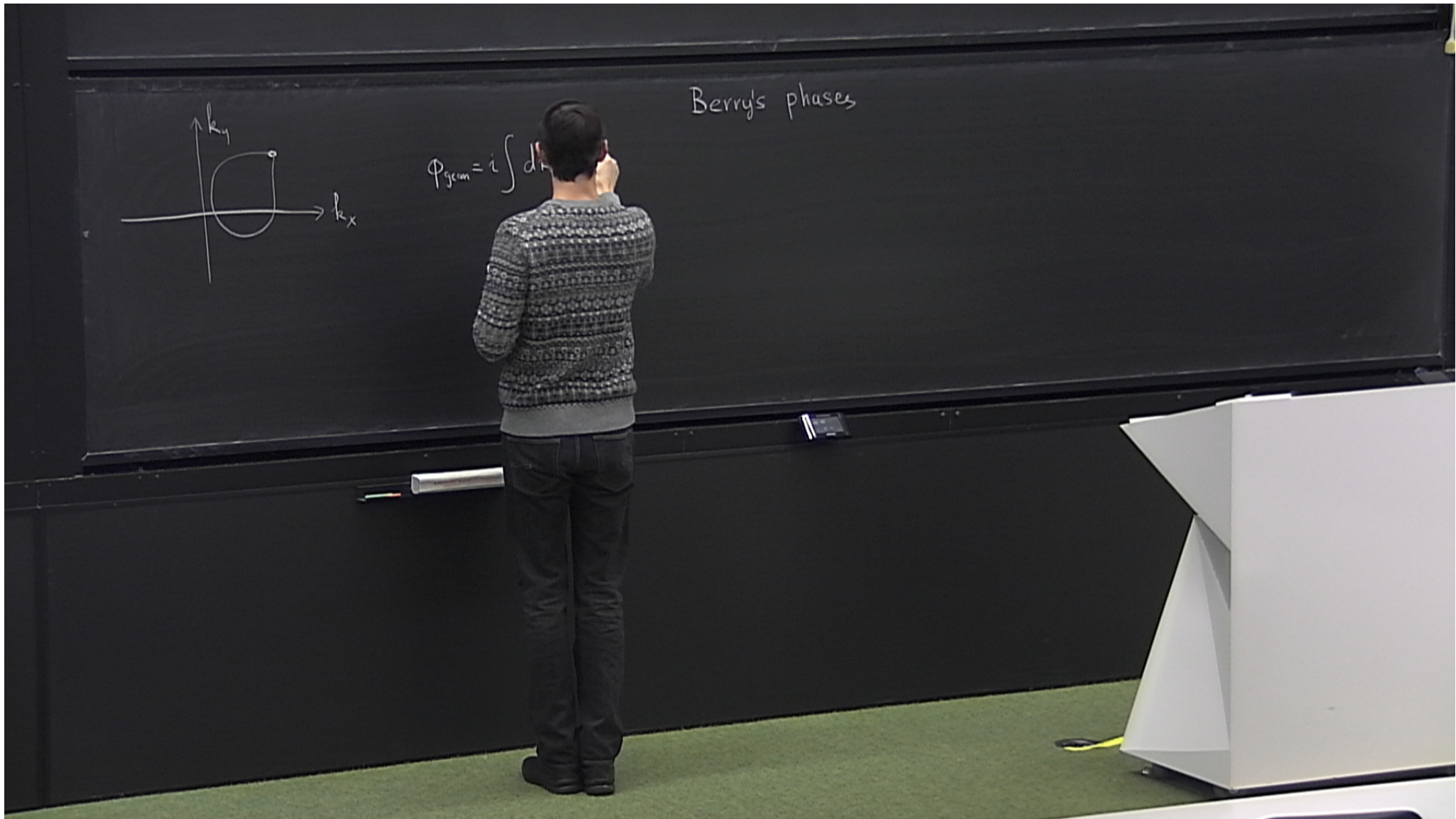


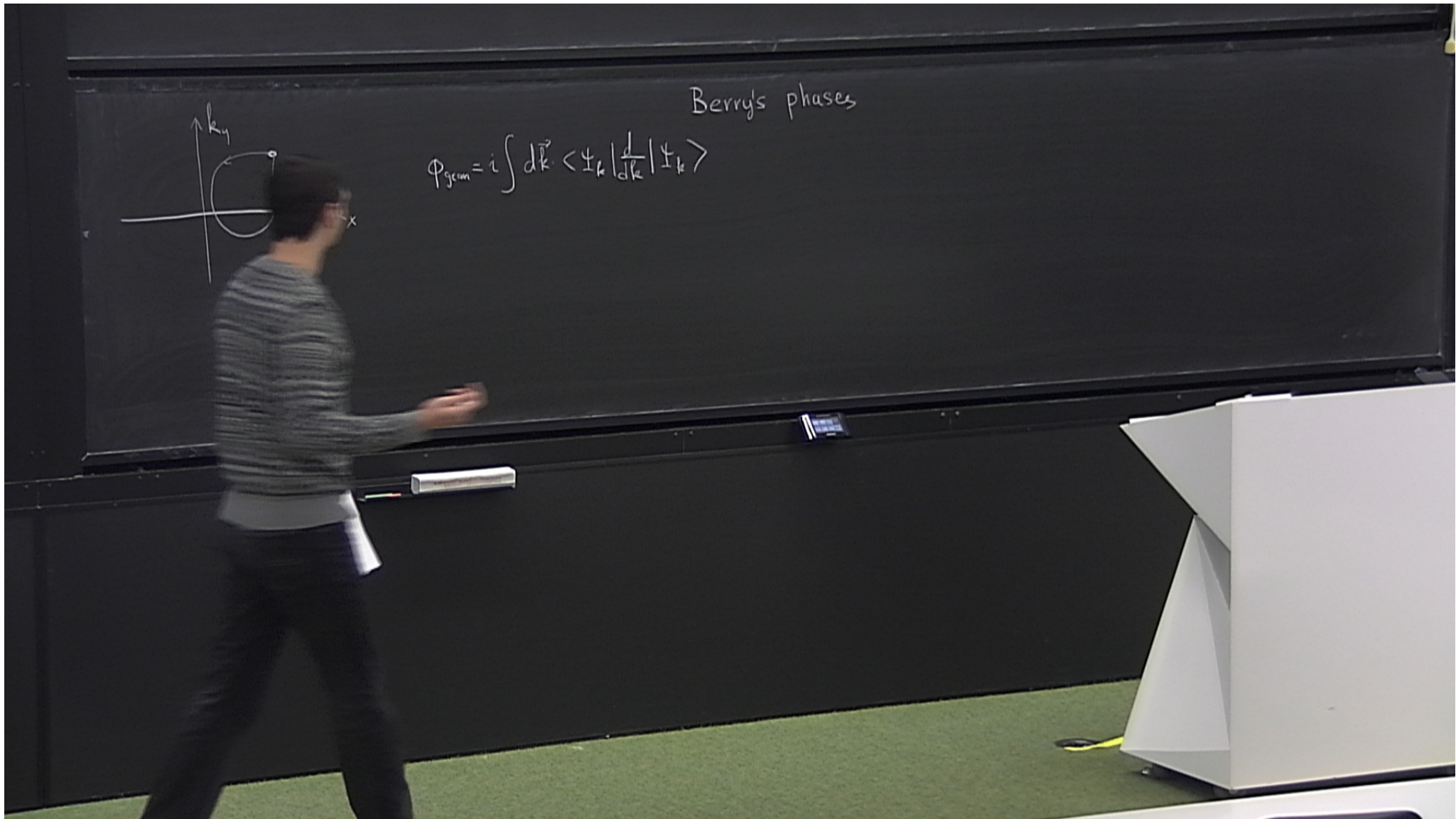
Title: 12/13 PSI - Condensed Matter Review Lecture 12

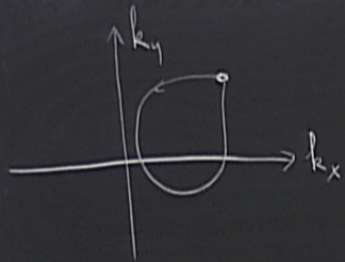
Date: Feb 12, 2013 11:30 AM

URL: <http://pirsa.org/13020085>

Abstract:



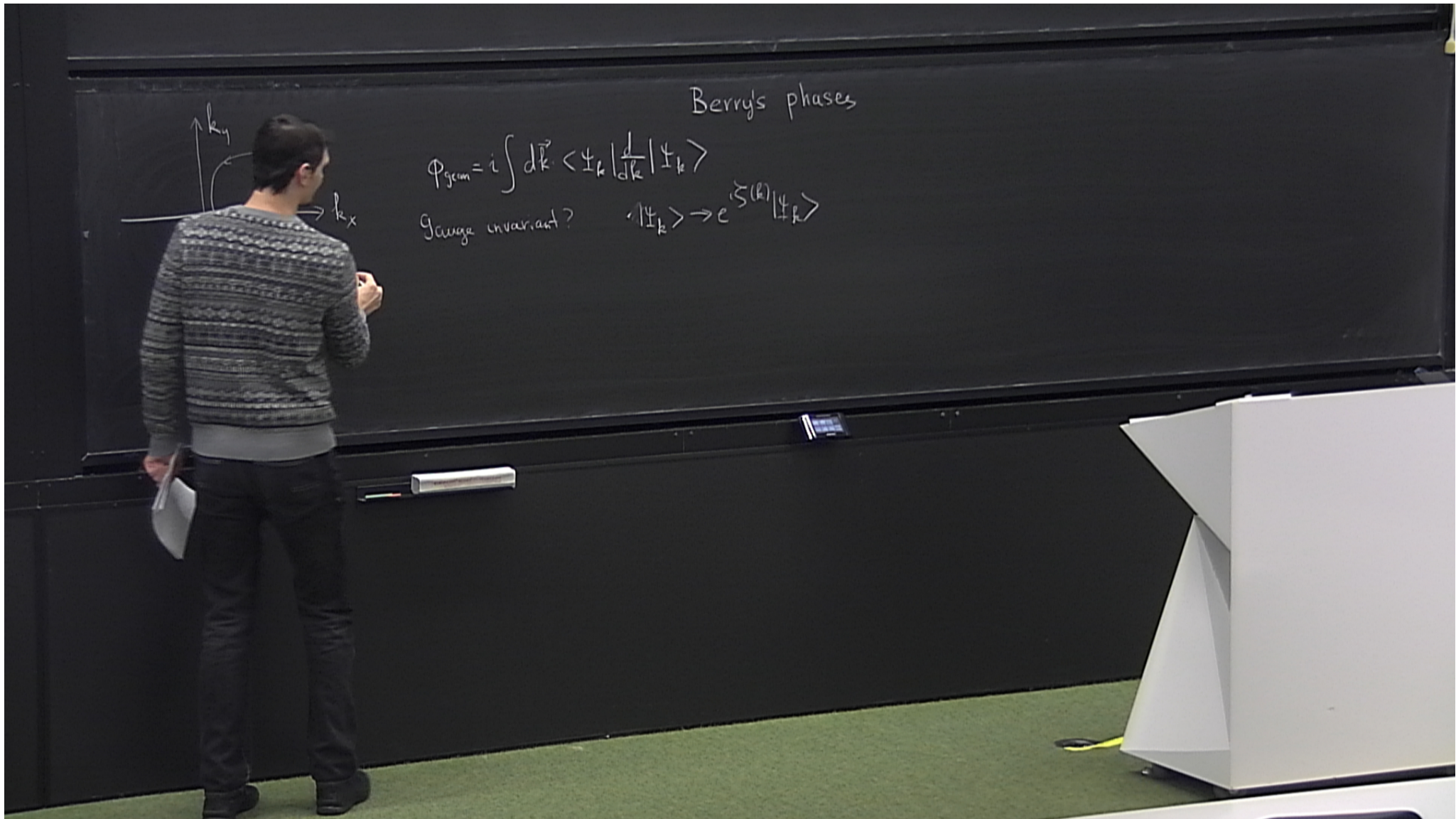




Berry's phases

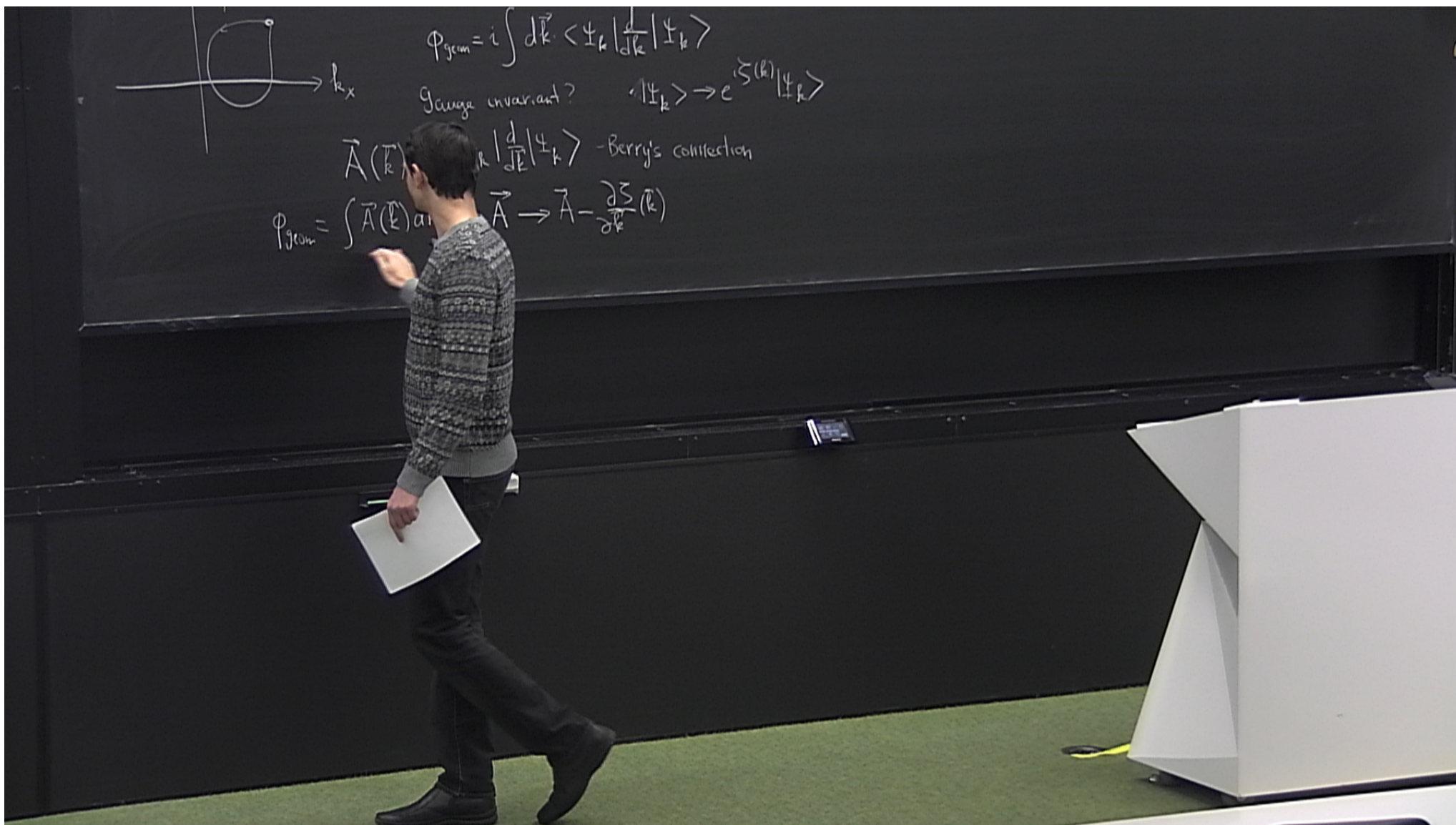
$$\phi_{\text{geom}} = i \int d\vec{k} \cdot \langle \psi_k | \frac{d}{d\vec{k}} | \psi_k \rangle$$

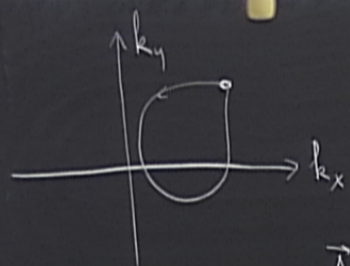
Gauge invariant? $|\psi_k\rangle \rightarrow e^{i\zeta(k)} |\psi_k\rangle$





$\phi_{\text{geom}} = i \int d\vec{k} \cdot \langle \psi_k | \frac{d}{d\vec{k}} | \psi_k \rangle$
Gauge invariant? $|\psi_k\rangle \rightarrow e^{i\zeta(k)} |\psi_k\rangle$
 $\vec{A}(\vec{k}) = i \langle \psi_k | \frac{d}{d\vec{k}} | \psi_k \rangle$ - Berry's connection
 $\phi_{\text{geom}} = \int \vec{A}(\vec{k}) d\vec{k}$





Berry's phases

$$\phi_{\text{geom}} = i \int d\vec{k} \langle \psi_k | \frac{d}{d\vec{k}} | \psi_k \rangle$$

Gauge invariant?

$$|\psi_k\rangle \rightarrow e^{i\zeta(k)} |\psi_k\rangle$$

$$\vec{A}(\vec{k}) = i \langle \psi_k | \frac{d}{d\vec{k}} | \psi_k \rangle$$

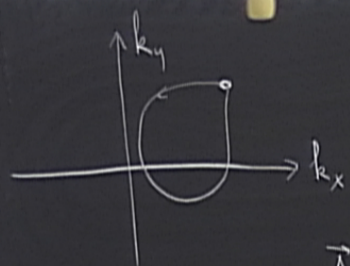
connection

$$\phi_{\text{geom}} = \int \vec{A}(\vec{k}) d\vec{k} \quad \vec{A} \rightarrow \vec{A} -$$

$$\gamma_{12} = \int_1^2 d\vec{k} \vec{A}(\vec{k}) \rightarrow \int_1^2 d\vec{k} \vec{A}(\vec{k}) - (\zeta(2) - \zeta(1))$$



$$\zeta(2) = \zeta(1) + 2\pi n, \quad n - \text{integer}$$



Berry's phases

$$\phi_{\text{geom}} = i \int d\vec{k} \cdot \langle \psi_k | \frac{d}{d\vec{k}} | \psi_k \rangle$$

Gauge invariant? $|\psi_k\rangle \rightarrow e^{i\zeta(k)} |\psi_k\rangle$

$$\vec{A}(\vec{k}) = i \langle \psi_k | \frac{d}{d\vec{k}} | \psi_k \rangle \quad \text{-- Berry's connection}$$

$$\phi_{\text{geom}} = \int \vec{A}(\vec{k}) d\vec{k} \quad \vec{A} \rightarrow \vec{A} - \frac{\partial \zeta(\vec{k})}{\partial \vec{k}}$$

$$\gamma_{12} = \int_1^2 d\vec{k} \cdot \vec{A}(\vec{k}) \rightarrow \int_1^2 d\vec{k} \cdot \vec{A}(\vec{k}) - (\zeta(2) - \zeta(1))$$



$$\zeta(2) = \zeta(1) + 2\pi n, \quad n \text{ - integer}$$

γ

$$\phi_{\text{geom}} = \int \vec{A}(\vec{k}) d\vec{k} \quad \vec{A} \rightarrow \vec{A} - \underbrace{\frac{\partial \chi}{\partial \vec{k}}(\vec{k})}$$

$\oint_{\text{loop}} \vec{A} \cdot d\vec{k} = \oint_{\text{loop}} \vec{A} \cdot d\vec{k} - \oint_{\text{loop}} \frac{\partial \chi}{\partial \vec{k}} \cdot d\vec{k}$ - gauge-independent

$$\vec{\Omega} = \vec{\nabla} \times \vec{A} \quad \text{Berry's curvature}$$

$$\phi_{\text{geom}} = \int_C \vec{A} \cdot d\vec{k} = \int_C \vec{\Omega} \cdot d\vec{S}$$

$$\Omega_{\mu\nu} = i \left(\left\langle \frac{\partial \psi_L}{\partial k_\mu} \right| \frac{\partial \psi_R}{\partial k_\nu} \right) - \left(\left\langle \frac{\partial \psi_R}{\partial k_\nu} \right| \frac{\partial \psi_L}{\partial k_\mu} \right)$$

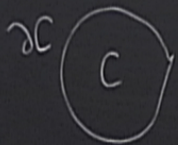
Ω_{42}

$$\phi_{\text{geom}} = \int \vec{A}(\vec{k}) d\vec{k} \quad \vec{A} \rightarrow \vec{A} - \underbrace{\frac{\partial \chi}{\partial \vec{k}}(\vec{k})}$$

$\oint_{\text{loop}} \phi^{\text{geom}}$ - gauge-independent

$$\vec{\Omega} = \vec{\nabla} \times \vec{A} \quad \text{Berry's curvature}$$

$$\phi_{\text{geom}} = \int_{\mathcal{C}} \vec{A} d\vec{k} = \int_{\mathcal{C}} \vec{\Omega} \cdot d\vec{S} = \int_{\mathcal{C}} \Omega_{\mu} dk_{\mu}$$



$$\Omega_{\mu\nu} = i \left(\left\langle \frac{\partial \psi_L}{\partial k_{\mu}} \right| \frac{\partial \psi_R}{\partial k_{\nu}} \right) - \left(\left\langle \frac{\partial \psi_R}{\partial k_{\nu}} \right| \frac{\partial \psi_L}{\partial k_{\mu}} \right)$$

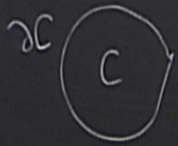
$$\Omega_{42}$$

$$\phi_{g.m} = \int \vec{A}(\vec{k}) d\vec{k} \quad \vec{A} \rightarrow \vec{A} - \frac{\partial \chi}{\partial \vec{k}}(\vec{k})$$

$\bigcup_{\text{loop}} \text{Closed } \phi^{g.m} - \text{gauge-independent}$

$$\vec{\Omega} = \vec{\nabla} \times \vec{A} \quad - \text{Berry's curvature}$$

$$\phi_{g.m} = \int_{\mathcal{C}} \vec{A} d\vec{k} = \int_{\mathcal{C}} \vec{\Omega} \cdot d\vec{S} = \int_{\mathcal{C}} \Omega_{ij} dk_i dk_j$$



$$\Omega_{\mu\nu} = i \left(\left\langle \frac{\partial \psi_L}{\partial k_\mu} \left| \frac{\partial \psi_R}{\partial k_\nu} \right\rangle - \left\langle \frac{\partial \psi_R}{\partial k_\nu} \left| \frac{\partial \psi_L}{\partial k_\mu} \right\rangle \right)$$

$$\Omega_{42} \quad \Omega_{11} = \Omega_{22} = 0$$

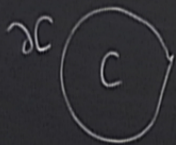


$$\phi_{\text{geom}} = \int \vec{A}(\vec{k}) d\vec{k} \quad \vec{A} \rightarrow \vec{A} - \frac{\partial \chi}{\partial \vec{k}}(\vec{k})$$

$\oint_{\text{loop}} \vec{A} \cdot d\vec{k} = \oint_{\text{loop}} \vec{A} \cdot d\vec{k} - \oint_{\text{loop}} \frac{\partial \chi}{\partial \vec{k}} \cdot d\vec{k} = \oint_{\text{loop}} \vec{A} \cdot d\vec{k} - \chi(\vec{k}_{\text{end}}) + \chi(\vec{k}_{\text{start}})$
 Closed loop - gauge-independent

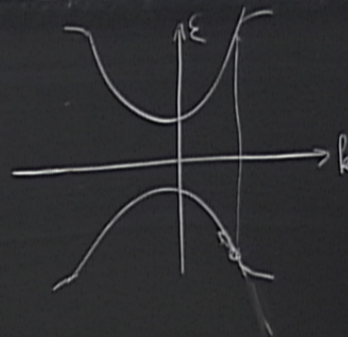
$$\vec{\Omega} = \vec{\nabla} \times \vec{A} \quad \text{Berry's curvature}$$

$$\phi_{\text{geom}} = \int_{\mathcal{C}} \vec{A} \cdot d\vec{k} = \int_{\mathcal{C}} \vec{\Omega} \cdot d\vec{S} = \int_{\mathcal{C}} \Omega_{ij} dk_i dk_j$$



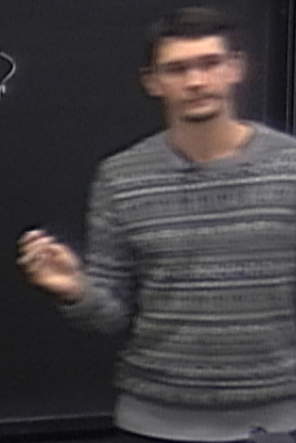
$$\Omega_{ij} = i \left(\left\langle \frac{\partial \psi_i}{\partial k_j} \right| \frac{\partial \psi_j}{\partial k_i} \right) - \left(\left\langle \frac{\partial \psi_j}{\partial k_i} \right| \frac{\partial \psi_i}{\partial k_j} \right)$$

$$\Omega_{42} \quad \Omega_{11} = \Omega_{22} = 0$$



$$|\psi(t)\rangle = \sum d_n(t) |\psi_{k(n)}\rangle$$

$$d_n = \begin{cases} 0, & n \geq 2 \\ e^{i\phi}, & n=1 \end{cases}$$

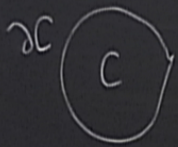


$$\phi_{\text{g.m.}} = \int \vec{A}(\vec{k}) d\vec{k} \quad \vec{A} \rightarrow \vec{A} - \frac{\partial \chi}{\partial \vec{k}}(\vec{k})$$

closed loop $\phi^{\text{g.m.}}$ - gauge-independent

$$\vec{\Omega} = \vec{\nabla} \times \vec{A} \quad \text{Berry's curvature}$$

$$\phi_{\text{g.m.}} = \int_{\mathcal{C}} \vec{A} d\vec{k} = \int_{\mathcal{C}} \vec{\Omega} \cdot d\vec{S} = \int_{\mathcal{C}} \Omega_{\mu} dk_{\mu}$$



$$\Omega_{\mu\nu} = i \left(\left\langle \frac{\partial \psi_{\vec{k}}}{\partial k_{\mu}} \right| \frac{\partial \psi_{\vec{k}}}{\partial k_{\nu}} \right\rangle - \left\langle \frac{\partial \psi_{\vec{k}}}{\partial k_{\nu}} \right| \frac{\partial \psi_{\vec{k}}}{\partial k_{\mu}} \right\rangle$$

$$\Omega_{42} \quad \Omega_{11} = \Omega_{22} = 0$$

$$\phi_{\text{geom}} = \int \vec{A}(\vec{k}) d\vec{k} \quad \vec{A} \rightarrow \vec{A} - \frac{\partial \chi}{\partial \vec{k}}(\vec{k})$$

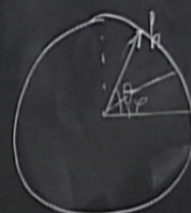
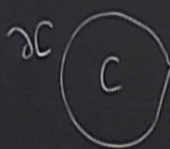
Closed loop ϕ^{geom} - gauge-independent

$$\vec{\Omega} = \vec{\nabla} \times \vec{A} \quad \text{Berry's curvature}$$

$$\phi_{\text{geom}} = \int_{\mathcal{C}} \vec{A} d\vec{k} = \int_{\mathcal{C}} \vec{\Omega} \cdot d\vec{S} = \int_{\mathcal{C}} \Omega_{ij} dk_i dk_j$$

$$\Omega_{ij} = i \left(\left\langle \frac{\partial \psi_i}{\partial k_j} \middle| \frac{\partial \psi_j}{\partial k_i} \right\rangle - \left\langle \frac{\partial \psi_j}{\partial k_i} \middle| \frac{\partial \psi_i}{\partial k_j} \right\rangle \right)$$

$$\Omega_{42} \quad \Omega_{11} = \Omega_{22} = 0$$



An example: spin $\frac{1}{2}$

$$H = \vec{h} \cdot \vec{\sigma}, \quad \vec{h} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$

Eigenstate

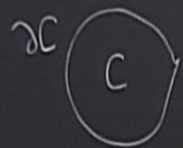
$$|u_{\pm}\rangle = \begin{pmatrix} \sin \frac{\theta}{2} e^{-i\phi} \\ \pm \cos \frac{\theta}{2} \end{pmatrix}$$

Berry's connection:

$$A_{\theta} = \langle u_{\pm} | i \partial_{\theta} | u_{\pm} \rangle$$

$$\vec{\Omega} = \vec{\nabla} \times \vec{A} \quad \text{Berry's curvature}$$

$$q_{\text{geom}} = \oint_C \vec{A} \cdot d\vec{k} = \int_C \vec{\Omega} \cdot d\vec{S} = \int_C \Omega_{12} dk_1 dk_2$$



$$\Omega_{\mu\nu} = i \left(\left\langle \frac{\partial \psi_k}{\partial k_\mu} \right| \frac{\partial \psi_k}{\partial k_\nu} \right) - \left(\left\langle \frac{\partial \psi_k}{\partial k_\nu} \right| \frac{\partial \psi_k}{\partial k_\mu} \right)$$

$$\Omega_{12} \quad \Omega_{11} = \Omega_{22} = 0$$

An example: spin $\frac{1}{2}$

$$H = \vec{h} \cdot \vec{\sigma}, \quad \vec{h} = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$$

Eigenstate

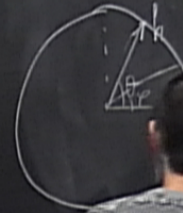
$$|u_{-}\rangle = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\phi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

Berry's connection:

$$A_{\theta} = \langle u_{-} | \partial_{\theta} | u_{-} \rangle = 0$$

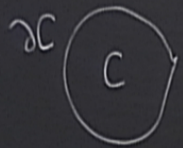
$$A_{\phi} = \langle u_{-} | \partial_{\phi} | u_{-} \rangle =$$

$$= \begin{pmatrix} \sin\frac{\theta}{2} e^{i\phi} & -\cos\frac{\theta}{2} \end{pmatrix} \cdot i \begin{pmatrix} -i \sin\frac{\theta}{2} e^{-i\phi} \\ 0 \end{pmatrix} = \sin^2\frac{\theta}{2}$$



$$\vec{\Omega} = \vec{\nabla} \times \vec{A} \quad \text{Berry's curvature}$$

$$q_{\text{anom}} = \int_C \vec{A} \cdot d\vec{k} = \int_C \vec{\Omega} \cdot d\vec{S} = \int_C \Omega_{12} dk_1 dk_2$$



$$\Omega_{\mu\nu} = i \left(\left\langle \frac{\partial \psi_k}{\partial k_\mu} \right| \frac{\partial \psi_k}{\partial k_\nu} \right) - \left(\left\langle \frac{\partial \psi_k}{\partial k_\nu} \right| \frac{\partial \psi_k}{\partial k_\mu} \right)$$

$$\Omega_{12} \quad \Omega_{11} = \Omega_{22} = 0$$

$$H = \vec{h} \cdot \vec{\sigma}, \quad \vec{h} = (\sin\theta \cos\phi, \sin\theta \sin\phi, \cos\theta)$$

Eigenstate

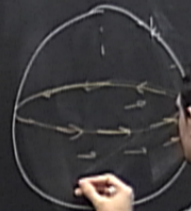
$$|u_-\rangle = \begin{pmatrix} \sin\frac{\theta}{2} e^{-i\phi} \\ -\cos\frac{\theta}{2} \end{pmatrix}$$

Berry's connection:

$$A_\theta = \langle u_- | \partial_\theta | u_- \rangle = 0$$

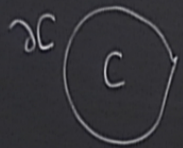
$$A_\phi = \langle u_- | \partial_\phi | u_- \rangle =$$

$$= \left(\sin\frac{\theta}{2} e^{i\phi} \quad -\cos\frac{\theta}{2} \right) \cdot i \cdot \begin{pmatrix} -i \sin\frac{\theta}{2} e^{-i\phi} \\ 0 \end{pmatrix} = \boxed{\sin\frac{\theta}{2}}$$



$$\vec{\Omega} = \vec{\nabla} \times \vec{A} \quad \text{Berry's curvature}$$

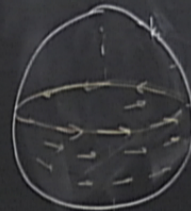
$$q_{\text{anom}} = \oint_C \vec{A} \cdot d\vec{k} = \int_C \vec{\Omega} \cdot d\vec{k} = \int_C \Omega_{12} dk_x dk_y$$



$$\Omega_{\mu\nu} = i \left(\left\langle \frac{\partial \psi_k}{\partial k_\mu} \right| \frac{\partial \psi_k}{\partial k_\nu} \right) - \left(\left\langle \frac{\partial \psi_k}{\partial k_\nu} \right| \frac{\partial \psi_k}{\partial k_\mu} \right)$$

$$\Omega_{12} \quad \Omega_{11} = \Omega_{22} = 0$$

$$H = \vec{h} \cdot \vec{\sigma}, \quad \vec{h} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$$



Eigenstate

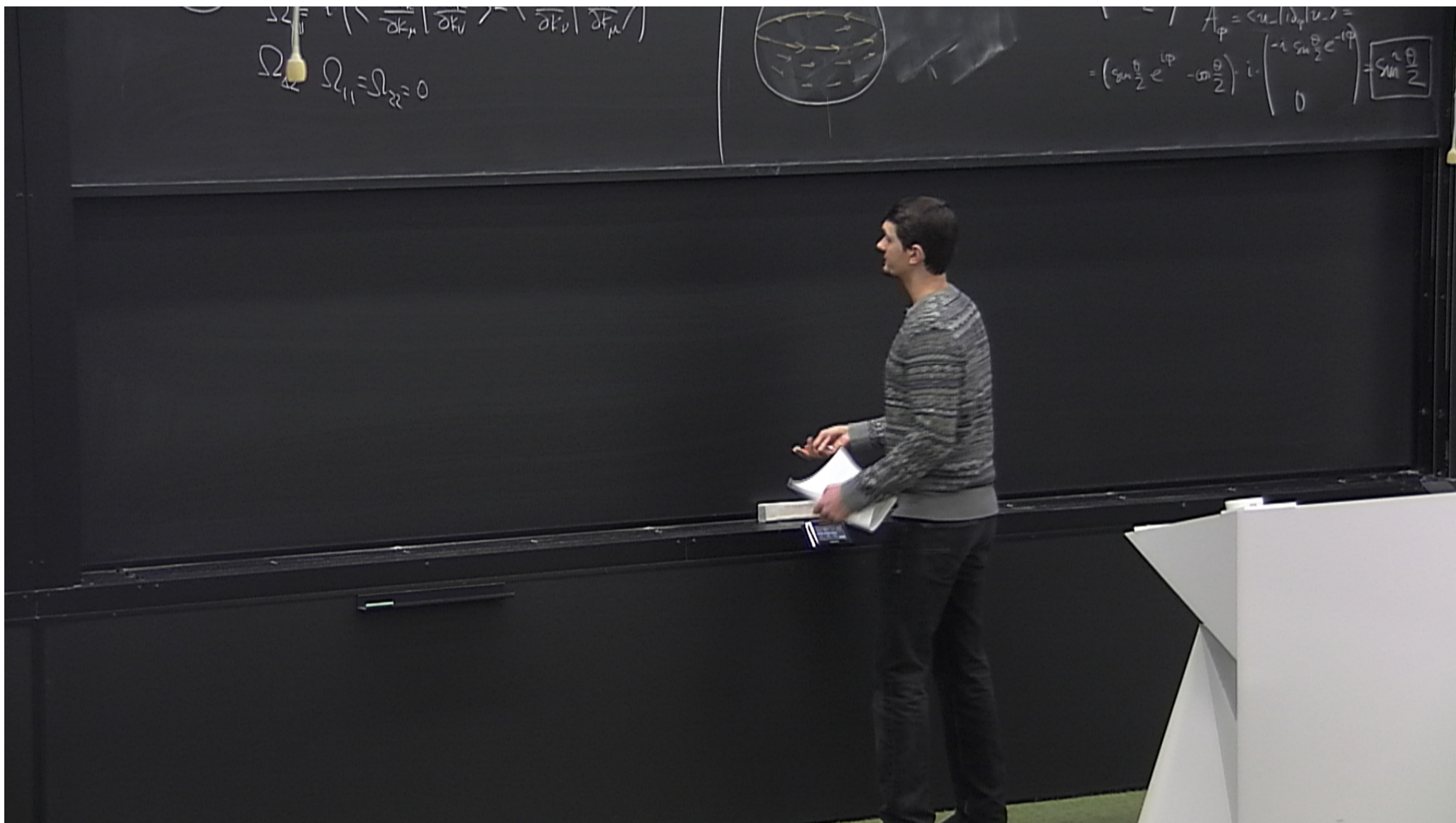
$$|u_-\rangle = \begin{pmatrix} \sin \frac{\theta}{2} e^{-i\phi} \\ -\cos \frac{\theta}{2} \end{pmatrix}$$

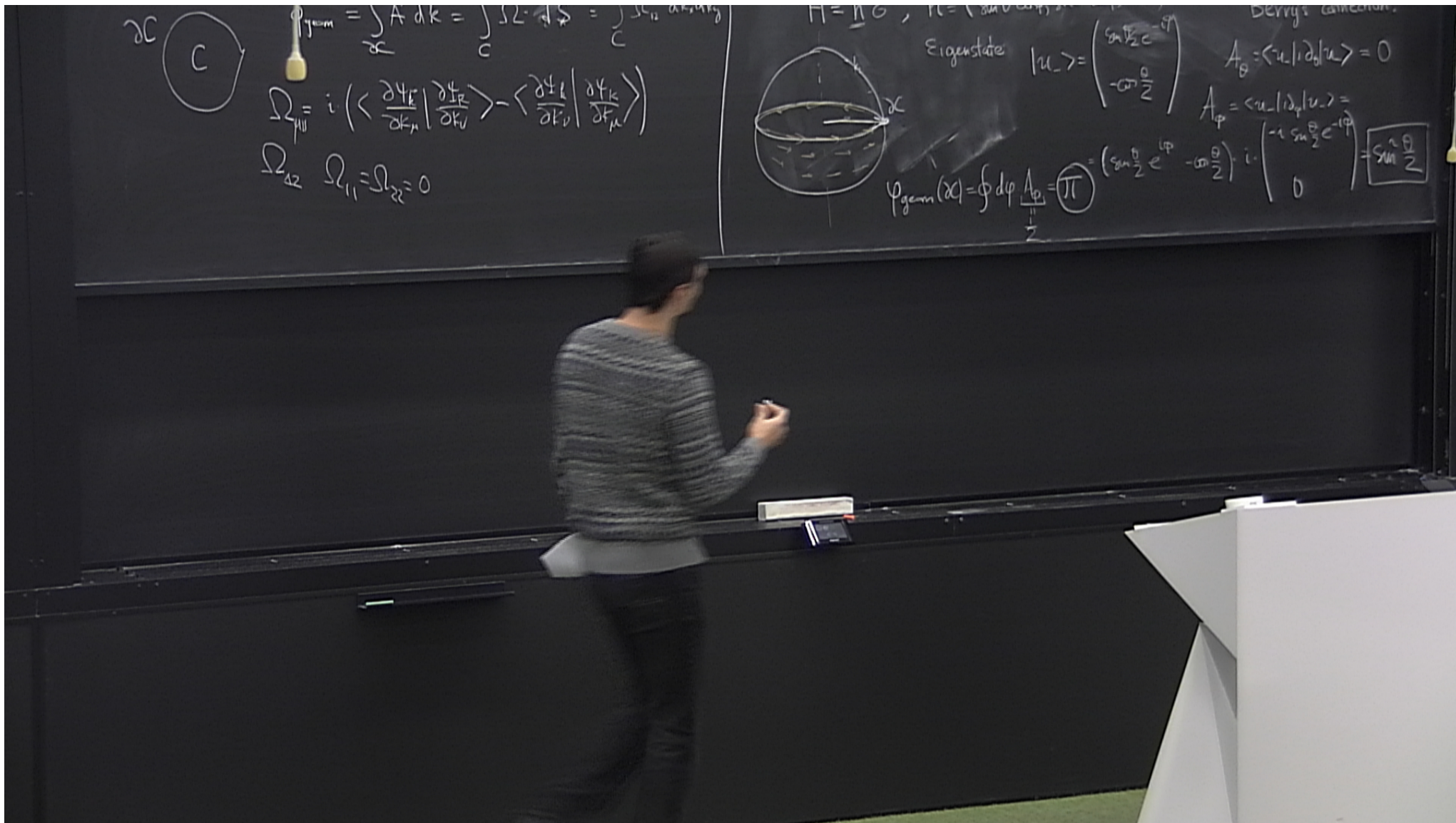
Berry's connection:

$$A_\theta = \langle u_- | i \partial_\theta | u_- \rangle = 0$$

$$A_\phi = \langle u_- | i \partial_\phi | u_- \rangle =$$

$$= \begin{pmatrix} \sin \frac{\theta}{2} e^{i\phi} & -\cos \frac{\theta}{2} \end{pmatrix} \cdot i \cdot \begin{pmatrix} -i \sin \frac{\theta}{2} e^{-i\phi} \\ 0 \end{pmatrix} = \sin \frac{\theta}{2}$$



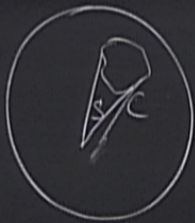


$$\Omega_{11} = \Omega_{22} = 0$$



$$\varphi_{\text{geom}}(\alpha) = \oint_C d\varphi \frac{A_\varphi}{\hbar} = \frac{\pi}{2} = \left(\sin \frac{\theta}{2} e^{i\varphi} - \cos \frac{\theta}{2} \right) \cdot i \cdot \begin{pmatrix} -i \sin \frac{\theta}{2} e^{-i\varphi} \\ 0 \end{pmatrix} = \sin \frac{\theta}{2}$$

Berry's curvature: $\Omega_{\varphi\theta} = \partial_\theta A_\varphi - \partial_\varphi A_\theta = \partial_\theta \sin^2 \frac{\theta}{2} = \frac{\sin \theta}{2}$



$$\varphi_{\text{geom}}(\alpha) = \int_C \frac{\sin \theta}{2} d\varphi d\theta = \frac{S}{2}$$

$$\Omega_{11} = \Omega_{22} = 0$$



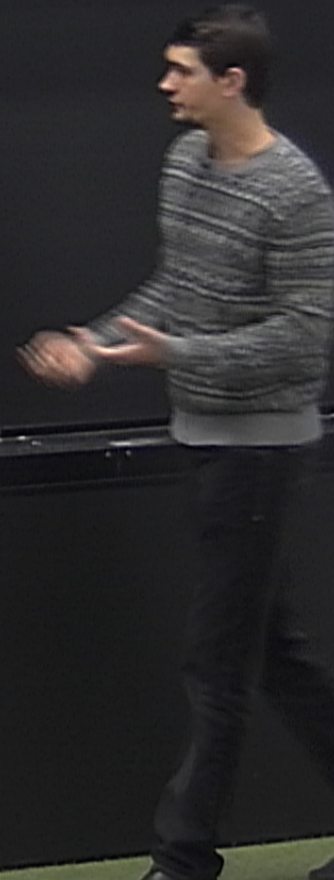
$$\varphi_{\text{geom}}(\alpha) = \oint_C d\varphi \frac{A_\varphi}{\hbar} = \pi = \left(\sin \frac{\theta}{2} e^{i\varphi} - \cos \frac{\theta}{2} \right) \cdot i \cdot \begin{pmatrix} -i \sin \frac{\theta}{2} e^{-i\varphi} \\ 0 \end{pmatrix} = \sin \frac{\theta}{2}$$

Berry's curvature: $\Omega_{\varphi\theta} = \partial_\theta A_\varphi - \partial_\varphi A_\theta = \partial_\theta \sin^2 \frac{\theta}{2} = \frac{\sin \theta}{2}$



$$\varphi_{\text{geom}}(\alpha) = \int_C \frac{\sin \theta}{2} d\varphi d\theta = \frac{S}{2}$$

Equator trajectory: $S = 2\pi$ $\varphi_{\text{geom}} = \pi$

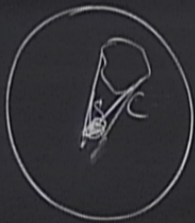


$$\Omega_{11} = \Omega_{22} = 0$$



$$\varphi_{\text{geom}}(\alpha) = \oint_{\frac{1}{2}} d\varphi \frac{A_p}{2} = \pi = \left(\sin \frac{\theta}{2} e^{i\varphi} - \cos \frac{\theta}{2} \right) \cdot i \cdot \begin{pmatrix} -i \sin \frac{\theta}{2} e^{-i\varphi} \\ 0 \end{pmatrix} = \sin \frac{\theta}{2}$$

Berry's curvature: $\Omega_{\varphi\theta} = \partial_{\varphi} A_{\theta} - \partial_{\theta} A_{\varphi} = \partial_{\varphi} \sin^2 \frac{\theta}{2} = \frac{\sin \theta}{2}$



$$\varphi_{\text{geom}}(\partial C) = \int_C \frac{\sin \theta}{2} d\varphi d\theta = \frac{S}{2}$$

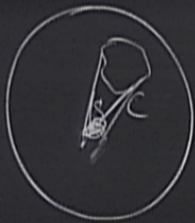
Equator trajectory: $S = 2\pi$ $\varphi_{\text{geom}} = \pi$

$$\Omega_{11} = \Omega_{22} = 0$$



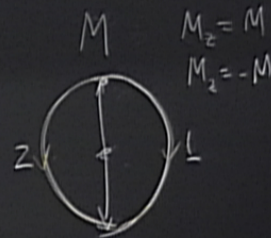
$$\varphi_{\text{geom}}(\alpha) = \oint \frac{1}{2} d\varphi \frac{A_\varphi}{\hbar} = \frac{\pi}{2} = \left(\sin \frac{\theta}{2} e^{i\varphi} - \cos \frac{\theta}{2} \right) \cdot i \cdot \begin{pmatrix} -i \sin \frac{\theta}{2} e^{-i\varphi} \\ 0 \end{pmatrix} = \sin \frac{\theta}{2}$$

Berry's curvature: $\Omega_{\varphi\varphi} = \partial_\varphi A_\varphi - \partial_\varphi A_\varphi = \partial_\varphi \sin^2 \frac{\varphi}{2} = \frac{\sin \varphi}{2}$



$$\varphi_{\text{geom}}(\alpha C) = \int_C \frac{\sin \varphi}{2} d\varphi d\vartheta = \frac{S}{2}$$

Equator trajectory: $S' = 2\pi$ $\varphi_{\text{geom}} = \pi$

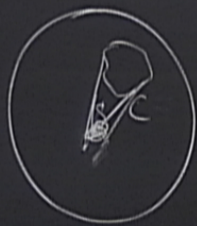


$$\Omega_{\theta} \quad \Omega_{11} = \Omega_{22} = 0$$



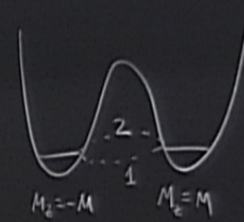
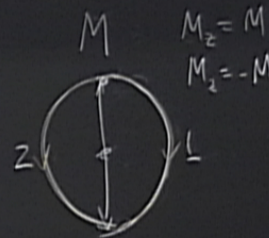
$$\varphi_{\text{geom}}(\alpha) = \oint d\varphi \frac{A_{\varphi}}{2} = \frac{\pi}{2} = \left(\sin \frac{\theta}{2} e^{i\varphi} - \cos \frac{\theta}{2} \right) \cdot i \cdot \begin{pmatrix} -i \sin \frac{\theta}{2} e^{-i\varphi} \\ 0 \end{pmatrix} = \sin \frac{\theta}{2}$$

Berry's curvature: $\Omega_{\varphi\varphi} = \partial_{\varphi} A_{\varphi} - \partial_{\varphi} A_{\varphi} = \partial_{\varphi} \sin^2 \frac{\varphi}{2} = \frac{\sin \varphi}{2}$



$$\varphi_{\text{geom}}(\partial C) = \int_C \frac{\sin \varphi}{2} d\varphi d\theta = \frac{S}{2}$$

Equator trajectory: $S' = 2\pi \quad \varphi_{\text{geom}} = \pi$



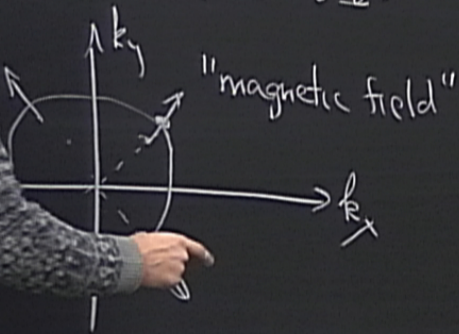
$P = P_1 + P_2$ - classically

$P = |t_1 + t_2|^2$

$|t_1 + t_2|^2 = \begin{cases} 0, \text{ half-integer} \\ 4|t_1|^2, \text{ integer} \end{cases}$

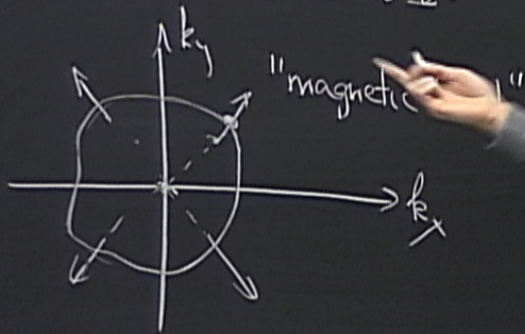
Example 2: graphene

$$\hbar k = \hbar v_F (G_x \underline{k}_x + G_y \underline{k}_y)$$



Example 2: graphene

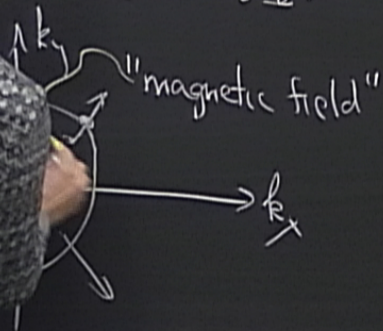
$$\hbar v_F (G_x \underline{k}_x + G_y \underline{k}_y)$$



Tray enclosing Dirac node $-\pi$

Example 2: graphene

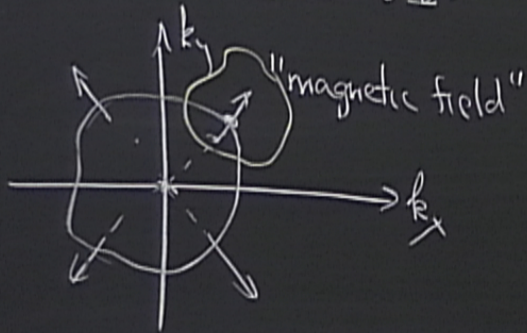
$$\hbar v_F (G_x \underline{k}_x + G_y \underline{k}_y)$$



Trajectory enclosing Dirac node $-\pi$
not enclosing Dir. p

Example 2: graphene

$$\hbar_k = \hbar v_F (G_x \underline{k}_x + G_y \underline{k}_y)$$

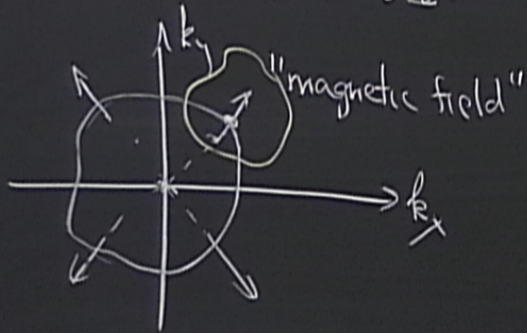


Trajectory enclosing Dirac node $-\pi$
not enclosing Dir. point: 0



Example 2: graphene

$$\hbar \mathbf{k} = \hbar v_F (\mathbf{G}_x \mathbf{k}_x + \mathbf{G}_y \mathbf{k}_y)$$

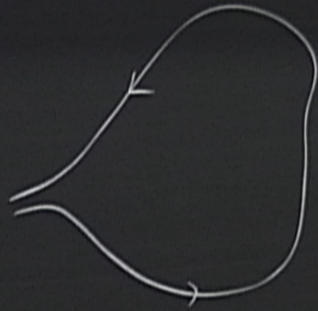


Trajectory enclosing Dirac node $-\pi$
not enclosing Dir. point: 0



node - π

Dir. point: 0

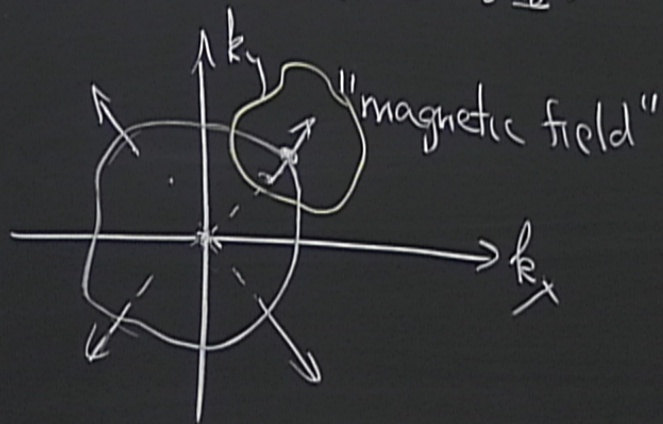


Berry's phase contribution:
weak localization $\rightarrow$ weak antiblocalization!



Example 2: graphene

$$\hbar_k = \hbar v_F (G_x \underline{k}_x + G_y \underline{k}_y)$$

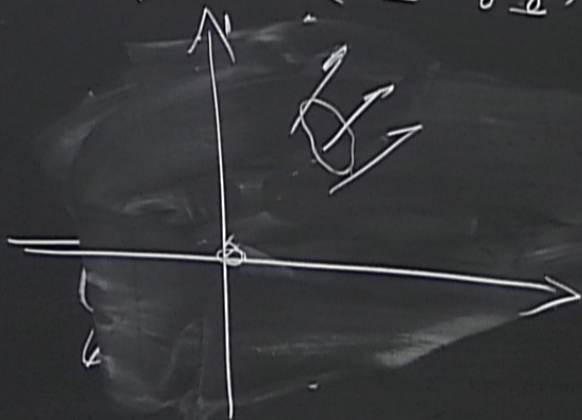


Trajectory enclosing Dirac node $-\pi$
not enclosing Dir. point: 0



Example 2: graphene

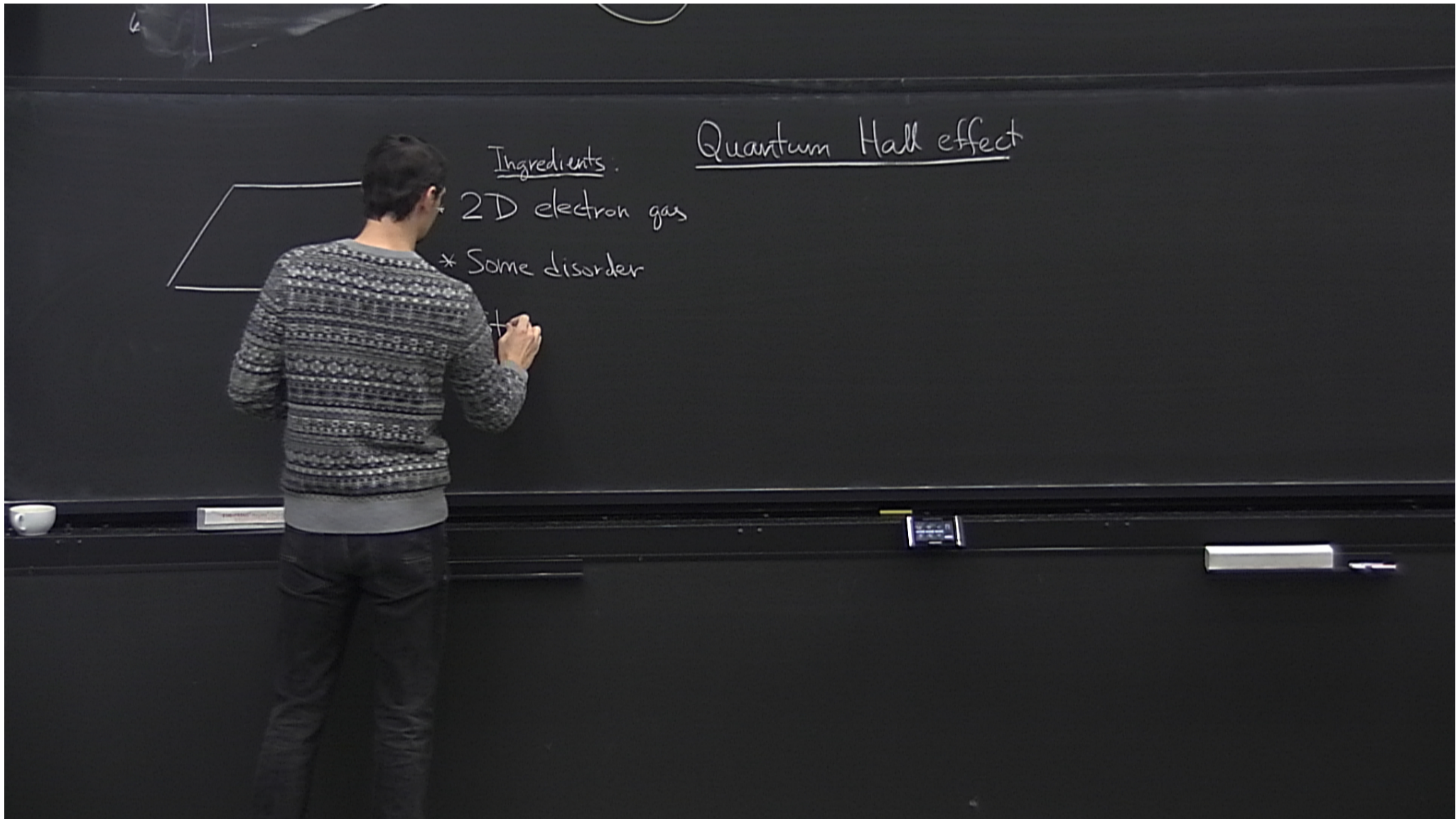
$$\hbar v_F (G_x \underline{k}_x + G_y \underline{k}_y)$$

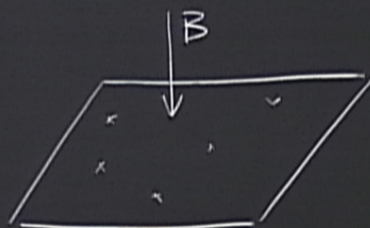


Trajectory enclosing Dirac node $-\pi$
not enclosing Dir. point: 0



Quantum Hall effect





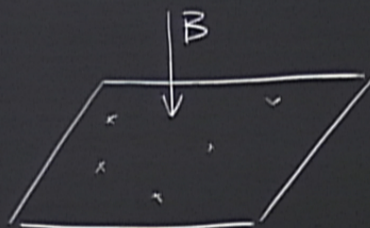
Ingredients

* 2D electron gas

* Some disorder

* Strong B-field. $\omega_c = \frac{eB}{mc}$

Quantum Hall effect



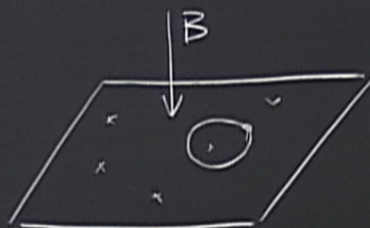
Ingredients

- * 2D electrons
- * Some disorder
- * Strong B

Quantum Hall effect

$$\omega_c = \frac{eB}{m^*}$$

$$\omega_c \tau \gg 1$$



Ingredients.

* 2D electron gas

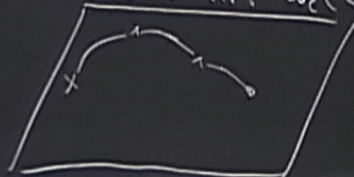
* Some disorder

* Strong B-field.

$$\omega_c = \frac{eB}{mc}$$

$$\omega_c \tau \gg 1$$

Drude limit: $\omega_c \tau \ll 1$



Quantum Hall effect

Ess of QH

Hall effect

nice of QHE.

$\rightarrow y$

$$\underline{\omega_c \tau \gtrsim 1}$$

Hall effect

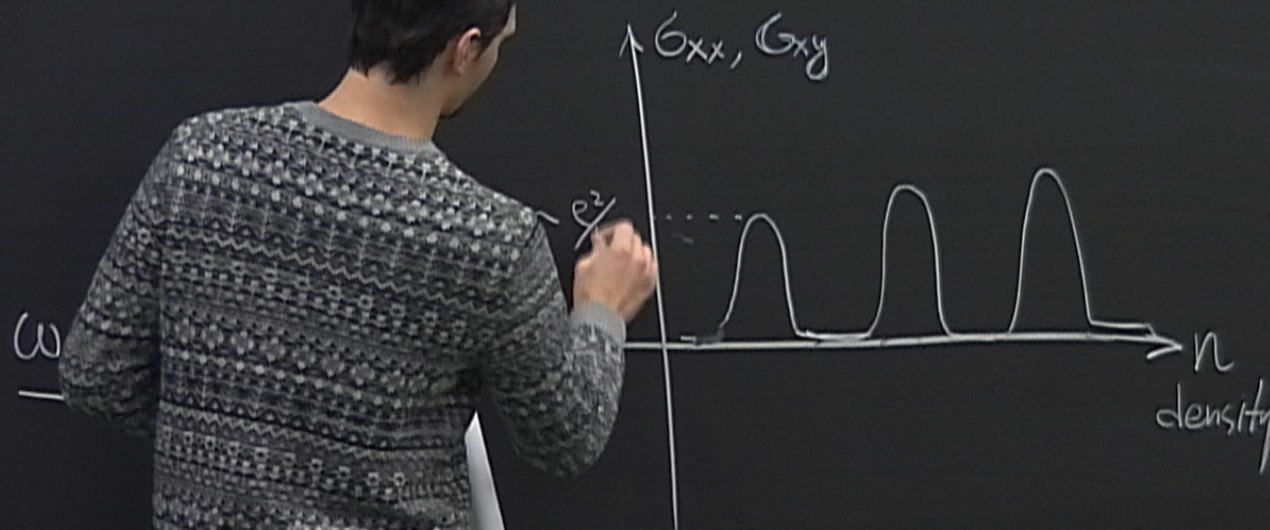
Essence of QHE.

$\uparrow G_{xx}, G_{xy}$

$\rightarrow n$
density

Hall effect

Essence of QHE.



Hall effect

$$\underline{\omega_c \tau \gtrsim 1}$$

Ess of QHE.

$\uparrow \sigma_{xx}$

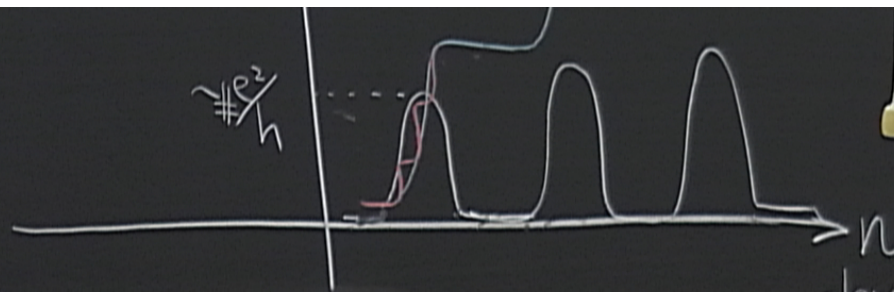
σ_{xy} quantized at plateaus

$$\sigma_{xy} = n \left(\frac{e^2}{h} \right) - \text{integer QHE}$$

h - integer 10^{-10}

n
density

$$\omega_c \tau \gtrsim 1$$

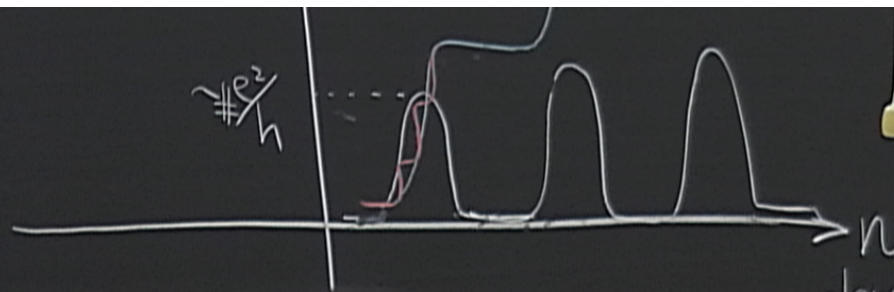


$$R_{xy} = n \left(\frac{e}{h} \right) - \text{integer QHE}$$

$h \sim \text{integer } 10^{-10}$

Stronger B/cleaner samples $\rightarrow$ fractional plateaus $R_{xy} = f \frac{e^2}{h}$

$$\omega_c \tau \gtrsim 1$$



$$\sigma_{xy} = n \left(\frac{e}{h} \right) - \text{integer QHE}$$

h -integer 10^{-10}

Stronger B/cleaner samples $\rightarrow$ fractional plateaus $\sigma_{xy} = f \frac{e^2}{h}$
F-QHE