma^\sigma_{\beta\alpha}$
$\delta\Gamma^\alpha_{\beta\nu;\alpha} - \delta\Gamma^\alpha_{\beta\alpha;\nu}$
$(\sqrt{-g}\, g^{\alpha\beta})_{,\gamma}$
$g_{\alpha\beta;\gamma} = 0 \rightarrow$
$\delta S_{EH} = \int d^4$

$$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu) V^\alpha = R^\alpha{}_{\beta\mu\nu} V^\beta \Rightarrow R^\alpha{}_{\beta\alpha\nu} = \Gamma^\alpha_{\beta\nu,\alpha} - \Gamma^\alpha_{\beta\alpha,\nu} + \Gamma^\alpha_{\alpha\sigma}\Gamma^\sigma_{\beta\nu} - \Gamma^\alpha_{\nu\sigma}\Gamma^\sigma_{\beta\alpha}$$

$$\delta\Gamma^\alpha_{\beta\nu;\alpha} - \delta\Gamma^\alpha_{\beta\alpha;\nu}$$

$$\underline{(\sqrt{-g}\, g^{\alpha\beta})_{,\gamma}}$$

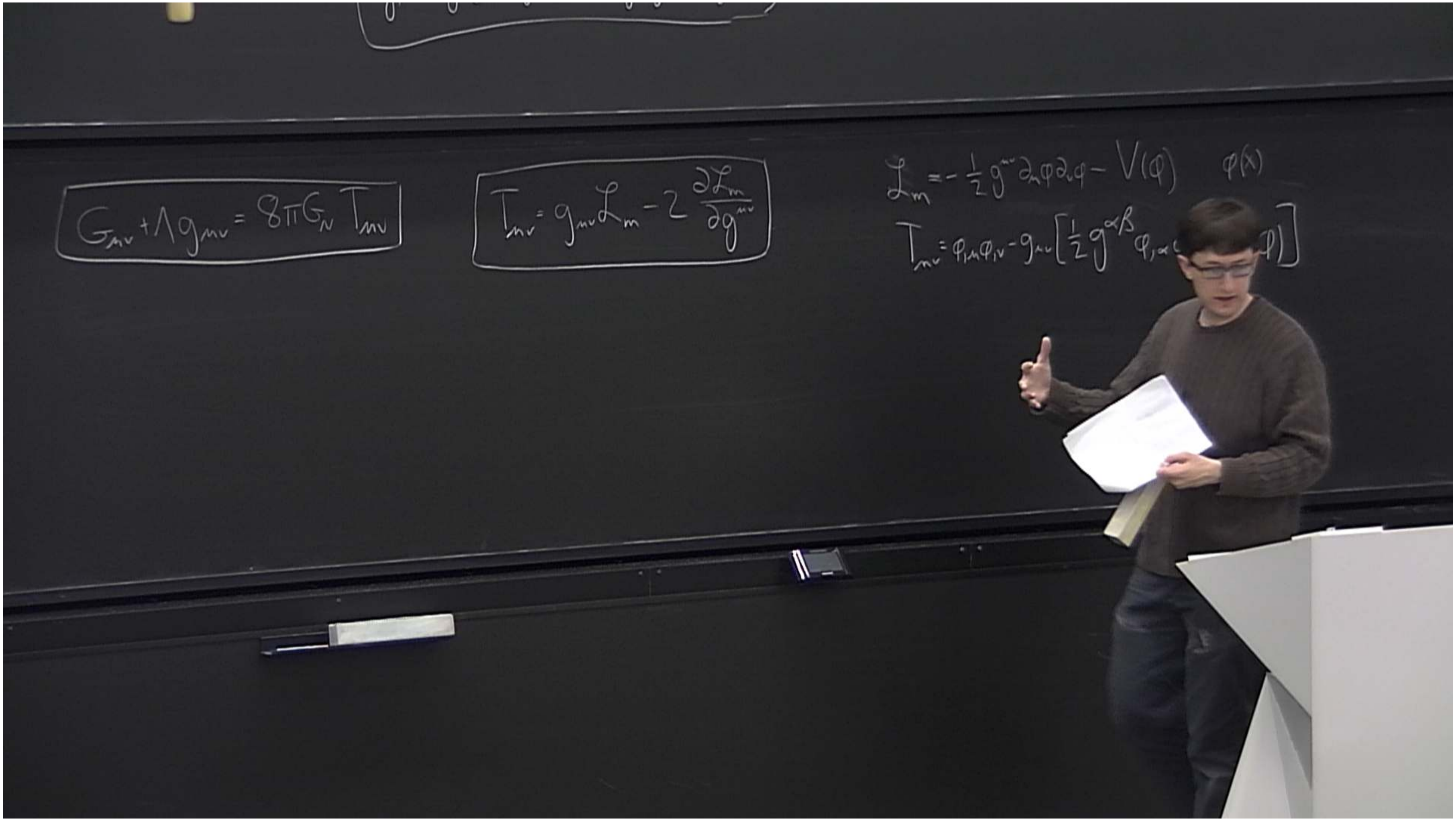
$$\boxed{g_{\alpha\beta;\gamma} = 0} \longrightarrow$$

$$\delta S_{EH} = \int d^4x \sqrt{-g} \{ \ldots\ldots \} \delta g^{\alpha\beta}$$

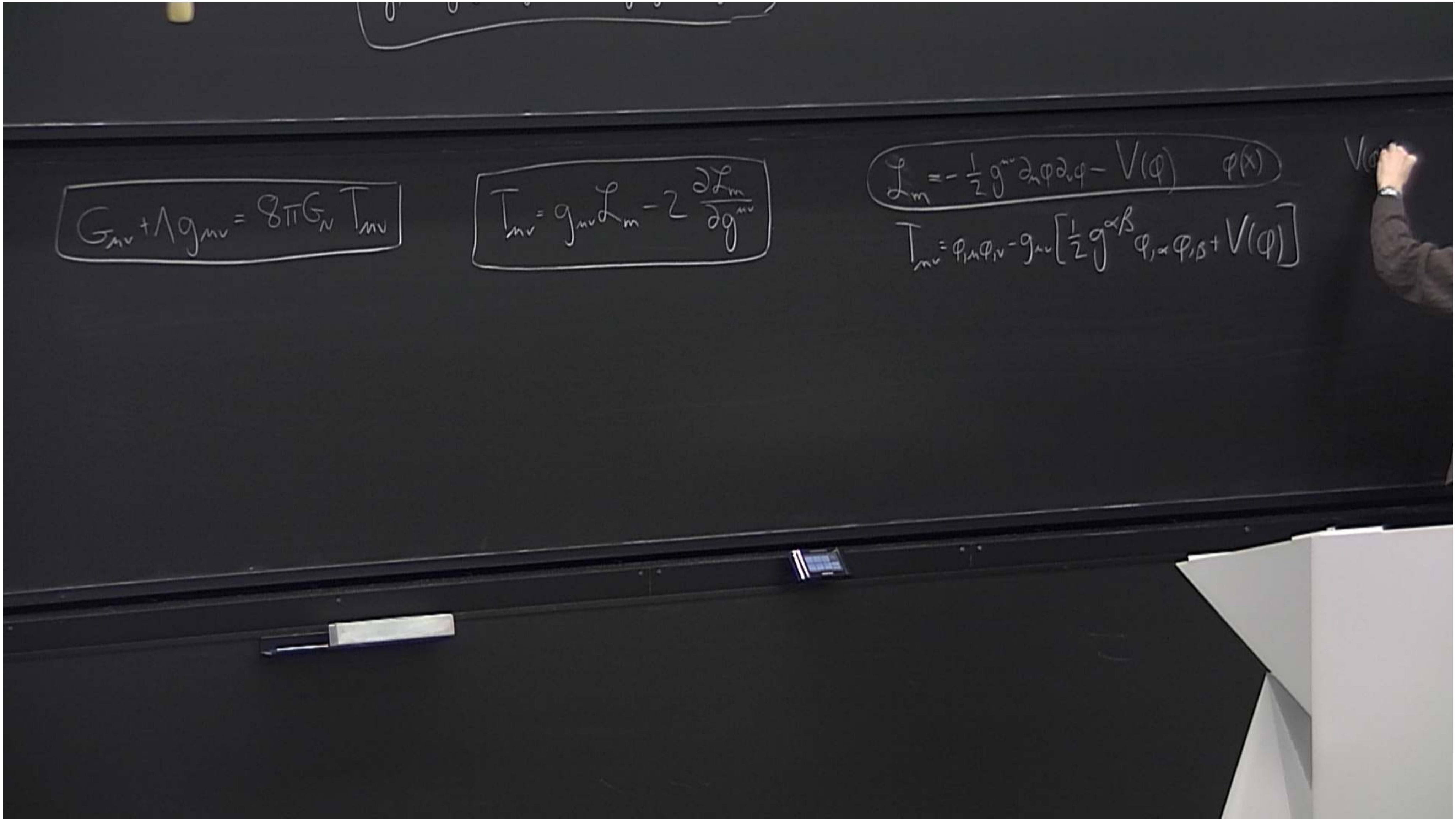
α

$\beta\gamma$

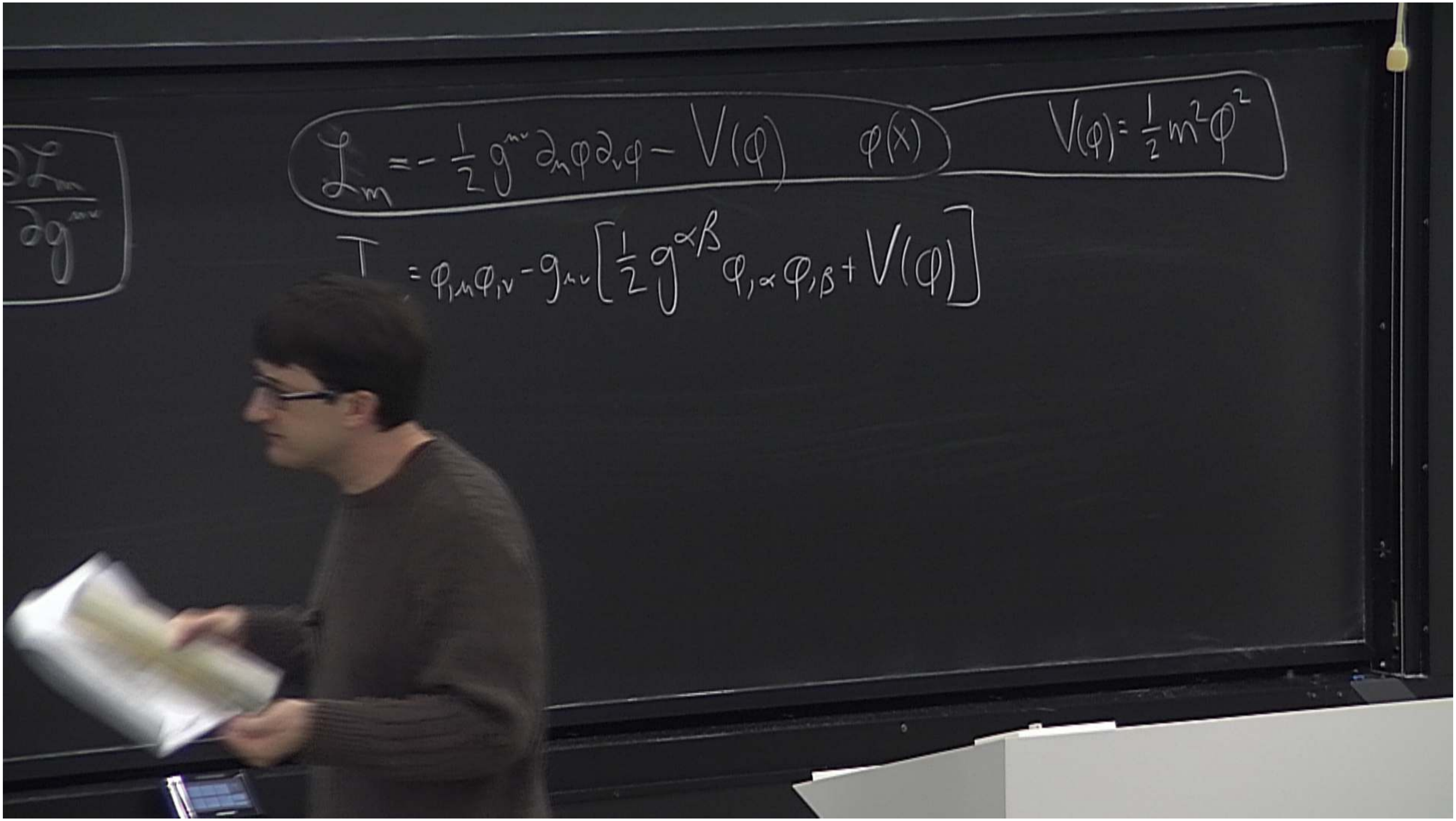
$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}} \qquad T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}$$



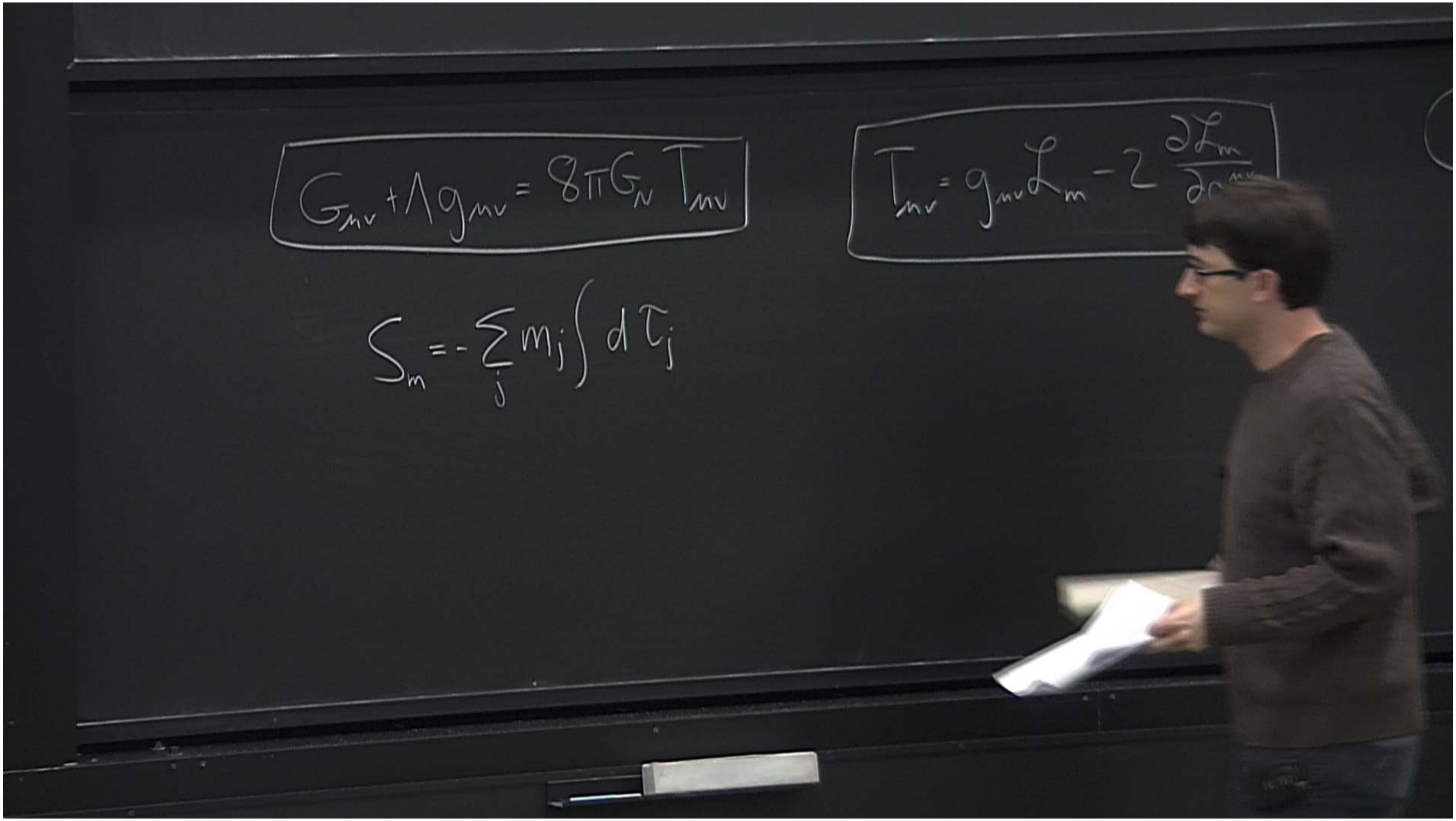
$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}$
$\mathcal{L}_m = -\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)$
$\varphi(x)$

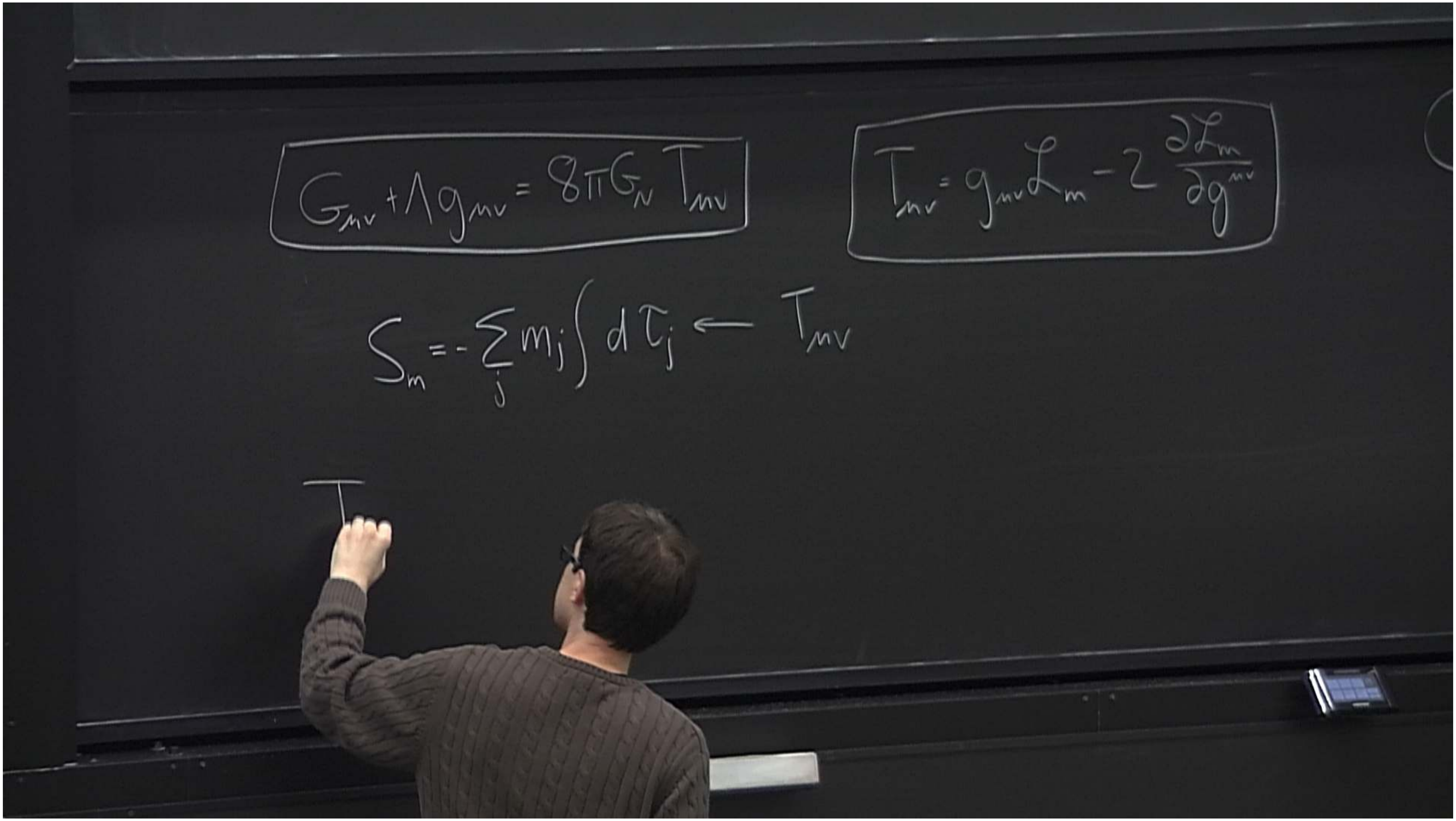


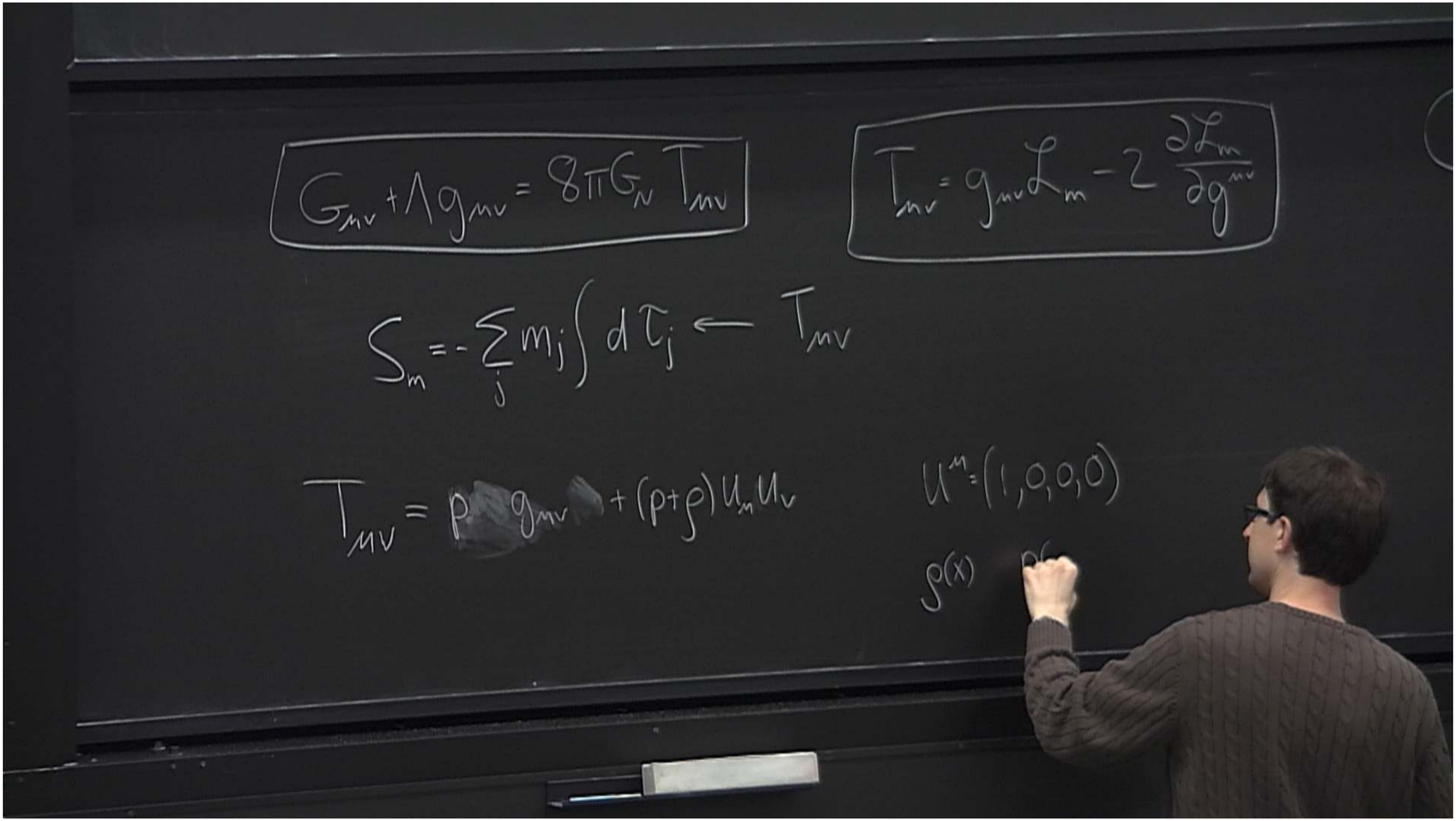
$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}$
$\mathcal{L}_m = -\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)$
$\varphi(x)$
$T_{\mu\nu} = \varphi_{,\mu}\varphi_{,\nu} - g_{\mu\nu}\left[\frac{1}{2} g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta} + V(\varphi)\right]$



$\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}$
$\mathcal{L}_m = -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)$
$\varphi(x)$
$V(\varphi) = \frac{1}{2} m^2 \varphi^2$
$T \;\; = \varphi_{,\mu} \varphi_{,\nu} - g_{\mu\nu} \left[\frac{1}{2} g^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta} + V(\varphi) \right]$


$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}$
$S_m = -\sum_j m_j \int d\tau_j$


$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}}$
$\boxed{T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}}$
$S_m = -\sum_j m_j \int d\tau_j \leftarrow T_{\mu\nu}$
T


$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}$
$S_m = -\sum_j m_j \int d\tau_j \leftarrow T_{\mu\nu}$
$T_{\mu\nu} = p\, g_{\mu\nu} + (p+\rho) u_\mu u_\nu$
$u^\mu = (1,0,0,0)$
$\rho(x)$

$$\boxed{G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}}$$

$$\boxed{T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}}$$

$$\boxed{\mathcal{L}_m = -\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\,\partial_\nu\varphi - V(\varphi)}$$

$$T_{\mu\nu} = \varphi_{,\mu}\varphi_{,\nu} - g_{\mu\nu}\left[\frac{1}{2} g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta} + \right.$$

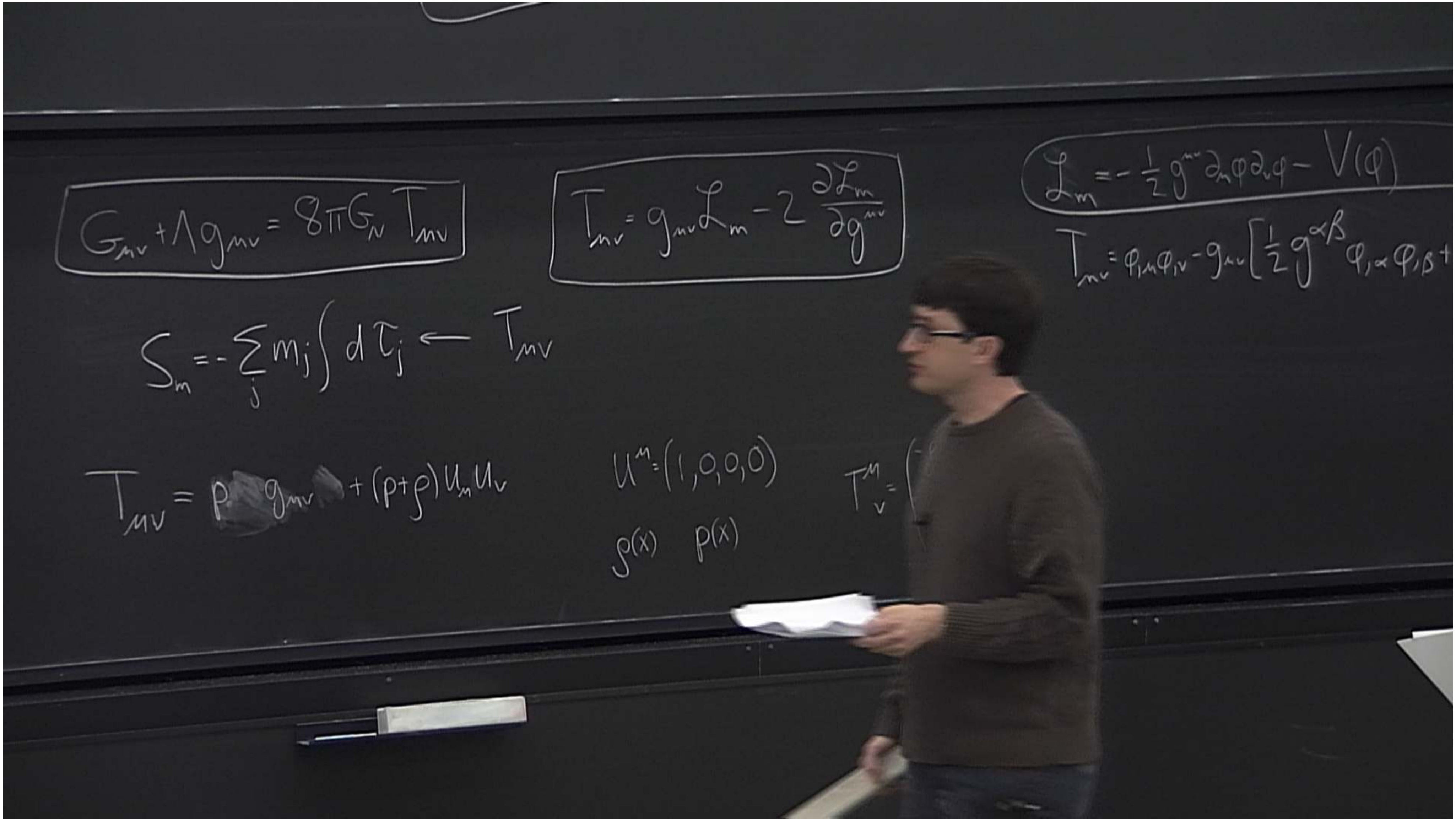
$$S_m = -\sum_j m_j \int d\tau_j \leftarrow T_{\mu\nu}$$

$$T_{\mu\nu} = p\, g_{\mu\nu} + (p+\rho)u_\mu u_\nu$$

$$u^\mu = (1,0,0,0)$$

$$\rho(x) \quad p(x)$$

$$T^\mu{}_\nu = \begin{pmatrix} -\rho & & 0 \\ & p & \\ 0 & p & \\ & & p \end{pmatrix}$$


$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\partial \mathcal{L}_m}{\partial g^{\mu\nu}}$
$\mathcal{L}_m = -\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)$
$T_{\mu\nu} = \varphi_{,\mu}\varphi_{,\nu} - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\varphi_{,\alpha}\varphi_{,\beta} +\right.$
$S_m = -\sum_j m_j \int d\tau_j \leftarrow T_{\mu\nu}$
$T_{\mu\nu} = p\, g_{\mu\nu} + (p+\rho)U_\mu U_\nu$
$U^\mu = (1,0,0,0)$
$\rho(x) \quad p(x)$
$T^\mu_\nu =$