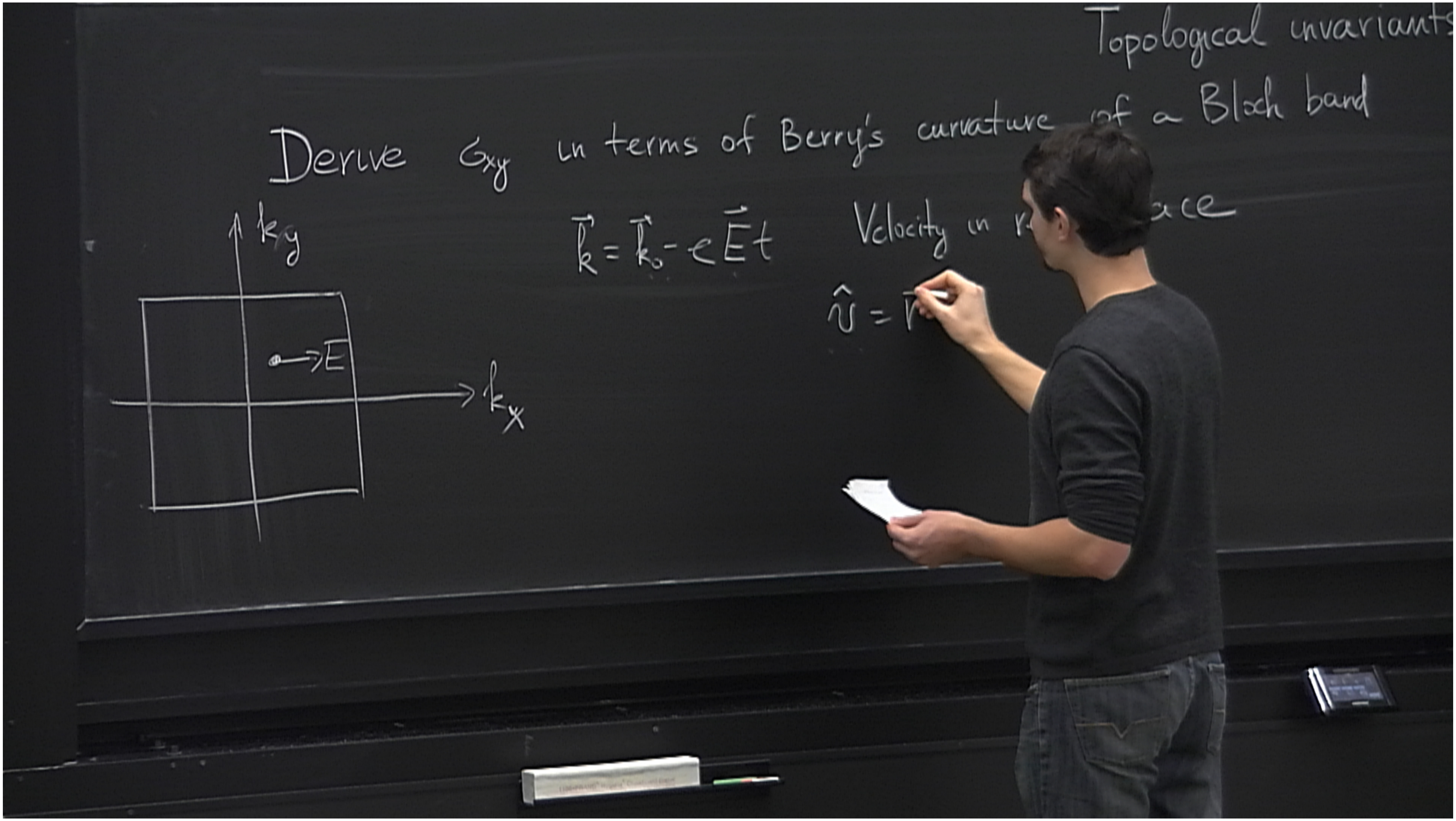
Title: 12/13 PSI - Condensed Matter Review Lecture 15

Date: Feb 15, 2013 11:30 AM

URL: http://pirsa.org/13020088

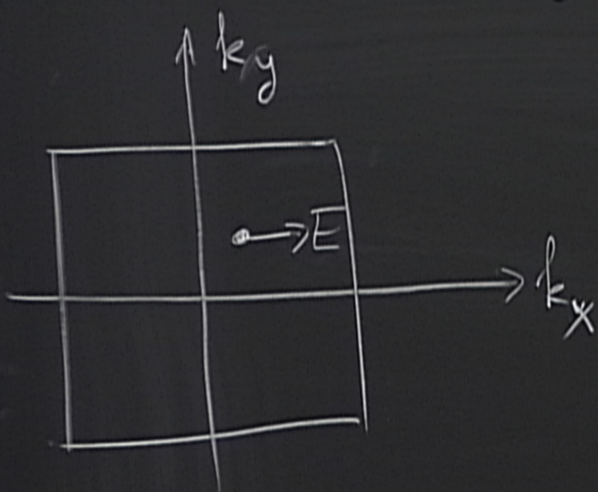
Abstract:

Topological invariants & Hall conductivity


Topological invariants
of a Bloch band
Derive σxy in terms of Berry's curvature
k = k0 - eEt
Velocity in
E
ky
kx

Topological invariants &

Derive σ_{xy} in terms of Berry's curvature of a Bloch band

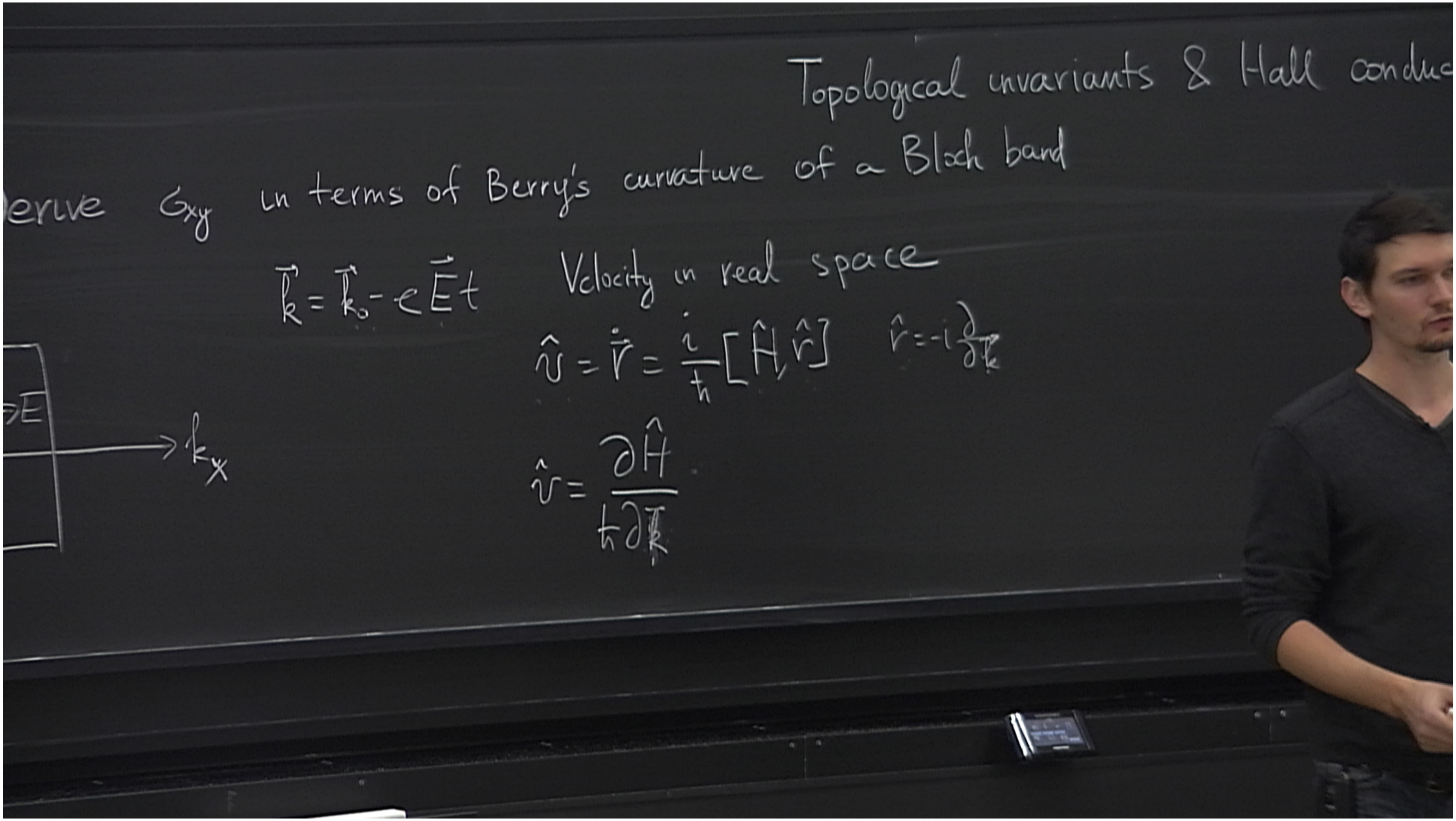


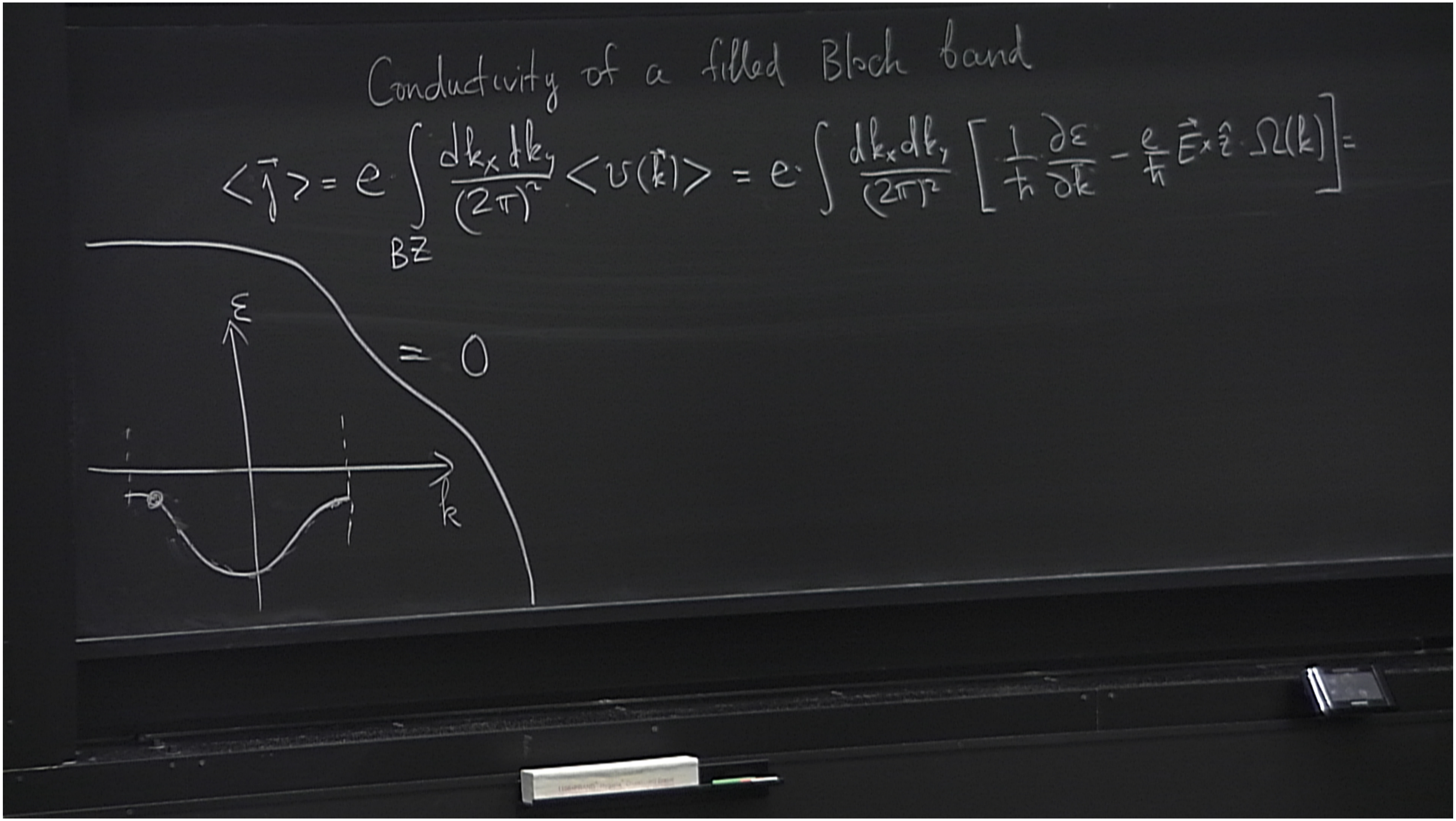
$$\vec{k} = \vec{k}_0 - e\vec{E}t$$

Velocity in real space

$$\hat{v} = \dot{\hat{r}} = \frac{i}{\hbar}[\hat{H}, \hat{r}] \qquad \hat{r} = -i\frac{\partial}{\partial k}$$

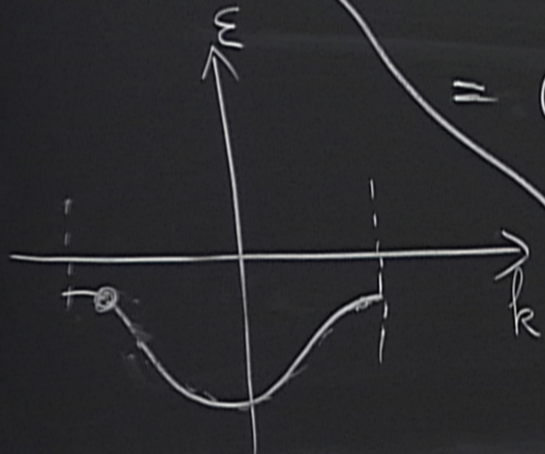
$$\hat{v} =$$


Topological invariants & Hall conduc
Bloch band
Derive σ_{xy} in terms of Berry's curvature of a Bloch band
$\vec{k} = \vec{k}_0 - e\vec{E}t$
Velocity in real space
$\hat{v} = \dot{\hat{r}} = \frac{i}{\hbar}[\hat{H}, \hat{r}]$
$\hat{r} = -i\frac{\partial}{\partial \vec{k}}$
$\hat{v} = \frac{\partial \hat{H}}{\hbar \partial \vec{k}}$
E
k_x


Conductivity of a filled Bloch band
$\langle \vec{j} \rangle = e \int_{BZ} \frac{dk_x dk_y}{(2\pi)^2} \langle v(\vec{k}) \rangle = e \int \frac{dk_x dk_y}{(2\pi)^2} \left[\frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k} - \frac{e}{\hbar} \vec{E} \times \hat{z} \cdot \Omega(k) \right] =$
$= 0$
ε
k

Conductivity of a filled Bloch band

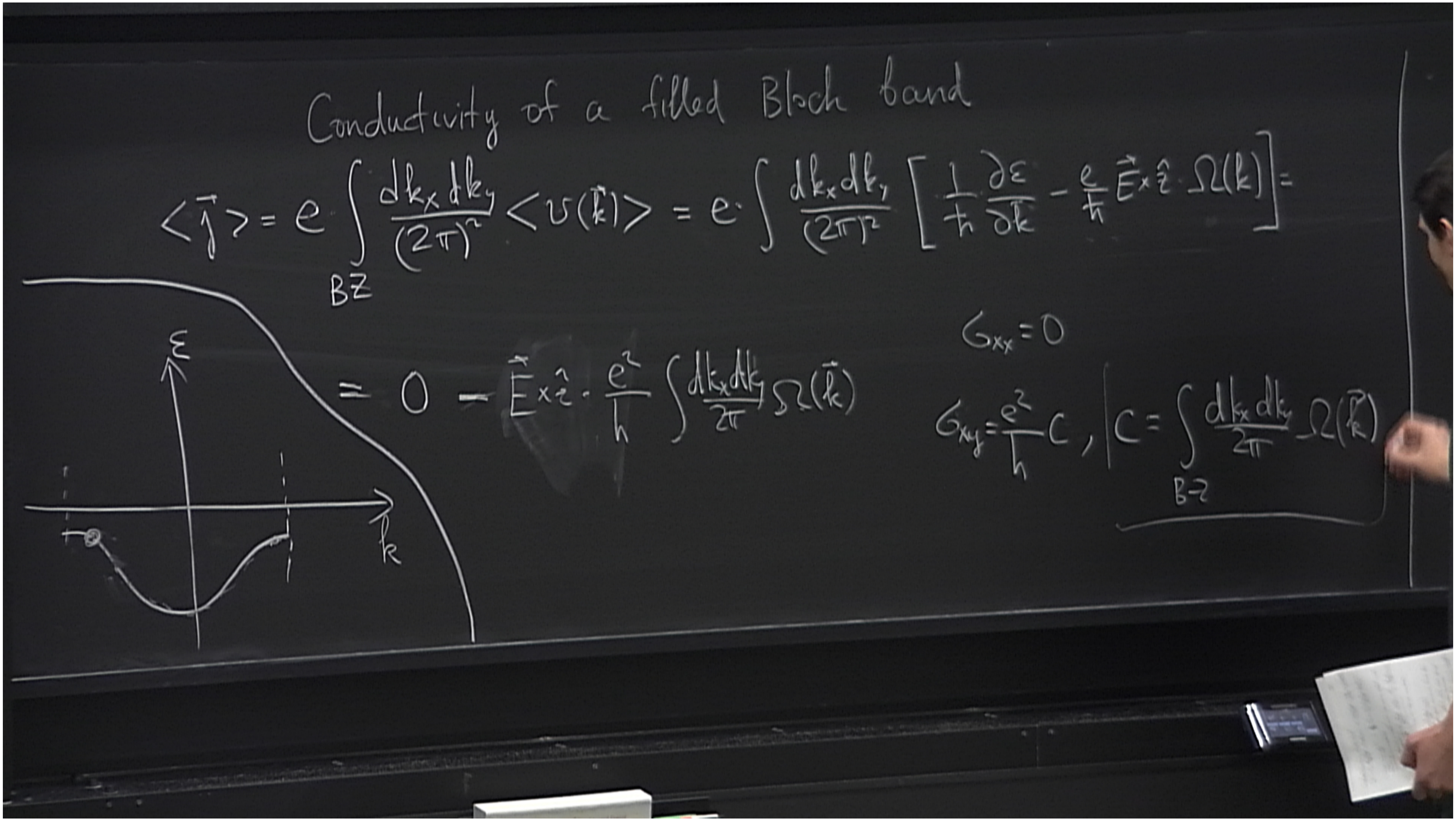
$$\langle \vec{j} \rangle = e \int_{BZ} \frac{dk_x\, dk_y}{(2\pi)^2} \langle v(\vec{k}) \rangle = e \int \frac{dk_x\, dk_y}{(2\pi)^2} \left[\frac{1}{\hbar} \frac{\partial \varepsilon}{\partial k} - \frac{e}{\hbar} \vec{E} \times \hat{z} \cdot \Omega(k) \right] =$$



$$= 0 - \vec{E} \times \hat{z} \cdot \frac{e^2}{h} \int \frac{dk_x dk_y}{2\pi} \Omega(\vec{k})$$

$$\sigma_{xx} = 0$$

$$\sigma_{xy} = \frac{e^2}{h} C, \quad C = \int_{BZ} \frac{dk_x\, dk_y}{2\pi} \Omega(\vec{k})$$


Conductivity of a filled Bloch band
$\langle \vec{j} \rangle = e \int_{BZ} \frac{dk_x\, dk_y}{(2\pi)^2} \langle v(\vec{k}) \rangle = e \int \frac{dk_x\, dk_y}{(2\pi)^2} \left[\frac{1}{\hbar}\frac{\partial \varepsilon}{\partial \vec{k}} - \frac{e}{\hbar} \vec{E}\times\hat{z}\cdot\Omega(k) \right] =$
$= 0 - \vec{E}\times\hat{z}\cdot\frac{e^2}{h}\int\frac{dk_x\, dk_y}{2\pi}\Omega(\vec{k})$
$\sigma_{xx} = 0$
$\sigma_{xy} = \frac{e^2}{h} C, \quad \boxed{C = \int_{BZ} \frac{dk_x\, dk_y}{2\pi}\Omega(\vec{k})}$
ε
k

$$C = \frac{1}{2\pi}\left[\theta_1 + \theta_2 + \theta_3 + \theta_4\right] =$$

$$= \frac{1}{2\pi} 2\pi n = \underline{\underline{n}}$$

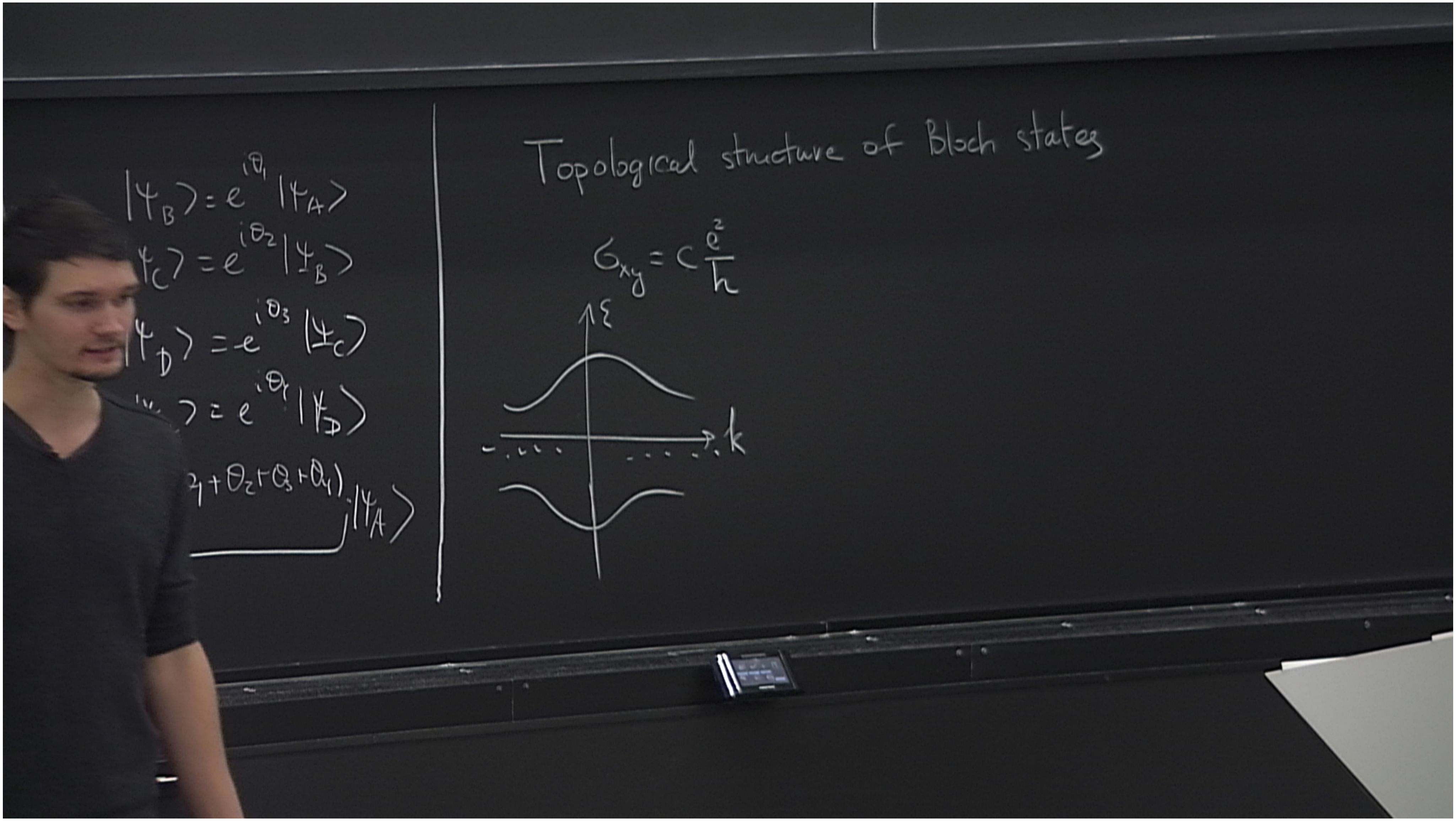
$$|\psi_B\rangle = e^{i\theta_1}|\psi_A\rangle$$

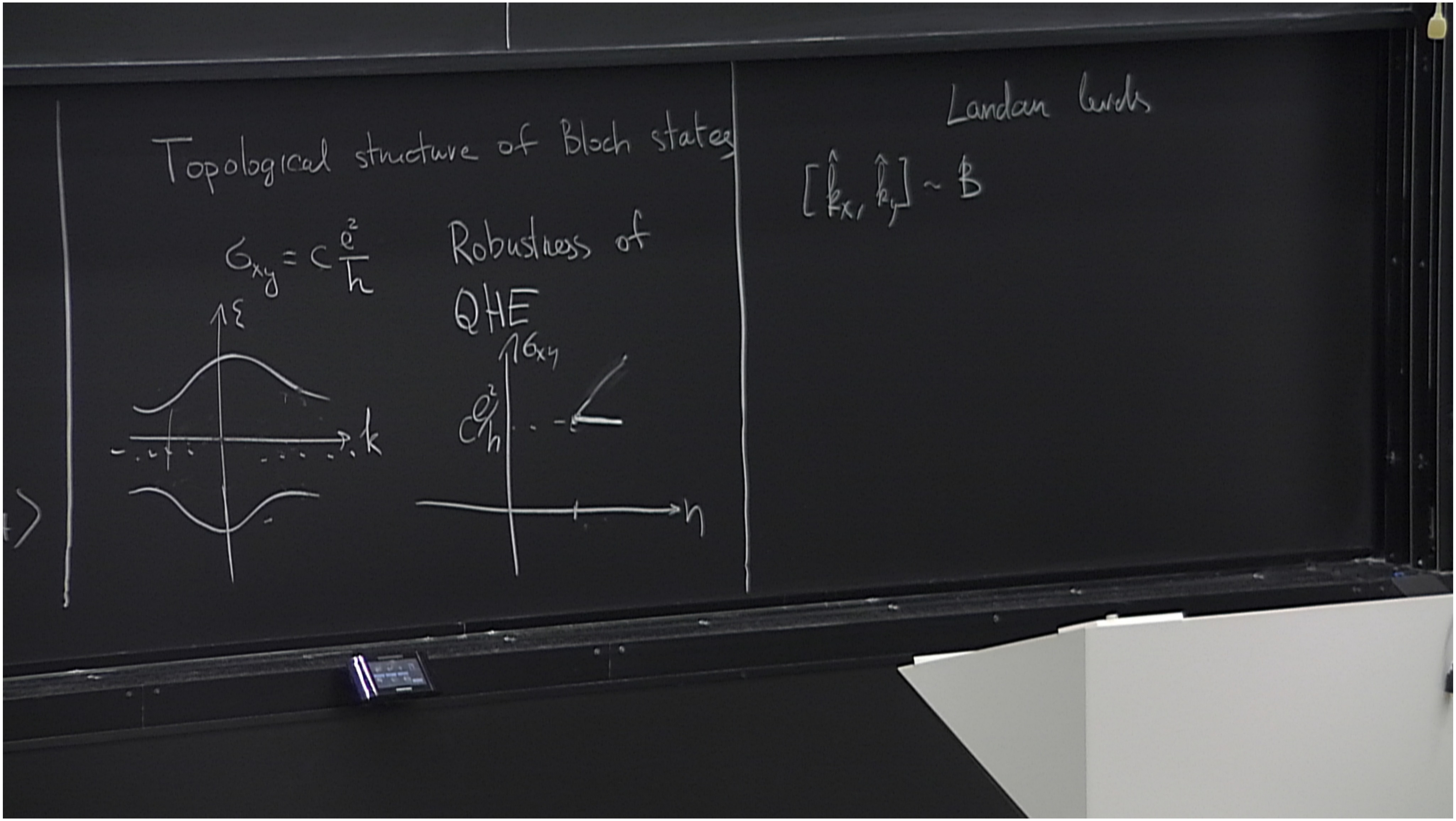
$$|\psi_C\rangle = e^{i\theta_2}|\psi_B\rangle$$

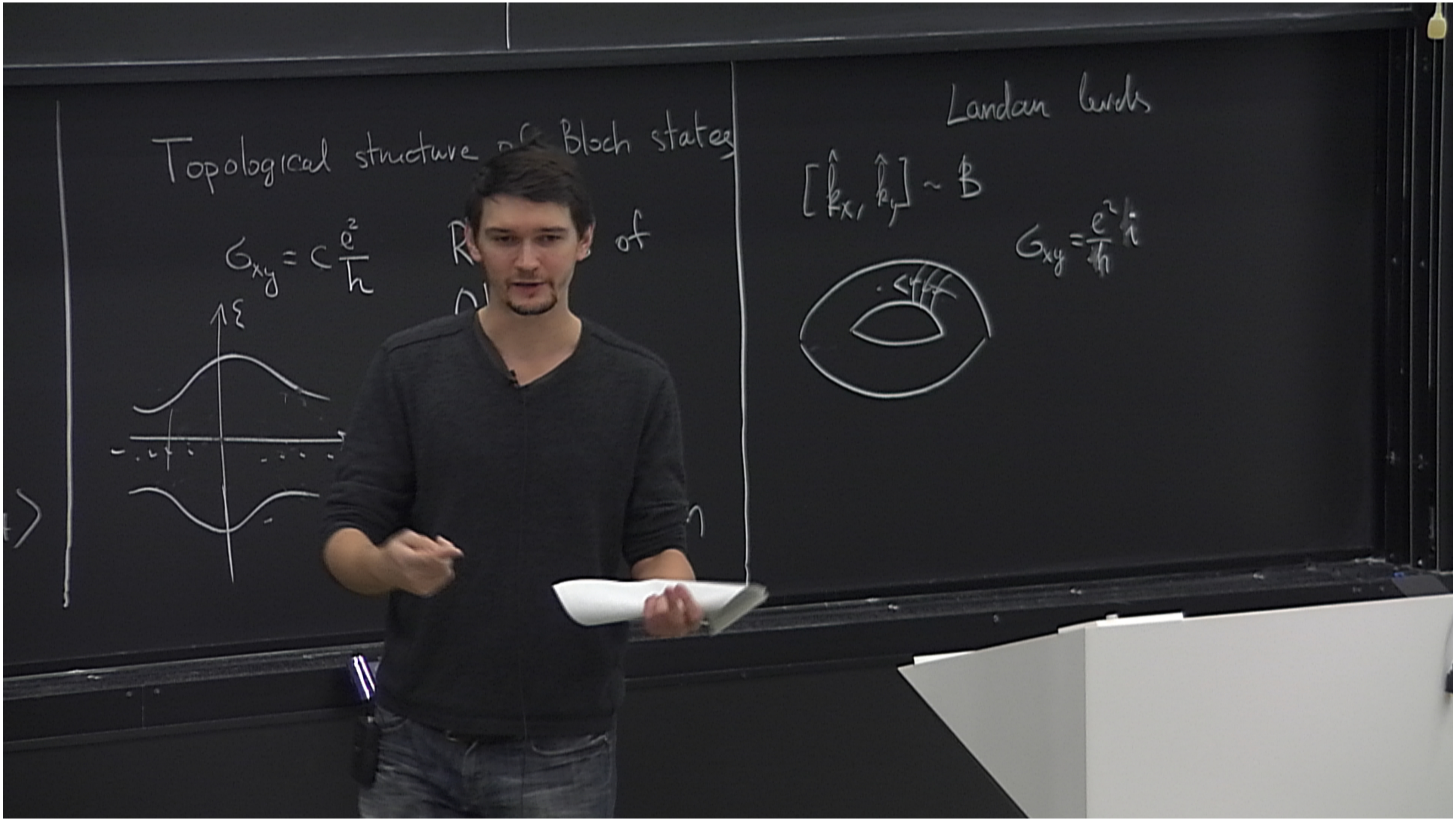
$$|\psi_D\rangle = e^{i\theta_3}|\psi_C\rangle$$

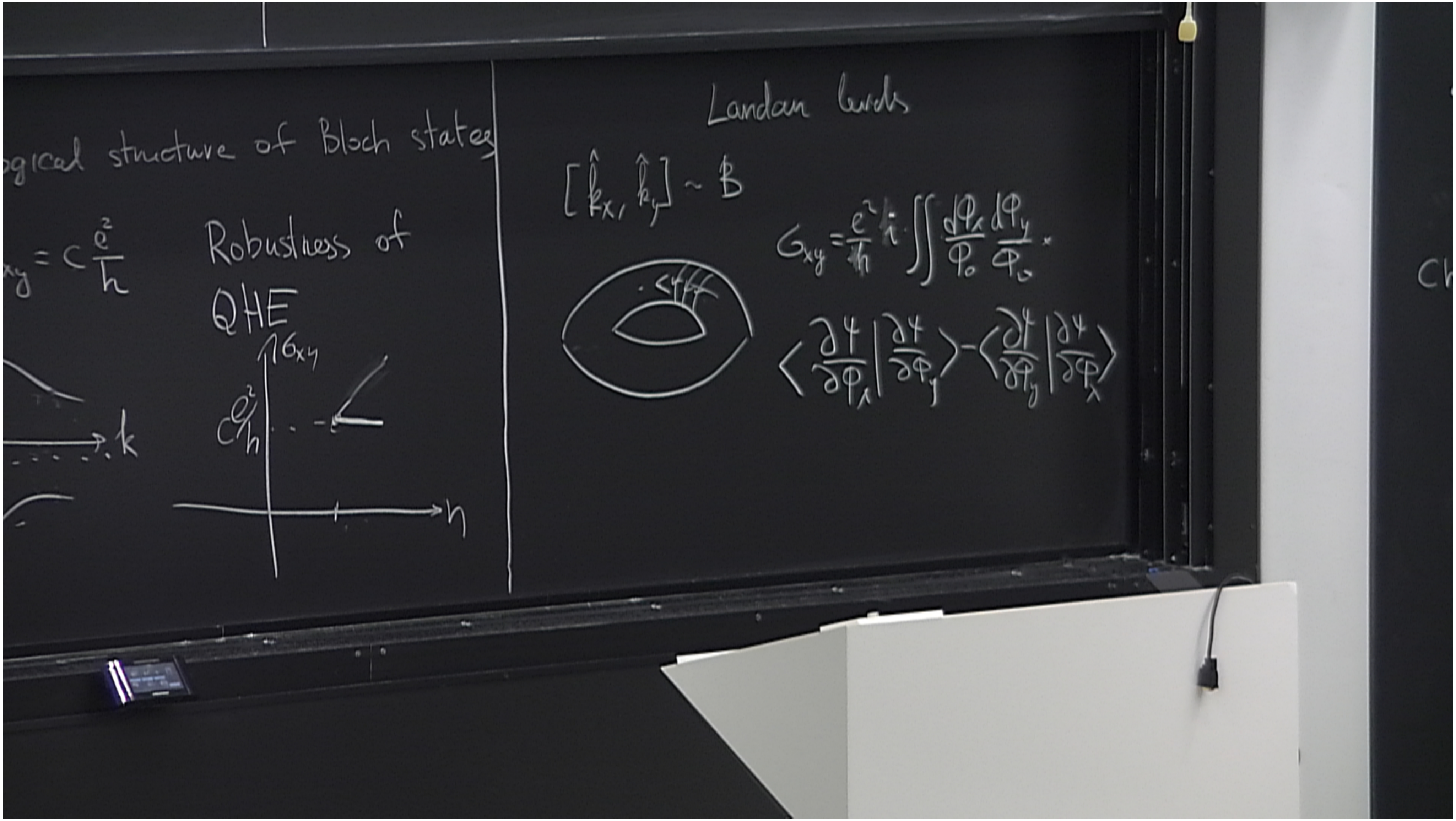
$$|\psi_A\rangle = e^{i\theta_4}|\psi_D\rangle$$

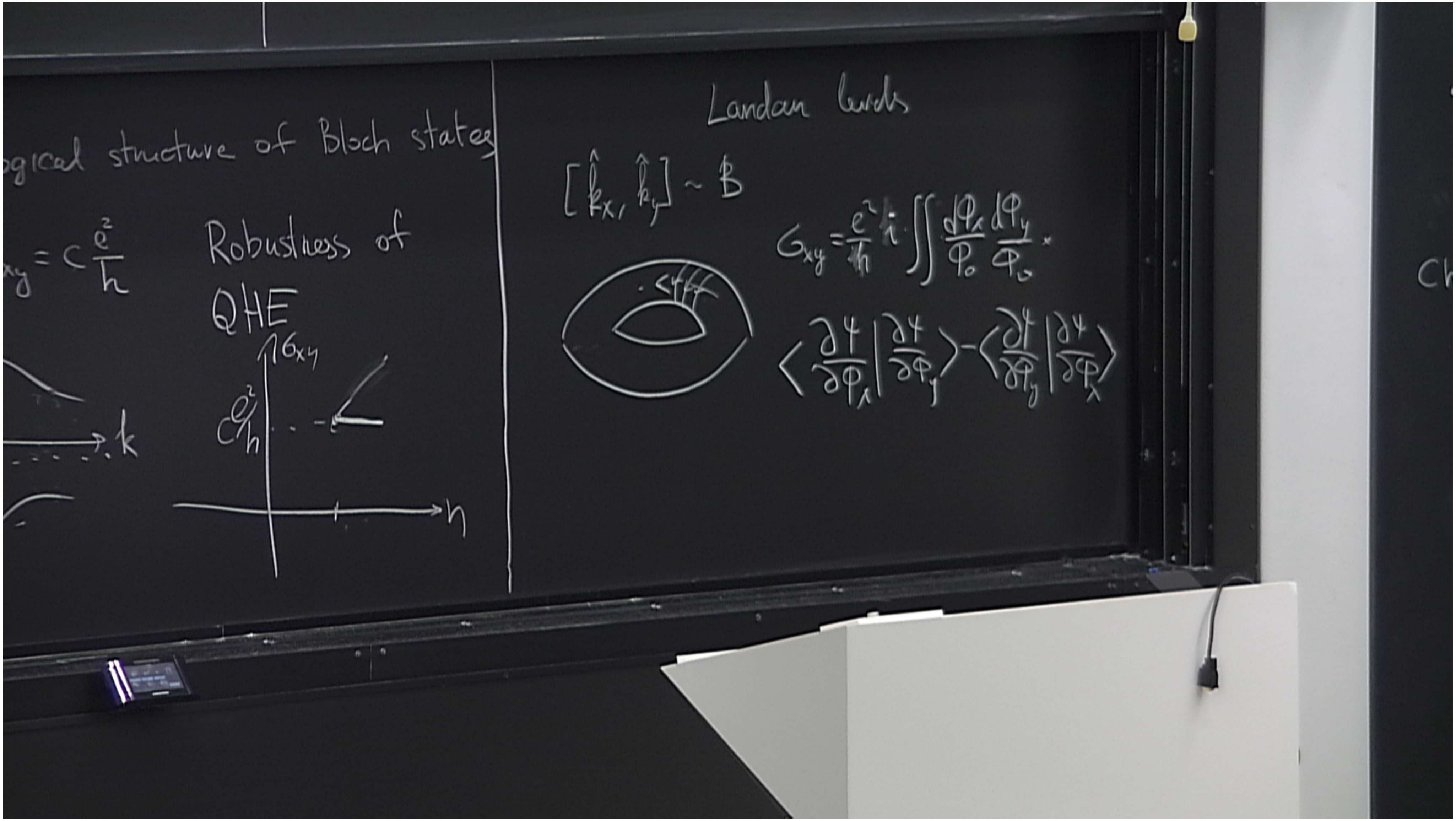
$$|\psi_A\rangle = \underbrace{e^{i(\theta_1+\theta_2+\theta_3+\theta_4)}}\,|\psi_A\rangle$$

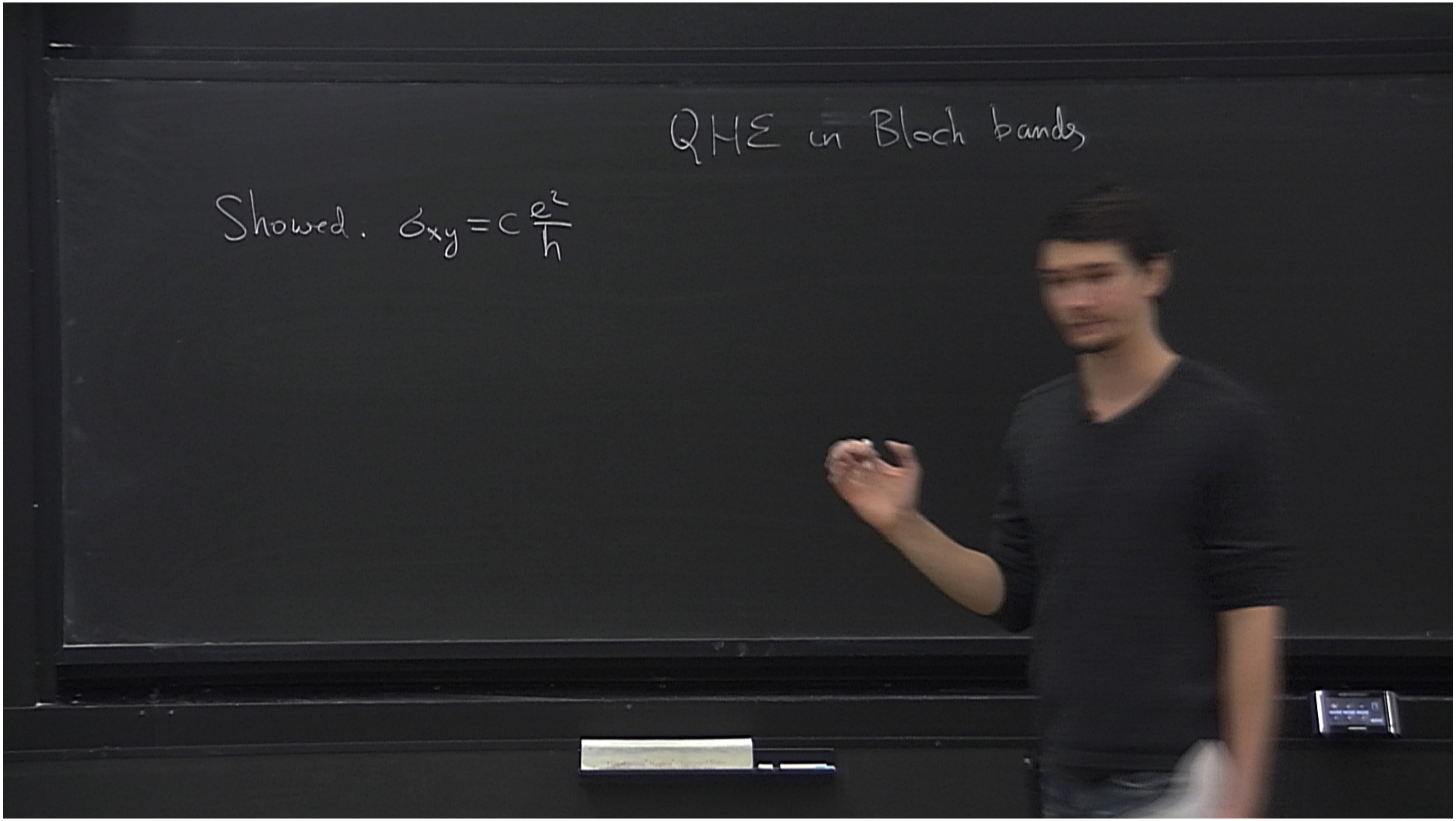

Topological structure of Bloch states
$|\psi_B\rangle = e^{i\theta_1}|\psi_A\rangle$
$\sigma_{xy} = C\frac{e^2}{h}$
ε
k


Topological structure of Bloch states
$\sigma_{xy} = c\,\frac{e^2}{h}$
Robustness of QHE
Landau levels
$[\hat{k}_x, \hat{k}_y] \sim B$
ε
k
σ_{xy}
$c\frac{e^2}{h}$
n


Topological structure of Bloch states
$G_{xy} = c\frac{e^2}{h}$
of
Landau levels
$[\hat{k}_x, \hat{k}_y] \sim B$
$G_{xy} = \frac{e^2}{h} i$


ogical structure of Bloch states
Robustness of QHE
Landau levels
[k̂x, k̂y] ~ B

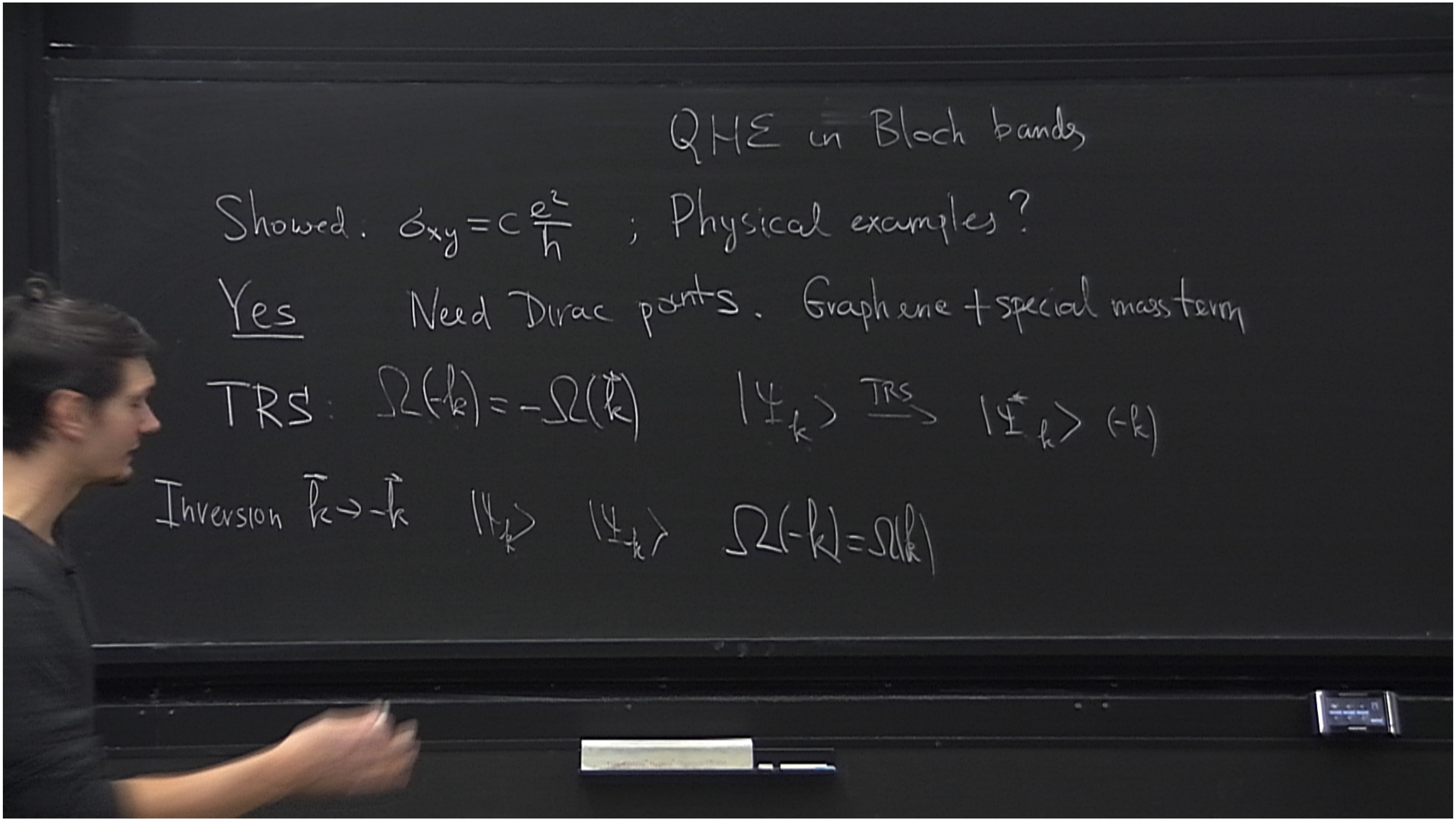

ogical structure of Bloch states
Robustness of QHE
Landau levels
$[\hat{k}_x, \hat{k}_y] \sim B$
$\sigma_{xy} = \frac{e^2}{h} \frac{1}{i} \iint \frac{d\varphi_x}{\varphi_0} \frac{d\varphi_y}{\varphi_0}$
$\left\langle \frac{\partial \psi}{\partial \varphi_x} \middle| \frac{\partial \psi}{\partial \varphi_y} \right\rangle - \left\langle \frac{\partial \psi}{\partial \varphi_y} \middle| \frac{\partial \psi}{\partial \varphi_x} \right\rangle$

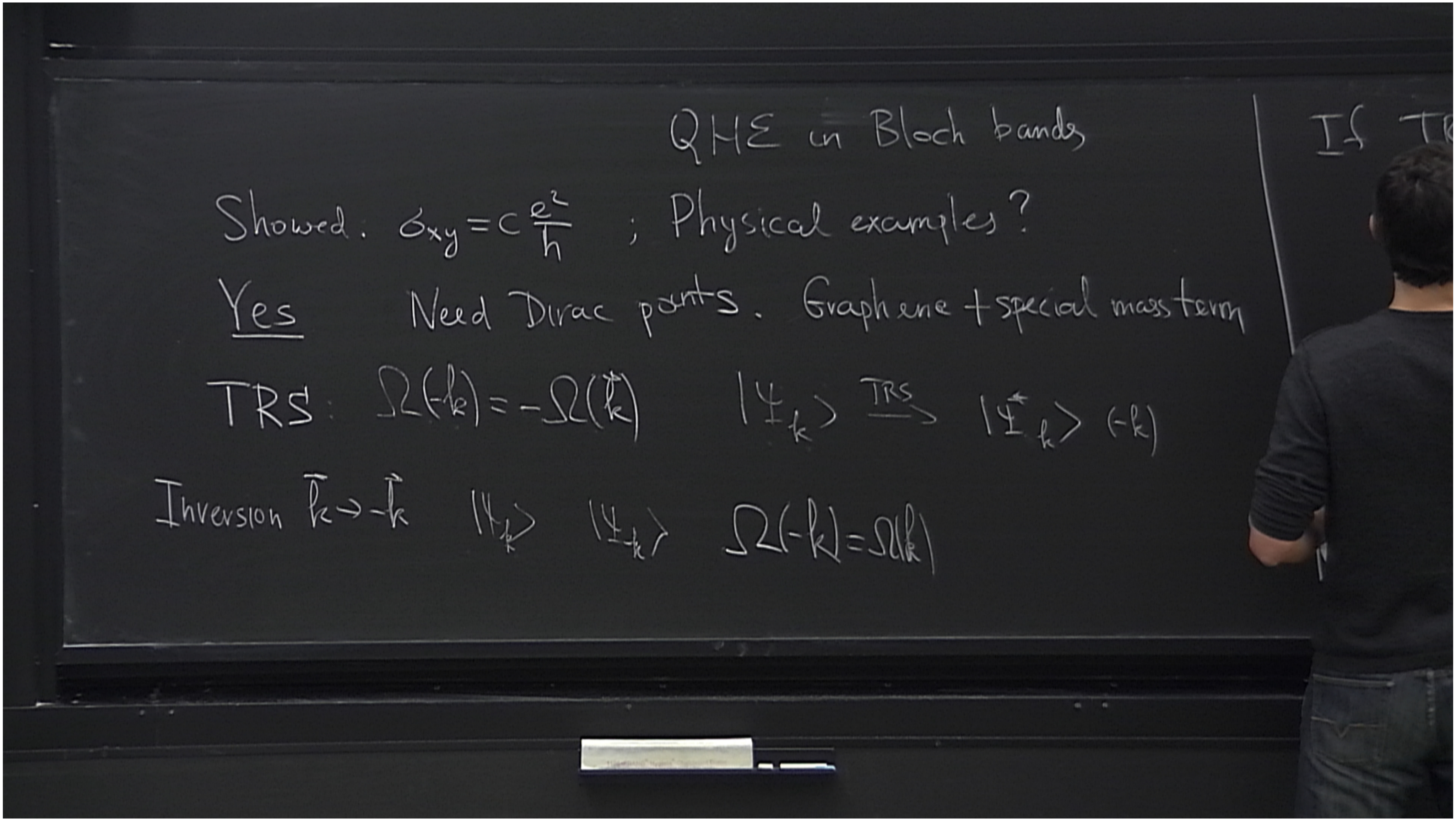

QHE in Bloch bands
Showed. $\sigma_{xy} = C\frac{e^2}{h}$

QHE in Bloch bands

Showed: $\sigma_{xy} = C\frac{e^2}{h}$; Physical examples?

<u>Yes</u>


QHE in Bloch bands
Showed: $\sigma_{xy} = C\frac{e^2}{h}$; Physical examples?
Yes Need Dirac points. Graphene + special mass term
TRS: $\Omega(-\vec{k}) = -\Omega(\vec{k})$ $|\Psi_k\rangle \xrightarrow{TRS} |\Psi^*_k\rangle$ $(-k)$
Inversion $\vec{k} \to -\vec{k}$ $|\Psi_k\rangle$ $|\Psi_{-k}\rangle$ $\Omega(-k) = \Omega(k)$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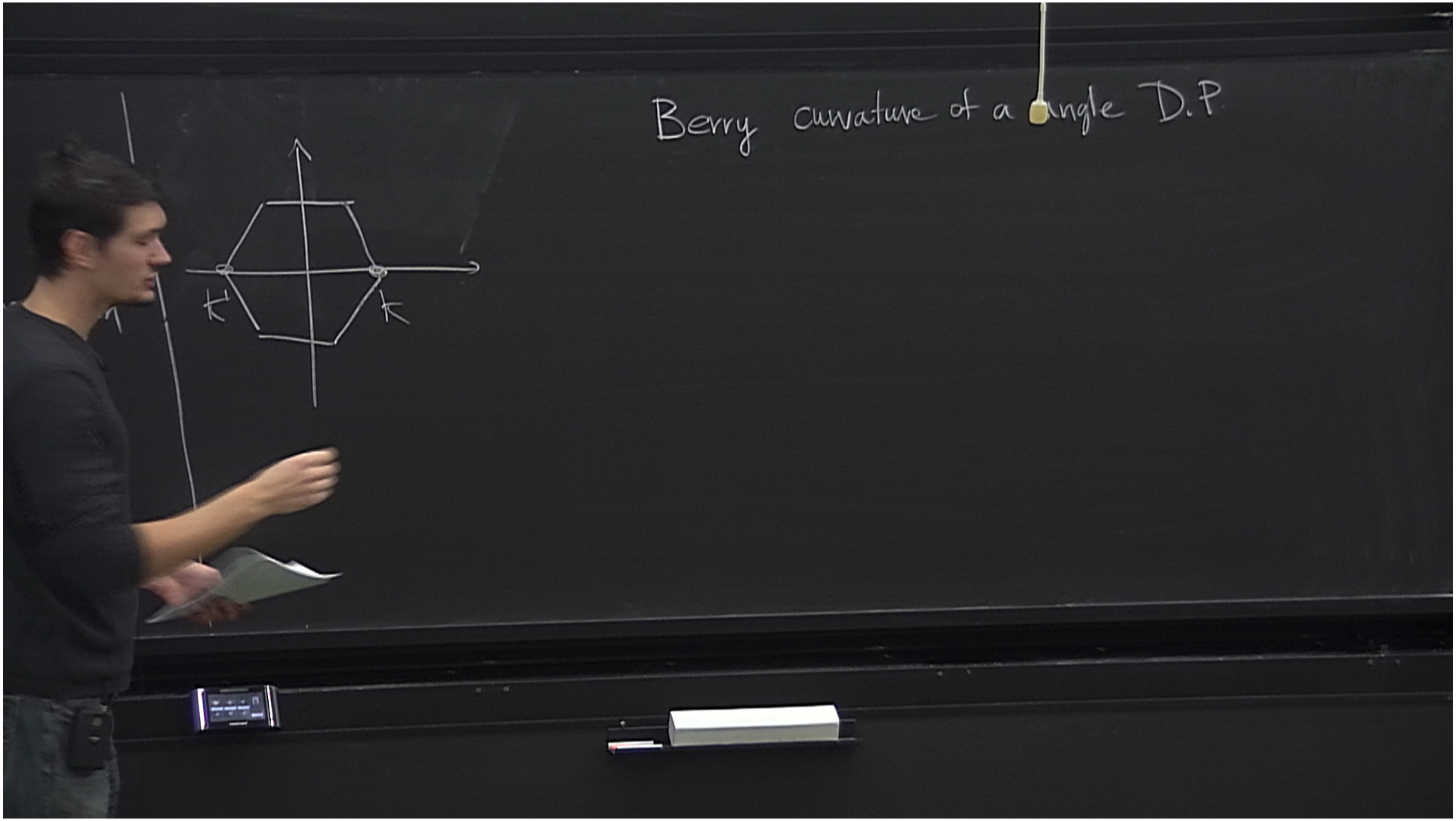
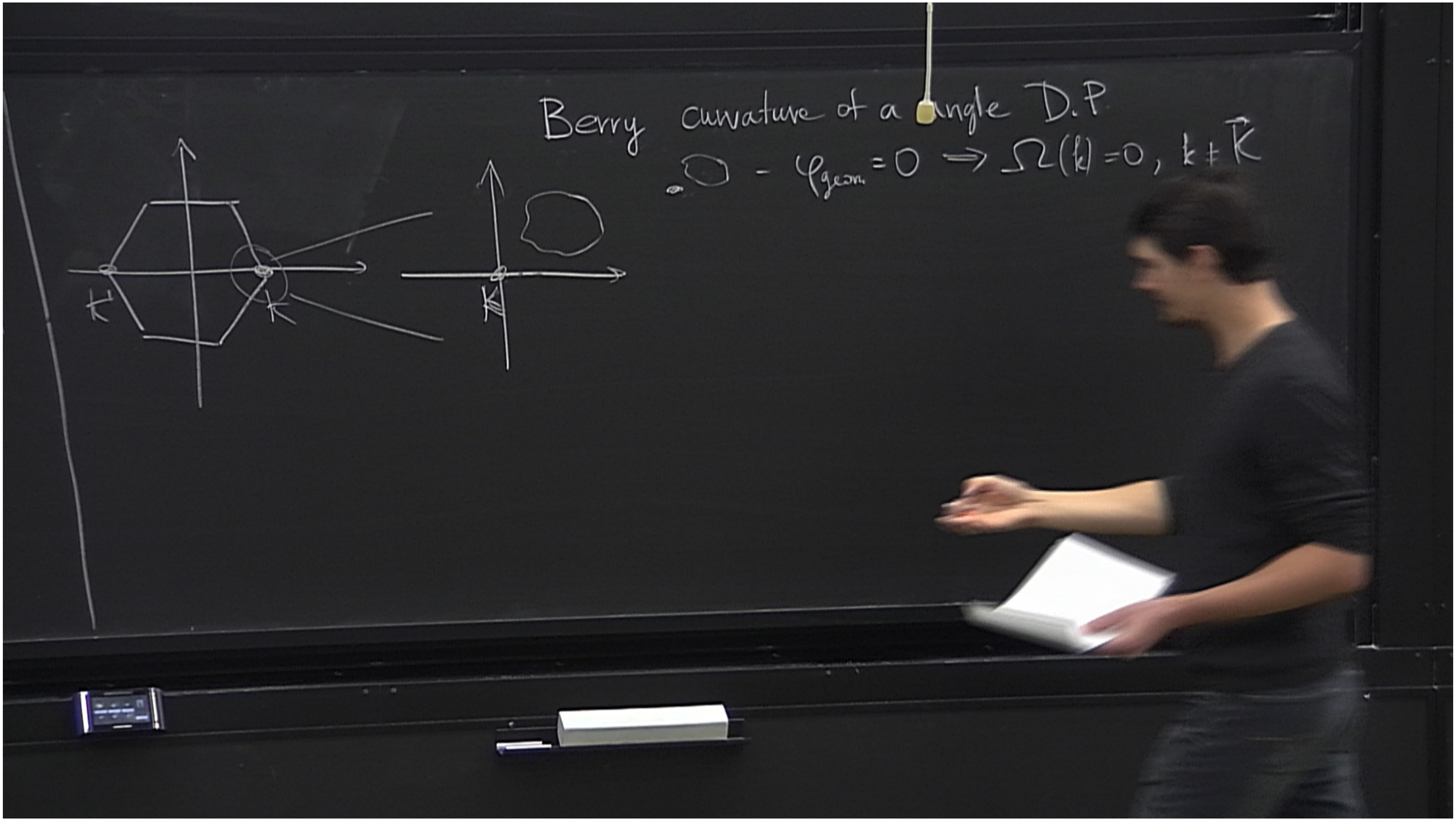
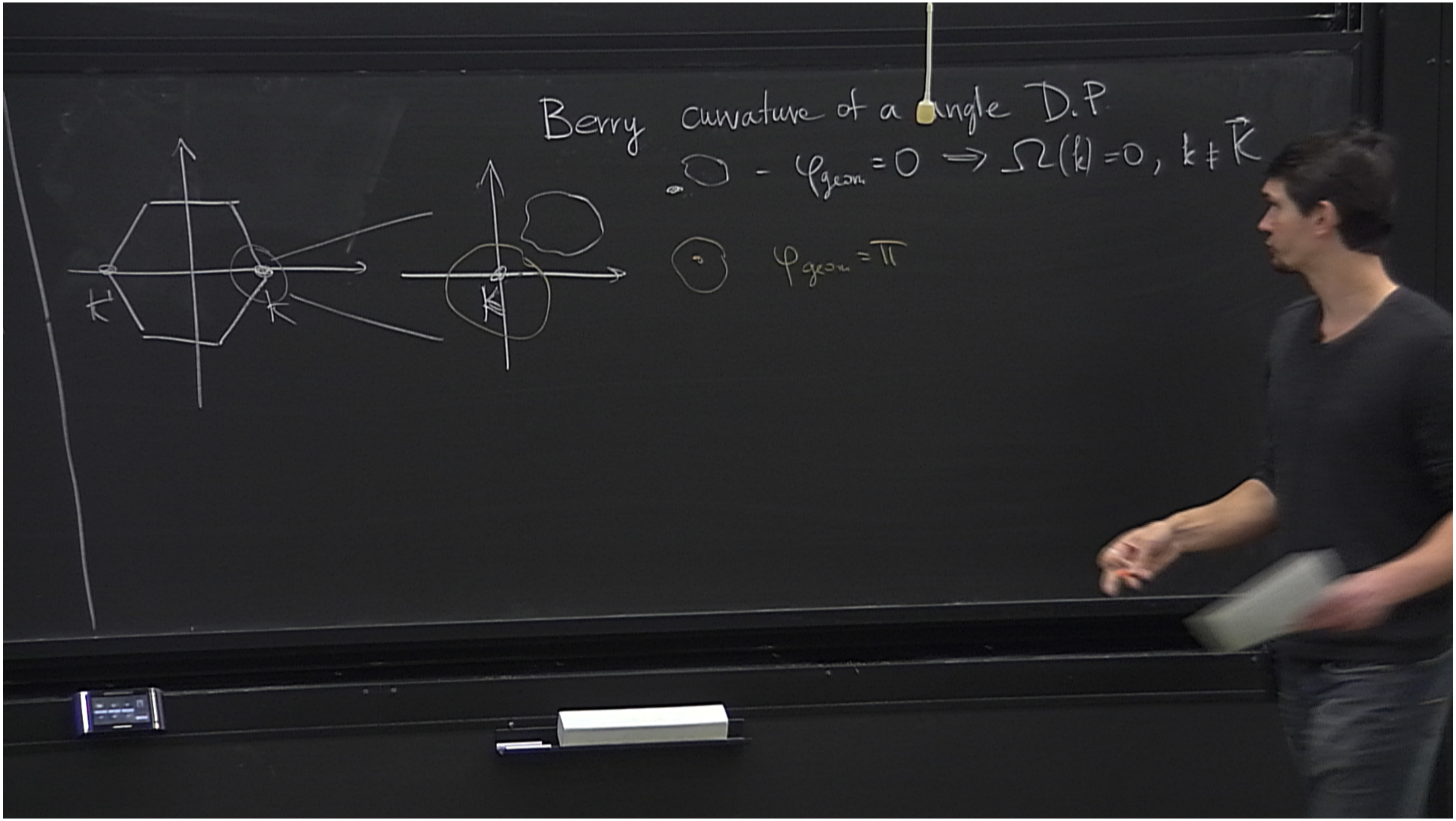
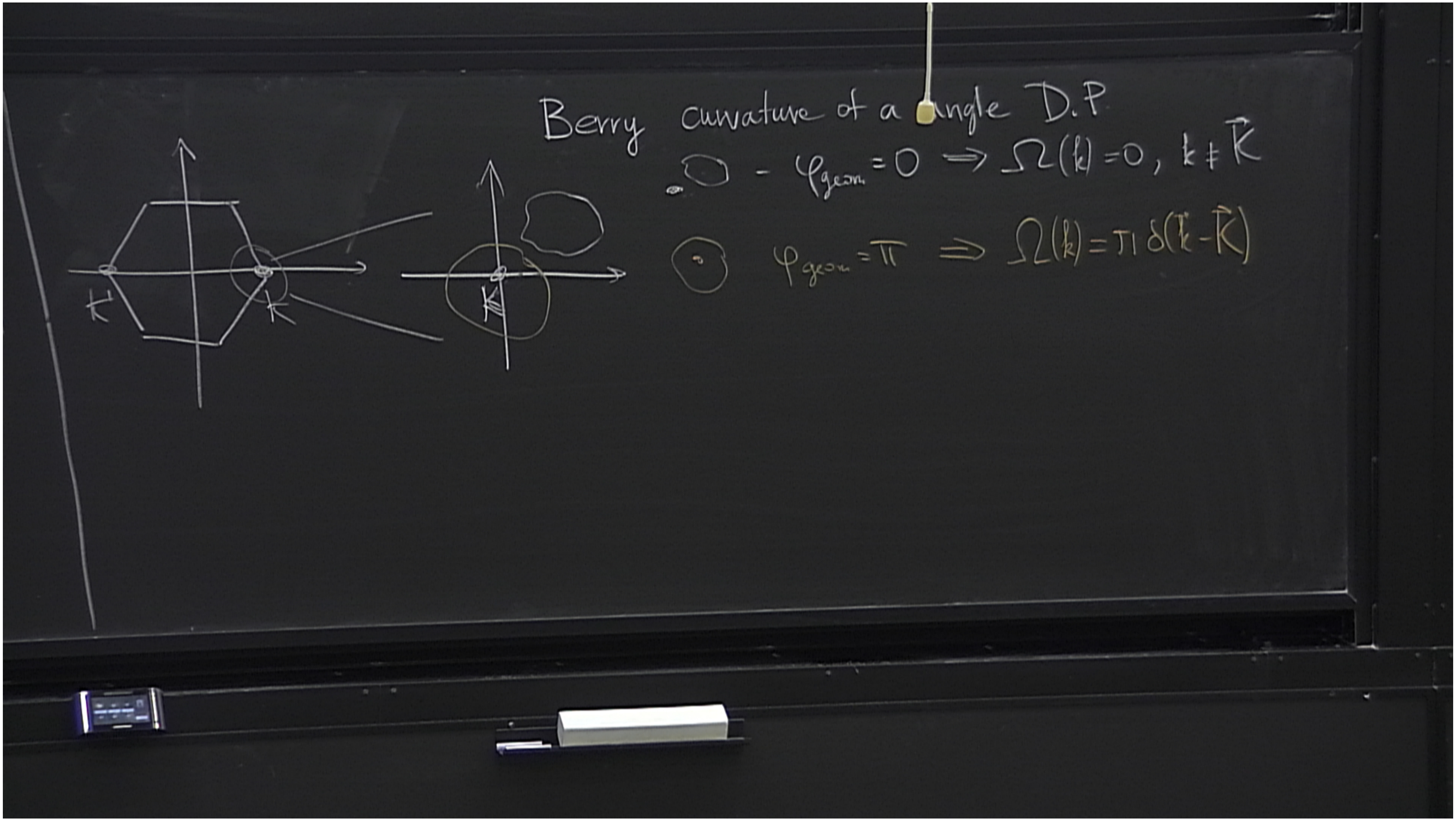
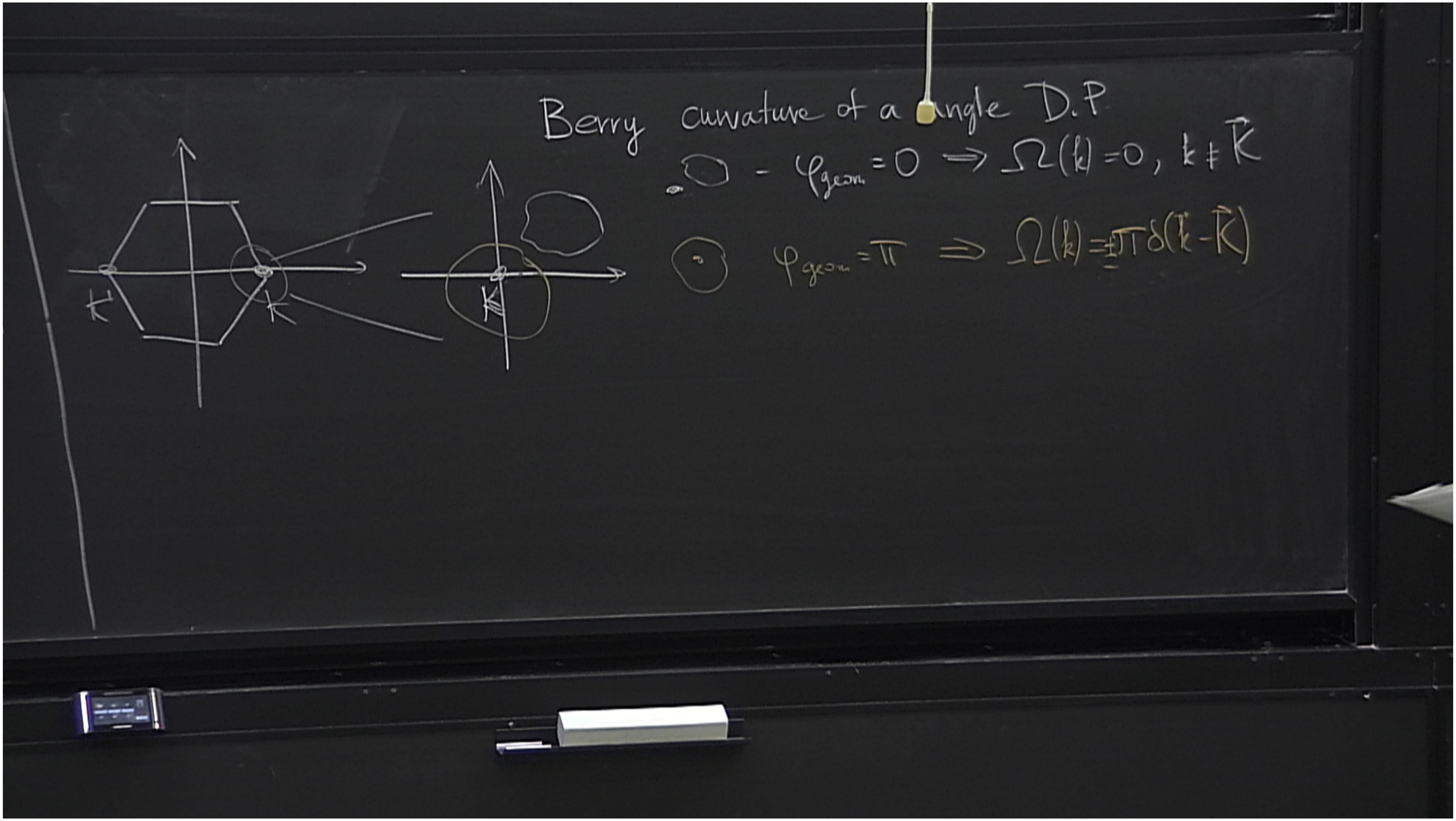
If TR

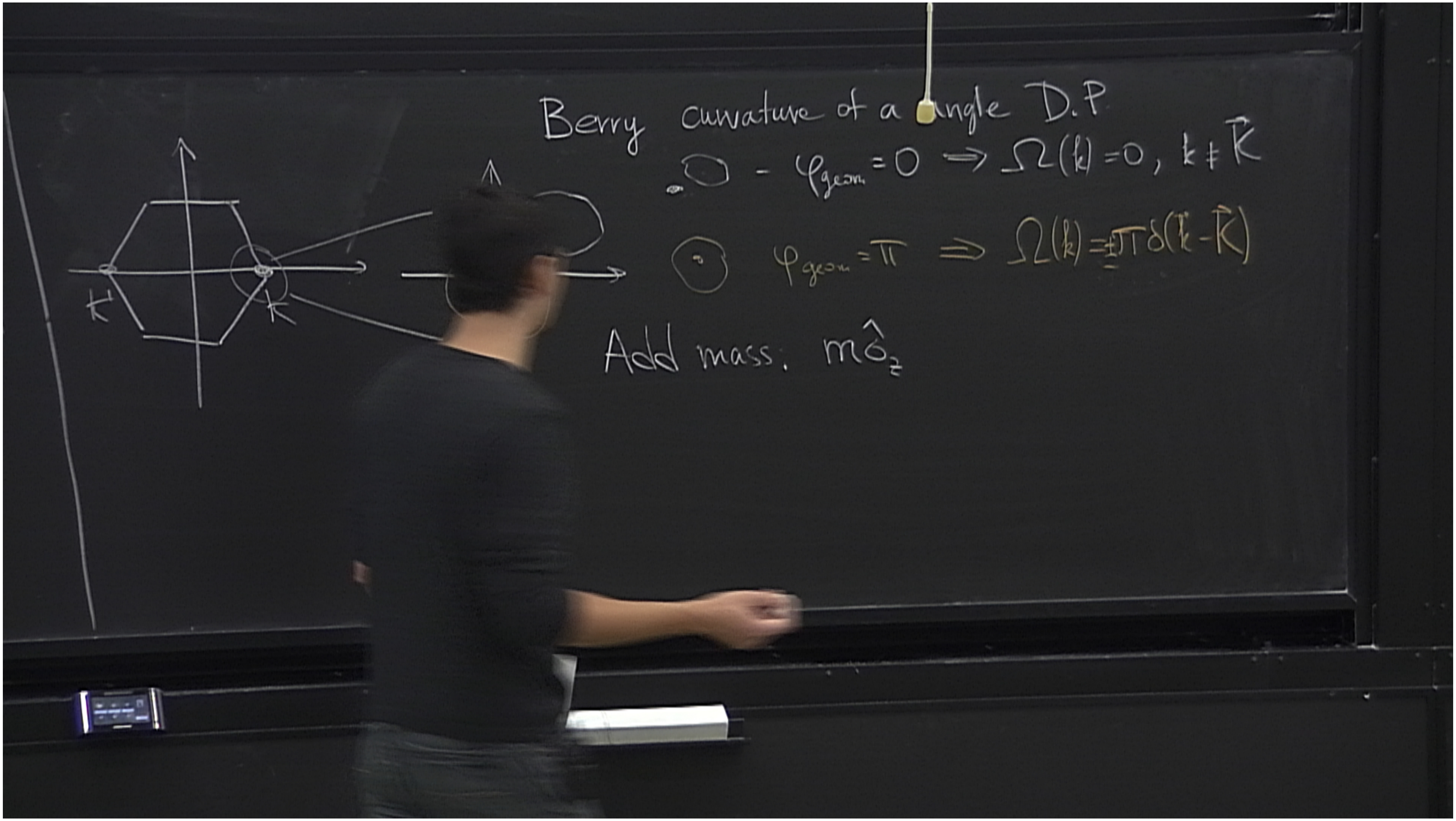

Berry curvature of a
D.P
K'
K

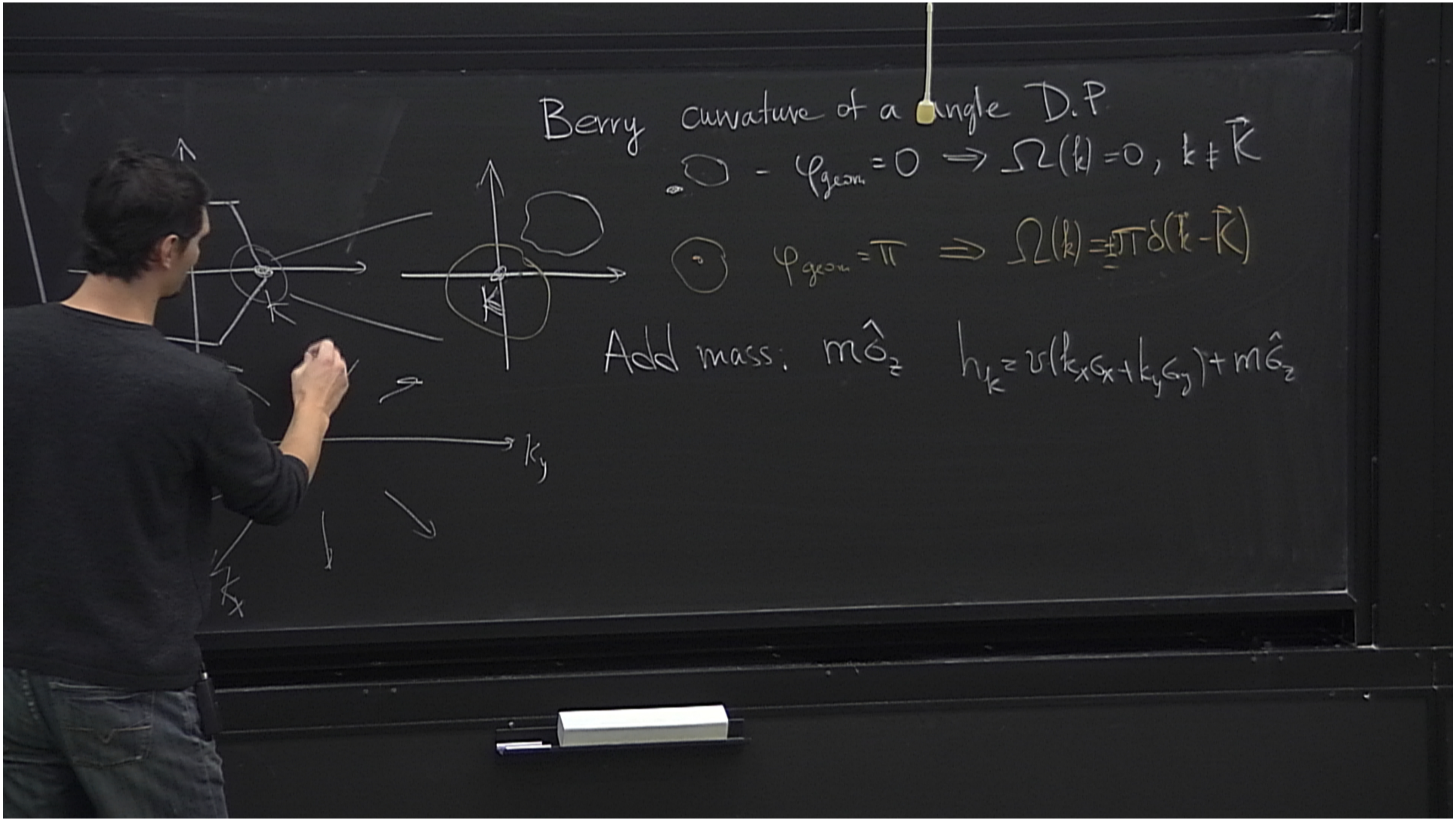

Berry curvature of a ingle D.P
$\oint - \varphi_{geom} = 0 \Rightarrow \Omega(k) = 0,\ k \neq \vec{K}$
K'
K
K

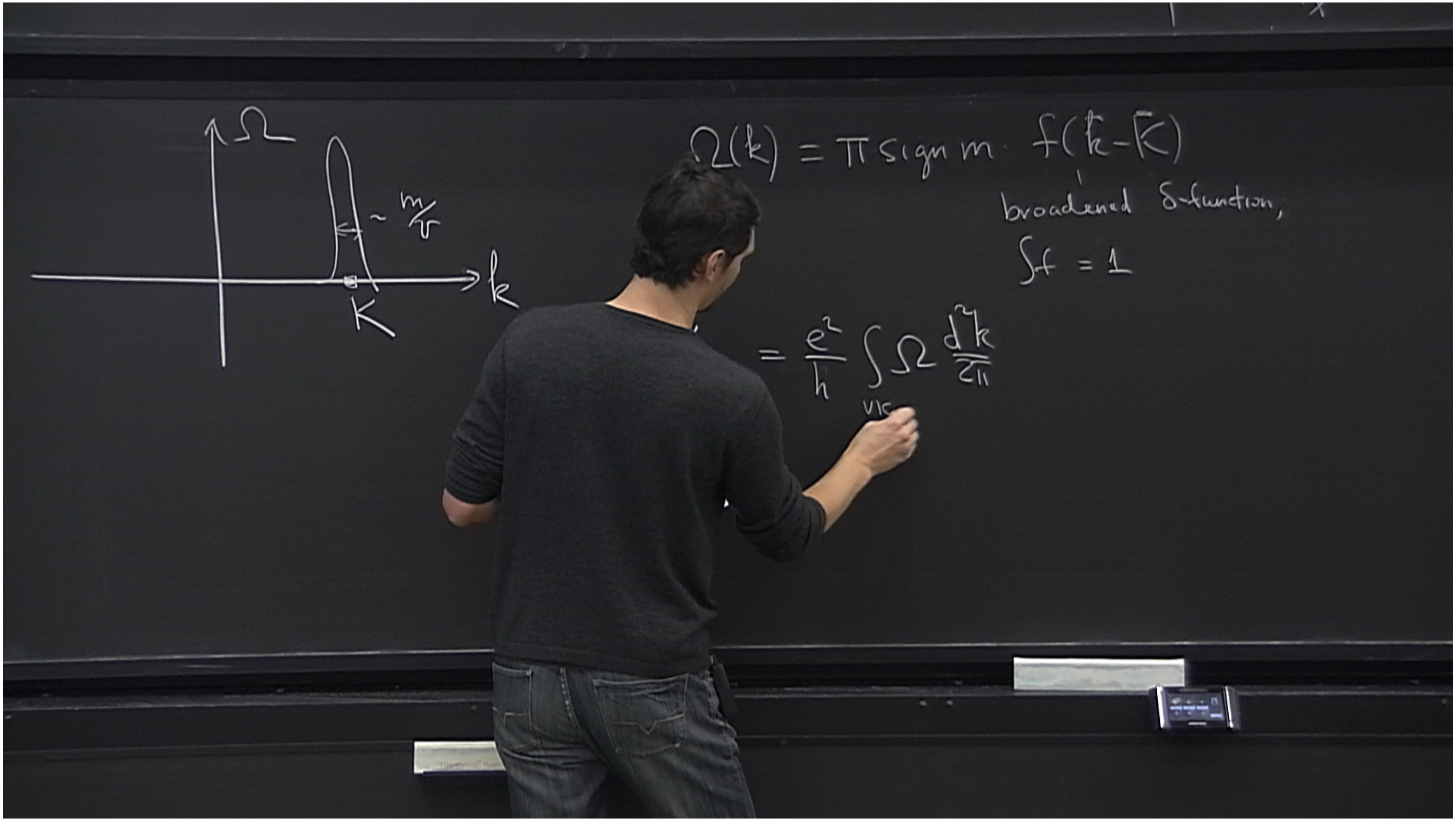

Berry curvature of a ingle D.P
- $\varphi_{geom} = 0 \Rightarrow \Omega(k) = 0, \; k \neq \vec{K}$
$\varphi_{geom} = \pi$
K'
K
K

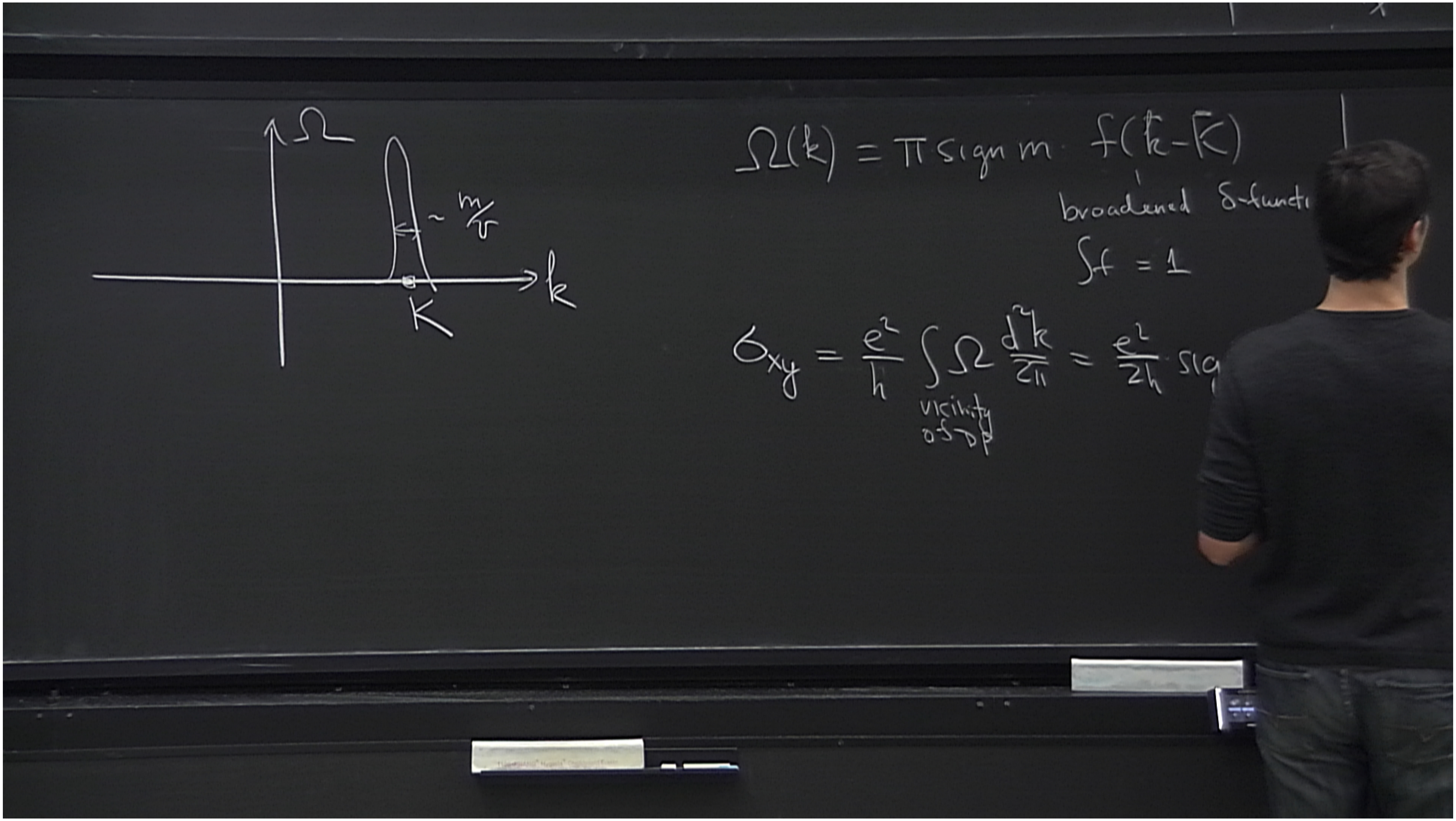

Berry curvature of a single D.P
$- \varphi_{geom} = 0 \Rightarrow \Omega(k) = 0, \; k \neq \vec{K}$
$\varphi_{geom} = \pi \Rightarrow \Omega(\vec{k}) = \pi \, \delta(\vec{k} - \vec{K})$
K'
K
K

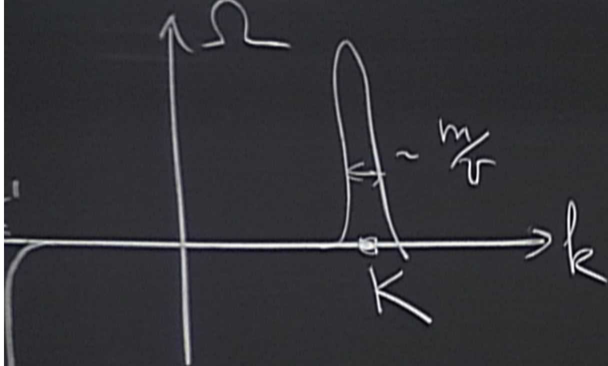

Berry curvature of a ingle D.P
$- \varphi_{geom} = 0 \Rightarrow \Omega(k) = 0, \; k \neq \vec{K}$
$\varphi_{geom} = \pi \Rightarrow \Omega(k) = \pm\pi\,\delta(\vec{k} - \vec{K})$
K'
K
K


Berry curvature of a ngle D.P
$\varphi_{geom} = 0 \Rightarrow \Omega(k) = 0, \; k \neq \vec{K}$
$\varphi_{geom} = \pi \Rightarrow \Omega(k) = \pm\pi\delta(\vec{k} - \vec{K})$
Add mass: $m\hat{\sigma}_z$
K'
K


Berry curvature of a ingle D.P
$- \varphi_{geom} = 0 \Rightarrow \Omega(k) = 0,\ k \neq \vec{K}$
$\varphi_{geom} = \pi \Rightarrow \Omega(k) = \pm\pi\,\delta(\vec{k} - \vec{K})$
Add mass: $m\hat{\sigma}_z$
$h_k = v(k_x\sigma_x + k_y\sigma_y) + m\hat{\sigma}_z$
K
K
k_y
k_x


Ω
~ m/v
k
K
Ω(k) = π sign m · f(k − K)
broadened δ-function,
∫f = 1
= e²/h ∫ Ω d²k/2π
vic


Ω
~ m/v
k
K
Ω(k) = π sign m · f(k − K)
broadened δ-functi
∫f = 1
σxy = e²/h ∫ Ω d²k/2π = e²/2h sig
vicinity
of Dp

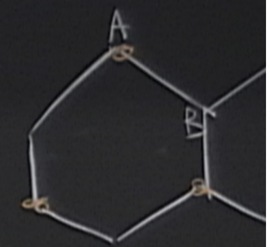


$$\Omega(\vec{k}) = \pi \,\mathrm{sign}\, m \cdot f(\vec{k}-\vec{K})$$

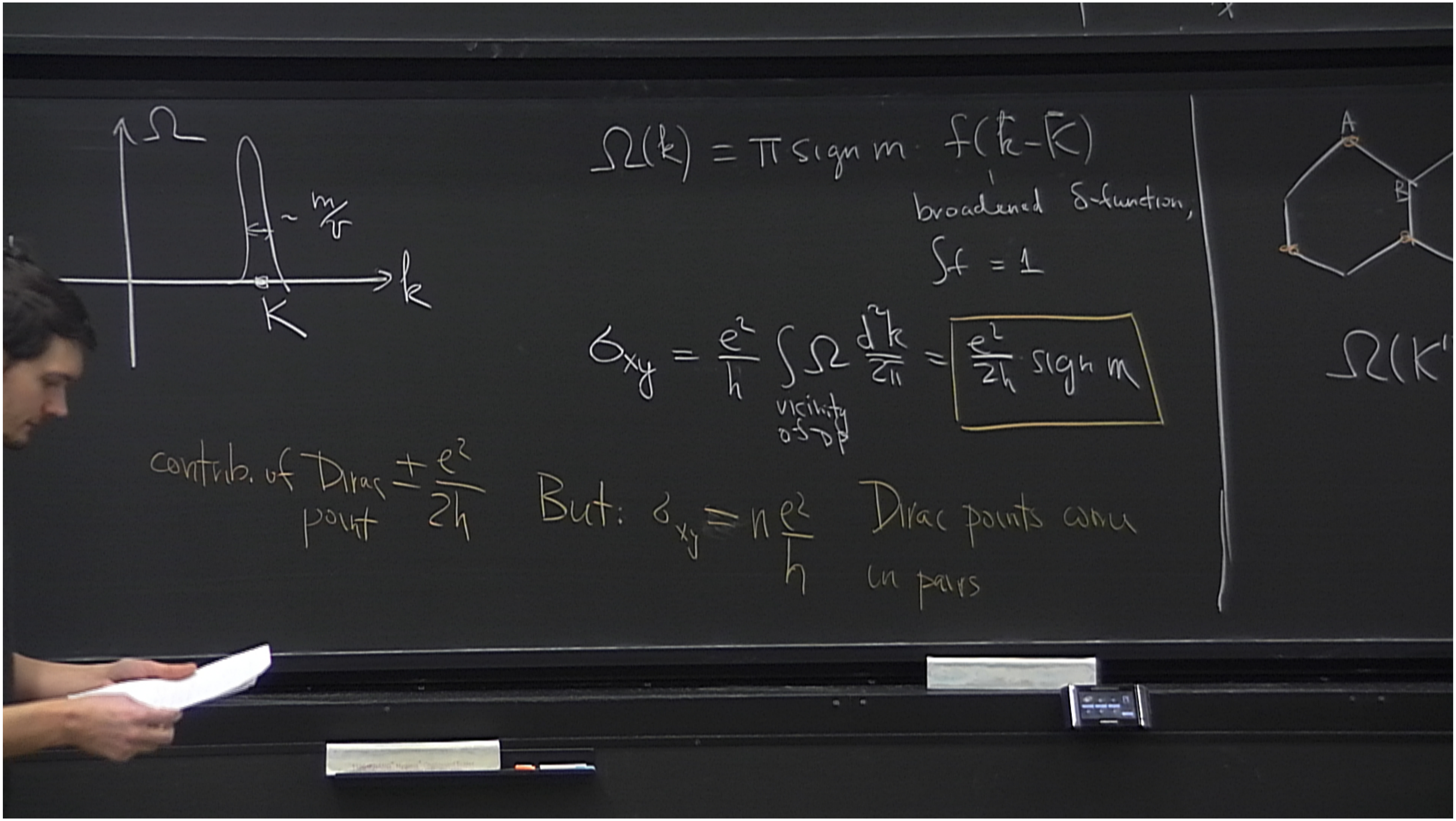
broadened δ-function, $\int f = 1$

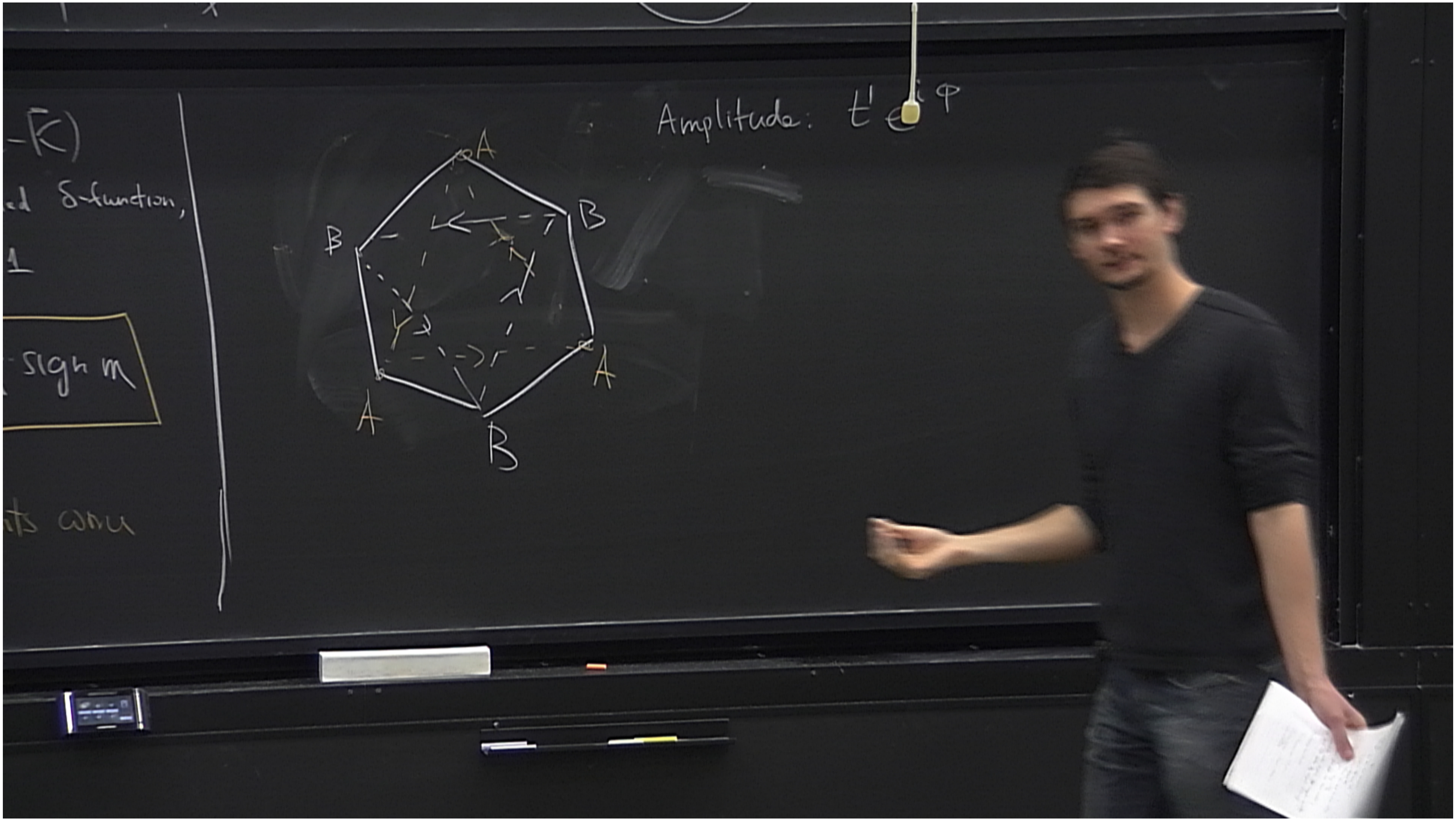
$$\sigma_{xy} = \frac{e^2}{h}\int_{\text{vicinity of DP}} \Omega \,\frac{d^2k}{2\pi} = \boxed{\frac{e^2}{2h}\,\mathrm{sign}\, m}$$

contrib. of Dirac point $\pm \frac{e^2}{2h}$ But: $\sigma_{xy} = n\frac{e^2}{h}$

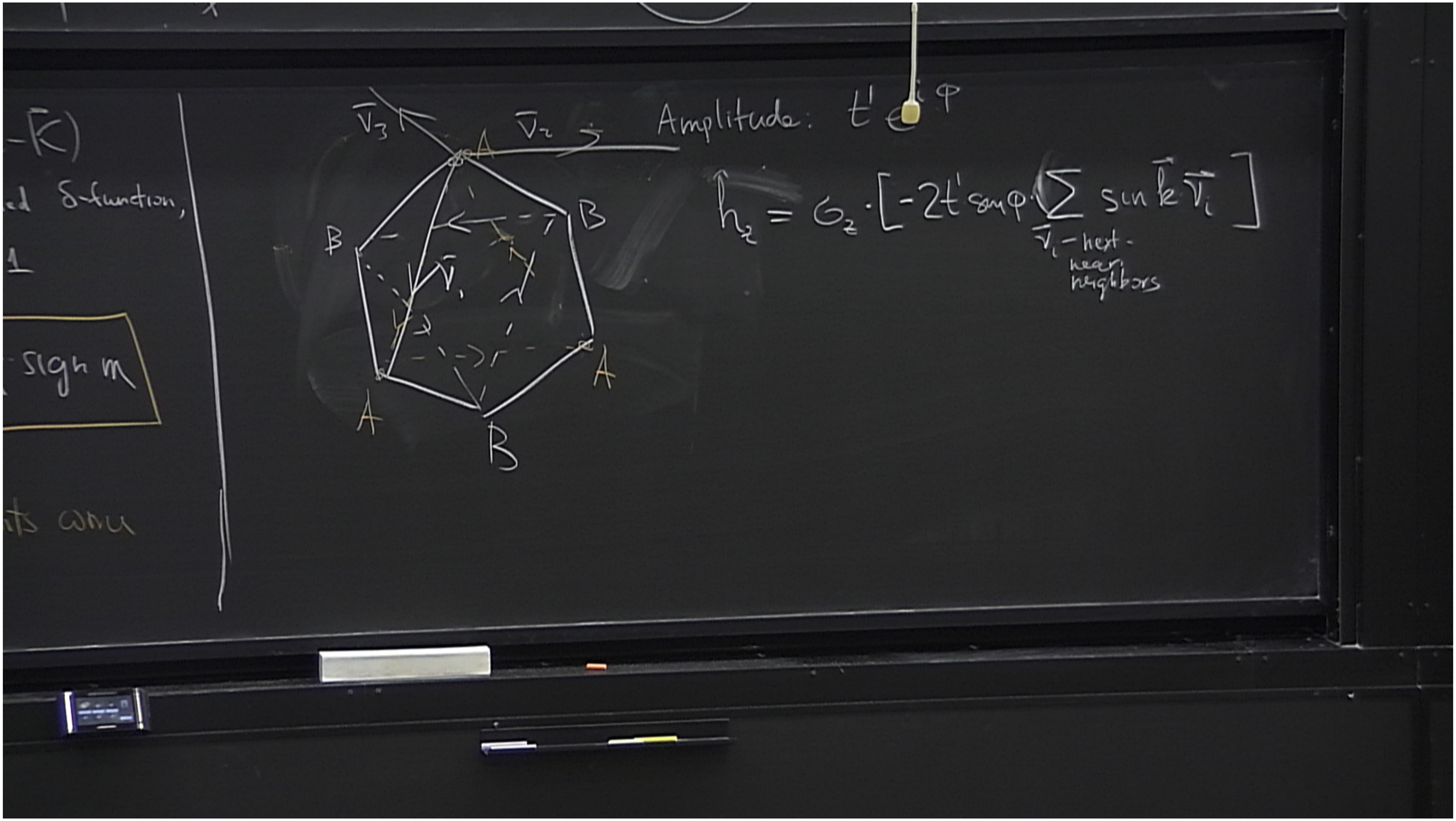


$\Omega(K'$

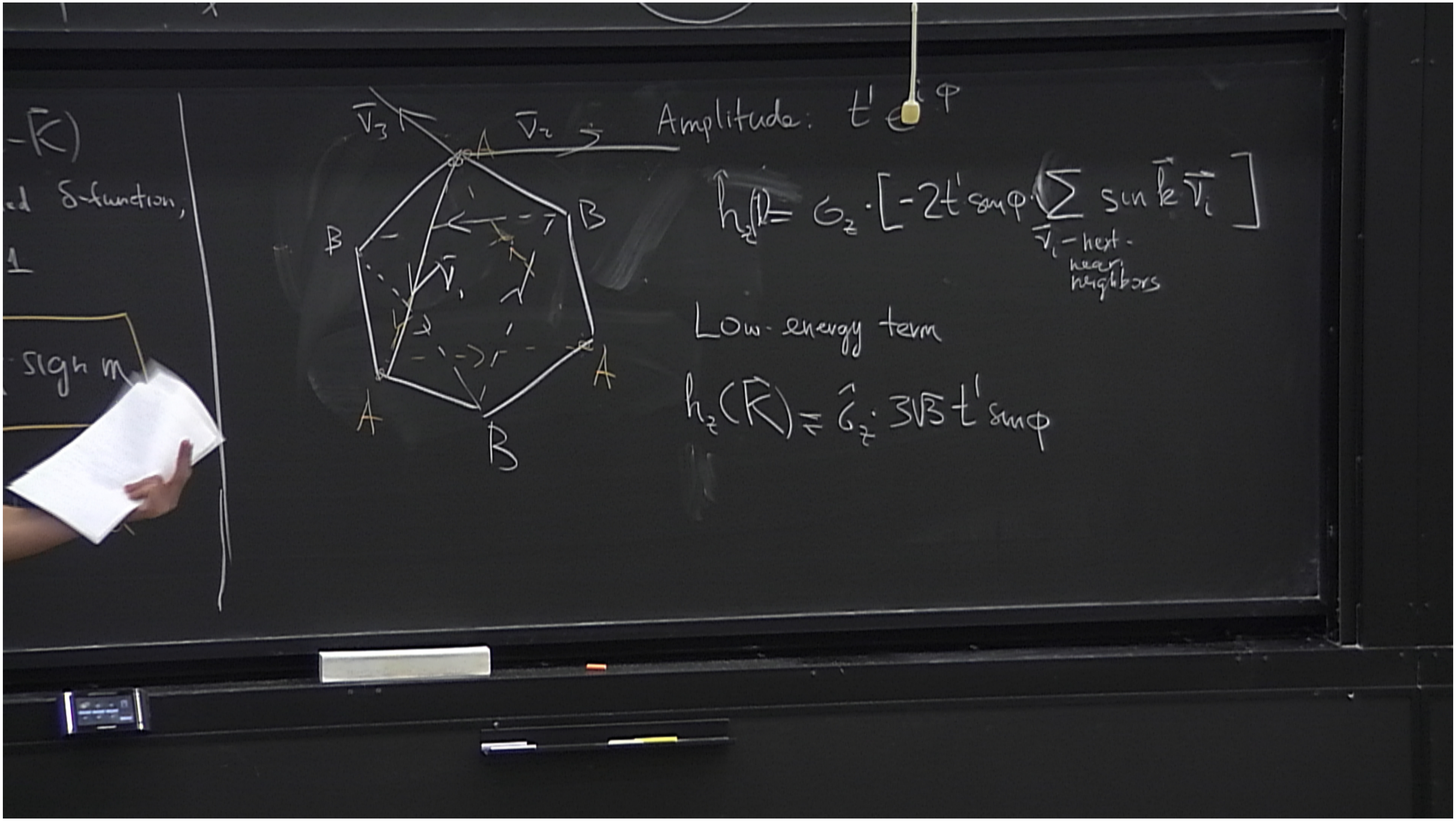

Ω
~ m/v
k
K
Ω(k) = π sign m · f(k − K)
broadened δ-function,
∫f = 1
σxy = e²/h ∫ Ω d²k/2π = e²/2h sign m
vicinity of DP
contrib. of Dirac point ± e²/2h
But: σxy = n e²/h
Dirac points come in pairs
A
B
Ω(K'



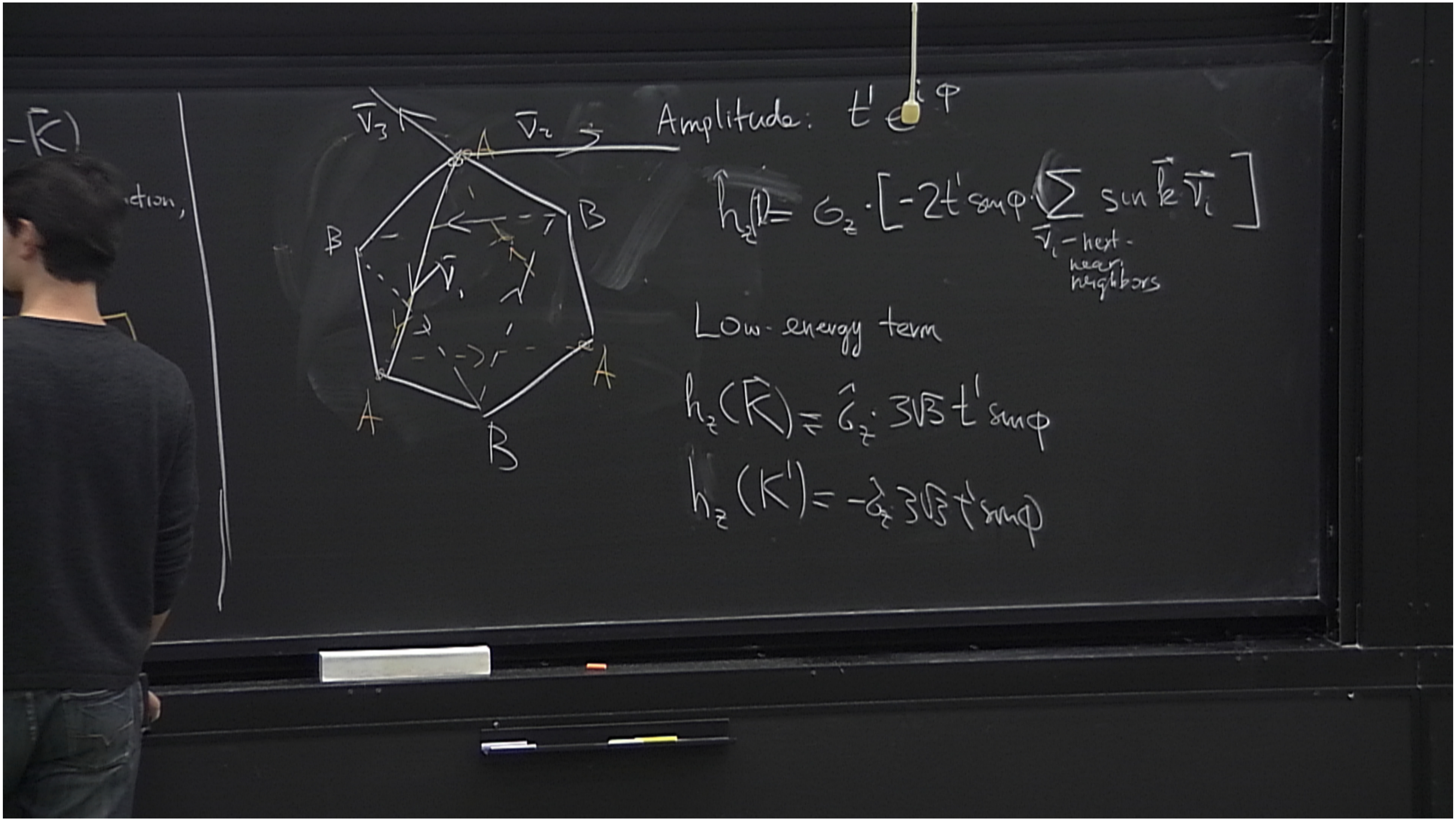
-K)
δ-function,
sign m
Amplitude: t' e^iφ
A
B
B
A
A
B

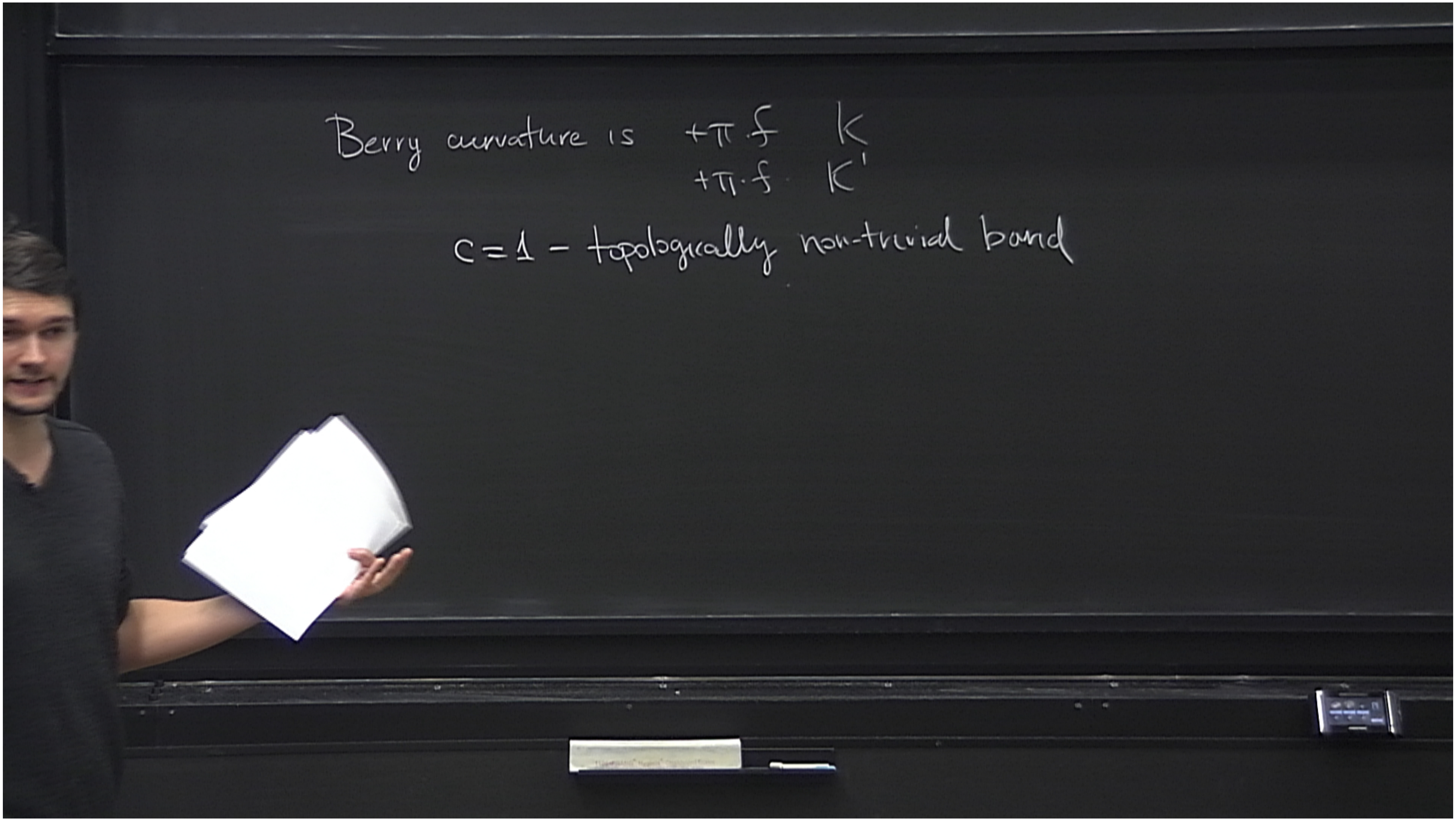


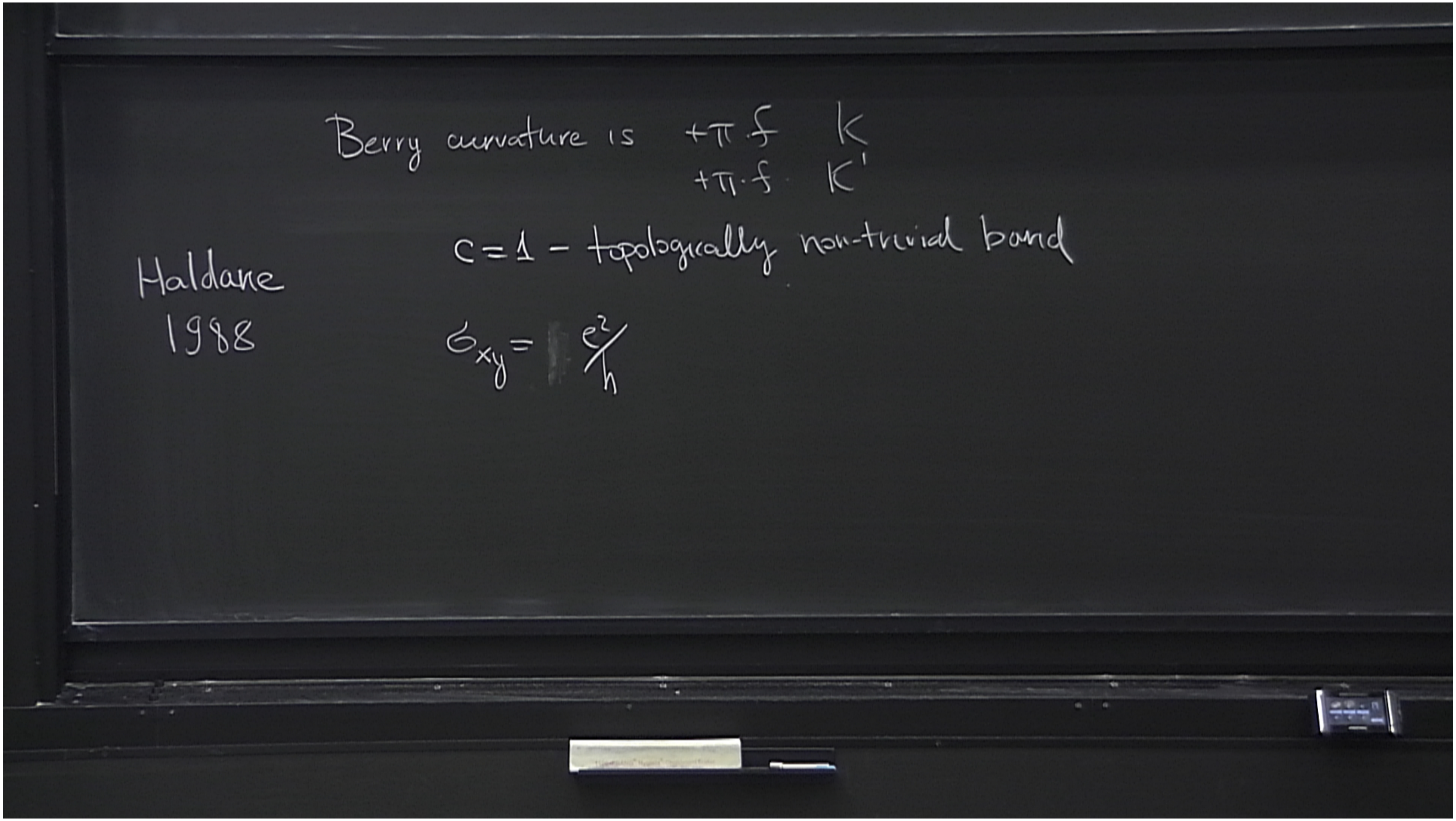
Amplitude: $t' e^{i\varphi}$
$\hat{h}_z = \sigma_z \cdot [-2t' \sin\varphi \cdot \sum \sin \vec{k}\cdot\vec{v}_i]$
$\vec{v}_i$ - next-nearest neighbors
$\vec{v}_1$
$\vec{v}_2$
$\vec{v}_3$
A
B
δ-function,
sign m



-K)
δ-function,
sign m
v3
v2
v1
A
B
Amplitude: t' e^{iφ}
h_z(k) = σ_z · [-2t' sinφ · Σ sin k·v_i]
v_i - next-nearest neighbors
Low-energy term
h_z(K) = σ_z · 3√3 t' sinφ


Amplitude: $t' e^{i\varphi}$
$\vec{v}_3$
$\vec{v}_2$
$\vec{v}_1$
A
B
B
B
A
A
$h_z(\vec{k}) = \sigma_z \cdot [-2t' \sin\varphi \cdot \sum \sin \vec{k}\vec{v}_i]$
$\vec{v}_i$ - next-near neighbors
Low-energy term
$h_z(\vec{K}) = \hat{\sigma}_z \cdot 3\sqrt{3}\, t' \sin\varphi$
$h_z(K') = -\hat{\sigma}_z\, 3\sqrt{3}\, t' \sin\varphi$


Berry curvature is $+\pi \cdot f$ K
$+\pi \cdot f$ K'
$c=1$ - topologically non-trivial band


Berry curvature is $+\pi \cdot f$ K
$+\pi \cdot f$ K'
Haldane
1988
$c = 1$ - topologically non-trivial band
$\sigma_{xy} = e^2/h$

Berry curvature is $+\pi \cdot f$ K

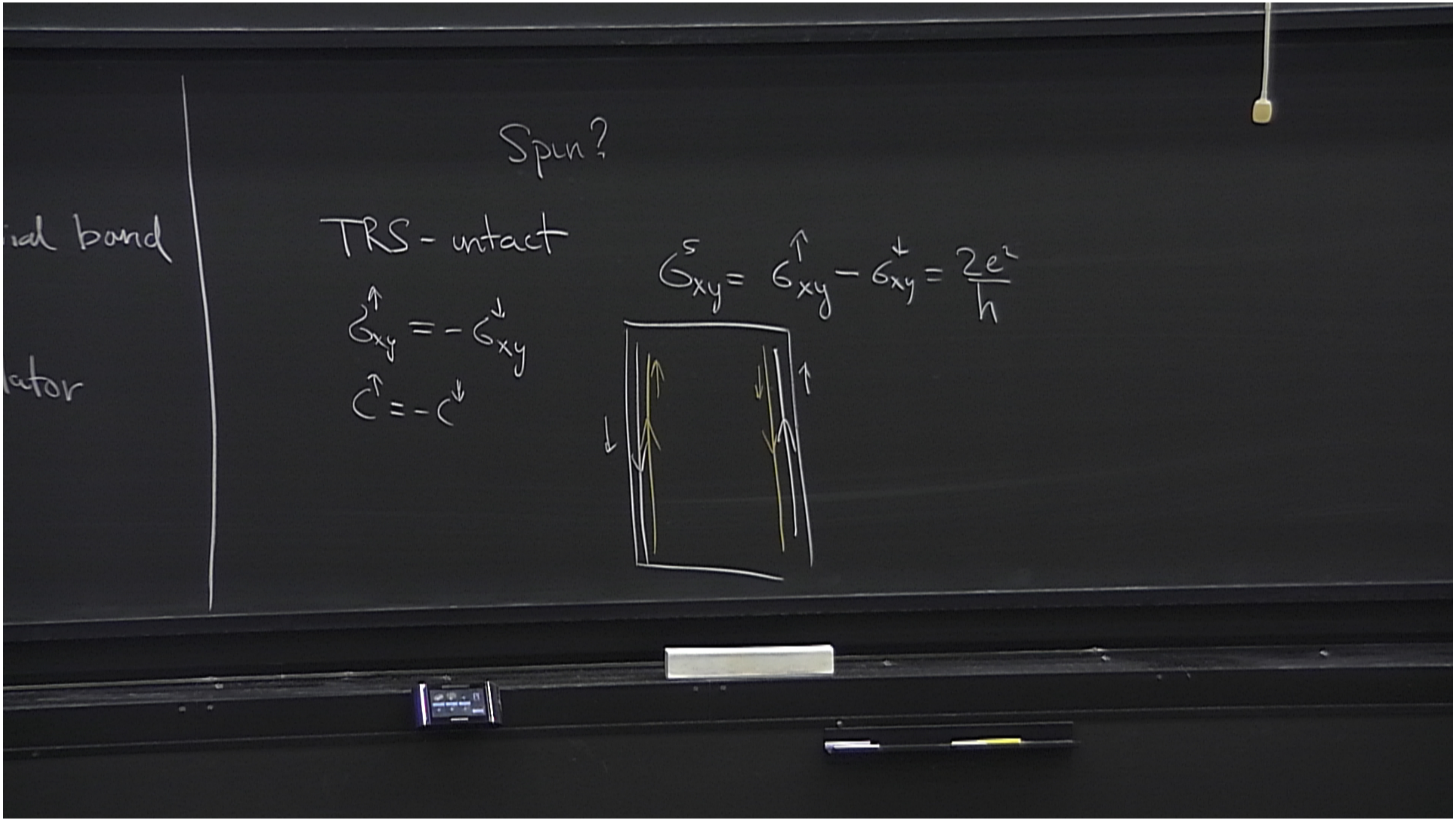
$+\pi \cdot f$ K'

Haldane
1988

$c = 1$ – topologically non-trivial band

$\sigma_{xy} = e^2/h$ Chern insulator

Candidate: magnetic insulators


Spin?
TRS - intact
$\sigma^{\uparrow}_{xy} = -\sigma^{\downarrow}_{xy}$
$C^{\uparrow} = -C^{\downarrow}$
$\sigma^{s}_{xy} = \sigma^{\uparrow}_{xy} - \sigma^{\downarrow}_{xy} = \frac{2e^2}{h}$
ial band
lator