

Title: 12/13 PSI - Condensed Matter Review Lecture 6

Date: Feb 04, 2013 11:30 AM

URL: <http://pirsa.org/13020077>

Abstract:

Low-dimensional electronic systems

1D

carbon nanotubes
nanowires
Quantum point contacts
Edges of certain 2D

2D

Graphene

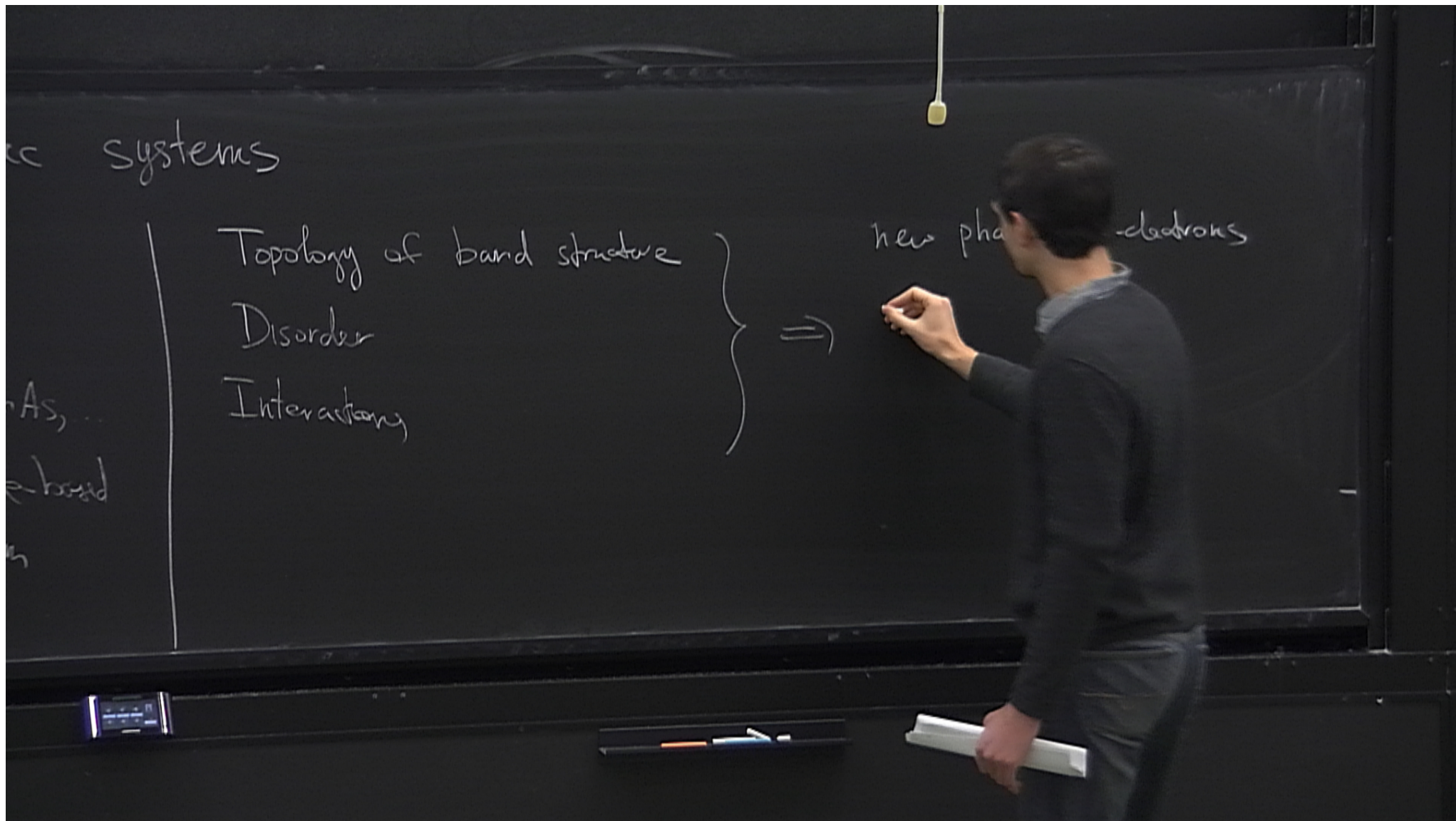
Low-dimensional electronic systems

1D

Carbon nanotubes
nanowires
Quantum point contacts
Edges of certain 2D

2D

Graphene
Heterostructures GaAs, InAs,
Topological insulators, HgTe-based
Edge of certain 3D system



ic systems

Topology of band structure

Disorder

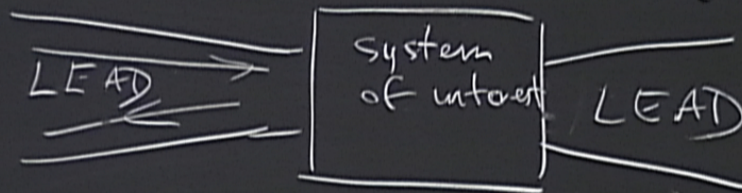
Interactions

} $\Rightarrow$ new phases of electrons
transport...

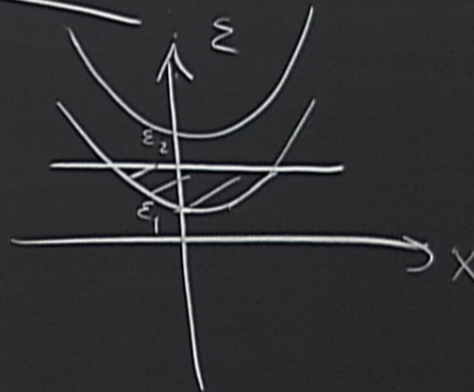
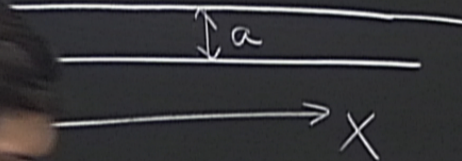
Quantum-coherent transport in 1D systems.

Quantum-coherent transport in 1D systems.

scattering matrix determines transport



Ideal 1D wire



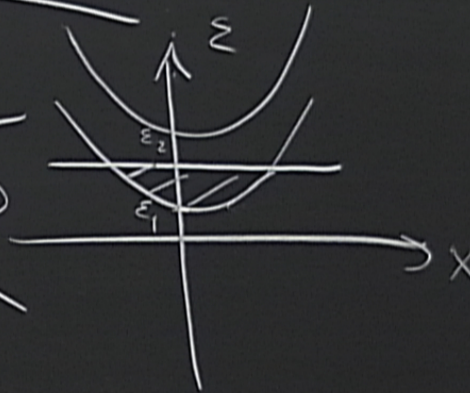
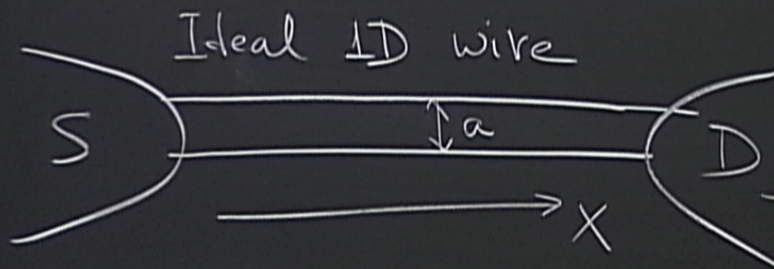
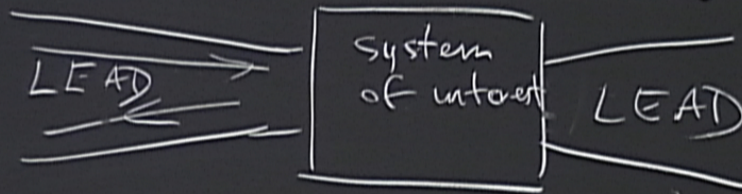
Transverse motion

$$\epsilon_{n+1} - \epsilon_n \sim \frac{\hbar^2}{ma^2}$$

$$\epsilon(k_x) = \epsilon_n + \frac{k_x^2}{2m}$$

Quantum-coherent transport in 1D systems.

scattering matrix determines transport



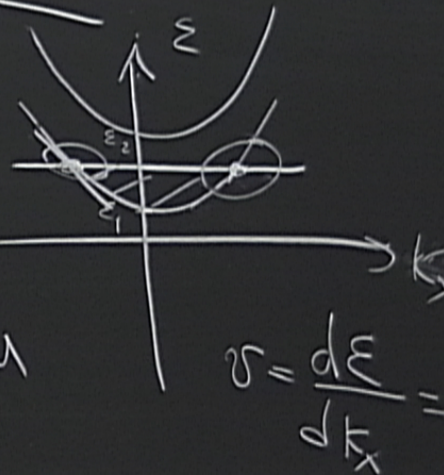
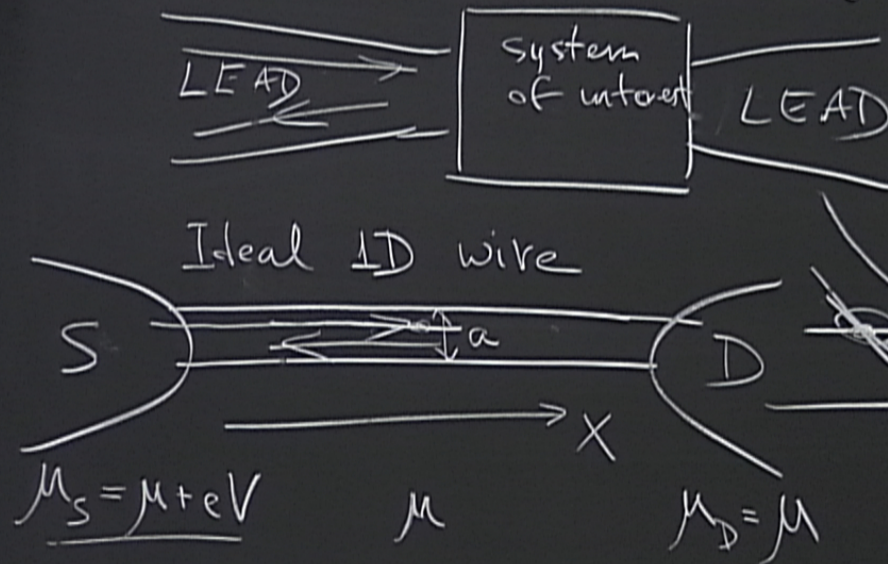
Transverse motion

$$\epsilon_{n+1} - \epsilon_n \sim \frac{\hbar^2}{ma^2}$$

$$\epsilon(k_x) = \epsilon_n + \frac{k_x^2}{2m}$$

Quantum-coherent transport in 1D systems.

scattering matrix determines transport



Transverse motion
 $\epsilon_{n+1} - \epsilon_n \sim \frac{\hbar^2}{ma^2}$

$\epsilon(k_x) = \epsilon_n$

$v = \frac{d\epsilon}{dk_x} = v_F \approx v_0$

transport in 1D systems.

matrix determines transport

reverse motion
 $\varepsilon_n \sim \frac{\hbar^2}{ma^2}$

$$\varepsilon(k_x) = \varepsilon_n + \frac{k_x^2}{2m}$$

$$\underline{I} = \underline{I}_R - \underline{I}_L = e \cdot \underbrace{v}_{\text{Density of states}} \times v \times \mu_S - e \cdot v \cdot v \cdot \mu_D$$

transport in 1D systems.

matrix determines transport

free motion
 $\epsilon_n \sim \frac{\hbar^2}{ma^2}$

$\epsilon(k_x) = \epsilon_n$

$$T = I_R - I_L = e \cdot v \times v \cdot \mu_S - e \cdot v \cdot v \cdot \mu_D$$

↓
Density of states

$$v(\epsilon) = \frac{1}{d\epsilon} = \int \delta(\epsilon - v \cdot k) \cdot \frac{dk}{2\pi\hbar}$$

1D systems.

determines transport

$$I = I_R - I_L = e \cdot \underbrace{v}_{\text{Density of states}} \cdot \mu_S - e \cdot v \cdot \mu_D = \frac{e^2}{h} \cdot V$$

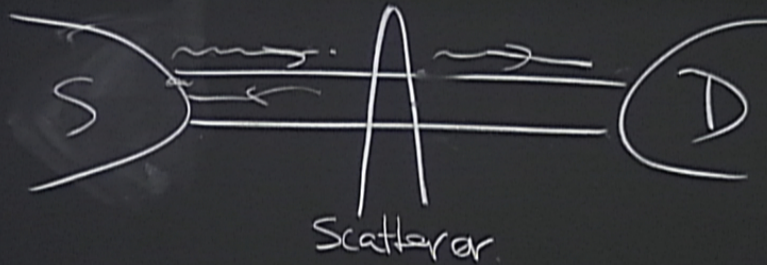
$$v(\epsilon) = \frac{1}{h} \frac{dN}{d\epsilon} = \int \delta(\epsilon - v \cdot k) \cdot \frac{dk}{2\pi h} = \frac{1}{h v}$$

$$R = \frac{V}{I} = \frac{h}{e^2}$$

Conductance

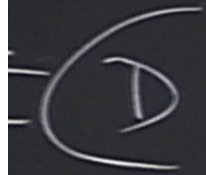
$$G = R^{-1} = \frac{e^2}{h}$$

Quantum wire with scatterer



$$\Psi = \begin{cases} a_{in} e^{ikx} & \text{out} e^{-ikx} \\ b_{in} e^{-ikx} & \text{in} e^{ikx} \end{cases}$$

th scatterer



$$\Psi = \begin{cases} a_{\text{in}} e^{ikx} + a_{\text{out}} e^{-ikx} \\ b_{\text{in}} e^{-ikx} + b_{\text{out}} e^{ikx} \end{cases}$$

$$\begin{pmatrix} a_{\text{out}} \\ b_{\text{out}} \end{pmatrix} = \hat{S} \begin{pmatrix} a_{\text{in}} \\ b_{\text{in}} \end{pmatrix} = \begin{pmatrix} r & t \\ t & r \end{pmatrix} \begin{pmatrix} a_{\text{in}} \\ b_{\text{in}} \end{pmatrix}$$

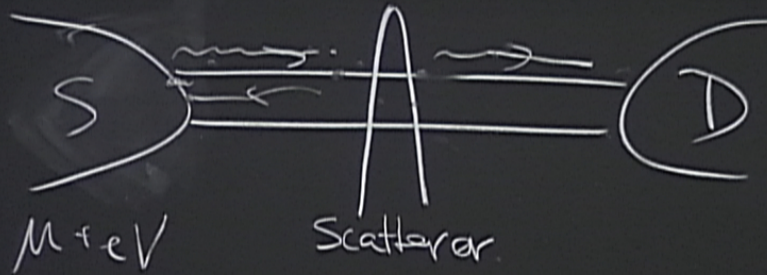
$$e^{ikx} + a_{out} e^{-ikx}$$

$$e^{-ikx} + b_{out} e^{ikx}$$

$$\begin{pmatrix} a_{out} \\ b_{out} \end{pmatrix} = \hat{S} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix} = \begin{pmatrix} z & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix}$$

$$\hat{S} = \hat{S}^T$$

Quantum wire with scatterer



$$\Psi = \begin{cases} a_{in} e^{ikx} + a_{out} e^{-ikx} \\ b_{in} e^{-ikx} + b_{out} e^{ikx} \end{cases}$$

$$I = \frac{e^2}{h} V T$$

$$T = 1$$

Landauer formula

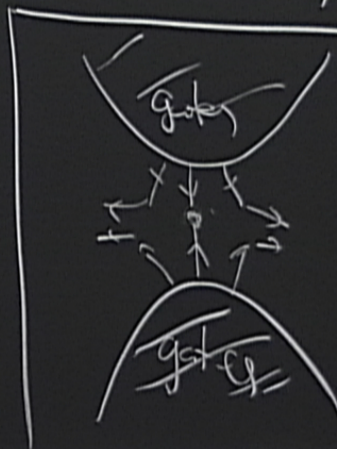
$$\Psi = \begin{cases} a_{in} e^{ikx} + a_{out} e^{-ikx} \\ b_{in} e^{-ikx} + b_{out} e^{ikx} \end{cases}$$

$$I = \frac{e^2}{h} V \cdot T$$

use formula

$$T = |t|^2$$

$$\begin{pmatrix} a_{out} \\ b_{out} \end{pmatrix} = \hat{S} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix} = \begin{pmatrix} z & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix}$$



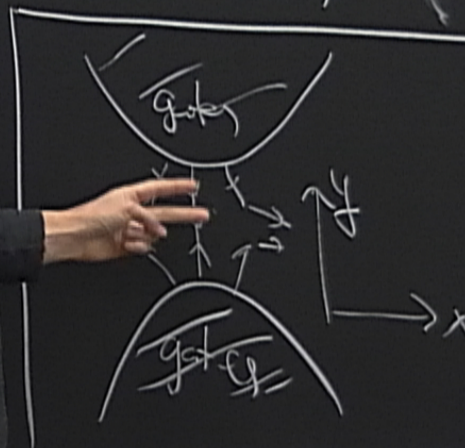
Quantum point contact

$$\Psi = \begin{cases} a_{in} e^{ikx} + 0 \\ b_{in} e^{-ikx} + t \end{cases}$$

$$I = \frac{e^2}{h} V \cdot T$$

Landauer formula

$$\begin{pmatrix} a_{out} \\ b_{out} \end{pmatrix} = \hat{S} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix}$$

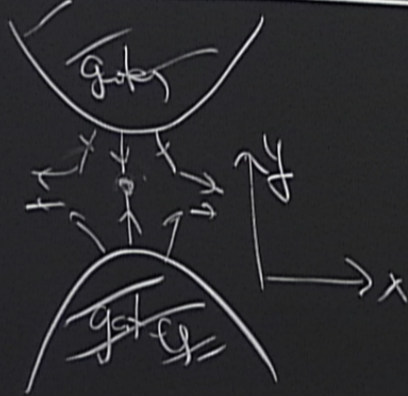


Quantum point contact

$$V(x,y) = V_0 - \frac{1}{2} m \omega_x^2 x^2 + \frac{1}{2} m \omega_y^2 y^2$$

$$\begin{pmatrix} a_{out} \\ b_{out} \end{pmatrix} = \hat{S} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix} = \begin{pmatrix} z & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix}$$

$$\underline{\hat{S}} = \underline{\hat{S}}^T$$



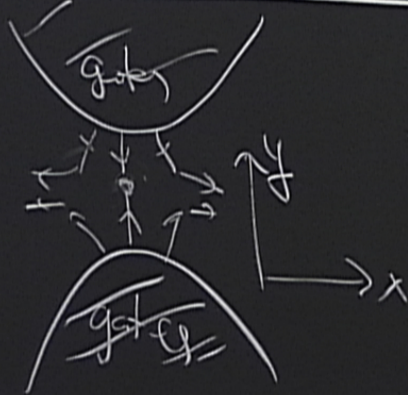
Quantum point contact

$$V(x,y) = V_0 - \frac{1}{2} m \omega_x^2 x^2 + \frac{1}{2} m \omega_y^2 y^2$$

$$E_n = \hbar \omega_x$$

$$\begin{pmatrix} a_{out} \\ b_{out} \end{pmatrix} = \hat{S} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix} = \begin{pmatrix} z & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in} \end{pmatrix}$$

$$\underline{\hat{S}} = \underline{\hat{S}}^T$$



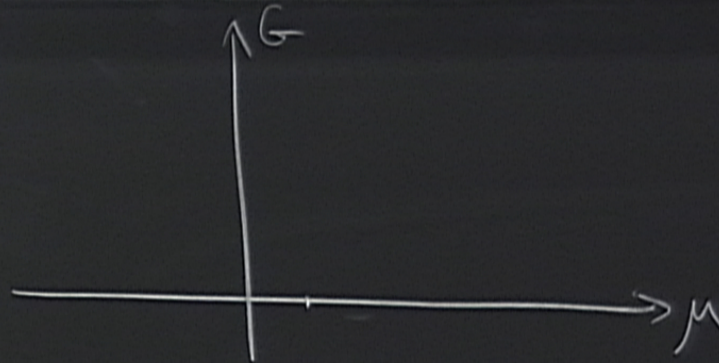
Quantum point contact

$$V(x,y) = V_0 - \frac{1}{2} m \omega_x^2 x^2 + \frac{1}{2} m \omega_y^2 y^2$$

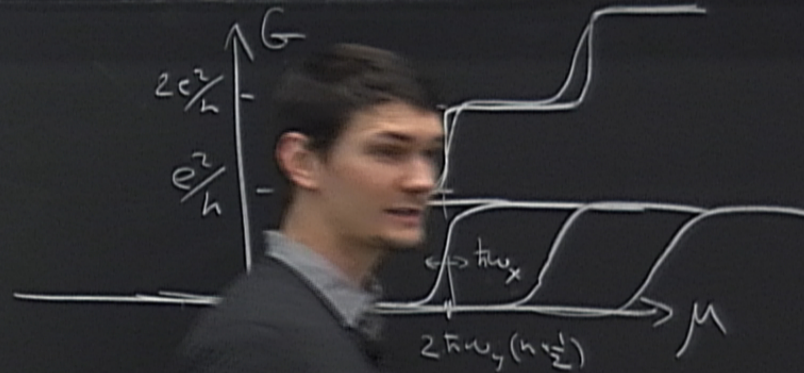
$$E_n = \hbar \omega_y \left(n + \frac{1}{2} \right)$$

$\omega_x < \omega_y$

$$E_n = \frac{2(\mu - V_0 - \hbar\omega_y(n + \frac{1}{2}))}{\hbar\omega_x}$$

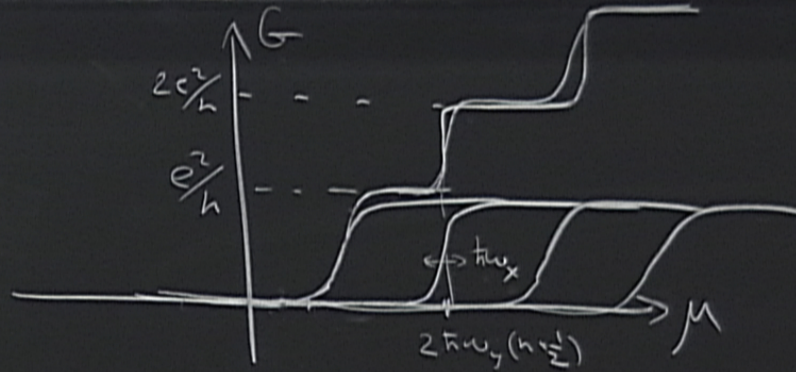


$$E_n = \frac{2(\mu - V_0 - \hbar\omega_y(n + \frac{1}{2}))}{\hbar\omega_x}$$

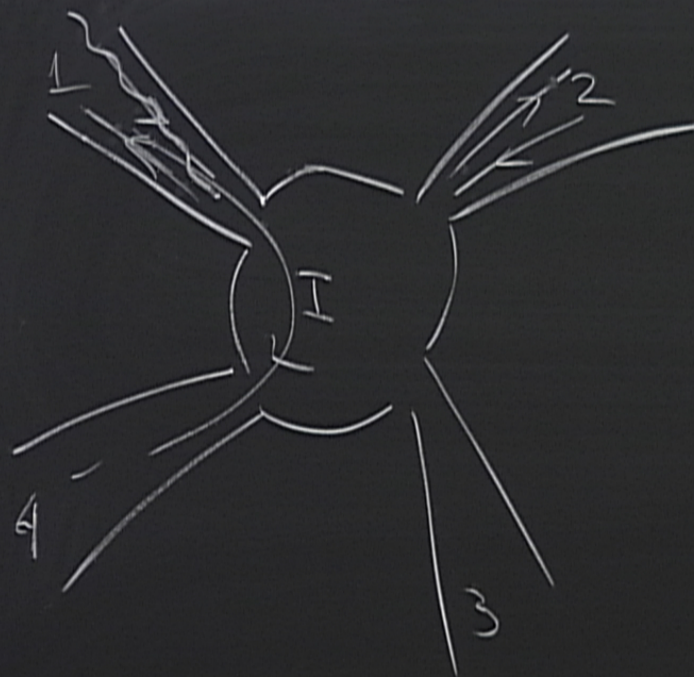


$$\frac{dG}{d\mu}$$

$$E_n = \frac{2(\mu - V_0 - \hbar\omega_y(n + \frac{1}{2}))}{\hbar\omega_x}$$



$$\frac{dG}{d\mu} \rightarrow \text{large}$$



Multi-terminal Landauer formula

$$\hat{S} = \begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} & \hat{S}_{13} & \hat{S}_{14} \\ \hat{S}_{21} & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \hat{S}_{41} & \dots & \dots & \hat{S}_{44} \end{bmatrix}$$

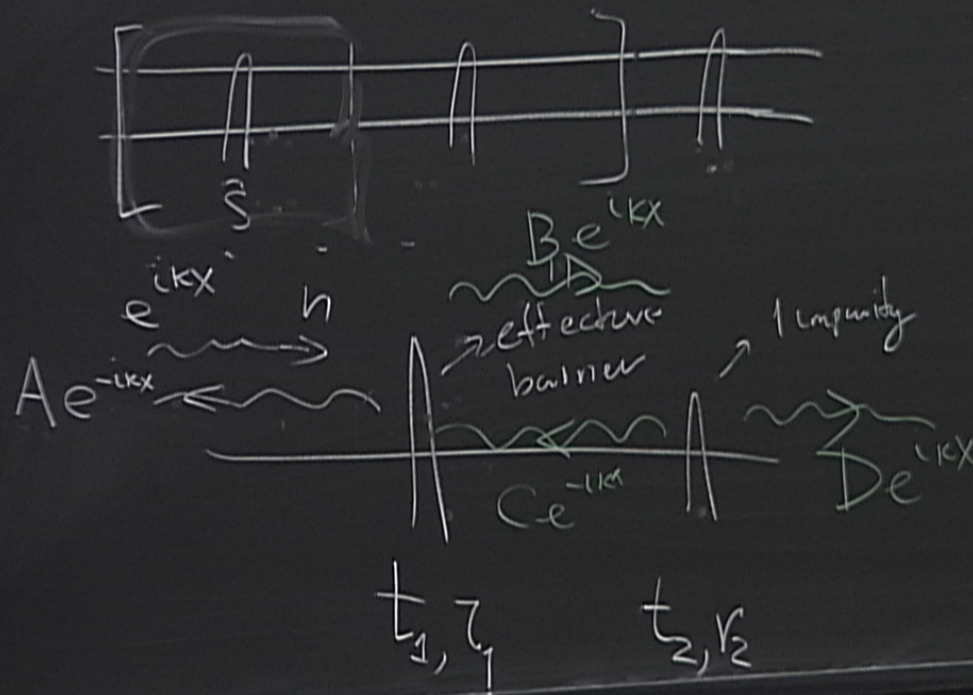
Multi-terminal Landauer formula

$$\begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} & \hat{S}_{13} & \hat{S}_{14} \\ \hat{S}_{21} & \dots & \dots & \dots \\ \vdots & \vdots & \vdots & \vdots \\ \hat{S}_{41} & \dots & \dots & \hat{S}_{44} \end{bmatrix}$$

$$T_{\beta\alpha} = \sum_{mn} T_{\beta\alpha, mn} = \sum_{mn} |S_{\beta\alpha, mn}|^2$$

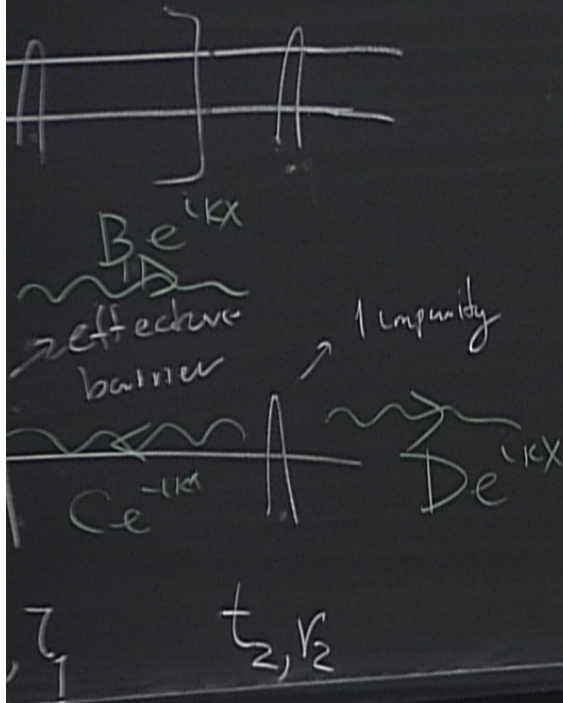
$$R_{\alpha\alpha} = \sum_{mn} R_{\alpha\alpha, mn} = \sum_{mn} |S_{\alpha\alpha, mn}|^2$$

Disordered 1D conductor



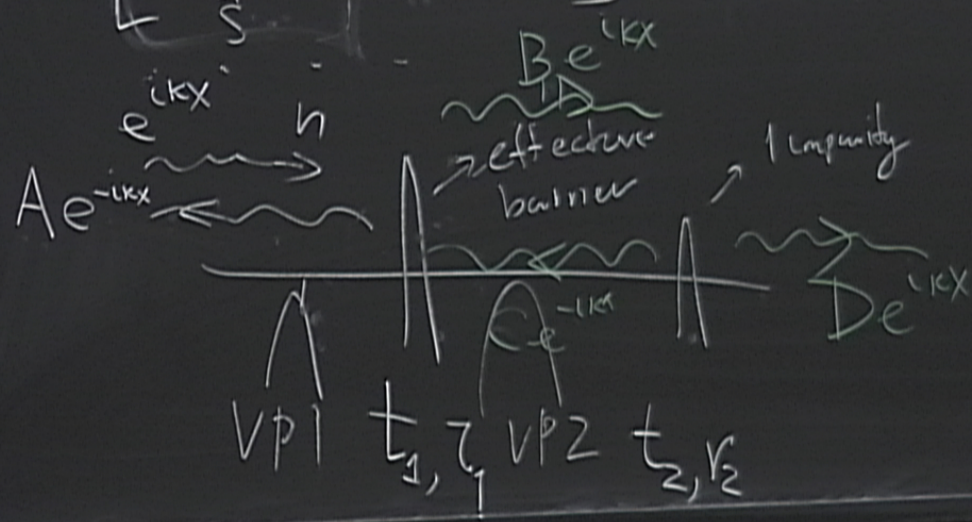
$$D = \frac{e^{i\phi} t_1 t_2}{1 - e^{2i\phi} r_2 r_1}, \quad \phi$$

Disordered 1D conductor



$$D = \frac{e^{i\phi} t_1 t_2}{1 - e^{2i\phi} r_2 r_1'}$$

ϕ - phase pickup
between barriers



$$1 - e^{2i\phi} r_2 r_1$$

$$R_0 = \frac{h}{e^2}$$

$$1 - e^{2i\phi} r_2 r_1'$$

between barriers
 $e^{i\phi} = e^{ikL}$

$$R_0 = \frac{h}{e^2} \frac{R}{T}$$

$$R_0 = \frac{R_1 + R_2 - 2\sqrt{R_1 R_2} \cos \tilde{\phi}}{(1 - R_1)(1 - R_2)}$$

$$1 - e^{2i\varphi} r_2 r_1'$$

between barriers
 $e^{i\varphi} = e^{ikL}$

$$g_4^{-1} = \frac{h}{e^2} \frac{R}{T}$$

$$R_0 = \frac{R_1 + R_2 - 2\sqrt{R_1 R_2} \cos \tilde{\varphi}}{(1 - R_1)(1 - R_2)} \quad \text{-- for a resonant mode}$$