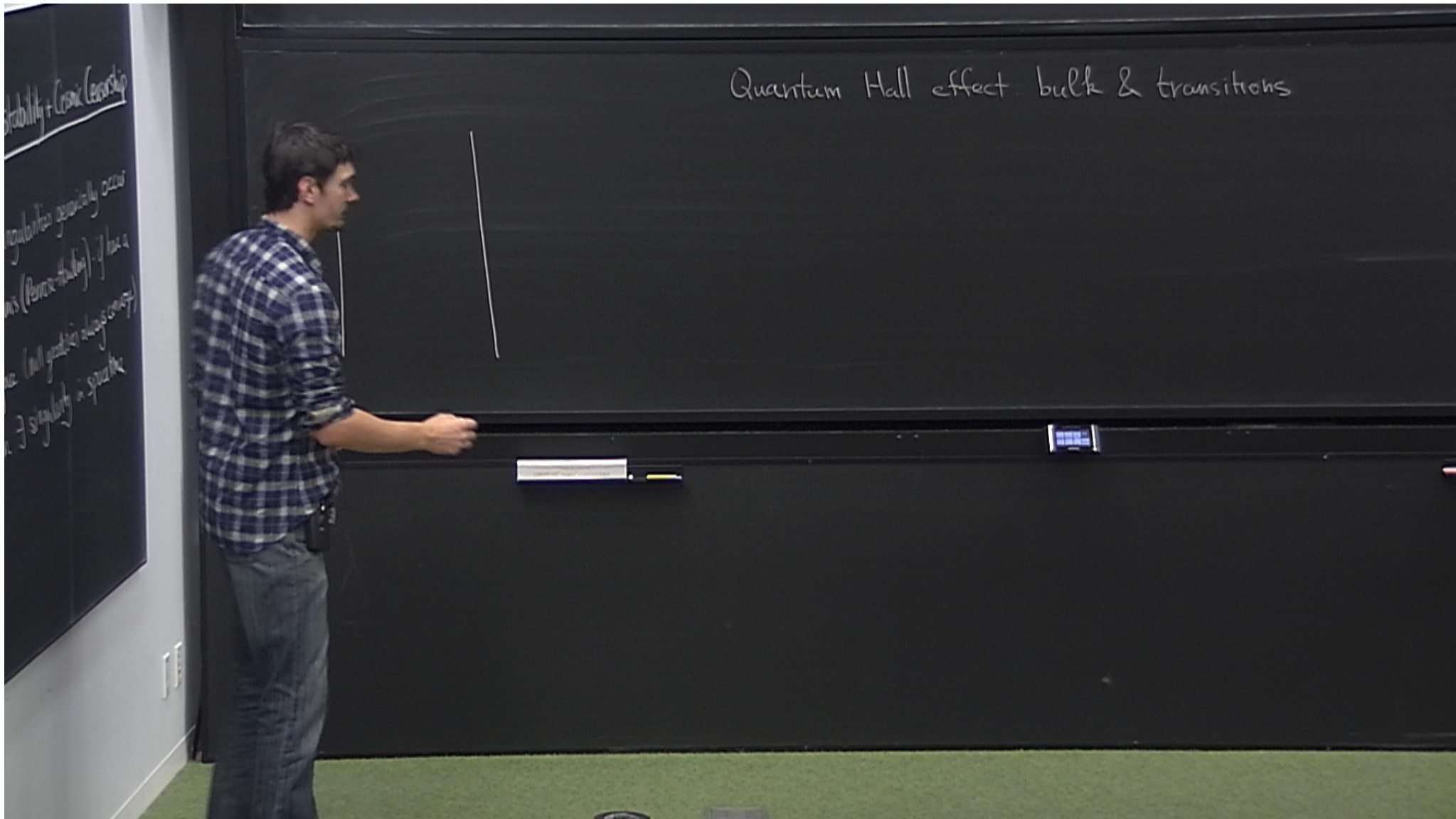


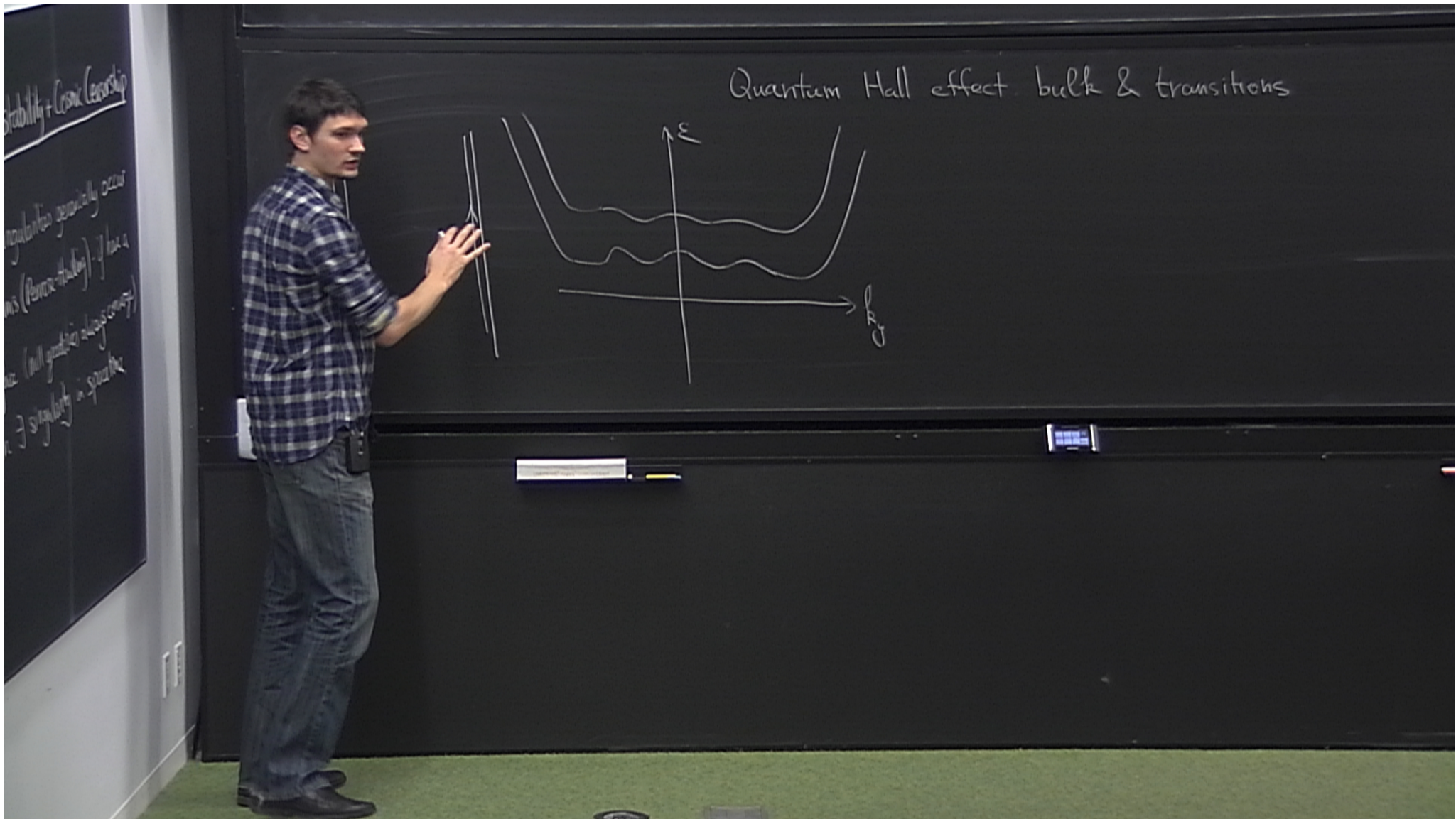
Title: 12/13 PSI - Condensed Matter Review Lecture 14

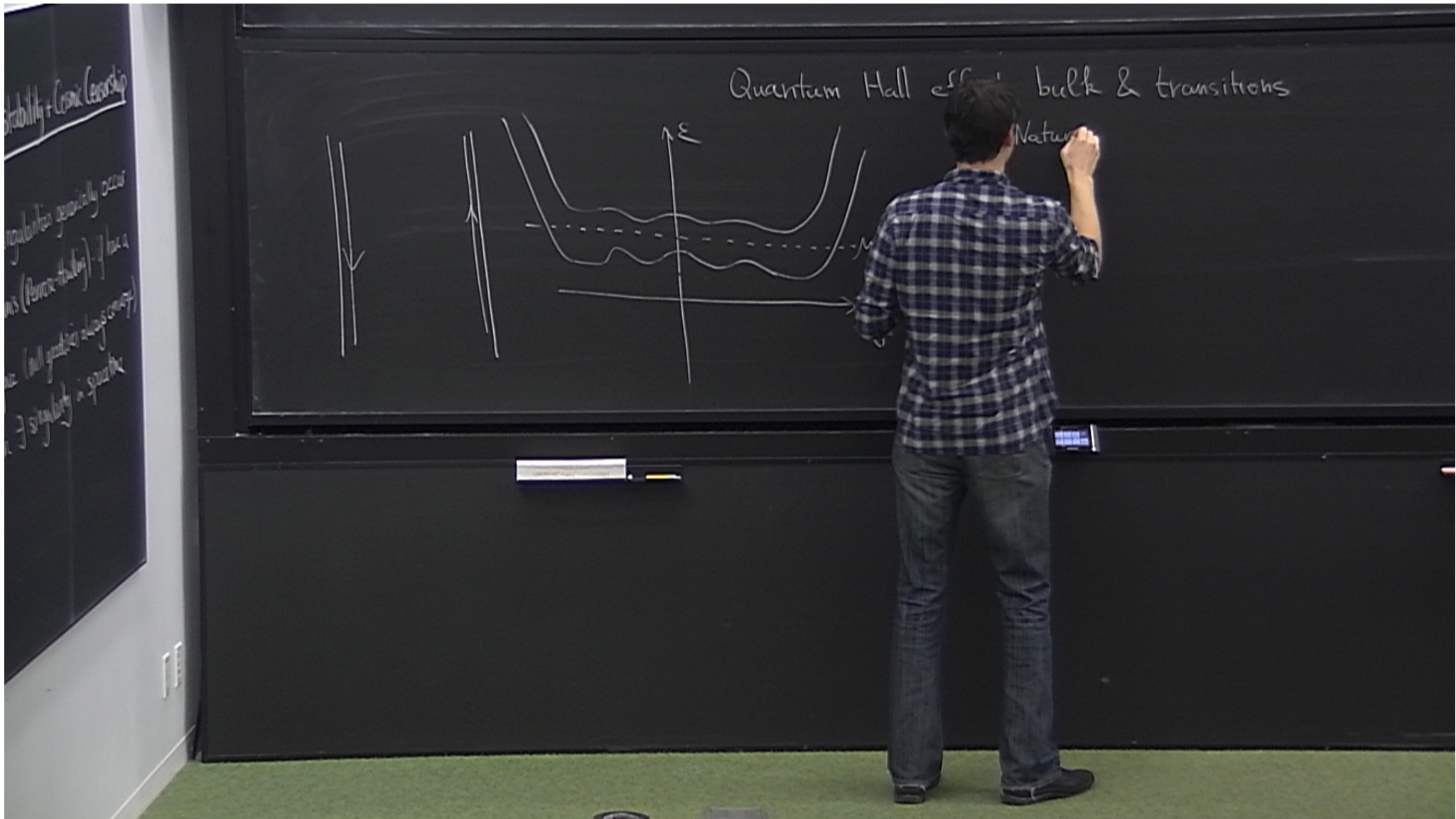
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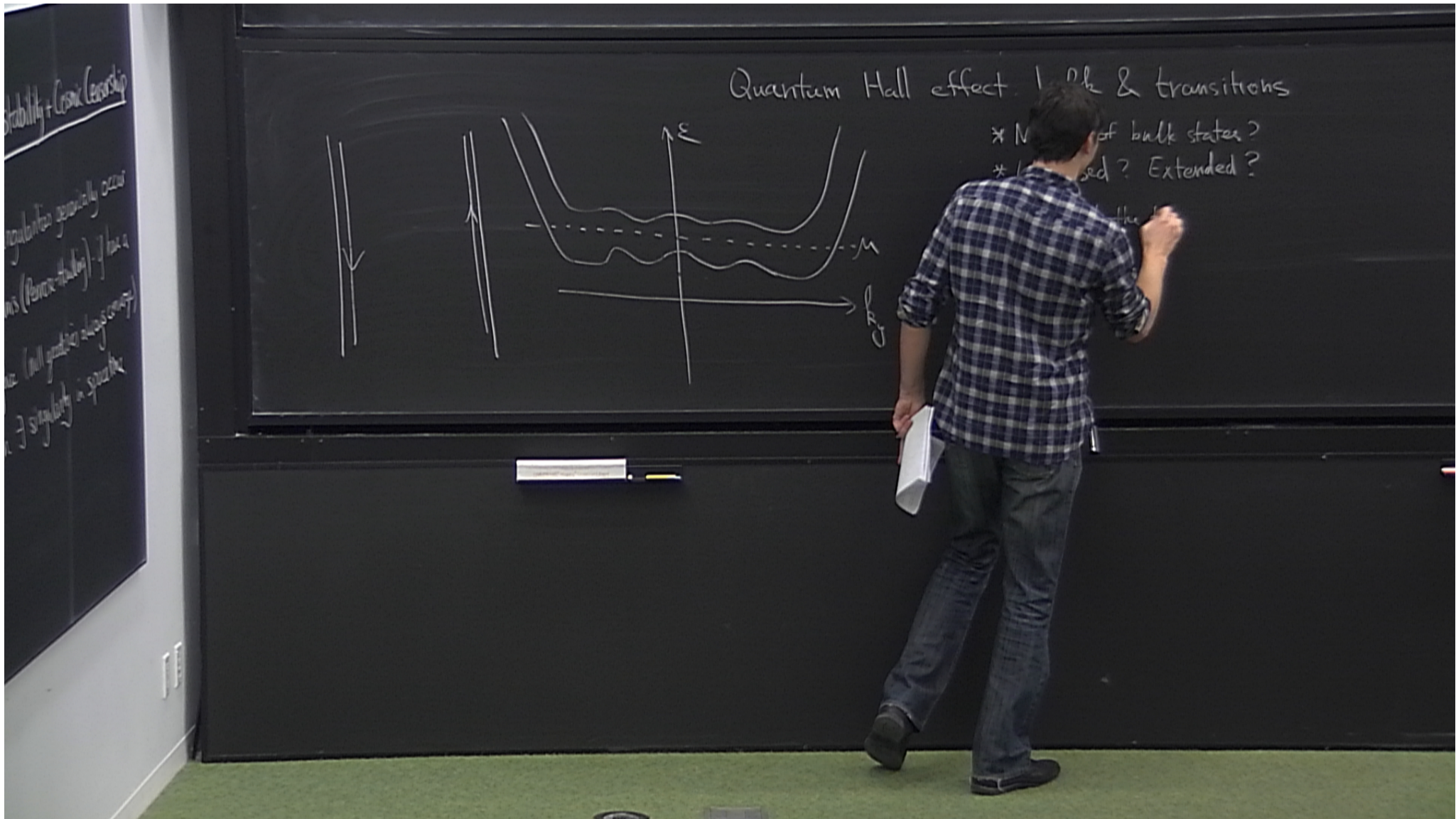
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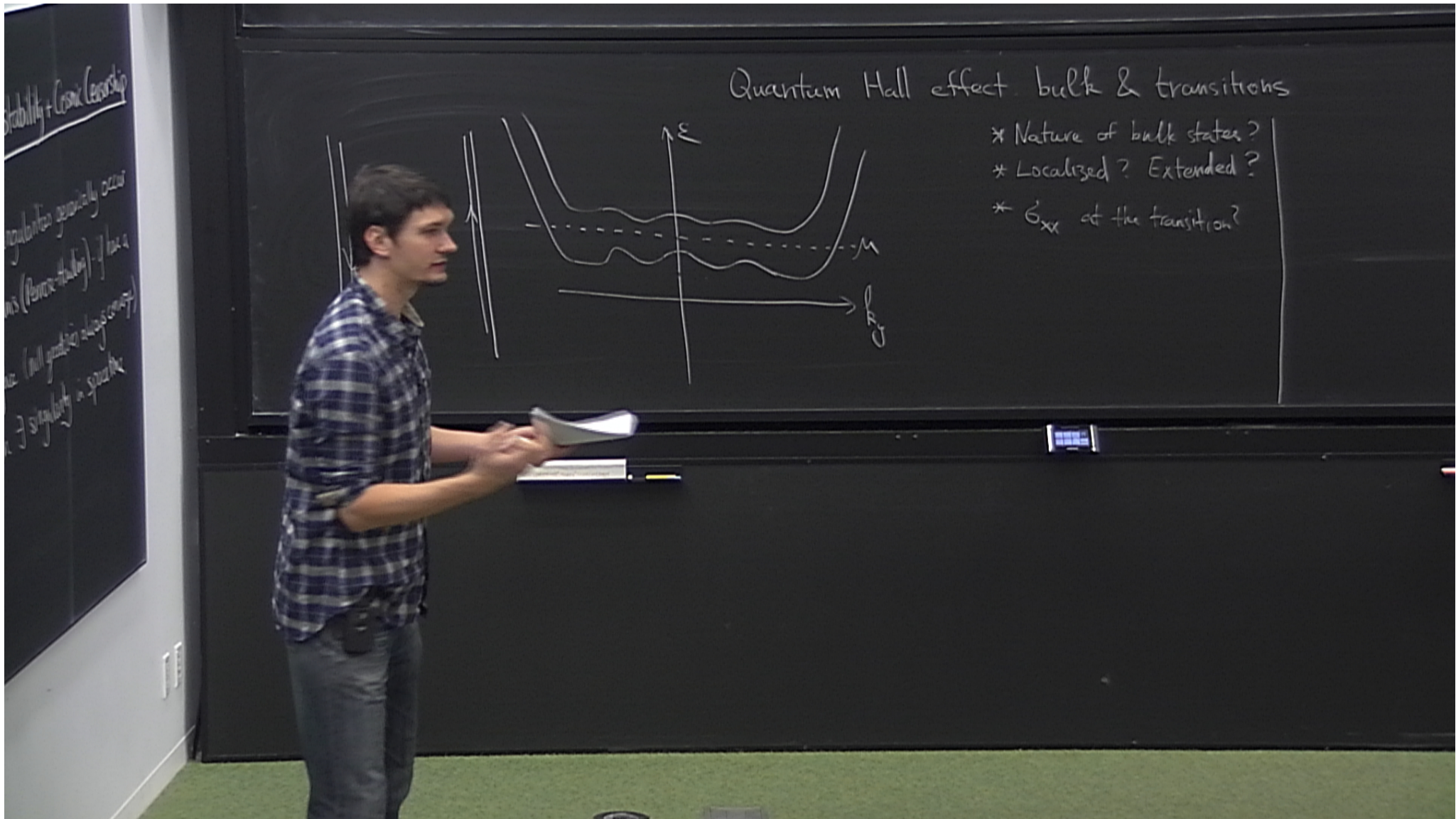
Abstract:











t. bulk & transitions

* Nature of bulk states?

* Localized? Extended?

* σ_{xx} at the transition?

Smooth disorder:

$$\hbar v_F \ll \hbar \omega_c$$

t. bulk & transitions

* Nature of bulk states?

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Smooth disorder:

$$|V(\vec{r})| \ll \hbar \omega_c, \quad \left| \frac{dV}{d\vec{r}} \right| \ll \frac{\hbar \omega_c}{\ell}$$

bulk & transitions

* Nature of bulk states?

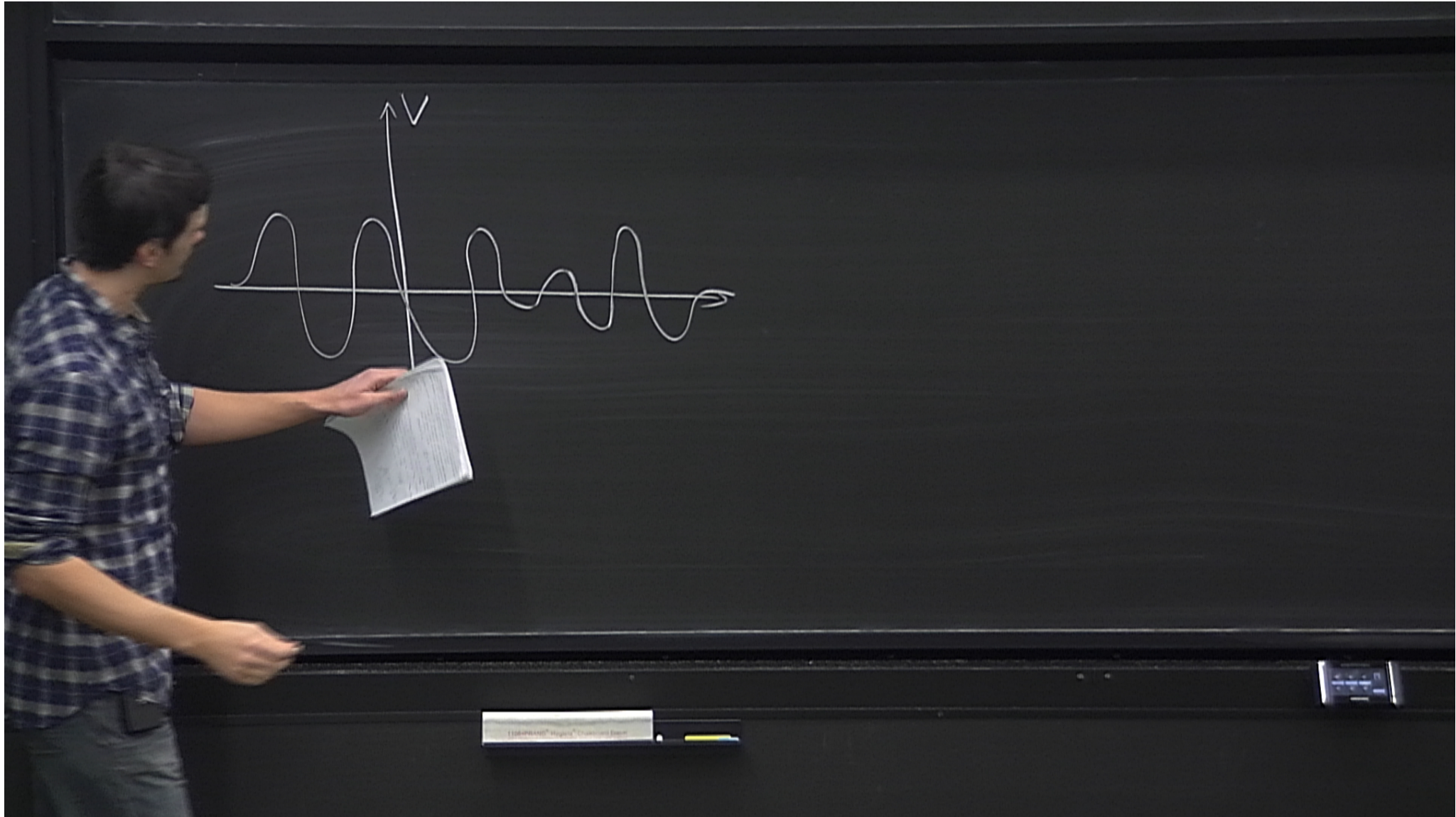
* Localized? Extended?

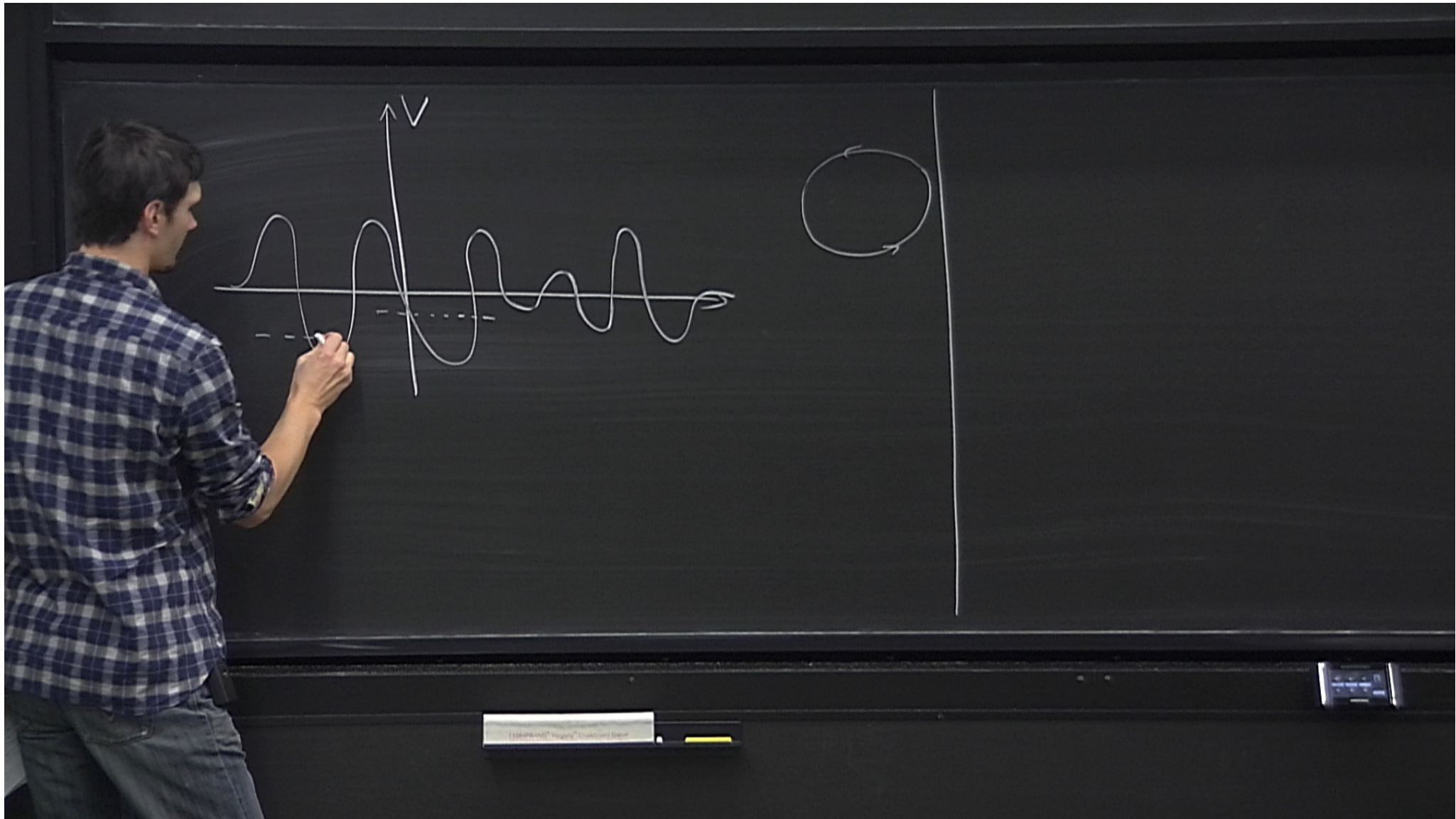
* σ_{xx} at the transition?

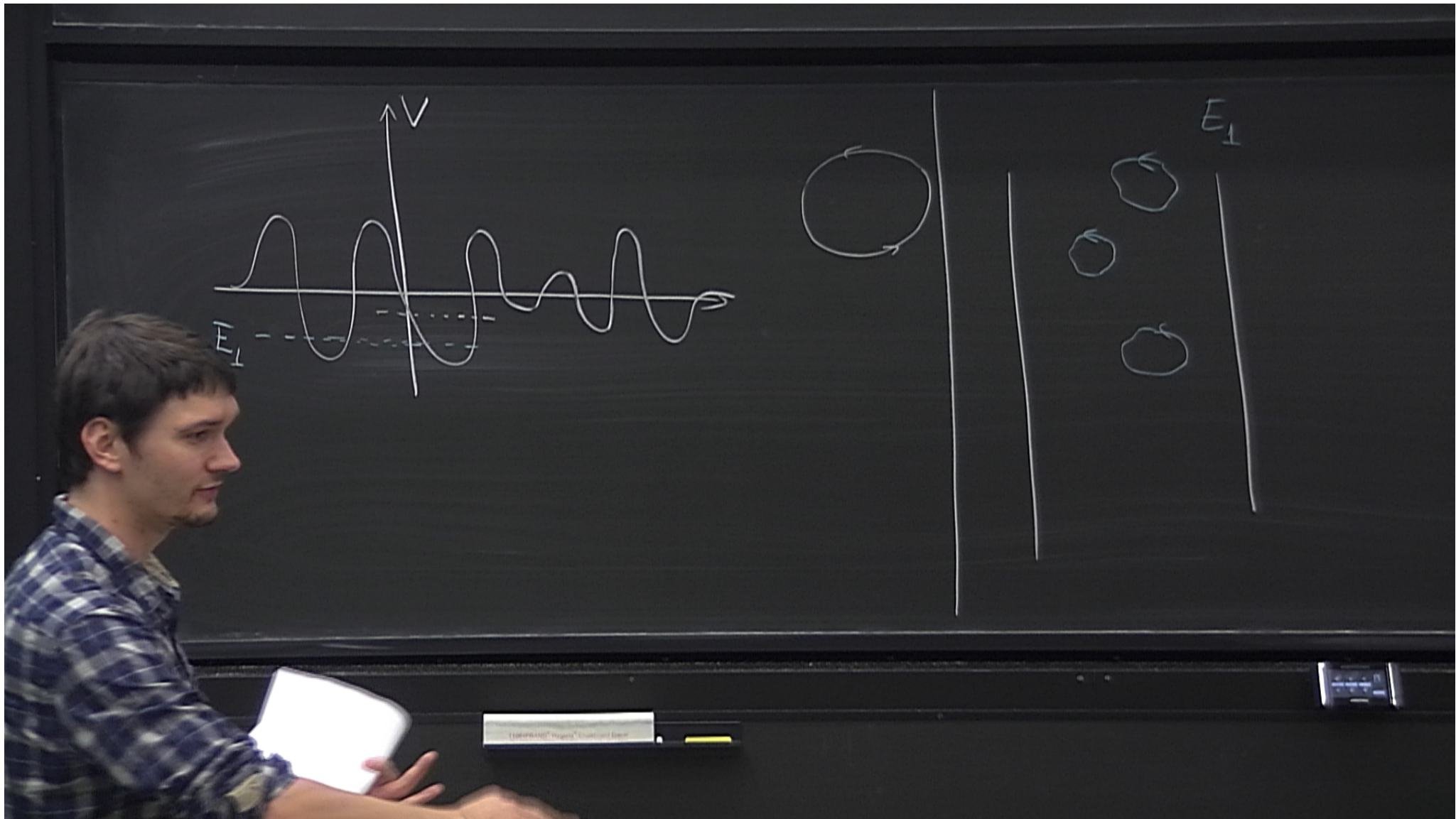
Smooth order:

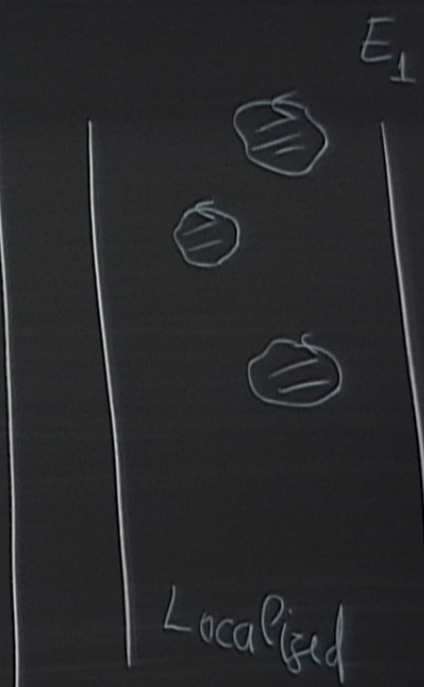
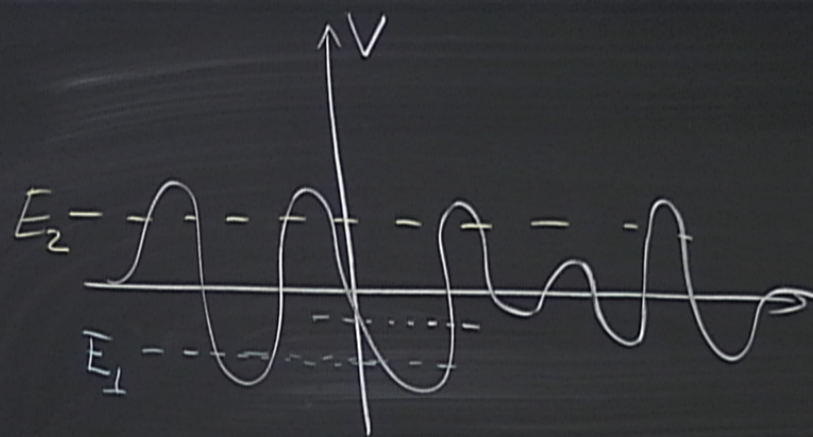
$$|V(\vec{r})| \ll \hbar \omega_c \quad \left| \frac{dV}{d\vec{r}} \right| \ll \frac{\hbar \omega_c}{\ell}$$

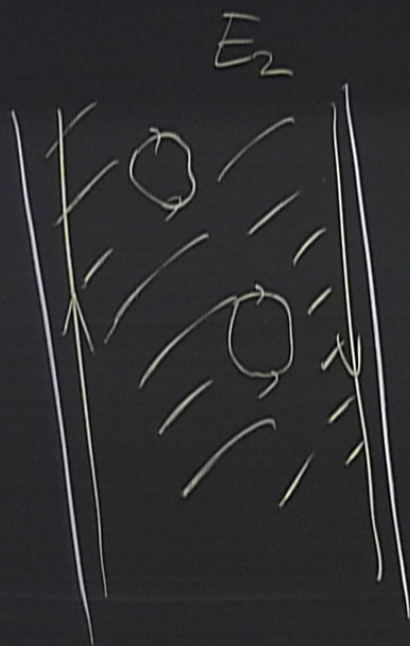
Electron length

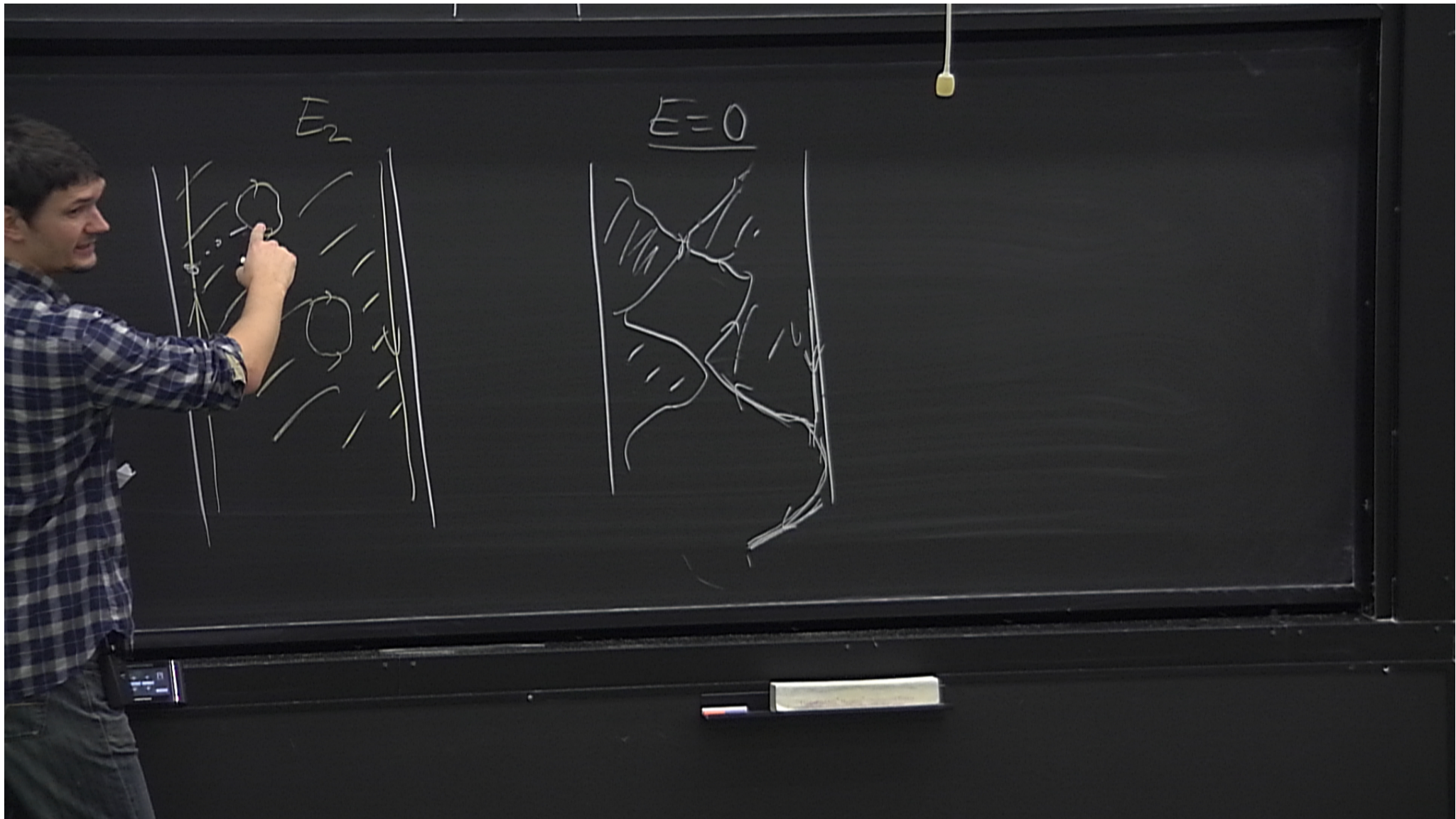


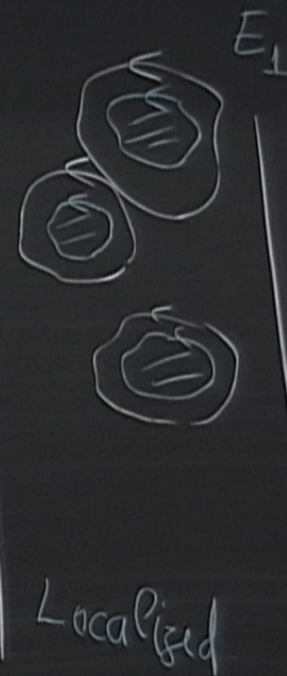
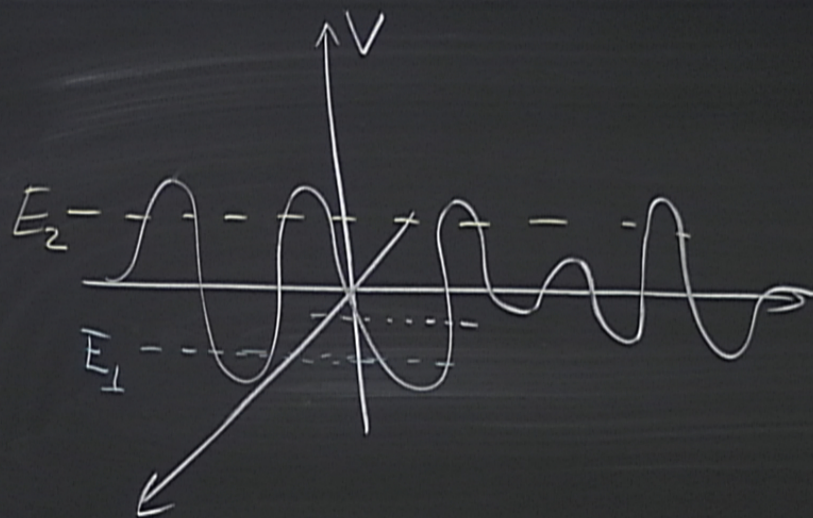


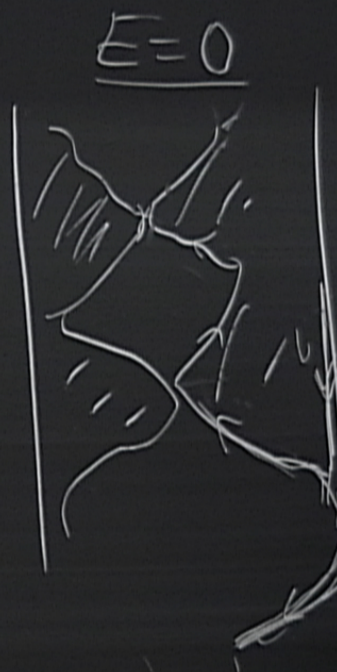
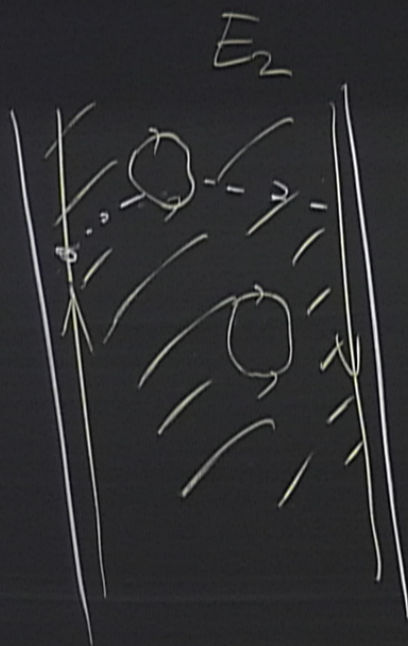






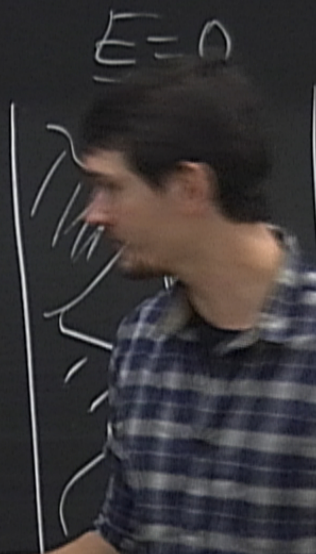
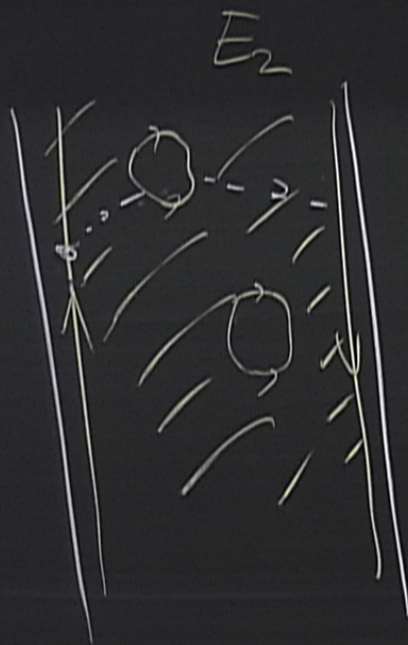






...ition occurs
 $= \hbar \omega_c (n + \frac{1}{2})$

* ...ed states in the bulk

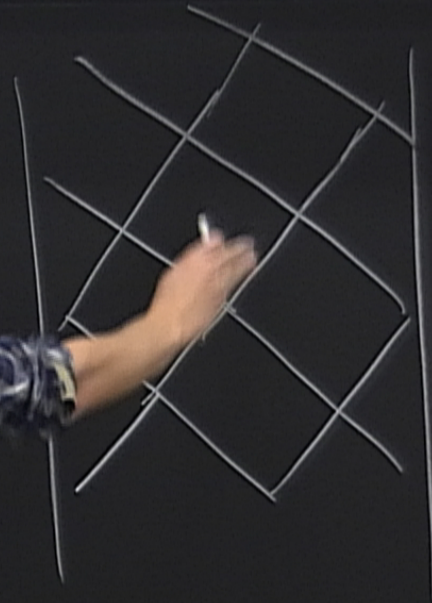
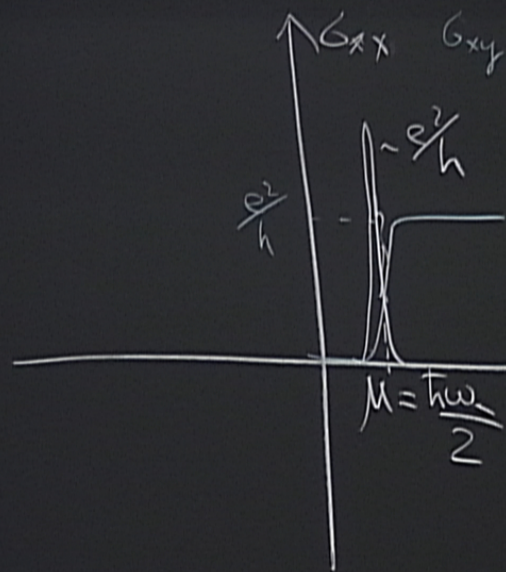


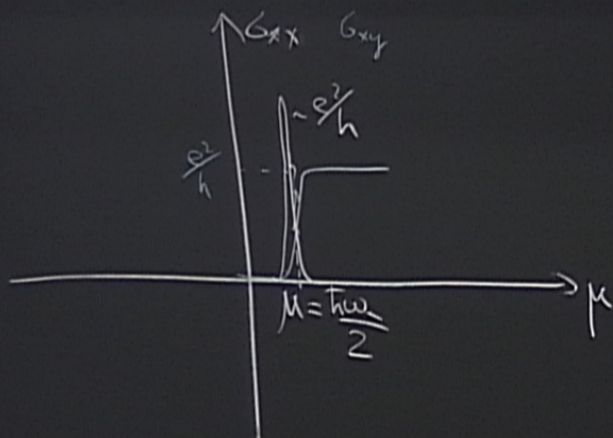
* Transition occurs
 $\mu_F = \hbar \omega_c (n + \frac{1}{2})$

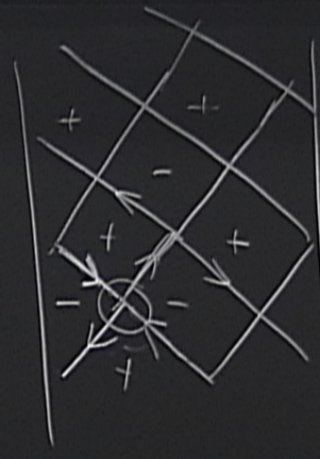
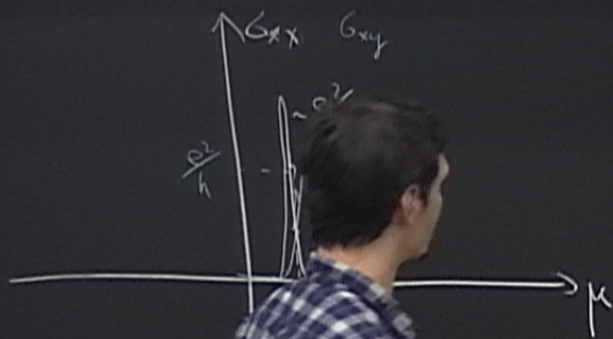
* Extended states in the bulk

* Percolating network

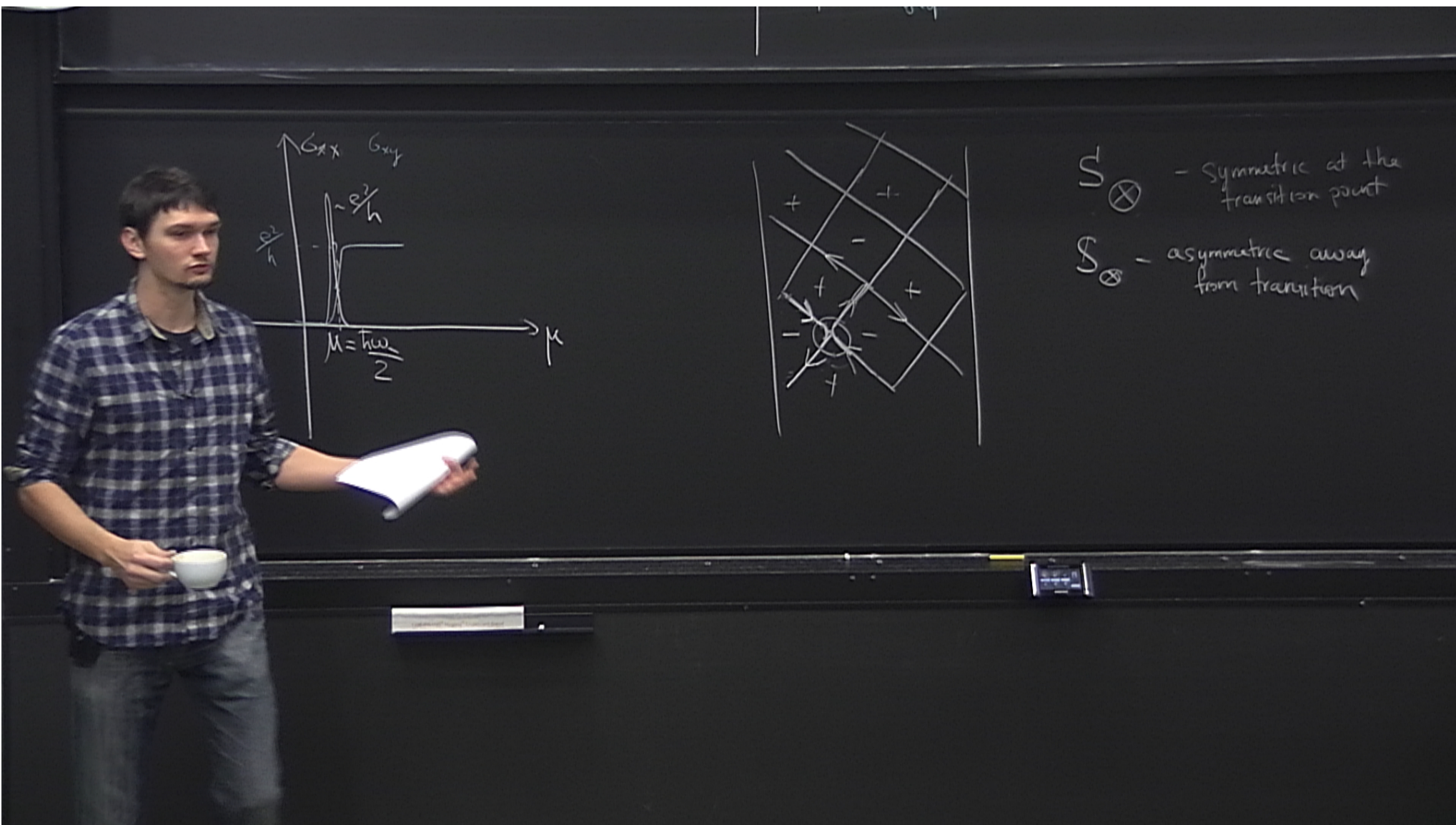
$$G_{xx} \sim \frac{e^2}{h}$$

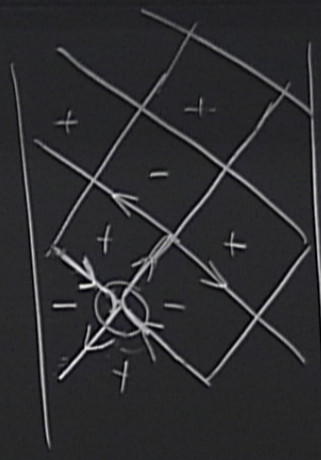
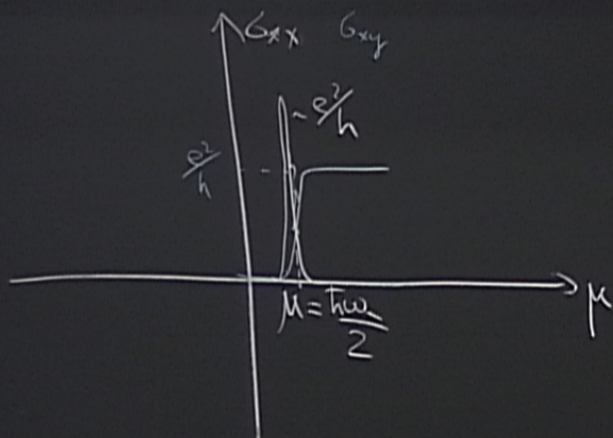






$S \otimes$ - Symmetric at the transition point

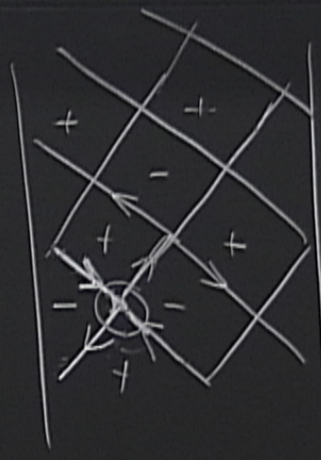
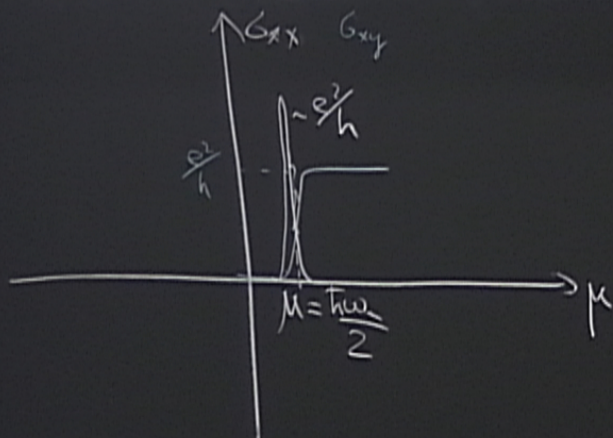




$S_{\otimes}$ - Symmetric at the transition point

$S_{\otimes}$ - asymmetric away from transition

Chalker-Coddington model



$S_{\otimes}$ - Symmetric at the transition point
 $S_{\otimes}$ - asymmetric away from transition
 Chalker-Coddington model

$S_{\otimes}$ - Symmetric at the transition point

$S_{\otimes}$ - asymmetric away from transition

Chalker-Coddington model

Summary



$S_{\otimes}$ - Symmetric at the transition point

$S_{\otimes}$ - asymmetric away from transition

Chalker-Coddington model

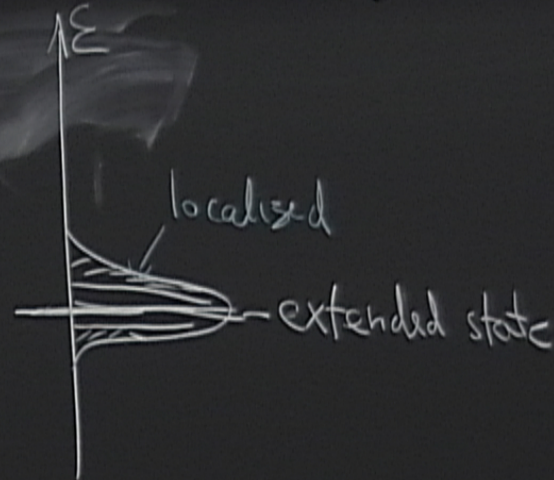
Summary

$S_{\otimes}$ - Symmetric at the transition point

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Chalker-Coddington model

Summary

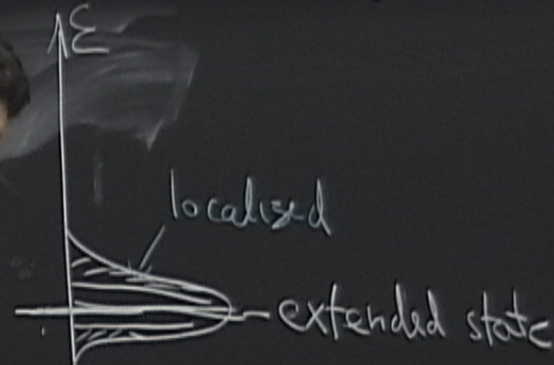


$S_{\otimes}$ - Symmetric at the transition point

$S_{\otimes}$ - asymmetric away from transition

Chalker-Coddington mod

Summary



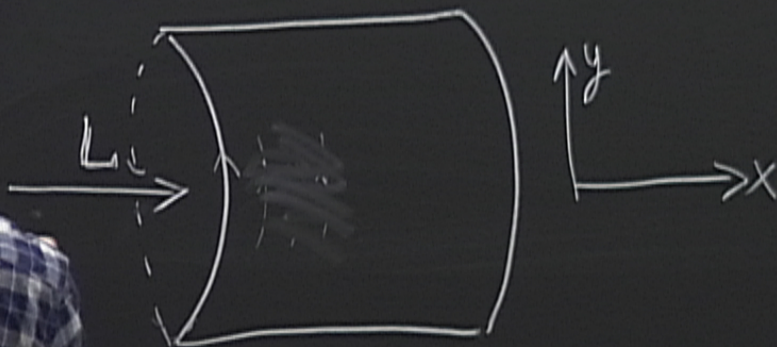
μ - between two Landau levels

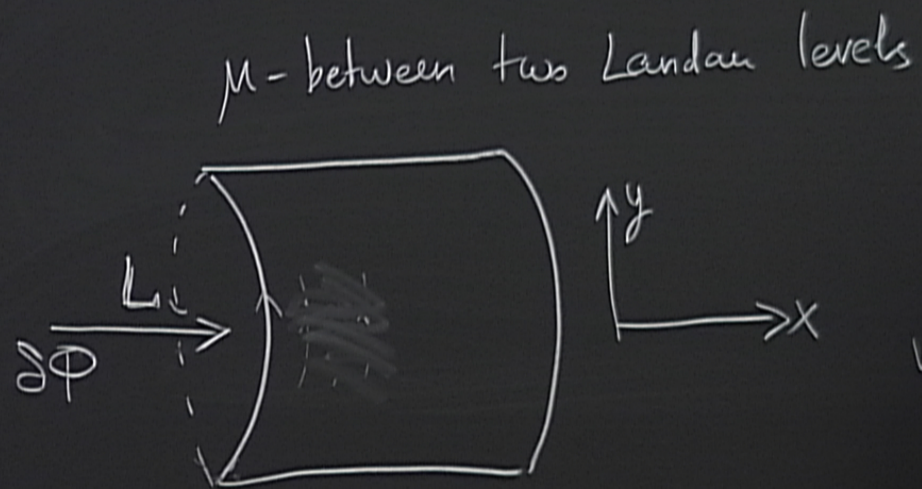
Another approach: μ

Another approach:

μ - between two Landau levels

$$k_y = \frac{2\pi n}{L}$$





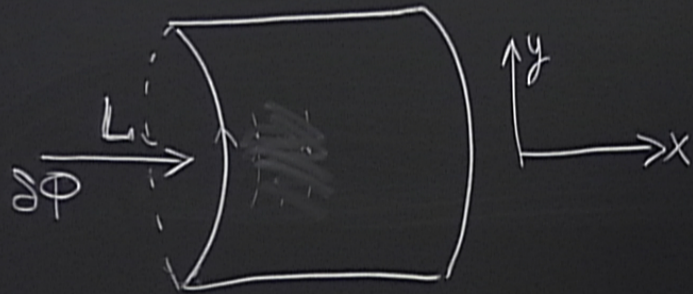
$$k_y = \frac{2\pi n}{L}$$

w/o flux: $\psi(L) = \psi(0)$

w/flux $\delta\phi$ $\delta\phi$

Another approach:

μ - between two Landau levels



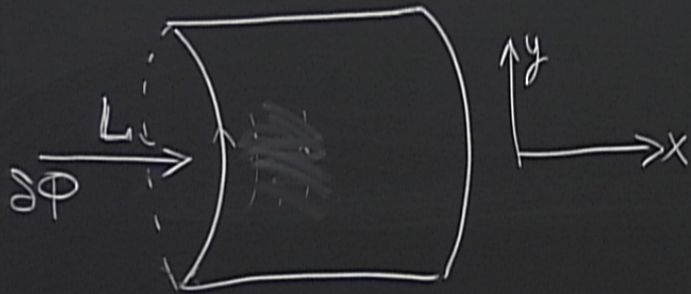
$$k_y = \frac{2\pi n}{L}$$

w/o flux: $\psi(L)$

w/flux $\delta\phi$ $\psi(L) = e^{i\delta\phi} \psi(0)$

Another approach: Flux insertion

μ - between two Landau levels



$$k_y = \frac{2\pi n}{L}$$

w/o flux: $\psi(L) = \psi(0)$

w/flux $\delta\phi$ $\delta\phi = \frac{\delta\phi}{\phi_0}$ $\therefore \psi(L) = e^{i\delta\phi} \psi(0)$
 $\psi \sim e^{ik_y y}$ $e^{ik_y' L + i\delta\phi} = 1$

Another approach: Flux insertion $k_y' L$

other approach: Flux insertion

$$k'_y L + \delta\varphi = 2\pi n$$

$$k'_y = \frac{2\pi n}{L} - \frac{\delta\varphi}{L} = k_y - \frac{\delta\varphi}{L}$$

(v)

$$\psi(L) = e^{i\delta\varphi} \psi(0)$$

$$\psi \sim e^{ik_y y} \quad e^{ik'_y L + i\delta\varphi} = 1$$

other approach: Flux insertion

$$k_y' L + \delta\phi = 2\pi$$

$$k_y' = \frac{2\pi}{L} - \frac{\delta\phi}{L}$$

$$\frac{\delta\phi}{L} \rightarrow k_y$$

(0)

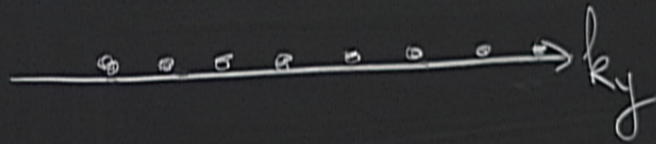
$$\psi(L) = e^{i\delta\phi} \psi(0)$$

$$\psi \sim e^{ik_y y} \quad e^{ik_y' L + i\delta\phi} = 1$$

Adiabatically insert a flux quantum

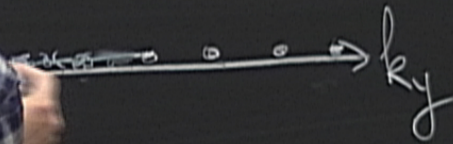
Adiabatically insert a flux quantum

$$0 < k_y l^2 < L_x$$



Adiabatically insert a flux quantum

$$0 < k_y l^2 < L_x$$



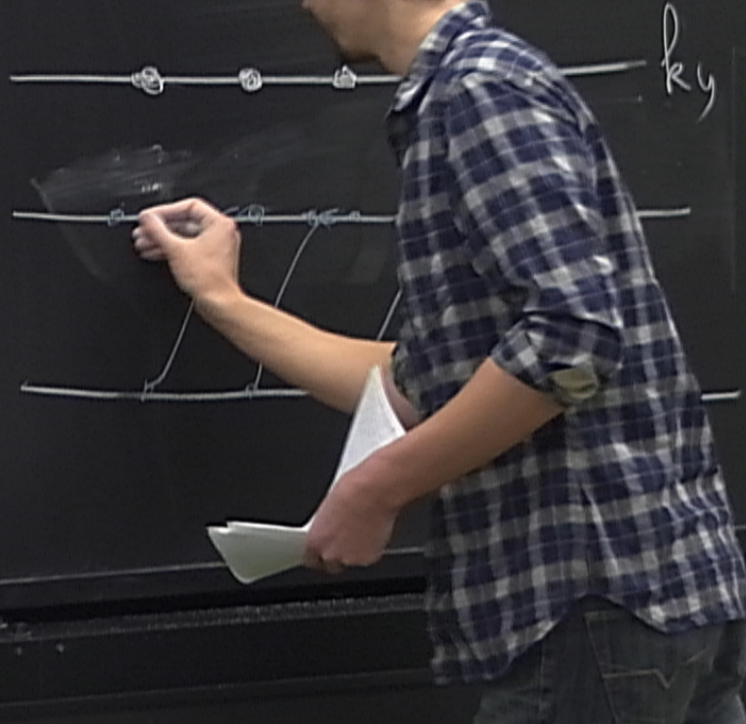
Adiabatically insert a flux quantum

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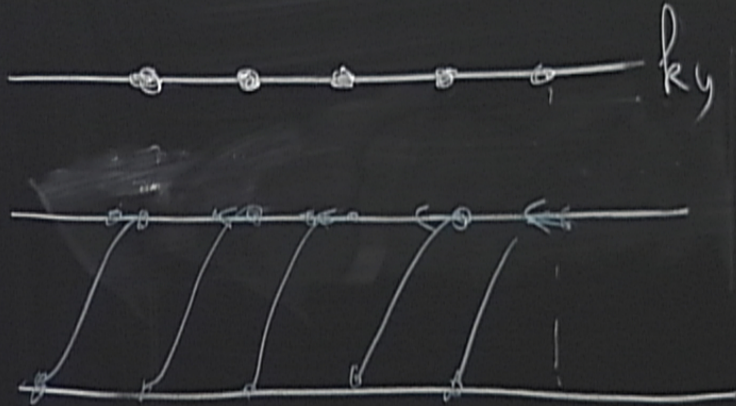
Adiabatically insert a flux quantum

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Adiabatically insert a flux quantum

$$0 < k_y l^2 < L_x$$



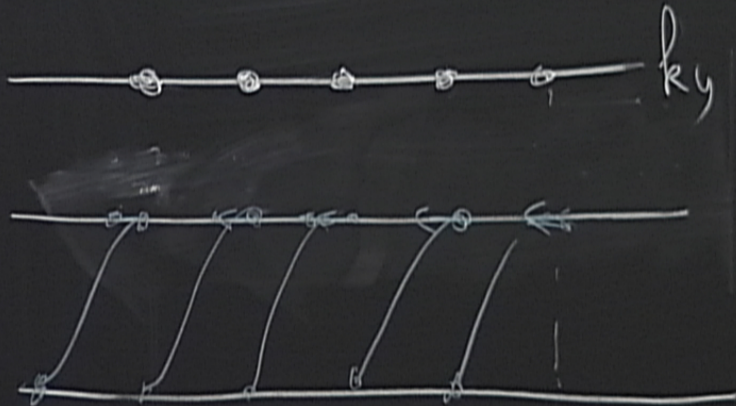
$$E = \frac{1}{cL} \frac{d\Phi}{dt}$$

$$\int j dt L = e$$

$$j = G_{xy} E$$

Adiabatically insert a flux quantum

$$0 < k_y l^2 < L_x$$



$$E = \frac{1}{cL} \frac{d\Phi}{dt}$$

$$\int j dt L = e$$

$$j = G_{xy} E$$

$$\int G_{xy} L \cdot \underline{E} dt = e$$

$$G_{xy} \times \frac{\Phi_0}{c} = e \quad \Phi_0 = \frac{hc}{e}$$

$$\frac{1}{c^2} \frac{d\phi}{dt}$$

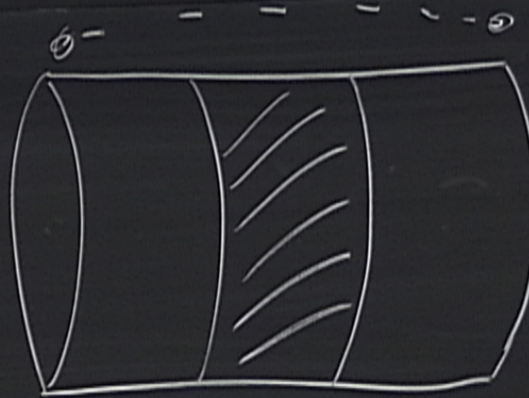
$$\frac{dL}{dt} = e$$

$$E$$

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$$e \phi_0 = \frac{hc}{e}$$

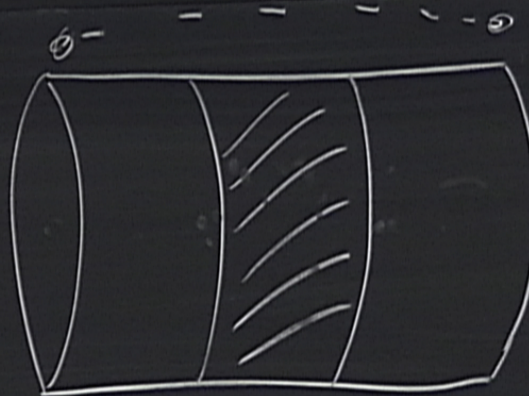
$$G_M = \frac{e^2}{h}$$



$$\frac{1}{c} \frac{d\phi}{dt}$$

$$\frac{dL}{dt} = e$$

$$G_M = \frac{e^2}{h}$$



$$\frac{1}{c} \frac{d\phi}{dt}$$

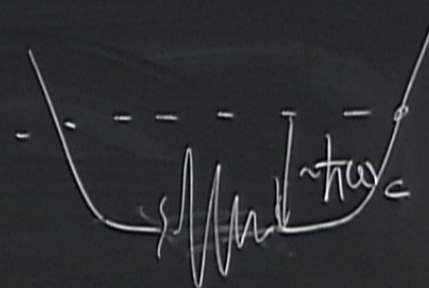
$$dL = e$$

$$E$$

$$dL = e$$

$$e \phi_0 = \frac{hc}{e}$$

$$G_M = \frac{e^2}{h}$$



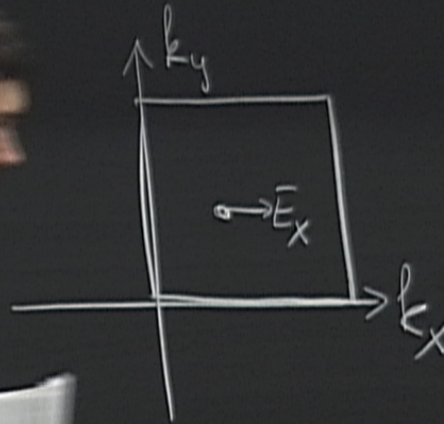
Hall conductivity as a topological invariant

2D Band stru
Bloch band

Hall conductivity as

2D Band str
Bloch band

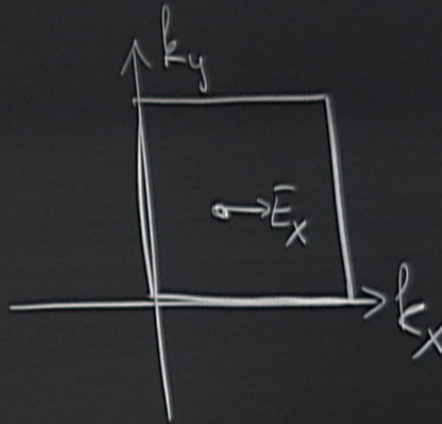
σ_{xy}



Hall conductivity as

2D Band structure
Bloch band

σ_{xy}

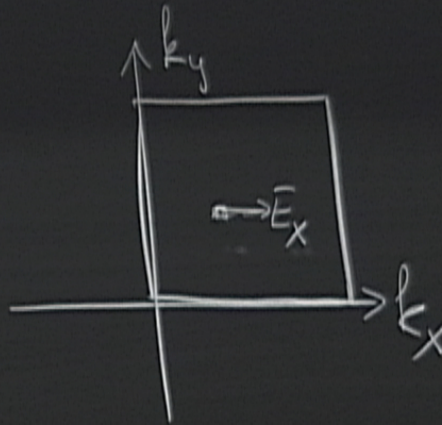


Hall conductivity as

$$v_o = \frac{\partial \epsilon}{\hbar \partial \vec{k}}$$

2D Band structure
Bloch band

σ_{xy}

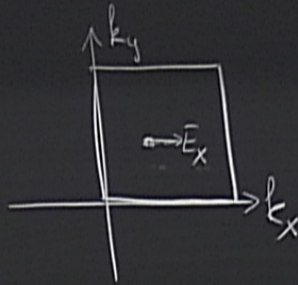


Hall conductivity as

$$v_0 = \frac{\partial \epsilon}{\hbar \partial \vec{k}}$$

$|\psi_n\rangle$

2D Band structure
 Bloch band
 G_{xy}



$$v_o = \frac{\partial \epsilon}{\hbar \partial k}$$

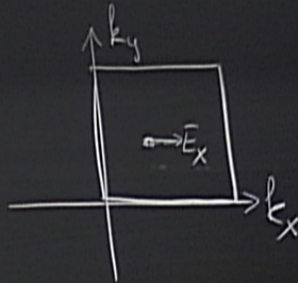
$$|\psi_n\rangle$$

Hall conductivity as a topological invariant

Wave function:

$$|\psi_n\rangle = i\hbar \sum_{n' \neq n} \frac{|\psi_{n'}\rangle \langle \psi_{n'} | \frac{\partial \psi_n}{\partial \epsilon} \rangle}{\epsilon_n - \epsilon_{n'}}$$

2D Band structure
 Bloch band
 G_{xy}



Hall conductivity as a topological invariant

$$v_0 = \frac{\partial \epsilon}{\hbar \partial k}$$

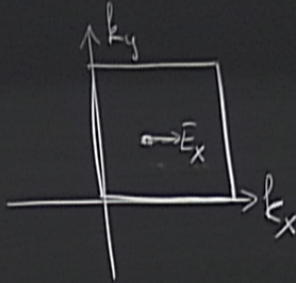
$$|\psi_n\rangle$$

$$\langle v \rangle$$

$$= \frac{|\psi_n\rangle \langle \psi_n| \frac{\partial \psi_n}{\partial \epsilon}}{\epsilon_n - \epsilon_{n'}}$$

2D Band structure
Bloch band

G_{xy}



Hall conductivity as a topological invariant

$$v_o = \frac{\partial \epsilon}{\hbar \partial k}$$

$$|\psi_n\rangle$$

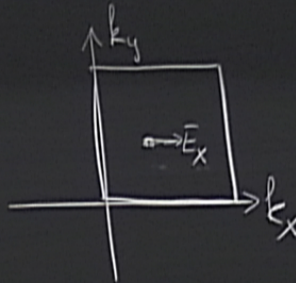
Wave function:

$$|\psi_n\rangle = -i\hbar \sum_{n' \neq n} \frac{|\psi_{n'}\rangle \langle \psi_{n'} | \frac{\partial \psi_n}{\partial \epsilon}}{\epsilon_n - \epsilon_{n'}}$$

$$\langle v_n(\vec{k}) \rangle = \left\langle \frac{\partial H}{\hbar \partial \vec{k}} \right\rangle$$

2D Band structure
Bloch band

G_{xy}



Hall conductivity as a topological invariant

$$v_o = \frac{\partial \epsilon}{\hbar \partial k}$$

$$|\psi_n\rangle$$

Wave function:

$$|\psi_n\rangle = i\hbar \sum_{n' \neq n} \frac{|\psi_{n'}\rangle \langle \psi_{n'} | \frac{\partial \psi_n}{\partial t} \rangle}{\epsilon_n - \epsilon_{n'}}$$

$$\langle U_n(\vec{k}) \rangle = \left\langle \frac{\partial H}{\hbar \partial \vec{k}} \right\rangle = \frac{\partial \epsilon_n}{\hbar \partial \vec{k}} - i \left[\frac{\langle \psi_n | \frac{\partial H}{\partial q} | \psi_{n'} \rangle \langle \psi_{n'} | \frac{\partial \psi_n}{\partial t} \rangle}{\epsilon_n - \epsilon_{n'}} \right] - CC$$

$$\uparrow G_{xx} \quad G_{xx} \quad \langle \psi_n | \frac{\partial H}{\partial k} | \psi_{n'} \rangle = \langle \psi_{n'} | \frac{\partial}{\partial k} H - H \frac{\partial}{\partial k} | \psi_n \rangle$$

↑ G_{xx} G_{yy}

$$\langle \psi_n | \frac{\partial H}{\partial k} | \psi_{n'} \rangle = \langle \psi_{n'} | \frac{\partial}{\partial k} H - H \frac{\partial}{\partial k} | \psi_n \rangle = (\epsilon_n - \epsilon_{n'}) \langle \frac{\partial \psi_n}{\partial k} | \psi_{n'} \rangle$$

↑ G_{xx} G_{xx}

$$\langle \psi_n | \frac{\partial H}{\partial k} | \psi_{n'} \rangle = \langle \psi_{n'} | \frac{\partial}{\partial k} H - H \frac{\partial}{\partial k} | \psi_n \rangle = (\epsilon_n - \epsilon_{n'}) \langle \frac{\partial \psi_n}{\partial k} | \psi_{n'} \rangle$$

$$\langle v_n(k) \rangle = \frac{\partial \epsilon_n(k)}{\hbar \partial k} = \langle \frac{\partial \psi_n}{\partial k} | \frac{\partial \psi}{\partial t} \rangle - \langle \frac{\partial \psi}{\partial t} | \frac{\partial \psi}{\partial k} \rangle$$

↑ G_{xx} G_{yy}

$$\langle \psi_n | \frac{\partial H}{\partial k} | \psi_{n'} \rangle = \langle \psi_{n'} | \frac{\partial}{\partial k} H - H \frac{\partial}{\partial k} | \psi_{n'} \rangle = (\epsilon_n - \epsilon_{n'}) \langle \frac{\partial \psi_n}{\partial k} | \psi_{n'} \rangle$$

$$\langle v_n(k) \rangle = \frac{\partial \epsilon_n(k)}{\hbar \partial k} - i \left[\langle \frac{\partial \psi_n}{\partial k} | \frac{\partial \psi_n}{\partial t} \rangle - \langle \frac{\partial \psi_n}{\partial t} | \frac{\partial \psi_n}{\partial k} \rangle \right]$$

$$\uparrow \text{Gex Gex} \quad \langle \psi_n | \frac{\partial H}{\partial k} | \psi_{n'} \rangle = \langle \psi_{n'} | \frac{\partial}{\partial k} H - H \frac{\partial}{\partial k} | \psi_n \rangle = (\epsilon_n - \epsilon_{n'}) \langle \frac{\partial \psi_n}{\partial k} | \psi_{n'} \rangle$$

$$\langle v_n(k) \rangle = \frac{\partial \epsilon_n(k)}{\hbar \partial k} - i \left[\langle \frac{\partial \psi_n}{\partial k} | \frac{\partial \psi_n}{\partial t} \rangle - \langle \frac{\partial \psi_n}{\partial t} | \frac{\partial \psi_n}{\partial k} \rangle \right] \Rightarrow \langle v_n(k) \rangle = \frac{\partial \epsilon_n}{\hbar \partial k} - e \vec{E} \times \vec{\Omega}(\vec{q})$$

$$\vec{k} = e \vec{E} \quad \left| \frac{\partial \psi_n}{\partial t} \right\rangle = \left| \frac{\partial \psi_n}{\partial k} \right\rangle \cdot e \vec{E}$$

Localized

$$\uparrow \text{Gex G...} \quad \langle \psi_n | \frac{\partial H}{\partial k} | \psi_n \rangle = \langle \psi_n | \frac{\partial}{\partial k} H - H \frac{\partial}{\partial k} | \psi_n \rangle = (\epsilon_n - \epsilon_n) \langle \frac{\partial \psi_n}{\partial k} | \psi_n \rangle$$

$$\langle v_n(k) \rangle = \frac{\partial \epsilon_n(k)}{\hbar \partial k} - i \left[\langle \frac{\partial \psi_n}{\partial k} | \frac{\partial \psi_n}{\partial t} \rangle - \langle \frac{\partial \psi_n}{\partial t} | \frac{\partial \psi_n}{\partial k} \rangle \right] \Rightarrow \langle v_n(k) \rangle = \frac{\partial \epsilon_n}{\hbar \partial k} - e \vec{E} \times \vec{\Omega}(k)$$

$$\vec{k} = e \vec{E} \quad \left| \frac{\partial \psi_n}{\partial t} \right\rangle = \left| \frac{\partial \psi_n}{\partial k} \right\rangle \cdot e \vec{E} \quad \vec{\Omega}(k) = i \left(\langle \frac{\partial \psi_n}{\partial k_x} | \frac{\partial \psi_n}{\partial k_j} \rangle - \langle \frac{\partial \psi_n}{\partial k_j} | \frac{\partial \psi_n}{\partial k_x} \rangle \right)$$