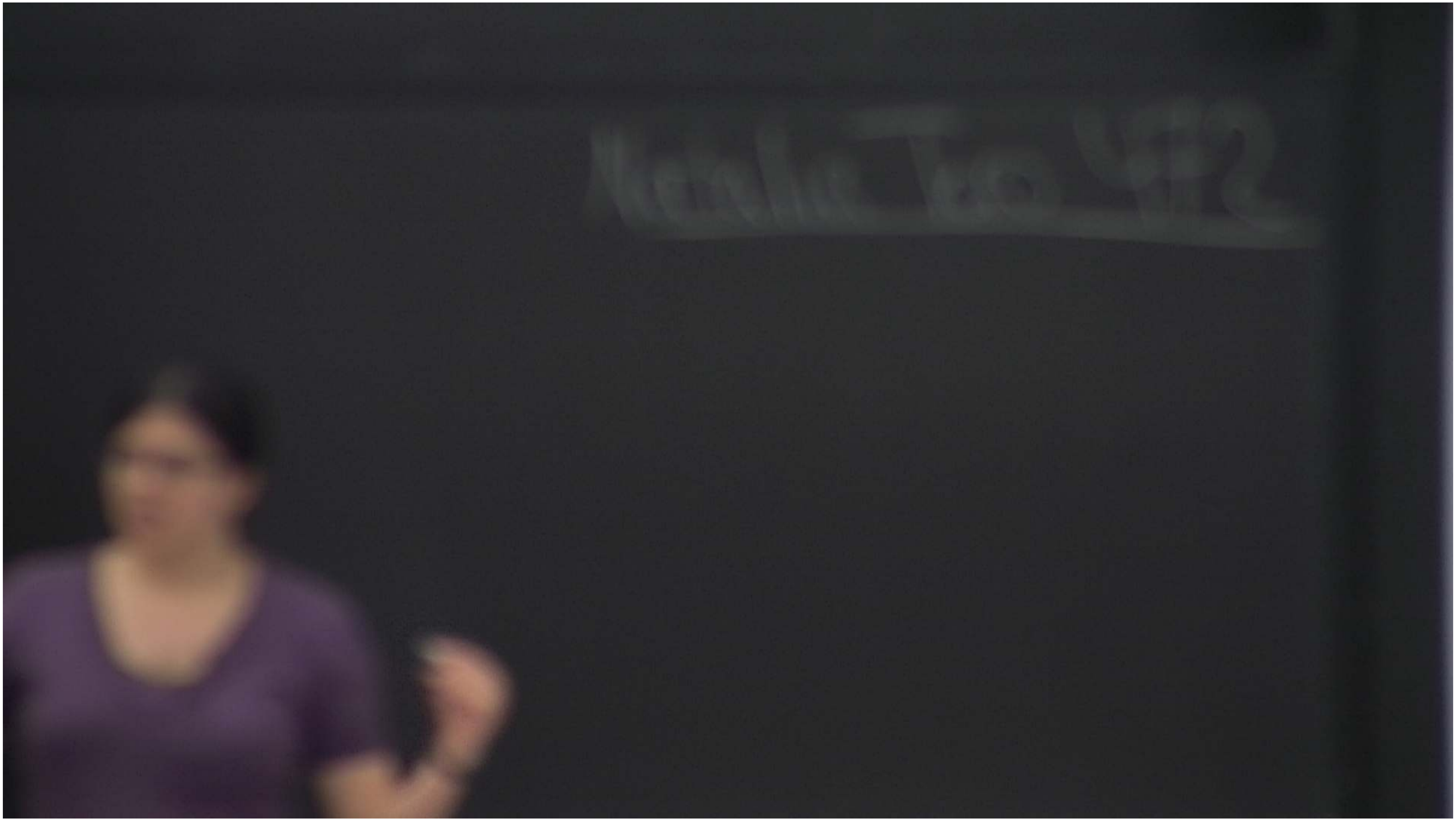
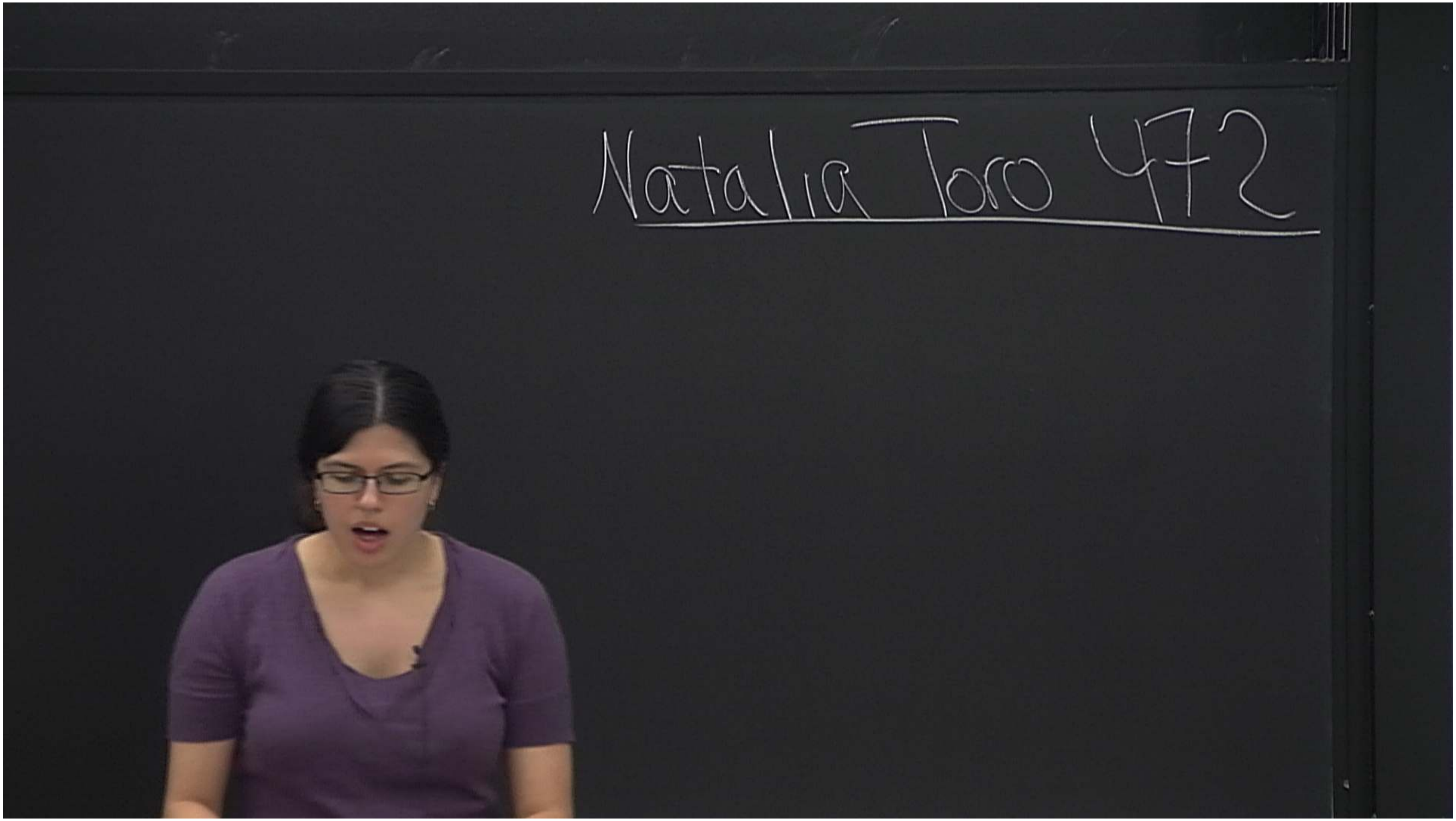
Title: 12/13 PSI - Beyond the Standard Model Lecture 8

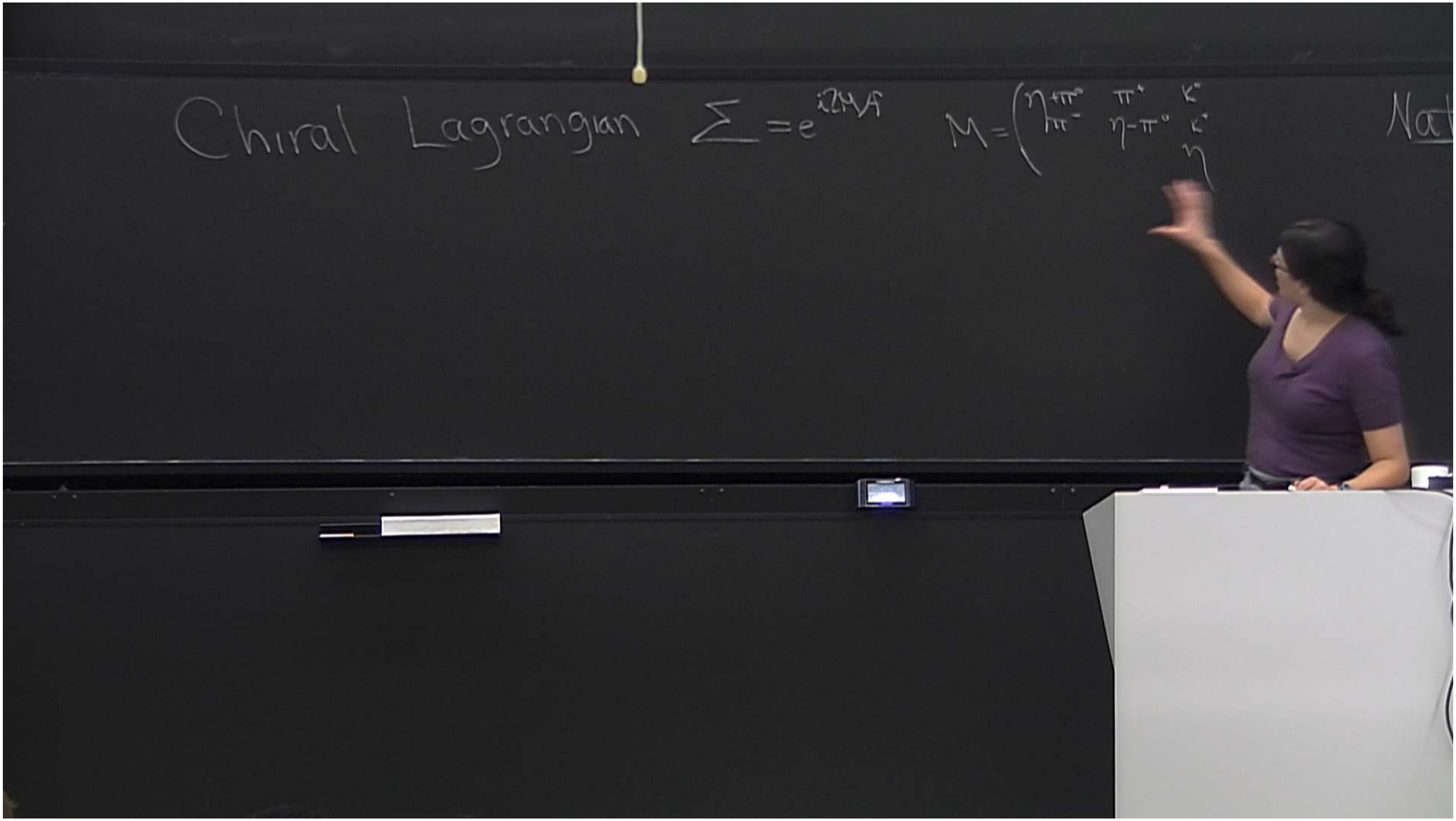
Date: Feb 28, 2013 09:00 AM

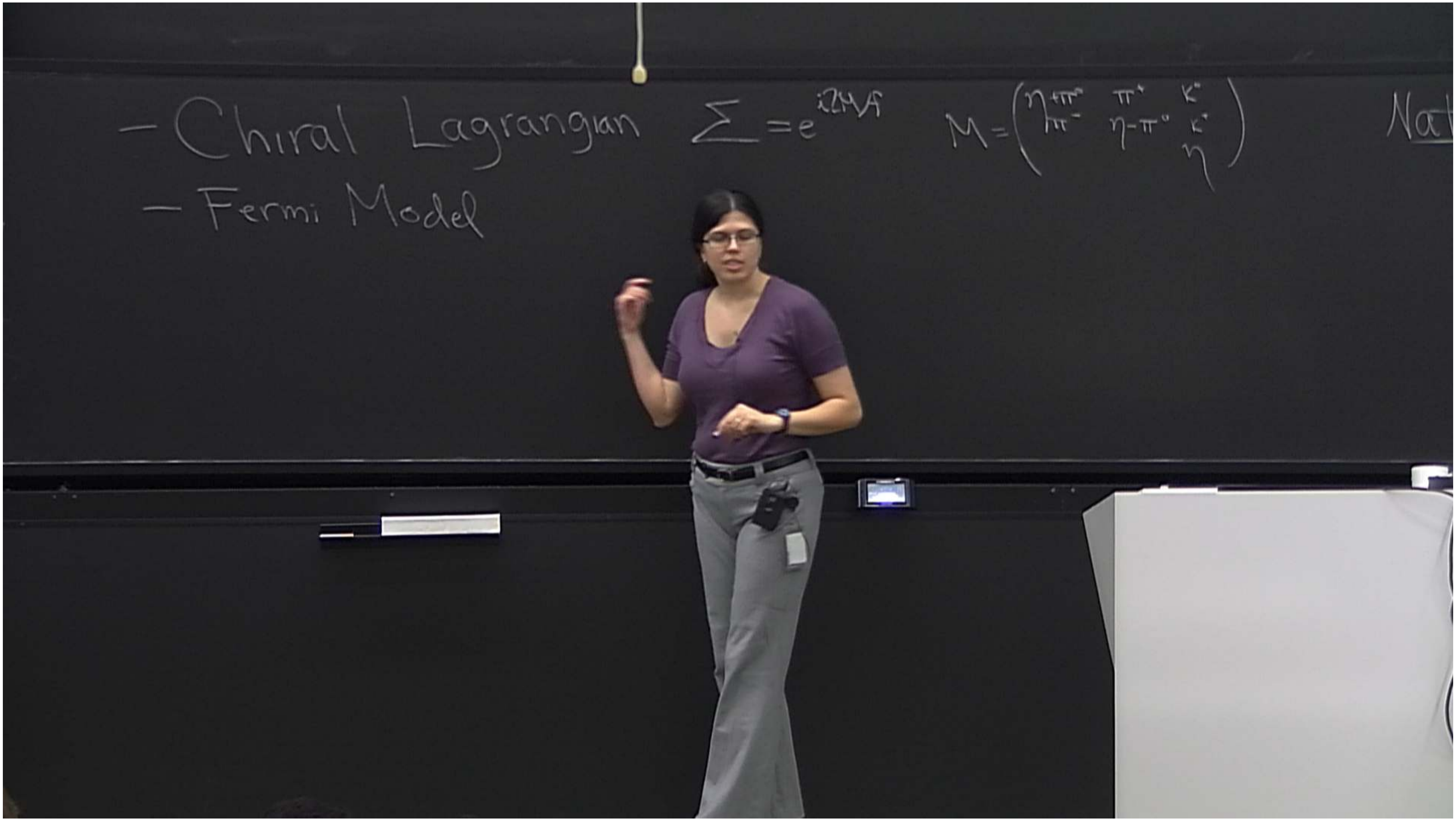
URL: http://pirsa.org/13020098

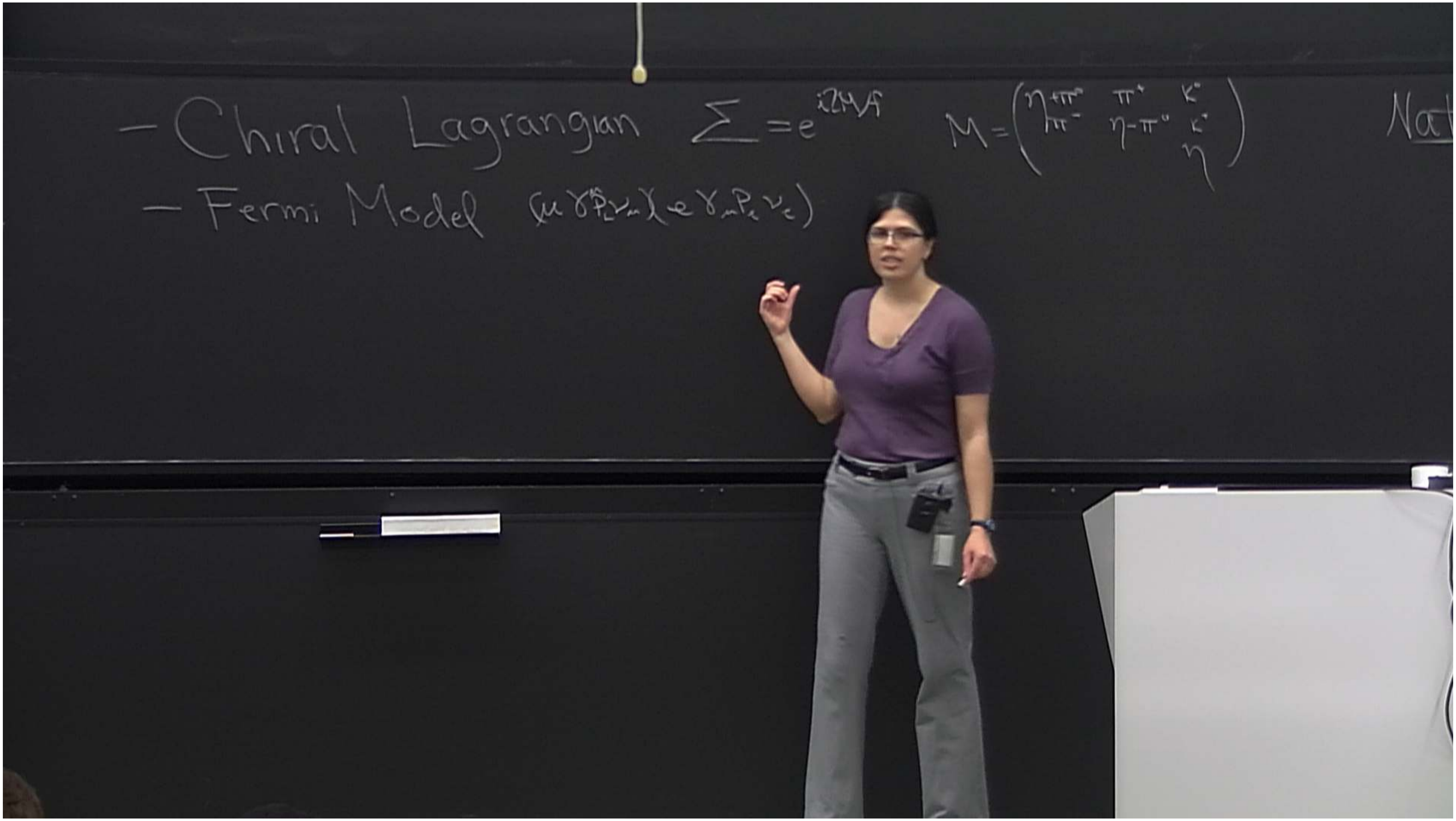
Abstract:

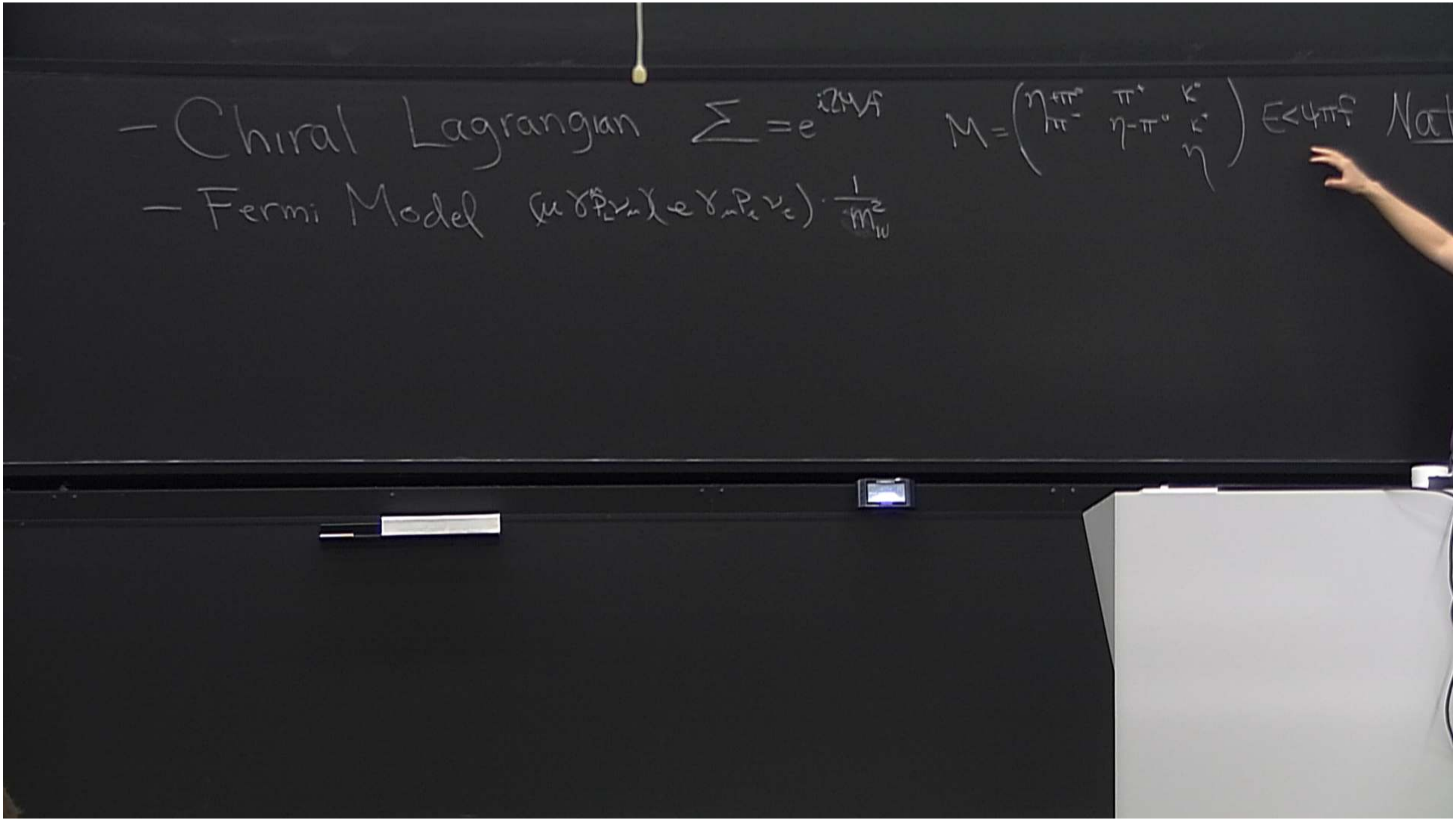


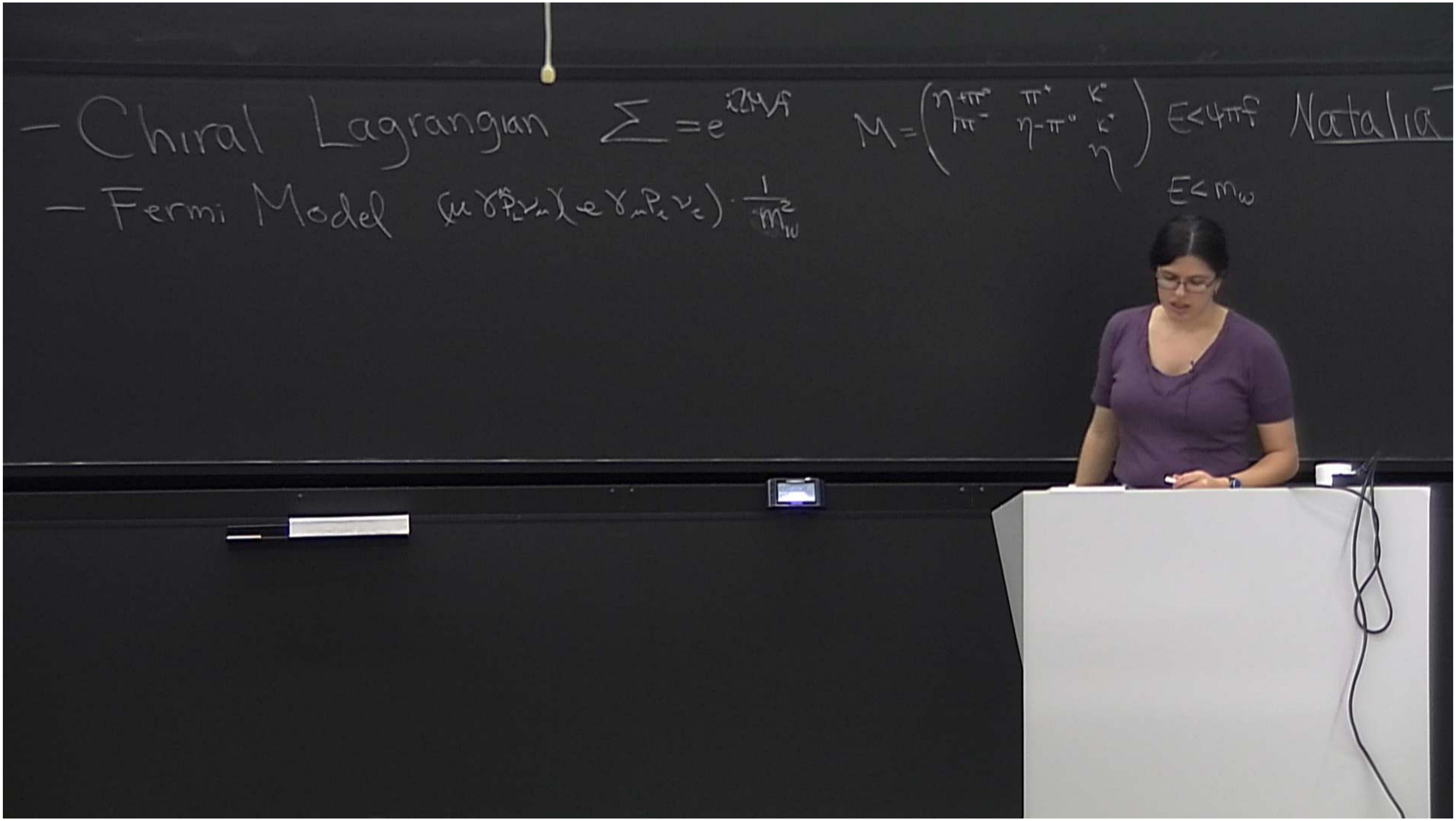

Natalia Toro 472

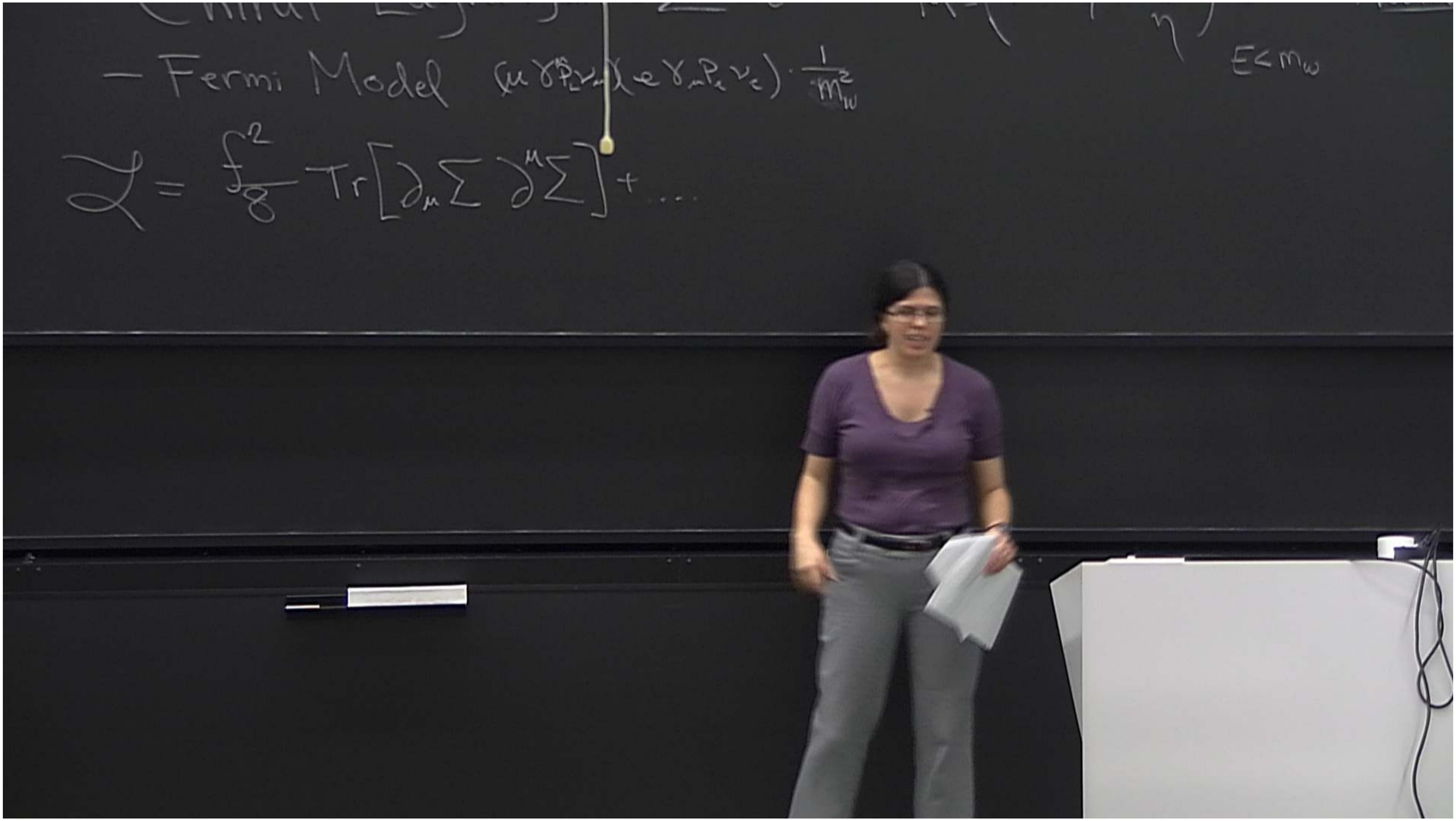

Chiral Lagrangian

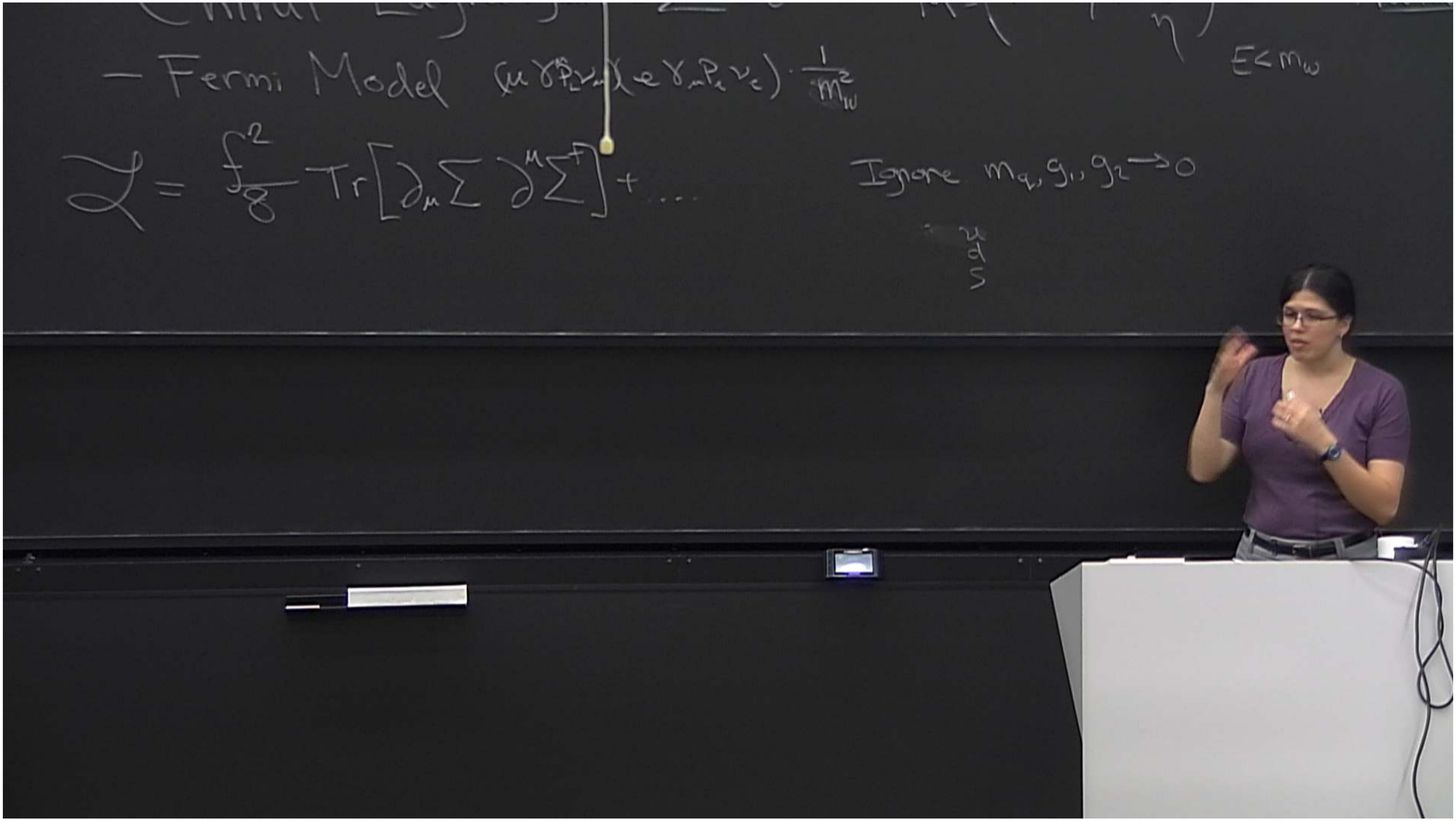

- Chiral Lagrangian
- Fermi Model

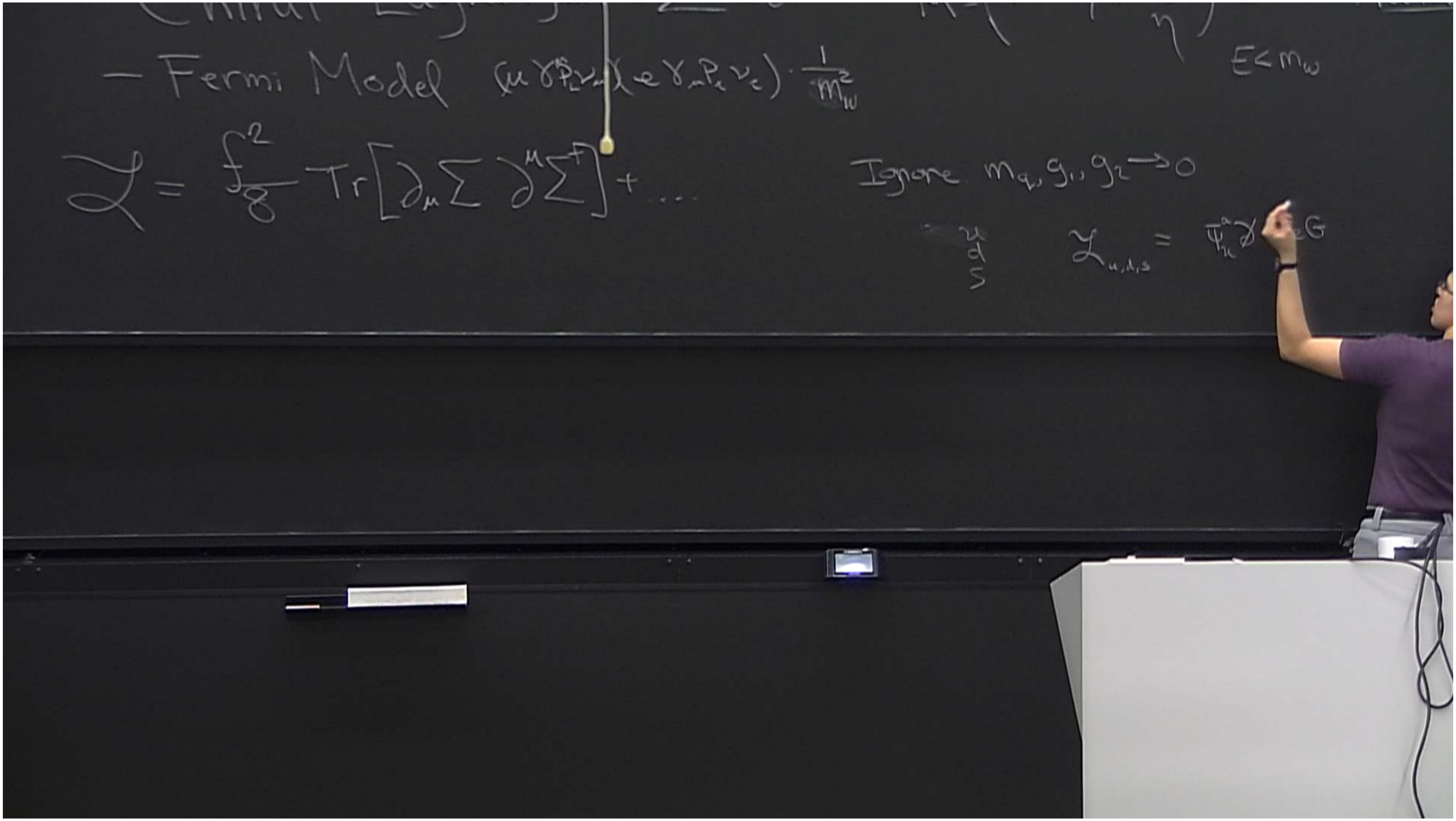

- Chiral Lagrangian
- Fermi Model

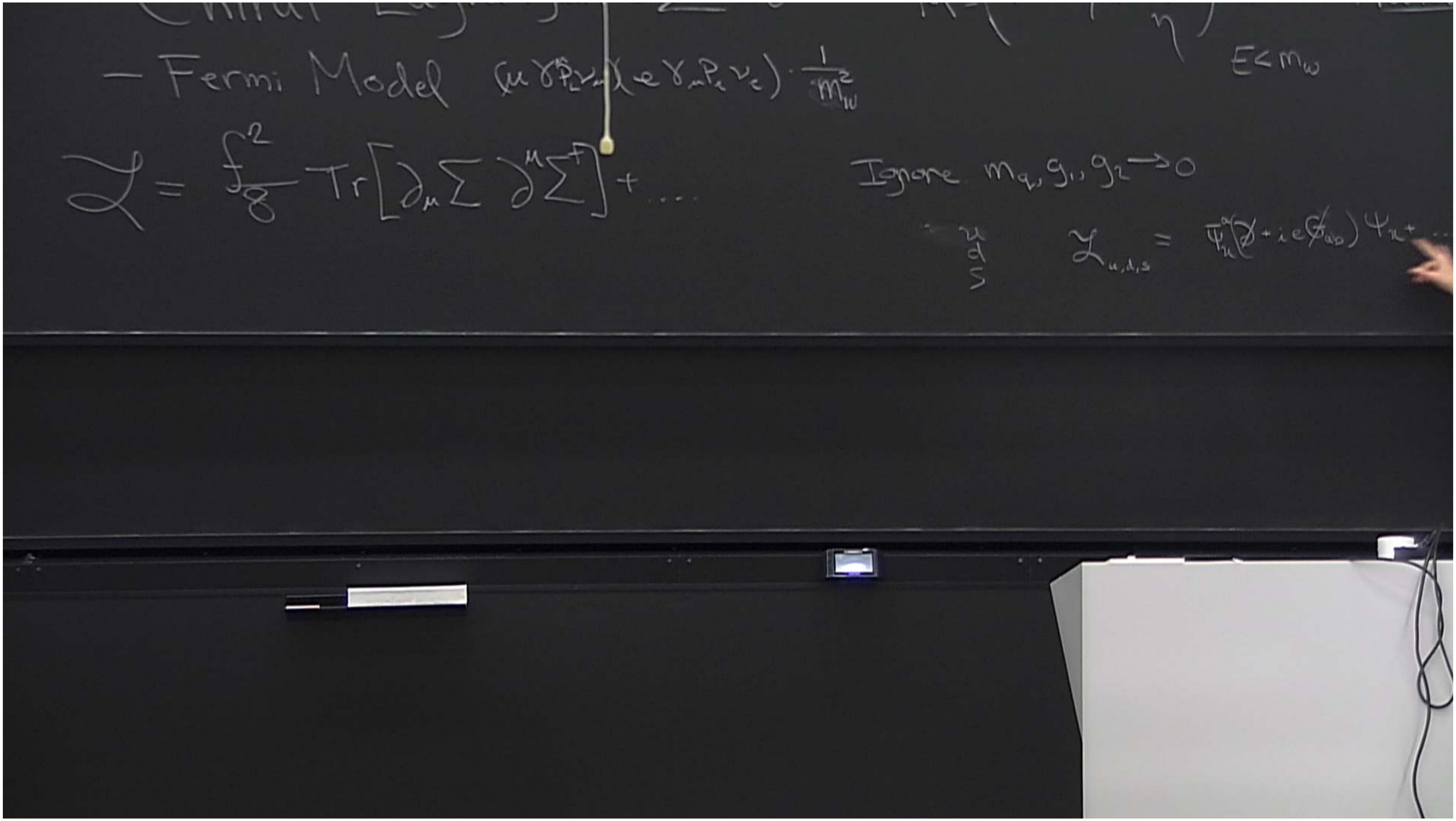

- Chiral Lagrangian Σ = e^{i2Π/f}
M = (η+π⁰ π⁺ K⁰ / π⁻ η−π⁰ K⁺ / η) E < 4πf
- Fermi Model (μ γ P_L ν_μ)(e γ_μ P_L ν_e) · 1/m_W²

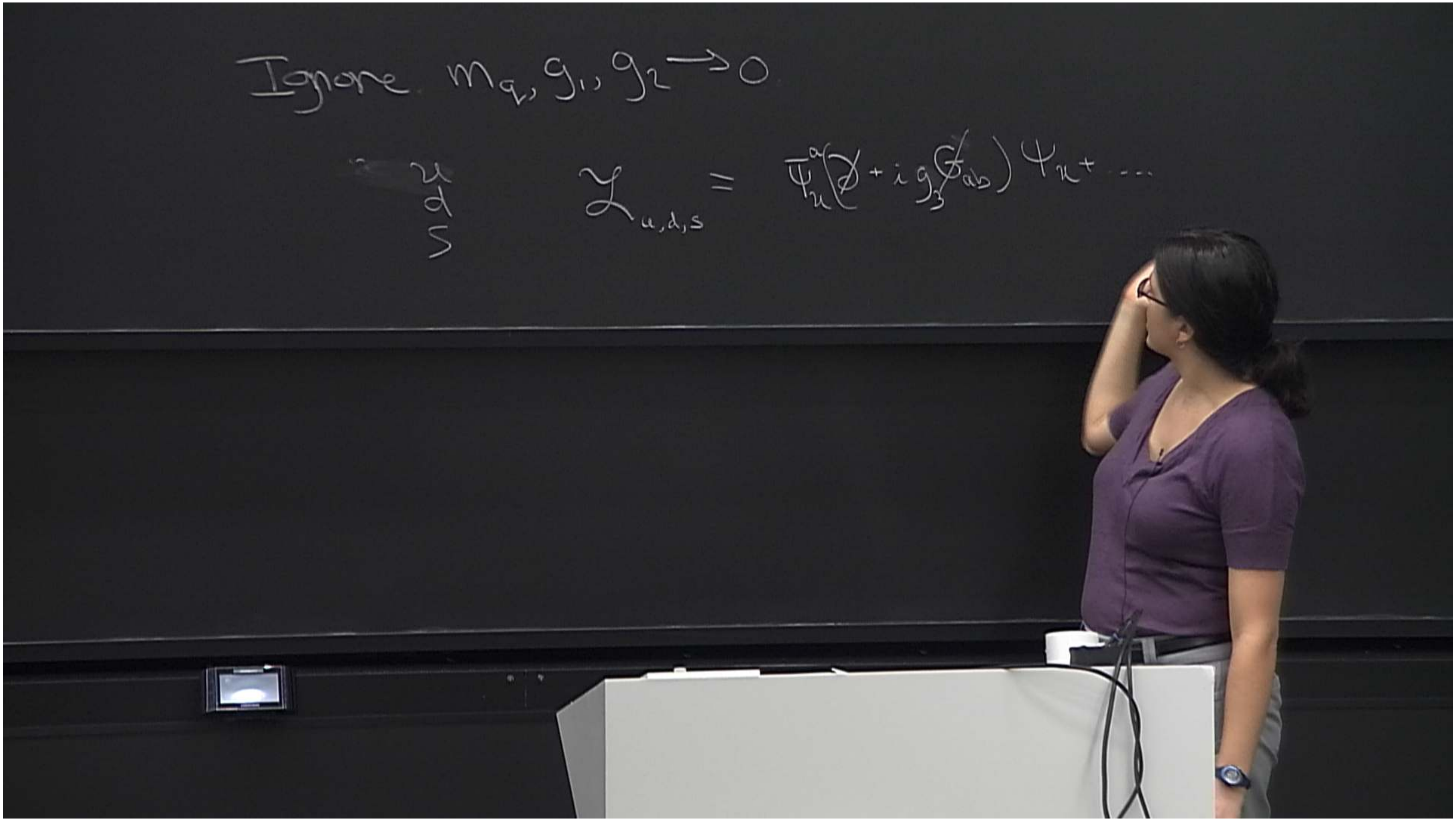

- Chiral Lagrangian
- Fermi Model
E < 4πf
E < mω
Natalia

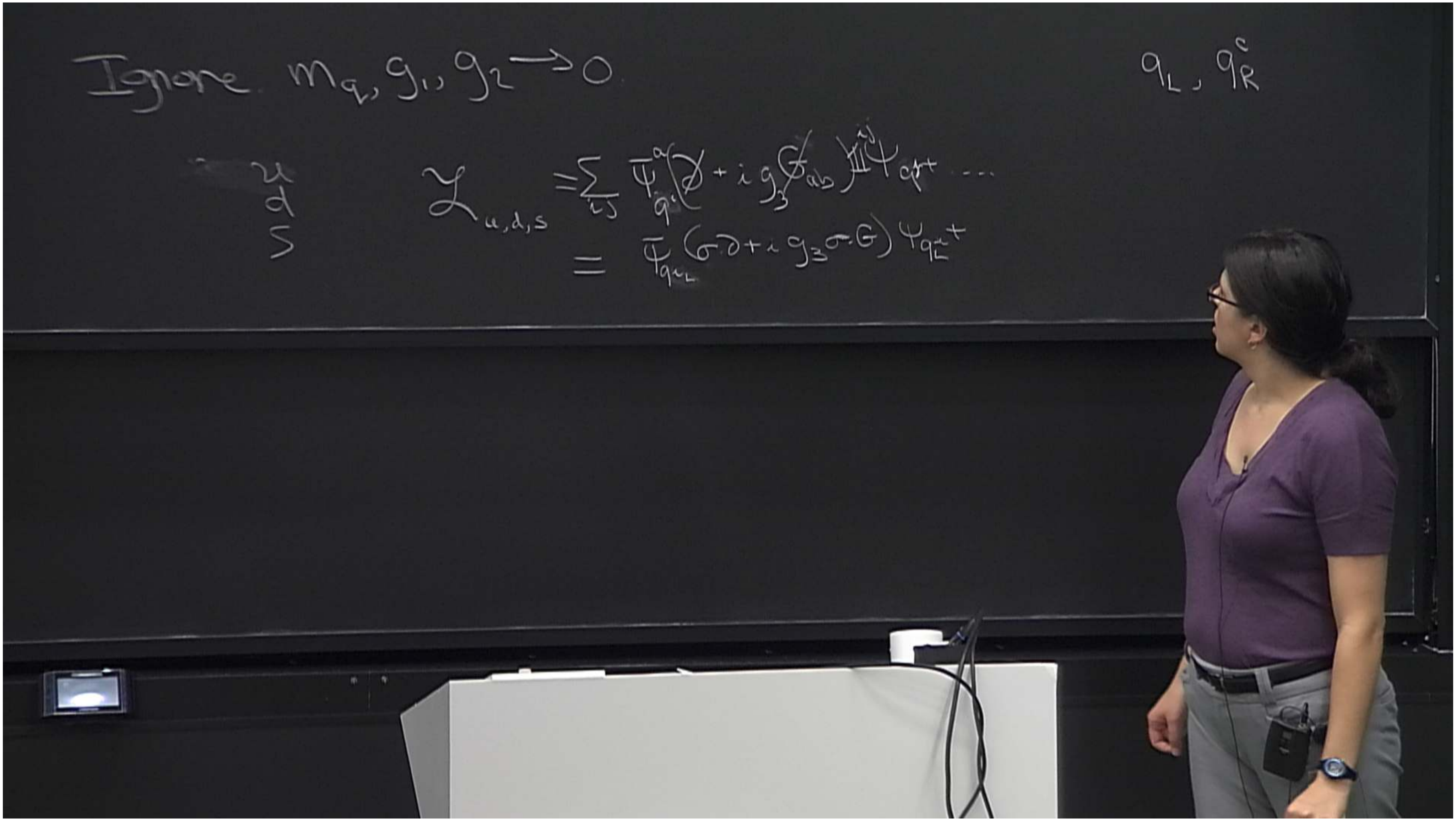

– Fermi Model
E < m_W
$\mathcal{L} = \frac{f^2}{8} \mathrm{Tr}[\partial_\mu \Sigma \partial^\mu \Sigma] + \ldots$

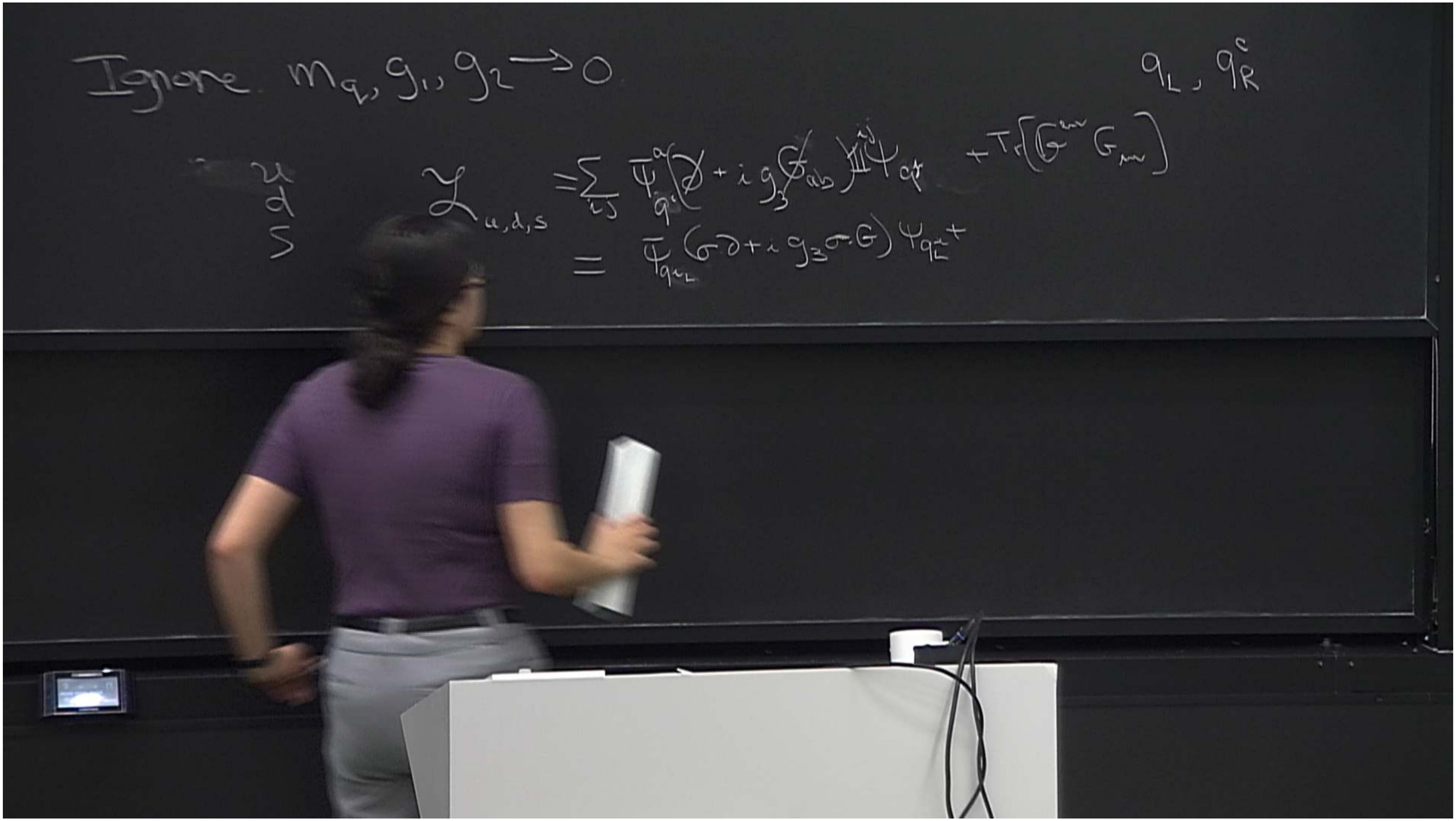

— Fermi Model
E < m_W
Ignore m_q, g_1, g_2 → 0
u
d
s

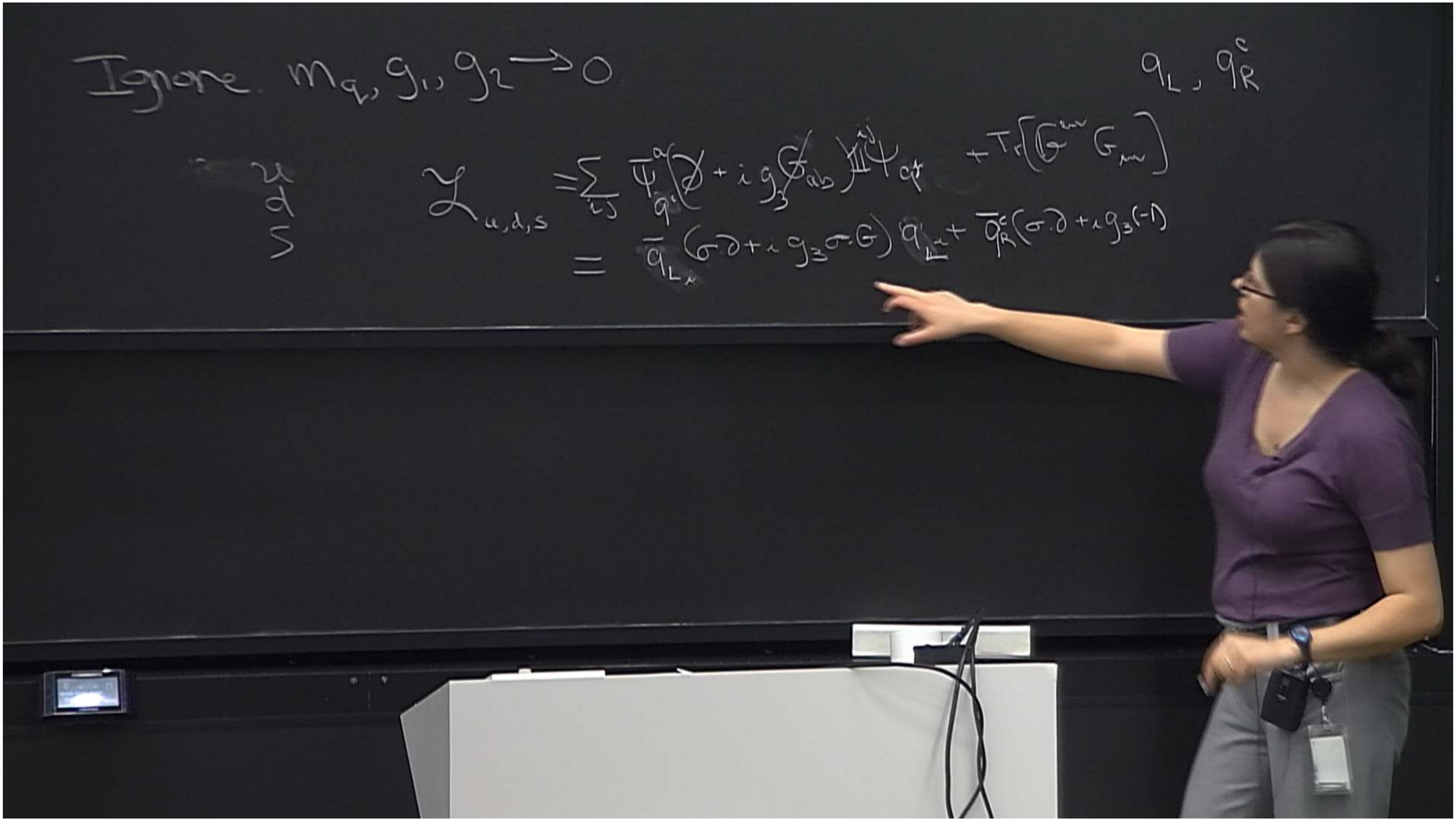

– Fermi Model $(\mu \gamma^\mu P_L \nu_\mu)(e \gamma_\mu P_L \nu_e) \cdot \frac{1}{m_W^2}$
$E < m_W$
$\mathcal{L} = \frac{f^2}{8} \mathrm{Tr}[\partial_\mu \Sigma \, \partial^\mu \Sigma^\dagger] + \ldots$
Ignore $m_q, g_1, g_2 \to 0$
u
d
s
$\mathcal{L}_{u,d,s} =$

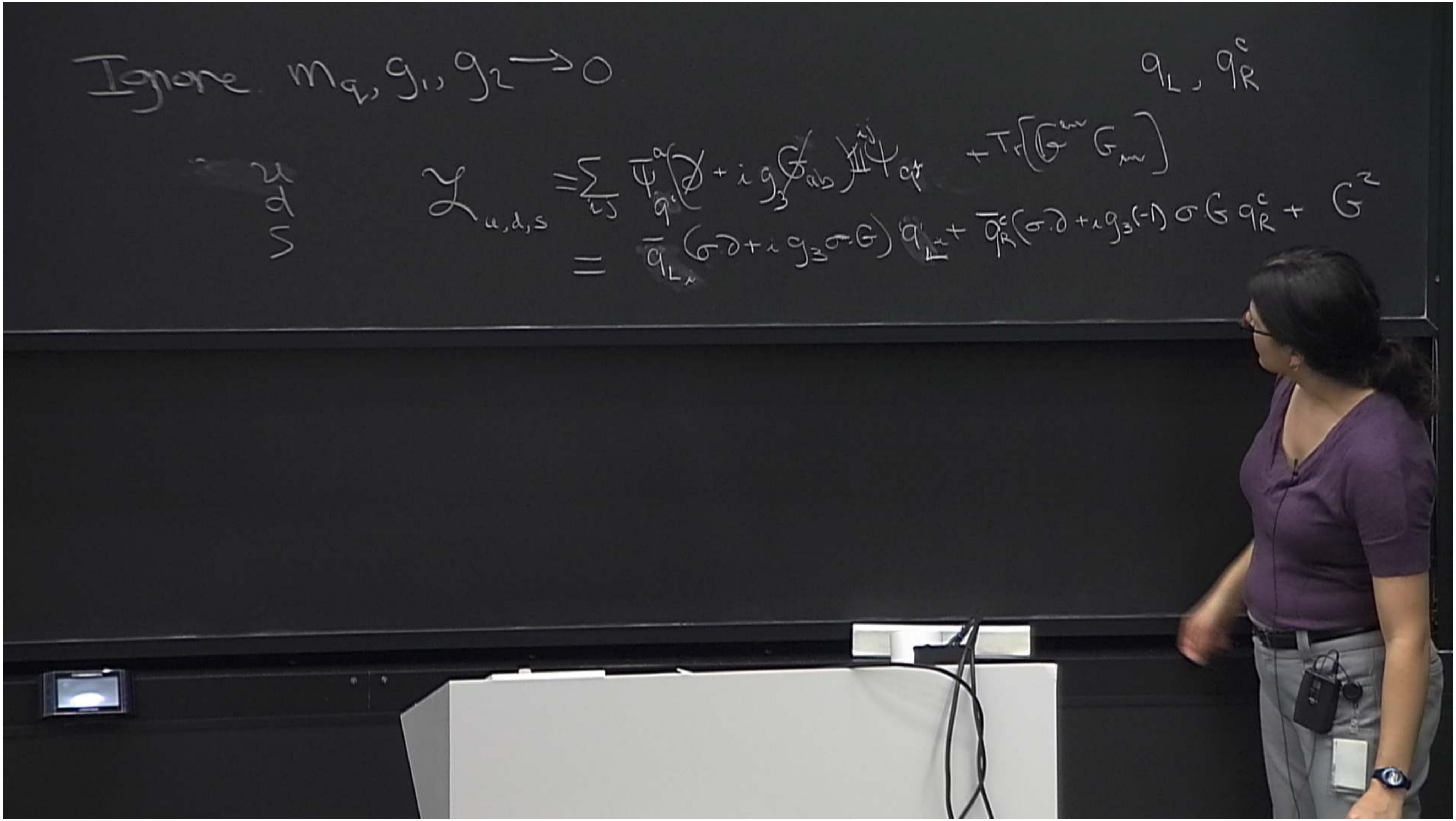

- Fermi Model
E < m_W
Ignore m_q, g_1, g_2 → 0
u
d
s

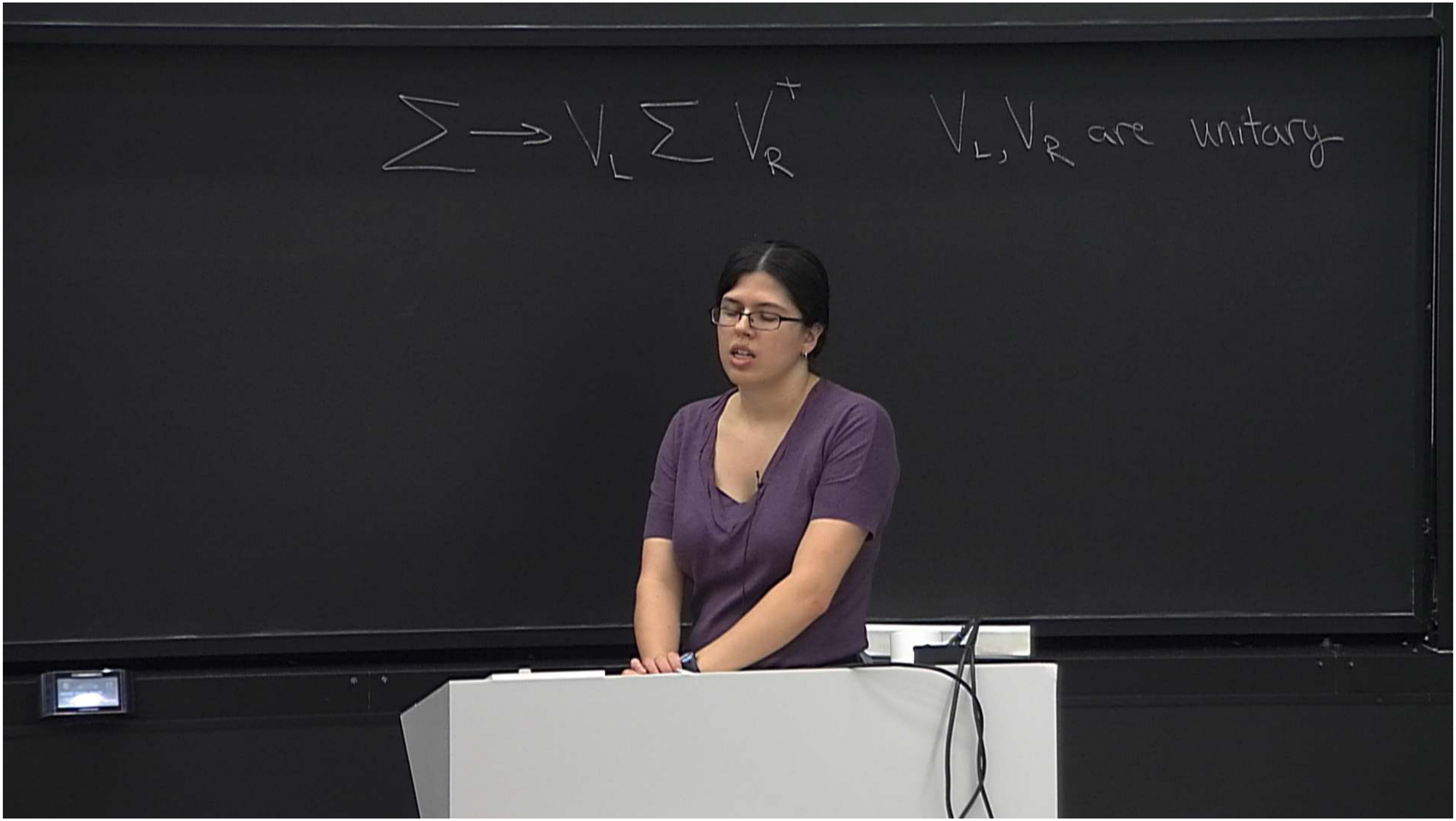

Ignore $m_q, g_1, g_2 \to 0$
u
d
s
$\mathcal{L}_{u,d,s} = \bar{\psi}^a_u (\not\partial + i g_3 \not G_{ab}) \psi_u + \dots$

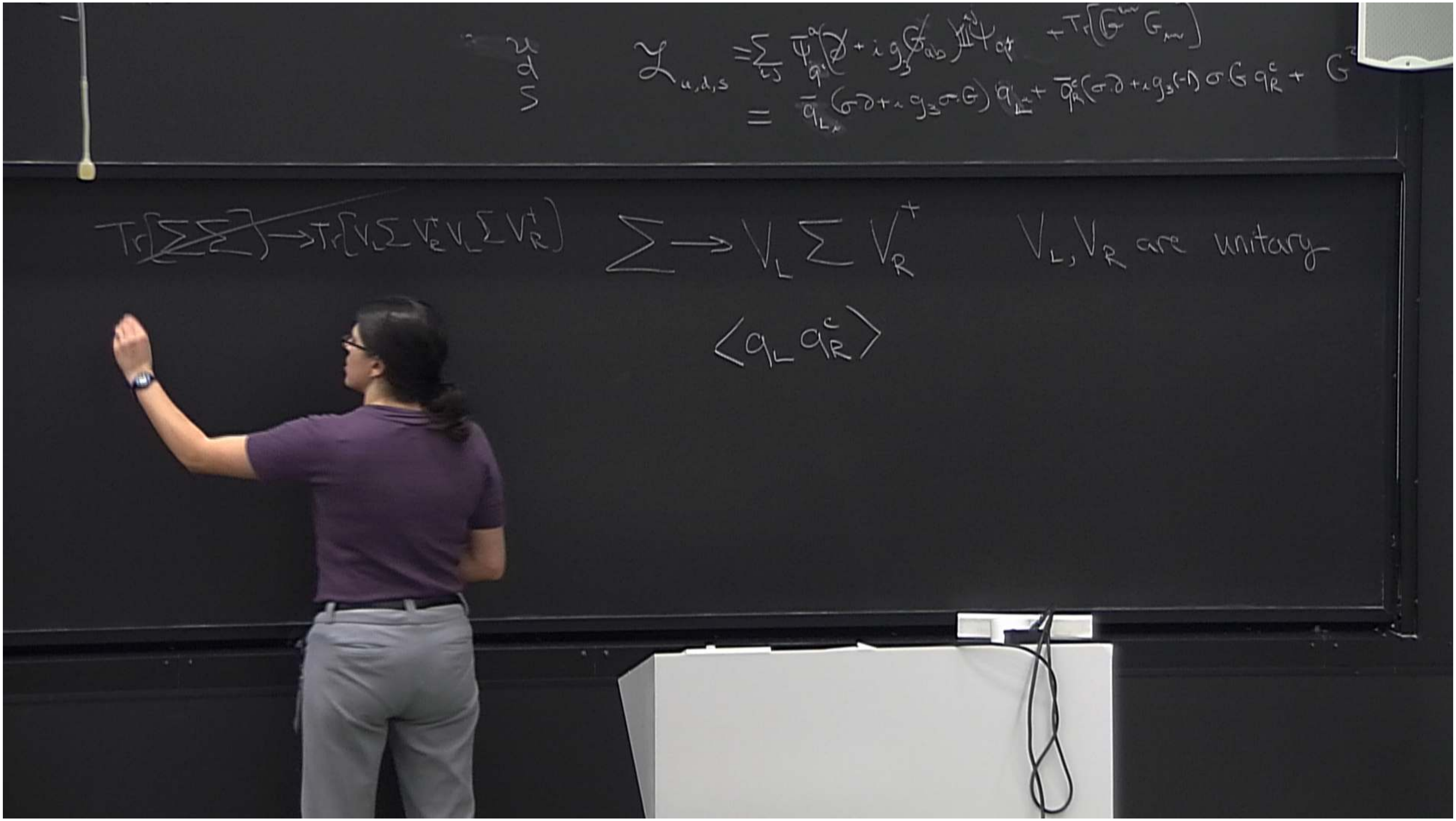

Ignore $m_q, g_1, g_2 \to 0$
q_L, q_R^c
u
d
s
$\mathcal{L}_{u,d,s} = \sum_{ij} \bar{\Psi}_{q^i}^{a} (\not{\partial} + i g_3 \not{G}_{ab})^{ij} \Psi_{q^j} + \ldots$
$= \bar{\Psi}_{q_L} (\sigma \cdot \partial + i g_3 \sigma \cdot G) \Psi_{q_L} +$


Ignore m_q, g_1, g_2 → 0
q_L, q_R^c
u
d
s
+ Tr[G^μν G_μν]


Ignore m_q, g_1, g_2 → 0
q_L , q_R^c

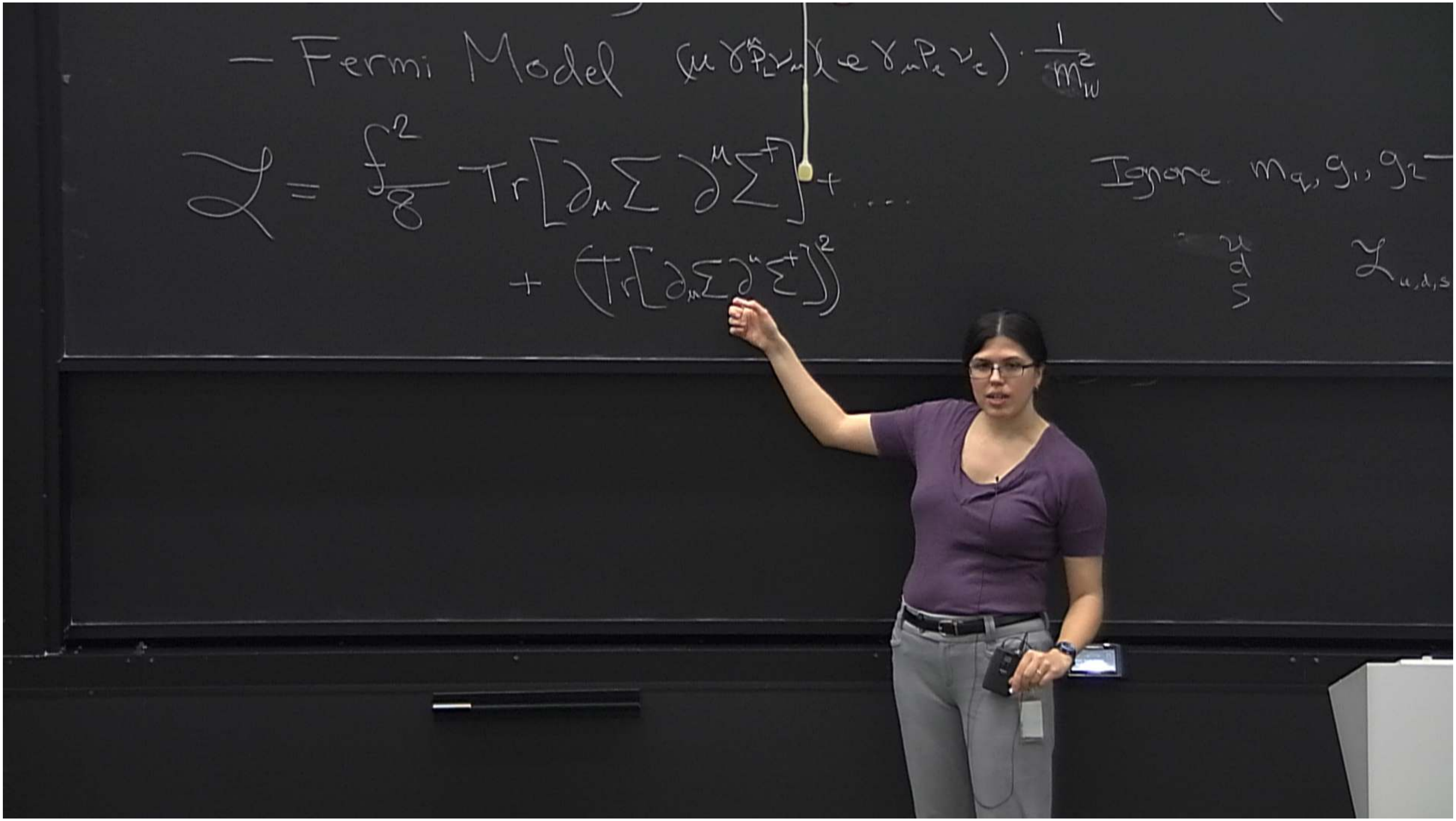

Ignore m_q, g_1, g_2 → 0
q_L , q_R^c
u
d
s

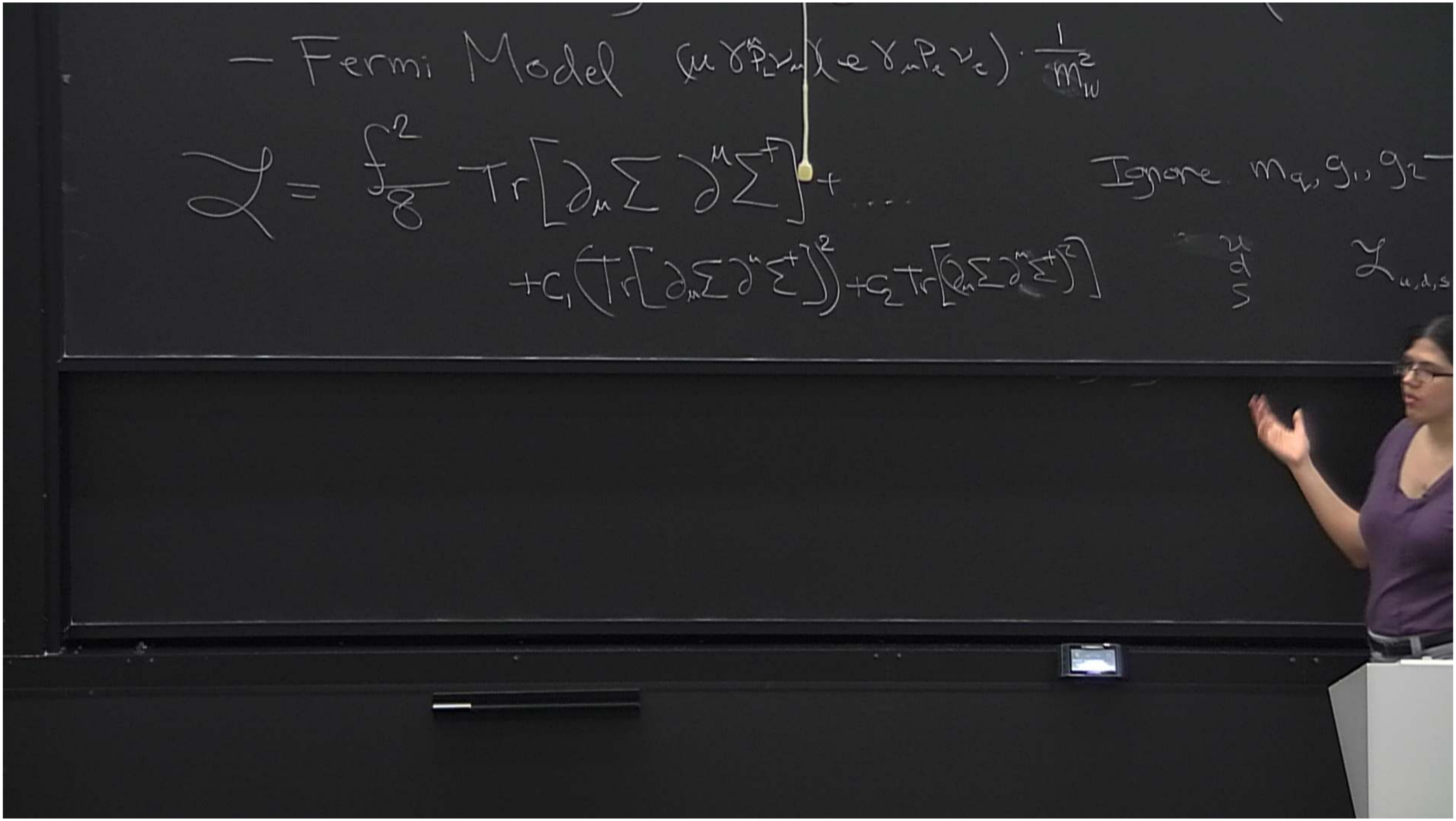

$\Sigma \rightarrow V_L \Sigma V_R^\dagger$
V_L, V_R are unitary

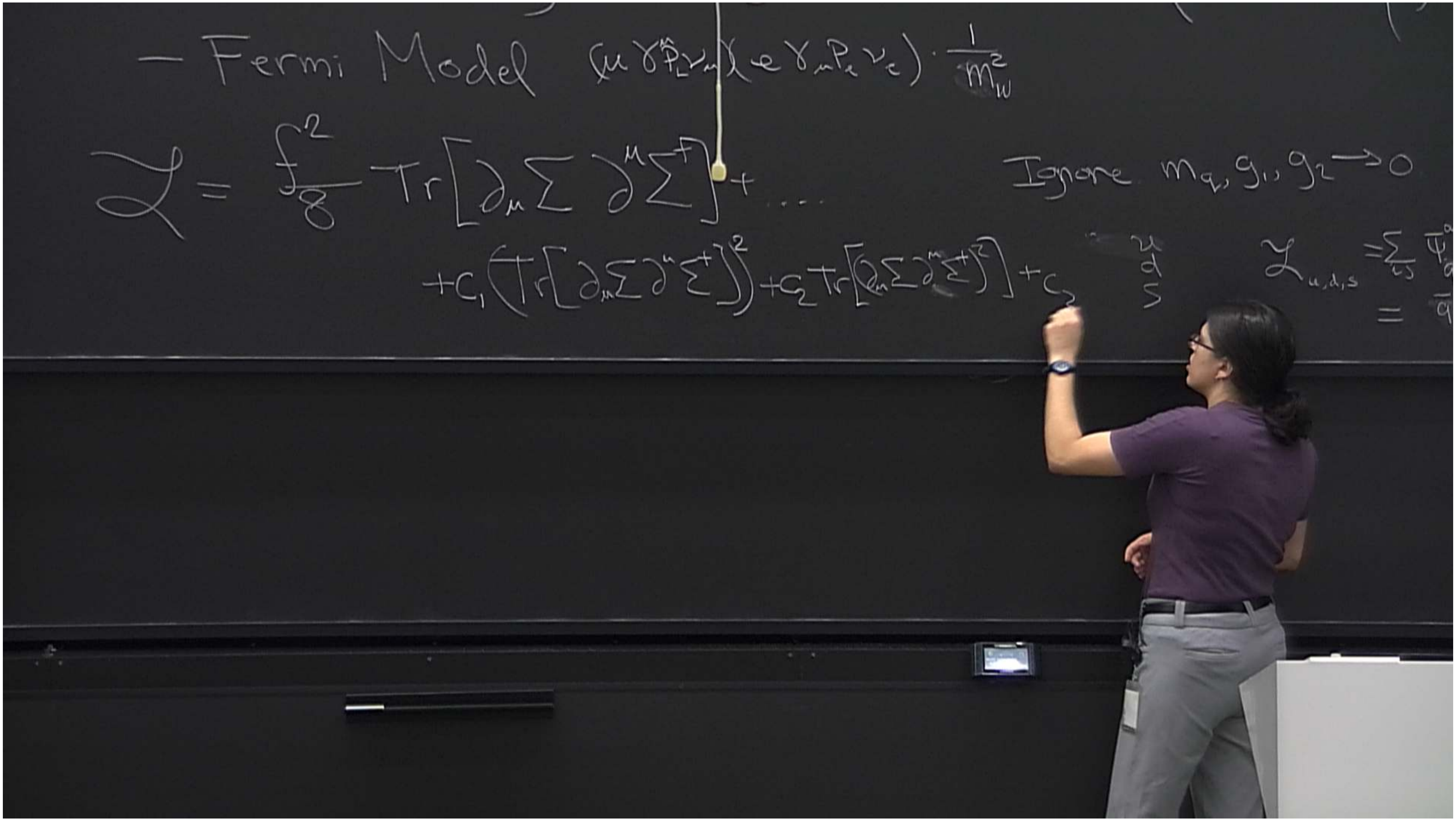

u
d
s
V_L, V_R are unitary

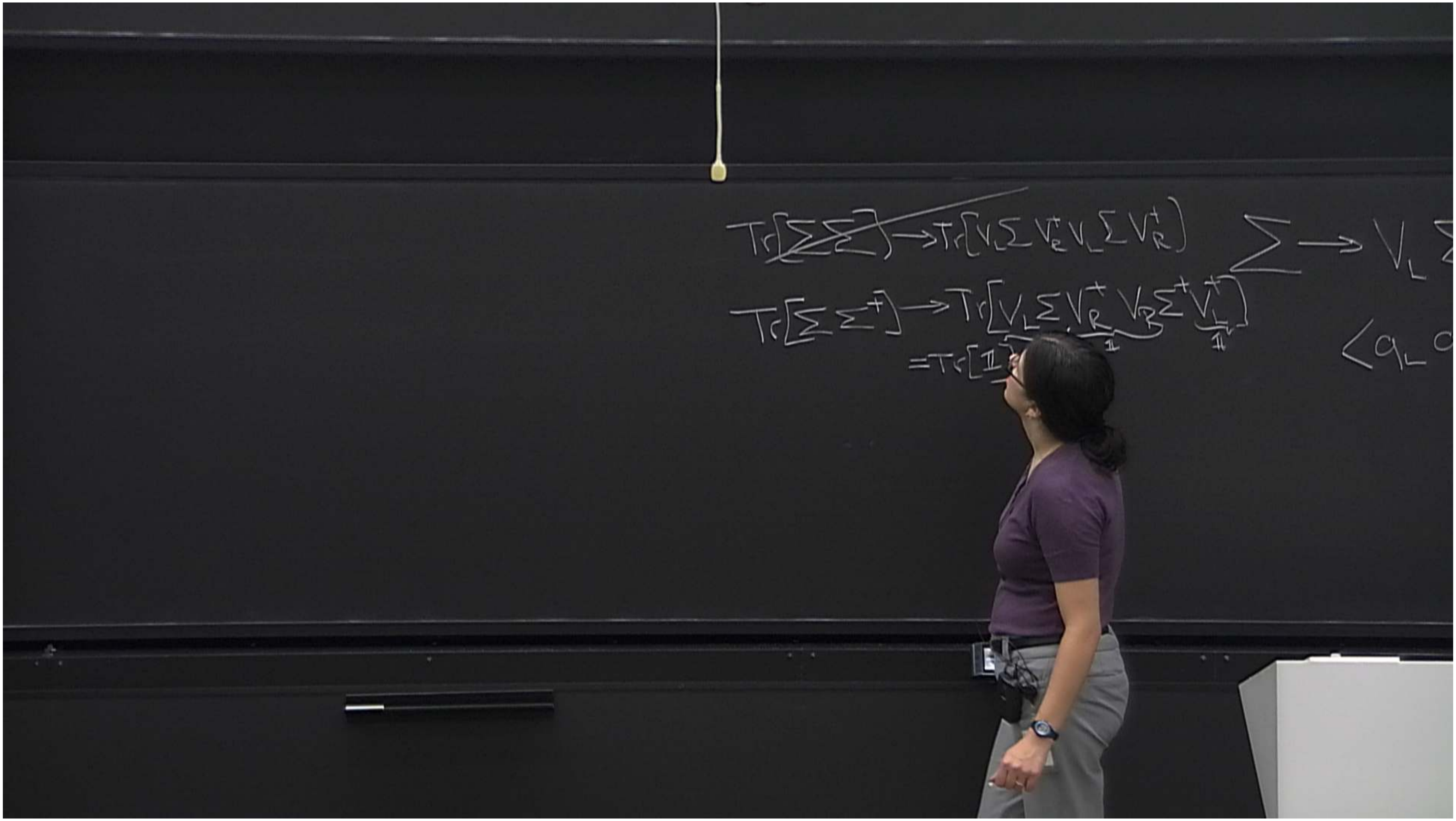

Σ → V_L Σ V_R^+
V_L, V_R are unitary

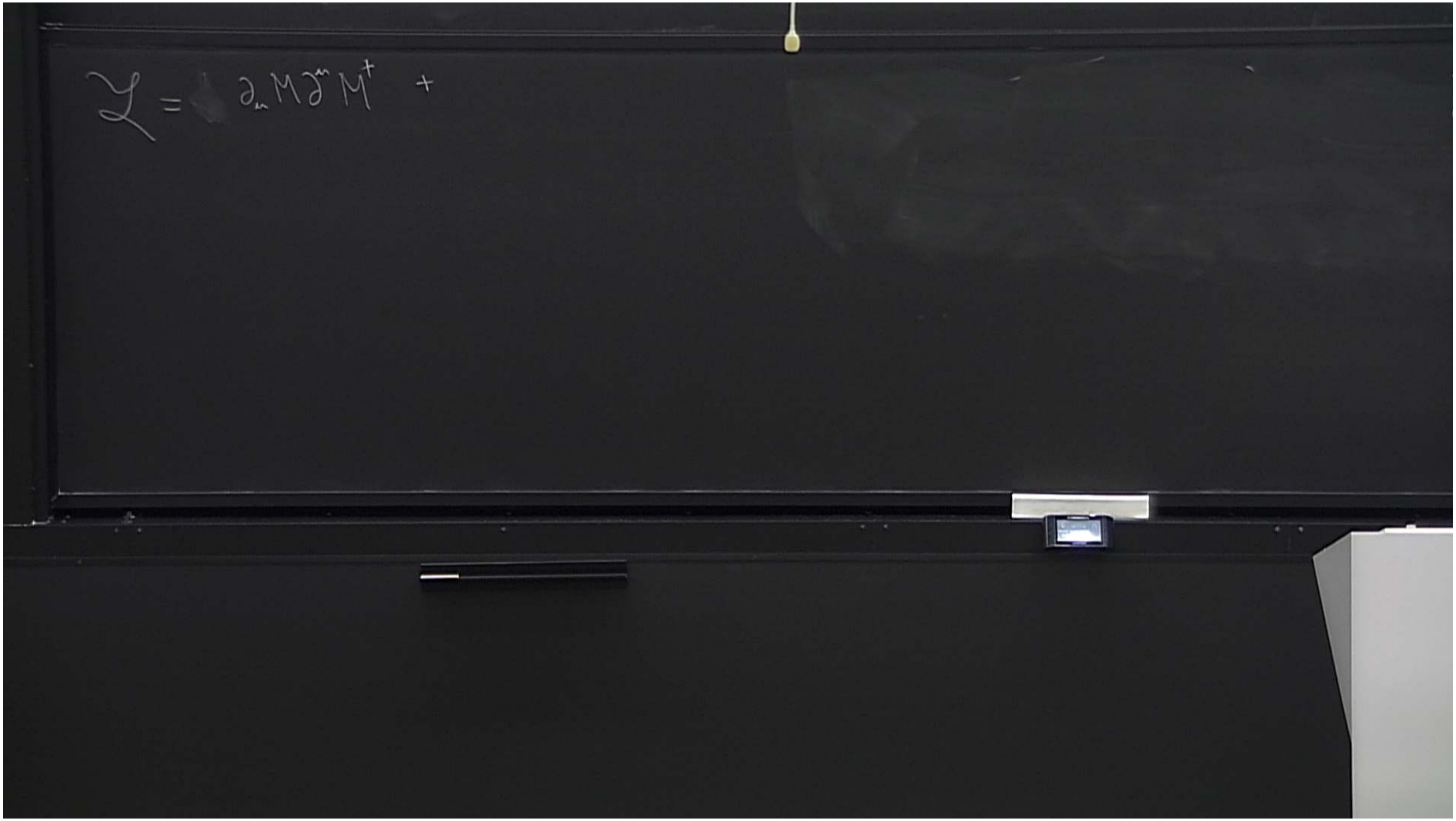

V_L, V_R are unitary
=Tr[1]=3

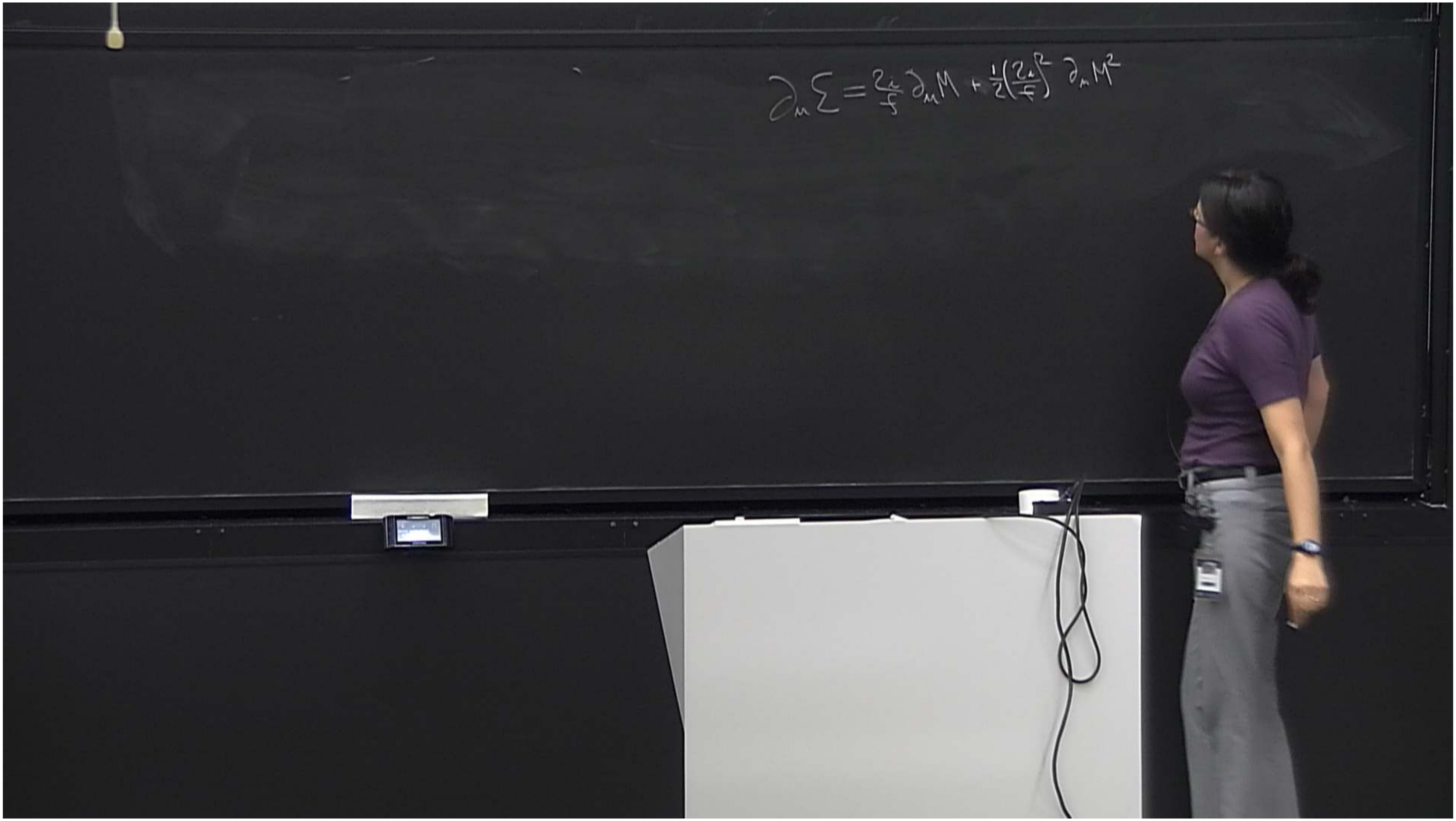

— Fermi Model
Ignore

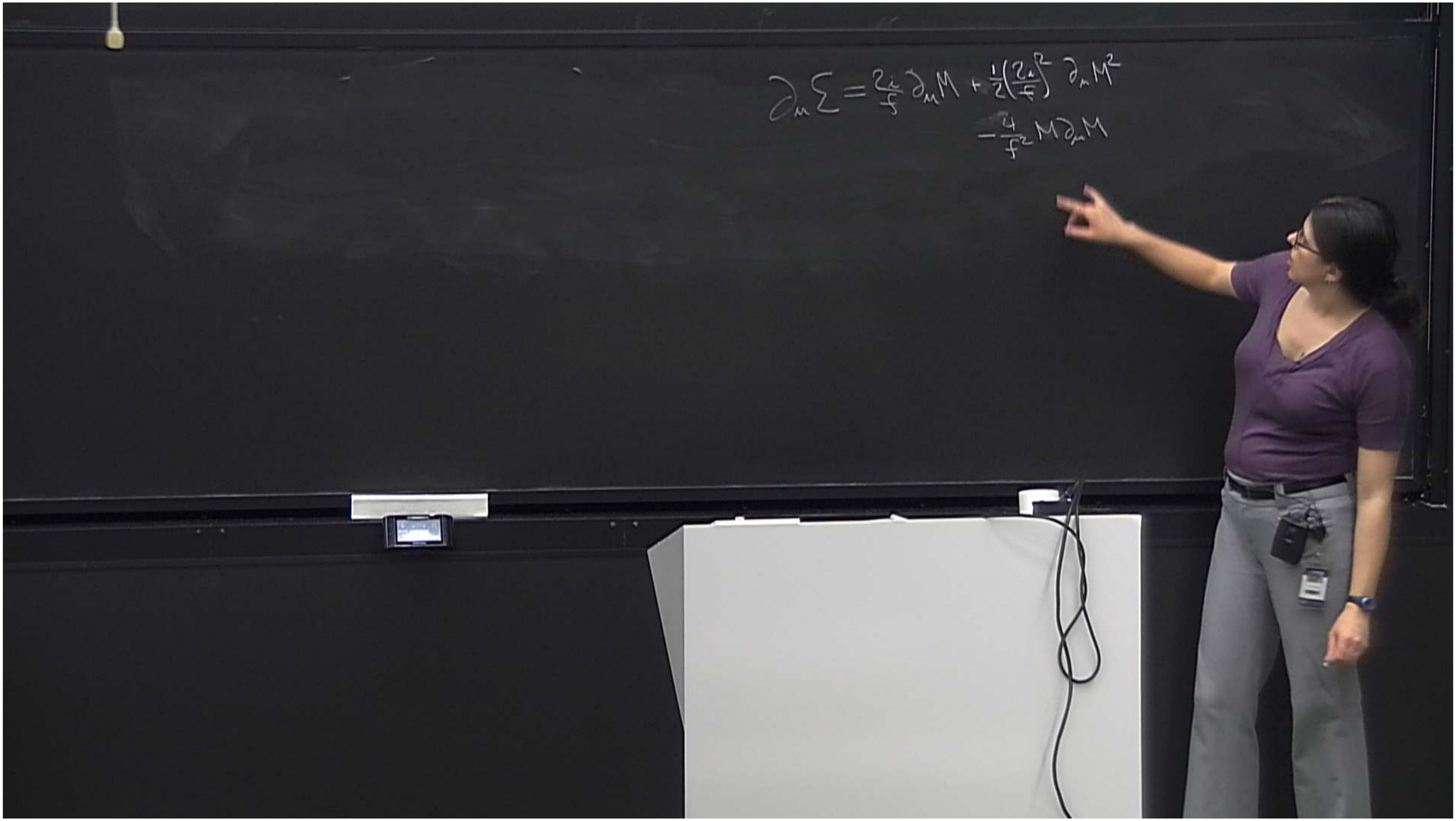

– Fermi Model $(\bar{\mu} \gamma^\mu P_L \nu_\mu)(\bar{e} \gamma_\mu P_L \nu_e) \cdot \frac{1}{m_W^2}$
$\mathcal{L} = \frac{f^2}{8} \mathrm{Tr}[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger] + \ldots$
$+c_1 (\mathrm{Tr}[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger])^2 + c_2 \mathrm{Tr}[(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger)^2]$
Ignore m_q, g_1, g_2
u
d
s
$\mathcal{L}_{u,d,s}$

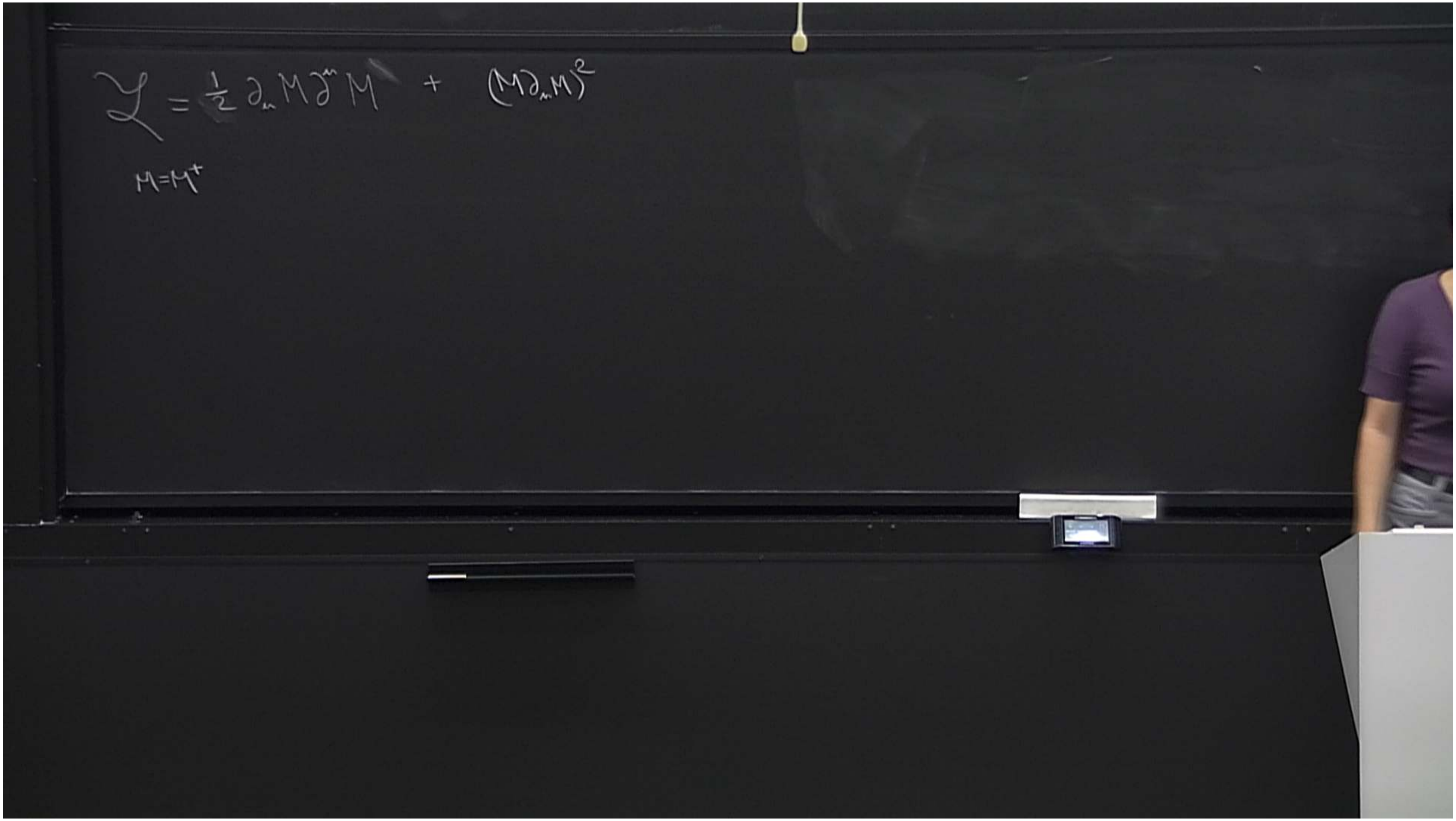

— Fermi Model
Ignore

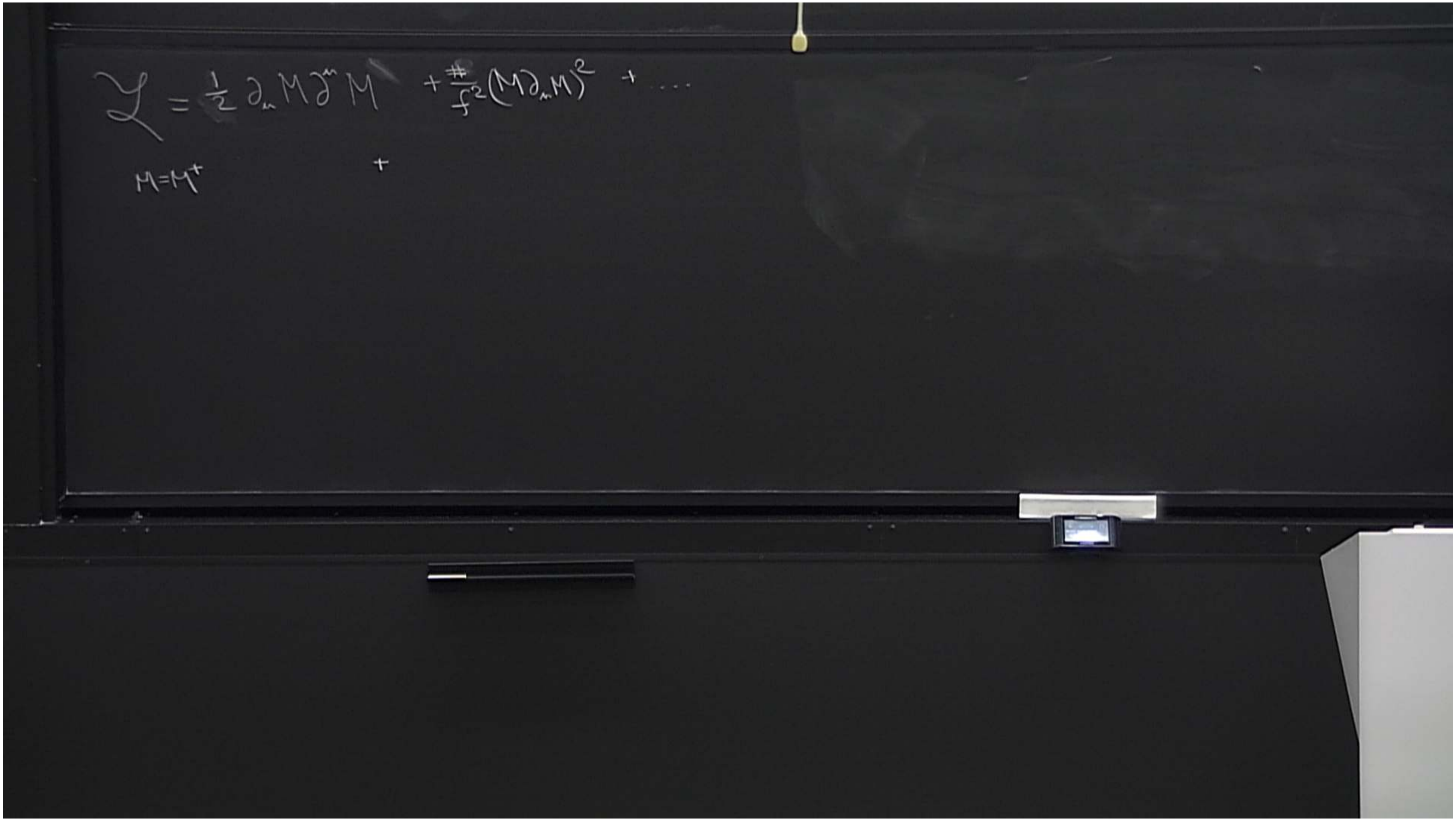

Tr[Σ Σ] → Tr[V_L Σ V_R^† V_L Σ V_R^†]
Σ → V_L Σ
Tr[Σ Σ^†] → Tr[V_L Σ V_R^† V_R Σ^† V_L^†]
= Tr[𝟙]

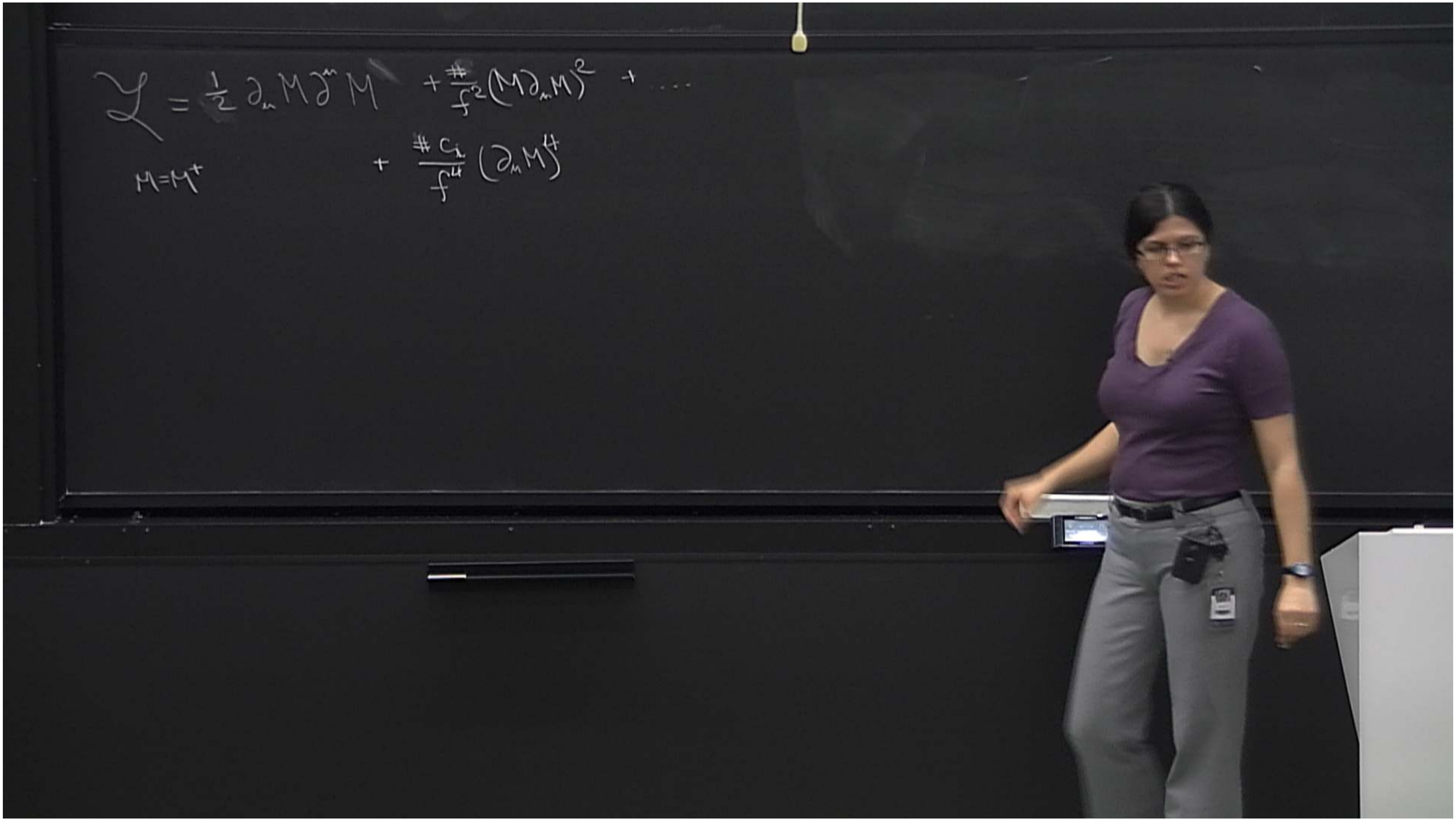

$\mathcal{L} = \partial_m M \partial^m M^\dagger +$

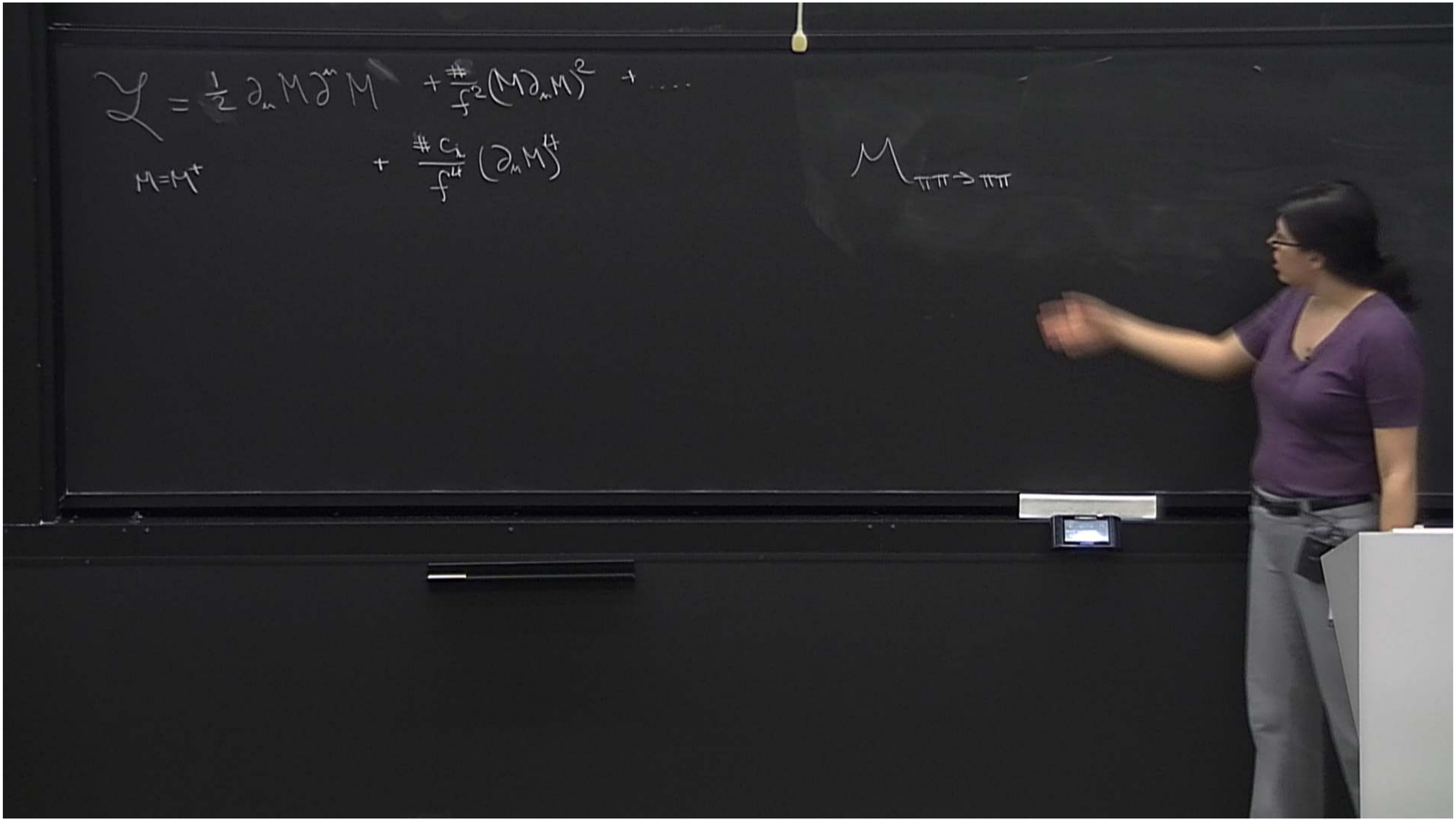

$\partial_\mu \Sigma = \frac{2i}{f} \partial_\mu M + \frac{1}{2}\left(\frac{2i}{f}\right)^2 \partial_\mu M^2$

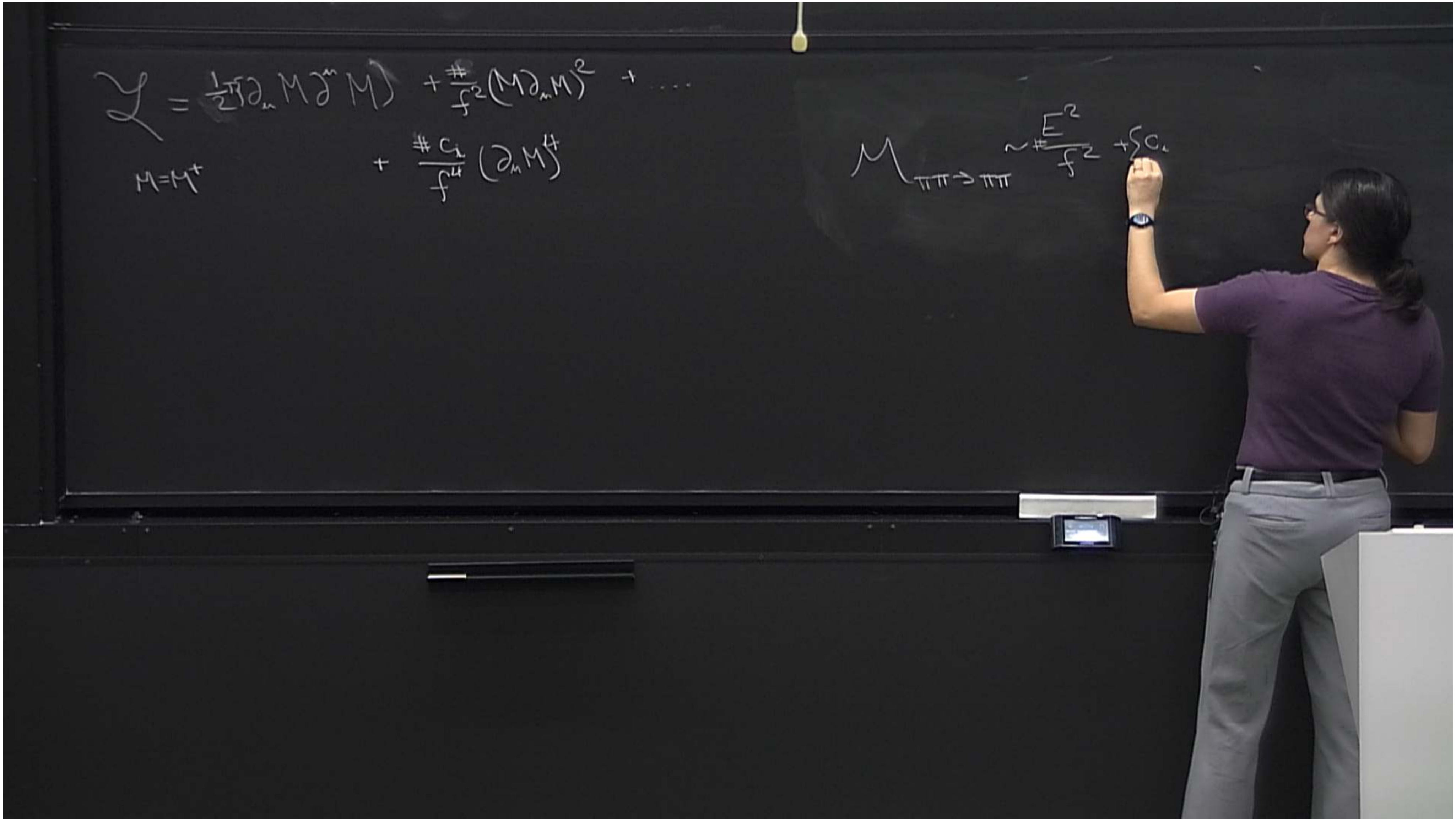

$\partial_\mu \Sigma = \frac{2i}{f} \partial_\mu M + \frac{1}{2}\left(\frac{2i}{f}\right)^2 \partial_\mu M^2$
$- \frac{4}{f^2} M \partial_\mu M$

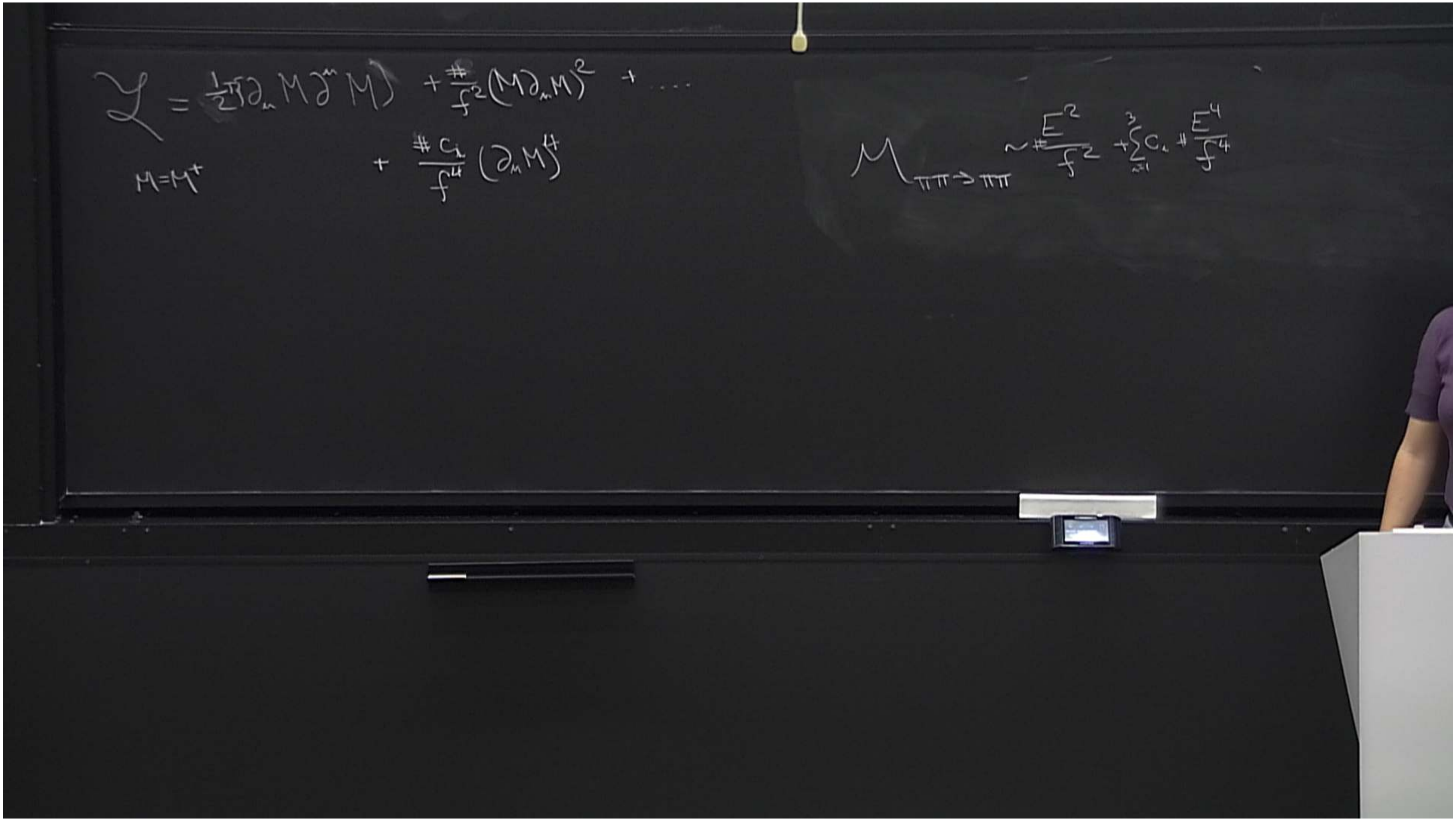

$\mathcal{L} = \frac{1}{2}\partial_\mu M \partial^\mu M + (M\partial_\mu M)^2$
$M = M^\dagger$

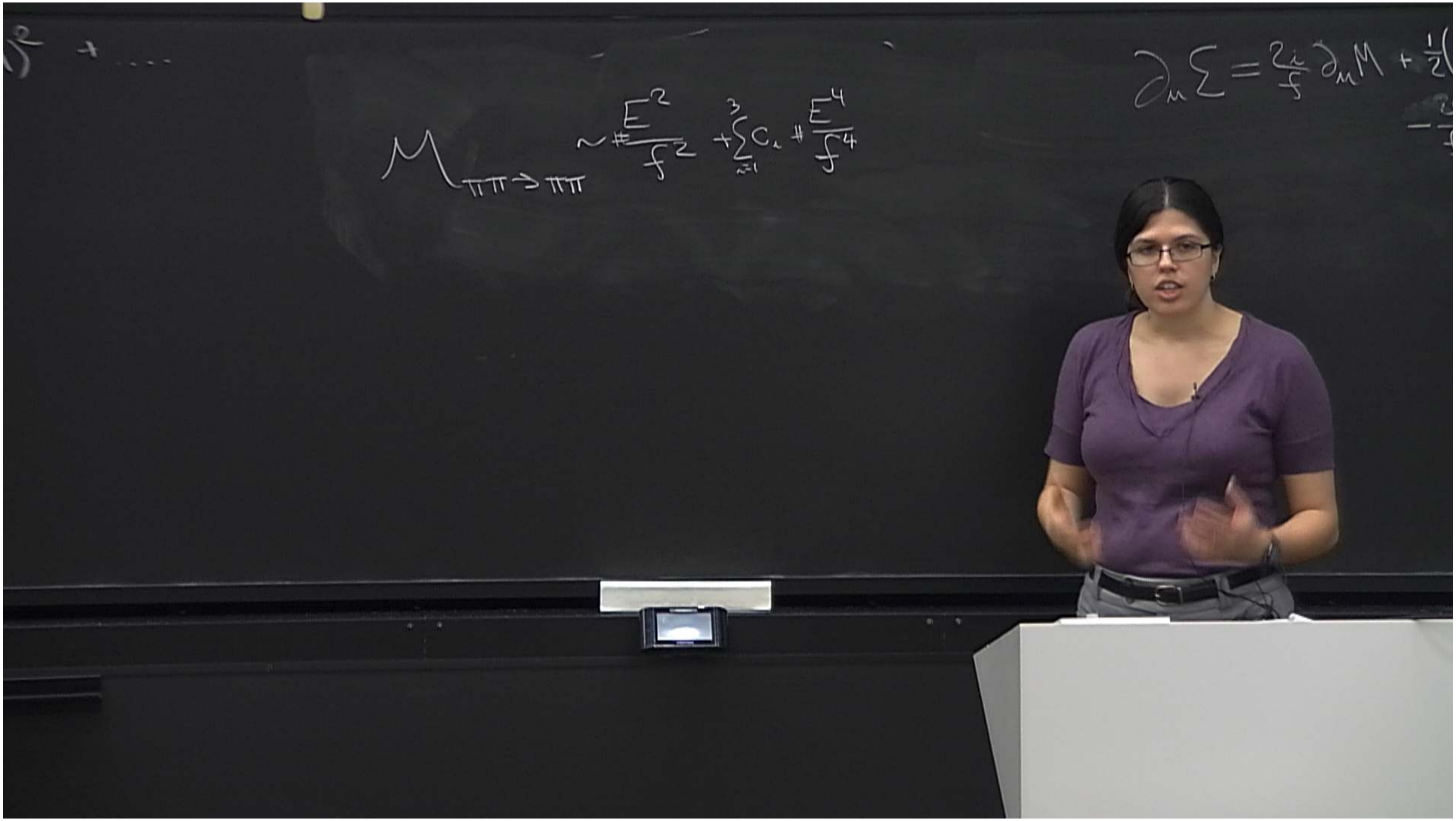

$\mathcal{L} = \frac{1}{2}\partial_\mu M \partial^\mu M + \frac{\#}{f^2}(M\partial_\mu M)^2 + \dots$
$M = M^\dagger$

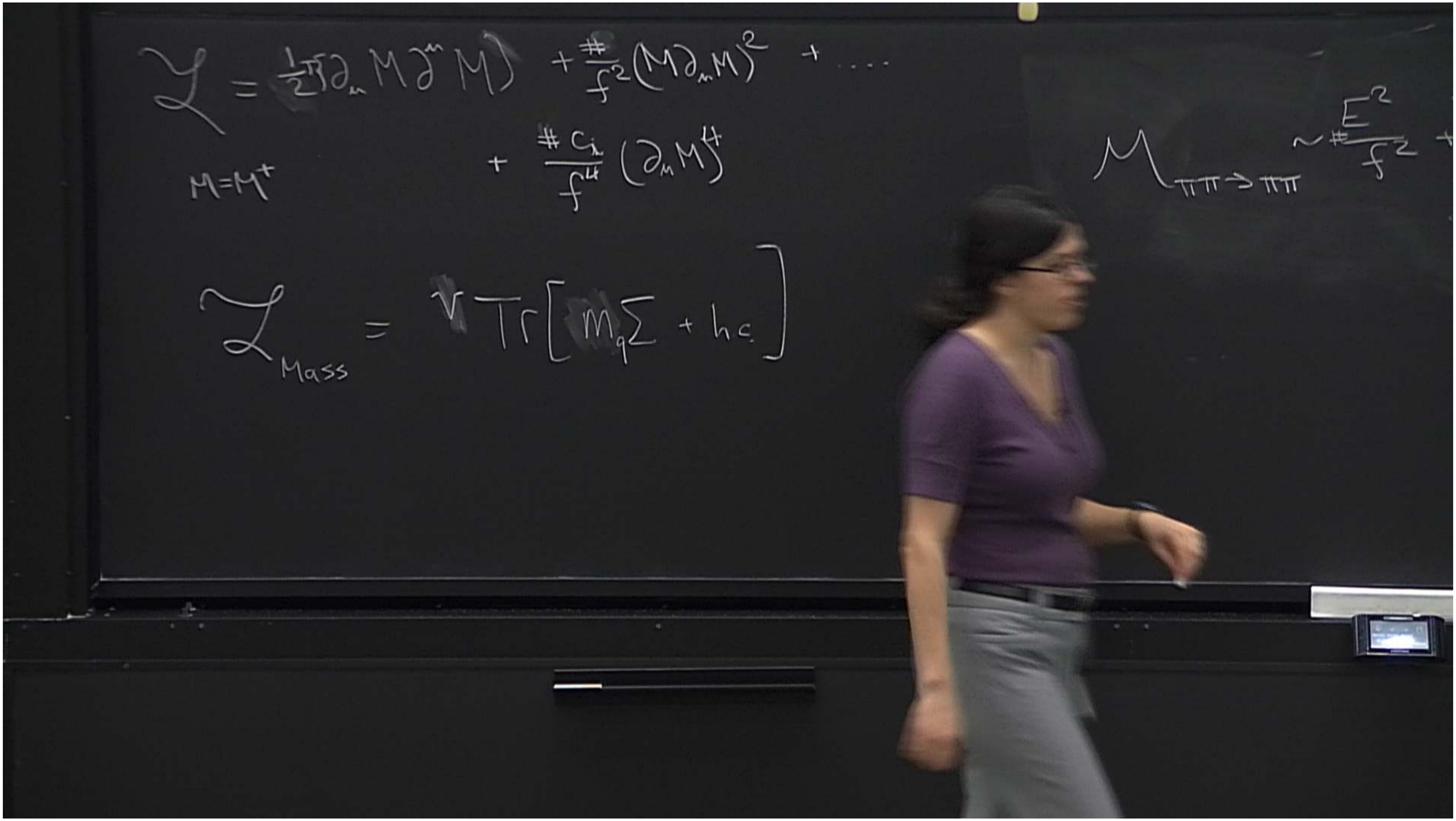

$\mathcal{L} = \frac{1}{2} \partial_\mu M \partial^\mu M + \frac{\#}{f^2} (M \partial_\mu M)^2 + \ldots$
$+ \frac{\# c_i}{f^4} (\partial_\mu M)^4$
$M = M^\dagger$


$\mathcal{L} = \frac{1}{2} \partial_\mu M \partial^\mu M + \frac{\#}{f^2} (M \partial_\mu M)^2 + \ldots$
$M = M^+$
$+ \frac{\# c_i}{f^4} (\partial_\mu M)^4$
$\mathcal{M}_{\pi\pi \to \pi\pi}$


$\mathcal{L} = \frac{1}{2}\mathrm{Tr}(\partial_\mu M \partial^\mu M) + \frac{\#}{f^2}(M\partial_\mu M)^2 + \ldots$
$M = M^+$
$+ \frac{\# c_i}{f^4}(\partial_\mu M)^4$
$\mathcal{M}_{\pi\pi \to \pi\pi} \sim \# \frac{E^2}{f^2} + \sum c_i$


L = ½ Tr(∂μ M ∂μ M) + #/f² (M∂μM)² + . . .
M=M†
+ # ci/f⁴ (∂μM)⁴
Mππ→ππ ~ # E²/f² + Σ ci # E⁴/f⁴


$\mathcal{M}_{\pi\pi\to\pi\pi} \sim \# \frac{E^2}{f^2} + \sum_{i=1}^{3} c_i \# \frac{E^4}{f^4}$



L = 1/2 Tr(∂μ M ∂^μ M) + #/f² (M∂μ M)² + ...
M=M⁺
+ # c_i/f⁴ (∂μ M)⁴
M ππ→ππ ~ # E²/f² +
L Mass = v Tr[m_q Σ + h.c.]

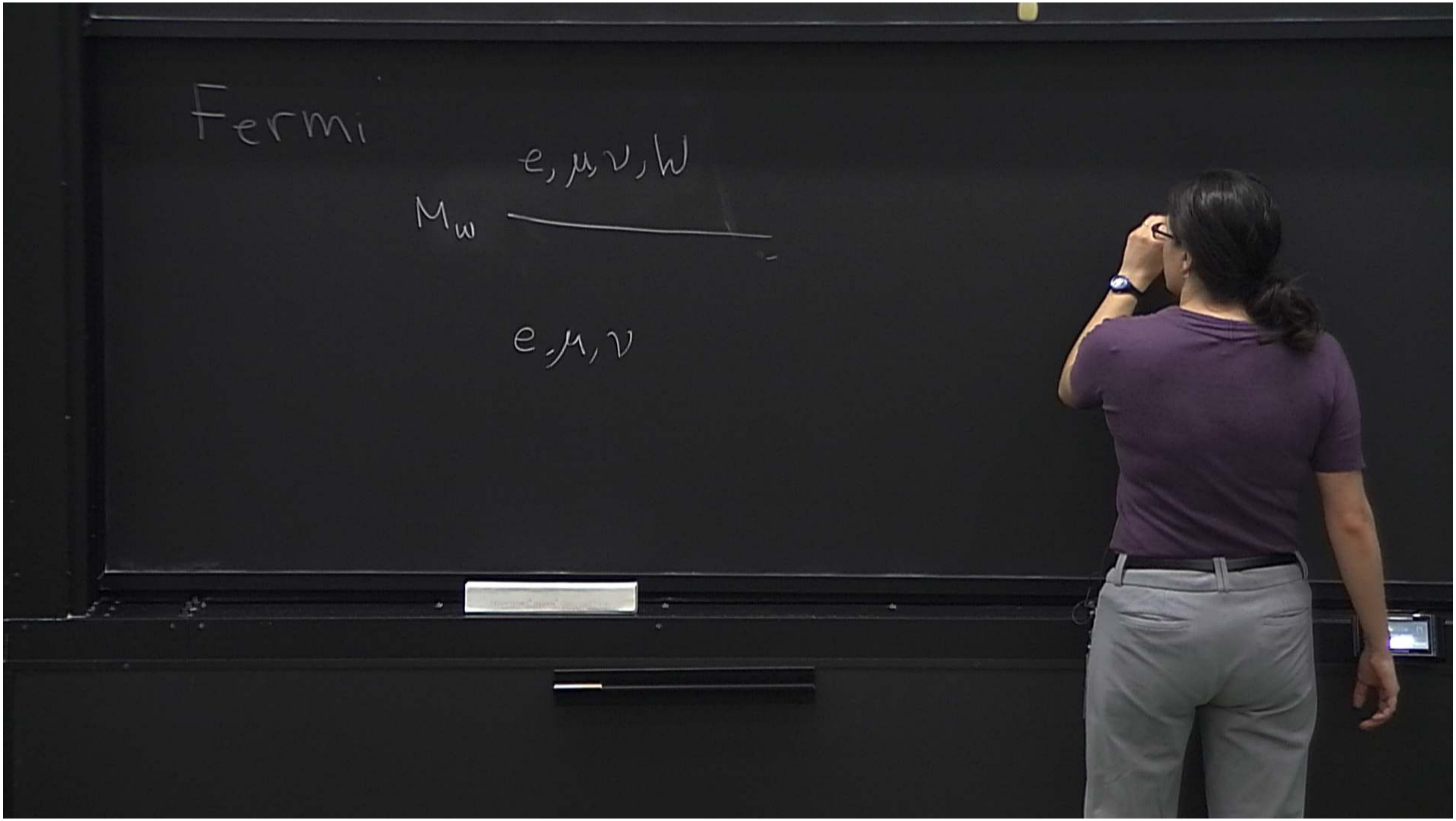
$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}(\partial_\mu M \partial^\mu M) + \frac{\#}{f^2}(M\partial_\mu M)^2 + \dots$$

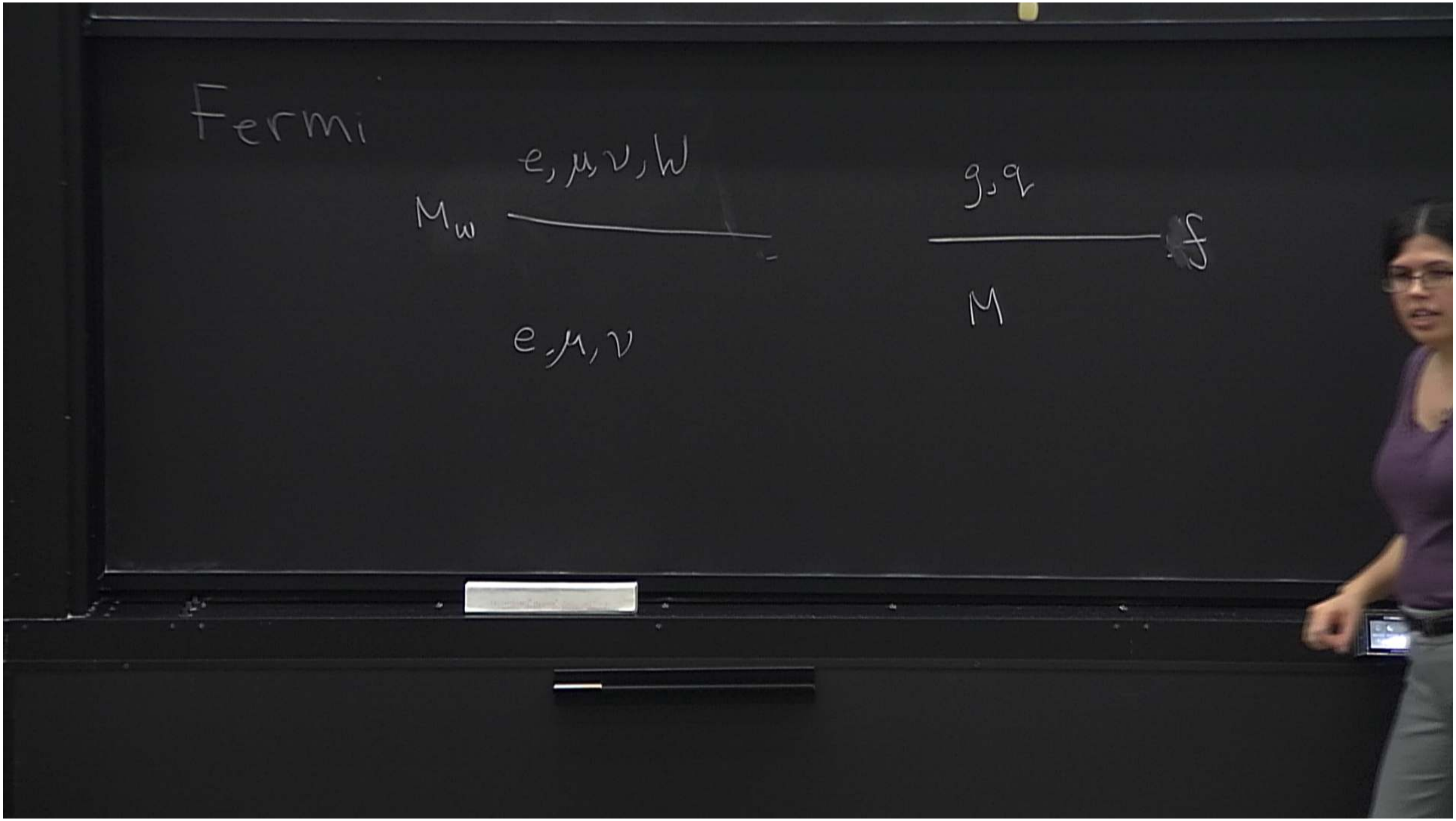
$$+ \frac{\# c_i}{f^4}(\partial_\mu M)^4$$

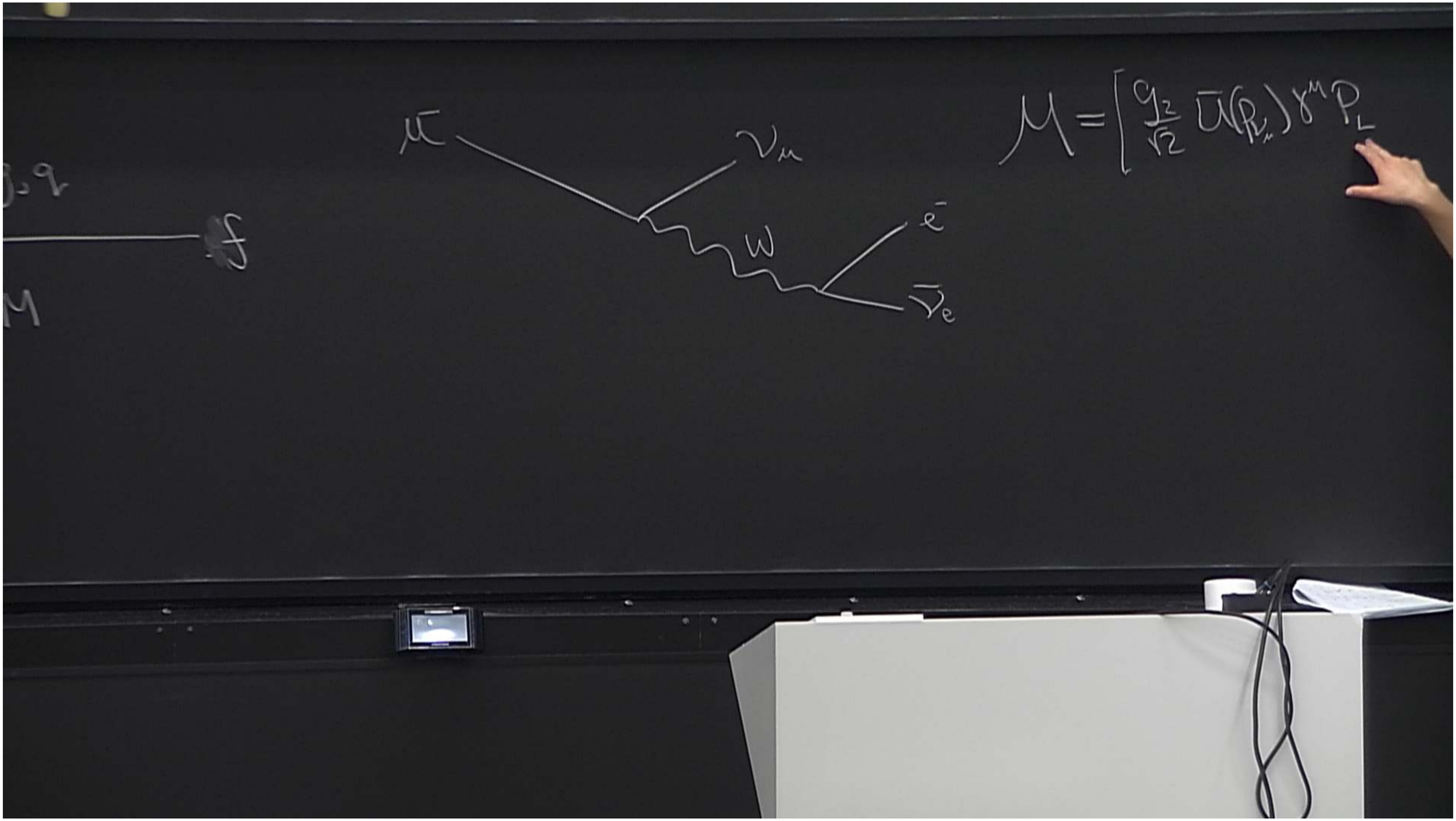
$M = M^+$

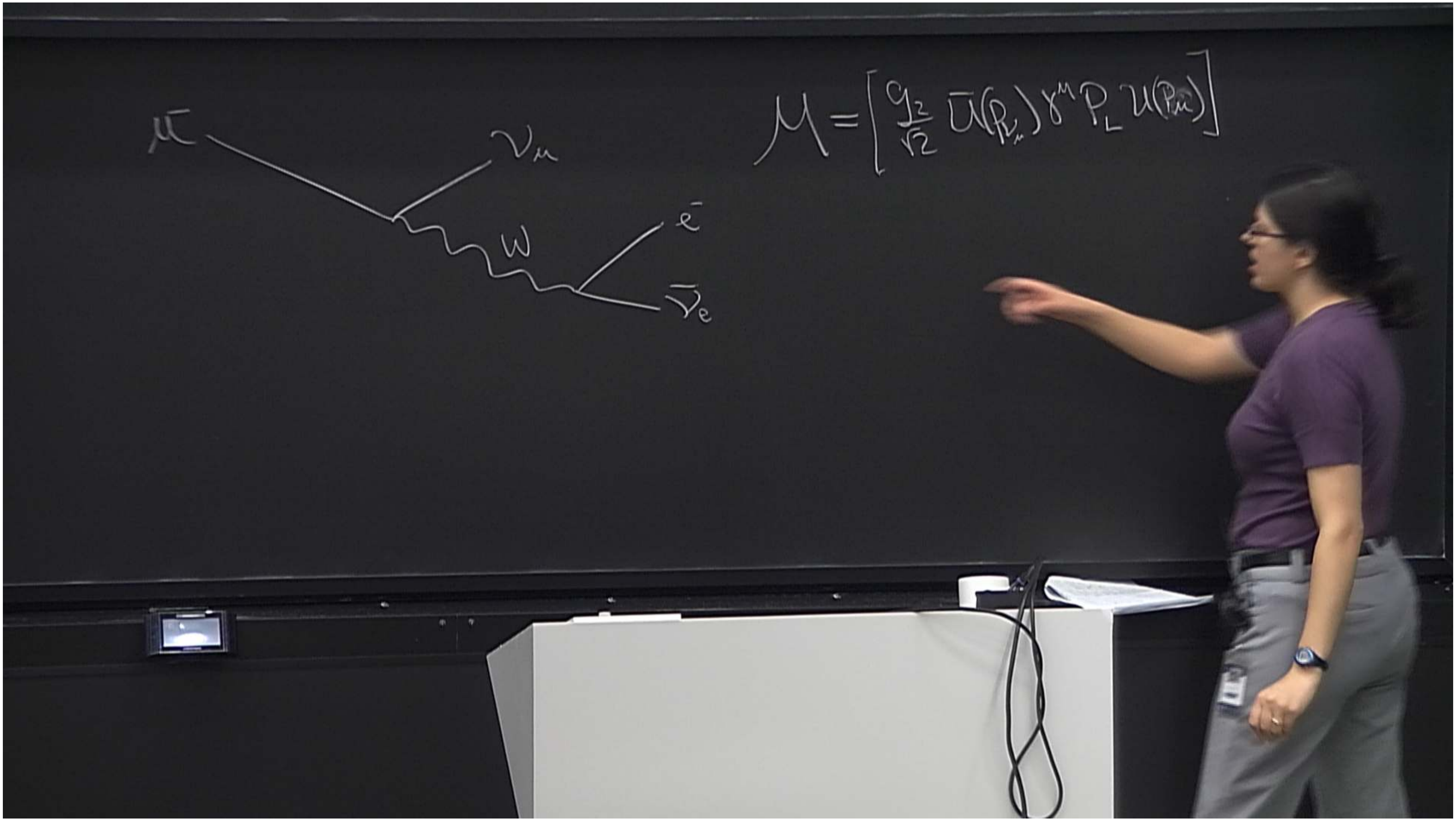
$$\mathcal{M}_{\pi\pi \to \pi\pi} \sim \# \frac{E^2}{f^2} +$$

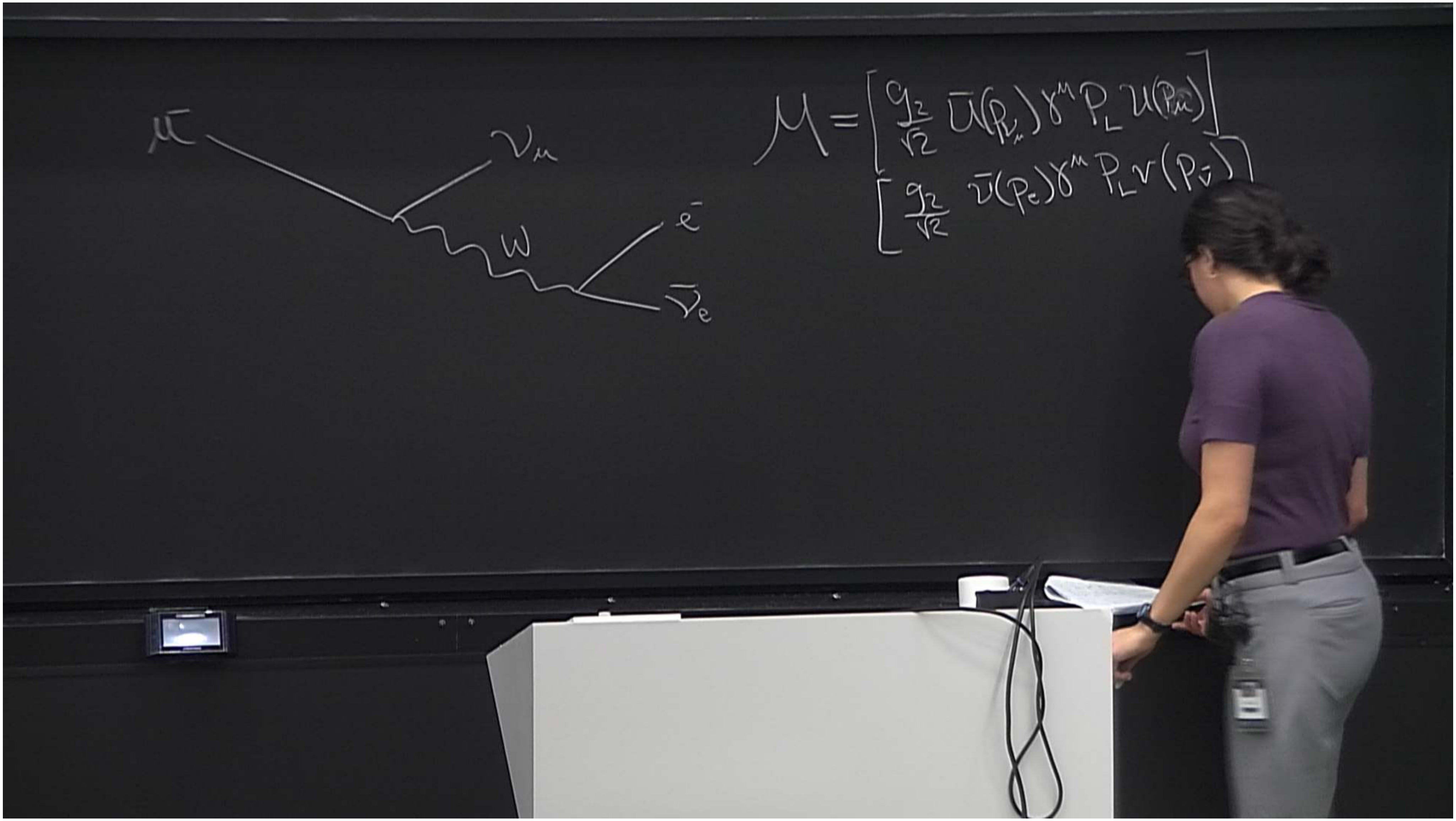
$$\mathcal{L}_{Mass} = \frac{f^3 v}{\#}\mathrm{Tr}\left[m_q \Sigma + h.c. \right]$$

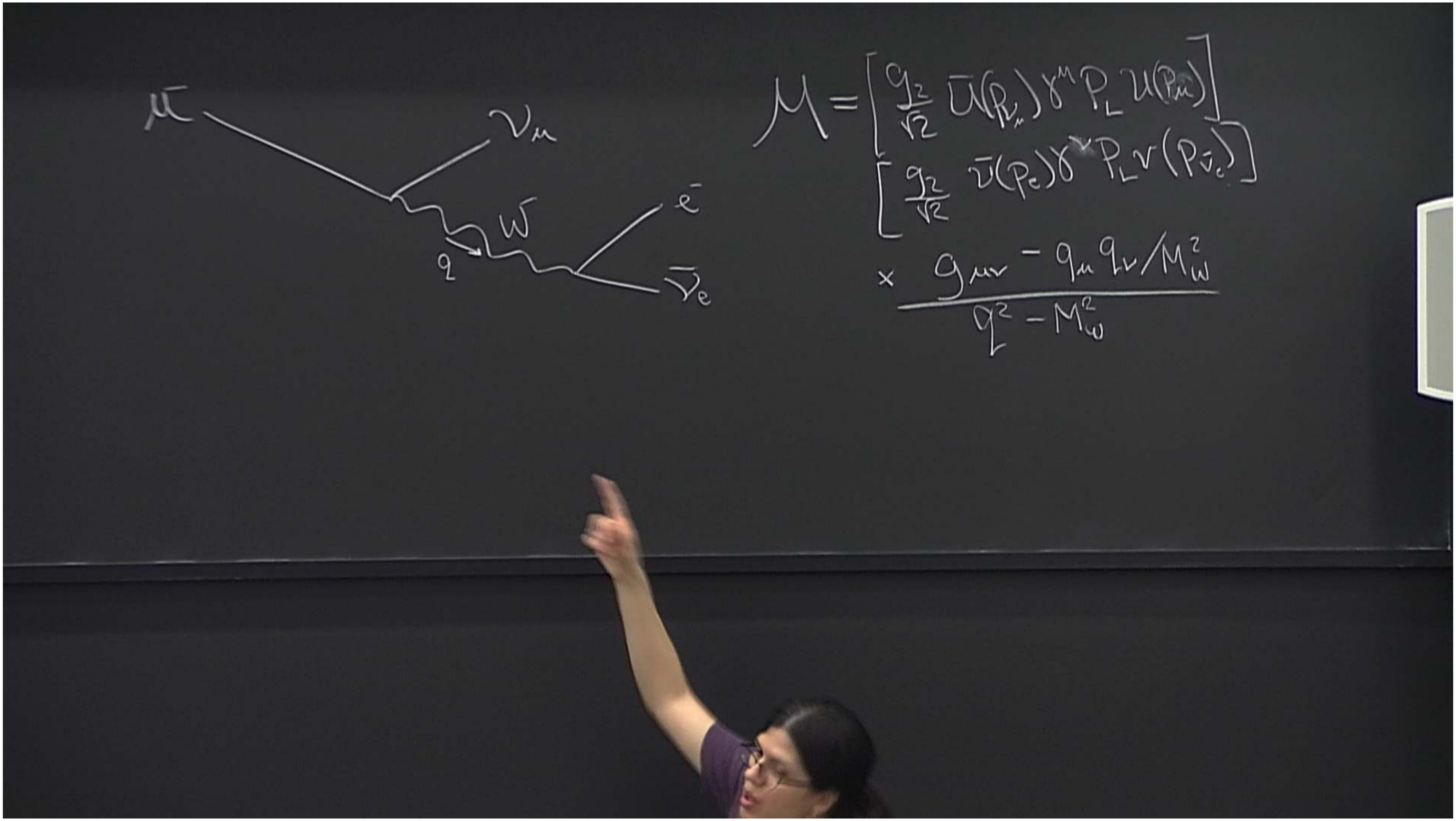

Fermi
e, μ, ν, W
M_W
e, μ, ν

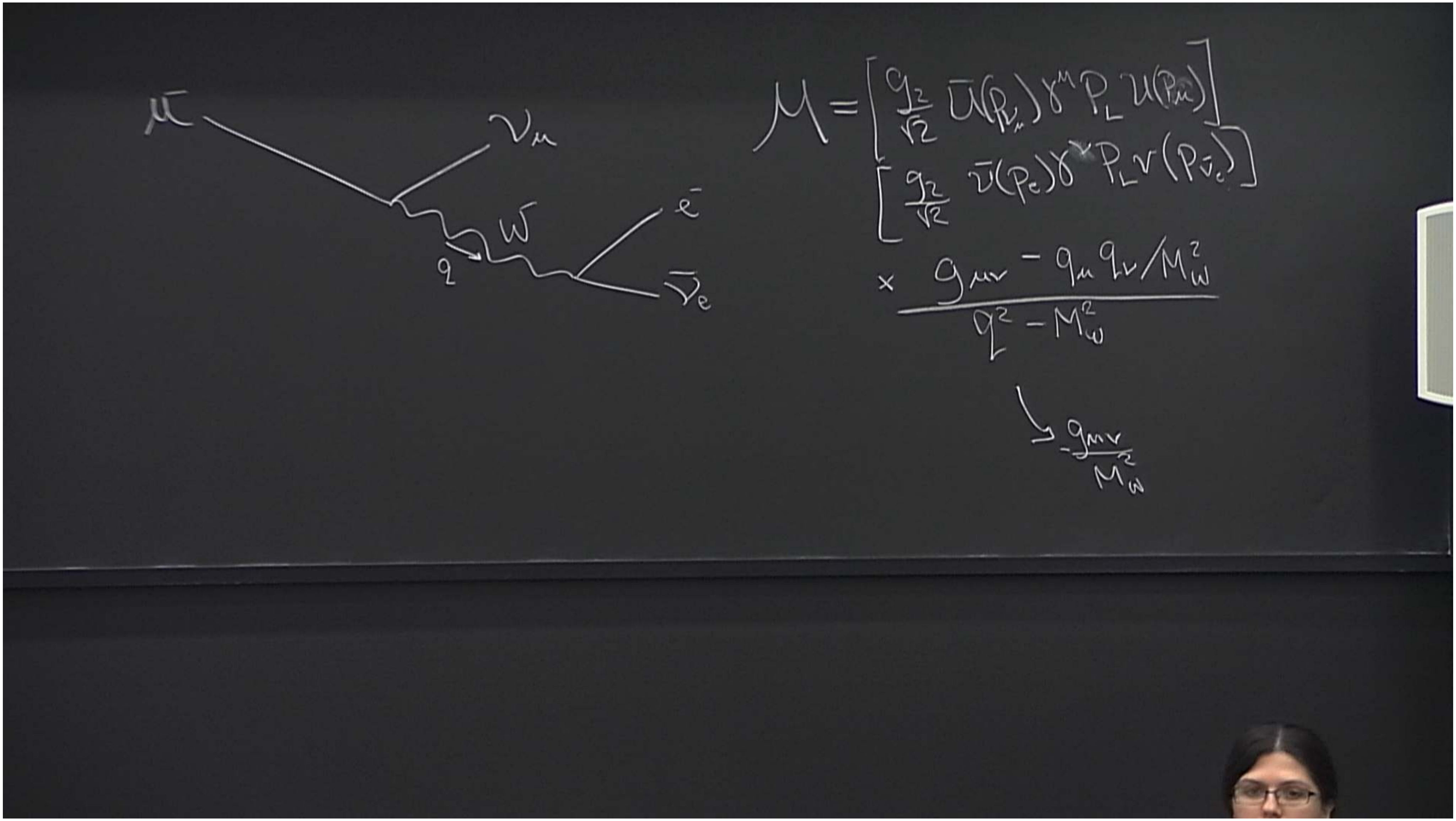

Fermi
e, μ, ν, W
M_W
g, q
f
M
e, μ, ν

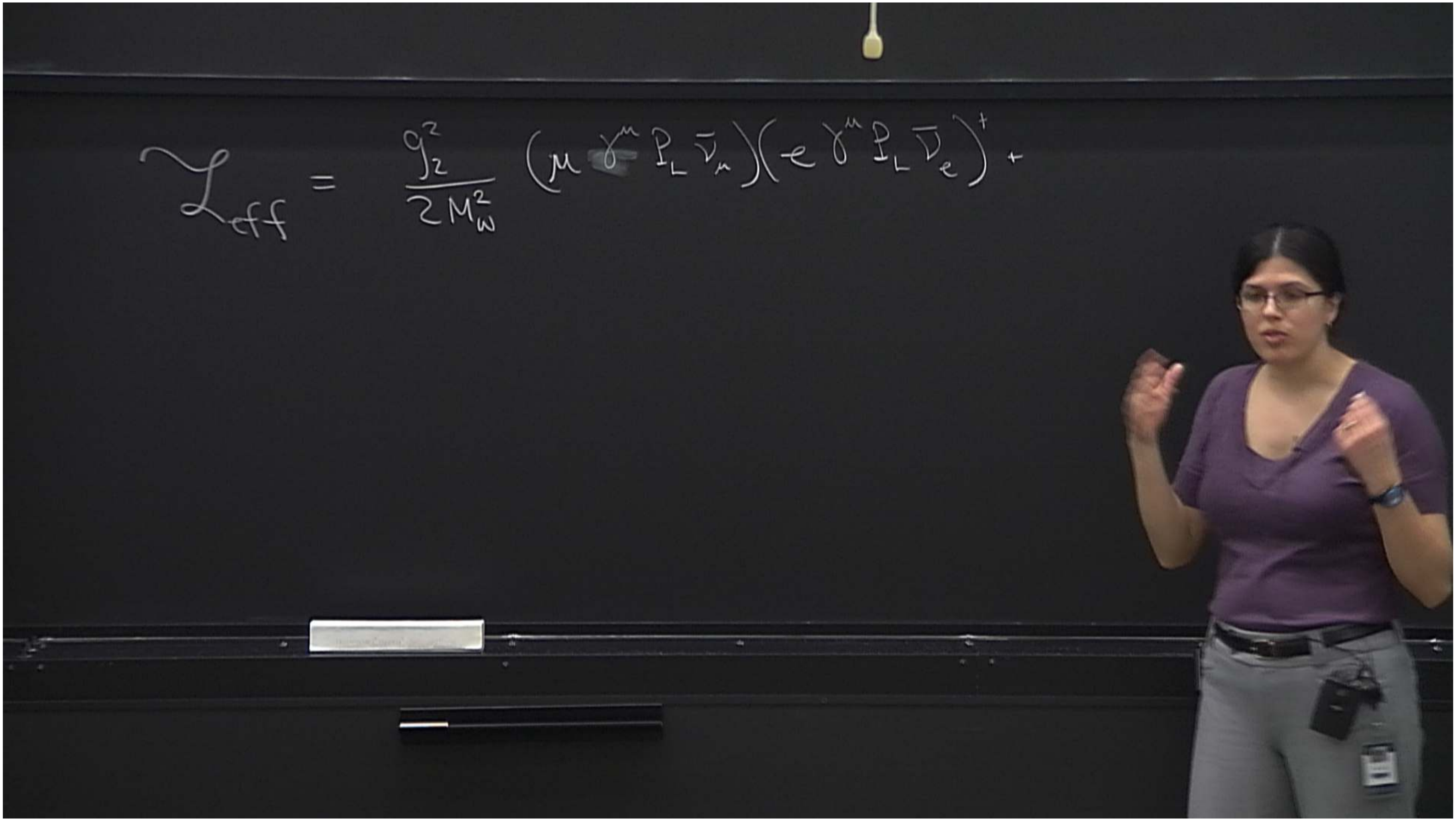

μ^-
ν_μ
W
e^-
$\bar{\nu}_e$
$\mathcal{M} = \left[\frac{g_2}{\sqrt{2}} \bar{u}(p_{\nu_\mu}) \gamma^\mu P_L \right.$


μ⁻
ν_μ
W
e⁻
ν̄_e
M = [g₂/√2 ū(p_ν_μ) γ^μ P_L u(p_μ)]


μ⁻
ν_μ
W
e⁻
ν̄_e
M = [g₂/√2 ū(p_νμ) γ^μ P_L u(p_μ)]
[g₂/√2 ū(p_e) γ^μ P_L v(p_ν̄)]

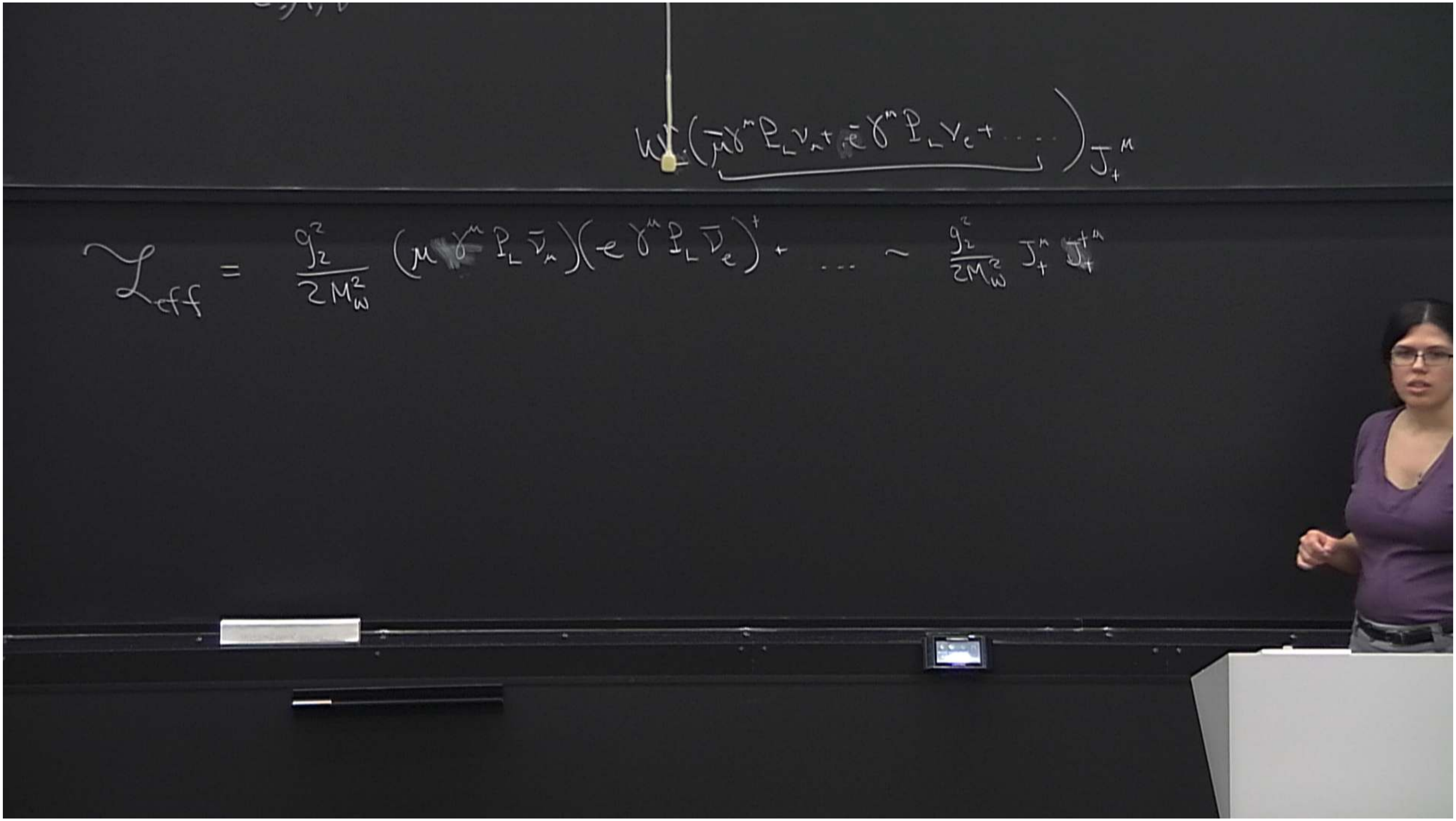

μ^-
ν_μ
W
q
e^-
$\bar{\nu}_e$
$\mathcal{M} = \left[\frac{g_2}{\sqrt{2}} \bar{u}(p_{\nu_\mu}) \gamma^\mu P_L u(p_\mu)\right]$
$\left[\frac{g_2}{\sqrt{2}} \bar{u}(p_e) \gamma^\nu P_L v(p_{\bar{\nu}_e})\right]$
$\times \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}$

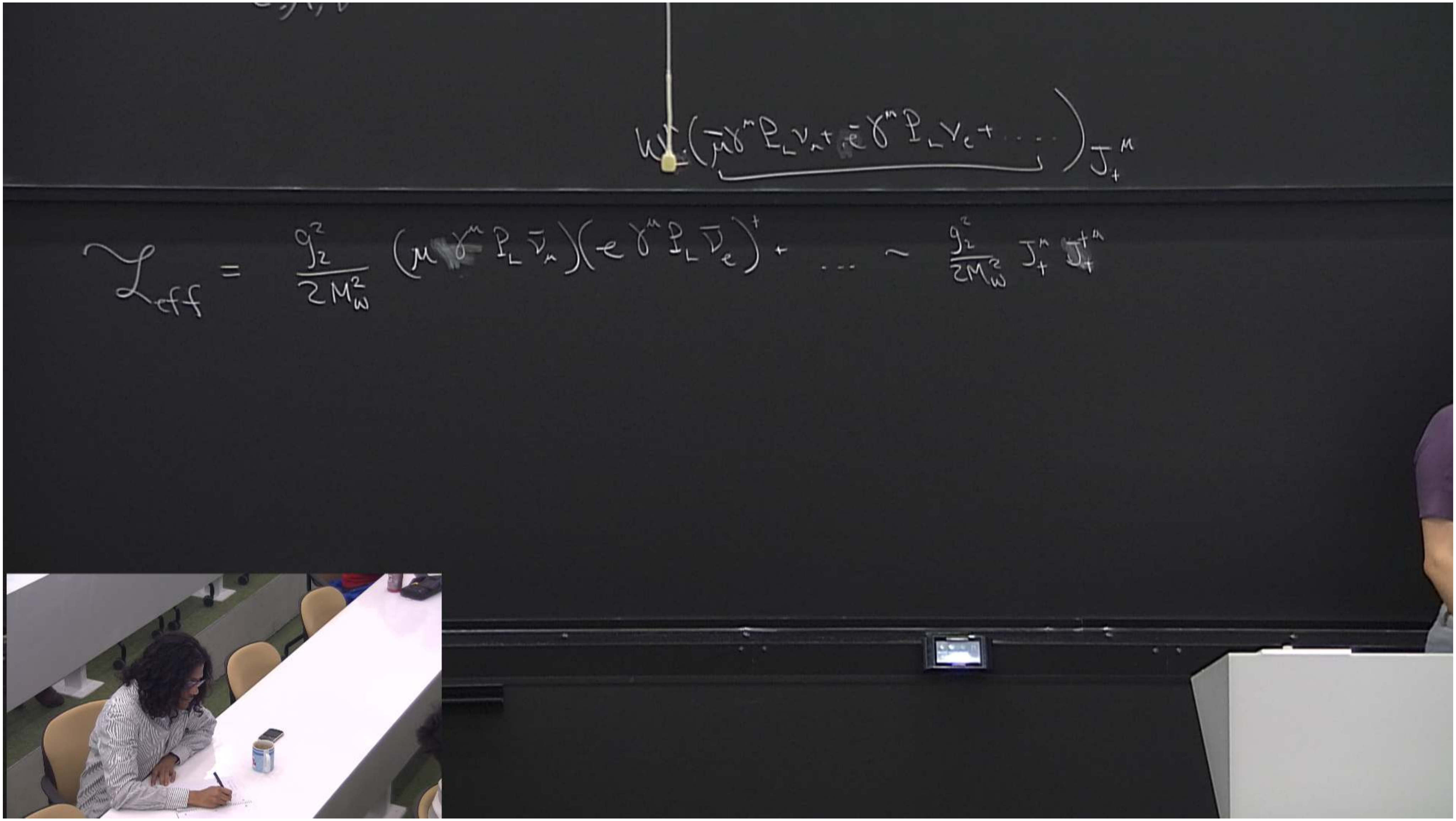

μ^-
ν_μ
W
q
e^-
$\bar{\nu}_e$
$\mathcal{M} = \left[\frac{g_2}{\sqrt{2}} \bar{u}(p_{\nu_\mu}) \gamma^\mu P_L u(p_\mu)\right]$
$\left[\frac{g_2}{\sqrt{2}} \bar{u}(p_e) \gamma^\nu P_L v(p_{\bar{\nu}_e})\right]$
$\times \frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}$
$\rightarrow -\frac{g_{\mu\nu}}{M_W^2}$


$\mathcal{L}_{eff} = \frac{g_2^2}{2M_W^2} (\mu \gamma^\mu P_L \bar{\nu}_\mu)(e \gamma^\mu P_L \bar{\nu}_e)^\dagger +$

$$W^+_\mu\left(\bar{\mu}\gamma^\mu P_L \nu_\mu + \bar{e}\gamma^\mu P_L \nu_e + \dots\right)$$

$$\mathcal{L}_{eff} = \frac{g_2^2}{2M_W^2}\left(\mu\gamma^\mu P_L \bar{\nu}_\mu\right)\left(e\gamma^\mu P_L \bar{\nu}_e\right)^\dagger +$$

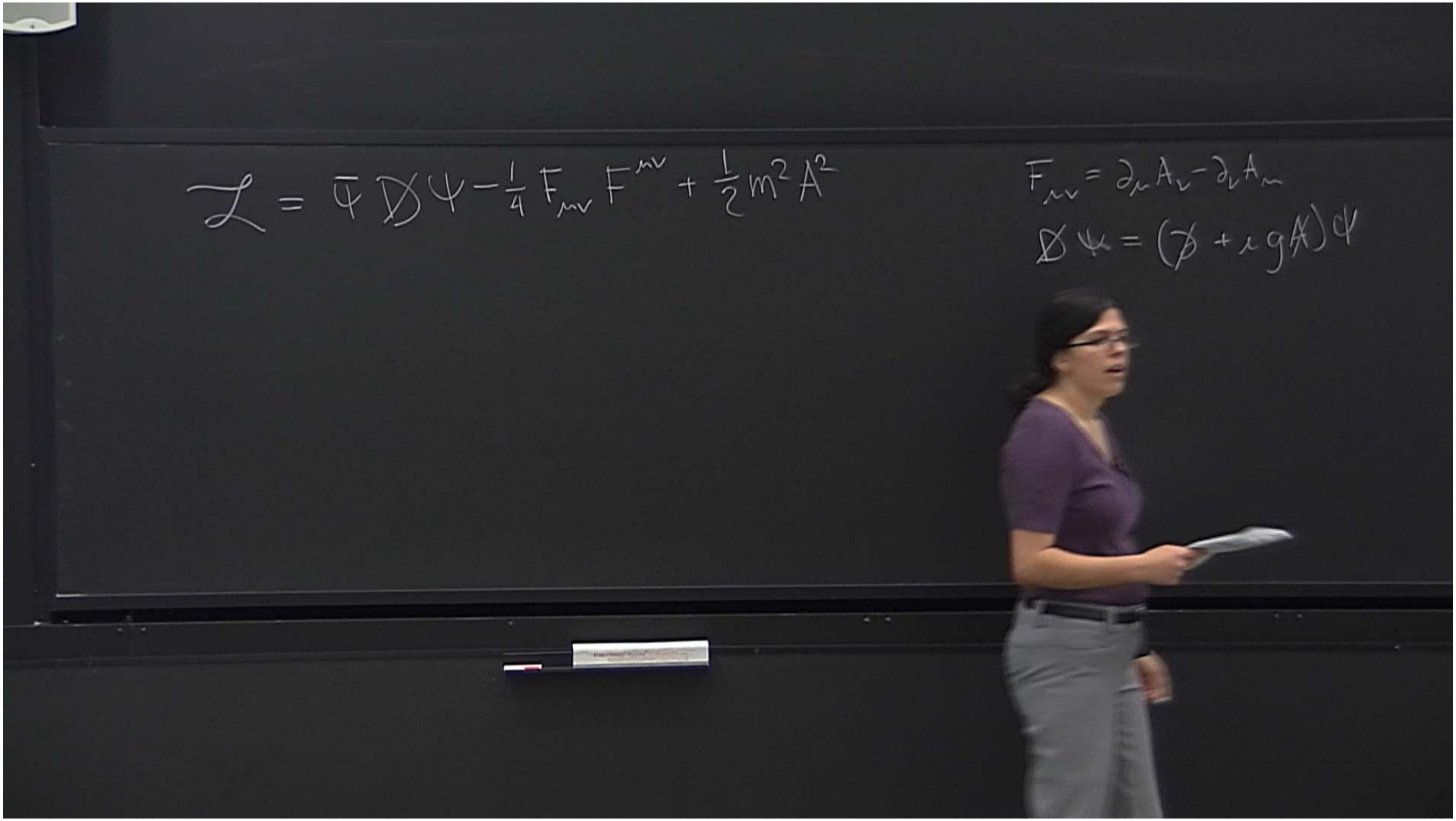

$W_+ \left(\bar{\mu} \gamma^\mu P_L \nu_\mu + \bar{e} \gamma^\mu P_L \nu_e + \dots \right) J_+^\mu$
$\mathcal{L}_{eff} = \frac{g_2^2}{2 M_W^2} \left(\mu \gamma^\mu P_L \bar{\nu}_\mu \right) \left(e \gamma^\mu P_L \bar{\nu}_e \right)^\dagger + \dots \sim \frac{g_2^2}{2 M_W^2} J_+^\mu J_+^{\dagger \mu}$

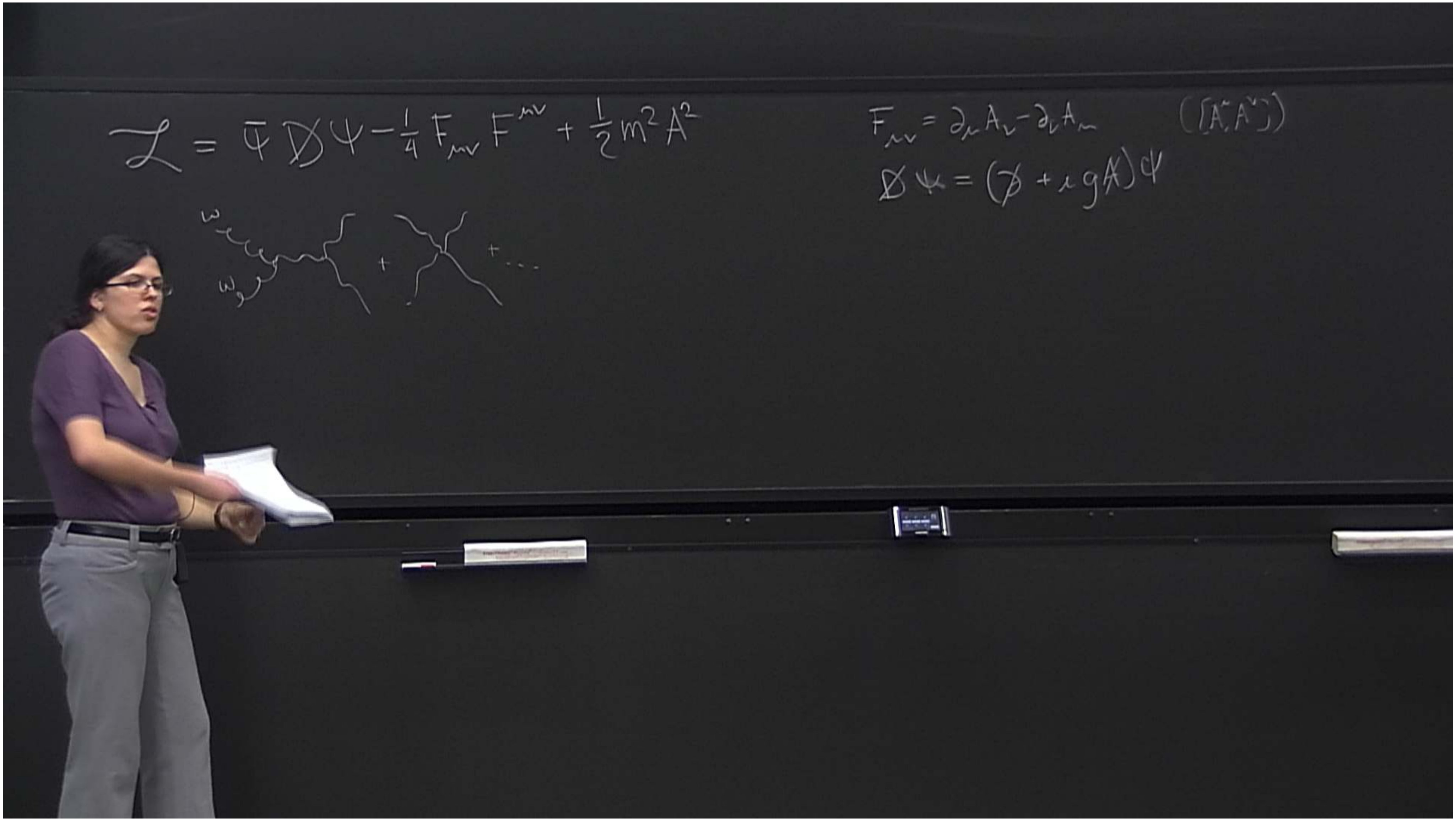


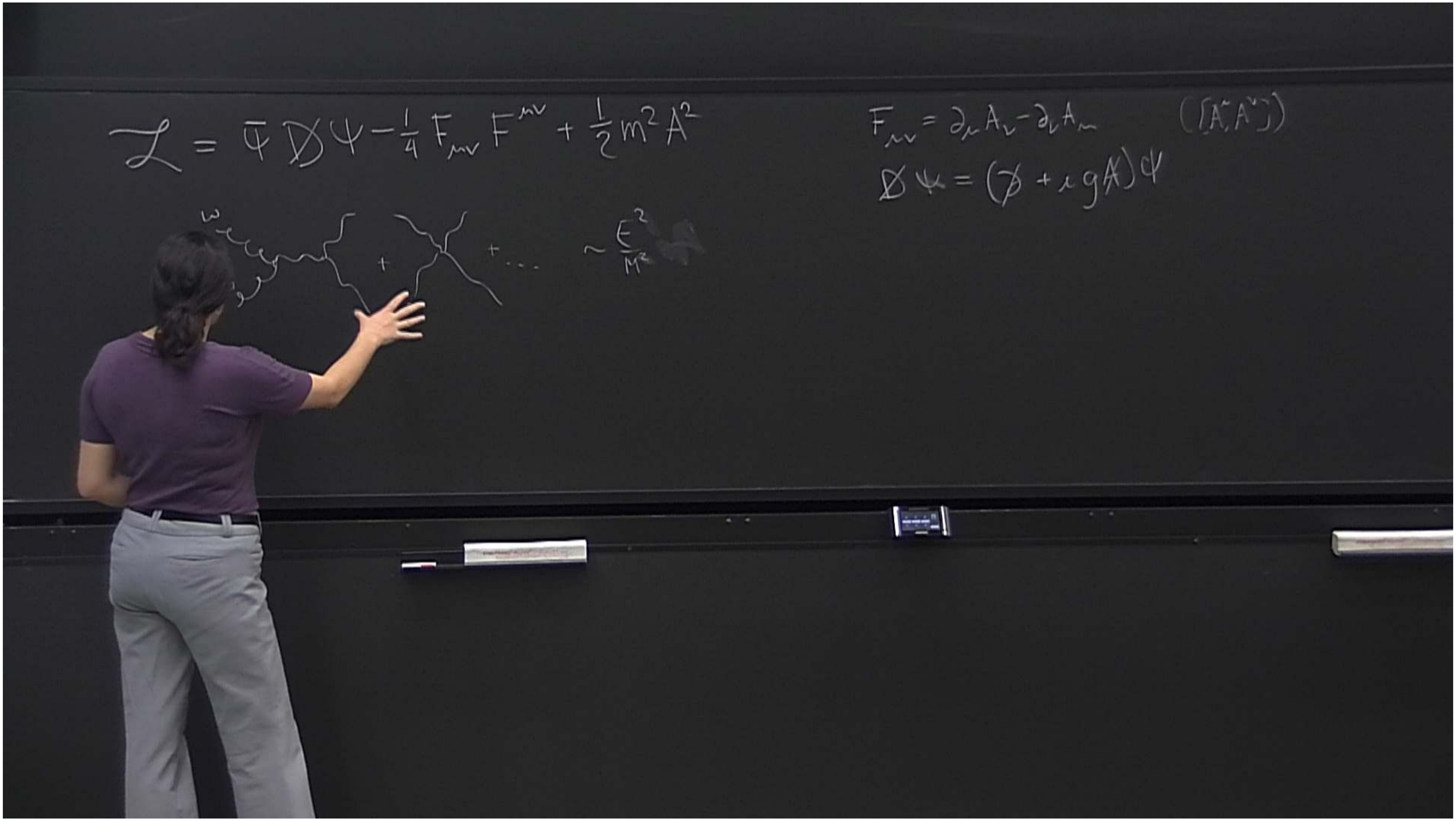
$W^-_\mu (\bar{\mu}\gamma^\mu P_L \nu_\mu + \bar{e}\gamma^\mu P_L \nu_e + \dots) J_+^\mu$
$\mathcal{L}_{eff} = \frac{g_2^2}{2M_W^2} (\mu \gamma^\mu P_L \bar{\nu}_\mu)(e \gamma^\mu P_L \bar{\nu}_e)^\dagger + \dots \sim \frac{g_2^2}{2M_W^2} J_+^\mu J_+^{\dagger\mu}$

$$W_\mu^-\,\underbrace{(\bar{\mu}\gamma^\mu P_L \nu_\mu + \bar{e}\gamma^\mu P_L \nu_e + \dots)}_{J_+^\mu}$$

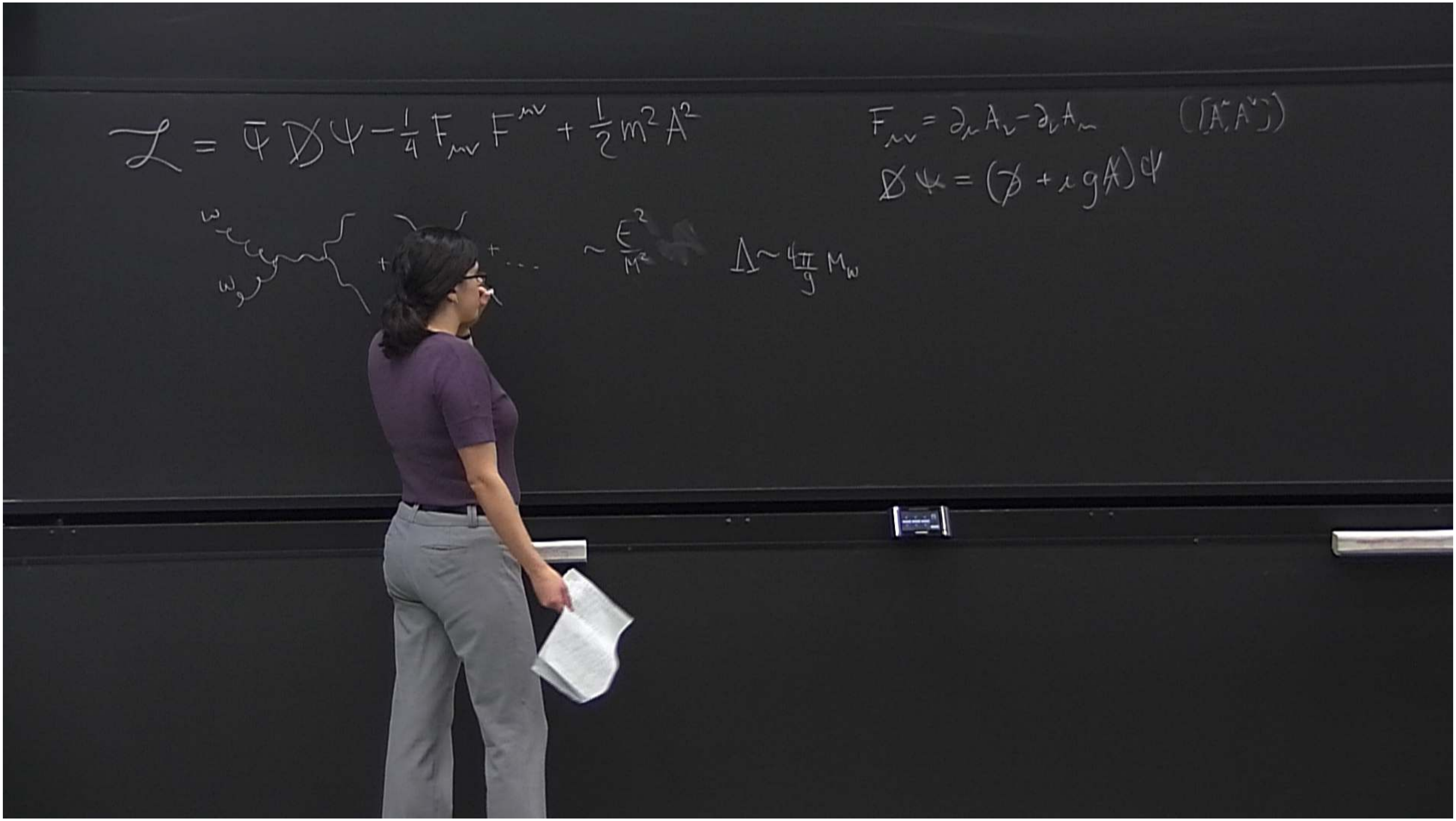
$$\mathcal{L}_{eff} = \frac{g_2^2}{2M_W^2}\,(\mu\,\gamma^\mu P_L \bar{\nu}_\mu)(e\,\gamma^\mu P_L \bar{\nu}_e)^\dagger + \dots \sim \frac{g_2^2}{2M_W^2}\, J_+^\mu\, J_+^{\dagger\mu}$$


$\mathcal{L} = \bar{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^2$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\not{D} \Psi = (\not{\partial} + i g \not{A}) \Psi$

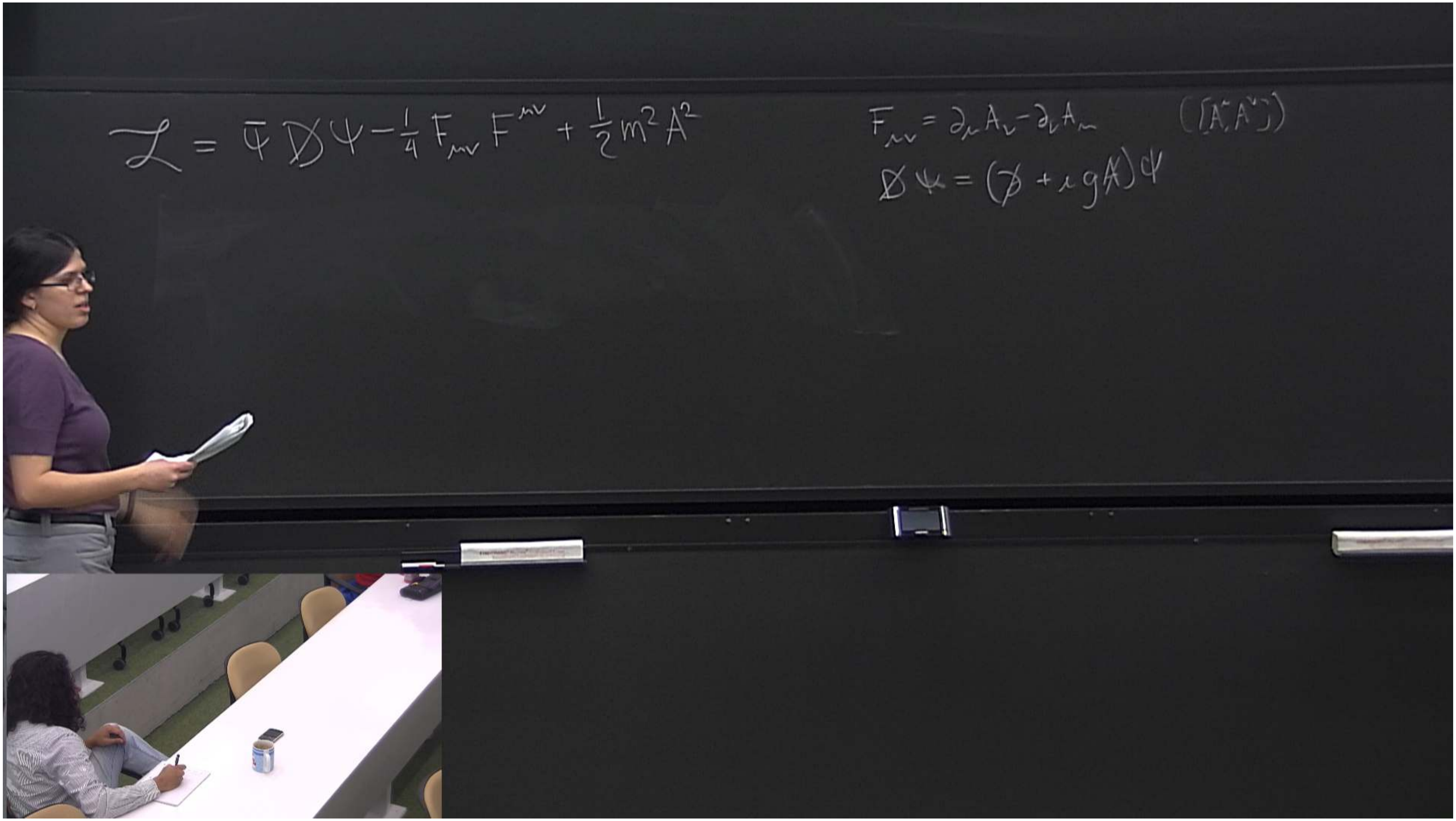

$\mathcal{L} = \bar{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^2$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$([A^\mu, A^\nu])$
$\not{D} \Psi = (\not{\partial} + i g \not{A}) \Psi$

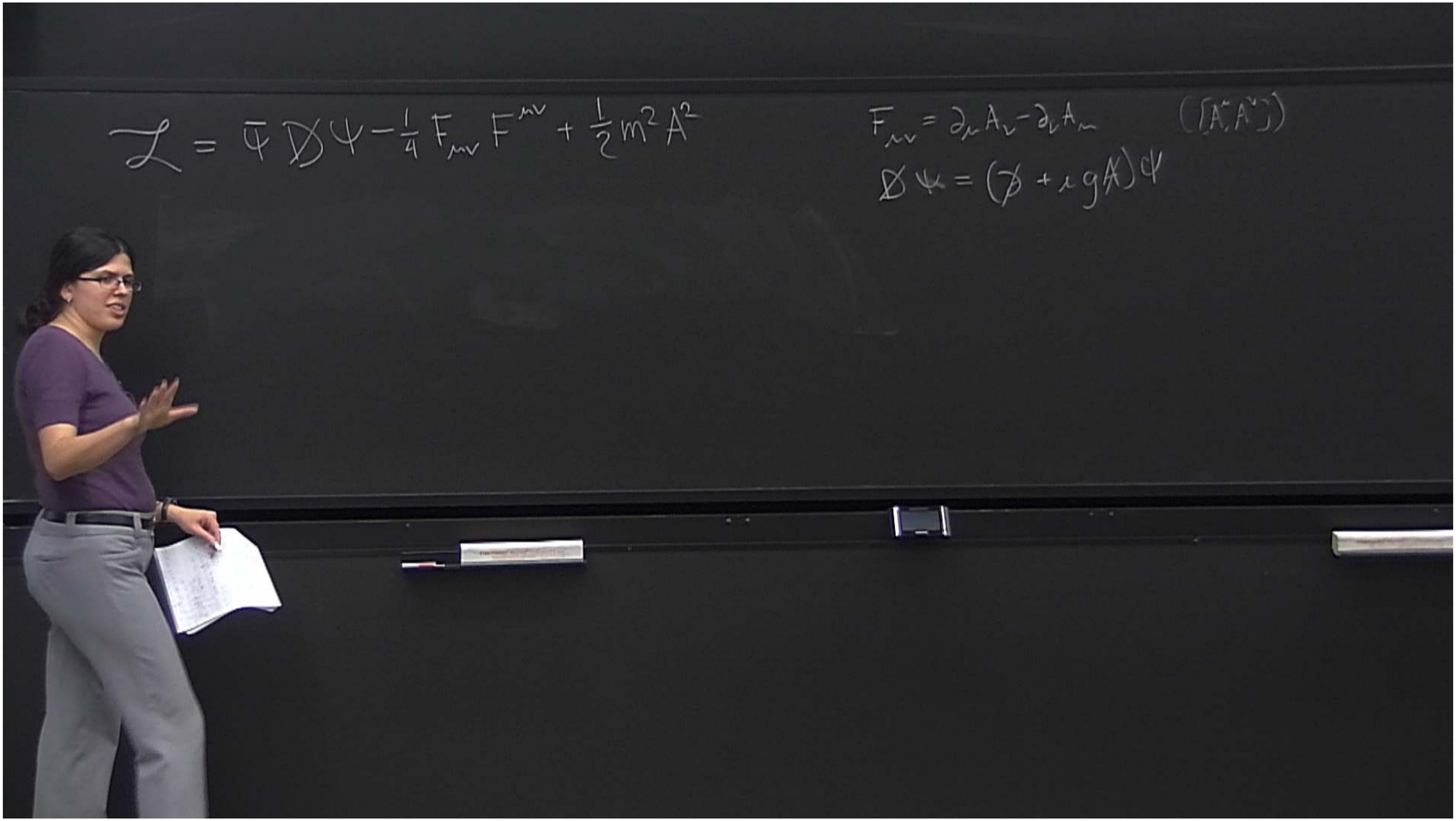


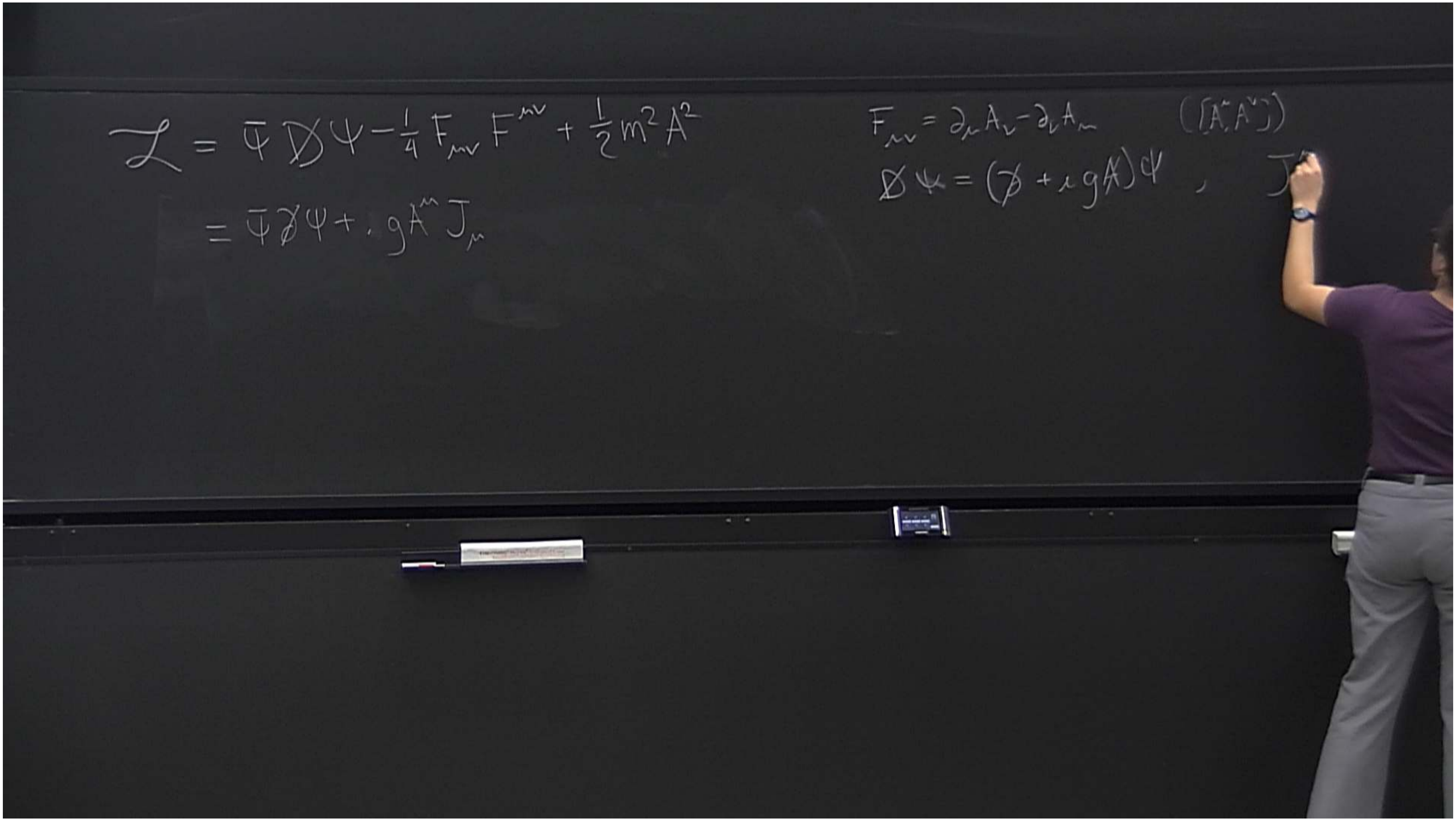
$\mathcal{L} = \bar{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^2$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$([A^\mu, A^\nu])$
$\not{D} \Psi = (\not{\partial} + i g \not{A}) \Psi$
$+ \dots \sim \frac{E^2}{M^2}$

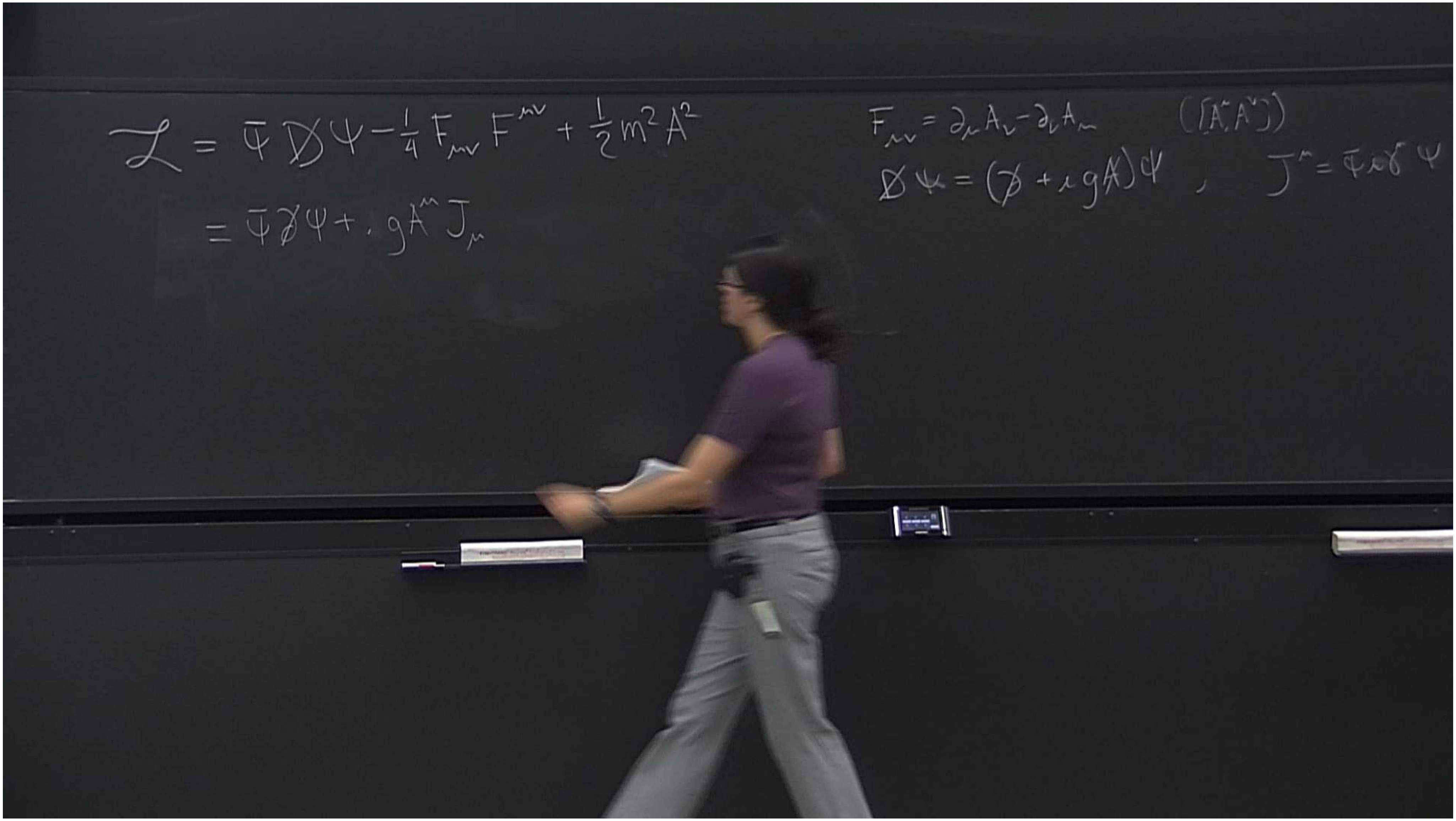


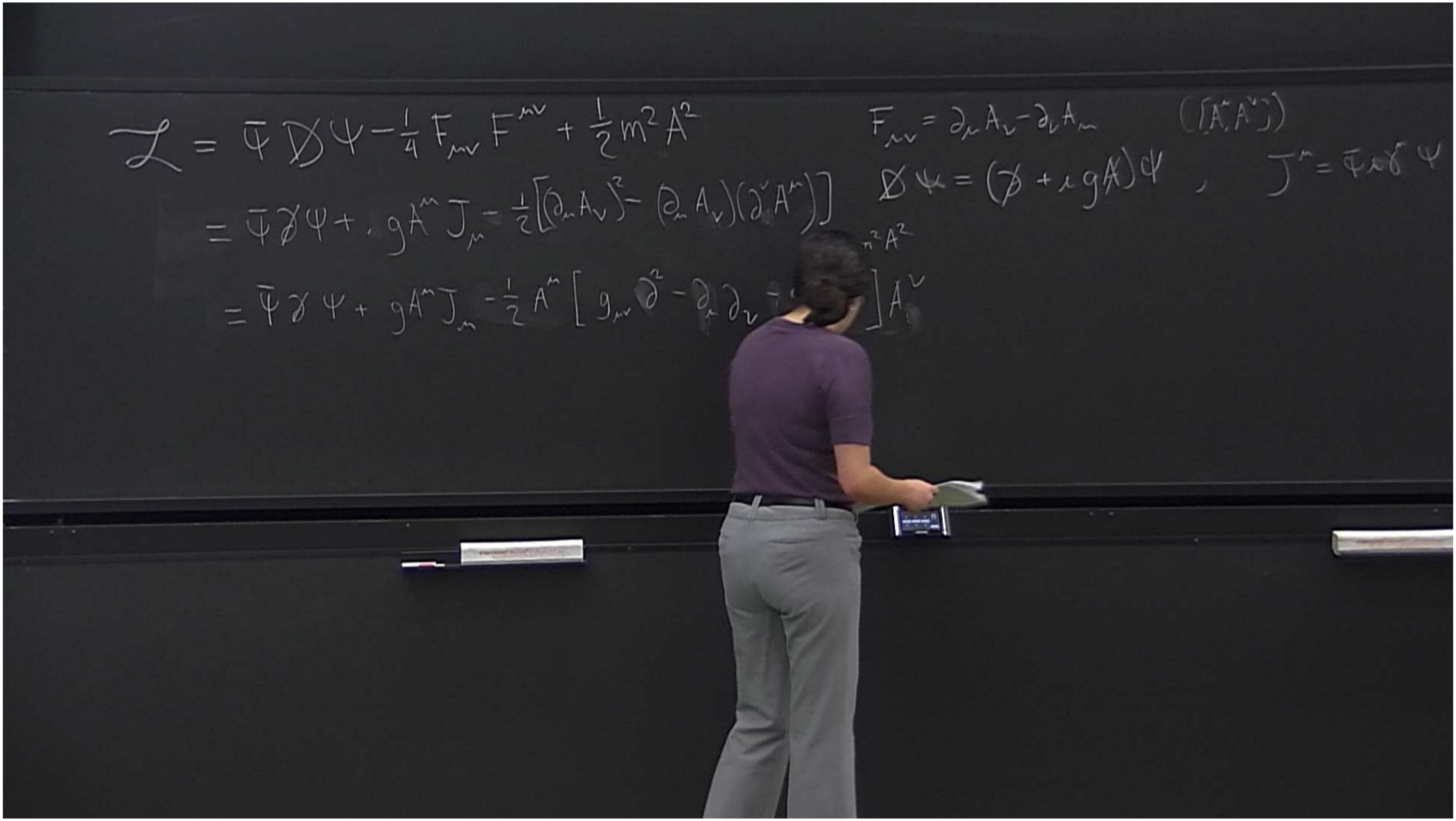
$\mathcal{L} = \bar{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^2$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ $([A^\mu, A^\nu])$
$\not{D} \Psi = (\not{\partial} + i g \not{A}) \Psi$
$+ \ldots \sim \frac{E^2}{M^2}$
$\Delta \sim \frac{4\pi}{g} M_W$

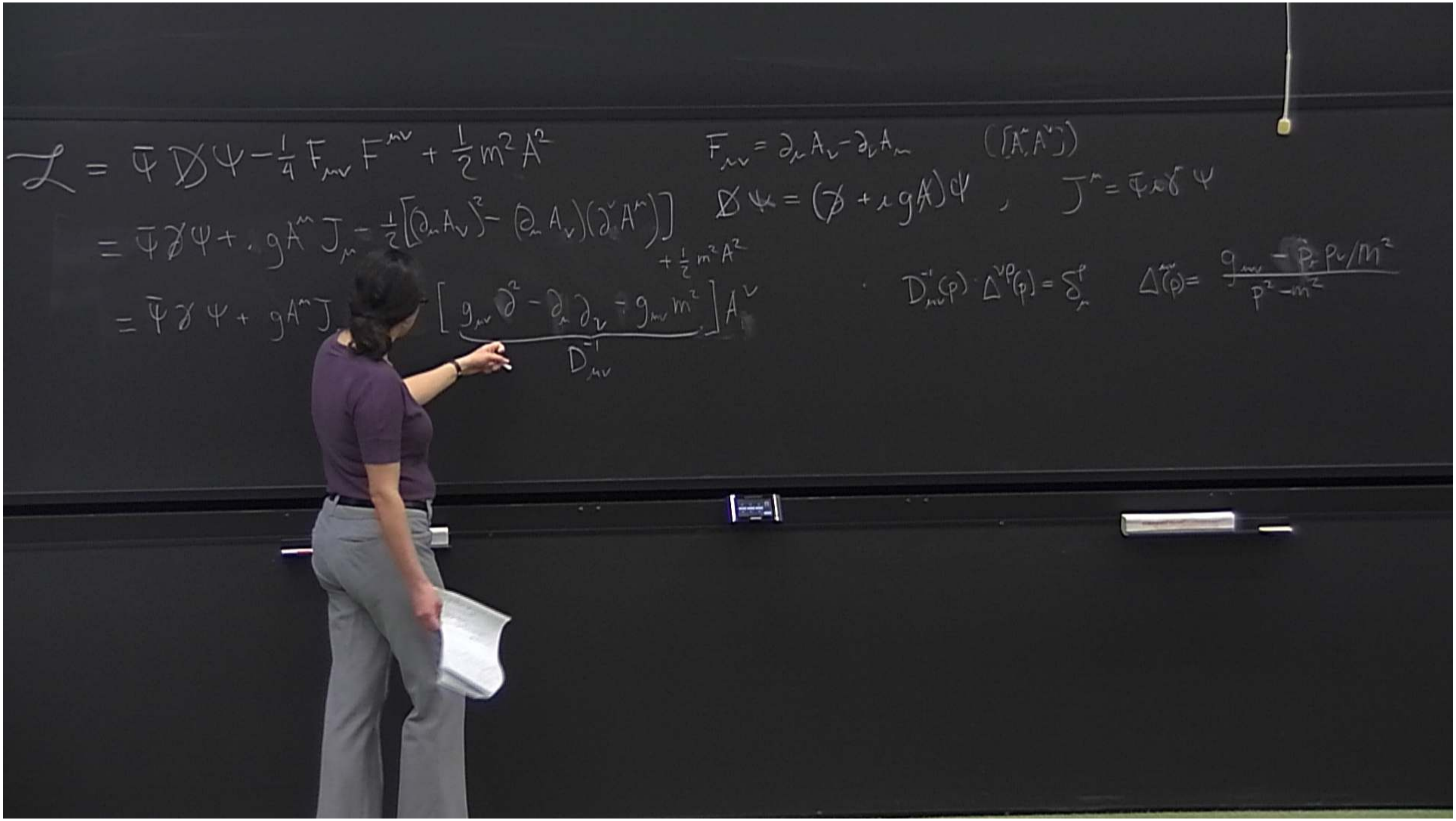

$\mathcal{L} = \bar{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^2$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$([A^\mu, A^\nu])$
$\not{D} \Psi = (\not{\partial} + i g \not{A}) \Psi$


$\mathcal{L} = \bar{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^2$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$([A^\mu, A^\nu])$
$\not{D} \Psi = (\not{\partial} + i g \not{A}) \Psi$

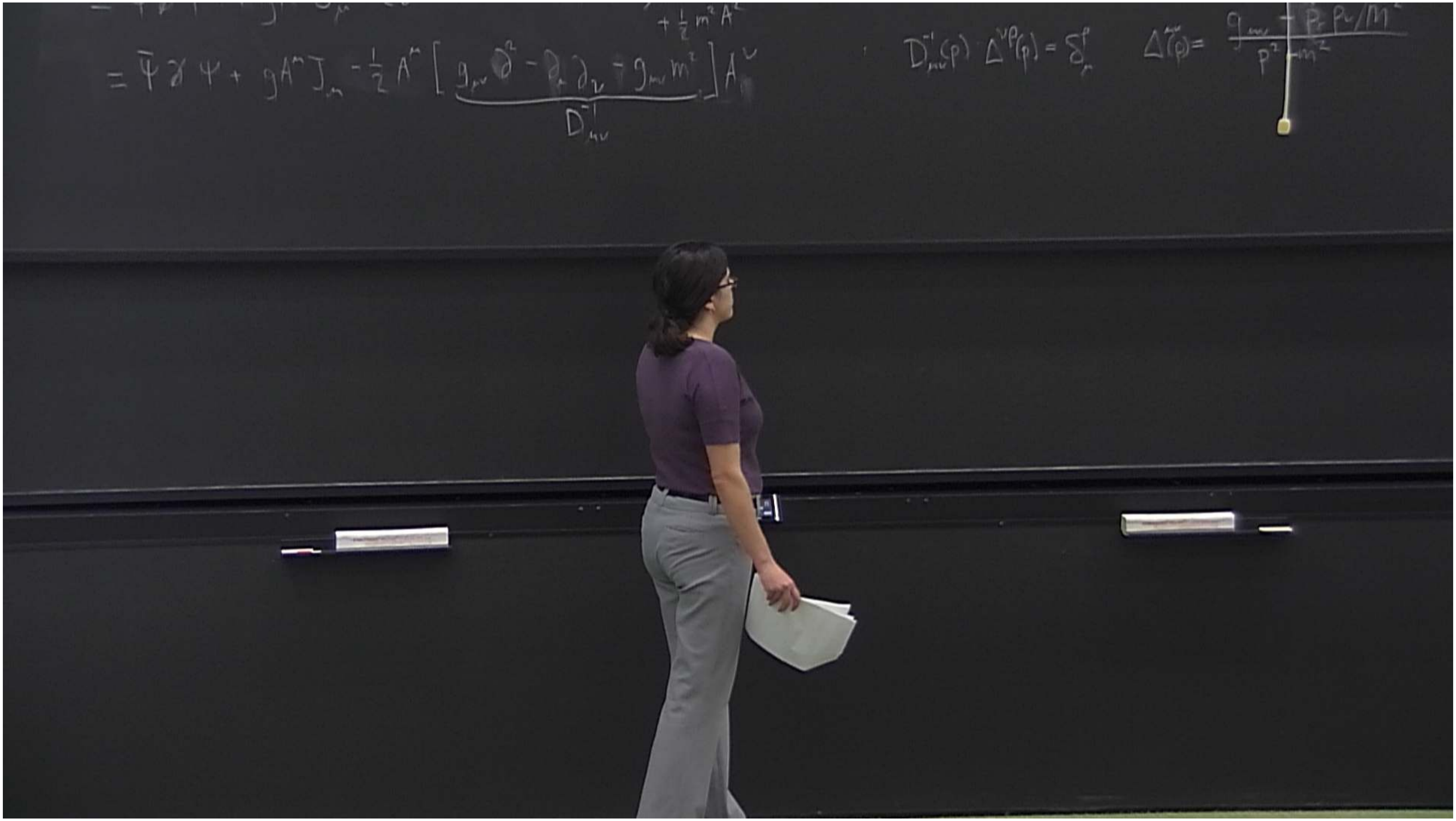

$\mathcal{L} = \bar{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^2$
$= \bar{\Psi} \not{\partial} \Psi + i g A^{\mu} J_{\mu}$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad ([A^\mu, A^\nu])$
$\not{D} \Psi = (\not{\partial} + i g \not{A}) \Psi \ , \quad J$

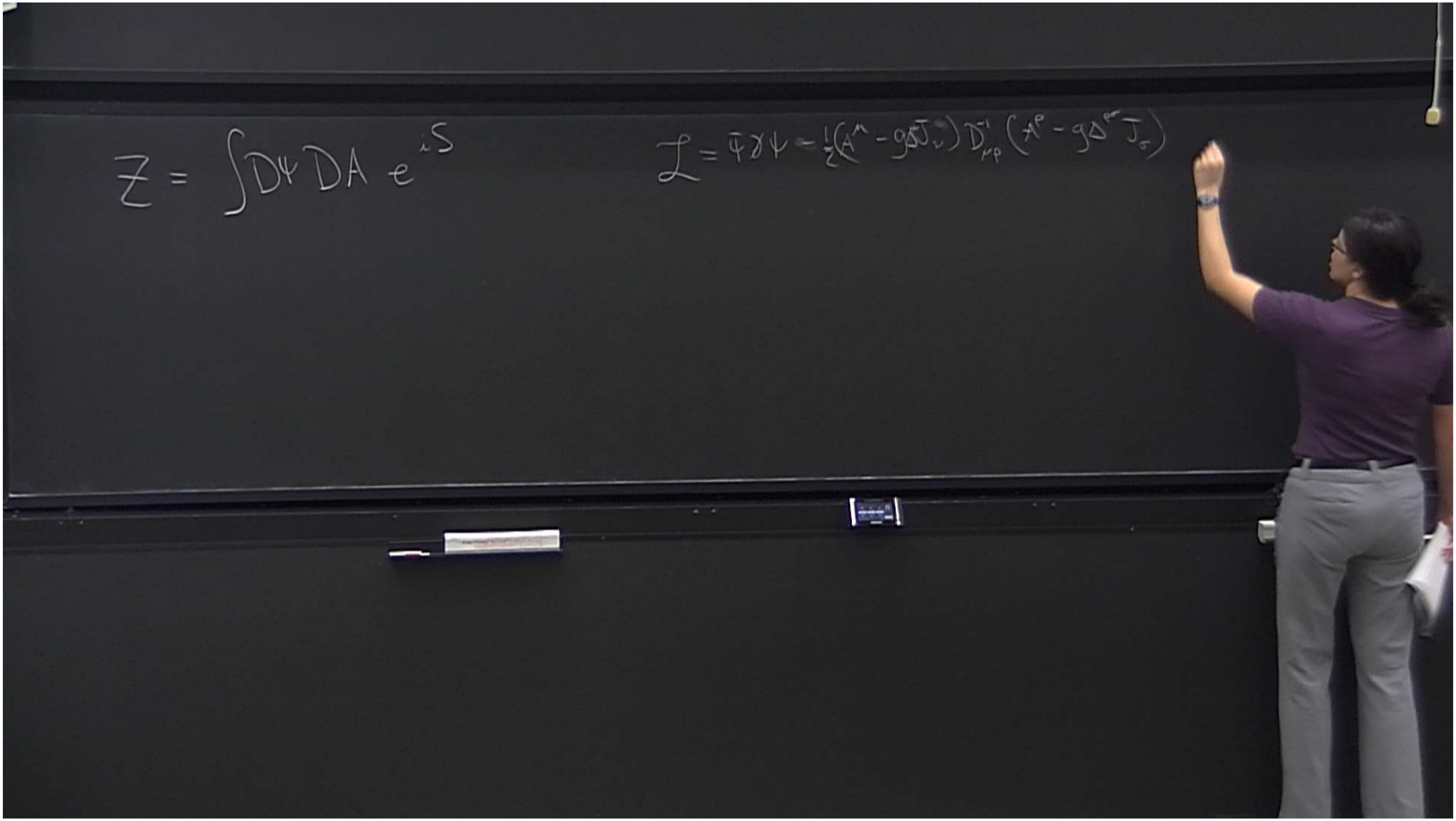

$\mathcal{L} = \bar{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^2$
$= \bar{\Psi} \not{\partial} \Psi + i g A^{\mu} J_{\mu}$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$([A^\mu, A^\nu])$
$\not{D} \Psi = (\not{\partial} + i g \not{A}) \Psi$,
$J^\mu = \bar{\Psi} i \gamma^\mu \Psi$

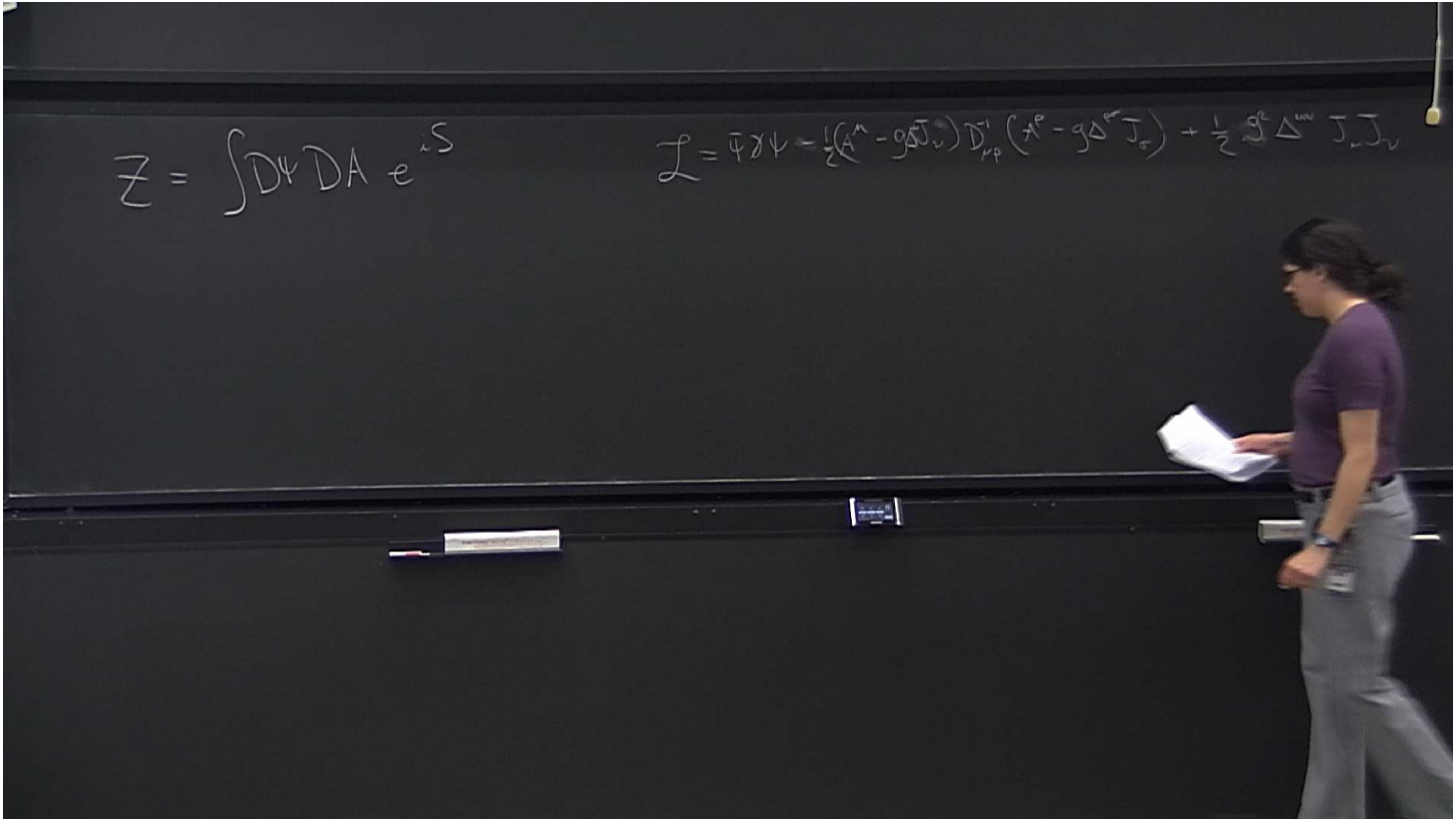

$\mathcal{L} = \bar{\Psi} \not{D} \Psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A^2$
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$([A^\mu, A^\nu])$
$= \bar{\Psi} \not{\partial} \Psi + g A^\mu J_\mu - \frac{1}{2}[(\partial_\mu A_\nu)^2 - (\partial_\mu A_\nu)(\partial^\nu A^\mu)]$
$\not{D} \Psi = (\not{\partial} + i g \not{A}) \Psi$, $J^\mu = \bar{\Psi} \gamma^\mu \Psi$
$= \bar{\Psi} \not{\partial} \Psi + g A^\mu J_\mu - \frac{1}{2} A^\mu [g_{\mu\nu} \partial^2 - \partial_\mu \partial_\nu] A^\nu$



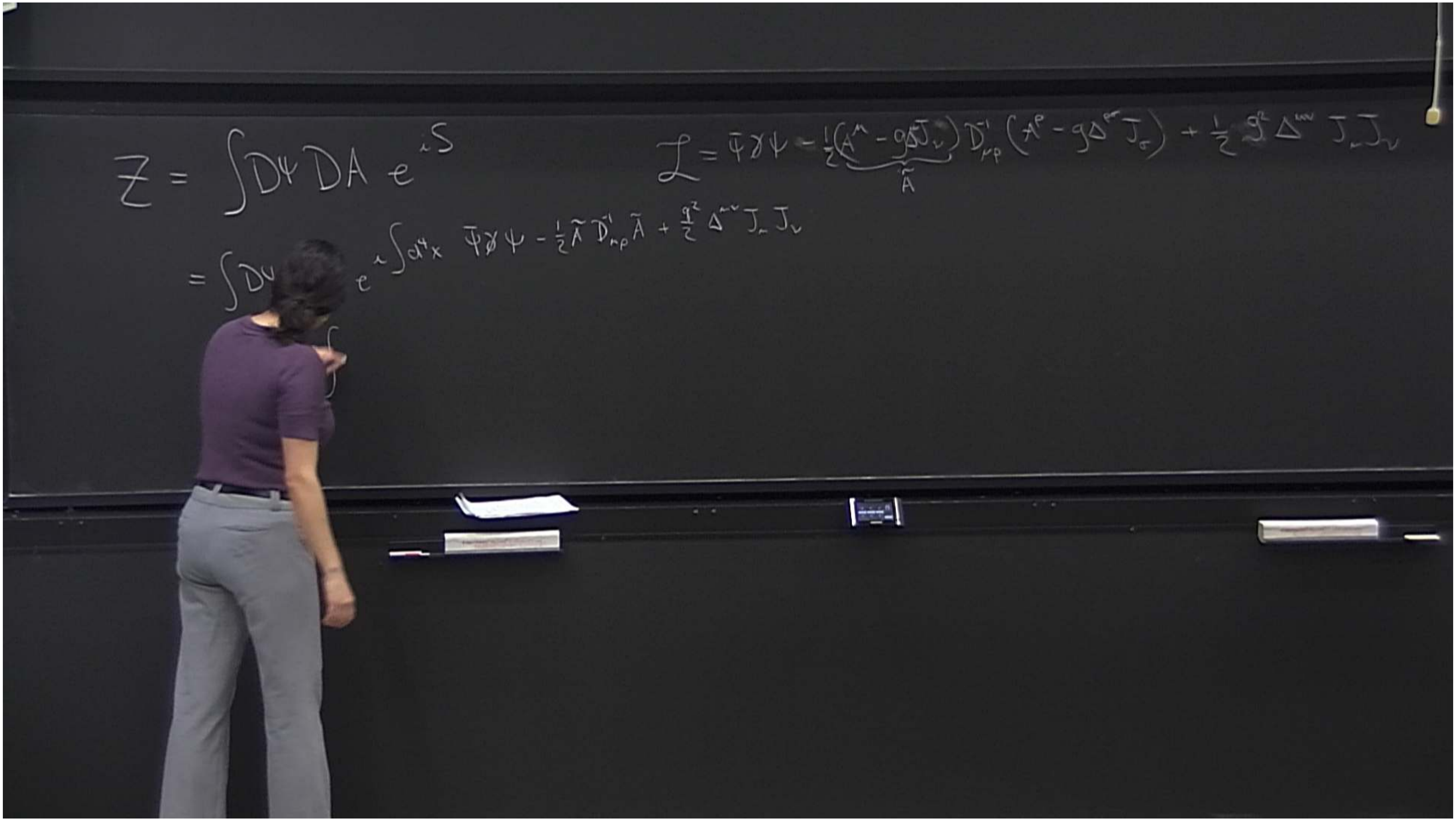
L = Ψ̄ D̸ Ψ − 1/4 F_μν F^μν + 1/2 m² A²
F_μν = ∂_μ A_ν − ∂_ν A_μ
D̸ Ψ = (∂̸ + i g A̸) Ψ ,
J^μ = Ψ̄ γ^μ Ψ
D^{-1}_μν(p) · Δ^{νρ}(p) = δ^ρ_μ
Δ^{μν}(p) = (g_μν − p_μ p_ν / m²) / (p² − m²)



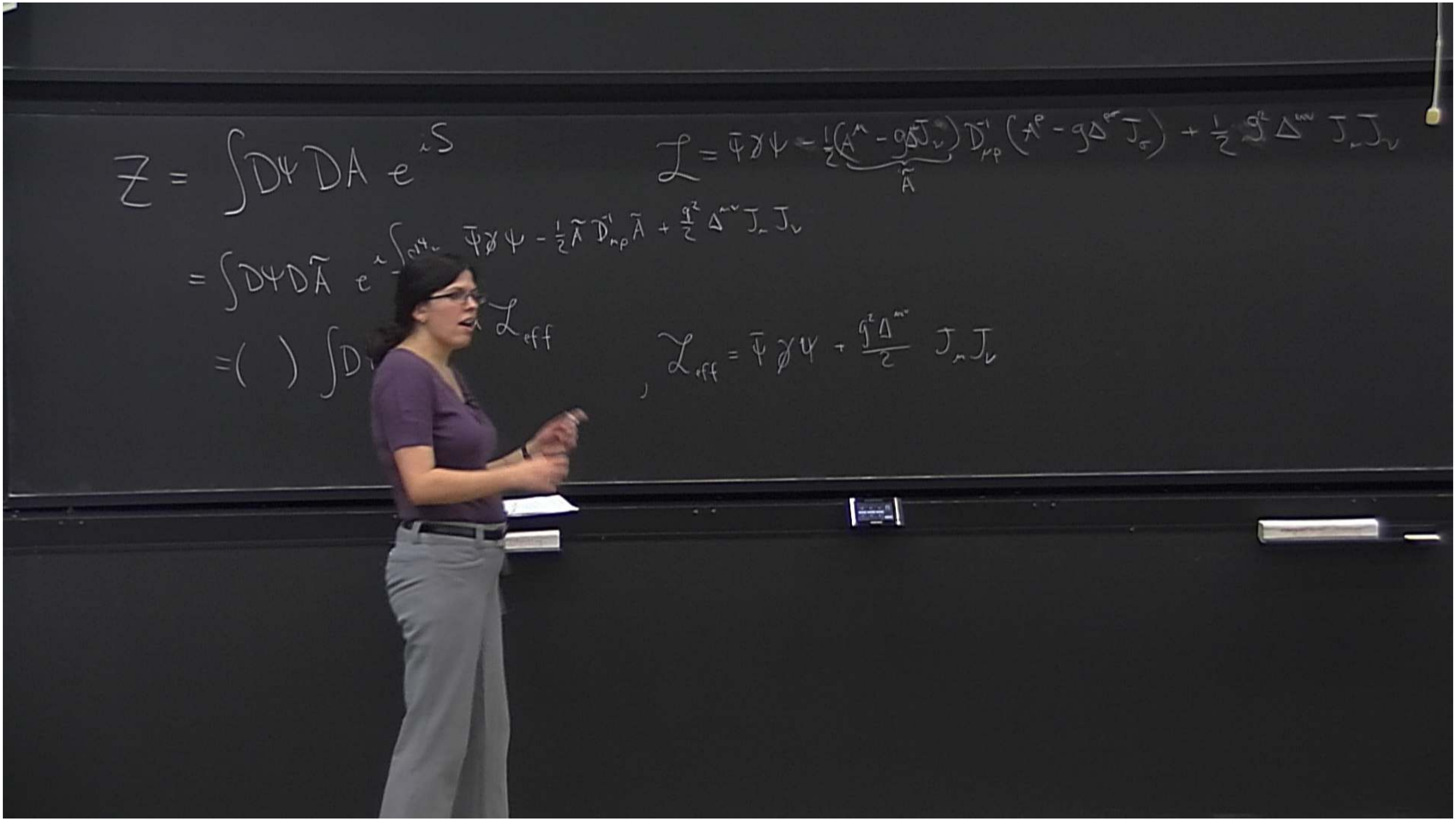

$Z = \int D\Psi \, DA \, e^{iS}$
$\mathcal{L} = \bar{\Psi} \gamma \Psi - \frac{1}{2}(A^{\mu} - g\Delta^{\mu\nu} J_{\nu}) D^{-1}_{\mu\rho} (A^{\rho} - g\Delta^{\rho\sigma} J_{\sigma})$



$Z = \int D\Psi\, DA\, e^{iS}$
$\mathcal{L} = \bar{\Psi}\gamma\Psi - \frac{1}{2}(A^{\mu} - g\Delta^{\mu\nu}J_{\nu}) D^{-1}_{\mu\rho} (A^{\rho} - g\Delta^{\rho\sigma}J_{\sigma}) + \frac{1}{2} g^2 \Delta^{\mu\nu} J_{\mu} J_{\nu}$



$Z = \int D\Psi\, DA\; e^{iS}$
$\mathcal{L} = \bar{\Psi}\gamma\Psi - \frac{1}{2}(A^{\mu} - g\Delta J_{\nu}) D^{-1}_{\mu\rho} (A^{\rho} - g\Delta^{\rho\sigma} J_{\sigma}) + \frac{1}{2} g^2 \Delta^{\mu\nu} J_{\mu} J_{\nu}$
$\tilde{A}$
$= \int D\Psi \; e^{i\int d^4x\; \bar{\Psi}\gamma\Psi - \frac{1}{2}\tilde{A} D^{-1}_{\mu\rho}\tilde{A} + \frac{g^2}{2}\Delta^{\mu\nu} J_{\mu} J_{\nu}}$


Z = ∫DΨ DA e^{iS}
L = Ψ̄γΨ − ½(A^μ − gΔJ_ν) D^{-1}_{μρ} (A^ρ − gΔ^{ρσ}J_σ) + ½ g² Δ^{μν} J_μ J_ν
Ã
= ∫DΨDÃ e^{i∫d⁴x Ψ̄γΨ − ½ÃD̃^{-1}_{μρ}Ã + g²/2 Δ^{μν} J_μ J_ν}
= ( ) ∫D L_eff
L_eff = Ψ̄γΨ + g²Δ^{μν}/2 J_μ J_ν

$$Z = \int D\Psi\, DA\; e^{iS}$$

$$\mathcal{L} = \bar{\Psi}\gamma\Psi - \frac{1}{2}\underbrace{\left(A^{\mu} - g\Delta^{\mu\nu}J_{\nu}\right)}_{\tilde{A}} D^{-1}_{\mu\rho}\left(A^{\rho} - g\Delta^{\rho\sigma}J_{\sigma}\right) + \frac{1}{2}g^{2}\Delta^{\mu\nu}J_{\mu}J_{\nu}$$

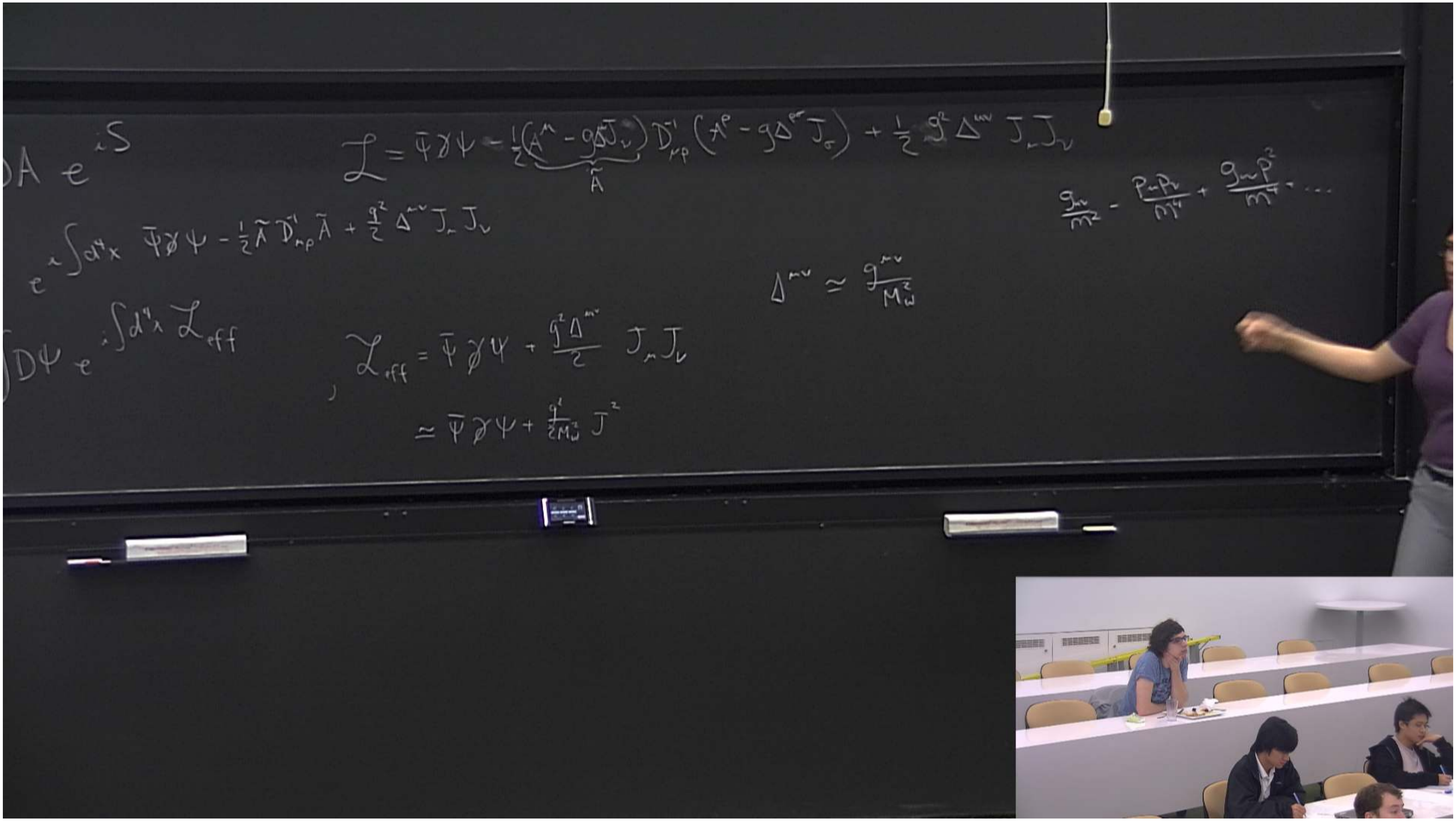
$$= \int D\Psi\, D\tilde{A}\; e^{i\int d^4x\; \bar{\Psi}\gamma\Psi - \frac{1}{2}\tilde{A}D^{-1}_{\mu\rho}\tilde{A} + \frac{g^2}{2}\Delta^{\mu\nu}J_{\mu}J_{\nu}}$$

$$\Delta^{\mu\nu} \simeq \frac{g^{\mu\nu}}{M_W^2}$$

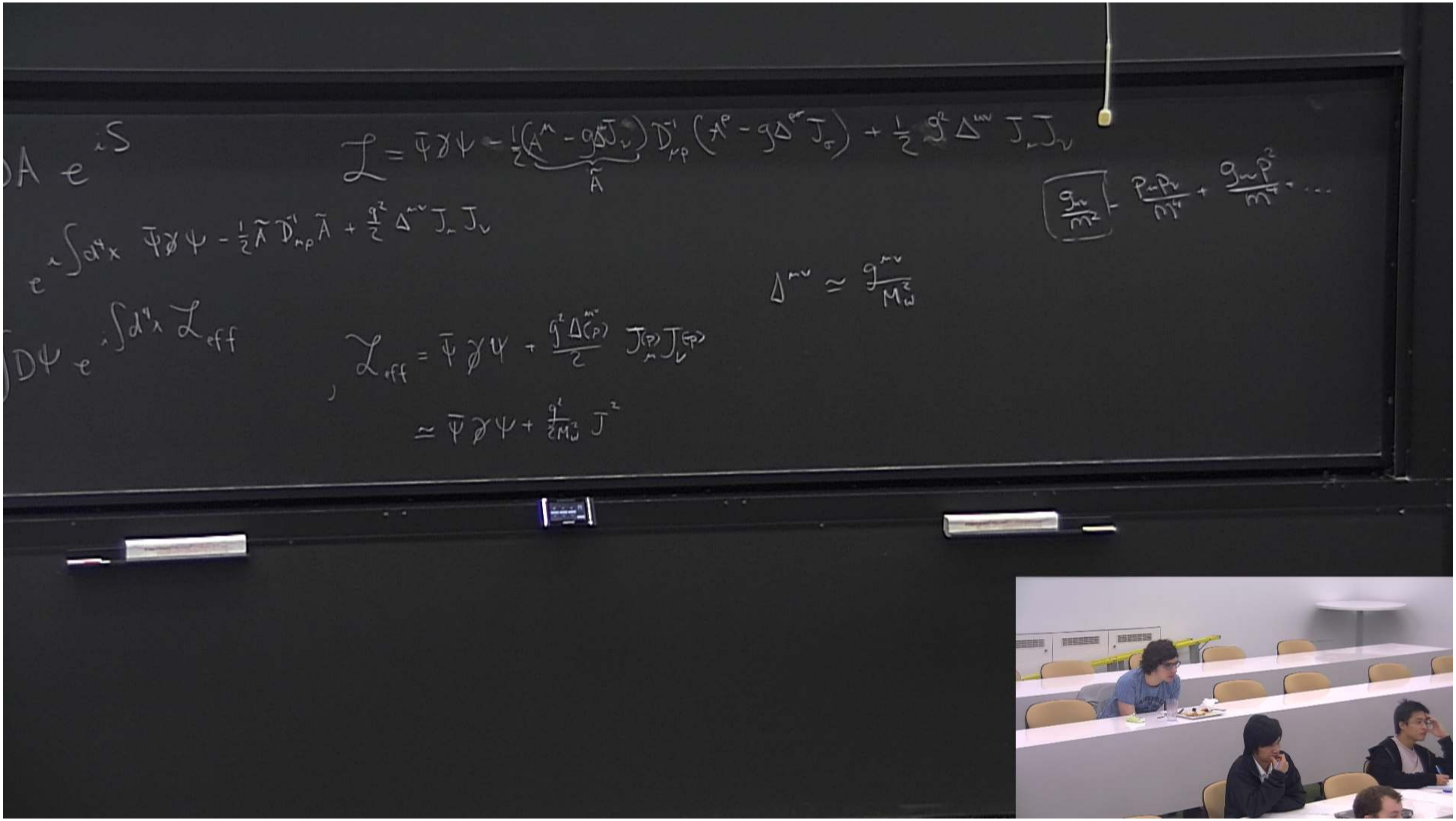
$$= (\quad)\int D\Psi\; e^{i\int d^4x\, \mathcal{L}_{eff}}$$

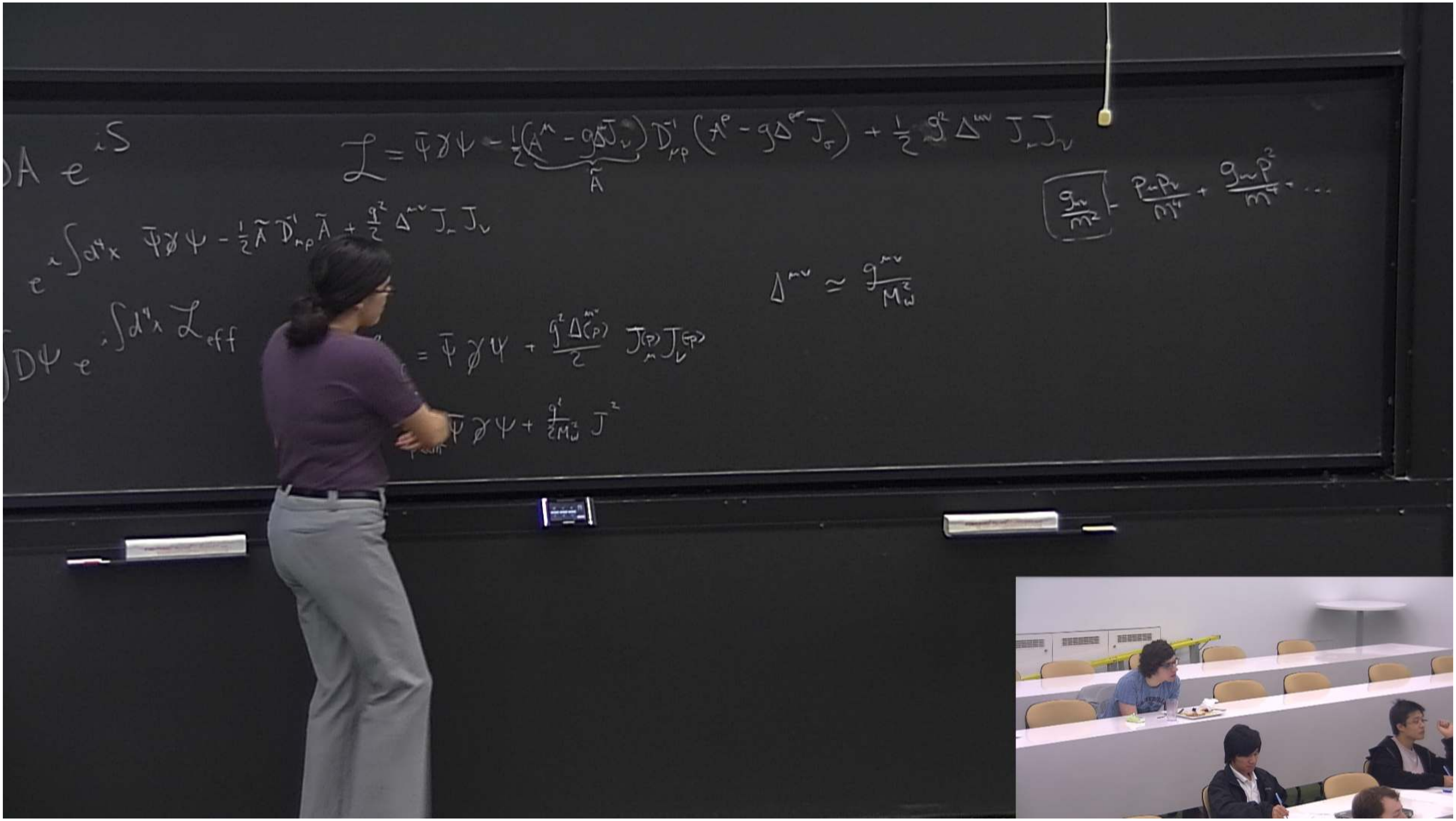
$$,\ \mathcal{L}_{eff} = \bar{\Psi}\not{\partial}\Psi + \frac{g^2\Delta^{\mu\nu}}{2}J_{\mu}J_{\nu}$$

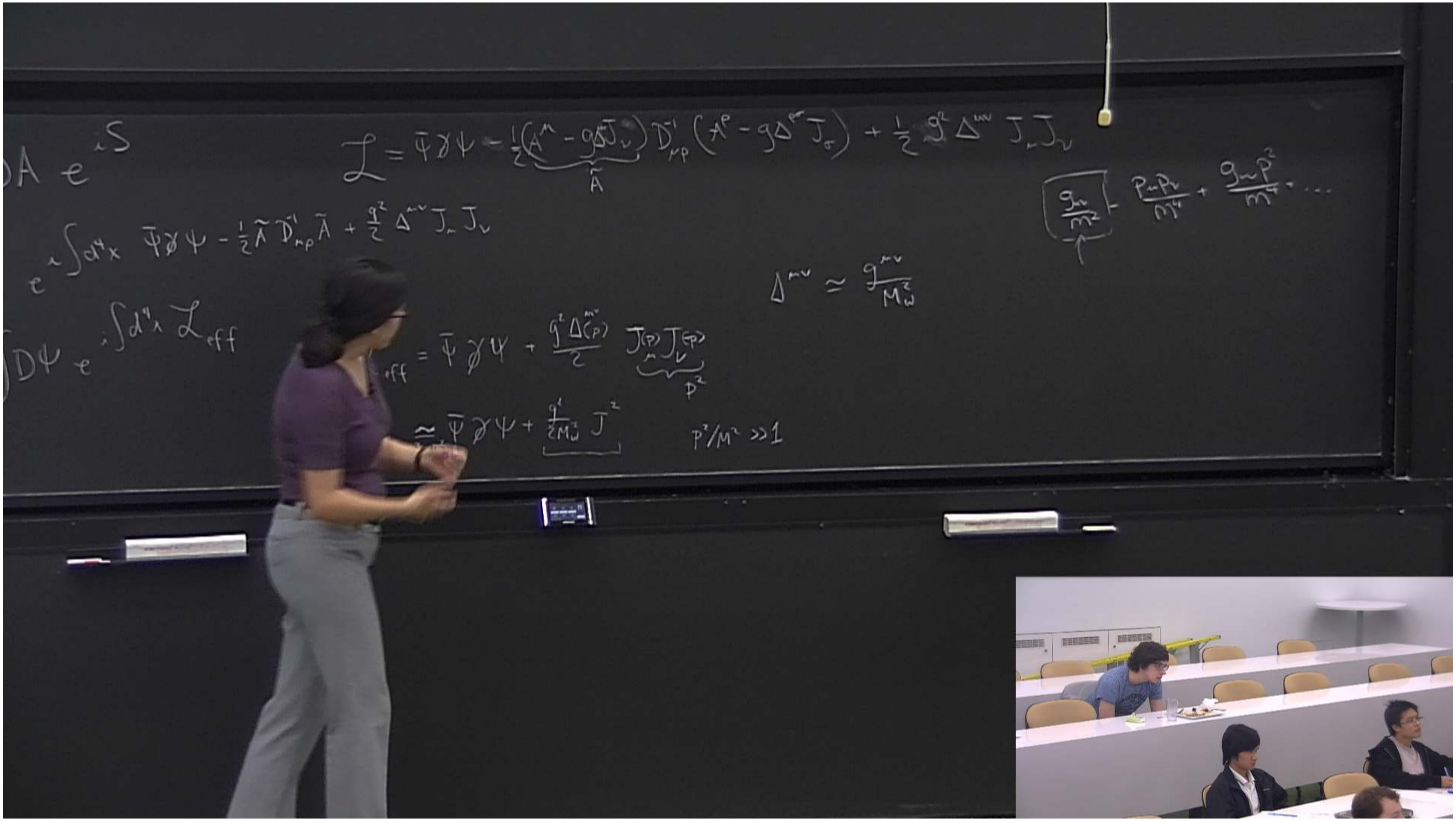
$$\simeq \bar{\Psi}\not{\partial}\Psi + \frac{g^2}{2M_W^2}J^2$$



$\mathcal{L} = \bar{\Psi}\gamma\Psi - \frac{1}{2}(A^\mu - g\Delta^{\mu\nu}J_\nu) D^{-1}_{\mu\rho} (A^\rho - g\Delta^{\rho\sigma}J_\sigma) + \frac{1}{2} g^2 \Delta^{\mu\nu} J_\mu J_\nu$
$e^{i\int d^4x\, \bar{\Psi}\gamma\Psi - \frac{1}{2}\tilde{A} D^{-1}_{\mu\rho}\tilde{A} + \frac{g^2}{2}\Delta^{\mu\nu}J_\mu J_\nu}$
$\int D\Psi\, e^{i\int d^4x\, \mathcal{L}_{eff}}$
$\mathcal{L}_{eff} = \bar{\Psi}\gamma\Psi + \frac{g^2\Delta^{\mu\nu}}{2} J_\mu J_\nu$
$\simeq \bar{\Psi}\gamma\Psi + \frac{g^2}{2M_W^2} J^2$
$\Delta^{\mu\nu} \simeq \frac{g^{\mu\nu}}{M_W^2}$
$\frac{g_{\mu\nu}}{m^2} - \frac{p_\mu p_\nu}{m^4} + \frac{g_{\mu\nu}p^2}{m^4} + \dots$


A e^{iS}
L = Ψ̄ ∂̸ Ψ − ½ (A^μ − gΔ J_ν) D^{-1}_{μρ} (A^ρ − gΔ J_σ) + ½ g² Δ^{μν} J_μ J_ν
Ã
g_{μν}/m² − p_μ p_ν/m⁴ + g_{μν} p²/m⁴ + ...
e^{i∫d⁴x Ψ̄ ∂̸ Ψ − ½ Ã D^{-1}_{μρ} Ã + g²/2 Δ^{μν} J_μ J_ν}
Δ^{μν} ≃ g^{μν}/M_W²
DΨ e^{i∫d⁴x L_eff}
L_eff = Ψ̄ ∂̸ Ψ + g² Δ^{μν}(p)/2 J_μ(p) J_ν(−p)
≃ Ψ̄ ∂̸ Ψ + g²/2M_W² J²


$\mathcal{L} = \bar\Psi \gamma \Psi - \frac{1}{2}(A^\mu - g\Delta^{\mu\nu}J_\nu) D^{-1}_{\mu\rho} (A^\rho - g\Delta^{\rho\sigma}J_\sigma) + \frac{1}{2} g^2 \Delta^{\mu\nu} J_\mu J_\nu$
$\Delta^{\mu\nu} \simeq \frac{g^{\mu\nu}}{M_W^2}$
$\frac{g_{\mu\nu}}{m^2} - \frac{p_\mu p_\nu}{m^4} + \frac{g_{\mu\nu} p^2}{m^4} + \dots$



L_eff
p²/M² >> 1