

Title: 12/13 PSI - Cosmology Review Lecture 1

Date: Feb 19, 2013 11:30 AM

URL: <http://www.pirsa.org/13020099>

Abstract:

Latham Boyle

Cosmology Review

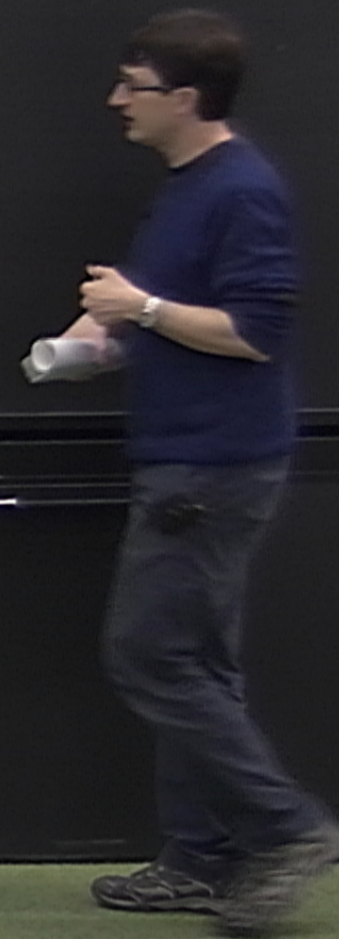
Latham Boyle Cosmology Review

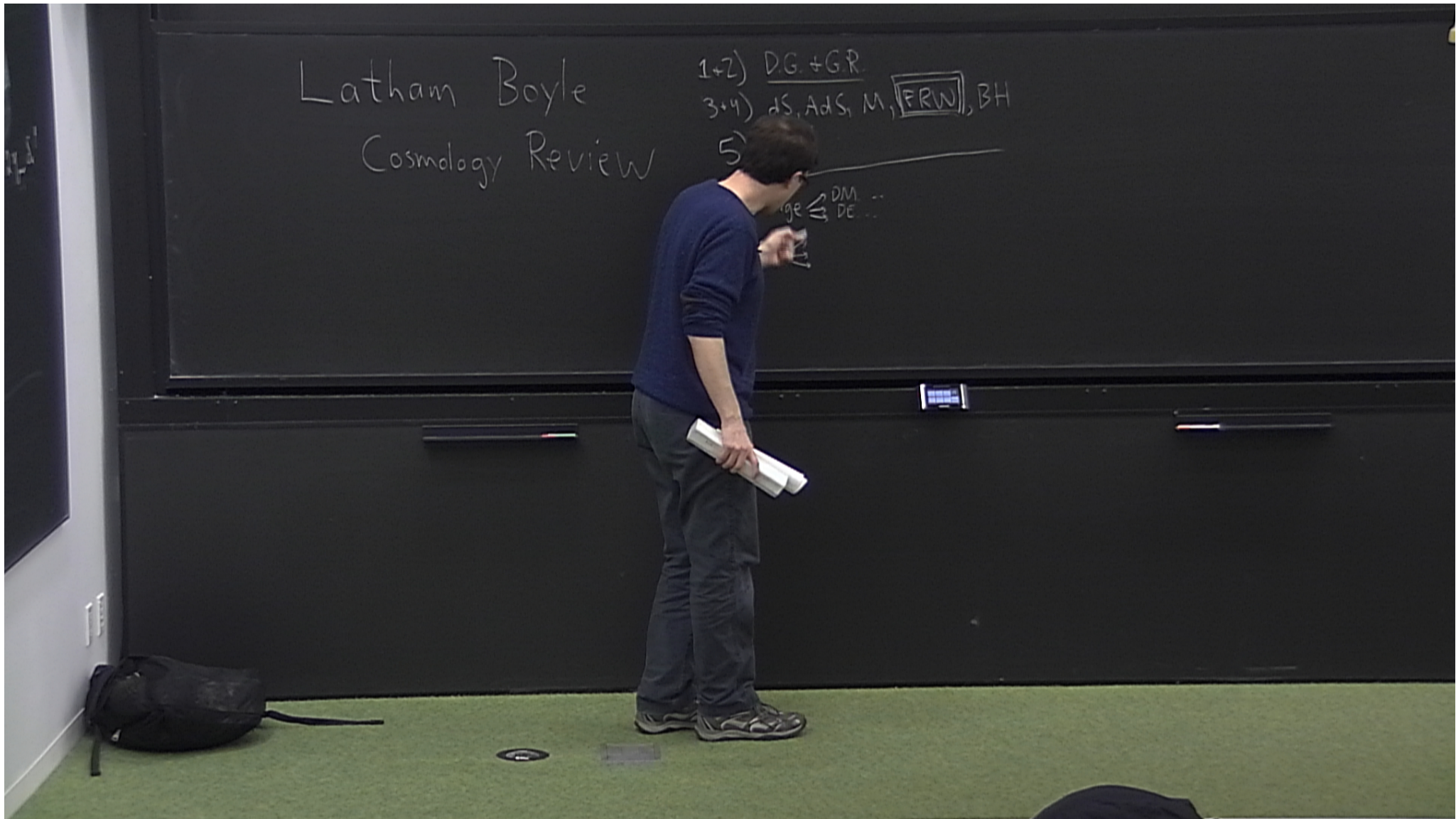
1+2) D.G. + G.R.

Latham Boyle

Cosmology Review

1+2) D.G. + G.R.
3+4) $dS, AdS, M, \boxed{FRW}, BH$
5) FRW



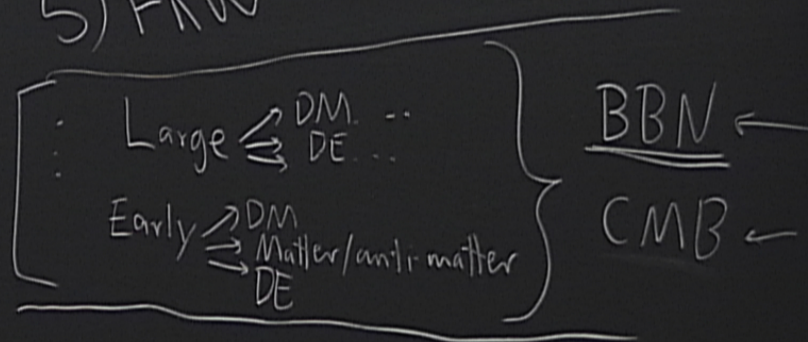


m Boyle
ology Review

1+2) D.G. + G.R.

3+4) dS , AdS , M , FRW , BH

5) FRW



W

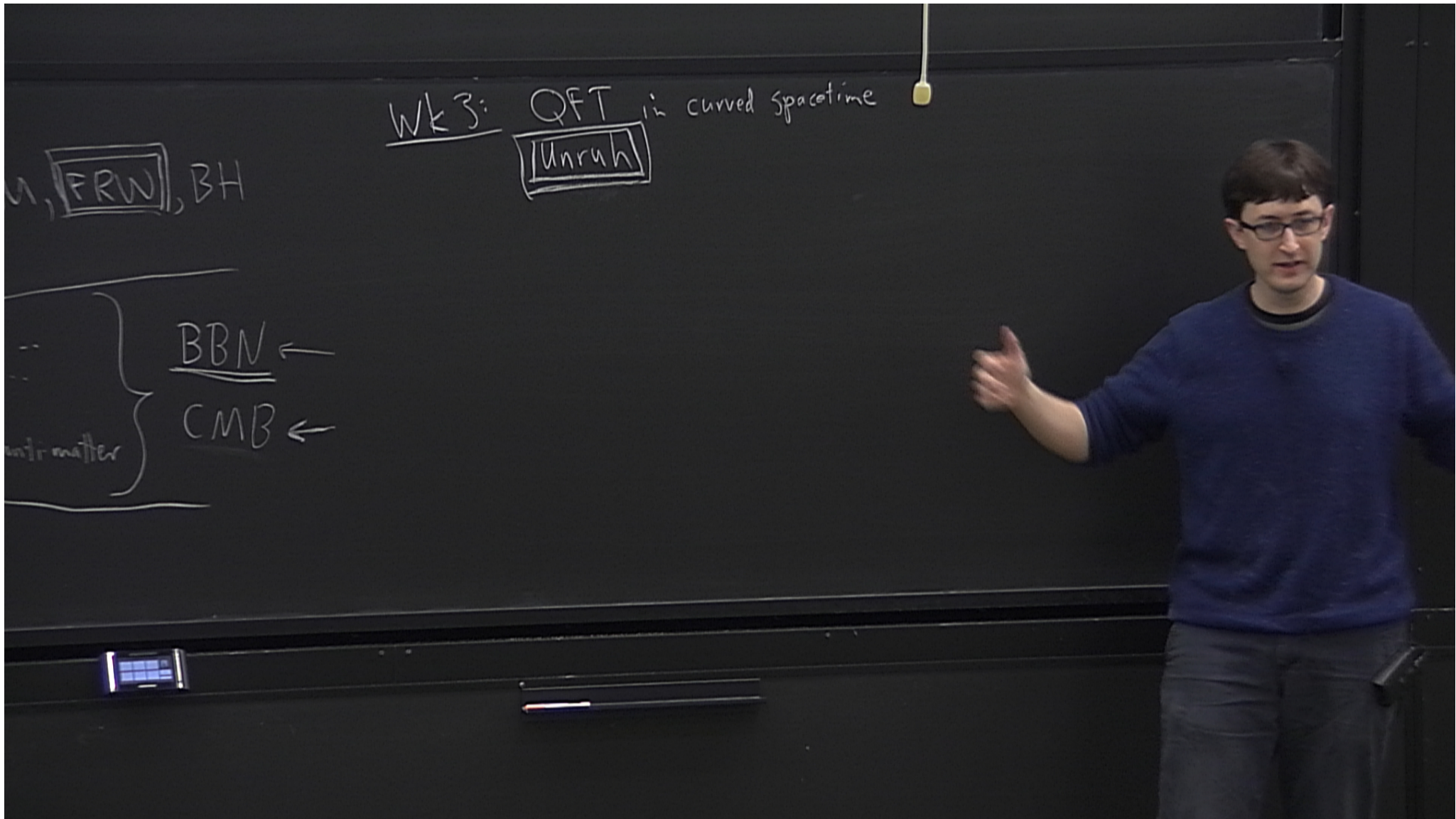
Wk 3: QFT in curved spacetime

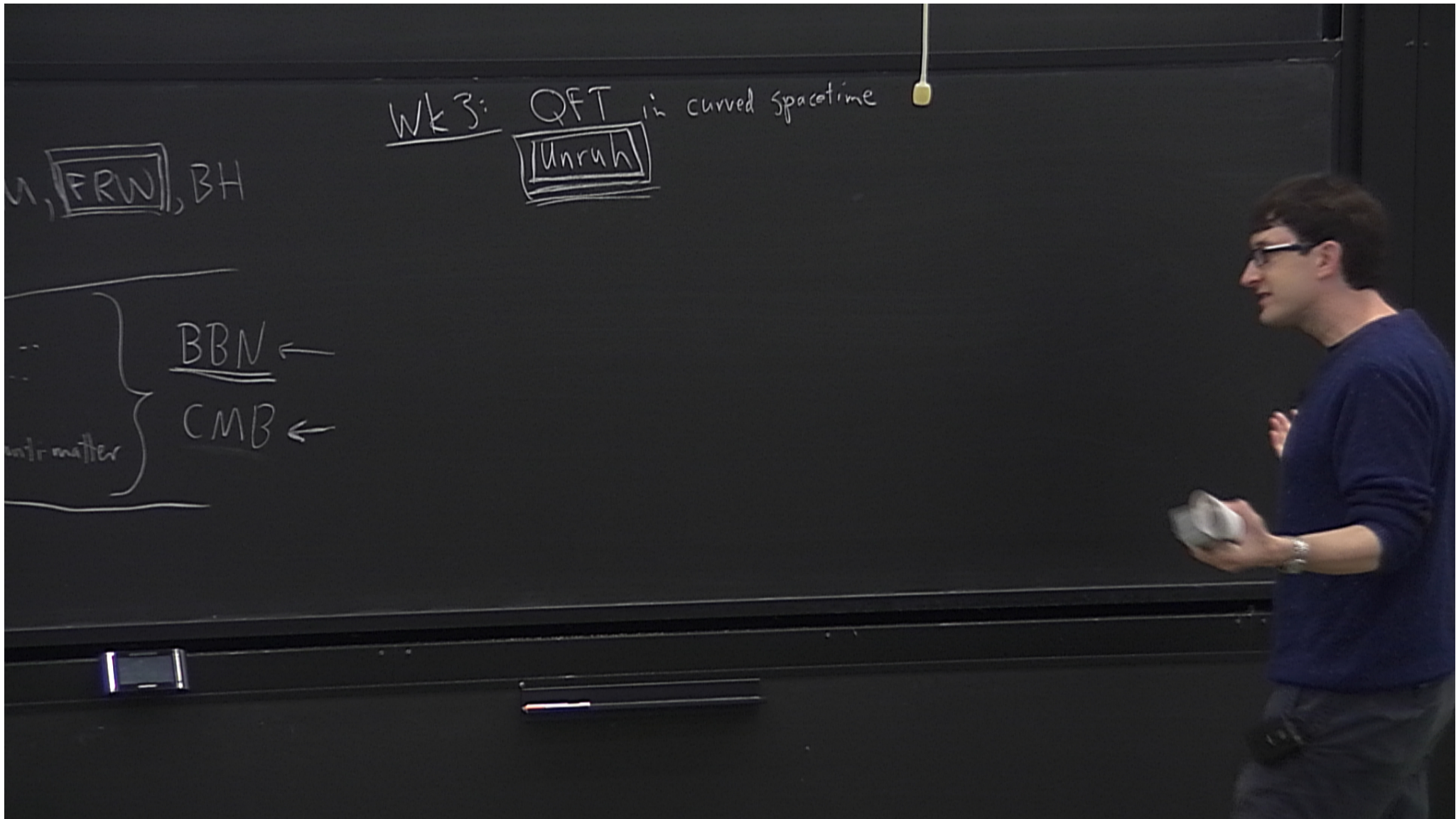
Λ , FRW, BH

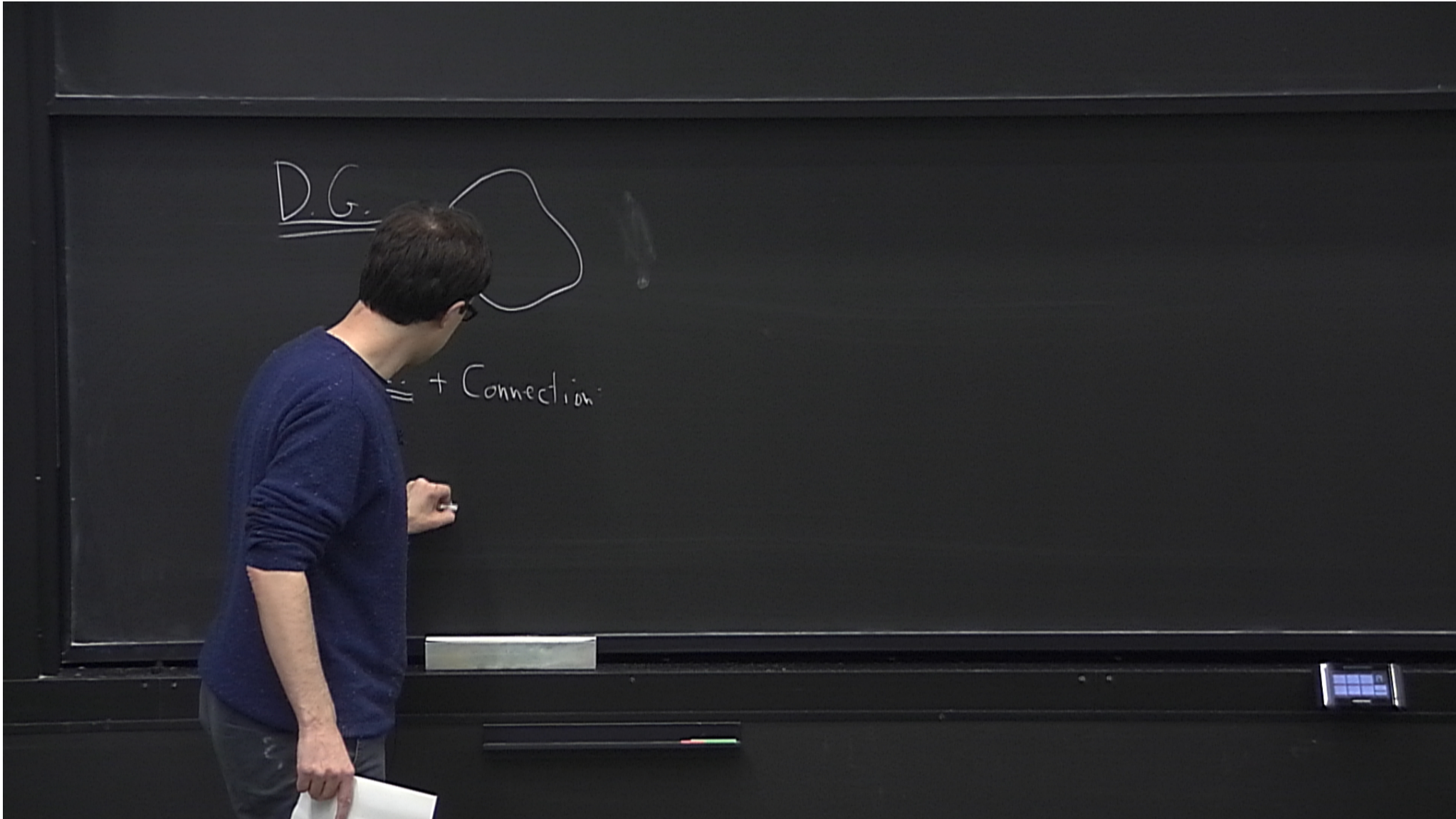
BBN ←

CMB ←

anti-matter

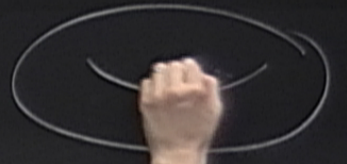






D.G. 1. M

n -dim



2. + Connection: $\Gamma_{\mu\nu}^{\alpha}$

3. + metric: $g_{\mu\nu}$

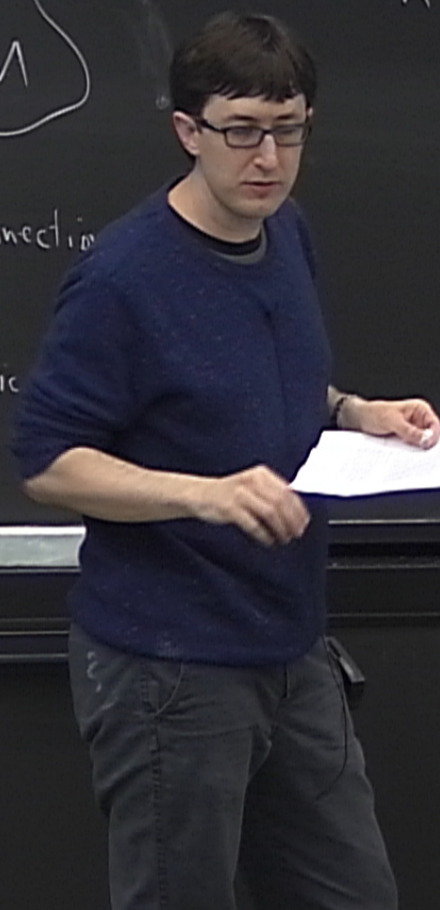
D.G. 1. M

n -dim



2. + Connection

3. + metric



D.G. 1. M

2. + Connection $\Gamma_{\mu\beta}^{\alpha}$

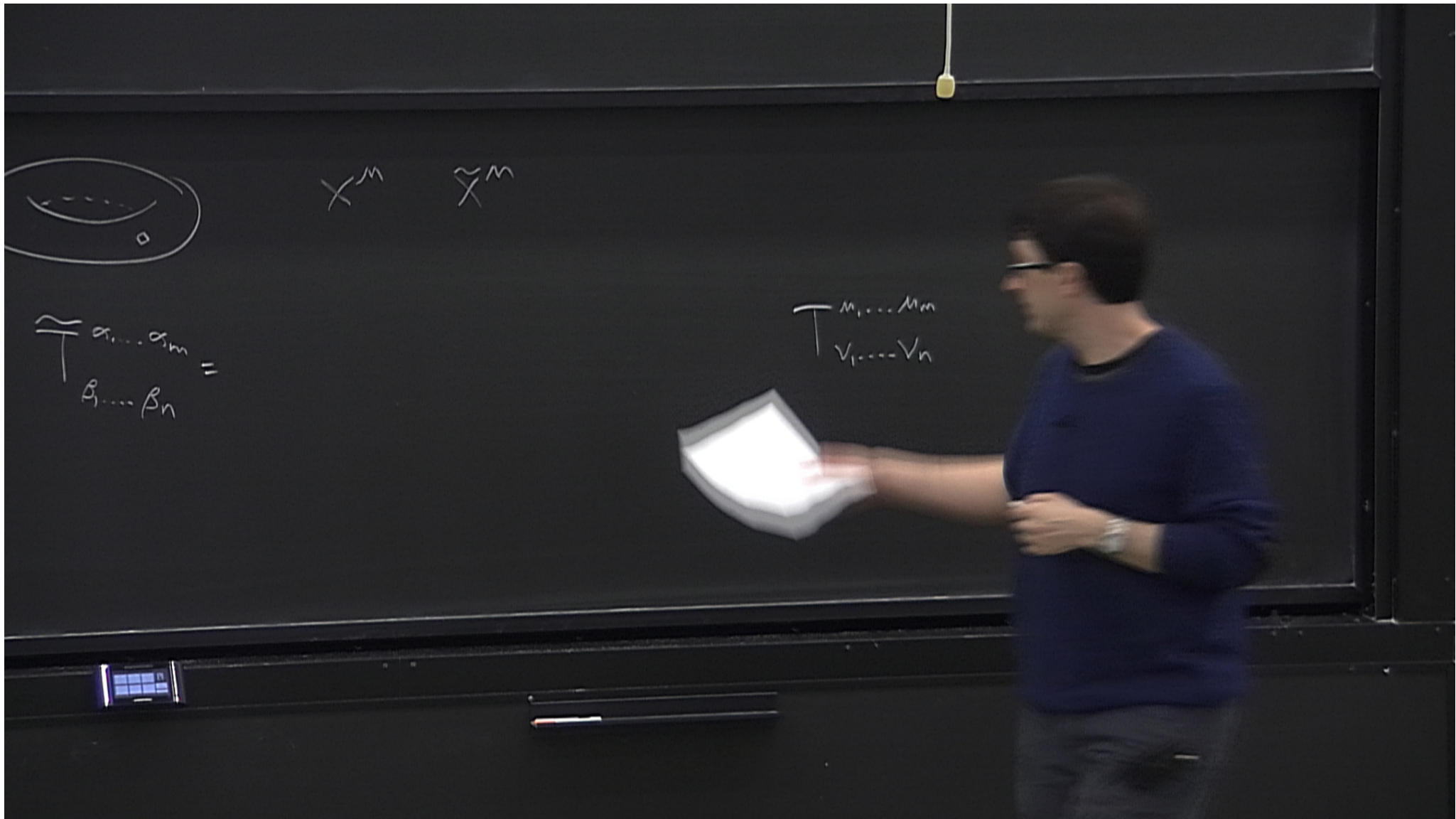
3. + metric $g_{\mu\nu}$

n -dim



$T_{\beta_1 \dots \beta_n}^{\alpha_1 \dots \alpha_m}$
 (m, n)

x^{μ}




 x^m
 $\tilde{x}^m$
 $(1,0): V^\alpha$
 contravariant

 $(0,1): V_\alpha$
 covariant

$$\tilde{T}_{\alpha_1 \dots \alpha_m}^{\beta_1 \dots \beta_n} = \frac{\partial \tilde{x}^{\alpha_1}}{\partial x^{\mu_1}} \dots \frac{\partial \tilde{x}^{\alpha_m}}{\partial x^{\mu_m}} \frac{\partial x^{\nu_1}}{\partial \tilde{x}^{\beta_1}} \dots \frac{\partial x^{\nu_n}}{\partial \tilde{x}^{\beta_n}}$$

 $T_{\nu_1 \dots \nu_n}^{\mu_1 \dots \mu_m}$



$$V^m \partial_m$$

$$\frac{\partial}{\partial x^m} = \partial_m$$

↓

$$\nabla_m$$

$$\partial_m V^\alpha = V^\alpha_{,m}$$

$$\nabla_m V^\alpha = V^\alpha_{;m}$$

$$V^m \partial_m = V^\alpha \partial_\alpha + \dots$$



$$V^m \partial_m = \tilde{V}^m \tilde{\partial}_m = V^m \frac{\partial \tilde{x}^m}{\partial x^m} \tilde{\partial}_m$$

∇^v

$$\frac{\partial}{\partial x^m} = \tilde{\partial}_m$$

$$\partial_m V^\alpha = V^\alpha_{,m}$$

$$\nabla_m V^\alpha = V^\alpha_{;m}$$

$$V^m \partial_m = V^\alpha \partial_\alpha + \dots$$



$$V^m \partial_m = \tilde{V}^m \tilde{\partial}_m = V^m \frac{\partial \tilde{x}^m}{\partial x^m} \tilde{\partial}_m$$

$$\tilde{\nabla}^v = \frac{\partial \tilde{x}^v}{\partial x^m} V^m$$

$$\frac{\partial}{\partial x^m} = \partial_m$$

$$\nabla_m$$

$$\partial_m V^\alpha = V^\alpha_{;m}$$

$$\alpha = V^\alpha_{;m}$$

$$\partial_m = V^\alpha \partial_\alpha + \dots$$

$$V^m \partial_m = \tilde{V}^m \tilde{\partial}_m = V^m \frac{\partial \tilde{x}^m}{\partial x^m} \tilde{\partial}_m$$

Diagram illustrating the transformation of the partial derivative operator ∂_m under a coordinate transformation. A large circle is drawn on the left, containing a small sketch of a coordinate system. An arrow points from the expression $\frac{\partial \tilde{x}^m}{\partial x^m} V^m$ to the transformed operator $\tilde{\partial}_m$ in the equation above.

$$\frac{\partial}{\partial x^m} = \partial_m$$

Diagram illustrating the transformation of the partial derivative operator ∂_m under a coordinate transformation. An arrow points from the expression $\frac{\partial}{\partial x^m}$ to the transformed operator ∂_m .

$$\partial_m V^\alpha = V^\alpha_{,m}$$

$$\nabla_m V^\alpha = V^\alpha_{;m}$$

$$V^m \partial_m = V^\sigma \partial_\sigma + \dots$$

$$\nabla_\mu d\tilde{x}^\mu = \tilde{x}^\mu_{,\nu} dx^\nu$$

$$\tilde{x}^\mu_{,\nu} = \frac{\partial \tilde{x}^\mu}{\partial x^\nu}$$

$$V^m \partial_m = \tilde{V}^m \tilde{\partial}_m = V^m \frac{\partial \tilde{x}^m}{\partial x^m} \tilde{\partial}_m$$

$$\tilde{\nabla}^v = \frac{\partial \tilde{x}^v}{\partial x^m} V^m$$

$$\tilde{\nabla}_v = \frac{\partial x^m}{\partial \tilde{x}^v} V_m$$

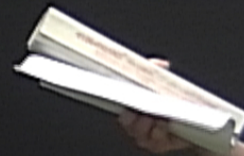
$$\frac{\partial}{\partial x^m} = \partial_m$$

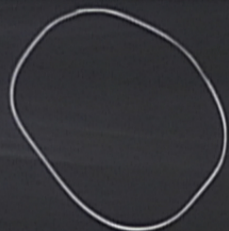
$$\downarrow$$

$$\tilde{\nabla}_m$$

$$\partial_m V^\alpha = V^\alpha_{,m}$$

$$\tilde{\nabla}_m V^\alpha = V^\alpha_{,m}$$

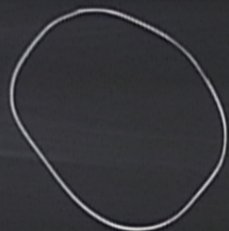




$$\partial_\nu V^\mu \quad \begin{matrix} (1,0) \\ (1,1) \end{matrix}$$

$$\tilde{\partial}_\nu \tilde{V}^\mu = \frac{\partial x^\alpha}{\partial \tilde{x}^\nu} \partial_\alpha \left(\frac{\partial \tilde{x}^\mu}{\partial x^\beta} V^\beta \right) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial \tilde{x}^\nu}{\partial x^\beta} \partial_\alpha V^\beta + \underbrace{\frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial^2 \tilde{x}^\mu}{\partial x^\nu \partial x^\alpha}}_{=0}$$

$$\nabla_\nu V^\mu = \partial_\nu V^\mu + \Gamma_{\nu\alpha}^\mu V^\alpha$$

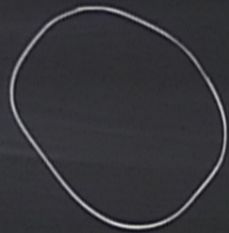


$$\partial_\nu V^\mu \quad \begin{matrix} (1,0) \\ (1,1) \end{matrix}$$

$$\tilde{\partial}_\nu \tilde{V}^\mu = \frac{\partial x^\alpha}{\partial \tilde{x}^\nu} \partial_\alpha \left(\frac{\partial \tilde{x}^\mu}{\partial x^\beta} V^\beta \right) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial \tilde{x}^\nu}{\partial x^\beta} \partial_\alpha V^\beta + \underbrace{\frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial^2 \tilde{x}^\mu}{\partial x^\alpha \partial x^\beta} V^\beta}_{=0}$$

$$\nabla_\nu V^\mu = \partial_\nu V^\mu + \Gamma_{\nu\alpha}^\mu V^\alpha$$

$$\tilde{\Gamma}_{\nu\alpha}^\mu = \frac{\partial \tilde{x}^\mu}{\partial x^\alpha}$$

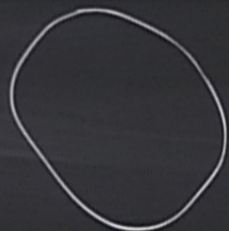


$$\partial_\nu V^\mu \quad \begin{matrix} (1,0) \\ (1,1) \end{matrix}$$

$$\tilde{\partial}_\nu \tilde{V}^\mu = \frac{\partial x^\alpha}{\partial \tilde{x}^\nu} \partial_\alpha \left(\frac{\partial \tilde{x}^\mu}{\partial x^\beta} V^\beta \right) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial \tilde{x}^\nu}{\partial x^\beta} \partial_\alpha V^\beta + \underbrace{\frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial^2 \tilde{x}^\mu}{\partial \tilde{x}^\nu \partial x^\beta} V^\beta}_{=0}$$

$$\nabla_\nu V^\mu = \partial_\nu V^\mu + \Gamma_{\nu\alpha}^\mu V^\alpha$$

$$\tilde{\Gamma}_{\beta\gamma}^\alpha = \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} \frac{\partial^2 \tilde{x}^\beta}{\partial \tilde{x}^\gamma \partial x^\alpha}$$

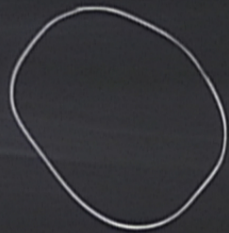


$$\partial_\nu V^\mu \quad \begin{matrix} (1,0) \\ (1,1) \end{matrix}$$

$$\tilde{\partial}_\nu \tilde{V}^\mu = \frac{\partial x^\alpha}{\partial \tilde{x}^\nu} \partial_\alpha \left(\frac{\partial \tilde{x}^\mu}{\partial x^\beta} V^\beta \right) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial \tilde{x}^\nu}{\partial x^\beta} \partial_\alpha V^\beta + \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial^2 \tilde{x}^\mu}{\partial x^\alpha \partial x^\beta} V^\beta$$

$$\tilde{\nabla}_\nu V^\mu = \partial_\nu V^\mu + \Gamma_{\nu\alpha}^\mu V^\alpha$$

$$\tilde{\Gamma}_{\beta\gamma}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\sigma}{\partial \tilde{x}^\beta} \frac{\partial x^\tau}{\partial \tilde{x}^\gamma} \Gamma_{\sigma\tau}^\alpha - \frac{\partial^2 \tilde{x}^\alpha}{\partial x^\beta \partial x^\tau} \frac{\partial x^\sigma}{\partial \tilde{x}^\beta} \frac{\partial x^\tau}{\partial \tilde{x}^\gamma}$$

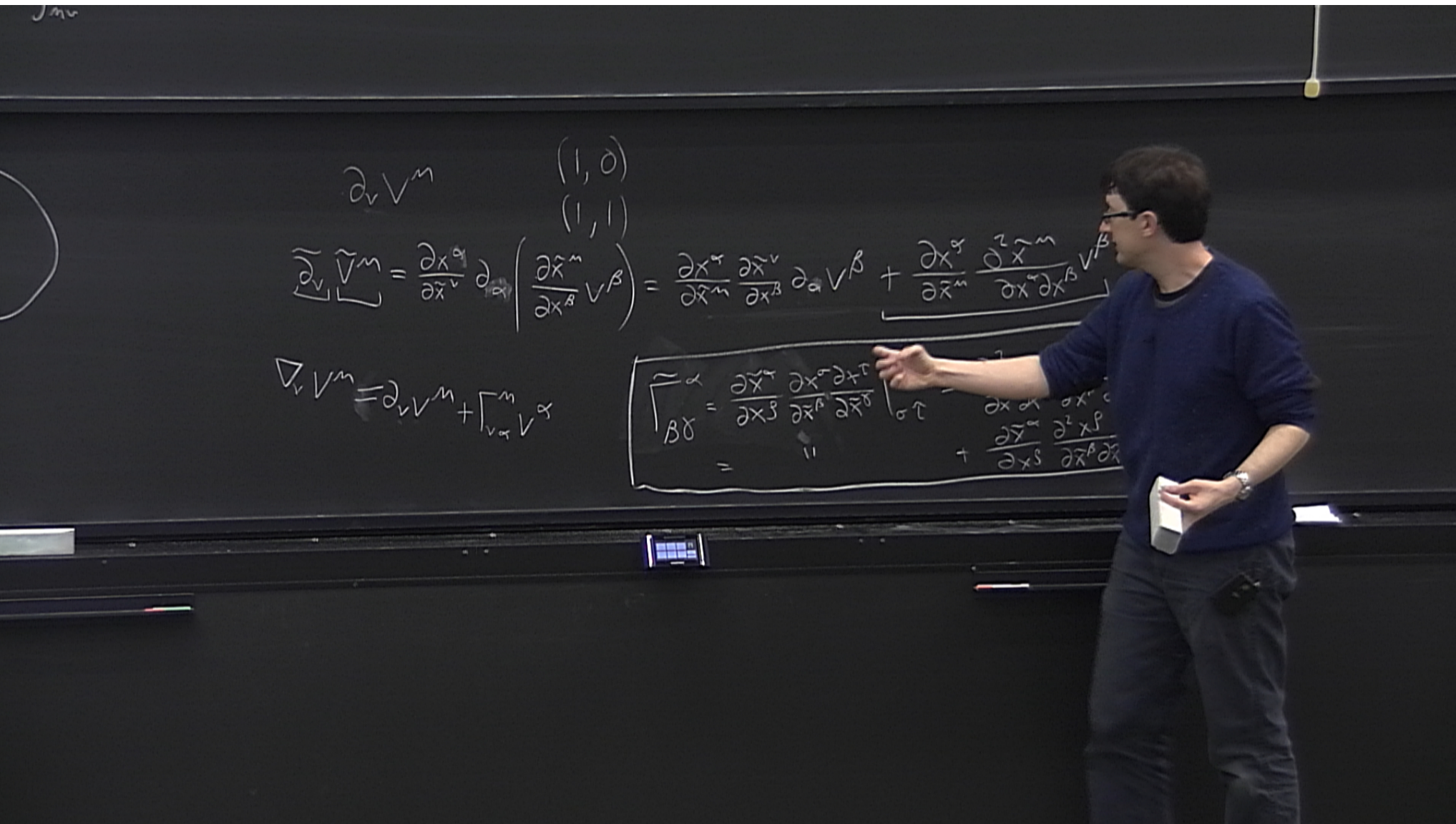


$$\partial_\nu V^\mu \quad \begin{matrix} (1,0) \\ (1,1) \end{matrix}$$

$$\tilde{\partial}_\nu \tilde{V}^\mu = \frac{\partial x^\alpha}{\partial \tilde{x}^\nu} \partial_\alpha \left(\frac{\partial \tilde{x}^\mu}{\partial x^\beta} V^\beta \right) = \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} \frac{\partial \tilde{x}^\nu}{\partial x^\beta} \partial_\alpha V^\beta + \frac{\partial x^\alpha}{\partial \tilde{x}^\mu} V^\beta$$

$$\nabla_\nu V^\mu = \partial_\nu V^\mu + \Gamma_{\nu\alpha}^\mu V^\alpha$$

$$\Gamma_{\beta\delta}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\sigma}{\partial \tilde{x}^\beta} \frac{\partial x^\tau}{\partial \tilde{x}^\delta} \Gamma_{\sigma\tau}^\beta - \frac{\partial}{\partial \tilde{x}^\delta} \left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \frac{\partial x^\beta}{\partial \tilde{x}^\delta} \right)$$



$$\nabla_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n} = \partial_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n} + \Gamma^{\alpha_1}_{m\delta} T^{\delta \dots \alpha_m}_{\beta_1 \dots \beta_n} + \dots - m$$

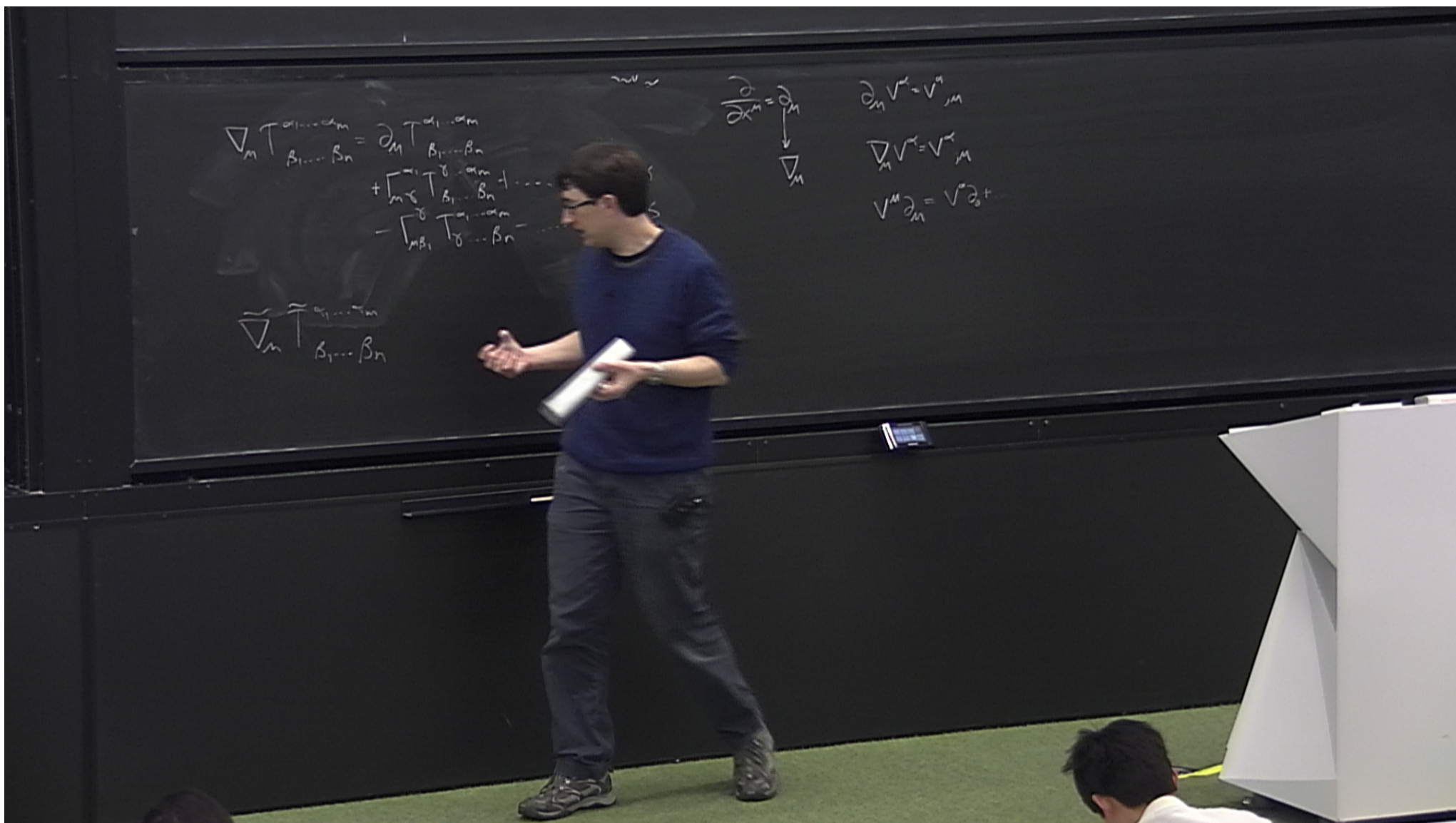
$\sim \sim$

$$\frac{\partial}{\partial x^m} = \partial_m \quad \downarrow \quad \nabla_m$$

$$\partial_m V^\alpha = V^\alpha_{,m}$$

$$\nabla_m V^\alpha = V^\alpha_{;m}$$

$$V^\mu \partial_\mu = V^\mu \partial_\mu$$



$$\nabla_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n} = \partial_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n} + \Gamma^{\alpha_1}_{m\gamma} T^{\gamma \dots \alpha_m}_{\beta_1 \dots \beta_n} + \dots - \Gamma^{\beta_1}_{m\gamma} T^{\alpha_1 \dots \alpha_m}_{\gamma \dots \beta_n} - \dots$$

m terms n terms

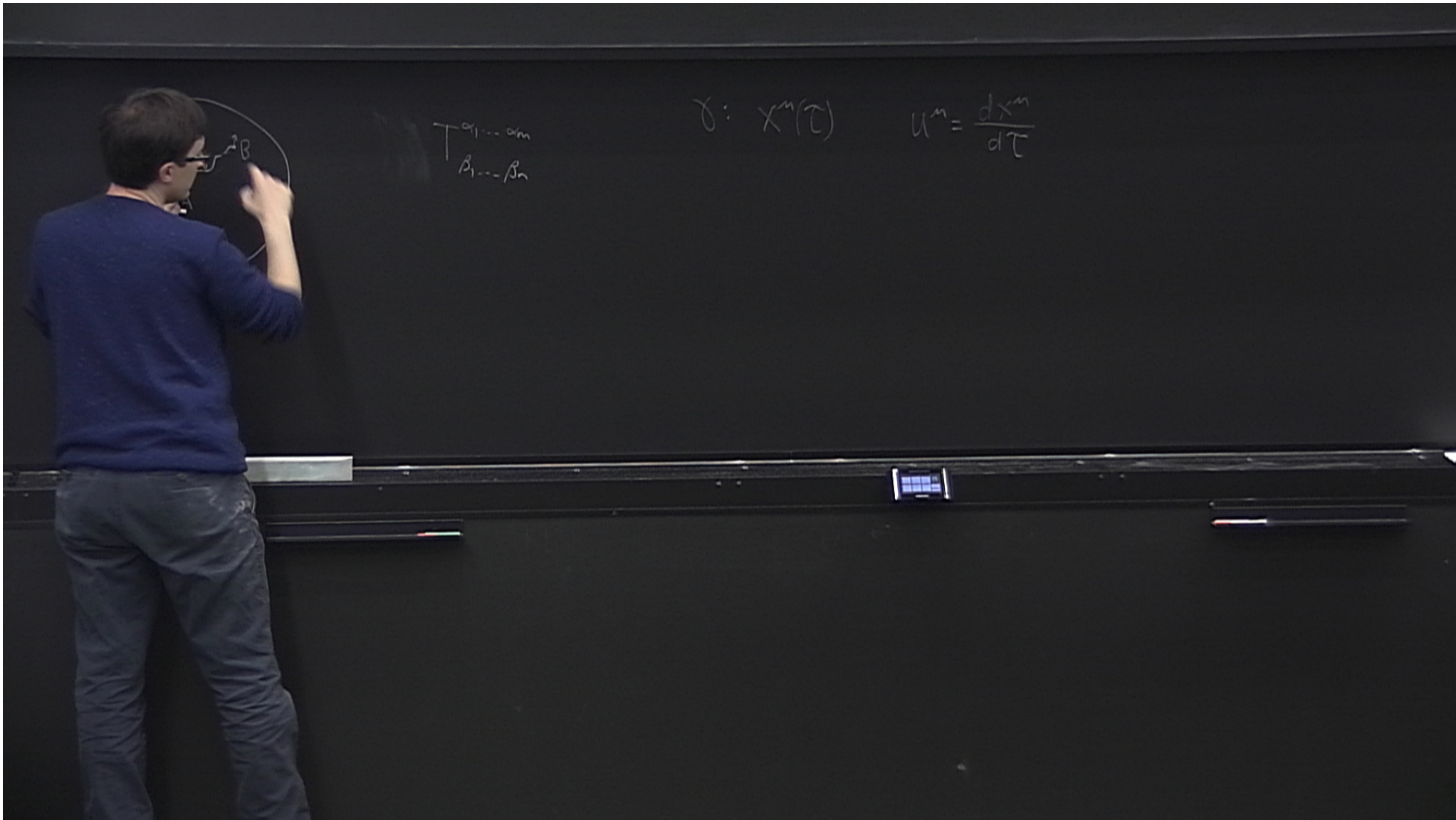
$$\tilde{\nabla}_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n}$$

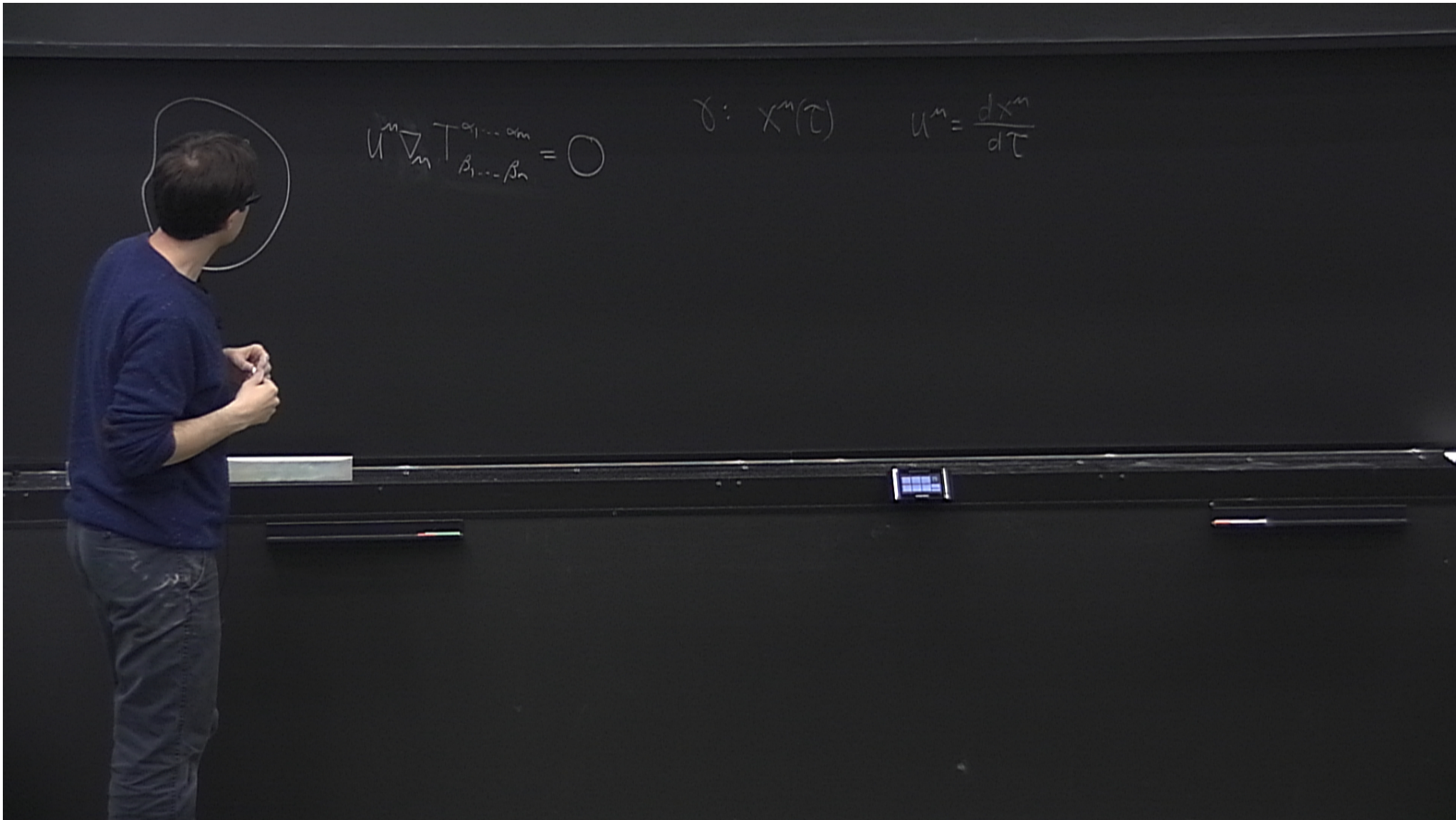
$$\frac{\partial}{\partial x^m} = \partial_m \quad \downarrow \quad \nabla_m$$

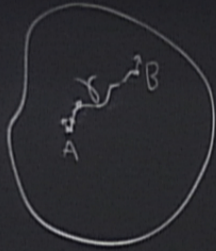
$$\partial_m V^\mu = V^\mu_{,m}$$

$$\nabla_m V^\mu = V^\mu_{;m}$$

$$V^\mu \partial_m = V^\mu \partial_\mu + \dots$$





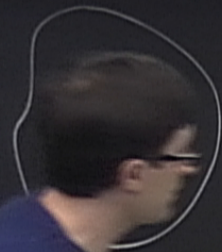


$$U^m \nabla_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m} = 0$$

$$U^m \nabla_m U^\nu = 0 \quad (\text{geodesic})$$

$$\gamma: X^m(\tau)$$

$$U^m = \frac{dx^m}{d\tau}$$

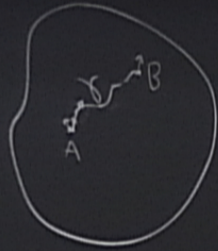


$$U^m \nabla_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m} = 0$$

$$U^m \nabla_m U^\nu = 0 \quad (\text{geodesic})$$

$$\gamma: X^m(\tau)$$

$$U^m = \frac{dx^m}{d\tau}$$



$$U^m \nabla_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m} = 0$$

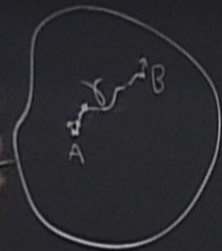
$$U^m \nabla_m U^\nu = 0 \quad (\text{geodesic})$$

$$\gamma: X^m(\tau)$$

$$U^m = \frac{dx^m}{d\tau}$$

$$(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu)$$

$$(\nabla_\mu \nabla_\nu - \nabla_\nu \nabla_\mu)$$



$$U^m \nabla_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m} = 0$$

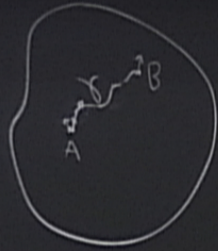
$$U^m \nabla_m U^\nu = 0 \quad (\text{geodesic})$$

$$\gamma: X^m(\tau)$$

$$U^m = \frac{dx^m}{d\tau}$$

$$(\partial_n \partial_\nu - \partial_\nu \partial_n) f = 0$$

$$(\nabla_n \nabla_\nu - \nabla_\nu \nabla_n) v^\alpha = R^\alpha_{\beta \mu \nu} v^\beta$$



$$U^m \nabla_m T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_m} = 0$$

$$U^m \nabla_m U^\nu = 0 \quad (\text{geodesic})$$

$$\gamma: X^m(\tau) \quad U^m = \frac{dx^m}{d\tau}$$

$$(\partial_n \partial_\nu - \partial_\nu \partial_n) f = 0$$

$$(\nabla_n \nabla_\nu - \nabla_\nu \nabla_n) v^\alpha = R^\alpha_{\beta \mu \nu} v^\beta$$

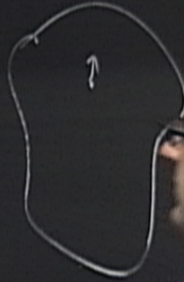
$$u^m = \frac{dx^m}{d\tau}$$

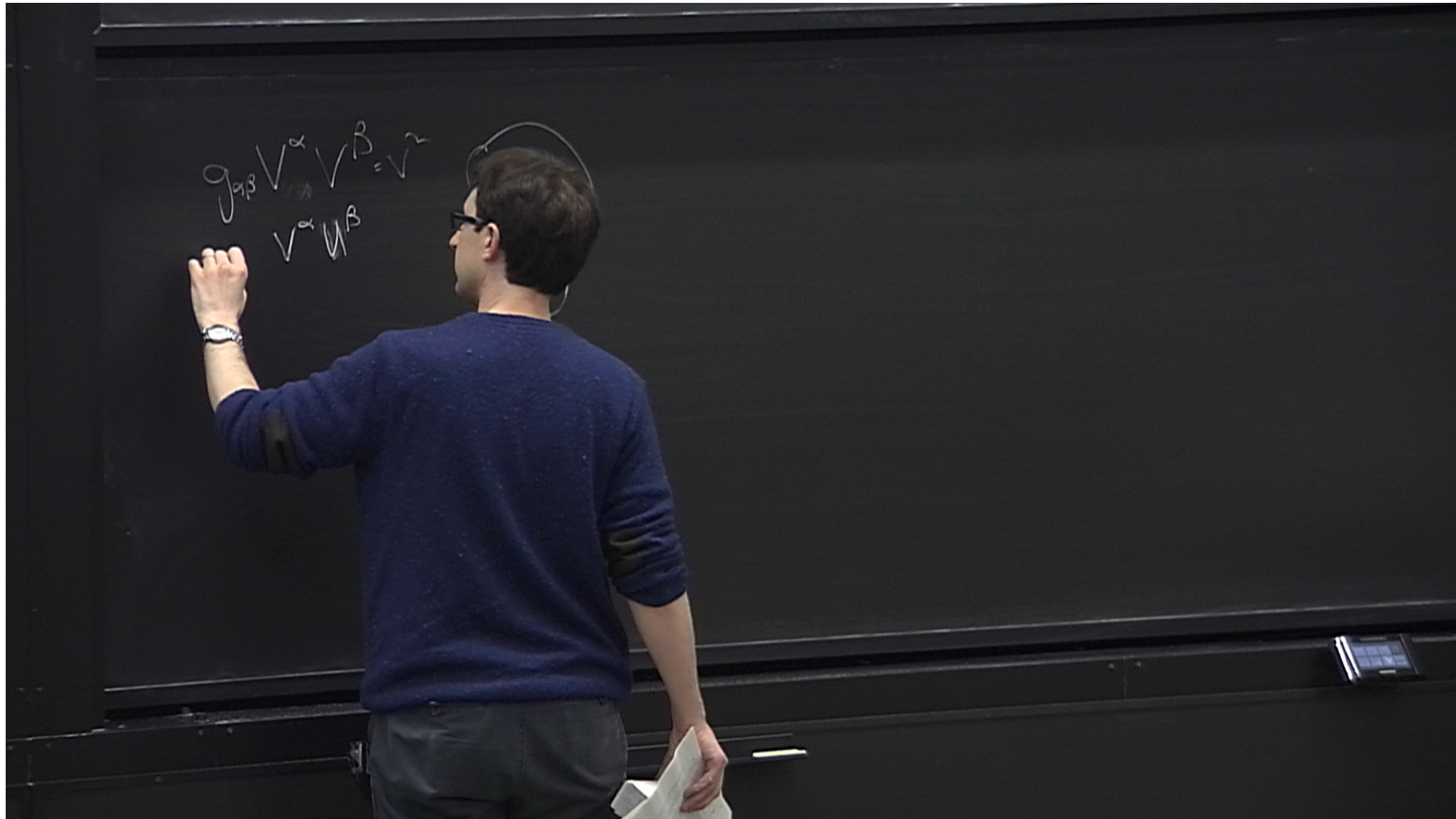
$$R_{\beta\delta} = R^\alpha_{\delta} \delta_{\alpha\beta}$$

$$(\partial_m \partial_\nu - \partial_\nu \partial_m) f = 0$$

$$(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) v^\alpha = R^\alpha_{\beta\mu\nu} v^\beta$$

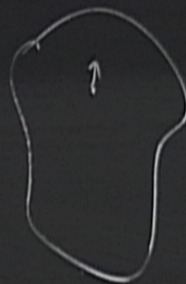
$$g_{\alpha\beta} V^\alpha V^\beta$$



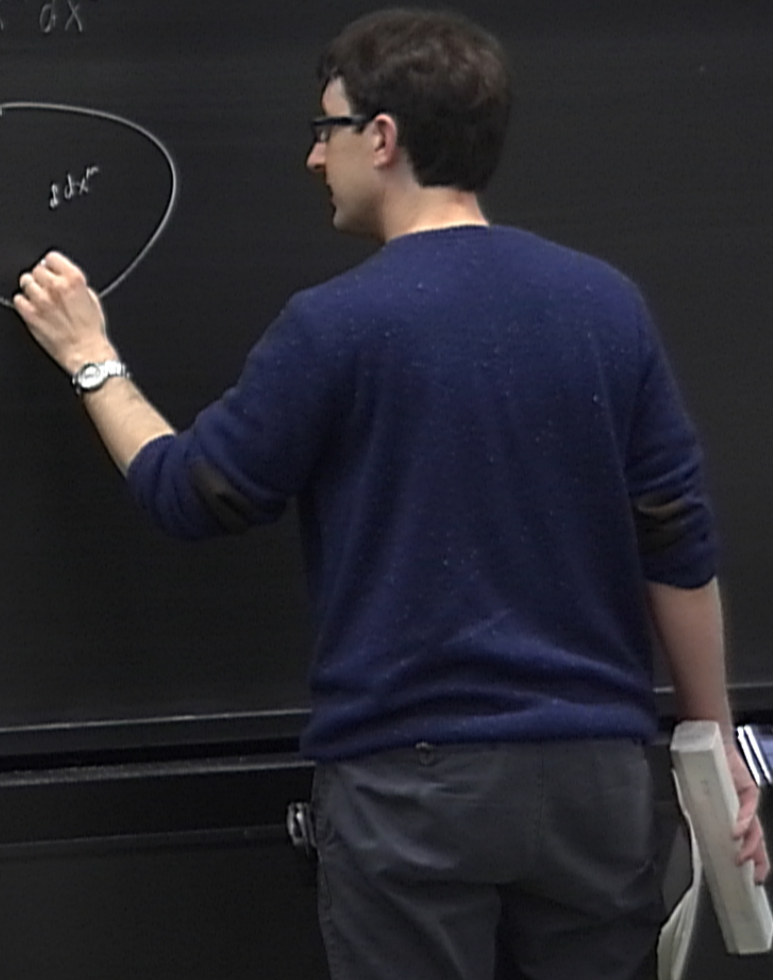


$$g_{\alpha\beta} V^\alpha V^\beta = V^2$$

$$g_{\alpha\beta} V^\alpha U^\beta$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

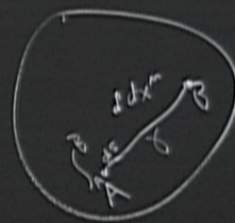


$$g_{\alpha\beta} V^\alpha V^\beta = V^2$$

$$g_{\alpha\beta} V^\alpha U^\beta$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



$$\int_A^B ds$$

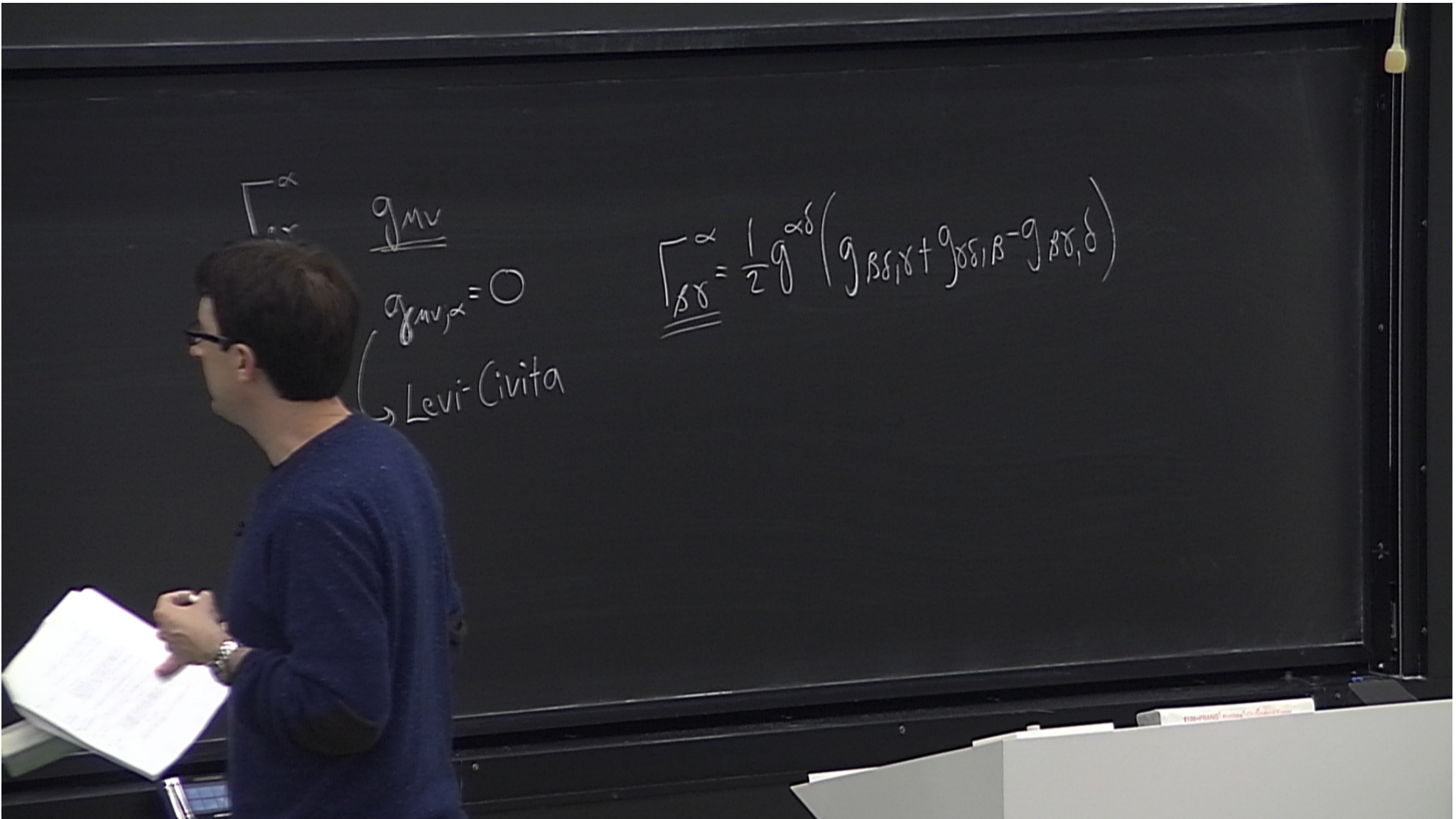
$$u^m = \frac{dx^m}{d\tau}$$

$$R_{\beta\delta} = R^{\alpha}{}_{\beta\alpha\delta}$$

$$(\partial_m \partial_\nu - \partial_\nu \partial_m) f = 0$$

$$\boxed{R} = g^{\beta\delta} R_{\beta\delta}$$

$$(\partial_\nu \partial_m - \partial_m \partial_\nu) v^\alpha = R^{\alpha}{}_{\beta\mu\nu} v^\beta$$

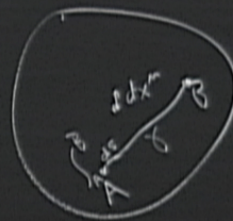


$$g_{\alpha\beta} V^\alpha V^\beta = V^2$$

$$g_{\alpha\beta} V^\alpha U^\beta$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



$$\int_A^B ds$$

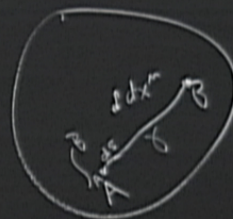
p.t. U^μ

$$g_{\alpha\beta} V^\alpha V^\beta = V^2$$

$$g_{\alpha\beta} V^\alpha U^\beta$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



$$\int_A^B ds$$

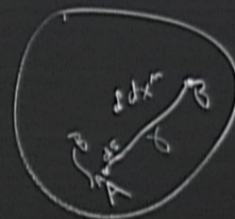
p.t. - U^μ
stationary length

$$g_{\alpha\beta} V^\alpha V^\beta = V^2$$

$$g_{\alpha\beta} V^\alpha U^\beta$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



$$\int_A^B ds$$

p.t. U^μ $\longleftarrow$ $U^\mu \nabla_\mu U^\nu = 0$

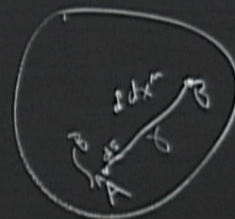
stationary length

$$g_{\alpha\beta} V^\alpha V^\beta = V^2$$

$$g_{\alpha\beta} V^\alpha U^\beta$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



$$\int_A^B ds$$

p.t. U^μ $\leftarrow U^\mu \nabla_\mu U^\nu = 0$

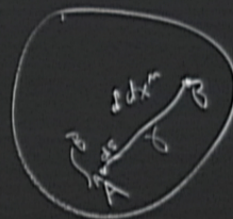
stationary length $\leftarrow$

$$g_{\alpha\beta} V^\alpha V^\beta = V^2$$

$$g_{\alpha\beta} V^\alpha U^\beta$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



$$\int_A^B ds$$

p.t. $U^\mu \leftarrow U^\mu \nabla_\mu U^\nu = 0$

stationary length $\leftarrow L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$

$$\dot{x}^\mu \equiv \frac{dx^\mu}{d\tau}$$

$$u^m = \frac{dx^m}{d\tau}$$

$$R_{\beta\delta} = R^{\alpha}{}_{\beta\alpha\delta}$$

$$(\partial_m \partial_\nu - \partial_\nu \partial_m) f = 0$$

$$\boxed{R} = g^{\beta\delta} R_{\beta\delta}$$

$$L(q, \dot{q}) \quad \dot{q} = \frac{dq}{d\tau}$$

$$\frac{\partial L}{\partial \dot{q}} = \frac{d}{d\tau}$$

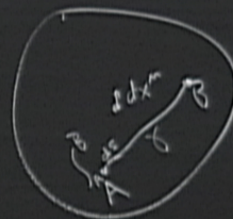
$$(\partial_\nu \partial_m - \partial_m \partial_\nu) v^\alpha = R^{\alpha}{}_{\beta\mu\nu} v^\beta$$

$$g_{\alpha\beta} V^\alpha V^\beta = V^2$$

$$g_{\alpha\beta} V^\alpha U^\beta$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



$$\int_A^B ds$$

p.t. $U^\mu \leftarrow U^\mu \nabla_\mu U^\nu = 0$

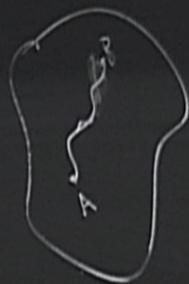
stationary length $\leftarrow L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$

$$\frac{\partial L}{\partial x^\mu}$$

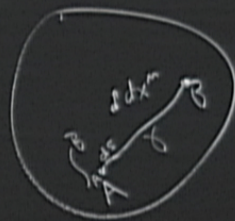
$$\dot{x}^\mu = \frac{dx^\mu}{dt}$$

$$g_{\alpha\beta} V^\alpha V^\beta = V^2$$

$$g_{\alpha\beta} V^\alpha U^\beta$$



$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$



$$\int_A^B ds$$

p.t. $U^\mu \leftarrow U^\mu \nabla_\mu U^\nu = 0$

stationary length $\leftarrow L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$

$$\dot{x}^\mu \equiv \frac{dx^\mu}{d\tau}$$

$$\frac{\partial L}{\partial x^\mu} = \frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{x}^\mu} \right)$$

$$u^m = \frac{dx^m}{d\tau}$$

$$(\partial_m \partial_\nu - \partial_\nu \partial_m) f = 0$$

$$(\partial_\nu \partial_m - \partial_m \partial_\nu) v^\alpha = R^\alpha_{\beta\mu\nu} v^\beta$$

$$R_{\beta\delta} = R^\alpha_{\beta\alpha\delta}$$

$$R = g^{\beta\delta} R_{\beta\delta}$$

$$L(q, \dot{q}) \quad \dot{q} = \frac{dq}{d\tau}$$

$$\frac{\partial L}{\partial q} = \frac{d}{d\tau} \left(\frac{\partial L}{\partial \dot{q}} \right)$$