

Title: 12/13 PSI - Condensed Matter Review Lecture 8

Date: Feb 07, 2013 11:30 AM

URL: <http://pirsa.org/13020080>

Abstract:

"Perturbative" quantum correction

$$\left\{ \begin{array}{l} -\frac{2v_F}{D} (L_{\max} - L_D), \quad d=1, \quad L_{\max} = \min(\sqrt{D\tau_p}, L) \\ -2 \end{array} \right.$$

"Perturbative" quantum correction

$$-\frac{2v_F}{D}(L_{\max} - L_D), d=1, \quad L_{\max} = \min(\sqrt{D\tau_p}, L)$$

$$-\frac{2v_F\lambda_F}{D} \ln \frac{L_{\max}}{L_D}, d=2$$

$$\frac{2v_F\lambda_F^2}{D} \left(\frac{1}{L_D} - \frac{1}{L_{\max}} \right)$$

"Perturbative" quantum correction

$$\left\{ \begin{array}{l} -\frac{2v_F}{D} (L_{\max} - L_D), \quad d=1, \quad L_{\max} = \min(\sqrt{D\tau_p}, L) \\ -\frac{2v_F \lambda_F}{D} \ln \frac{L_{\max}}{L_D}, \quad d=2 \\ \frac{2v_F \lambda_F^2}{D} \left(\frac{1}{L_D} - \frac{1}{L_{\max}} \right), \quad d=3 \end{array} \right. \quad \text{"weak localization correction"}$$

"Perturbative" quantum correction

$$\left\{ \begin{array}{l} -\frac{2v_F}{D} (L_{\max} - L_D), d=1, \quad L_{\max} = \min(\sqrt{D\tau_p}, L) \end{array} \right.$$

$$-\frac{2v_F \lambda_F}{D} \ln \frac{L_{\max}}{L_D}, d=2$$

$$\frac{2v_F \lambda_F^2}{D} \left(\frac{1}{L_D} - \frac{1}{L_{\max}} \right), d=3$$

"weak localization correction"

tion

$\overline{\sigma}_p, L)$

ak localization correction'

1) Ohm's law breaks down $\beta(L)$

tion

$\overline{\sigma}_p, L)$

ak localization correction'

+ 1) Ohm's law breaks down $\beta(L)$

2) $G(T)$

"Perturbative" quantum correction

$$\frac{\delta G}{G} = \begin{cases} -\frac{1}{2} \frac{L_{\max} - L_D}{L_D}, d=1, & L_{\max} = \min(\sqrt{D\tau_p}, L) \\ -\ln \frac{L_{\max}}{L_D}, d=2 & \text{"weak localization correction"} \\ \frac{1}{2} \left(\frac{1}{L_D} - \frac{1}{L_{\max}} \right), d=3 \end{cases}$$

- + 1) Ohm's law breaks down $\propto(L)$
 2) $G(T)$, $\tau_p \sim \frac{1}{T^\alpha}$

tion

$\overline{\tau_p}, L)$

ak localization correction'

+ 1) Ohm's law breaks down $\beta(L)$

2) $G(T)$, $\tau_p \sim \frac{1}{T^\alpha}$ $|\delta G| \nearrow T \searrow$

tion

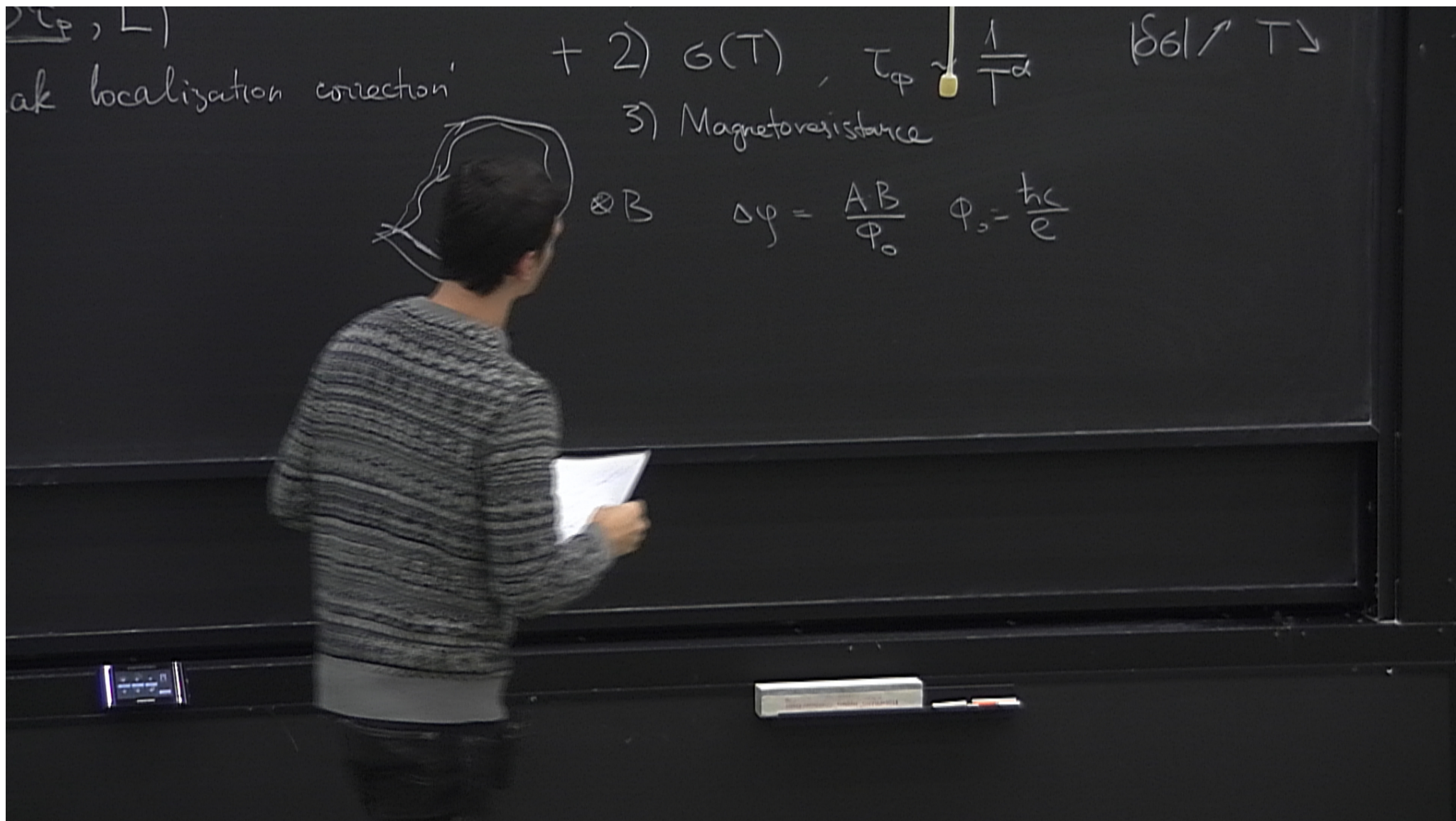
$\overline{\tau_p}, L)$

ak localization correction'

+ 1) Ohm's law breaks down $\propto(L)$

+ 2) $G(T)$, $\tau_p \sim \frac{1}{T^\alpha}$ $|\delta G| \propto T^\alpha$

3) Magnetoresistance



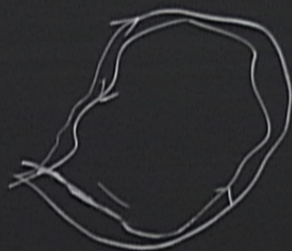
$2\tau_F, L)$

ak localization correction

+ 2) $G(T)$, $\tau_F \sim \frac{1}{T^\alpha}$

$|\delta G| \propto T^\alpha$

3) Magnetoresistance



$\otimes B$

$$\Delta\varphi = \frac{A \cdot B}{\Phi_0} \quad \Phi_0 = \frac{hc}{e}$$

$$\Delta\varphi \ll 2\pi$$

$$G = \begin{cases} -\frac{2v_F \lambda_F}{D} \ln\left(\frac{\sqrt{D} L_D}{L_D}\right), d=2 \\ \frac{2v_F \lambda_F^2}{D} \left(\frac{1}{L_D} - \frac{1}{L_{\max}}\right), d=3 \end{cases}$$

"Weak localization correction"

$$\frac{G(B) - G(0)}{G(0)} = - \int_0^{L_{\max}} \frac{v_F \lambda_F}{(D t)^{d/2}} \times \left[\cos \right]$$

3) Magnetoresistance

$\otimes B$

$\Delta \rho = A$
 $\Delta \rho \ll \rho$

turn correction

$$l_{\text{max}} = \min(\sqrt{D\tau_F}, L)$$

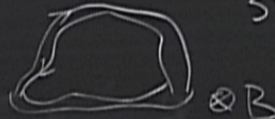
"weak localization correction"

+ 1) Ohm's law breaks down $\propto(L)$

+ 2) $G(T)$, $\tau_F \sim \frac{1}{T^\alpha}$ $|\delta G| \propto T^\alpha$

3) Magnetoresistance

$$\frac{G(B) - G(0)}{G(0)} = - \int_0^{\tau_{\text{max}}} \frac{v_F \lambda_F}{D} d\tau$$



$$\Delta\varphi = \frac{AB}{\Phi_0} \quad \Phi_0 = \frac{h c}{e}$$

$$\Delta\varphi \ll 2\pi$$

turn correction

$$L_{\text{max}} = \min(\sqrt{D\tau_p}, L)$$

"weak localization correction"

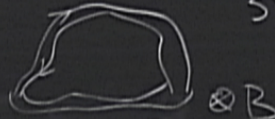
+ 1) Ohm's law breaks down $\propto(L)$

+ 2) $G(T)$, $\tau_p \sim \frac{1}{T^\alpha}$ $|\delta G| \propto T^\alpha$

3) Magnetoresistance

$$\frac{G(B) - G(0)}{G(0)}$$

$$= \frac{\int_0^L \frac{dL}{D\tau_p} \times [1 - \cos(\frac{2eB + D}{\hbar c} L)]}{\int_0^L \frac{dL}{D\tau_p}}$$



$$\Delta\varphi = \frac{AB}{\Phi_0} \quad \Phi_0 = \frac{hc}{e}$$

$$\Delta\varphi \ll 2\pi \quad A(t) \sim D \cdot t$$

turn correction

$$l_{\text{max}} = \min(\sqrt{D\tau_F}, L)$$

"weak localization correction"

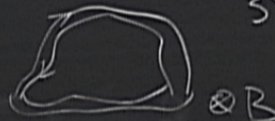
+ 1) Ohm's law breaks down $\propto(L)$

+ 2) $G(T)$, $\tau_F \sim \frac{1}{T^\alpha}$ $|\delta G| \propto T^\alpha$

3) Magnetoresistance

$$\frac{G(B) - G(0)}{G(0)}$$

$$= \frac{\lambda_F}{(Dl)^{1/2}} \frac{d}{dt} \left[\frac{1 - \cos\left(\frac{2eB + D}{\hbar c}\right)}{2} \right]$$



$$\Delta\varphi = \frac{AB}{\Phi_0} \quad \Phi_0 = \frac{hc}{e}$$

$$\Delta\varphi \ll 2\pi \quad A(t) \sim D \cdot t$$

turn correction

$$l_{\text{max}} = \min(\sqrt{D\tau_p}, L)$$

"weak localization correction"

+ 1) Ohm's law breaks down $\propto(L)$

+ 2) $G(T)$, $\tau_p \sim \frac{1}{T^\alpha}$ $|\delta G| \propto T^\alpha$

3) Magnetoresistance



$$\frac{G(B) - G(0)}{G(0)} = - \int_0^{\tau_{\text{max}}} \frac{v_F \lambda_F}{(D t)^{3/2}} dt \left[1 - \cos \left(\frac{2eB + D}{\hbar c} \right) \right]$$

$$\Delta \varphi = \frac{AB}{\Phi_0} \quad \Phi_0 = \frac{hc}{e}$$

$$\Delta \varphi \ll 2\pi \quad A(t) \sim D \cdot t$$

$$L \ll \frac{\hbar c}{2eB D} = l_B \quad \frac{G(B) - G(0)}{G(0)} = -$$

turn correction

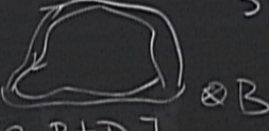
$$L_{\max} = \min(\sqrt{D\tau_F}, L)$$

"weak localization correction"

+ 1) Ohm's law breaks down $\propto(L)$

+ 2) $G(T)$, $\tau_F \sim \frac{1}{T^\alpha}$ $|\delta G| \propto T^\gamma$

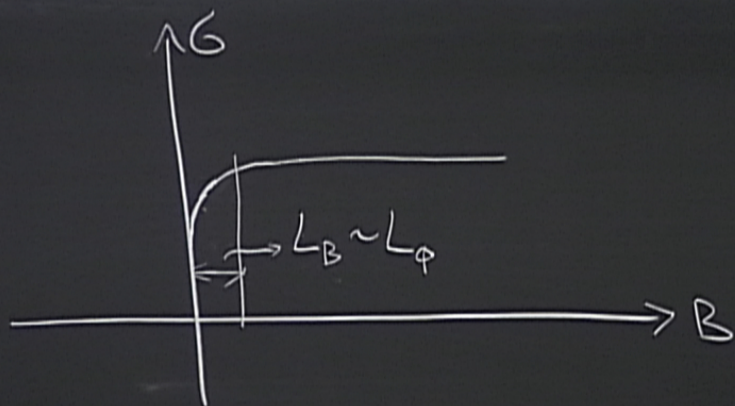
3) Magnetoresistance



$$\frac{G(B) - G(0)}{G(0)} = - \int_0^{\tau_{\max}} \frac{v_F \lambda_F}{(Dt)^{d/2}} \times \left[1 - \cos \frac{2eB + D}{\hbar c} \right] dt$$

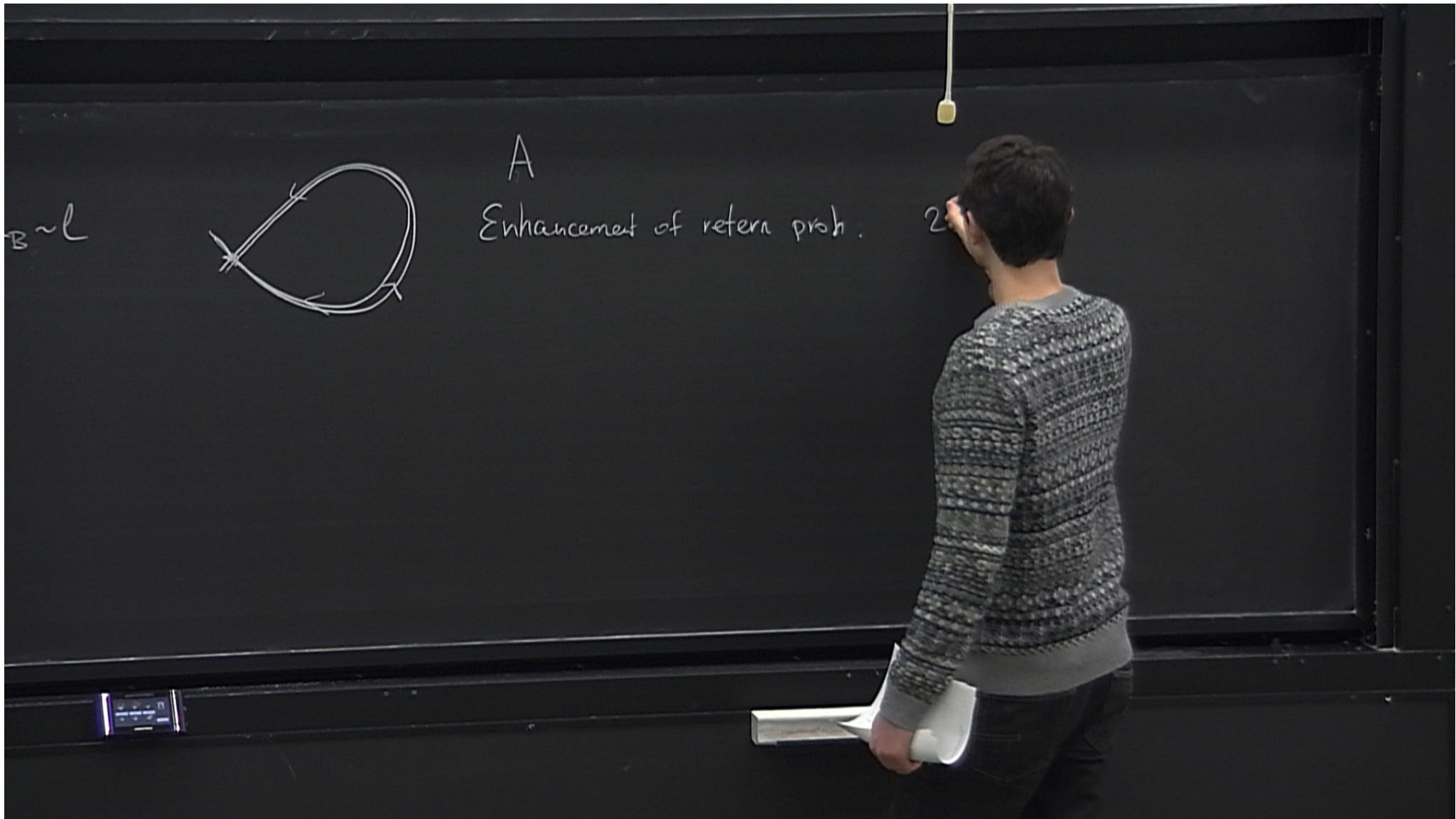
$$L \ll \frac{\hbar c}{2eB D} = t_B \quad \frac{G(B) - G(0)}{G(0)} = - v_F \lambda_F \int_{t_B}^{\tau_{\max}} \frac{dt}{(Dt)^{d/2}} \stackrel{d=2}{=} \frac{v_F \lambda_F}{D} \log \frac{L_{\max}^2}{L_B^2}$$

$\Delta \varphi = \frac{AB}{\Phi_0}$ $\Phi_0 = \frac{\hbar c}{e}$ $A(t) \sim D \cdot t$ magnetic length
 $L_B = \sqrt{\frac{\hbar c}{eB}}$

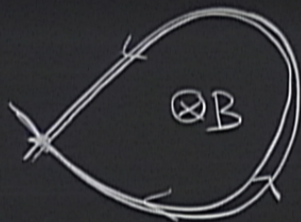


L Drude formula
magnetoconductance

L

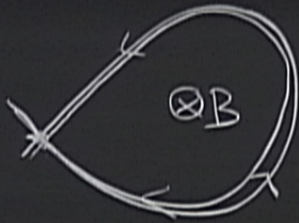


$B \sim l$



A
Enhancement of prob. $2|A|^2$
 $A e^{i\varphi}$

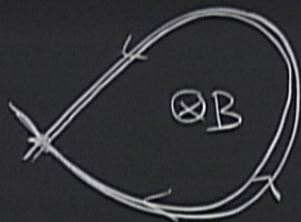
$B \sim l$



A
Enhancement of return prob. $2|A|^2$

$$Ae^{i\varphi} \quad Ae^{-i\varphi} \quad |Ae^{i\varphi} + Ae^{-i\varphi}|^2 = 2|A|^2 + 2|A|^2 \cos \varphi$$

$B \sim l$



A
Enhancement

etern prob.

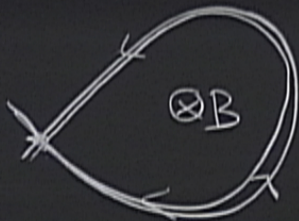
$$2|A|^2$$

$$A e^{i\varphi}$$

$$A e^{-i\varphi}$$

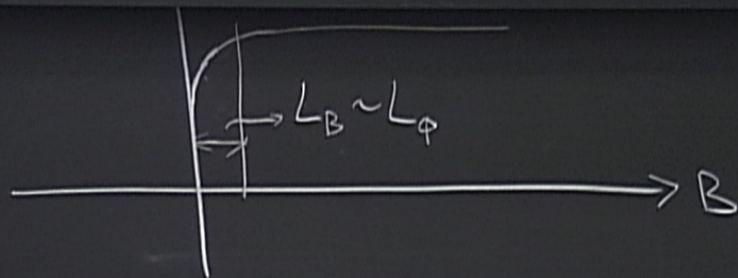
$$|A e^{i\varphi} + A e^{-i\varphi}|^2 = 2|A|^2 + 2|A|^2 \underline{\underline{\cos \varphi}}$$

$B \sim l$



A
Enhancement of h $2|A|^2$

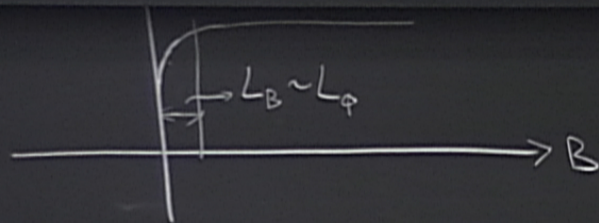
$$A e^{i\varphi} + A e^{-i\varphi} = 2|A|^2 \cos \varphi$$
$$\varphi \sim \frac{B \cdot A \cdot 2\pi r}{\Phi_0}$$



magneto-resistance $L_B \sim L$

$$\underline{\underline{d=2}}$$

$$\frac{\delta G}{G} = -\frac{2v_F \hbar \tau}{D} \log D =$$



$$\underline{d=2}$$

magneto-resistance $L_B \sim L$

$$\frac{\delta G}{G} = -\frac{2v_F \lambda_F}{D} \log \frac{L_{max}}{L_D}$$

$$D = \frac{v_F^2 \tau}{2}$$

$$\frac{\delta G}{G} = -\frac{2v_F \lambda_F}{v_F^2 \tau / 2} \log \frac{L_{max}}{L_D} \sim \frac{\lambda_F}{\tau} \log \frac{L_{max}}{L_D} \sim \left(\frac{1}{k_F L} \right) \log \frac{L_{max}}{L_D}$$

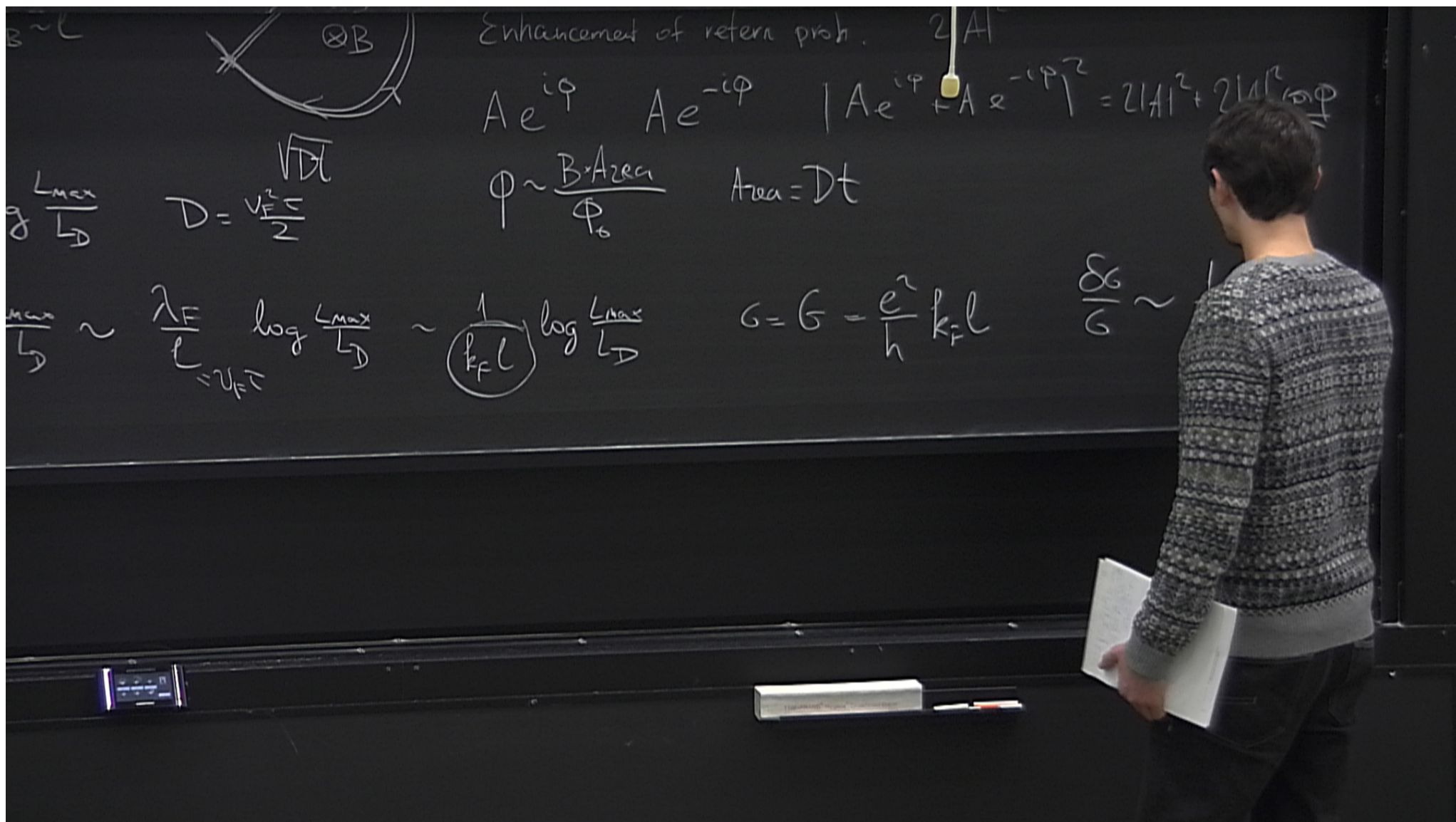
$$\frac{t_c}{2eB D} = t_B \quad \frac{G(B) - G(0)}{G(0)} = -v_F \lambda_F$$

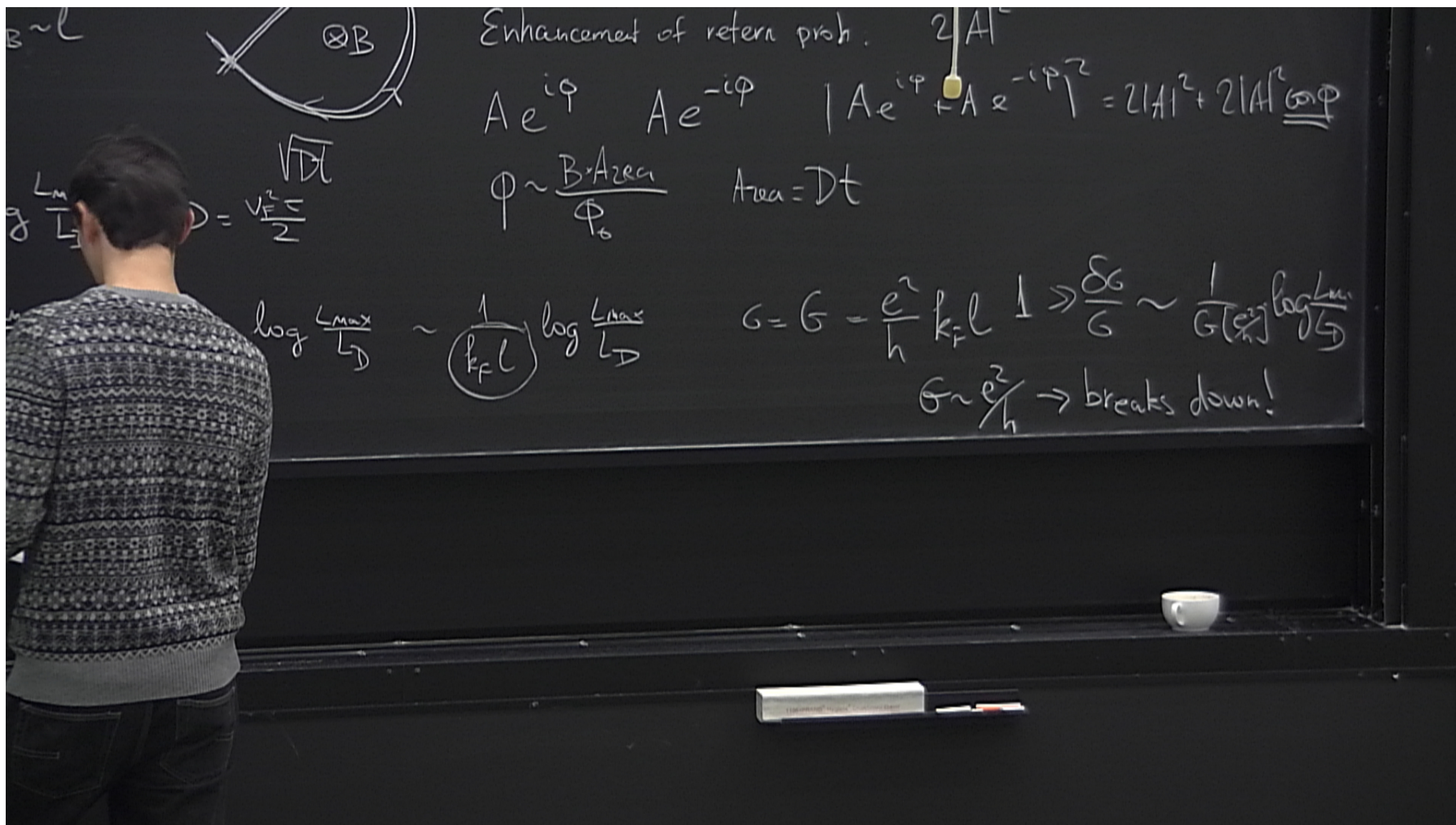


Enhancement

$$A e^{i\varphi}$$

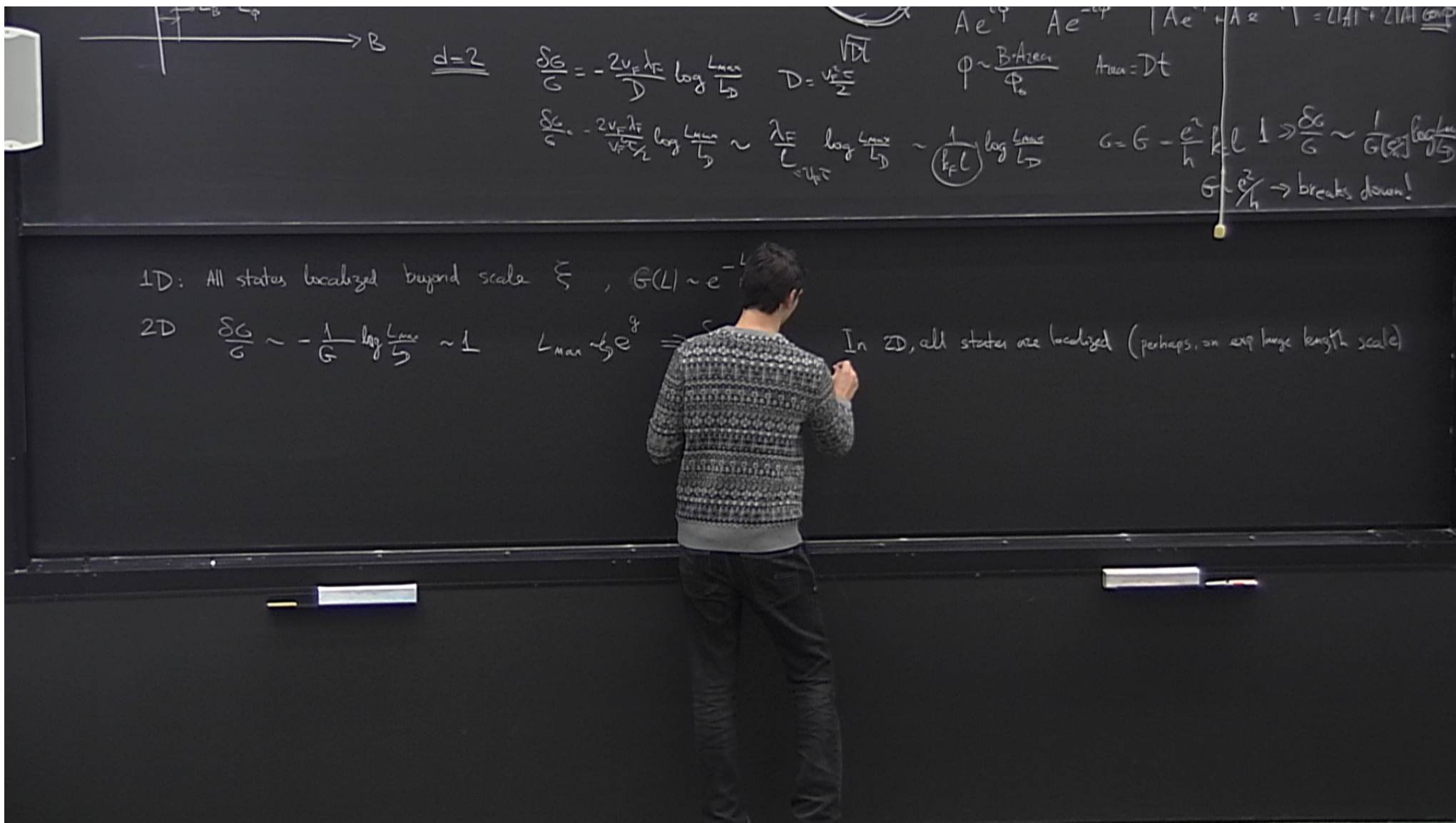
$$\varphi \sim \frac{B \cdot A_{2D}}{\Phi_0}$$

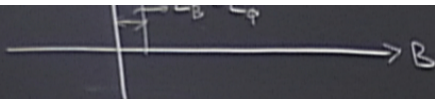




1D: All states localized beyond scale ξ , $G(L) \sim e^{-L/\xi}$

2D $-\frac{1}{G} \log \frac{L_{\max}}{L} \sim 1$ $L_{\max} \sim e^g \Rightarrow \frac{\delta G}{G} \sim -1$





d=2

$$\frac{\delta G}{G} = -\frac{2v_F \lambda_F}{D} \log \frac{L_{\max}}{L_D} \quad D = \frac{v_F^2 \tau}{2}$$

$$\frac{\delta G}{G} = -\frac{2v_F \lambda_F}{v_F^2 \tau} \log \frac{L_{\max}}{L_D} \sim \frac{\lambda_F}{L} \log \frac{L_{\max}}{L_D} \sim \frac{1}{k_F L} \log \frac{L_{\max}}{L_D}$$

$Ae^{i\varphi} \quad Ae^{-i\varphi} \quad |Ae^{i\varphi} + Ae^{-i\varphi}|^2 = 2|A|^2 + 2|A|^2 \cos \varphi$
 $\varphi \sim \frac{B \cdot A_{\text{area}}}{\Phi_0} \quad A_{\text{area}} = Dt$

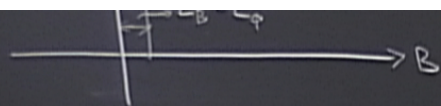
$G = G_0 - \frac{e^2}{h} k_F L \Rightarrow \frac{\delta G}{G} \sim \frac{1}{G_0} \log \frac{L_{\max}}{L_D}$
 $G \sim \frac{e^2}{h} \rightarrow \text{breaks down!}$

1D: All states localized beyond scale ξ , $G(L) \sim e^{-L/\xi}$

2D $-\frac{1}{G} \log \frac{L_{\max}}{L_D} \sim 1 \quad L_{\max} \sim e^8 \Rightarrow \frac{\delta G}{G} \sim -1$

3D

In 2D, all states are localized (perhaps, on exp large length scale)
Metal - unstable!



d=2

$$\frac{\delta G}{G} = -\frac{2v_F \lambda_F}{D} \log \frac{L_{max}}{L_D}$$

$$D = \frac{v_F^2 \tau}{2}$$

$$Ae^{i\varphi} Ae^{-i\varphi} |Ae^{i\varphi} + Ae^{-i\varphi}|^2 = 2|A|^2 + 2|A|^2 \cos \varphi$$

$$\varphi \sim \frac{B \cdot A_{area}}{\Phi_0} \quad A_{area} = Dt$$

$$\frac{\delta G}{G} = -\frac{2v_F \lambda_F}{v_F^2 \tau} \log \frac{L_{max}}{L_D} \sim \frac{\lambda_F}{L} \log \frac{L_{max}}{L_D} \sim \frac{1}{k_F L} \log \frac{L_{max}}{L_D}$$

$$G = G_0 - \frac{e^2}{h} k_F L \quad 1 \Rightarrow \frac{\delta G}{G} \sim \frac{1}{G_0} \log \frac{L_{max}}{L_D}$$

$G \sim \frac{e^2}{h} \rightarrow \text{breaks down!}$

1D: states localized beyond scale ξ , $G(L) \sim e^{-L/\xi}$

2D: $\frac{\delta G}{G} \sim -\frac{1}{G} \log \frac{L_{max}}{L_D} \sim 1 \quad L_{max} \sim e^8 \Rightarrow \frac{\delta G}{G} \sim -1$

In 2D, all states are localized (perhaps, on exp large length scale)
Metal - unstable!

is stable. Strong disorder would cause localization; Anderson

$\frac{\delta G}{G} = -\frac{2v_F \lambda_F}{D} \log \frac{L_{max}}{L_D}$
 $D = \frac{v_F^2 \tau}{2}$
 $\phi \sim \frac{B \cdot A_{area}}{\Phi_0}$
 $A_{area} = Dt$
 $Ae^{i\phi} A e^{-i\phi} |Ae^{i\phi} + A e^{-i\phi}|^2 = 2|A|^2 + 2|A|^2 \cos \phi$

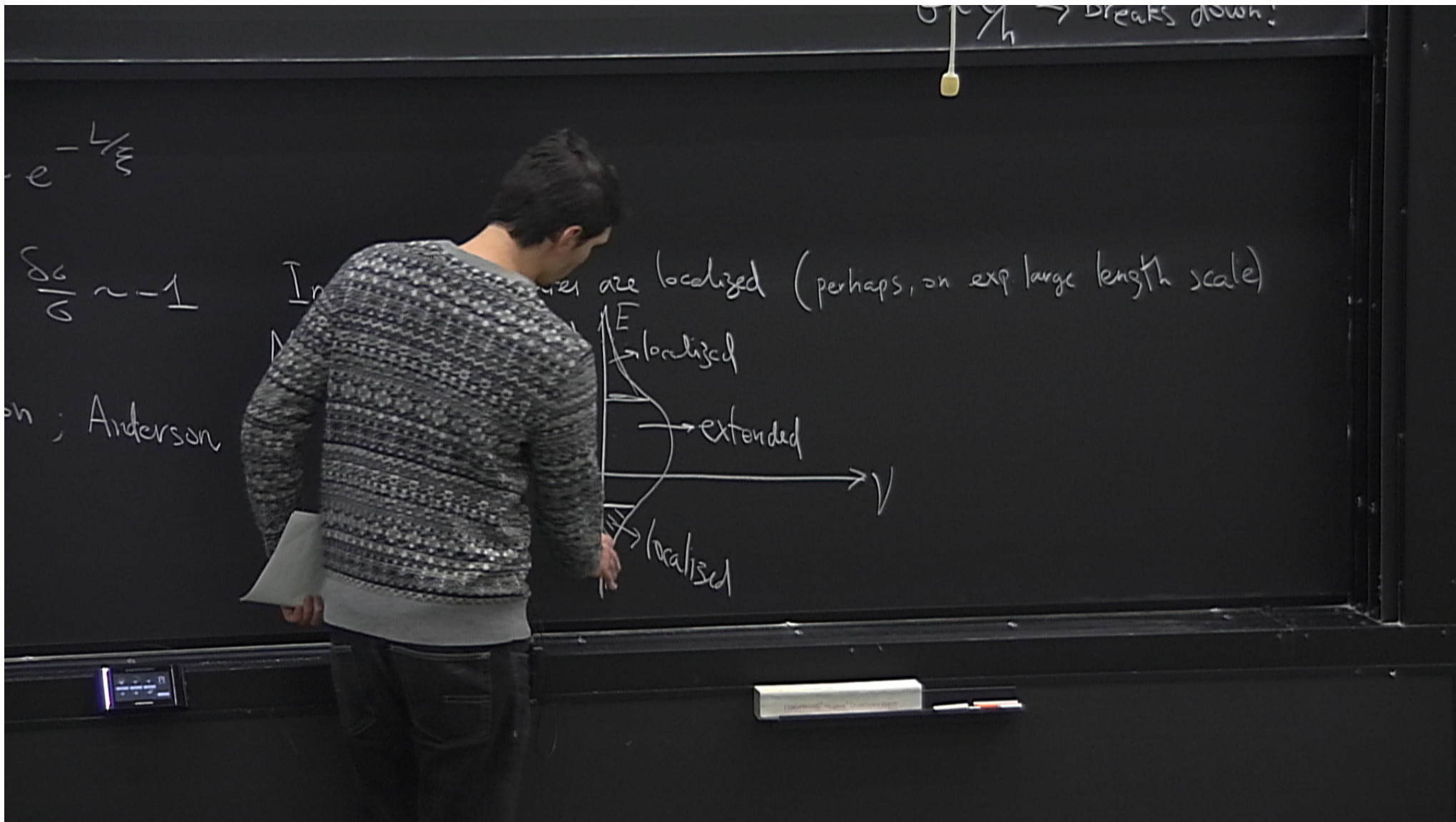
$\frac{\delta G}{G} = -\frac{2v_F \lambda_F}{v_F^2 \tau} \log \frac{L_{max}}{L_D} \sim \frac{\lambda_F}{L} \log \frac{L_{max}}{L_D} \sim \frac{1}{k_F L} \log \frac{L_{max}}{L_D}$
 $G = G_0 - \frac{e^2}{h} k_F L \Rightarrow \frac{\delta G}{G} \sim \frac{1}{G_0} \log \frac{L_{max}}{L_D}$
 $G \sim \frac{e^2}{h} \rightarrow \text{breaks down!}$

1D: All states localized beyond scale ξ , $G(L) \sim e^{-L/\xi}$

2D $\frac{\delta G}{G} \sim -\frac{1}{G} \log \frac{L_{max}}{L_D} \sim 1$ $L_{max} \sim e^8 \Rightarrow \frac{\delta G}{G} \sim -1$

3D: Metal is stable. Strong disorder would cause localization; Anderson
 Metal-insulator transition $\begin{cases} \text{Vary disorder strength} \\ \text{Vary chem potential } E_F \end{cases}$

All states are localized (perhaps, on exp large length scale) stable!



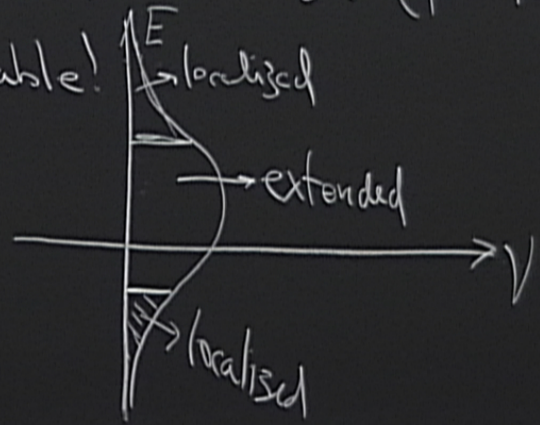
$\frac{1}{h} \rightarrow$ breaks down!

$$e^{-L/\xi}$$

$$\frac{\delta c}{c} \sim -1$$

In 2D, all states are localized (perhaps, on exp large length scale)
Metal - unstable!

on ; Anderson

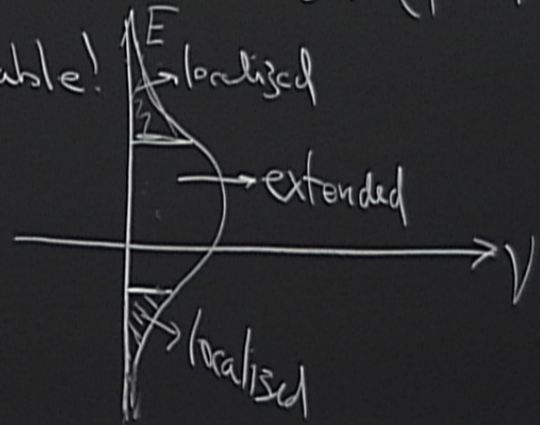


$g \sim 1/h \rightarrow$ breaks down!

$$e^{-L/\xi} \quad g = G/e^2/h$$

$\frac{\delta G}{G} \sim -1$ In 2D, all states are localized (perhaps, on exp large length scale)
Metal - unstable!

on ; Anderson

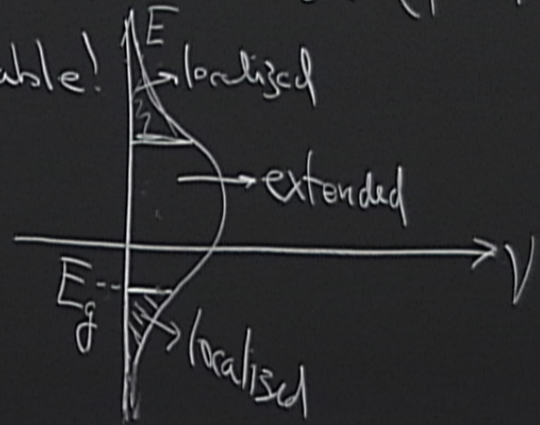


$g \sim 1/h \rightarrow$ breaks down!

$$e^{-L/\xi} \quad g = G/e^2/h$$

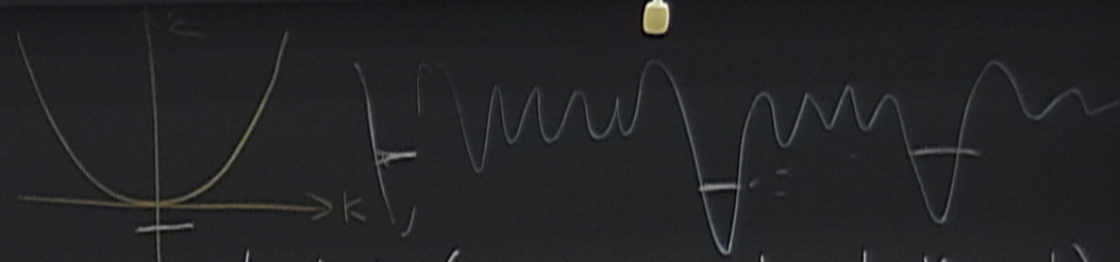
$\frac{\delta G}{G} \sim -1$ In 2D, all states are localized (perhaps, on exp large length scale)
Metal - unstable!

on ; Anderson



$g \propto 1/h \rightarrow$ breaks down!

$e^{-L/\xi} \quad g = G/e^2/h$

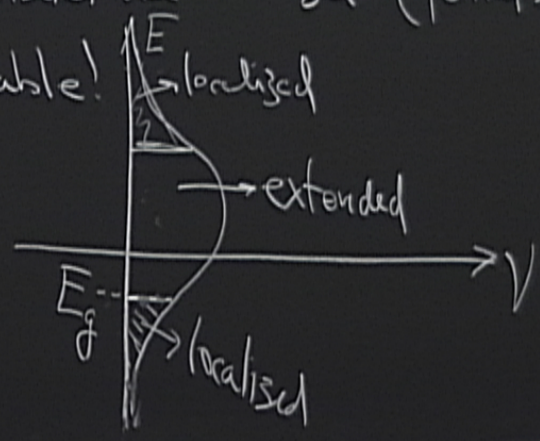


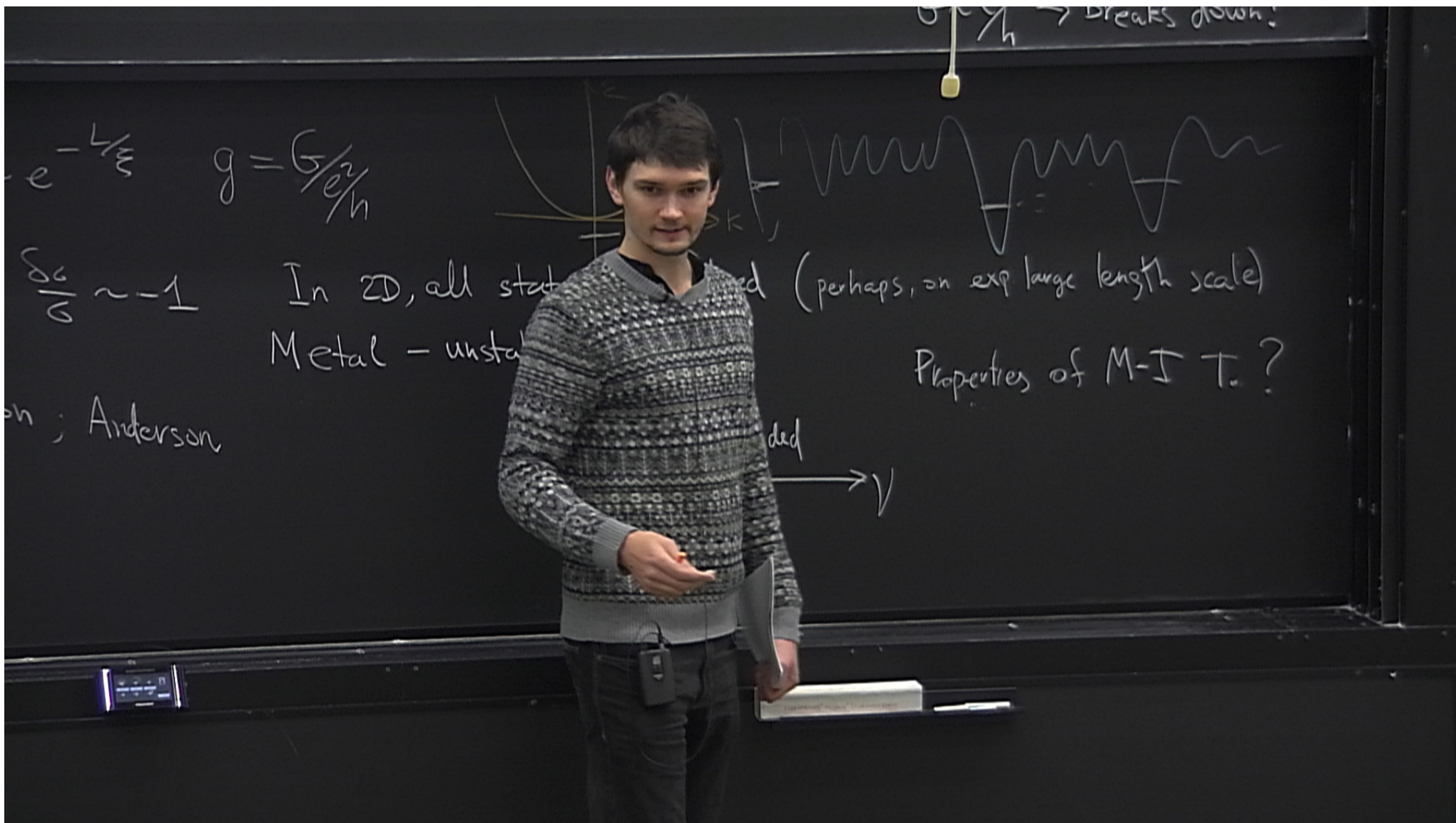
$\frac{\delta G}{G} \sim -1$

In 2D, all states are localized (perhaps, on exp large length scale)
Metal - unstable!

Properties of M-I T.?

on ; Anderson



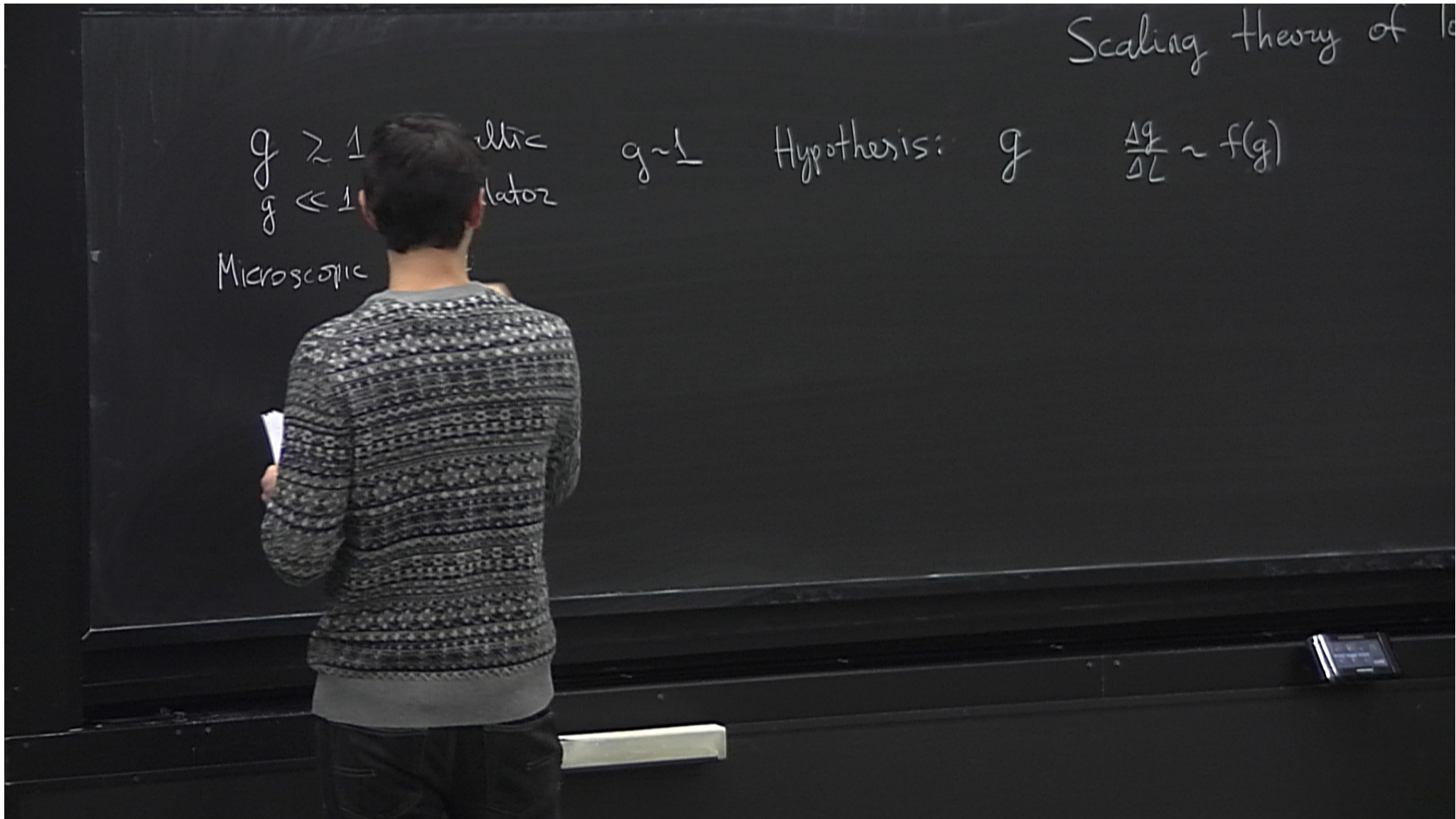


Scaling theory of ν

$g \gtrsim 1$ - metallic
 $g \ll 1$ - insulator

$g \sim 1$

Hypothesis



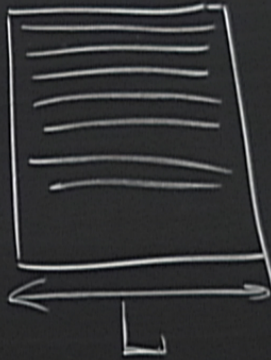
Scaling theory of

$g \gtrsim 1$ - metallic
 $g \ll 1$ - insulator

$g \sim 1$

Hypothesis: $g \frac{\Delta g}{\Delta L} \sim f(g)$

Microscopic picture



discrete spectrum if isolated

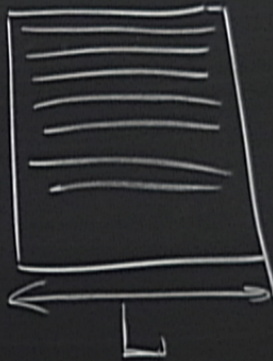
Scaling theory of Γ

$g \gtrsim 1$ - metallic
 $g \ll 1$ - insulator

$g \sim 1$

Hypothesis: $\alpha \quad \frac{\Delta g}{\Delta L} \sim f(g)$

Microscopic picture



discrete spectrum if

Finite width if α to a bigger system, Γ

g theory of localization

$\sim f(g)$

a bigger system, Γ

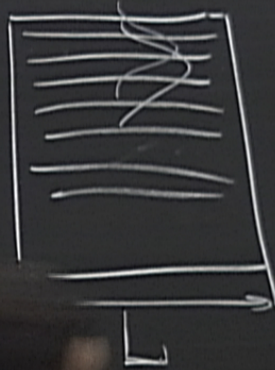
Relevant scales: 1) Level spacing, $\Delta E(L) = \frac{1}{\sqrt{L}} L^d$
2) Level width Γ

$g \gtrsim 1$ - metallic
 $g \ll 1$ - insulator

$g \sim 1$

Hypothesis: $g \frac{\Delta g}{\Delta L} \sim f(g)$

Microscopic picture



discrete spectrum if isolated

finite width if coupled to a bigger system, Γ

$$\frac{\Gamma}{\Delta E(L)} \gg 1$$

Scaling Theory of Localization

hypothesis: $g \quad \frac{\Delta g}{\Delta L} \sim f(g)$

ete spectrum if isolated

te width if coupled to a bigger system, Γ

Relevant scales: 1) Level spacing $\Delta E(L) = \frac{1}{V \cdot L^d}$
2) Level width $\Gamma \sim \frac{\hbar}{\tau_{\text{esc}}} \quad \tau_{\text{esc}} \sim$

$$\frac{\Gamma}{\Delta E(L)} = \hbar L^{d-2} v \cdot D(L)$$

$\frac{\Gamma}{\Delta E(L)} \gg 1$ - extended states $\frac{\Gamma}{\Delta E(L)} \ll 1$

$d=2$ Physical meaning: conductance

$$\nu = \int \delta(\epsilon - \frac{p^2}{2m}) \frac{d^2p}{(2\pi\hbar)^2} \sim \frac{m}{\hbar^2}$$

$$\frac{\Gamma}{\Delta E(L)} \sim \hbar \frac{m}{\hbar^2} \cdot \frac{V_F^2}{2} \sim \frac{P_F}{\hbar} \downarrow$$

Hypothesis $\frac{d \log g(L)}{d \log L} = \beta_2(g)$

Deeply localized: $g \sim e^{-L}$

Hypothesis $\frac{d \log g(L)}{d \log L} = \beta_d(g)$

Deeply localized: $\beta_d(g) = -\frac{L}{\xi} g g + o(1)$

Hypothesis $\frac{d \log g(L)}{d \log L} = \beta_d(g)$

Deeply localized: $g \sim e^{-\frac{L}{\xi}}$
 $\beta_d(g) = \log g + o(1)$

Almost metallic: $d=2$ $\frac{\delta g}{g} = -\frac{1}{g}$

Hypothesis $\frac{d \log g(L)}{d \log L} = \beta_d(g)$

Deeply localized: $g \sim e^{-\frac{L}{\xi}}$
 $\beta_d(g) = \log g + o(1)$

Almost metallic: $d=2$ $\frac{\delta g}{g} = -\frac{1}{g} \Delta \log L$

$\frac{\Delta \log g}{\Delta \log L} = -\frac{1}{g} \quad g \gg 1 \quad \beta_2(g) = -\frac{1}{g}$

$\Rightarrow \log g$

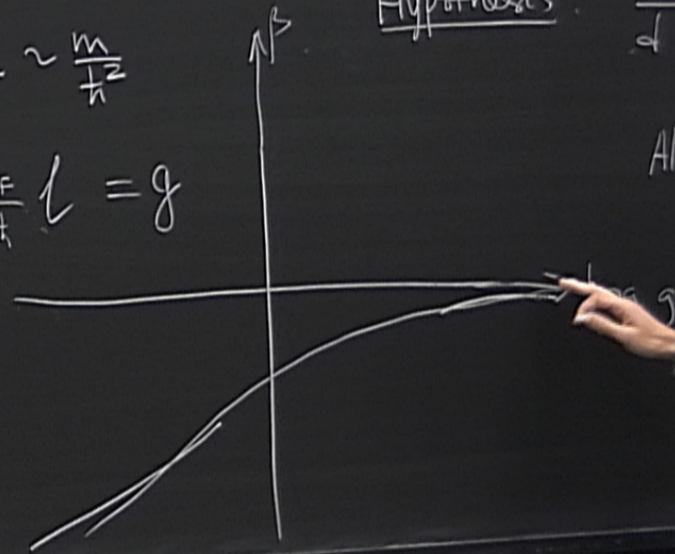
$d=2$ Physical meaning: conductance

$$\nu = \int \delta(\epsilon - \frac{p^2}{2m}) \frac{d^2p}{(2\pi\hbar)^2} \sim \frac{m}{\hbar^2}$$

$$\frac{\Gamma}{\Delta E(L)} \sim \hbar \frac{m}{\hbar^2} \frac{v_F^2}{2} \sim \frac{p_F}{\hbar} L = g$$

Hypothesis: $\frac{d \log g(L)}{d \log L} = \beta$

Almost metallic



$$-\frac{\log g(L)}{\log L} = \beta_d(g)$$

Deeply localized: $g \sim e^{-\frac{L}{\xi}}$

$$\beta_d(g) = \log g + o(1)$$

Almost metallic: $d=2$ $\frac{\delta g}{g} = -\frac{1}{g} \Delta \log L$ $\beta_3(g) = 1 - \frac{1}{g} + \dots$

$$\frac{\Delta \log g}{\Delta \log L} = -\frac{1}{g}$$

$$g \gg 1 \quad \beta_2(g) = -\frac{1}{g}$$

$$\beta_1(g) = -1 - \frac{1}{g} + \dots$$

$d=2$ Physical meaning: conductance

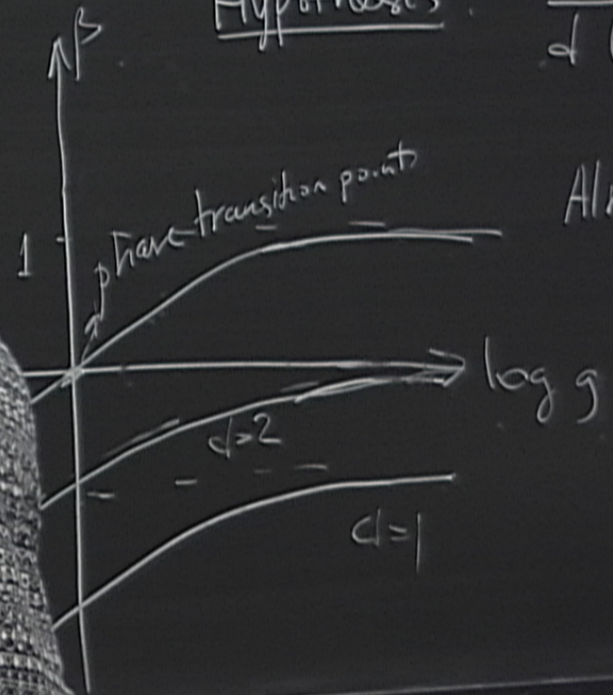
$$\nu = \int \delta(\epsilon - \frac{p^2}{2m}) \frac{d^2 p}{(2\pi\hbar)^2} \sim \frac{m}{\hbar^2}$$

$$\frac{\Gamma}{\Delta E(L)} \sim \hbar \frac{m}{\hbar^2} \frac{V_{FC}^2}{2} \sim \frac{P_F}{\hbar} g$$

$$\mathcal{O}(g - g_c) \sim (g - g_c)$$

$$\sum (g_c - g) \sim (g_c - g)$$

Hypothesis: $\frac{d \log g(L)}{d \log L}$



$d=2$ Physical meaning: conductance

$$\nu = \int \delta(\varepsilon - \frac{p^2}{2m}) \frac{d^2 p}{(2\pi\hbar)^2} \sim \frac{m}{\hbar^2}$$

$$\frac{\Gamma}{\Delta E(L)} \sim \hbar \frac{m}{\hbar^2} \frac{V_F^2 L}{2} \sim \frac{p_F}{\hbar} L = g$$

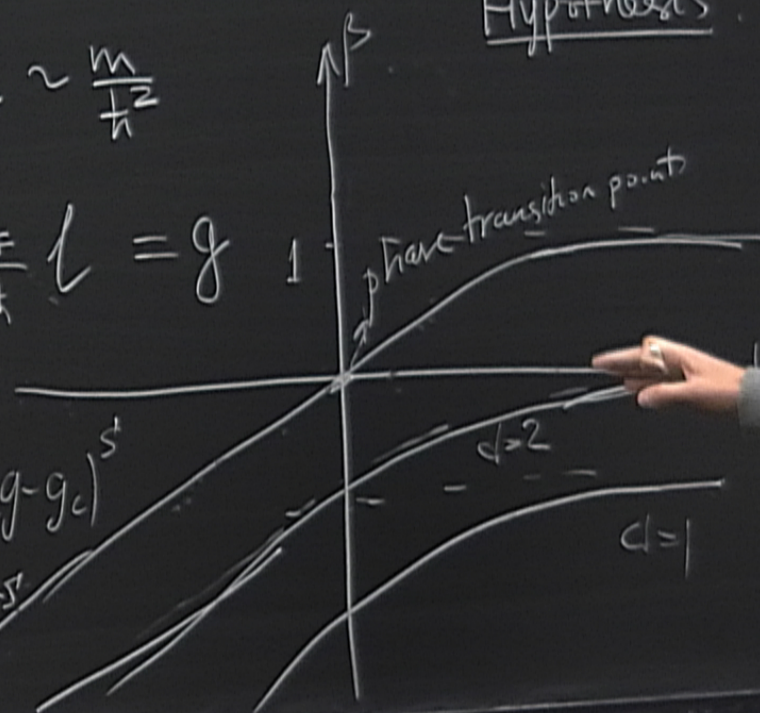
$$G(g - g_c) \sim (g - g_c)^5$$

$$\Sigma(g_c - g) \sim (g_c - g)^{5/2}$$

Hypothesis

$$\frac{d \log g}{d \log L}$$

Almost



$d=2$ Physical meaning: conductance

$$\nu = \int \delta(\varepsilon - \frac{p^2}{2m}) \frac{d^2 p}{(2\pi\hbar)^2} \sim \frac{m}{\hbar^2}$$

$$\frac{\Gamma}{\Delta E(L)} \sim \hbar \frac{m}{\hbar^2} \frac{V_F^2 L}{2} \sim \frac{p_F}{\hbar} L = g$$

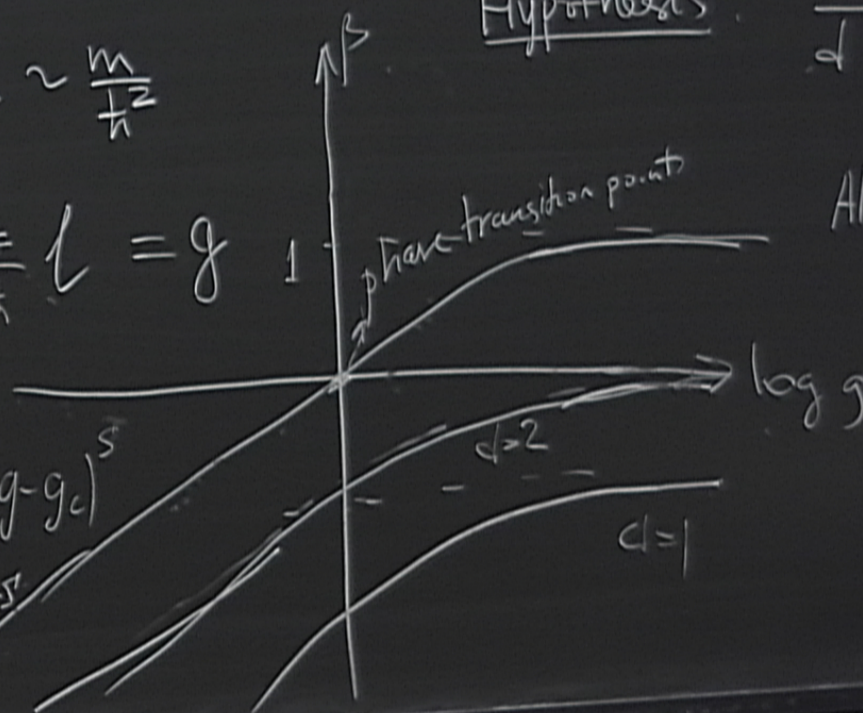
$$\mathcal{O}(g - g_c) \sim (g - g_c)^5$$

$$\sum (g_c - g) \sim (g_c - g)^{5/2}$$

Hypothesis

$$\frac{d \log g(L)}{d \log L}$$

Almost me



conductance

$$\left(\frac{p^2}{2m}\right) \frac{d^2 p}{(2\pi\hbar)^2} \sim \frac{m}{\hbar^2}$$

$$\frac{v_F^2}{2} \sim \frac{p_F}{\hbar} l = g$$

$$\sigma(g-g_c) \sim (g-g_c)^5$$

$$\sigma(g) \sim (g-g_c)^{5/2}$$

Hypothesis

$$\frac{d \log g(L)}{d \log L} = \beta$$

Deep

Almost metallic

$d=2$

$$\frac{\delta g}{g} =$$

$$\frac{\Delta \log g}{\Delta \log L} =$$

