

Title: 12/13 PSI - Condensed Matter Review Lecture 7

Date: Feb 06, 2013 11:30 AM

URL: <http://www.pirsa.org/13020079>

Abstract:

Lecture 2

electrons in disorder potential, APPROACH 1: Landauer formalism, APPR. 2:

ν - dens. of states, D - diff. coefficient

$$G_{\text{Dnole}} = e^2 \nu \cdot D, \quad D = \frac{v_F^2 L}{d}$$

COACH 1: Landauer formalism; APPR 2: PERTURBATION th. in disorder strength

$$G_{\text{Dude}} = e^2 \nu \cdot D, \quad D = \frac{v_F^2 L}{d} \quad \text{System of size } L: \quad R(L) = \frac{L^{2-d}}{G}$$

Drude theory

ν - dens. of states, D - diff. coefficient

$$\sigma_{\text{Drude}} = e^2 \nu D$$

Experimental observations: AT low T

- 1) Ohm's law violated $R(L)$ decreases fast 1D, 2D
- 2)

Drude theory

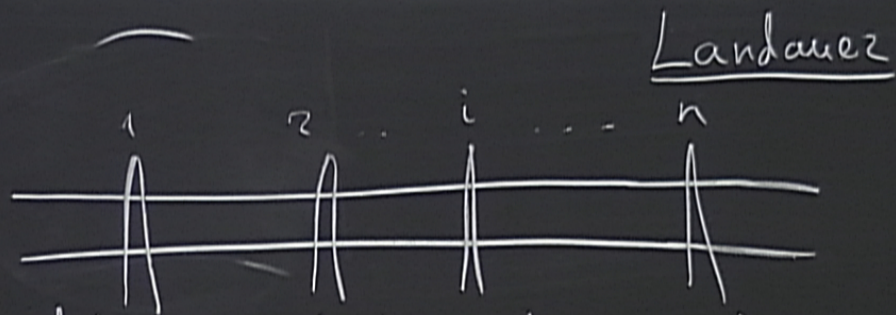
ν - dens. of states, D - diff. coefficient

$$\sigma_{\text{Drude}} = e^2 \nu D$$

Experimental observations: AT low T

- 1) Ohm's law violated $R(L)$ decreases fast. 1D, 2D
- 2) Anomalous $\sigma(T)$

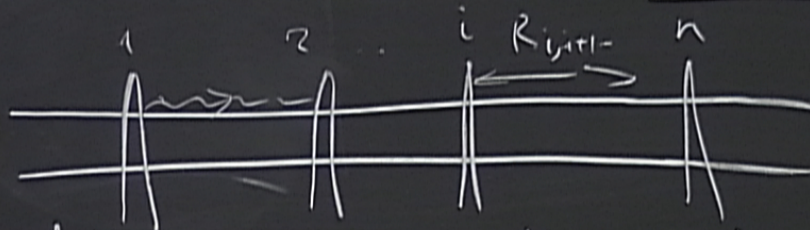




Model: 1D conductor with potential barriers

$$\hat{S}_i = \begin{pmatrix} r_i & t_i \\ t_i & r_i \end{pmatrix}$$

Landauer

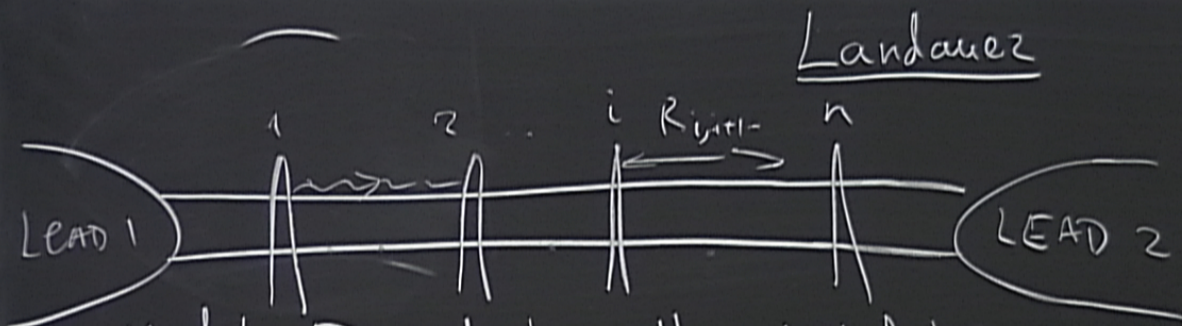


Model: 1D conductor with potential barriers

$$\hat{S}_i = \begin{pmatrix} r_i & t_i \\ t_i & r_i \end{pmatrix}$$

Barriers weak: $R = |r_i|^2 \ll 1$

$$e^{ikR_{i,i+1}} = e^{i\phi_{i,i+1}} \rightarrow \text{random}$$

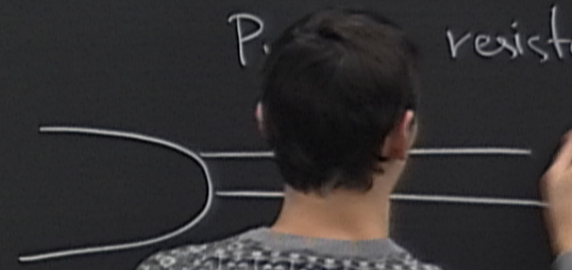


Model: 1D conductor with potential barriers

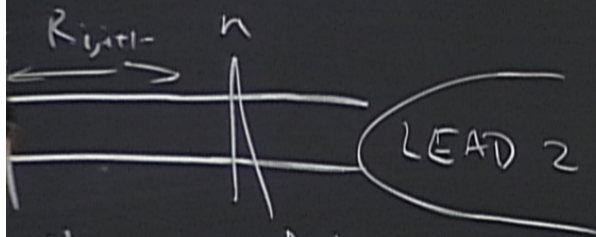
$$\hat{S}_1 = \begin{pmatrix} r_1 & t_1 \\ t_1 & r_1 \end{pmatrix}$$

Barriers weak: $R = |r_1|^2 \ll 1$

$$e^{ikR_{i,i+1}} = e^{i\phi_{i,i+1}} \rightarrow \text{random}$$



Landauer

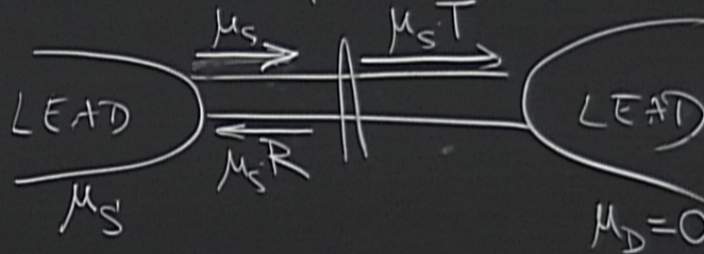


th potential barriers

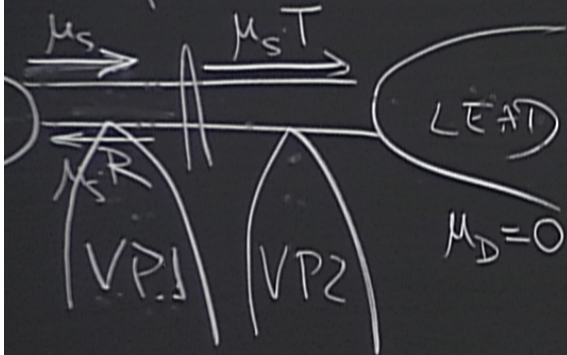
as weak: $R = |r_c|^2 \ll 1$

$= e \langle P_{\text{in}} \rangle \rightarrow \text{random}$

Proper resistance



Proper resistance



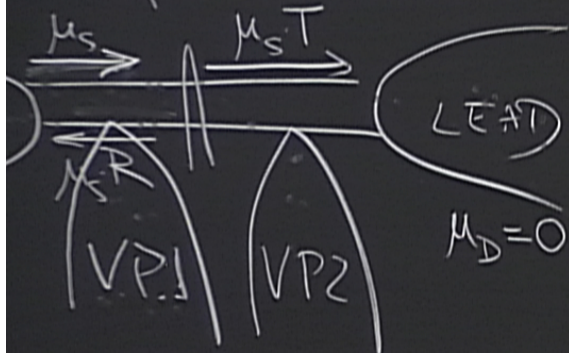
$$= \frac{\mu_S + \mu_S R}{2}, \quad V_2 = \frac{\mu_S T}{2}$$

$$V_1 - V_2 = \frac{\mu_S (1 + R - T)}{2} = \mu_S R$$

$$I = \frac{e^2}{h} \mu_S T$$

$$g_1^{-1}$$

Proper resistance



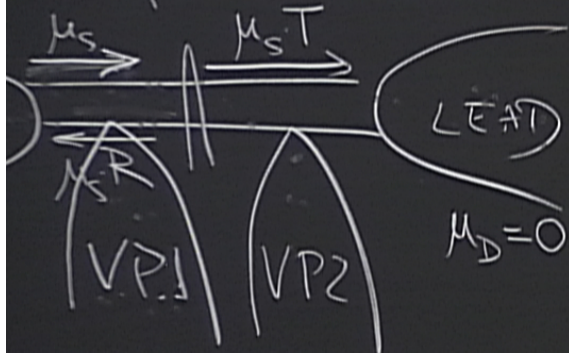
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$$g^{-1} = \frac{h}{e^2} \cdot \frac{R}{T}$$

Proper resistance



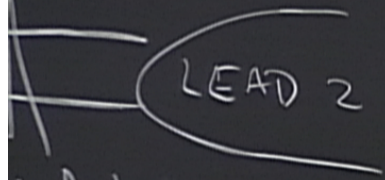
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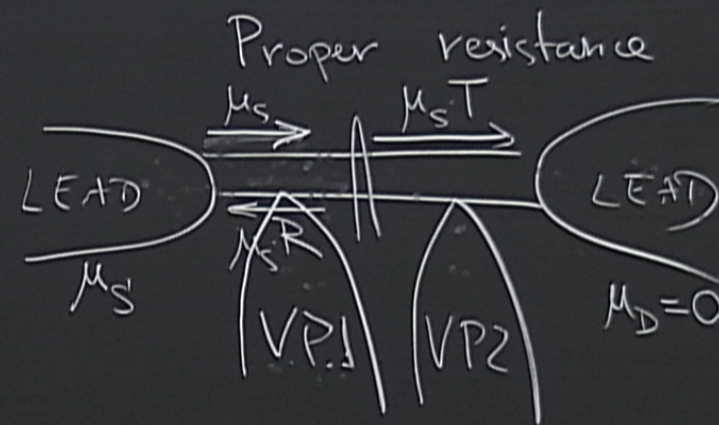
Landauer



potential barriers

weak: $R = |r_c|^2 \ll 1$

$e^{i\phi_{c,12}} \rightarrow \text{random}$



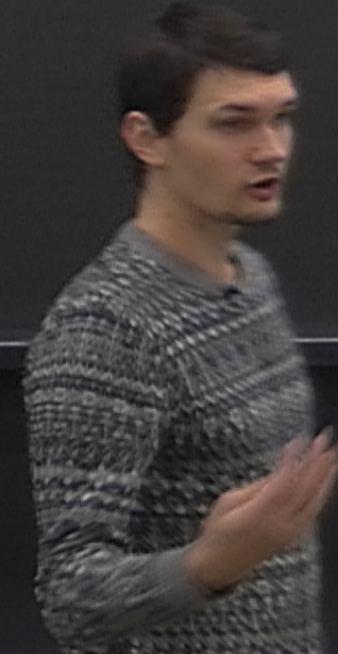
Proper resistance

$$V_1 = \frac{\mu_S + \mu_S R}{2}, \quad V_2 = \frac{\mu_S T}{2}$$

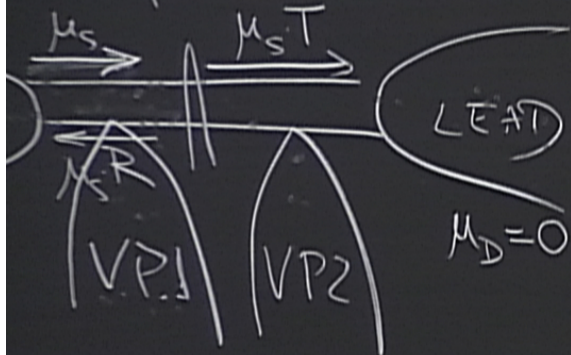
$$\Delta V = V_1 - V_2 = \frac{\mu_S (1 + R - T)}{2} = \mu_S R$$

$$I = \frac{e^2}{h} \mu_S T$$

$$g^{-1}$$



Proper resistance

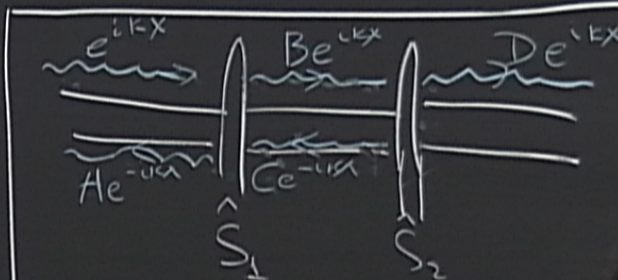


$$= \frac{\mu_S + \mu_S R}{2}, V_2 = \frac{\mu_S T}{2}$$

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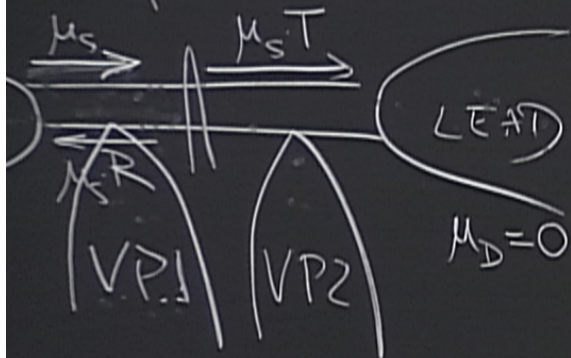
$$g_1^{-1} = \frac{h}{e^2} \frac{R}{T}$$



$$A = r_1 + C t_1$$

$$B = t_1 + C r_1$$

Proper resistance

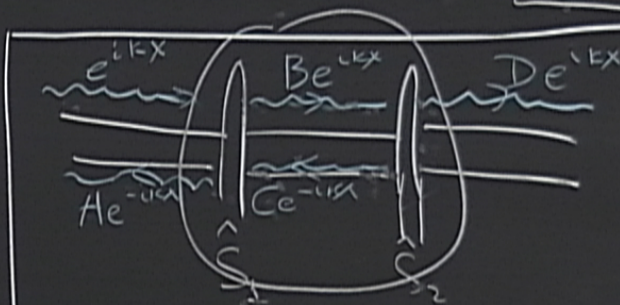


$$= \frac{M_S + M_S R}{2}, V_2 = \frac{M_S T}{2}$$

$$V_1 - V_2 = \frac{M_S (1 + R - T)}{2} = M_S R$$

$$I = \frac{e^2}{h} \mu_S T$$

$$g_1^{-1} = \frac{h}{e^2} \frac{R}{T}$$



$$D =$$

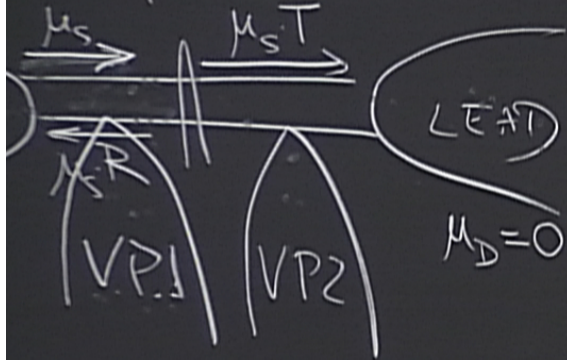
$$A = r_1 + C t_1$$

$$B = t_1 + C r_1$$

$$D = B e^{i\phi} t_2$$

$$C e^{-i\phi} = B e^{i\phi} r_2$$

Proper resistance

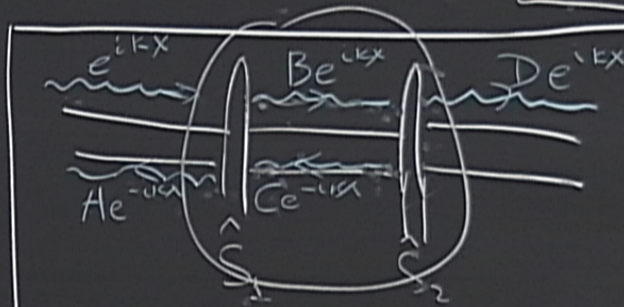


$$= \frac{\mu_S + \mu_S R}{2}, V_2 = \frac{\mu_S T}{2}$$

$$V_1 - V_2 = \frac{\mu_S (1 + R - T)}{2} = \mu_S R$$

$$I = \frac{e^2}{h} \mu_S T$$

$$g_1^{-1} = \frac{h}{e^2} \frac{R}{T}$$



$$D \equiv t_{12} = \frac{e^{i\phi} t_1 t_2}{1 - e^{2i\phi} r_1 r_2}$$

$$A = r_1 + C t_1$$

$$B = t_1 + C r_1$$

$$D = B e^{i\phi} t_2$$

$$C e^{-i\phi} = B e^{i\phi} r_2$$

Resistance

$$g_1^{-1} = \frac{h}{e^2} \frac{|A|^2}{|D|^4} = \frac{h}{e^2} \frac{1 + R_1 R_2 - 2 \cos \varphi \cdot \sqrt{R_1 R_2}}{(1 - R_1)(1 - R_2)}$$

Ohm's law.

$$\frac{v_1 v_2 - \frac{1}{2} \frac{v_1^2 + v_2^2}{2}}{2} = M_S R$$

$$\frac{g \cdot \sqrt{R_1 R_2}}{2}$$

Ohm's law: $\bar{g}_{\text{class}} = \frac{h}{e^2} \left(\frac{R_1}{1-R_1} + \frac{R_2}{1-R_2} \right)$

Resistance

$$g_4^{-1} = \frac{h}{e^2} \frac{|A|^2}{|D|^4} = \frac{h}{e^2} \frac{R_1 + R_2 - 2 \cos \varphi \sqrt{R_1 R_2}}{(1-R_1)(1-R_2)}$$

Ohm

1) Quantum - "mesoscopic fluctuations"

2) Q-M average

$$\langle g_4^{-1} \rangle = \frac{h}{e^2} \frac{R_1 + R_2}{(1-R_1)(1-R_2)} = g_{\text{class}}^{-1} + \frac{2R_1 R_2}{(1-R_1)(1-R_2)}$$

$$\text{Ohm's law. } \bar{g}_{\text{class}}^{-1} = \frac{h}{e^2} \left(\frac{R_1}{1-R_1} + \frac{R_2}{1-R_2} \right)$$

Induction: $\langle \bar{g}_4^{-1} \rangle_n$,

$$\langle \bar{g}_4^{-1} \rangle_{n+1} = \langle \bar{g}_4^{-1} \rangle_n + R \left(\langle \bar{g}_4^{-1} \rangle_n + \frac{h}{e^2} \right)$$

$$\frac{d \langle \bar{g}_4^{-1} \rangle_n}{dn} = R \left(\langle \bar{g}_4^{-1} \rangle_n + \frac{h}{e^2} \right), R \ll 1$$

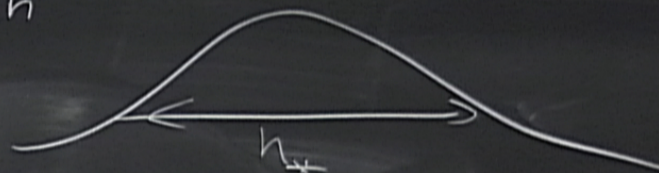
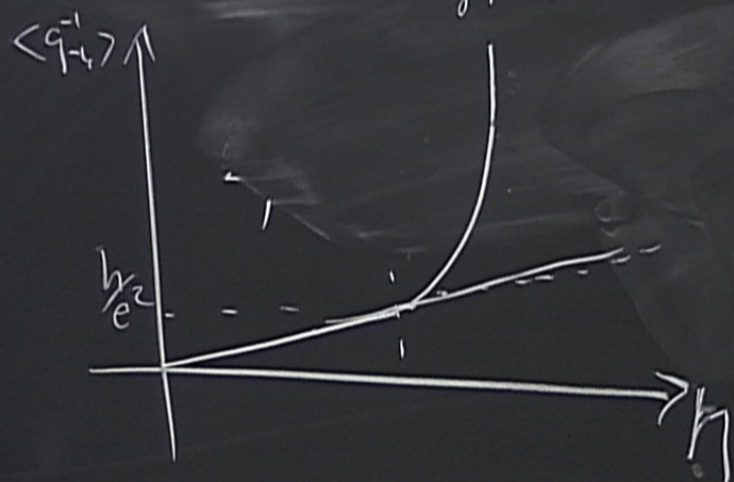
Regime 1: $\langle \bar{g}_4^{-1} \rangle_n \ll \frac{h}{e^2}$

$$\langle \bar{g}_4^{-1} \rangle_n \approx R \cdot \frac{h}{e^2} n$$

$$n_x \sim \frac{1}{R}$$

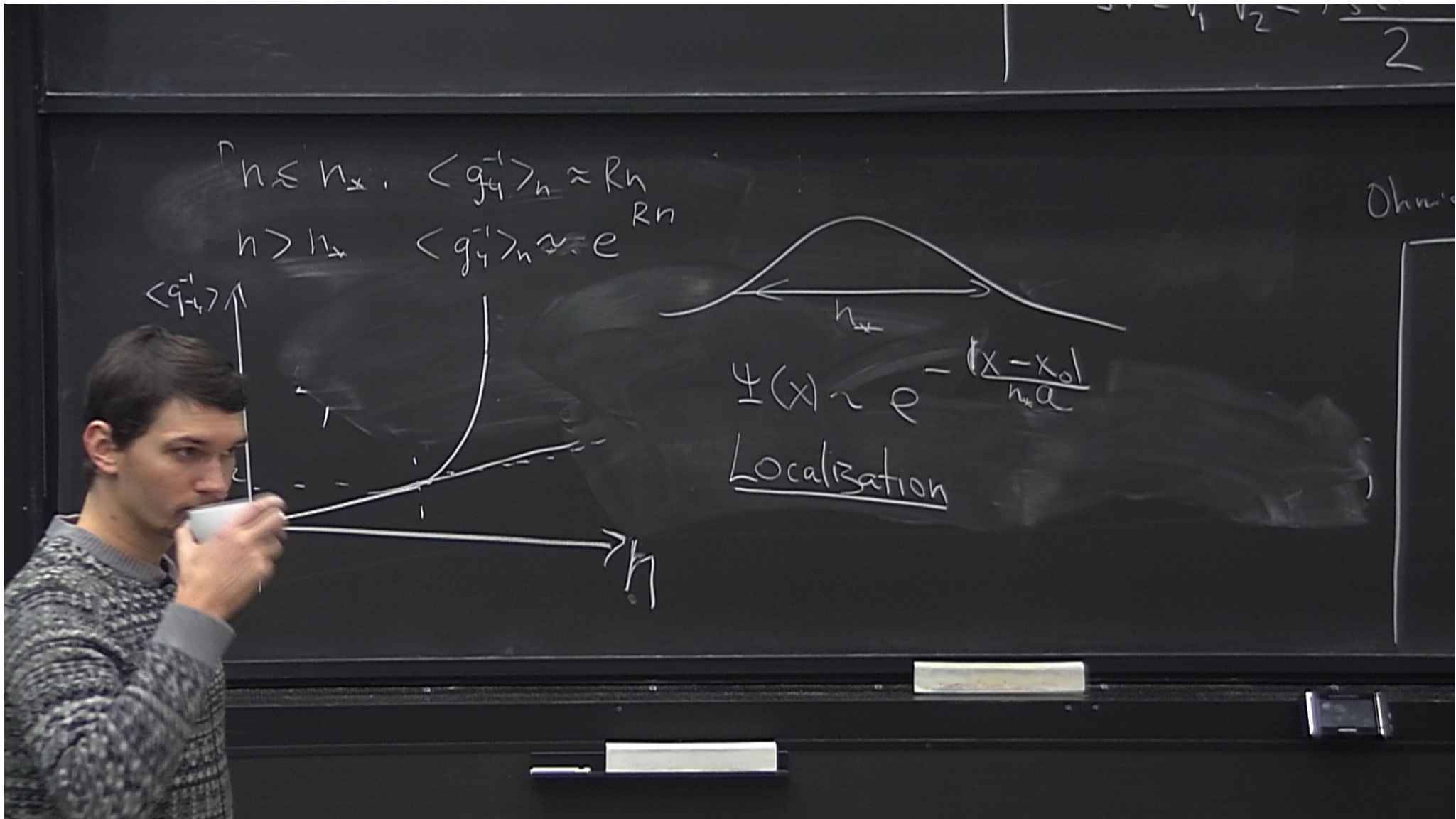
$$n \leq n_x, \quad \langle g_4^{-1} \rangle_n \approx R_n$$

$$n > n_x, \quad \langle g_4^{-1} \rangle_n \approx e^{R_n}$$

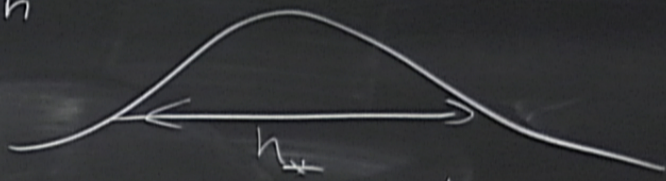
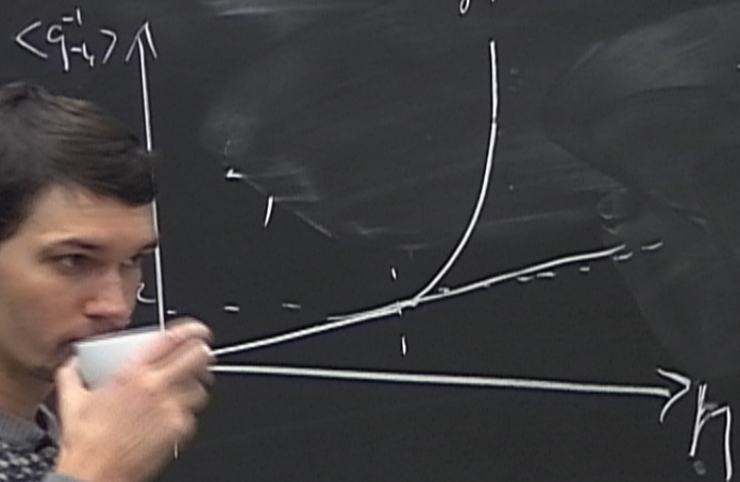


$$\psi(x) \sim e^{-\frac{|x-x_0|}{n_x a}}$$

Localization



$$n \leq n_x, \quad \langle g_y^{-1} \rangle_n \approx R_n$$
$$n > n_x, \quad \langle g_y^{-1} \rangle_n \approx e^{R_n}$$

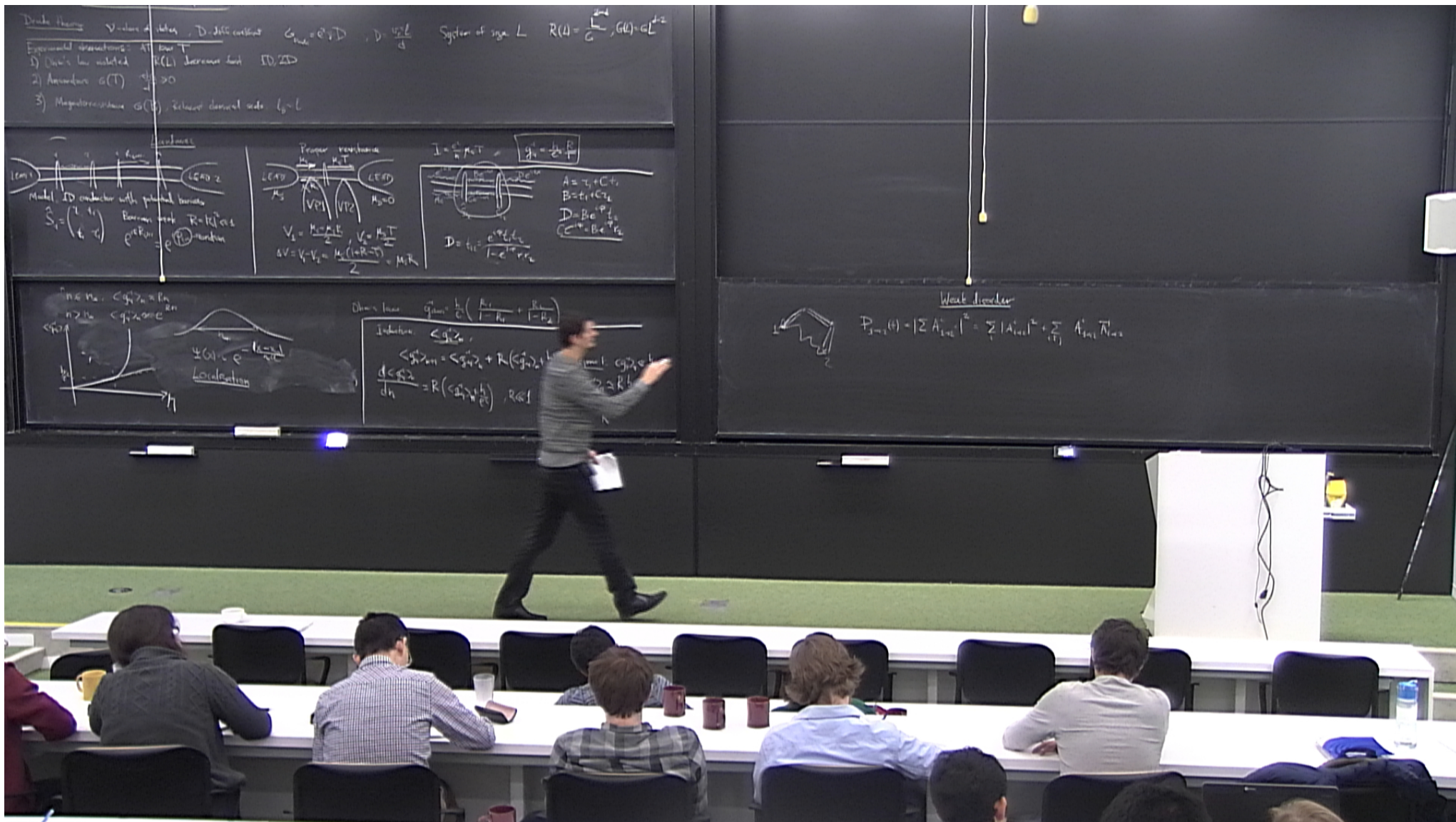


$$\psi(x) \sim e^{-\frac{|x-x_0|}{n_x a}}$$

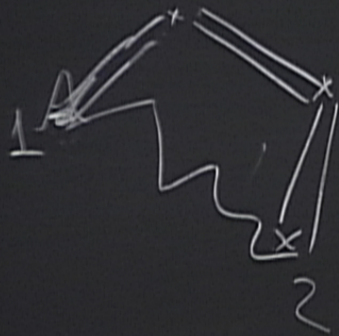
Localization

Ohmic





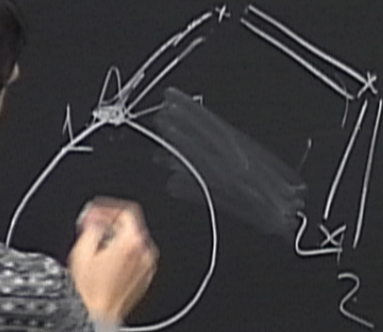
Weak disorder

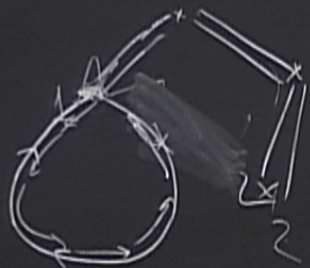


$$P_{1 \rightarrow 2}(t) = \left| \sum_i A_{1 \rightarrow 2}^i \right|^2 = \underbrace{\sum_i |A_{1 \rightarrow 2}^i|^2}_{\text{Diffusion}} + \sum_{i \neq j} A_{1 \rightarrow 2}^i \overline{A_{1 \rightarrow 2}^j}$$

Weak disorder

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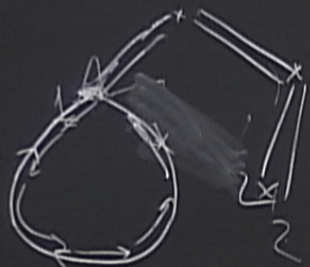




Weak disorder

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Time-reversed "pathway" $\rightarrow$ same phase.



Weak disorder

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Time-reversed "parhuni" $\rightarrow$ same phase.

$$t. \quad x(t) \sim \sqrt{Dt}$$

$$|z|^2 + \sum_{i \neq j} A_{1 \rightarrow i} \overline{A_{1 \rightarrow j}}$$

$$t: \quad x(t) \sim \sqrt{Dt} \quad , \quad V(t) \sim x^d(t) \sim (Dt)^{d/2}$$

$$\lambda_F \quad dt \quad \lambda_F^{d-1} v_F dt$$

$$\delta P = \frac{\lambda_F^{d-1} v_F dt}{(Dt)^{d/2}}$$

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$$\delta P_{dt} = \frac{\lambda_F^{d-1} v_F dt}{(Dt)^{d/2}} \quad ; \quad \delta P = \int dt \frac{\lambda_F^{d-1} v_F dt}{(Dt)^{d/2}}$$

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$$\delta P_{dt} = \frac{\lambda_F^{d-1} v_F dt}{(Dt)^{d/2}} \quad ; \quad \delta P = \int_0^{\tau_{max}} dt \frac{\lambda_F^{d-1} v_F dt}{(Dt)^{d/2}}$$

$$\tau_{max} = \max(\tau_p,$$

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$$|z|^2 + \sum_{i \neq j} A_{1 \rightarrow i} \overline{A_{1 \rightarrow j}}$$

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$$\tau_{max} = \max\left(\tau_p, \frac{L^2}{D}\right)$$

$$P \sim \frac{1}{(Dt)^{d/2}} \quad \delta P \sim -\frac{\delta D}{D}$$

$$\frac{\delta D}{D} = \frac{\delta G}{G} = \begin{cases} -\frac{2V_F}{D} (L_{\max} - L_D) & L_{\max} = \max(\sqrt{D\tau_p}, L), L_D = \sqrt{D\tau_p} \\ -\frac{2V_F \lambda_F}{D} \ln \frac{L_{\max}}{L_D} \end{cases}$$

$$\frac{D}{\delta} = \frac{\delta_G}{G} = \begin{cases} -\frac{2V_F}{D} (L_{\max} - L_D) & L_{\max} = \max(\sqrt{D\tau}, L), L_D = \sqrt{D\tau} \quad d=1 \\ -\frac{2V_F \lambda_F}{D} \ln \frac{L_{\max}}{L_D} & d=2 \\ \frac{2V_F \lambda_F^2}{D} \left(\frac{1}{L_D} - \frac{1}{L_{\max}} \right) & d=3 \end{cases}$$