

Title: 12/13 PSI - Condensed Matter Review Lecture 13

Date: Feb 13, 2013 11:30 AM

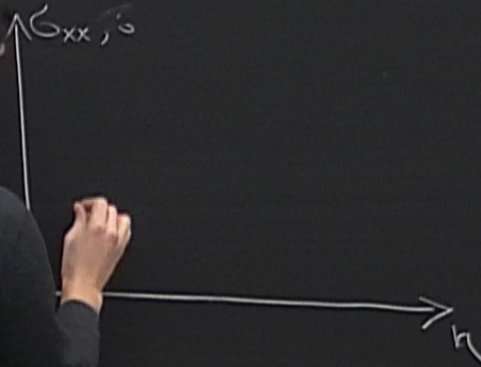
URL: <http://pirsa.org/13020086>

Abstract:

Quantum Hall Effect

Quantum Hall Effect

- * 2D electron gas
- * Disorder
- * Strong B-field

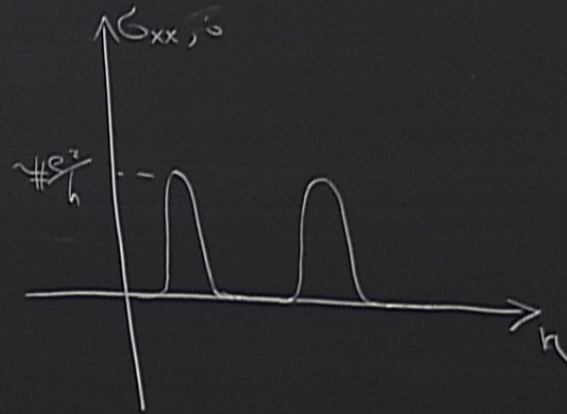


Quantum Hall Effect

* 2D electron gas

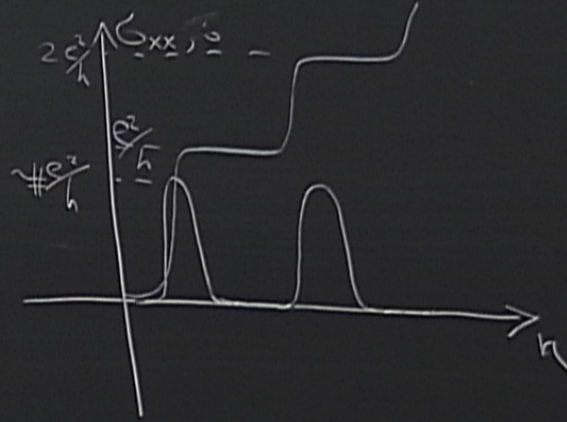
* Disordered

* Hall



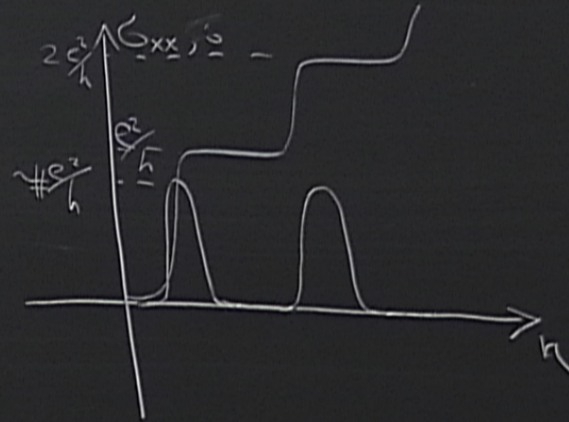
Quantum Hall Effect

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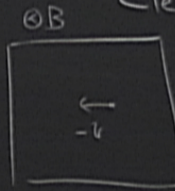
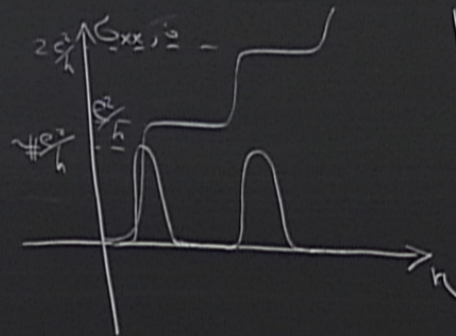
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Quantum Hall Effect

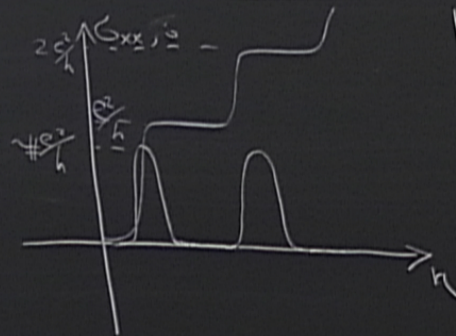
* 2D electron gas
Disorder
Strong B-field



Why is disorder important?
Clean 2DEG ; Frame moving with some velocity $\vec{v}$
 $\vec{j} = -n e \vec{v}$ - current density

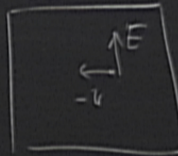
Quantum Hall Effect

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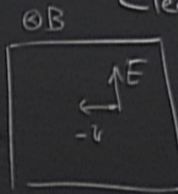
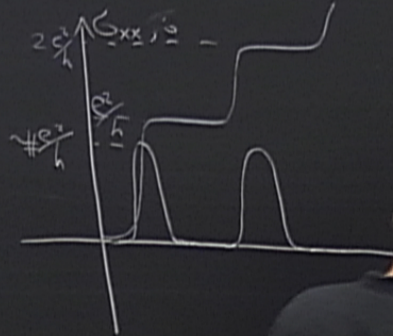


$$\vec{j} = -ne\vec{v} \text{ - current density}$$

Lorentz transformation $v \ll c$ $\vec{E} = -\frac{1}{c}\vec{v} \times \vec{B}$

Quantum Hall Effect

- * 2D electron gas
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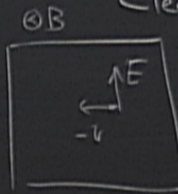
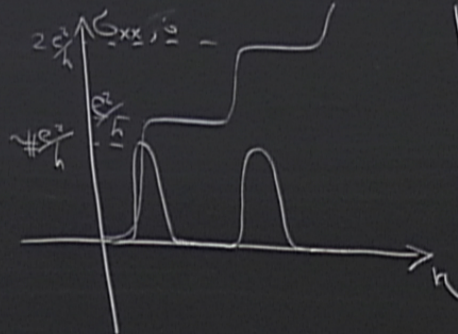
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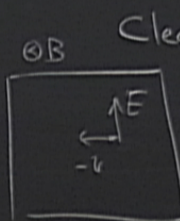
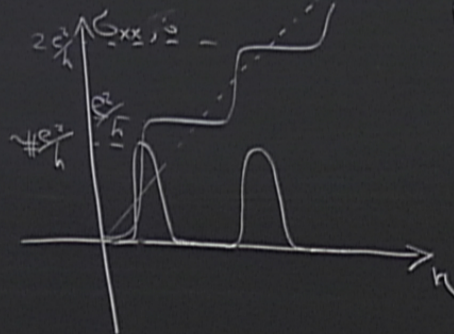
$$\vec{E} = \frac{B}{nc} \vec{j} \times \hat{z}$$

$$\vec{E} = -\frac{1}{c} \vec{v} \times \vec{B}$$

$$\vec{P} = \begin{bmatrix} 0 & \frac{B}{nc} \\ -\frac{B}{nc} & 0 \end{bmatrix}$$

Quantum Hall Effect

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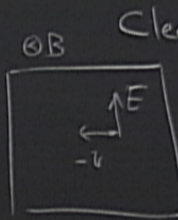
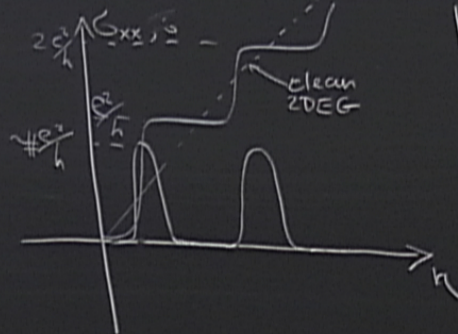
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Quantum dynamics in strong B-field

Quantum dynamics in strong B-field
Neglect disorder, find spectrum ; $A_y = Bx$, $A_z = 0$ Landau gauge
 $\nabla \times \mathbf{A} = B \hat{z}$

Quantum dynamics in strong B-field

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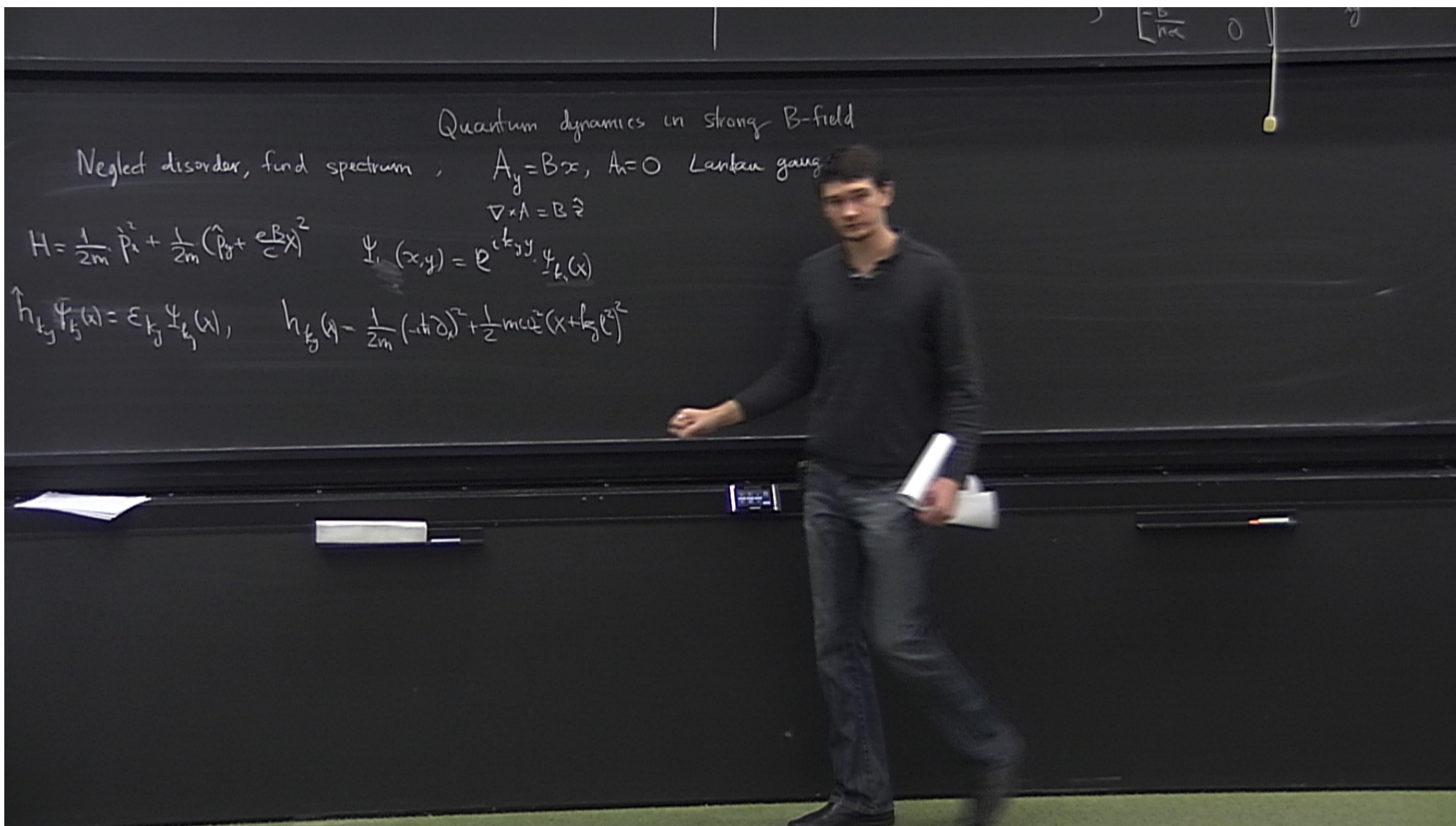
$$H = \frac{1}{2m} \hat{p}_z^2 + \frac{1}{2m} \left(\hat{p}_y + \frac{eBx}{c} \right)^2$$

Quantum dynamics in strong B-field

Neglect disorder, find spectrum ; $A_y = Bx$, $A_x = 0$ Landau gauge

$$\nabla \times \mathbf{A} = B \hat{z}$$

$$H = \frac{1}{2m} \hat{p}_x^2 + \frac{1}{2m} \left(\hat{p}_y + \frac{eBx}{c} \right)^2 \quad \Psi_i(x, y) = e^{ik_y y} \psi_{k_y}(x)$$



Neglect disorder, find spectrum, $A_y = Bx$, $A_x = 0$ Landau gauge

$$\nabla \times \mathbf{A} = B \hat{z}$$

$$H = \frac{1}{2m} \hat{p}_x^2 + \frac{1}{2m} \left(\hat{p}_y + \frac{eBx}{c} \right)^2 \quad \Psi(x, y) = e^{ik_y y} \psi_k(x)$$

$$\hat{H}_{k_y} \psi_k(x) = \epsilon_{k_y} \psi_k(x), \quad \hat{H}_{k_y} = \frac{1}{2m} (-i\hbar \partial_x)^2 + \frac{1}{2} m \omega_c^2 \left(x + \frac{k_y \hbar c}{eB} \right)^2,$$

$$\omega_c = \frac{eB}{mc}, \quad l = \sqrt{\frac{\hbar c}{eB}}$$

Neglect disorder, find spectrum, $A_y = Bx$, $A_x = 0$ Landau gauge

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$$\psi_k(x) = \langle x | \psi_k \rangle, \quad h_k(x) = \frac{1}{2m} (-i\hbar \partial_x)^2 + \frac{1}{2} m \omega_c^2 \left(x + \frac{\hbar k_y}{eB} \right)^2,$$

$$\omega_c = \frac{eB}{m c}, \quad l = \sqrt{\frac{\hbar c}{eB}}, \quad X = -\frac{\hbar k_y}{eB}$$

Quantum dynamics in strong B-field

Neglect disorder, find spectrum ; $A_y = Bx$, $A_x = 0$ Landau gauge

$$H = \frac{1}{2m} \hat{p}_x^2 + \frac{1}{2m} \left(\hat{p}_y + \frac{eBx}{c} \right)^2$$

$$\Psi(x, y)$$

$$\nabla \cdot A = B \hat{z}$$

$$\epsilon_{k_y, n} = (n + \frac{1}{2}) \hbar \omega_c$$

$$\Psi_{k_y, n}(x) = H_n(x - x_c) e^{-\frac{1}{2} (x - x_c)^2}$$

$$\hat{H}_{k_y} \Psi_k(x) = \epsilon_{k_y} \Psi_k(x)$$

$$h_{k_y} = \frac{1}{2}$$

$$L m \omega_c^2 (x + \frac{c}{B} k_y)^2$$

$$x_c = -\frac{c}{B} k_y \text{ - guiding center position}$$

Quantum dynamics in strong B-field

Neglect disorder, find spectrum ; $A_y = Bx$, $A_x = 0$ Landau gauge

$$H = \frac{1}{2m} \hat{p}_x^2 + \frac{1}{2m} \left(\hat{p}_y + \frac{eB}{c}x \right)^2$$

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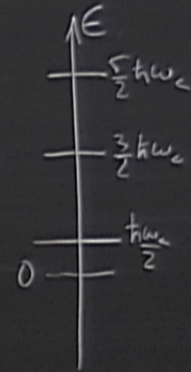
$$\Psi_i(x, y) = e^{ik_y y} \psi_{k_y}(x)$$

$$\epsilon_{k_y, n} = (n + \frac{1}{2}) \hbar \omega_c$$

$$\psi_{k_y, n}(x) = H_n(x-X) e^{-\frac{1}{2l^2}(x-X)^2}$$

$$\hat{h}_{k_y} \psi_{k_y}(x) = \epsilon_{k_y} \psi_{k_y}(x), \quad h_{k_y} = \frac{1}{2m} (-i\hbar \partial_x)^2 + \frac{1}{2} m \omega_c^2 (x - \frac{\hbar k_y}{eB})^2$$

$$\omega_c = \frac{eB}{mc}, \quad l = \sqrt{\frac{\hbar c}{eB}}, \quad X = -\frac{\hbar k_y}{eB} \text{ - guiding center position}$$



Quantum dynamics in strong B-field

ed disorder, find spectrum, $A_y = Bx$, $A_x = 0$ Landau gauge

$$\hat{p}_x^2 + \frac{1}{2m} (\hat{p}_y + \frac{eB}{c}x)^2 \quad \nabla \times A = B \hat{z}$$

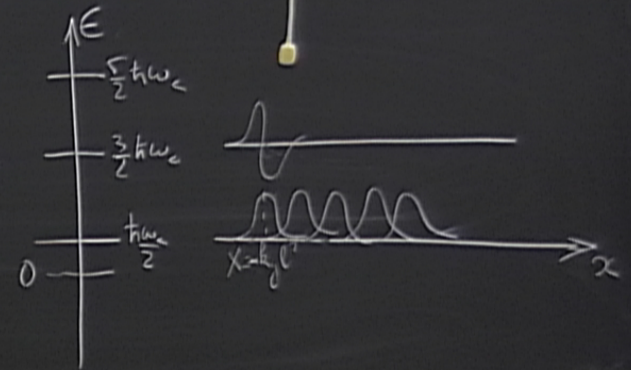
$$\Psi_1(x, y) = e^{ik_y y} \psi_{-k_y}(x)$$

$$\epsilon_{k_y, n} = (n + \frac{1}{2}) \hbar \omega_c$$

$$\psi_{k_y, n}(x) = H_n(x - X) e^{-\frac{1}{2l^2}(x - X)^2}$$

$$\epsilon_{k_y} \psi_{k_y}(x), \quad h_{k_y} = \frac{1}{2m} (-i\hbar \partial_x)^2 + \frac{1}{2} m \omega_c^2 (x + \frac{k_y l^2}{2})^2$$

$$\omega_c = \frac{eB}{mc}, \quad l = \sqrt{\frac{\hbar c}{eB}}, \quad X = -k_y l^2 - \text{guiding center position}$$



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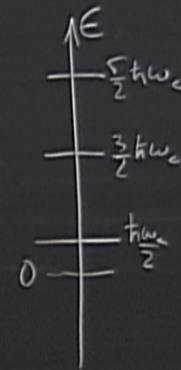
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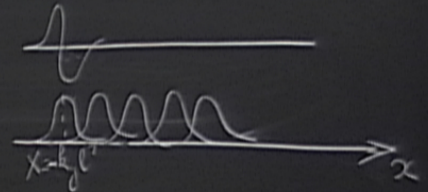
$$\omega_c = \frac{eB}{mc}, \quad \ell = \sqrt{\frac{\hbar c}{eB}}, \quad X = -k_y \ell^2 - \text{guiding center position}$$

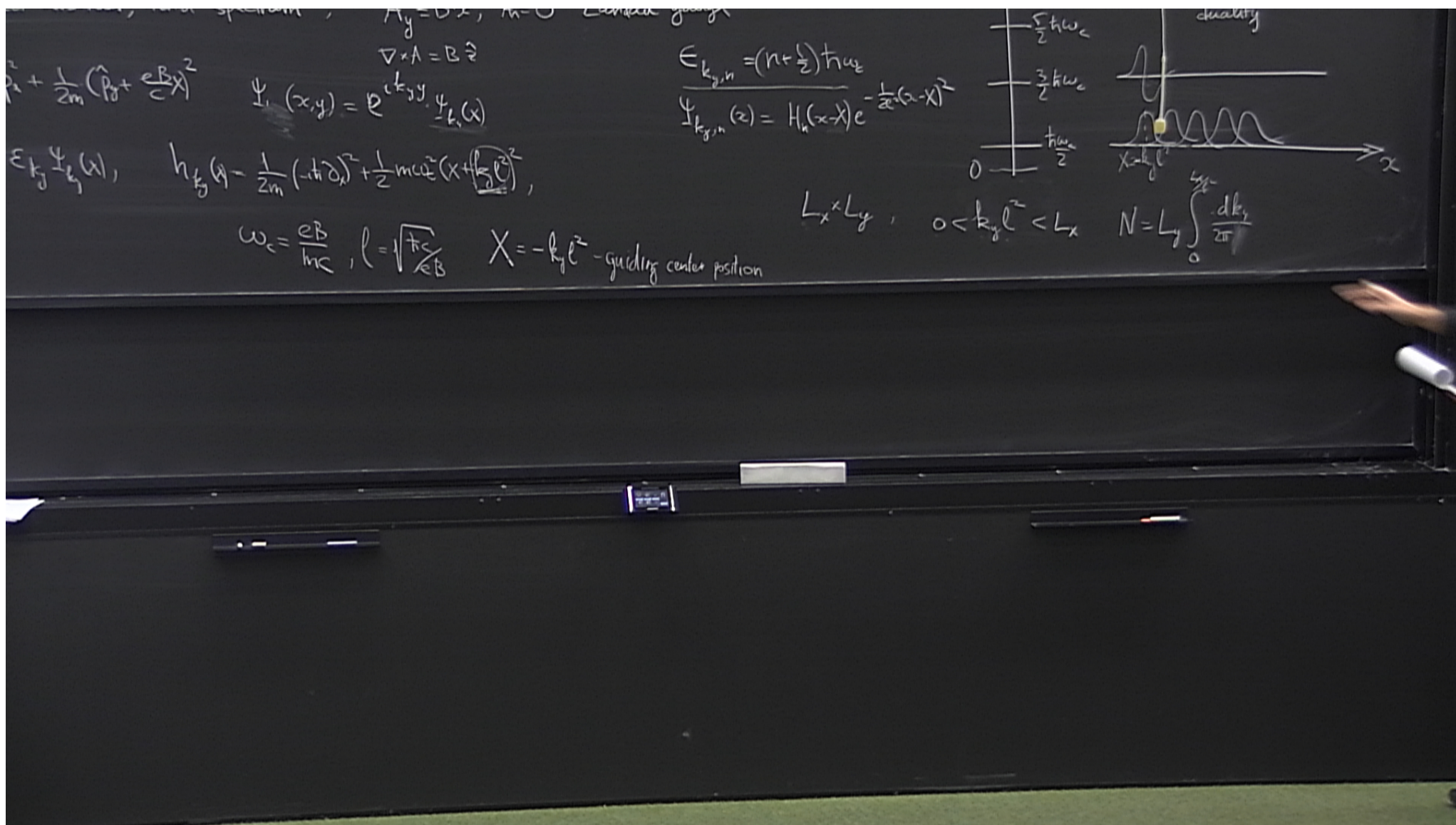
$$\epsilon_{k_y, n} = (n + \frac{1}{2}) \hbar \omega_c$$

$$\psi_{k_y, n}(x) = H_n(x - X) e^{-\frac{1}{2\ell^2}(x - X)^2}$$



momentum-real space duality





$$\hat{p}_x + \frac{1}{2m} (\hat{p}_y + \frac{eBx}{c})^2$$

$$\nabla \times \mathbf{A} = B \hat{z}$$

$$\Psi_1(x, y) = e^{ik_y y} \psi_L(x)$$

$$E_{k_y} \psi_L(x), \quad h_{k_y}^2 = \frac{1}{2m} (-i\hbar \partial_x)^2 + \frac{1}{2} m \omega_c^2 (x + \frac{1}{2} \ell^2)^2$$

$$\omega_c = \frac{eB}{mc}, \quad \ell = \sqrt{\frac{\hbar c}{eB}}$$

$$X = -k_y \ell^2 \text{ - guiding center position}$$

$$E_{k_y, n} = (n + \frac{1}{2}) \hbar \omega_c$$

$$\psi_{k_y, n}(z) = H_n(x - X) e^{-\frac{1}{2\ell^2}(x - X)^2}$$

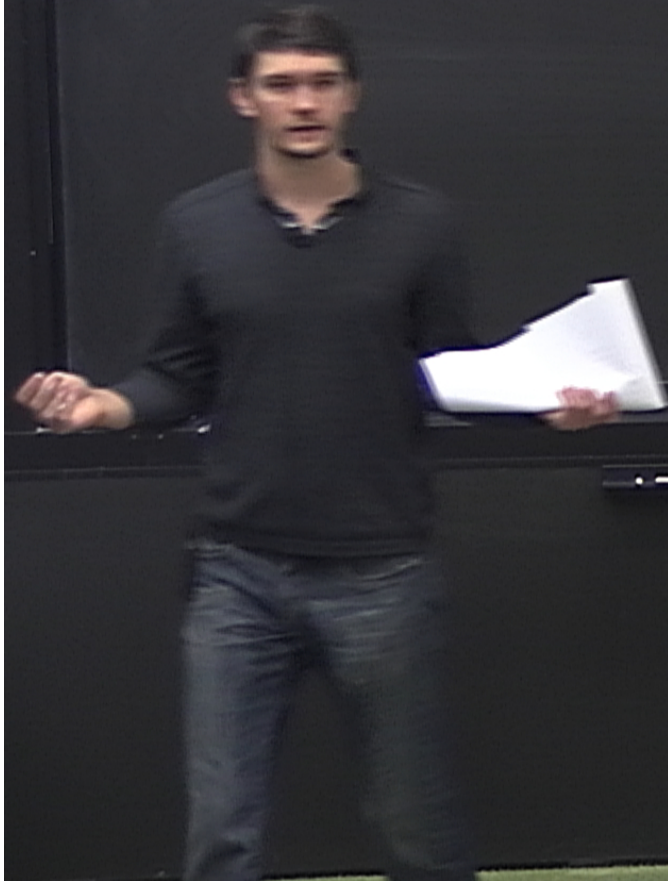
$$L_x \times L_y, \quad 0 < k_y \ell^2 < L_x$$

$$N = L_y \int_0^{L_x} \frac{dk_y}{2\pi} = \frac{L_x L_y}{2\pi \ell^2}$$

energy

$$\omega_c = \frac{eB}{\hbar c}, \quad l = \sqrt{\frac{\hbar c}{eB}}, \quad X = -k_y l^2 - \text{guiding center position}$$

$$\uparrow \text{state per area } 2\pi l^2 = 2\pi \frac{\hbar c}{eB} = \frac{\Phi_0}{B}, \quad \Phi_0 = \frac{h c}{e}$$



$$\omega_c = \frac{eB}{\hbar c}, \quad l = \sqrt{\frac{\hbar c}{eB}} \quad X = -k_y l^2 - \text{guiding center position}$$

$$\uparrow \text{state per area} \quad 2\pi l^2 = 2\pi \frac{\hbar c}{eB} = \frac{\Phi_0}{B}, \quad \Phi_0 = \frac{hc}{e}$$

Transport

Flat bands $\Rightarrow$ group velocity $\equiv 0$

$$\omega_c = \frac{eB}{\hbar c}, \quad l = \sqrt{\frac{\hbar c}{eB}} \quad X = -k_y l^2 - \text{guiding center position}$$

↑ state per area $2\pi l^2 = 2\pi \frac{\hbar c}{eB} = \frac{\Phi_0}{B}$, $\Phi_0 = \frac{h c}{e}$

Transport

Flat bands $\Rightarrow$ group velocity $\equiv 0$, Let's consider $k=0$ LL

$$\Psi_{k_y}(x, y) = \frac{1}{\sqrt{2\pi l^2 L_y l}}$$

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† state per area $2\pi l^2 = 2\pi \frac{\hbar c}{eB} = \frac{\Phi_0}{B}$, $\Phi_0 = \frac{h c}{e}$

Transport

Flat bands $\Rightarrow$ group velocity $\equiv 0$, Let's consider $\hbar=0$ LL

$$\Psi_{k_y}(x, y) = \frac{1}{\sqrt{2\pi} l_y l} e^{i k_y y} e^{-\frac{1}{2l^2}(x-X)^2}, \quad \vec{j} = e \vec{v} = \frac{e}{m} (\vec{p} + \frac{e}{c} \vec{A})$$

1 state per area $2\pi l^2 = 2\pi \frac{\hbar c}{eB} = \frac{\Phi_0}{B}$, $\Phi_0 = \frac{hc}{e}$
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$$\langle \Psi_{k_y} | j_y | \Psi_{k_y} \rangle = -\frac{e\hbar}{m k_y l^2} \int dx e^{-\frac{1}{2l^2}(x-X)^2} \left(\frac{\hbar k_y + \frac{eB}{c}x}{\frac{eB}{c}(x-X)} \right) e^{-\frac{1}{2l^2}(x-X)^2} = 0$$

Current density
asymmetric

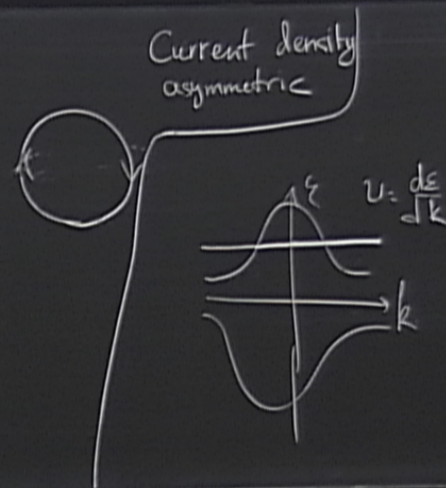


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$$H = \frac{1}{2m} \hat{p}_x^2 + \frac{1}{2m} \left(\hat{p}_y + \frac{eB}{c} x \right)^2$$

$$\nabla \times \mathbf{A} = B \hat{z}$$

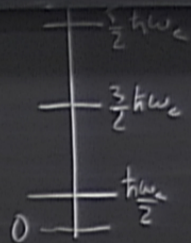
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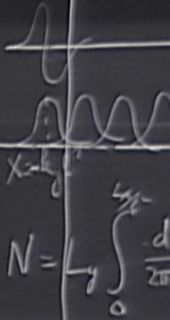
$$\omega_c = \frac{eB}{mc}, \quad l = \sqrt{\frac{\hbar c}{eB}}, \quad X = -k_y l^2 - \text{guiding center position}$$

$$\epsilon_{k_y, n} = (n + \frac{1}{2}) \hbar \omega_c$$

$$\psi_{k_y, n}(z) = H_n(x-X) e^{-\frac{1}{2l^2}(x-X)^2}$$



$$L_x \times L_y, \quad 0 < k_y l^2 < L_x$$



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$$\Psi_{k_y}(x, y) = \frac{1}{\sqrt{2\pi l^2} L_y} e^{ik_y y} e^{-\frac{1}{2l^2}(x-X)^2}, \quad \vec{j} = e \vec{v} = \frac{e}{m} \left(\vec{p} + \frac{e}{c} \vec{A} \right)$$

$$\langle \Psi_{k_y} | j_y | \Psi_{k_y} \rangle = -\frac{e}{m k_y l^2} \int dx e^{-\frac{1}{2l^2}(x-X)^2} \left(\frac{\hbar k_y + \frac{eB}{c} x}{\frac{eB}{c} (x-X)} \right) e^{-\frac{1}{2l^2}(x-X)^2} = 0$$

$$V(x) = eEx$$

Disorder

$$H = \frac{1}{2m} \hat{p}_x^2 + \frac{1}{2m} \left(\hat{p}_y + \frac{eB}{c} x \right)^2$$

$$\nabla \times \mathbf{A} = B \hat{z}$$

$$\Psi_1(x, y) = e^{ik_y y} \psi_{-k_y}(x)$$

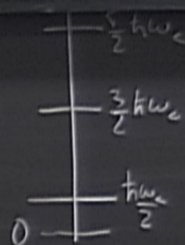
$$\hat{h}_{k_y} \psi_{k_y}(x) = \epsilon_{k_y} \psi_{k_y}(x), \quad h_{k_y}(x) = \frac{1}{2m} (-i\hbar \partial_x)^2 + \frac{1}{2} m \omega_c^2 \left(x + \frac{\hbar k_y}{eB} \right)^2,$$

$$\omega_c = \frac{eB}{\hbar m}, \quad l = \sqrt{\frac{\hbar c}{eB}}$$

$X = -k_y l^2$ - guiding center position

$$\epsilon_{k_y, n} = (n + \frac{1}{2}) \hbar \omega_c$$

$$\psi_{k_y, n}(z) = H_n(x-X) e^{-\frac{1}{2l^2}(x-X)^2}$$



$$L_x \times L_y, \quad 0 < k_y l^2 < L_x$$

$$N = L_y \int_0^{L_x} \frac{dx}{2\pi l^2}$$

1 state per area $2\pi l^2 = 2\pi \frac{\hbar c}{eB} = \frac{\Phi_0}{B}$, $\Phi_0 = \frac{h c}{e}$

Transport

Flat bands $\Rightarrow$ group velocity $\equiv 0$, Let's consider $\hbar=0$ LL

$$\Psi_{k_y}(x, y) = \frac{1}{\sqrt{2\pi l^2} L_y} e^{ik_y y} e^{-\frac{1}{2l^2}(x-X)^2}, \quad \vec{j} = e \vec{v} = \frac{e}{m} (\hat{p} + \frac{e}{c} \vec{A})$$

$$\langle \Psi_{k_y} | j_y | \Psi_{k_y} \rangle = -\frac{e c}{m \hbar k_y l^2} \int dx e^{-\frac{1}{2l^2}(x-X)^2} \left(\frac{\hbar k_y + \frac{eB}{c} x}{\frac{eB}{c} (x-X)} \right) e^{-\frac{1}{2l^2}(x-X)^2} = 0$$

Disorder

$$V(x) = eEx$$

$$h_{k_y}^{\text{dis}} = \frac{1}{2m} \hat{p}_x^2 + \frac{m \omega_c^2 (x-X)^2}{2} + eEx$$

$$l = \sqrt{\frac{\hbar c}{eB}}$$

$$X = -k_y l^2 \text{ - guiding center position}$$

$$L_x \times L_y, \quad 0 < k_y l^2 < L_x$$

$$N = L_y \int_0^{L_x} \frac{dk_y}{2\pi} = \frac{L_x L_y}{2\pi l^2}$$

$$\Phi_0 = \frac{h c}{e}$$

Transport

Let's consider $\hbar=0$ LL

$$\vec{j} = e \vec{v} = \frac{e}{m} (\vec{p} + \frac{e}{c} \vec{A})$$

$$\frac{(1/\hbar k_y + \frac{eB}{c} x)}{\frac{eB}{c} (x-X)} e^{-\frac{1}{2l^2}(x-X)^2} = 0$$

Disorder

$$V(x) = eEx$$

$$\hbar k_y = \frac{1}{2m} \hat{p}_x^2 + \frac{m\omega_c^2 (x-X)^2}{2} + eEx$$

$$X' = X - \frac{eE}{m\omega_c^2}$$

$$E_{k_y} = \frac{1}{2} \hbar \omega_c + eEX' + \frac{1}{2} m \bar{v}^2, \quad \bar{v} = -\frac{eE}{m\omega_c}$$

$$V(x,y) \text{ Disorder scale } l_{dis} \gg l$$

$$\langle j_y \rangle \sim \int dx e^{-\frac{1}{2l^2}(x-X)^2} (x-X) e^{-\frac{1}{2l^2}(x-X)^2} (X - \frac{1}{2} \frac{eE}{m\omega_c^2})$$

$$V(x,y) \approx V(x_0,y) + \nabla V(x_0,y) \cdot (x-x_0, y-y_0)$$

$$l = \sqrt{\frac{\hbar c}{eB}} \quad X = -k_y l^2 - \text{guiding center position} \quad L_x \times L_y, \quad 0 < k_y l^2 < L_x \quad N = L_y \int_0^{L_x} \frac{dk_y}{2\pi} = \frac{L_x L_y}{2\pi l^2}$$

$$\Phi_0 = \frac{h c}{e} \quad \text{Transport}$$

Let's consider $\hbar=0$ LL

$$\vec{j} = e \vec{v} = \frac{e}{m} (\vec{p} + \frac{e}{c} \vec{A})$$

$$\frac{(1/\hbar k_y + \frac{eB}{c} x)}{\frac{eB}{c} (x-X)} e^{-\frac{1}{2l^2}(x-X)^2} = 0$$

$$V(x) = eEx \quad \text{Disorder} \quad \hbar k_y = \frac{1}{2m} \hat{p}_x^2 + \frac{m\omega_c^2 (x-X)^2}{2} + eEx$$

$$X' = X - \frac{eE}{m\omega_c^2} \quad E_{k_y} = \frac{1}{2} \hbar \omega_c + eEX' + \frac{1}{2} m \bar{v}^2, \quad \bar{v} = -c \frac{E}{B}$$

$$V(x, y) \quad \text{Disorder scale} \quad \ell_{dis} \gg \ell \quad \langle j_y \rangle \sim \int dx e^{-\frac{1}{2l^2}(x-X)^2} (x-X) e^{-\frac{1}{2l^2}(x-X)^2} (x-X) \frac{eE}{m\omega_c^2}$$

$$V(x, y) = V(x_0, y) + \nabla V(x_0) \cdot (x - x_0, y - y_0)$$

$$l = \sqrt{\frac{\hbar c}{eB}} \quad X = -k_y l^2 - \text{guiding center position}$$

$$L_x \times L_y, \quad 0 < k_y l^2 < L_x \quad N = L_y \int_0^{L_x} \frac{dk_y}{2\pi} = \frac{L_x L_y}{2\pi l^2}$$

Let's consider $n=0$ LL

$$\vec{j} = e\vec{v} = \frac{e}{m}(\vec{p} + \frac{e}{c}\vec{A})$$

$$\int_{-\infty}^{\infty} dx \left(\frac{\hbar k_y + \frac{eB}{c}x}{\frac{eB}{c}(x-X)} \right) e^{-\frac{1}{2l^2}(x-X)^2} = 0$$

$$X' = X - \frac{eE}{m\omega_c^2}$$

$$E_{k_y} = \frac{1}{2}\hbar\omega_c + eEX' + \frac{1}{2}m\bar{v}^2, \quad \bar{v} = -c\frac{E}{B}$$

$$\langle j_y \rangle \sim \int dx e^{-\frac{1}{2l^2}(x-X')^2} (x-X') e^{-\frac{1}{2l^2}(x-X')^2}$$

$$\langle j_y \rangle = \bar{v}e$$

$$(x-X') \frac{eE}{m\omega_c^2}$$

$$\Phi_0 = \frac{hc}{e}$$

Transport

Let's consider $\hbar=0$ LL

$$\vec{j} = e\vec{v} = \frac{e}{m}(\vec{p} + \frac{e}{c}\vec{A})$$

$$\frac{(1/\hbar k_y + \frac{eB}{c}x)}{\frac{eB}{c}(x-X)} e^{-\frac{1}{2l^2}(x-X)^2}$$

Disorder

$$V(x) = eEx$$

$$H_{ky} = \frac{1}{2m}\hat{p}_x^2 + \frac{m\omega_c^2(x-X)^2}{2} + eEx$$

$$X' = X - \frac{eE}{m\omega_c^2}$$

$$E_{ky} = \frac{1}{2}\hbar\omega_c + eEX' + \frac{1}{2}m\bar{v}^2, \quad \bar{v} = -c\frac{E}{B}$$

$$\langle j_y \rangle \sim \int dx e^{-\frac{1}{2l^2}(x-X)^2} (x-X) e^{-\frac{1}{2l^2}(x-X)^2} \frac{eE}{m\omega_c^2}$$

$$\langle j_y \rangle = \bar{v}e$$

Drift in crossed fields

$$\bar{c} \cdot (x \rightarrow X)$$

$$v_y = \frac{1}{\hbar} \frac{\partial E_{k_y}}{\partial k_y} = \frac{eE}{\hbar} \frac{\partial X_{k_y}}{\partial k_y} = \bar{v}$$

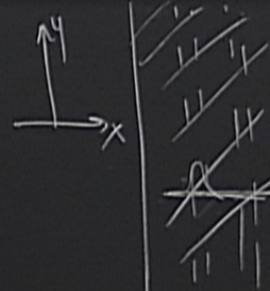
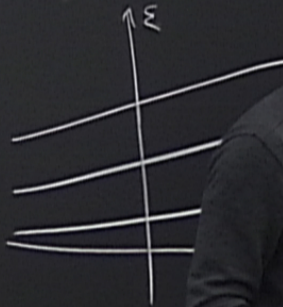
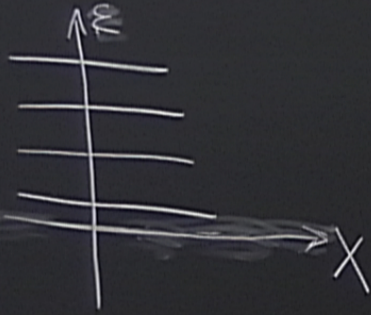


$$\langle \psi_{k_y} | j_y | \psi_{k_y} \rangle = -\frac{e}{m\sqrt{2\pi}l} \int dx e^{-\frac{1}{2l^2}(x-X)^2} \left(\frac{\hbar k_y + \frac{eB}{c}x}{\frac{eB}{c}(x-X)} \right) e^{-\frac{1}{2l^2}(x-X)^2} = 0$$

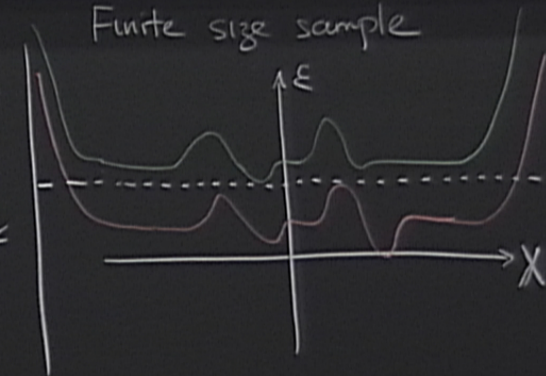
$$\langle j_y \rangle = \bar{v} e$$

Drift in crossed fields

$$v_y = \frac{1}{\hbar} \frac{\partial E_m}{\partial k_y} = \frac{eE}{\hbar} \frac{\partial X_{k_y}}{\partial k_y} = \bar{v}$$



Finite size sample

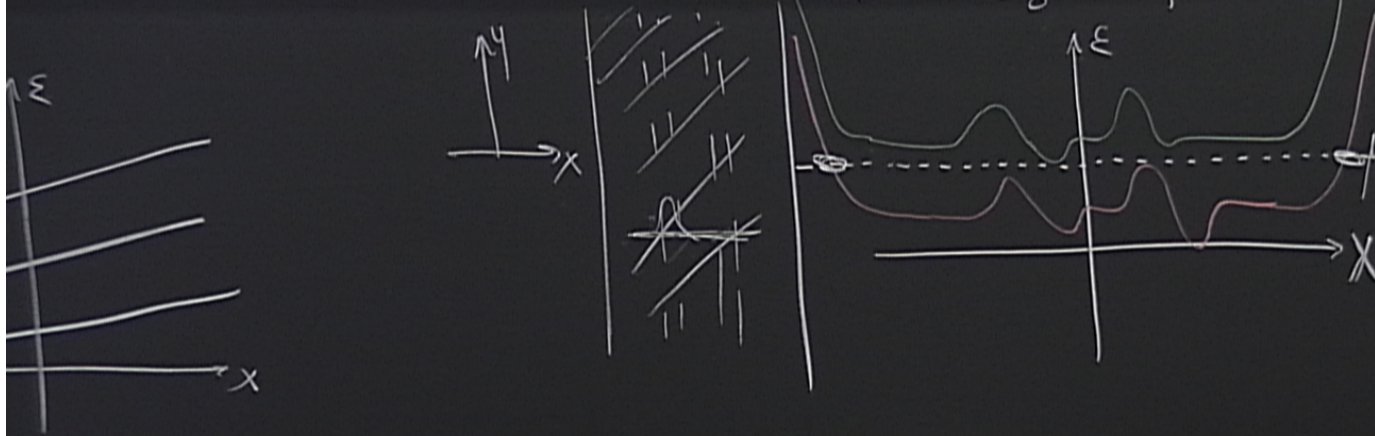


$$\frac{eB}{c} (x-X)$$

$\langle v_y \rangle = v_e$
Drift in crossed fields

$$(x-X) \frac{eE}{m\omega_c^2}$$

Finite size sample



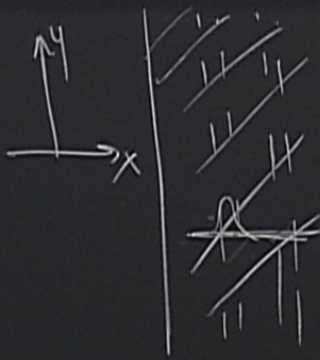
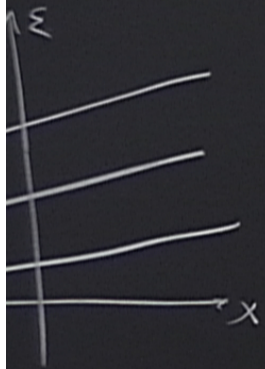
1) if $|\mu - \frac{\hbar\omega_c}{2}| \geq |V(x)|$

$$\frac{eB}{c} (x-X)$$

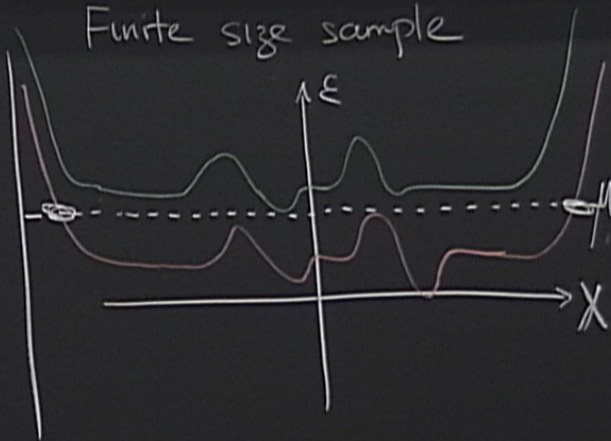
$$\langle v_y \rangle = v_e$$

Drift in crossed fields

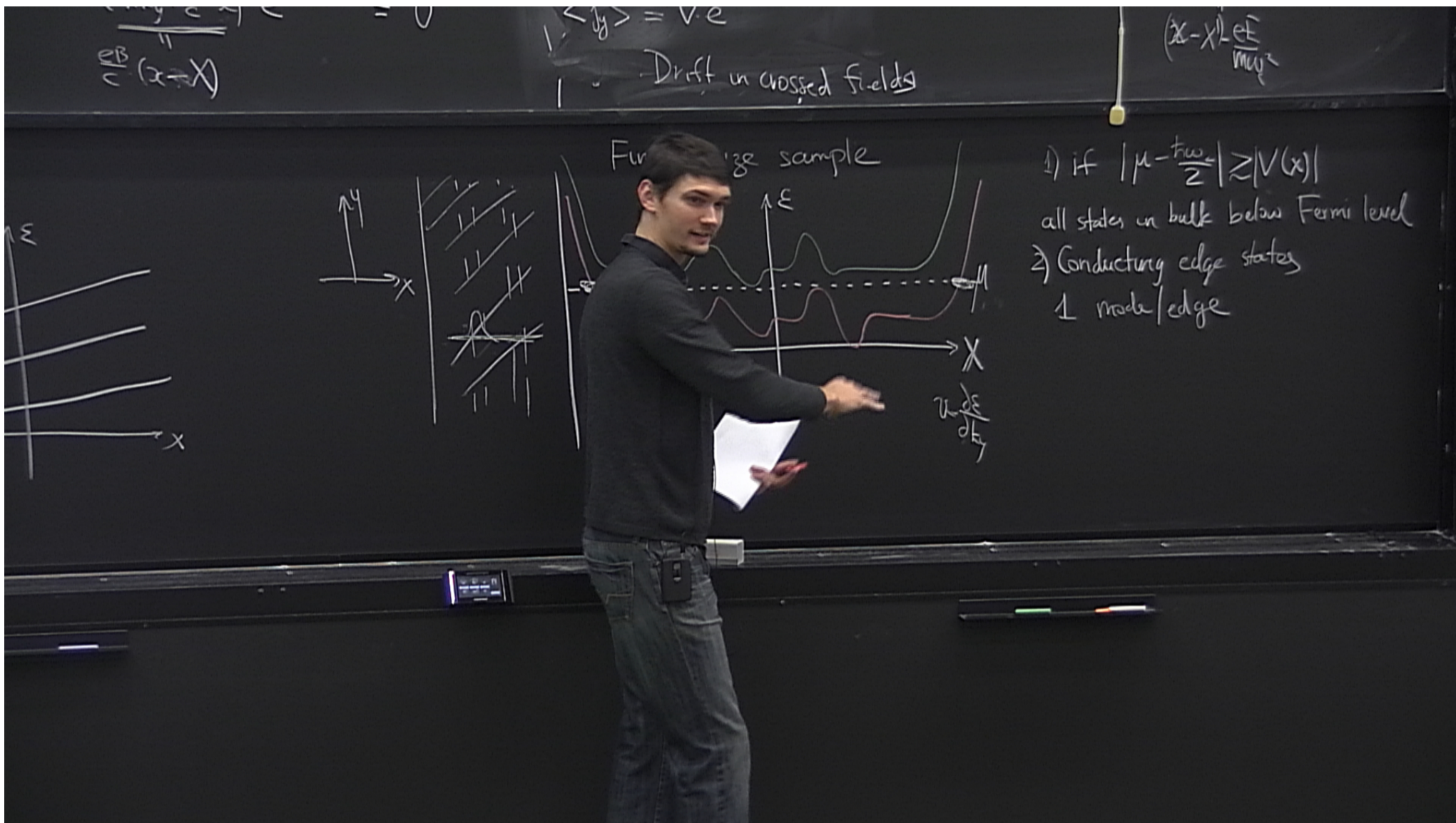
$$(x-X) \frac{eE}{m\omega_c^2}$$



Finite size sample



- 1) if $|\mu - \frac{\hbar\omega_c}{2}| \geq |V(x)|$
all states in bulk below Fermi level
- 2) Conducting edge states

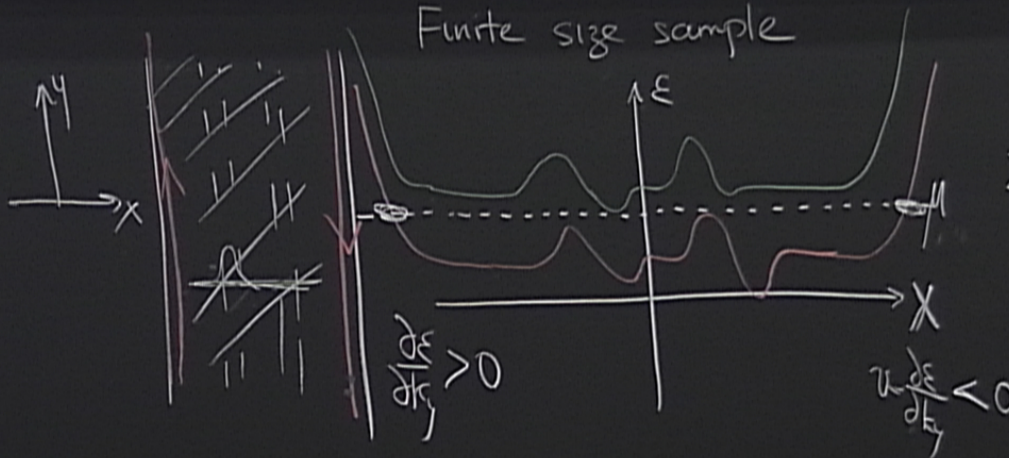


$$\frac{eB}{c} (x-X)$$

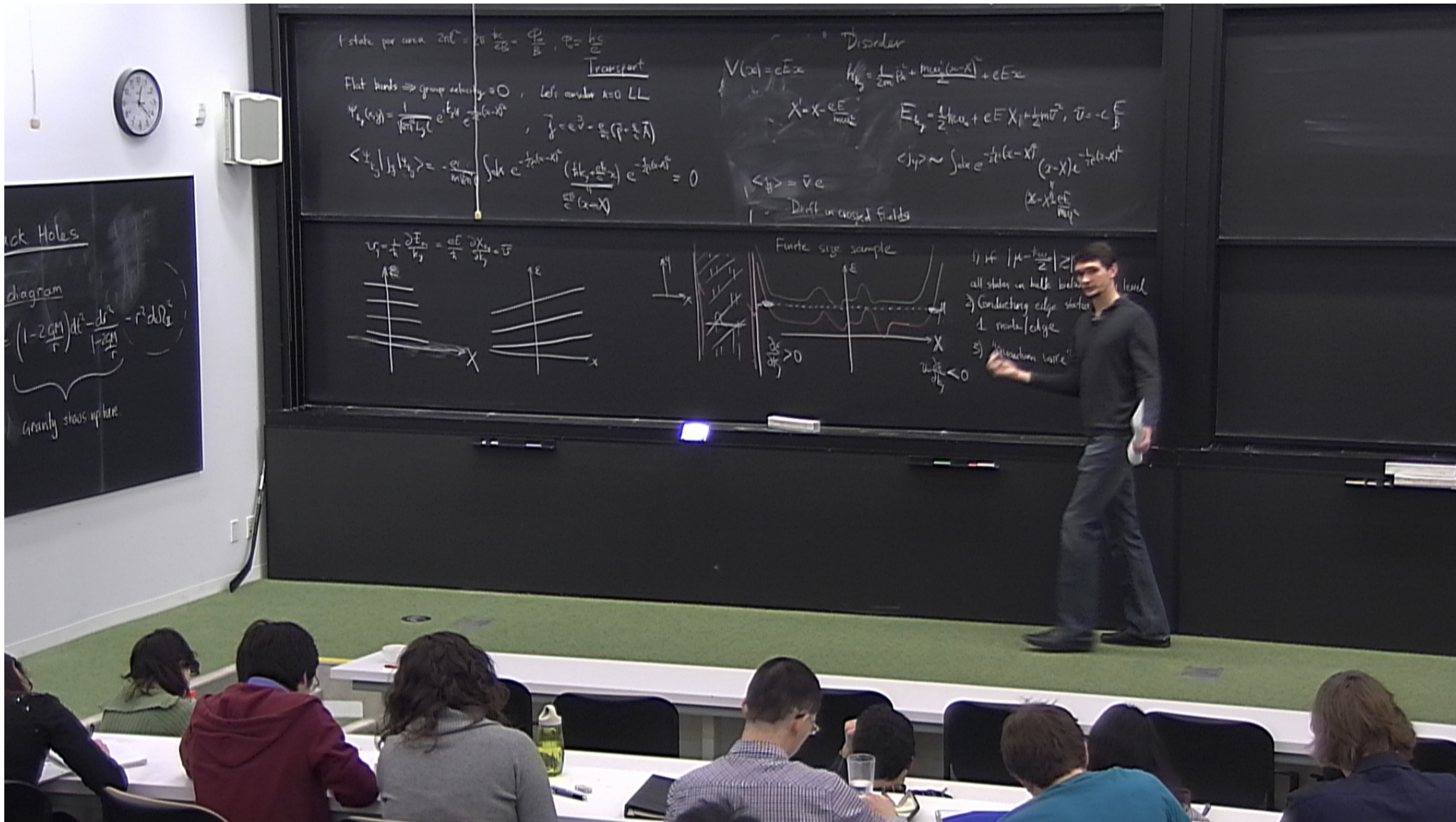
$\langle v_y \rangle = v_e$
Drift in crossed fields

$$(x-X) = \frac{e\hbar}{m\omega_c}$$

Finite size sample



- 1) if $|\mu - \frac{\hbar\omega_c}{2}| \geq |V(x)|$
all states in bulk below Fermi level
- 2) Conducting edge states
1 mode/edge



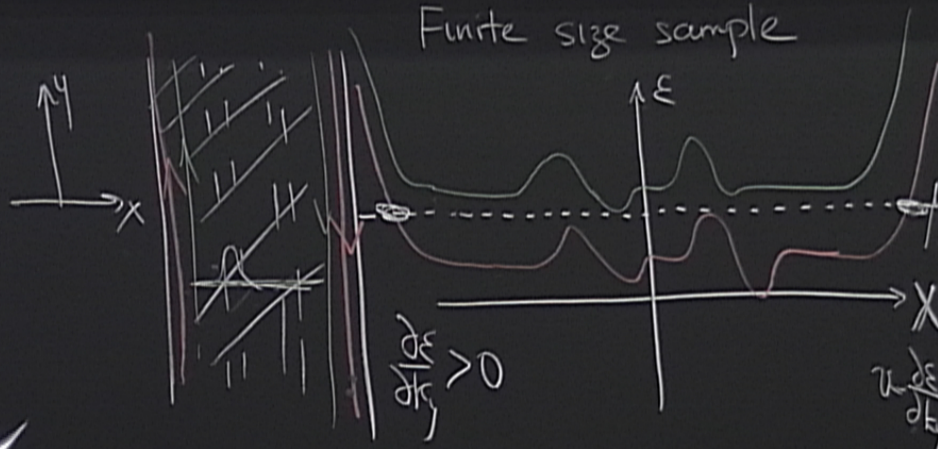
$$\frac{eB}{c} (x-X)$$

$$\langle v_y \rangle = v_e$$

Drift in crossed fields

$$(x-X) \frac{eE}{m\omega_c}$$

Finite size sample



- 1) if $|\mu - \frac{\hbar\omega_c}{2}| \geq |V(x)|$
all states in bulk below Fermi level
- 2) Conducting edge states
1 mode/edge
- 3) "Quantum wire"
counter-prop. modes spatially separated

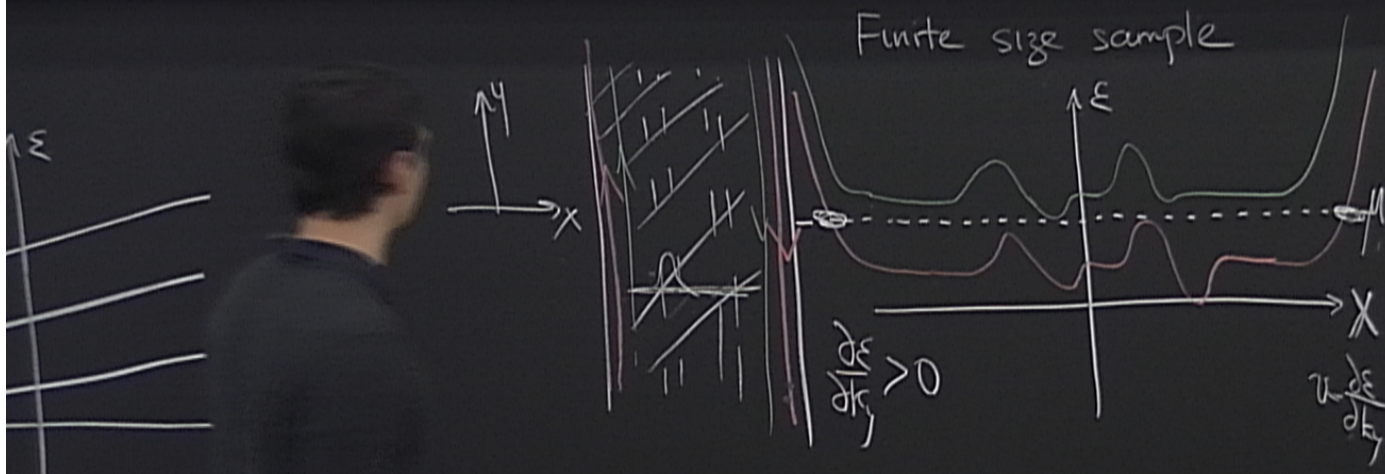
$$\frac{eB}{c} (x-X)$$

$$\langle v_y \rangle = v_e$$

Drift in crossed fields

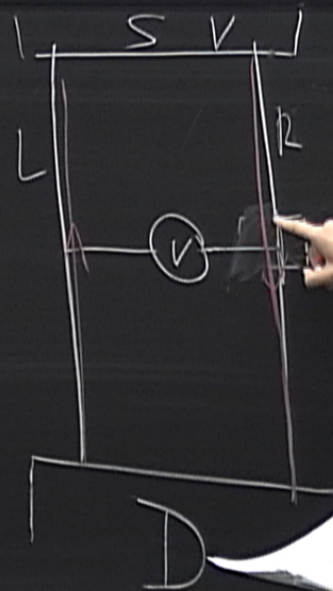
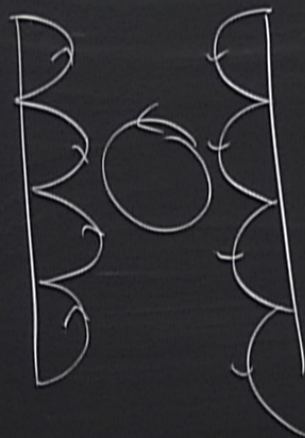
$$(x-X) \frac{eE}{m\omega_c}$$

Finite size sample

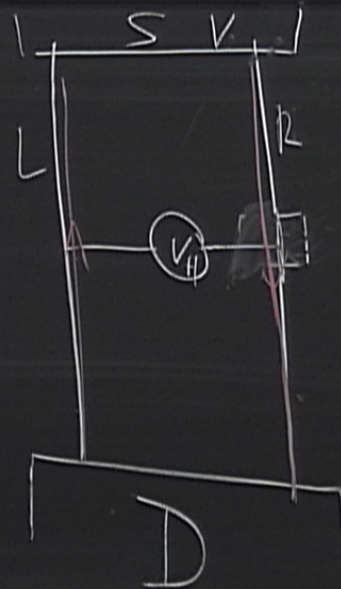
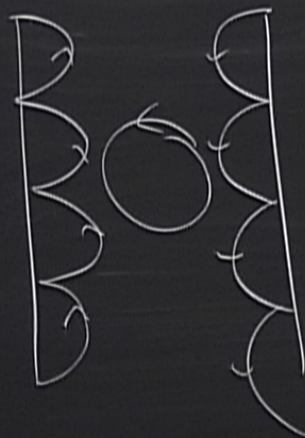


- 1) if $|\mu - \frac{\hbar\omega_c}{2}| \geq |V(x)|$
all states in bulk below Fermi level
- 2) Conducting edge states
1 mode/edge
- 3) "Quantum wire"
counter-prop. modes spatially separated

BO

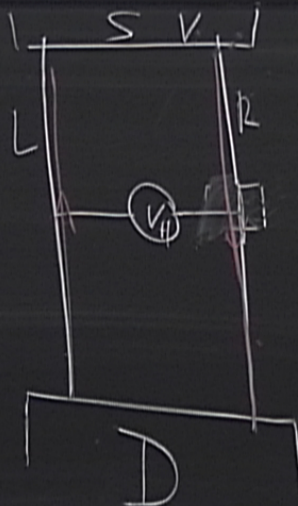
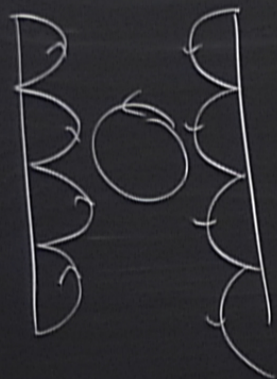


$$I = \int_{\mu_L}^{\mu_R} \frac{dk_y}{2\pi\hbar} \frac{\partial \varepsilon}{\partial k_y} = -\frac{e}{\hbar} (\mu_R - \mu_L)$$



$$I = -e \int_{\mu_L}^{\mu_R} \frac{dk_y}{2\pi\hbar} \frac{\partial \varepsilon}{\partial k_y} = -\frac{e}{h} (\mu_R - \mu_L) = -\frac{e}{h} V_H$$

$$V_H = \frac{h}{e^2} I$$



$$I = -e \int_{\mu_L}^{\mu_R} \frac{dk_y}{2\pi\hbar} \frac{\partial \varepsilon}{\partial k_y} = -\frac{e}{h} (\mu_R - \mu_L) = -\frac{e}{h} V$$

$$V_H = \frac{h}{e^2} I \quad \sigma_{xy} = \frac{e^2}{h}$$

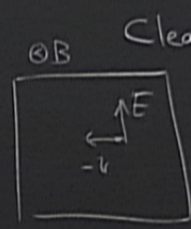
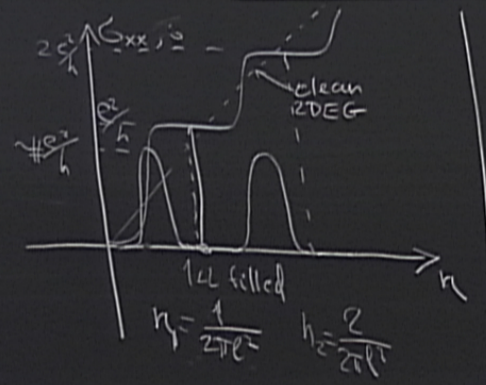
$$N \text{ channels: } \sigma_{xy} = N \frac{e^2}{h}$$

$$\langle \frac{1}{4} \rangle = -\frac{e}{\hbar} \int dx e^{-\frac{1}{2\ell^2}(z-X)^2} \left(\hbar k_x + \frac{eB}{2} x \right) e^{-\frac{1}{2\ell^2}(z-X)^2} = 0$$

$$\langle j_y \rangle \sim \int dx e^{-\frac{1}{2\ell^2}(z-X)^2} (z-X) e^{-\frac{1}{2\ell^2}(z-X)^2}$$

Quantum Hall Effect

electron gas
order
long eld



Why is disorder important?

Clean 2DEG ; Frame moving with some velocity $\vec{v}$

$\vec{j} = -ne\vec{v}$ - current density

Lorentz transformation $v \ll c$

$$\vec{E} = \frac{\Phi}{nc} \vec{j} \times \hat{z}$$

$$\vec{E} = -\frac{1}{c} \vec{v} \times \vec{B}$$

$$\vec{S} = \begin{bmatrix} 0 & \frac{B}{nc} \\ -\frac{B}{nc} & 0 \end{bmatrix}$$

$$\rightarrow G_{xy} \propto n$$

$$\langle \psi | \psi \rangle = -\frac{e}{\hbar} \int dx e^{-\frac{1}{2\ell^2}(x-X)^2} \left(\frac{\hbar k + eB}{2} x \right) e^{-\frac{1}{2\ell^2}(x-X)^2} = 0$$

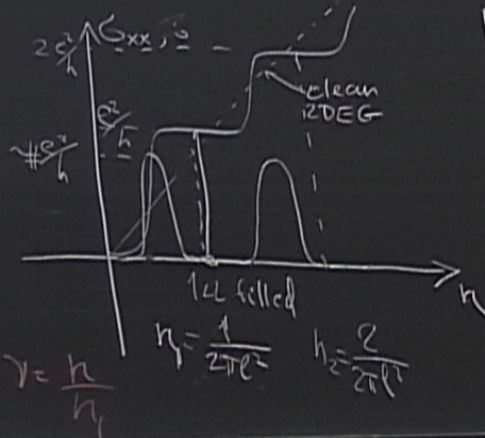
$$\langle J_y \rangle \sim \int dx e^{-\frac{1}{2\ell^2}(x-X)^2} (x-X) e^{-\frac{1}{2\ell^2}(x-X)^2}$$

Quantum Hall Effect

2D electron gas

Disorder

Strong B-field



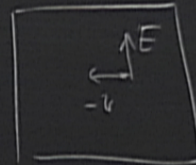
Filling factor $\nu = \frac{n}{n_f}$

$\otimes B$

Clean 2DEG

Why is disorder important?

Frame moving with some velocity $\vec{v}$



$$\vec{j} = -ne\vec{v} - \text{current density}$$

Lorentz transformation $v \ll c$

$$\vec{E} = \frac{B}{\hbar c} \vec{j} \times \hat{z}$$

$$\vec{E} = -\frac{1}{c} \vec{v} \times \vec{B}$$

$$\vec{E} = \begin{bmatrix} 0 & \frac{B}{\hbar c} \\ -\frac{B}{\hbar c} & 0 \end{bmatrix} \vec{j} \rightarrow G_{xy} \propto \nu$$

$$\langle \psi | \psi \rangle = -\frac{e}{\hbar} \int dx e^{-\frac{1}{2\ell^2}(x-X)^2} \left(\frac{\hbar k + eB}{2} x \right) e^{-\frac{1}{2\ell^2}(x-X)^2} = 0$$

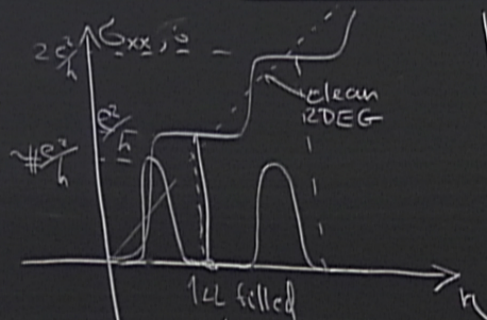
$$\langle J_y \rangle \sim \int dx e^{-\frac{1}{2\ell^2}(x-X)^2} (x-X) e^{-\frac{1}{2\ell^2}(x-X)^2}$$

Quantum Hall Effect

2D electron gas

Disorder

Strong B-field



Filling factor $\nu = \frac{n}{n_f}$

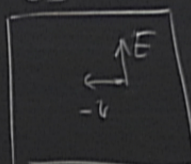
$$n_f = \frac{1}{2\pi\ell^2} \quad n = \frac{2}{2\pi\ell^2}$$

$\otimes B$

Clean 2DEG

Why is disorder important?

Frame moving with some velocity $\vec{v}$



$$\vec{j} = -ne\vec{v} \text{ - current density}$$

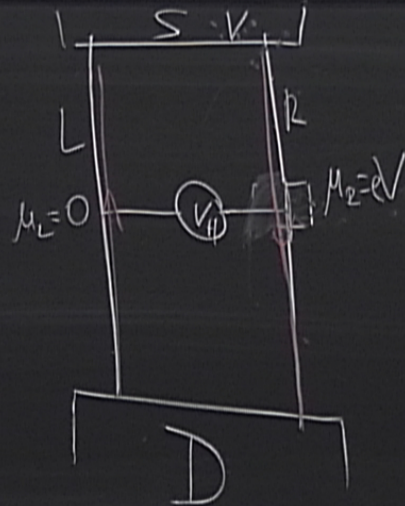
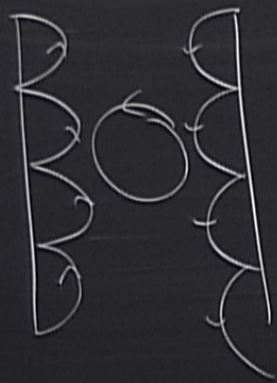
Lorentz transformation $v \ll c$

$$\vec{E} = -\frac{1}{c}\vec{v} \times \vec{B}$$

$$\vec{E} = \frac{\Phi}{nc} \vec{j} \times \hat{z}$$

$$\vec{F} = \begin{bmatrix} 0 & \frac{B}{nc} \\ -\frac{B}{nc} & 0 \end{bmatrix} \rightarrow G_{xy} \propto n$$





$$I = -e \int_{\mu_L}^{\mu_R} \frac{dk_y}{2\pi\hbar} \frac{\partial \varepsilon}{\partial k_y} = -\frac{e}{h} (\mu_R - \mu_L) = -\frac{e}{h} V$$

$$V_H = \frac{h}{e^2} I$$

N channels: $G_{xy} = N$

