

Title: 12/13 PSI - Condensed Matter Review Lecture 9

Date: Feb 11, 2013 11:30 AM

URL: <http://pirsa.org/13020084>

Abstract:

Graphene

* Carbon-based 2D material
C-atoms arranged in a honeycomb
lattice



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Graphene

* Carbon-based 2D material

C-atoms arranged in a honeycomb lattice



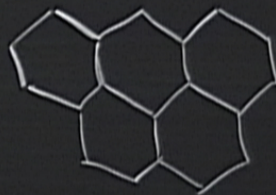
* 2004 . atomically flat .

Other examples: graphene multilayers, MoS_2

* Dirac-like low-energy excitations contrast $\epsilon = \frac{p^2}{2m}$

Graphene

- * Carbon-based 2D material
C-atoms arranged in a honeycomb lattice

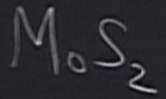
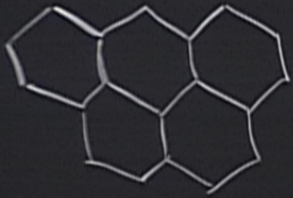


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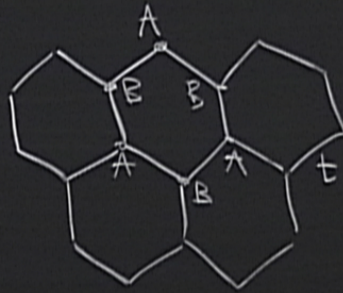
Other examples: graphene multilayers, MoS_2

- * Dirac-like low-energy excitations contrast $\epsilon = \frac{p^2}{2m}$
- * 3D Topological insulator, Weyl semimetals 3D

Graphene



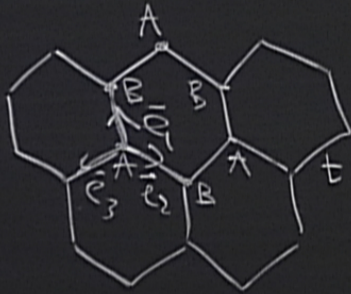
contrast $\epsilon = \frac{p^2}{2m}$
metals 3D



Band structure

$$H = t \sum_{\langle ij \rangle} (a_i^\dagger b_j + b_j^\dagger a_i)$$

k-space



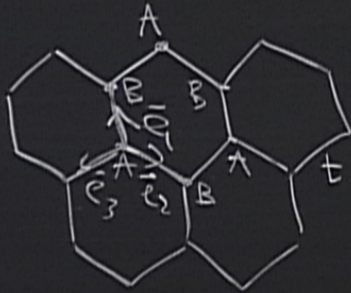
Band structure

$$H = t \sum_{\langle ij \rangle} (a_i^\dagger b_j + b_j^\dagger a_i)$$

$$H = \sum_k (a_k^\dagger \ b_k^\dagger) \hat{h}_k \begin{pmatrix} a_k \\ b_k \end{pmatrix}$$

$$\hat{h}_k = \begin{bmatrix} 0 & t^*(k) \\ t(k) & 0 \end{bmatrix}$$

go to k-space: $a_i = \sum_k a_k e^{i\vec{k} \cdot \vec{R}_i}$
 $b_i = \sum_k b_k e^{i\vec{k} \cdot \vec{R}_i + \vec{e} \cdot \vec{R}_i}$



Band structure

$$H = t \sum_{\langle ij \rangle} (a_i^\dagger b_j + b_j^\dagger a_i)$$

$$H = \sum_k (a_k^\dagger \ b_k^\dagger) \hat{h}_k \begin{pmatrix} a_k \\ b_k \end{pmatrix}$$

$$\hat{h}_k = \begin{bmatrix} 0 & t^*(k) \\ t(k) & 0 \end{bmatrix},$$

$$t(k) = t \left[e^{i\vec{k} \cdot \vec{e}_1} + e^{i\vec{k} \cdot \vec{e}_2} + e^{i\vec{k} \cdot \vec{e}_3} \right]$$

go to k-space:

$$a_i = \sum_k e^{i\vec{k} \cdot \vec{r}_i} a_k$$

$$b_i = \sum_k e^{i\vec{k} \cdot \vec{r}_i} b_k$$

Topological Insulator, Weyl Semimetals 3D

$$t(\vec{k}) = 0$$

$$\vec{k} = (k_x, 0)$$

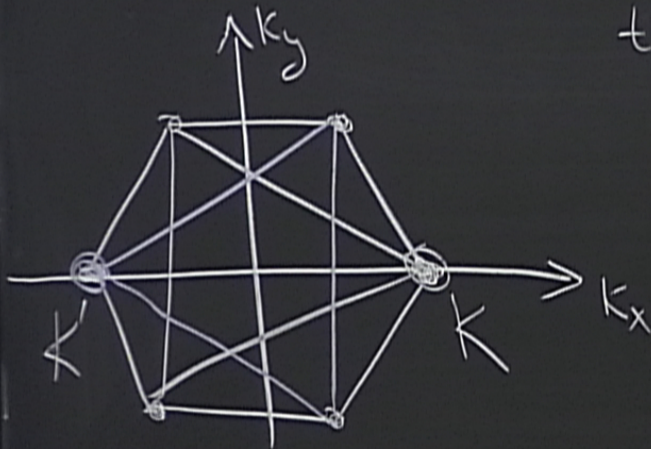
$$t(\vec{k}) = t \left[1 + e^{i\frac{\sqrt{3}}{2}ak_x} + e^{-i\frac{\sqrt{3}}{2}ak_x} \right] =$$

$$= t \left(1 + 2 \cos \left(\frac{\sqrt{3}}{2} ak_x \right) \right) = 0 \quad \frac{\sqrt{3}}{2} ak_x = \pm \frac{2\pi}{3}$$

$$K = \pm \frac{4\pi}{3\sqrt{3}a}$$

Symmetries: particle-hole
Symmetry

$$\begin{pmatrix} u_k \\ v_k \end{pmatrix} : \epsilon_k \quad \hat{\sigma}_z \begin{pmatrix} u_k \\ v_k \end{pmatrix} = -\epsilon_k$$



$$\vec{k} = \vec{K} + \delta\vec{k}, \quad |\delta k| \ll \frac{1}{a}$$

$$t(\vec{k}) = \frac{1}{2} \left(e^{i\delta k_y a} + e^{-i\delta k_y a} \right) 2 \cos\left(\vec{K}_x \frac{\sqrt{3}}{2} a + \delta k_x \frac{\sqrt{3}}{2} a\right) \approx \frac{3}{2} a [-\delta k_x + i\delta k_y]$$

$$\hat{h}_{\vec{k}+\delta\vec{k}} = \hbar v_F \begin{bmatrix} 0 & -\delta k_x - i\delta k_y \\ -\delta k_x + i\delta k_y & 0 \end{bmatrix} = \hbar v_F (-\delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y)$$

$-\epsilon_k$

$$\vec{k} = \vec{K} + \delta\vec{k}, \quad |\delta k| \ll \frac{1}{a}$$

$$t(\vec{k}) = \frac{1}{2} \left(e^{i\delta k_y a} + e^{-i\delta k_y a} \cdot 2 \cos\left(k_x \frac{\sqrt{3}}{2} a + \delta k_x \frac{\sqrt{3}}{2} a\right) \right) \approx \frac{3}{2} a [-\delta k_x + i\delta k_y]$$

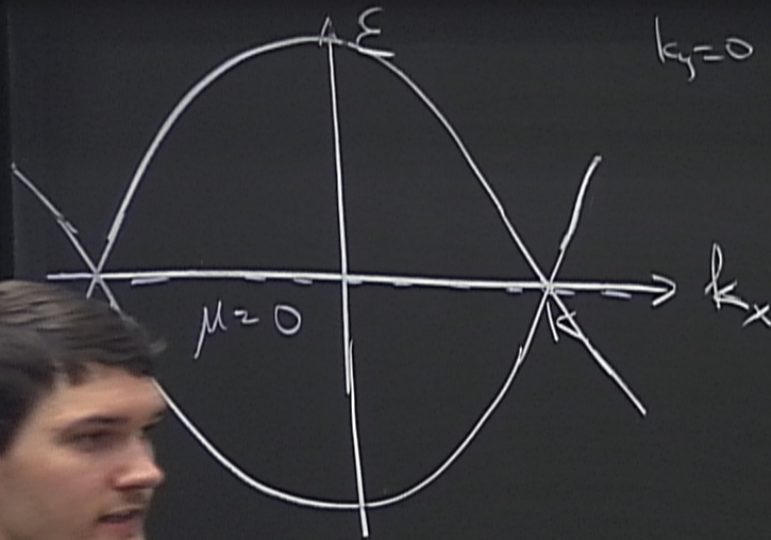
$$\hat{h}_{\vec{k}+\delta\vec{k}} = \hbar v_F \begin{bmatrix} 0 & -\delta k_x - i\delta k_y \\ -\delta k_x + i\delta k_y & 0 \end{bmatrix} = \hbar v_F (-\delta k_x \hat{\sigma}_x + \delta k_y \hat{\sigma}_y),$$

$$\hbar v_F = \frac{3}{2} \hbar t a, \quad v_F \approx 10^6 \text{ m/s}$$

$-\epsilon_k$

Symmetry

$$|u_k| = \sqrt{\frac{E_k}{2}} \quad |v_k| = \sqrt{\frac{E_k}{2}}$$

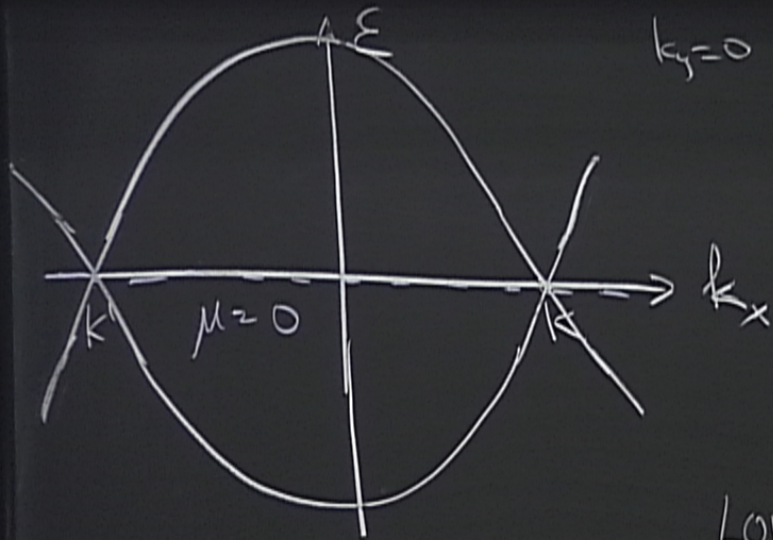


$k_y=0$

Fermi points

Symmetry

$$|u_k\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_k \\ \psi_k \end{pmatrix}$$



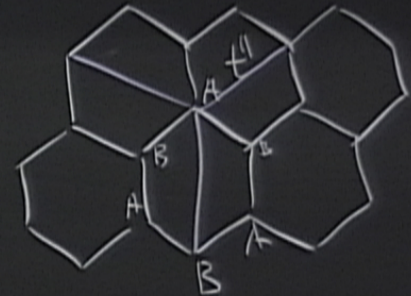
Fermi points

Are Dirac points robust?

$$A \leftrightarrow B$$

Longer-range hops

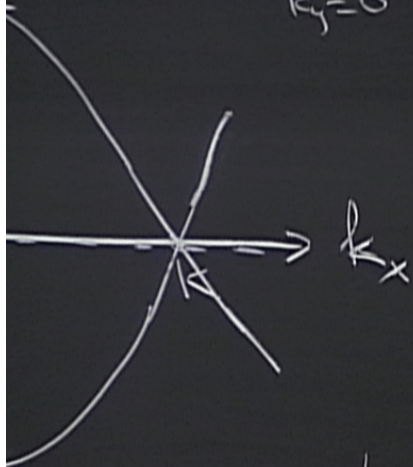
n.-n.n. hops $A \rightarrow B$



Symmetry

$$\begin{pmatrix} u_k \\ v_k \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon_k & 0 \\ 0 & -\epsilon_k \end{pmatrix} \begin{pmatrix} u_k \\ v_k \end{pmatrix} \quad \hbar \mathbf{v}_F = (\hbar v_F \hat{x} + \hbar v_F \hat{y})$$

$k_y = 0$

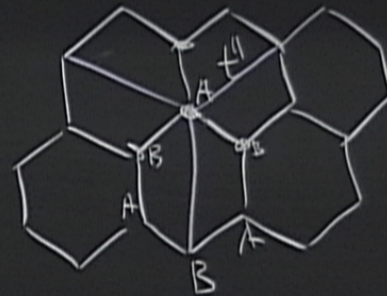


Fermi point

Are Dirac
robust?

A ←

Longer-range
h.-h.h. hops



K-point
Wave function
zero-energy

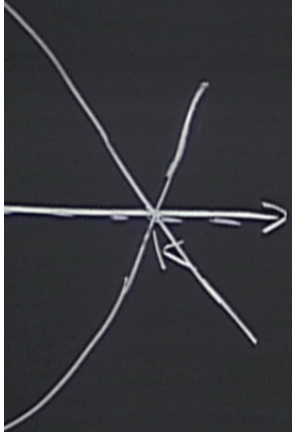
$$\psi(\mathbf{r}) = e^{i\mathbf{k} \cdot (\mathbf{R}_i + \mathbf{e}_i)}$$

$$\psi = \begin{cases} 0, & \text{A-sublattice} \end{cases}$$

Symmetry

$$|u_k\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_k \\ \phi_k \end{pmatrix} \quad G_z |u_k\rangle = -\epsilon_k |u_k\rangle \quad \hbar \mathbf{k} \cdot \mathbf{v}_F = (\hbar v_F)(g_x \delta k_x + g_y \delta k_y)$$

$$k_y = 0$$



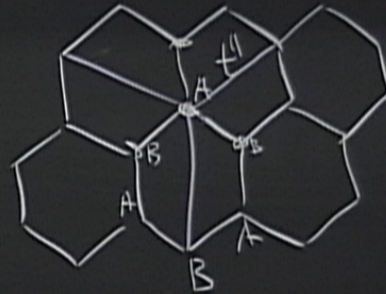
Fermi points

Are Dirac points robust?

$$A \longleftrightarrow B$$

Longer-range hops

n.-n.n. hops $A \rightarrow B$



K-point

Wave function

Zero-energy

$$\psi(\mathbf{r}) = e^{i\mathbf{k} \cdot \mathbf{r}}$$

$$\psi = \begin{cases} 0 \\ \dots \end{cases}$$

$$\begin{pmatrix} u_k \\ v_k \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon_k \\ \sigma_z(u_k) \end{pmatrix} \quad \epsilon_k = \hbar v_F (g_x \sigma_{kx} + g_y \sigma_{ky}) \quad \epsilon_k = \hbar v_F (g_x \sigma_{kx} + g_y \sigma_{ky}) \quad \begin{pmatrix} u_k \\ v_k \end{pmatrix} e$$

points

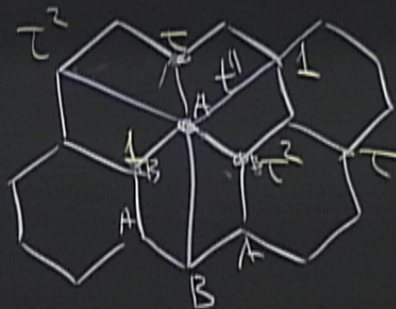
Dirac points

?

$\rightarrow B$

hops

$A \rightarrow B$



K-point
Wave function
zero-energy

$$\psi(r) = e^{i\vec{k}(\vec{R}_i + \vec{e}_i)}$$

$$\psi = \begin{cases} 0, & \text{A-sublattice} \\ \text{sum of amplitudes} \\ \text{for 3 sites} = 0 \end{cases}$$

$$\tau = e^{\frac{2\pi i}{3}}$$

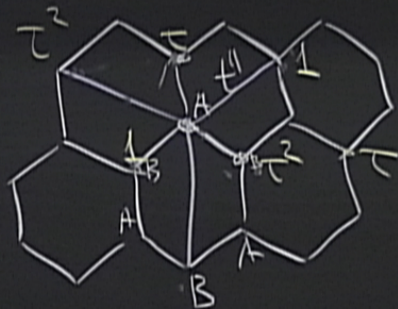
$$1, \tau, \tau^2$$

$$\begin{pmatrix} u_k \\ v_k \end{pmatrix} = \frac{1}{\epsilon_k} \begin{pmatrix} \epsilon_k \\ -\epsilon_k \end{pmatrix} \quad \epsilon_k = \hbar v_F (g_x \delta k_x + g_y \delta k_y) \quad \begin{pmatrix} u_k \\ v_k \end{pmatrix} e^{i k \cdot r}$$

points

Dirac points

?



$1, \tau, \tau^2$

K-point
Wave function
zero-energy

$$\psi(r) = e^{i \vec{k} \cdot (\vec{R}_i + \vec{e}_i)}$$

$$\psi = \begin{cases} 0, & \text{A-sublattice} \\ \text{sum of amplitudes} \\ \text{for 3 sites} = 0 \end{cases}$$

$$\tau = e^{\frac{2\pi i}{3}}$$

$1, \tau, \tau^2$

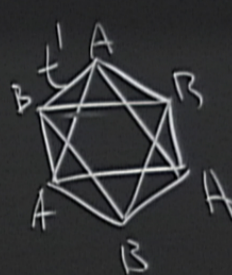
A diagram of a hexagonal lattice structure. Vertices are labeled with letters: 'A' at the top-left, 'B' at the bottom-left, and 'A' at the bottom-right. Edges are labeled with numbers: '1' on the top-right edge, '2' on the middle-right edge, '3' on the bottom-right edge, and '4' on the top-left edge. The lattice is composed of interconnected hexagons.

$$\psi(r) = e^{i\vec{k}(\vec{r}_1 + \vec{r}_2)}$$

$$\psi = \begin{cases} 0, & \text{A-sublattice} \\ \text{sum of amplitudes} \\ \text{for 3 sites} = 0 \end{cases}$$

$$T = \frac{2\pi i}{3}$$

$$1, T, T^2$$

$$A \leftrightarrow A, B \leftrightarrow B \text{ loops}$$


$$h_k = \begin{bmatrix} g(k) & t^*(k) \\ t(k) & g(k) \end{bmatrix}$$

$$E_k = \hbar \omega_k = \hbar v_F (g_x \delta k_x + g_y \delta k_y) \quad \left(\begin{pmatrix} v_k \end{pmatrix} e \right) \xrightarrow{\text{conjugate}}$$

K-point
Wave function
zero-energy

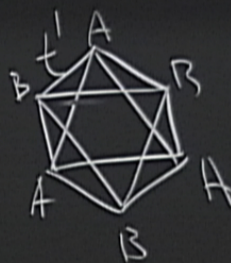
$$\psi(\vec{r}) = e^{i\vec{k}(\vec{R}_i + \vec{e}_x)}$$

$$\psi = \begin{cases} 0, & \text{A-sublattice} \\ \text{sum of amplitudes} \\ \text{for 3 sites} = 0 \end{cases}$$

$$\tau = e^{2\pi i/3}$$

$$1, \tau, \tau^2$$

$A \leftrightarrow A, B \leftrightarrow B$ hops



$$h_{\vec{k}} = \begin{bmatrix} g(k) & t^*(k) \\ t(k) & g(k) \end{bmatrix}$$

$$E = g(\vec{k}) \pm \hbar v_F \sqrt{\delta k_x^2 + \delta k_y^2} \approx$$

$$\approx g_0 + \frac{g_2 k^2}{2} \pm \hbar v_F \sqrt{\delta k_x^2 + \delta k_y^2}$$

$$ck \quad n \quad k' \leftrightarrow k \quad = \hbar v_F (g_x \delta k_x + g_y \delta k_y) + \hbar v_F \delta k_x + \hbar v_F \delta k_y \quad (v_k) e \quad \rightarrow \text{conjugate}$$

K-point

Wave function

Zero-energy

$$\psi(\vec{r}) = e^{i\vec{k} \cdot \vec{r}}$$

$$\psi = \begin{cases} 0 \\ 1 \end{cases}$$

$$\tau = e^{\frac{2\pi i}{3}}$$

$$1, \tau, \tau^2$$

$A \leftrightarrow A, B \leftrightarrow B$ hops



$$h_{\vec{k}} = \begin{bmatrix} g(k) & t^*(k) \\ t(k) & g(k) \end{bmatrix}$$

$$E = g(\vec{k}) \pm \hbar v_F \sqrt{\delta k_x^2 + \delta k_y^2} \approx$$

$$\approx g_0 + \frac{g_2 k^2}{2} \pm \hbar v_F \sqrt{\delta k_x^2 + \delta k_y^2}$$

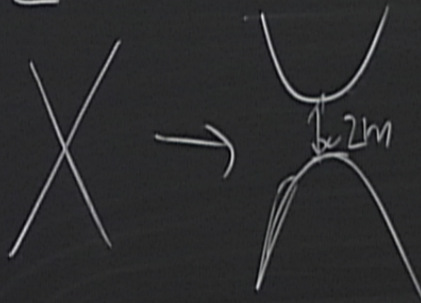
Break $A \leftrightarrow B$ symmetry

BN $\rightarrow$ large
band gap
 $\sim eV$

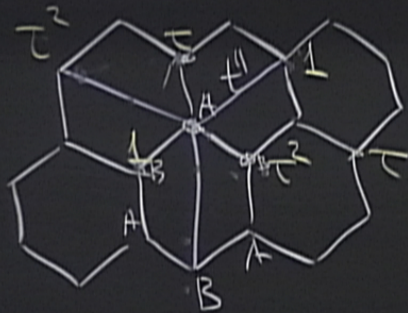
$$\epsilon_A = +m, \quad \epsilon_B = -m$$

$$h_k = \begin{bmatrix} m & t(k) \\ t(k) & -m \end{bmatrix}$$

$$\epsilon(k) \approx \sqrt{m^2 + (\hbar v_F k)^2}$$



$$|u_k\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} |u_k\rangle \\ |v_k\rangle \end{pmatrix} \quad \text{with } |u_k\rangle = \frac{1}{\sqrt{2}} (g_x \delta k_x + g_y \delta k_y) \quad \text{conjugate}$$



$1, \tau, \tau^2$

K-point
Wave function
zero-energy

$$\Delta U \sim t'' a$$

$$t' \ll t$$

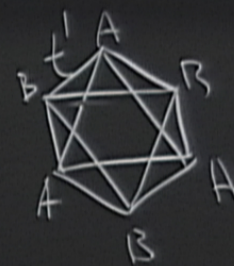
$$\psi(r) = e^{i\vec{k}(\vec{R}_i + \vec{e}_i)}$$

$$\psi = \begin{cases} 0, & \text{A-sublattice} \\ \text{sum of amplitudes} \\ \text{for 3 sites} = 0 \end{cases}$$

$$\tau = e^{\frac{2\pi i}{3}}$$

$1, \tau, \tau^2$

$$A \leftrightarrow A, B \leftrightarrow B \text{ lines}$$



$$h_k =$$

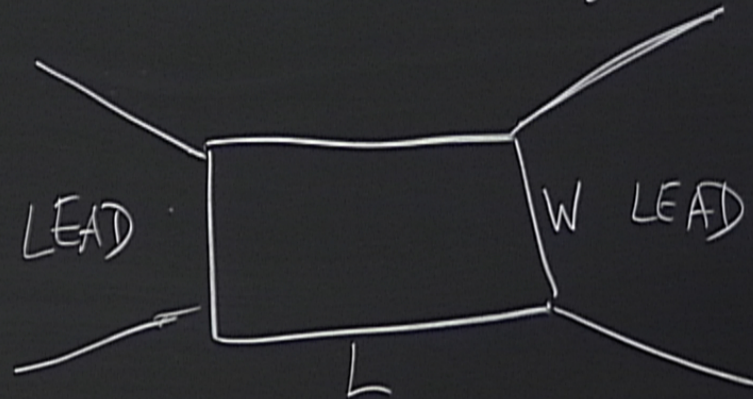
$$\epsilon = \sqrt{\delta k_x^2 + \delta k_y^2}$$

Conductivity

$$G = e^2 v D$$



$$v(\mu) \sim \mu = 0$$



Disorder

Introduces some carriers

$$\varepsilon_x \sim \frac{\hbar}{\tau}$$

$$\nu = \int \delta(\varepsilon - \nu p) \frac{d^2 p}{(2\pi\hbar)^2} = \frac{\varepsilon}{v_F^2 2\pi\hbar^2} \quad D = \frac{v_F^2 \tau}{2}$$

$$\sigma \sim e^2 \nu \cdot D \sim e^2 \cdot \frac{\varepsilon_x}{v_F^2 \hbar} \times \frac{v_F^2 \tau}{2} \sim \frac{e^2}{h}$$