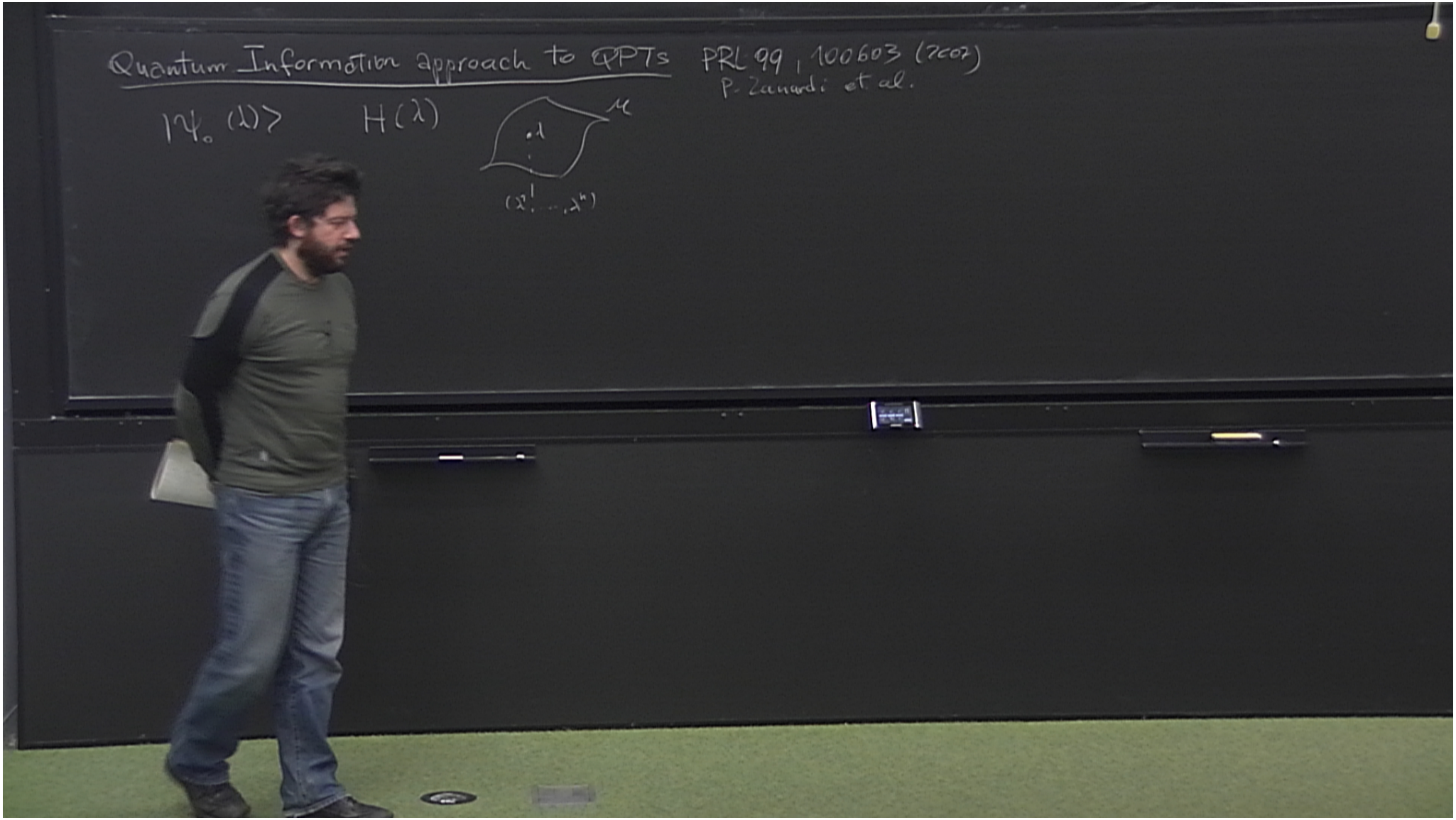
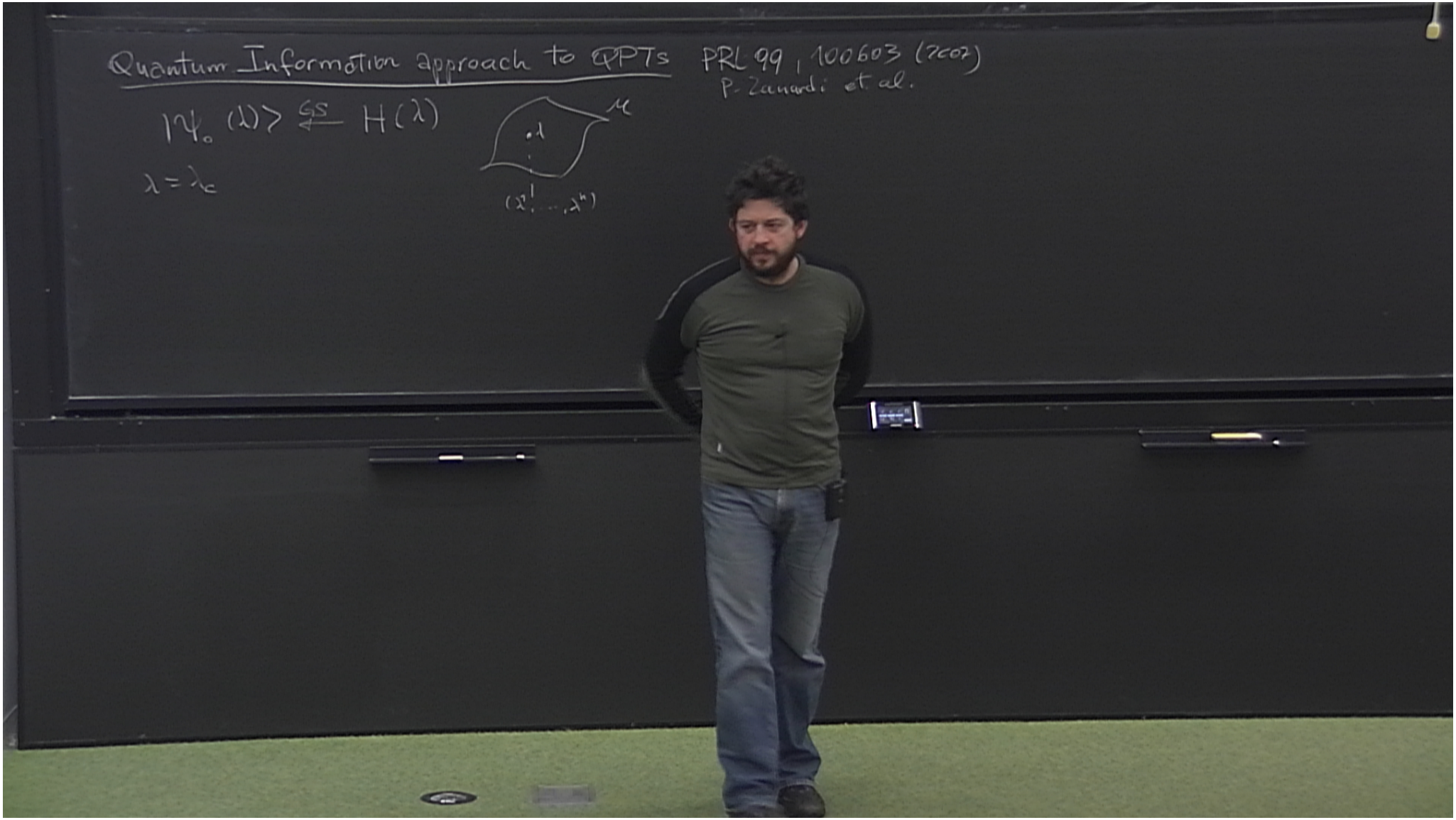
Title: 12/13 PSI - Condensed Matter Review Lecture 5

Date: Feb 01, 2013 11:30 AM

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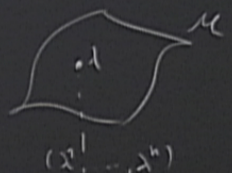
Abstract:


Quantum Information approach to QPTs
PRL 99, 100603 (2007)
P. Zanardi et al.
$|\psi_0(\lambda)\rangle$
$H(\lambda)$
λ
$(\lambda^1, \ldots \lambda^n)$


Quantum Information approach to QPTs
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$|\psi_0(\lambda)\rangle \xleftarrow{GS} H(\lambda)$
$\lambda = \lambda_c$
$\mathcal{M}$
$(x^1, \ldots, x^n)$

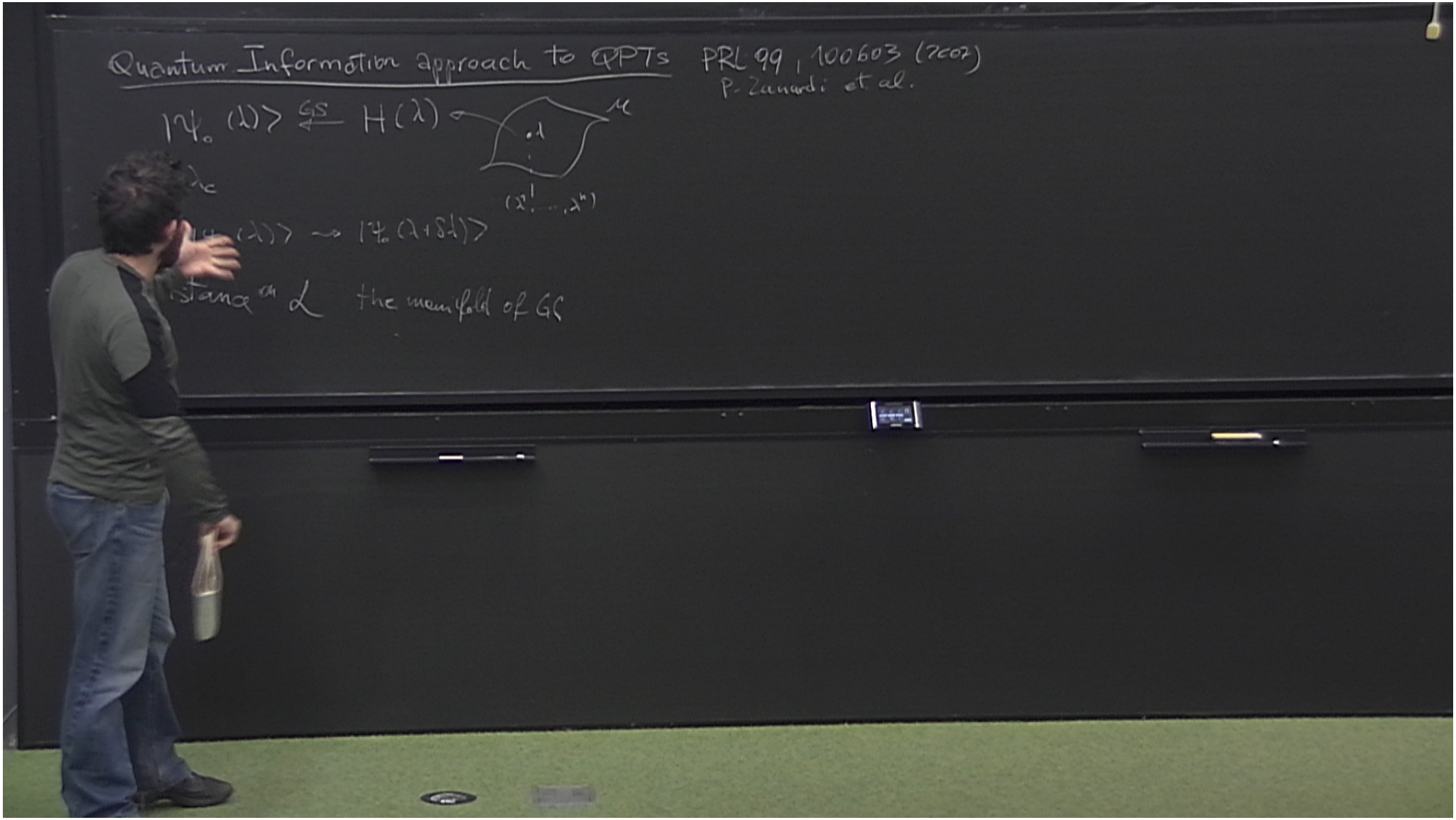
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$|\psi_0(\lambda)\rangle \rightsquigarrow |\psi_0(\lambda+\delta\lambda)\rangle$


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distance on $\mathcal{L}$ the manifold of GS

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of GS

Distance between prob. distributions.

P_1 of yielding heads

$$P_1 = \frac{\#\text{ of heads}}{n_{trials}}$$

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$P_1\ P_2 \cdots P_3 \cdots P_5$

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$d(P_1, P_5) =$ larger $\#$ indp

$\#$ intermediate mutually dist P_i

ch to QPTs PRL 99, 100603 (2007)
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$$d(P_1, P_2) = \frac{1}{\sqrt{n}} \quad \text{larger } n$$

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$\Delta P_1 = \sqrt{\frac{P_1(1-P_1)}{n}}$

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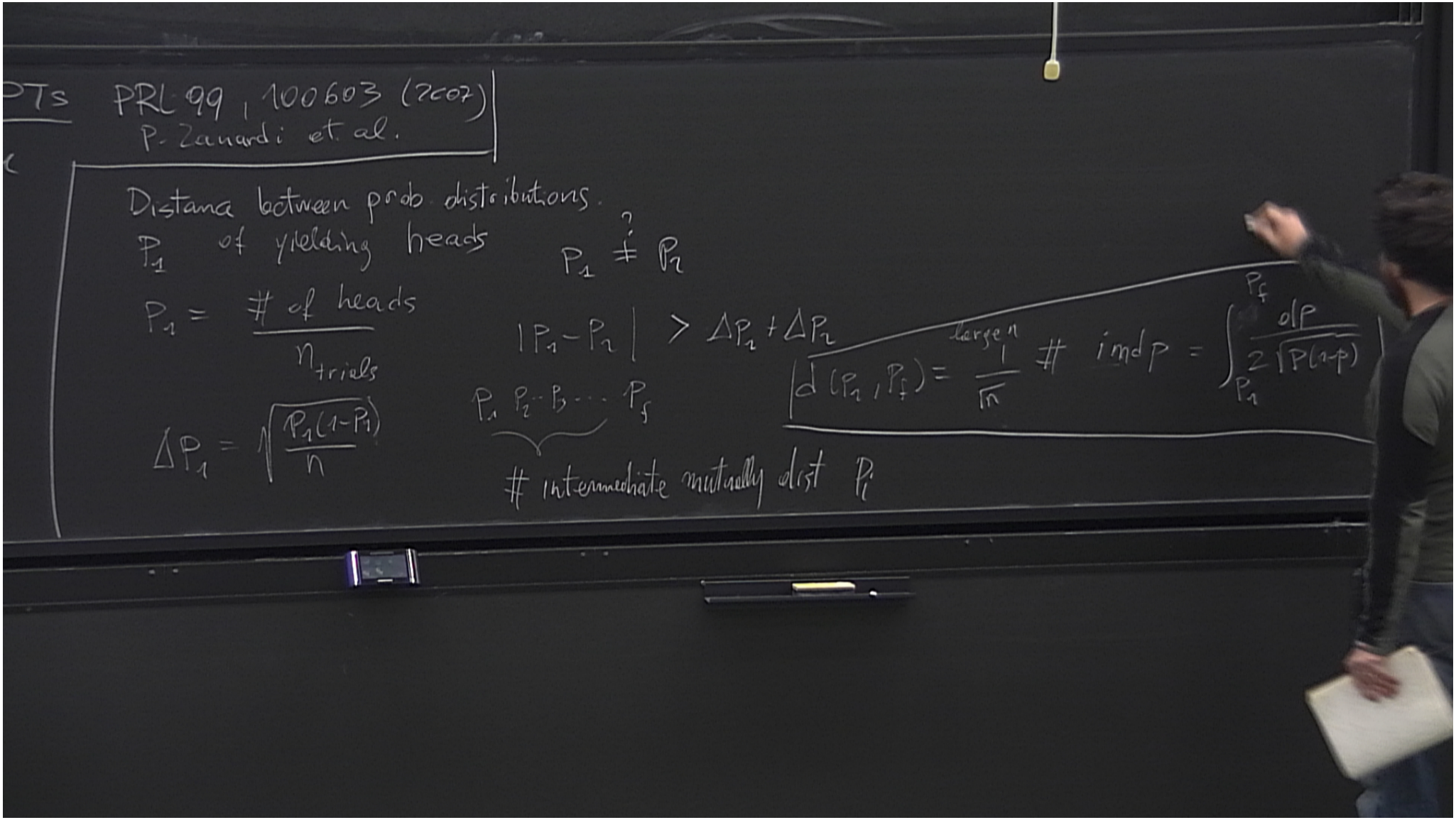
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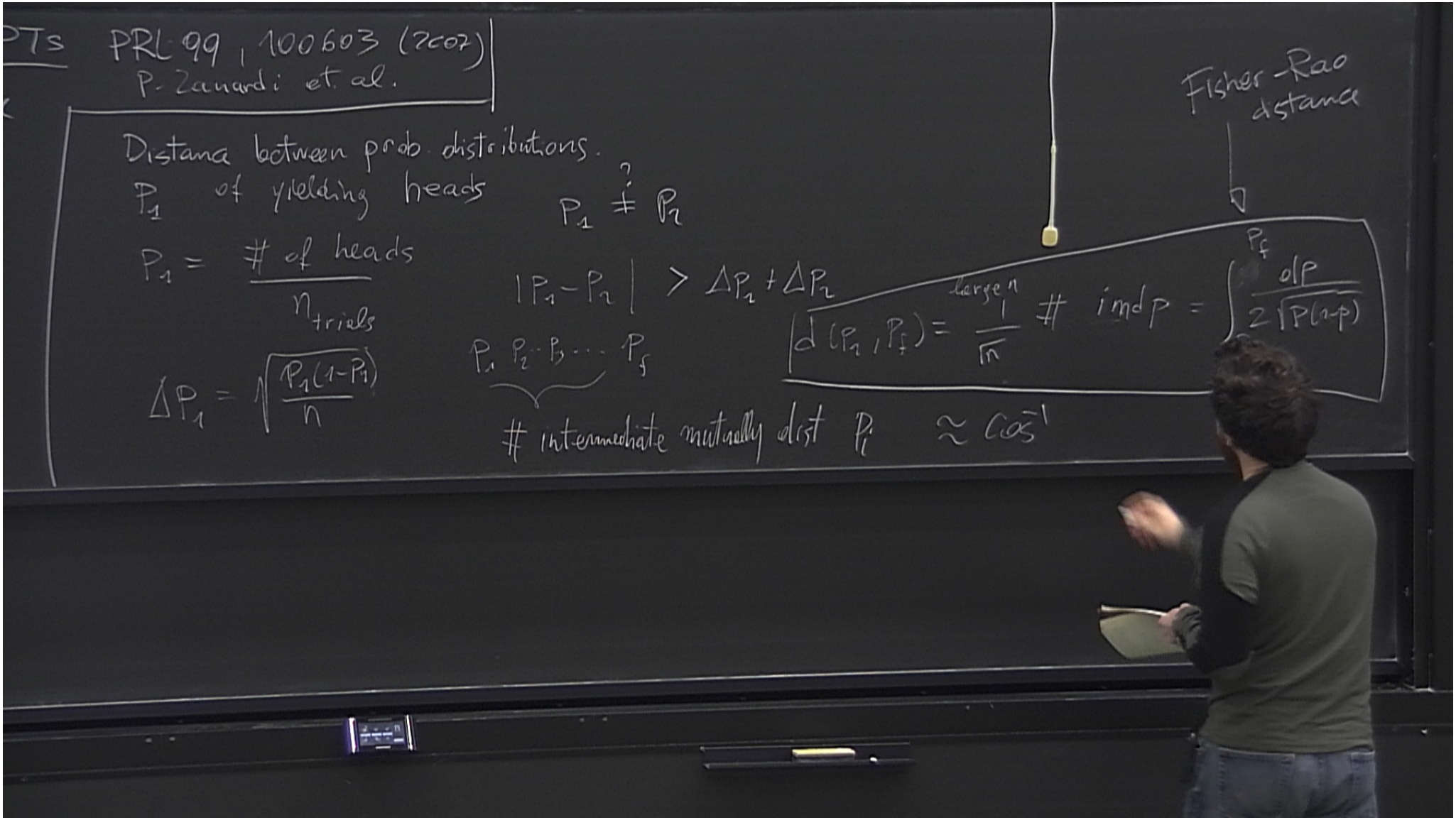
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$\#$ intermediate mutually dist P_i

$$d(P_1, P_f) = \frac{1}{\sqrt{n}} \#\text{ ind } p = \int_{P_1}^{P_f} \frac{dp}{2\sqrt{p(1-p)}}$$


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larger


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large n
intermediate mutually dist P_i $\approx \cos^{-1}$

PTs PRL 99, 100603 (2007)
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Fisher-Rao distance

$d(P_1, P_f) \overset{\text{large } n}{=} \frac{1}{\sqrt{n}} \#\text{ ind } p = \int_{P_1}^{P_f} \frac{dp}{2\sqrt{p(1-p)}}$

$\approx \cos^{-1}\left(P_1^{1/2} P_f^{1/2} + (1-P_1)^{1/2}(1-P_f)^{1/2}\right)$

PTs PRL 99, 100603 (2007)
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$P_i^{(1)}$

Fisher-Rao distance

$d(P_1, P_f) = \frac{1}{\sqrt{n}} \#\text{ ind } p = \int_{P_1}^{P_f} \frac{dp}{2\sqrt{p(1-p)}}$ (large n)

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PTs PRL 99, 100603 (2007)
P. Zanardi et al.

$P_i^{(1)}$ generalized weighted DIE

Fisher-Rao distance

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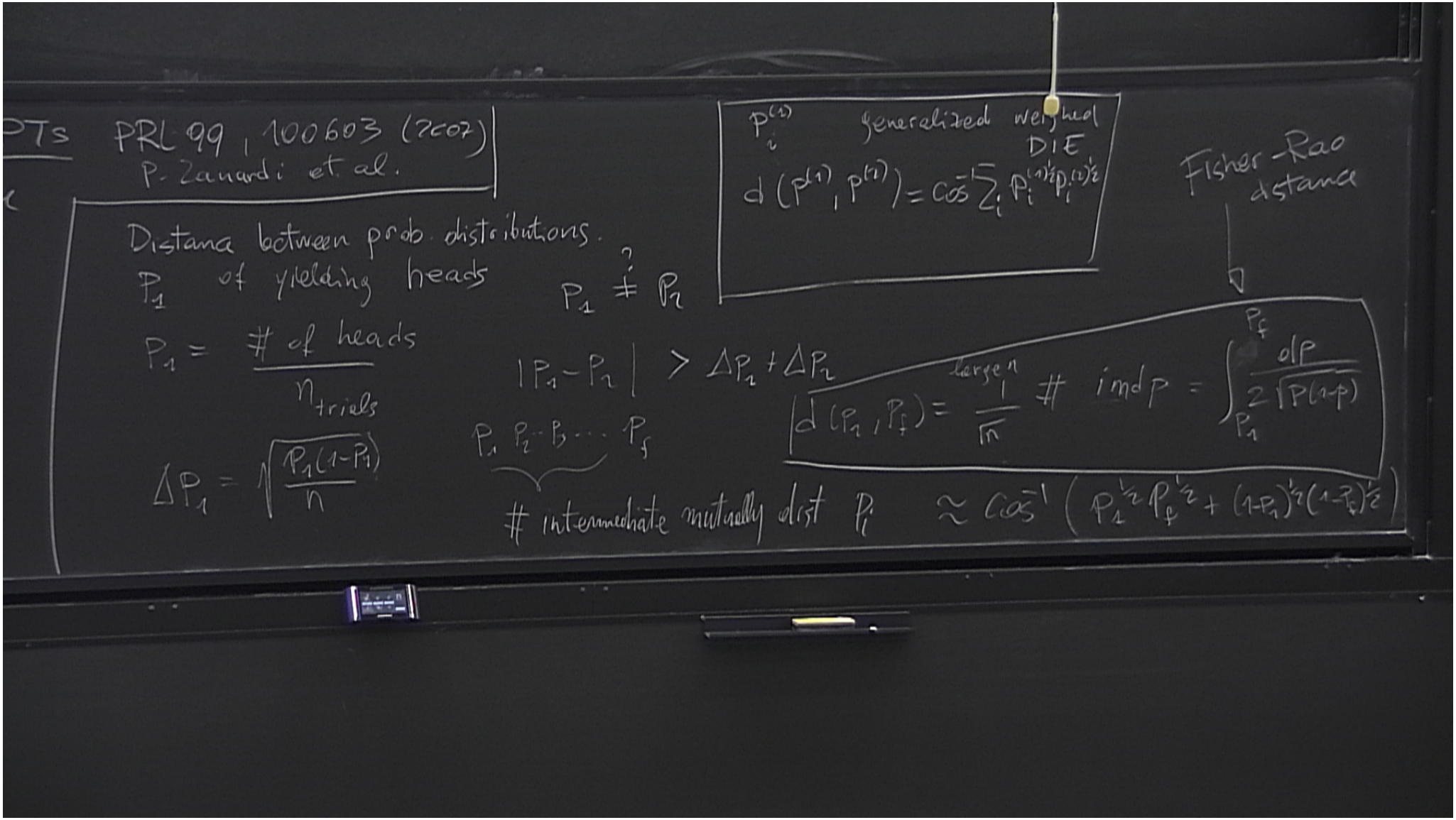
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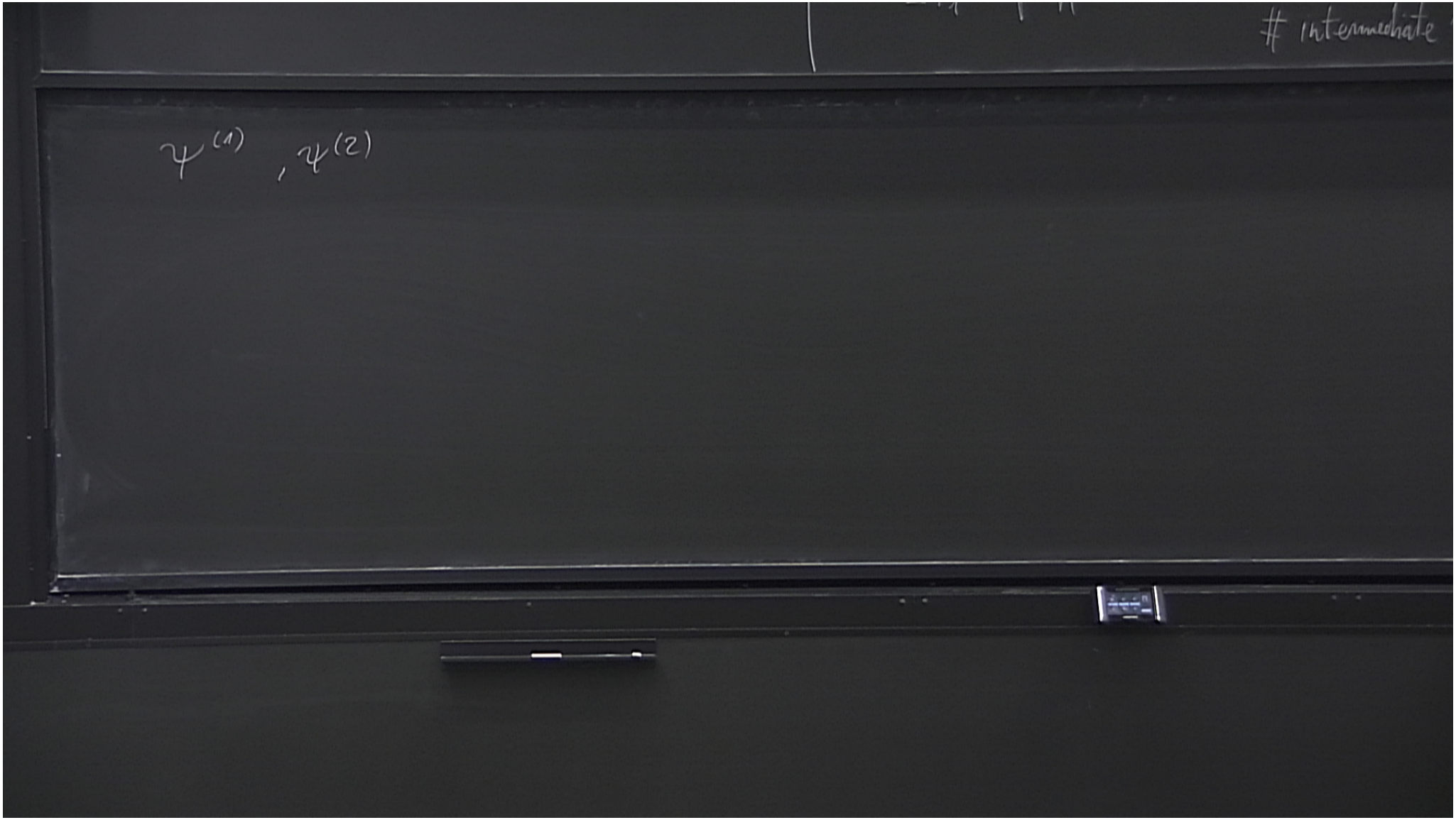
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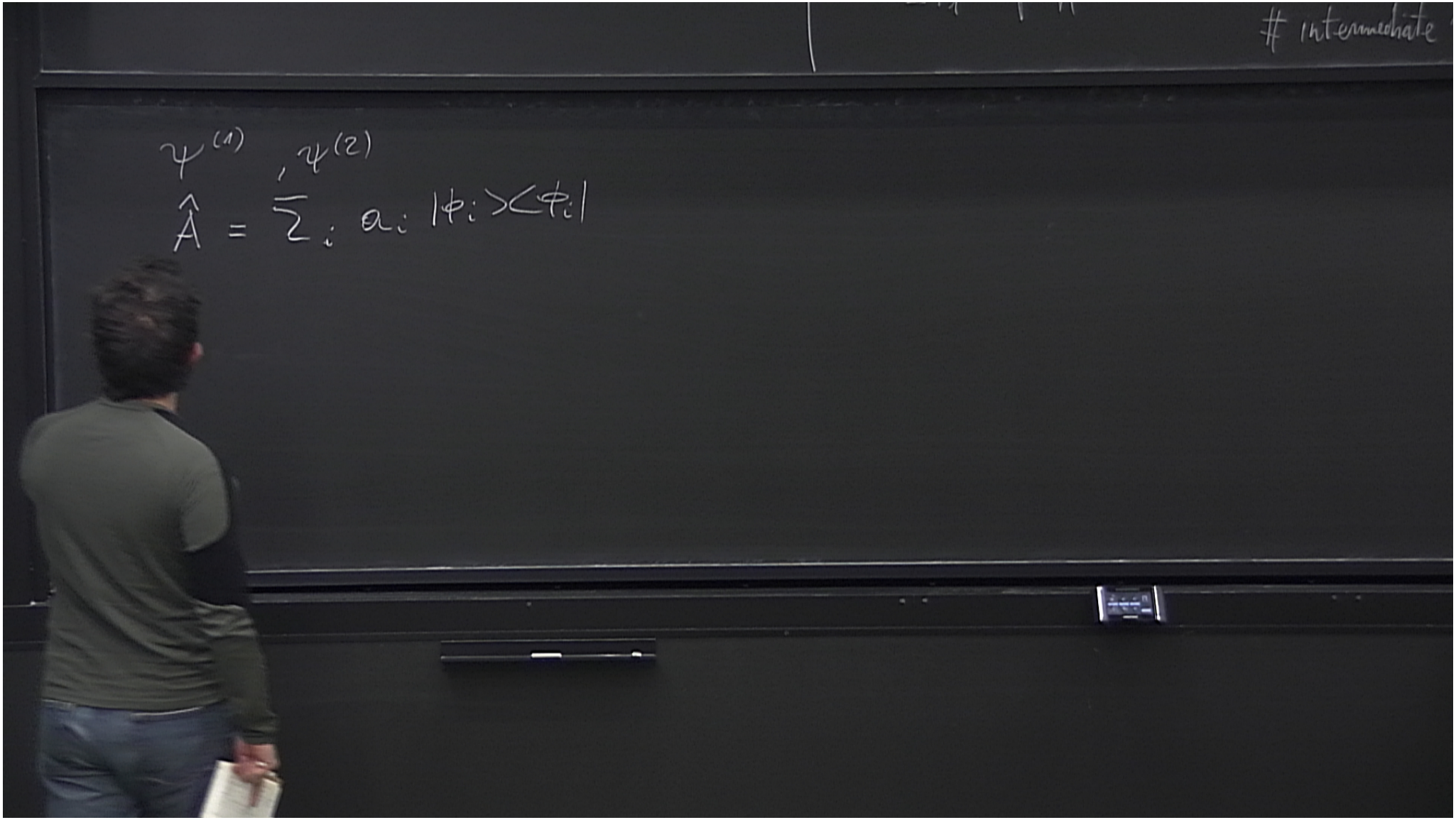
$P_1\ P_2\ P_3 \cdots P_f$

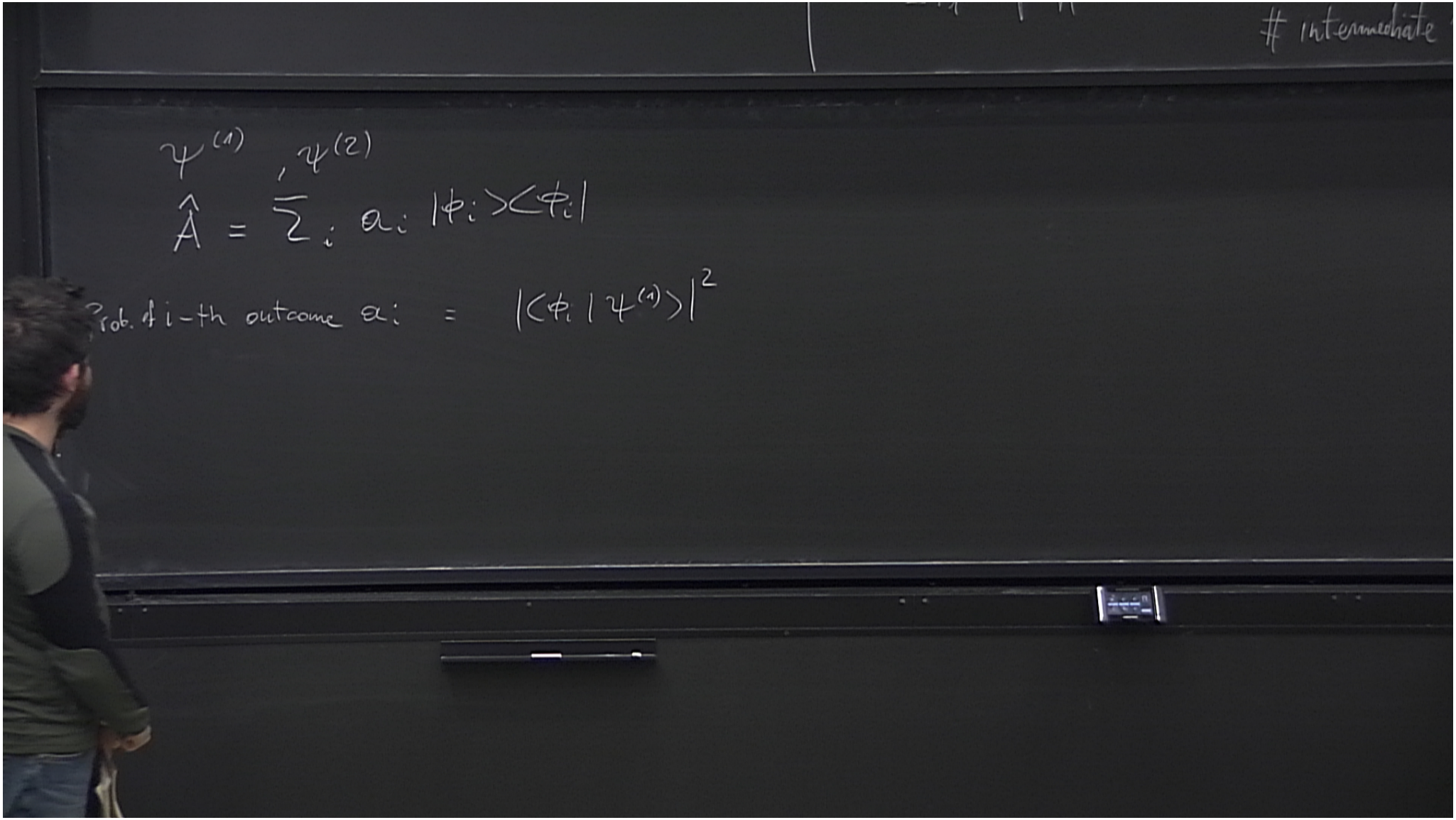
$d(P_1, P_f) = \frac{1}{\sqrt{n}} \#\text{ ind } p = \int_{P_1}^{P_f} \frac{dp}{2\sqrt{p(1-p)}}$ (large n)

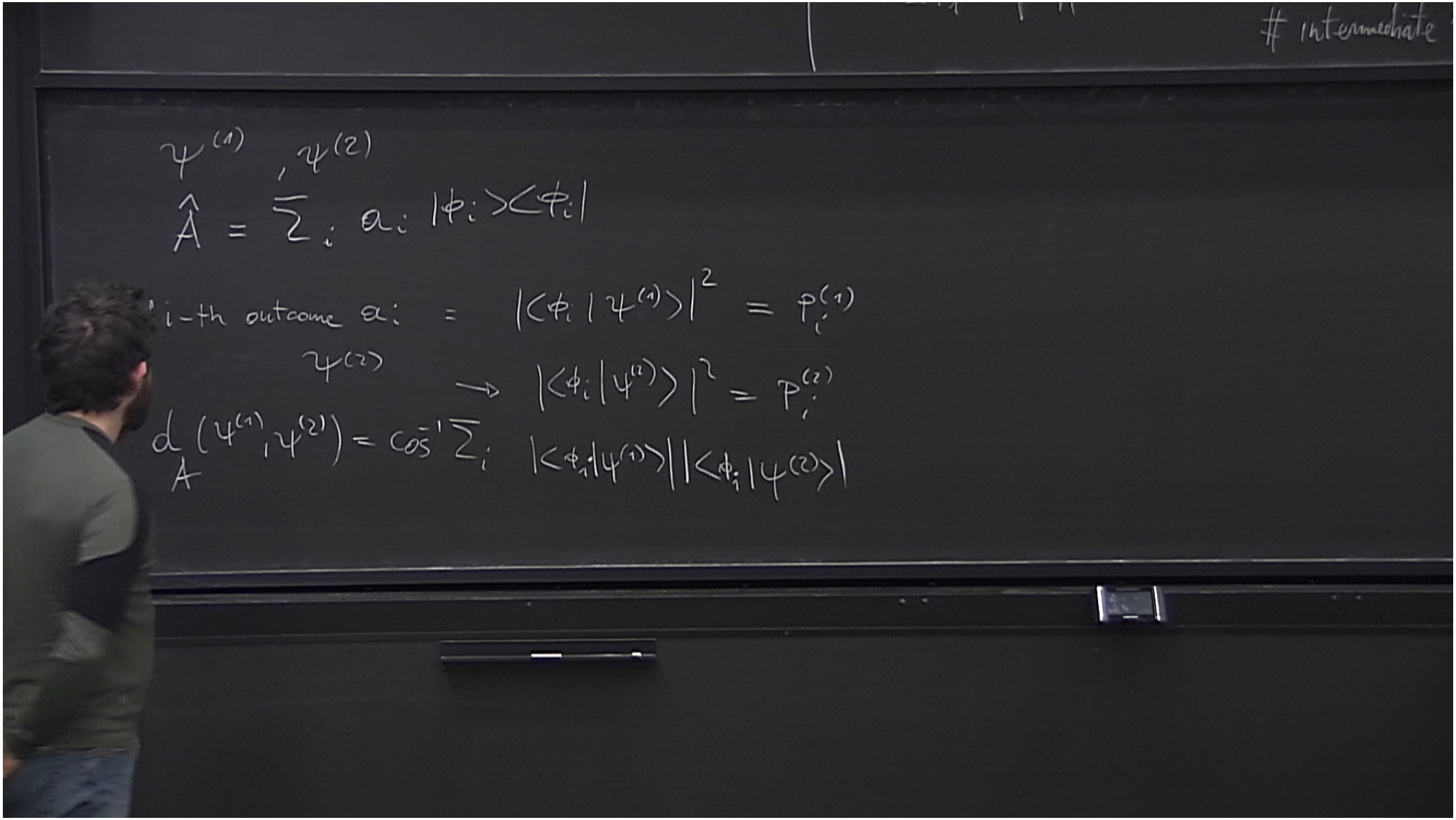
$\#$ intermediate mutually dist $P_i \approx \cos^{-1}\left(P_1^{1/2} P_f^{1/2} + (1-P_1)^{1/2}(1-P_f)^{1/2}\right)$


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intermediate
$\psi^{(1)}$, $\psi^{(2)}$


$\psi^{(1)}$, $\psi^{(2)}$
$\hat{A} = \sum_i a_i |\phi_i\rangle\langle\phi_i|$
intermediate


$\psi^{(1)}$, $\psi^{(2)}$
$\hat{A} = \sum_i a_i |\phi_i\rangle\langle\phi_i|$
Prob. of i-th outcome a_i = $|\langle\phi_i|\psi^{(1)}\rangle|^2$
intermediate


intermediate
$\psi^{(1)}$, $\psi^{(2)}$
$\hat{A} = \sum_i a_i |\phi_i\rangle\langle\phi_i|$
i-th outcome a_i = $|\langle\phi_i|\psi^{(1)}\rangle|^2 = p_i^{(1)}$
$\psi^{(2)}$
$\rightarrow |\langle\phi_i|\psi^{(2)}\rangle|^2 = p_i^{(2)}$
$d_A(\psi^{(1)}, \psi^{(2)}) = \cos^{-1} \sum_i |\langle\phi_i|\psi^{(1)}\rangle||\langle\phi_i|\psi^{(2)}\rangle|$

intermediate

$\psi^{(1)}, \psi^{(2)}$

$\hat{A} = \sum_i a_i |\phi_i\rangle\langle\phi_i|$

Prob. of i-th outcome a_i $= |\langle\phi_i|\psi^{(1)}\rangle|^2 = p_i^{(1)}$

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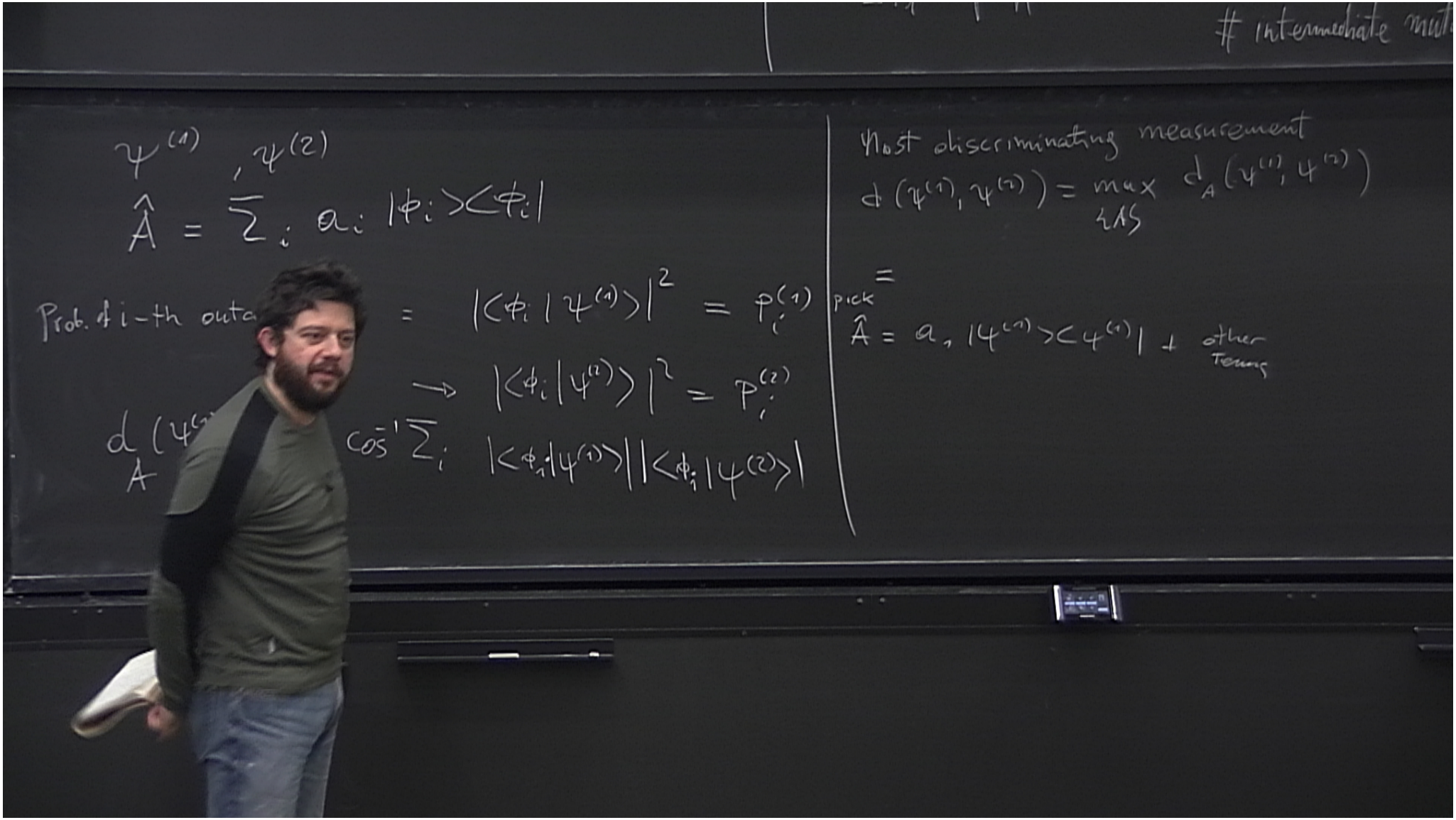
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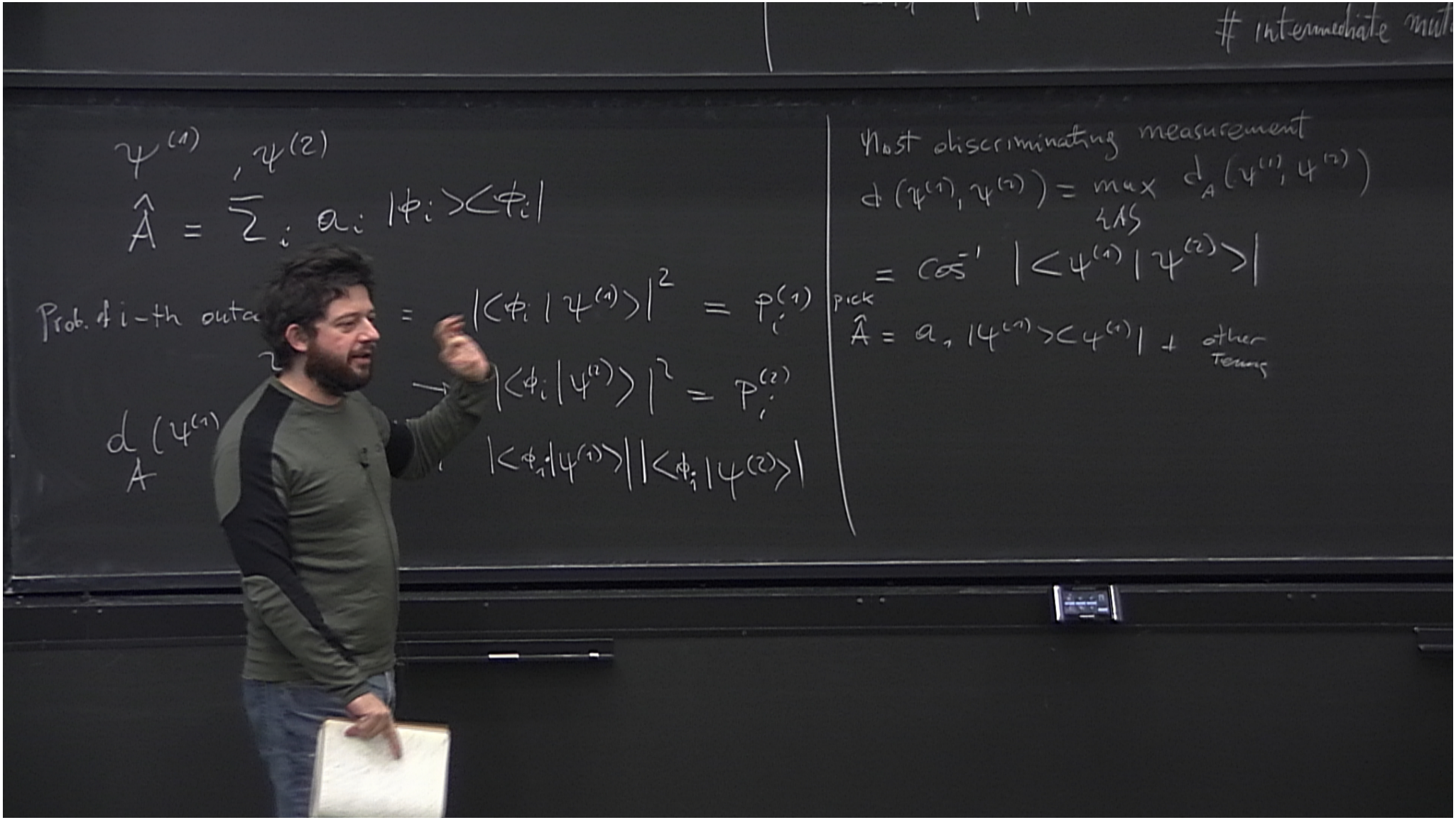
Prob. of i-th outcome a_i = $|\langle\phi_i|\psi^{(1)}\rangle|^2 = p_i^{(1)}$

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$d_A(\psi^{(1)}, \psi^{(2)}) = \cos^{-1} \sum_i |\langle\phi_i|\psi^{(1)}\rangle||\langle\phi_i|\psi^{(2)}\rangle|$

Most discriminating measurement


intermediate
$\psi^{(1)}, \psi^{(2)}$
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Most discriminating measurement
$d(\psi^{(1)}, \psi^{(2)}) = \max_{\{A\}} d_A(\psi^{(1)}, \psi^{(2)})$
=
pick
$\hat{A} = a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| +$ other terms


$\psi^{(1)}$, $\psi^{(2)}$
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$|\langle\phi_i|\psi^{(1)}\rangle||\langle\phi_i|\psi^{(2)}\rangle|$
$d_A(\psi^{(1)}$
Most discriminating measurement
$d(\psi^{(1)}, \psi^{(2)}) = \max_{\{A\}} d_A(\psi^{(1)}, \psi^{(2)})$
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intermediate

Most discriminating measurement

$$d(\psi^{(1)}, \psi^{(2)}) = \max_{\{A\}} d_A(\psi^{(1)}, \psi^{(2)}) \qquad \psi \\ \| \\ = \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle| \approx 2(1 - |\langle \psi^{(1)} | \psi^{(2)} \rangle|)$$

$p_i^{(1)}$

pick

$\hat{A} = a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| + \text{other Terms}$

$p_i^{(2)}$

$|\psi^{(2)}\rangle|$

Most discriminating measurement

$$d(\psi^{(1)}, \psi^{(2)}) = \max_{\{A\}} d_A(\psi^{(1)}, \psi^{(2)}) \qquad \psi \leftrightarrow \rho$$

$$= \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle| \approx 2\underbrace{(1 - |\langle \psi^{(1)} | \psi^{(2)} \rangle|)}_{d_{FS}}$$

d_{FS} Fubini-Study distance

$p_i^{(1)}$

$p_i^{(2)}$

$|\psi^{(2)}\rangle|$

pick

$\hat{A} = a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| + \text{other Terms}$

Most discriminating measurement

$$d(\psi^{(1)}, \psi^{(2)}) = \max_{\{A\}} d_A(\psi^{(1)}, \psi^{(2)}) \quad \text{(Fubini-Study distance)}$$

$$= \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle| \approx 2\left(1 - |\langle \psi^{(1)} | \psi^{(2)} \rangle|\right) = d_{FS}$$

pick $\hat{A} = a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| + \text{other terms}$

$|\psi_0(\lambda)\rangle$ GS of $H(\lambda)$

Most discriminating measurement

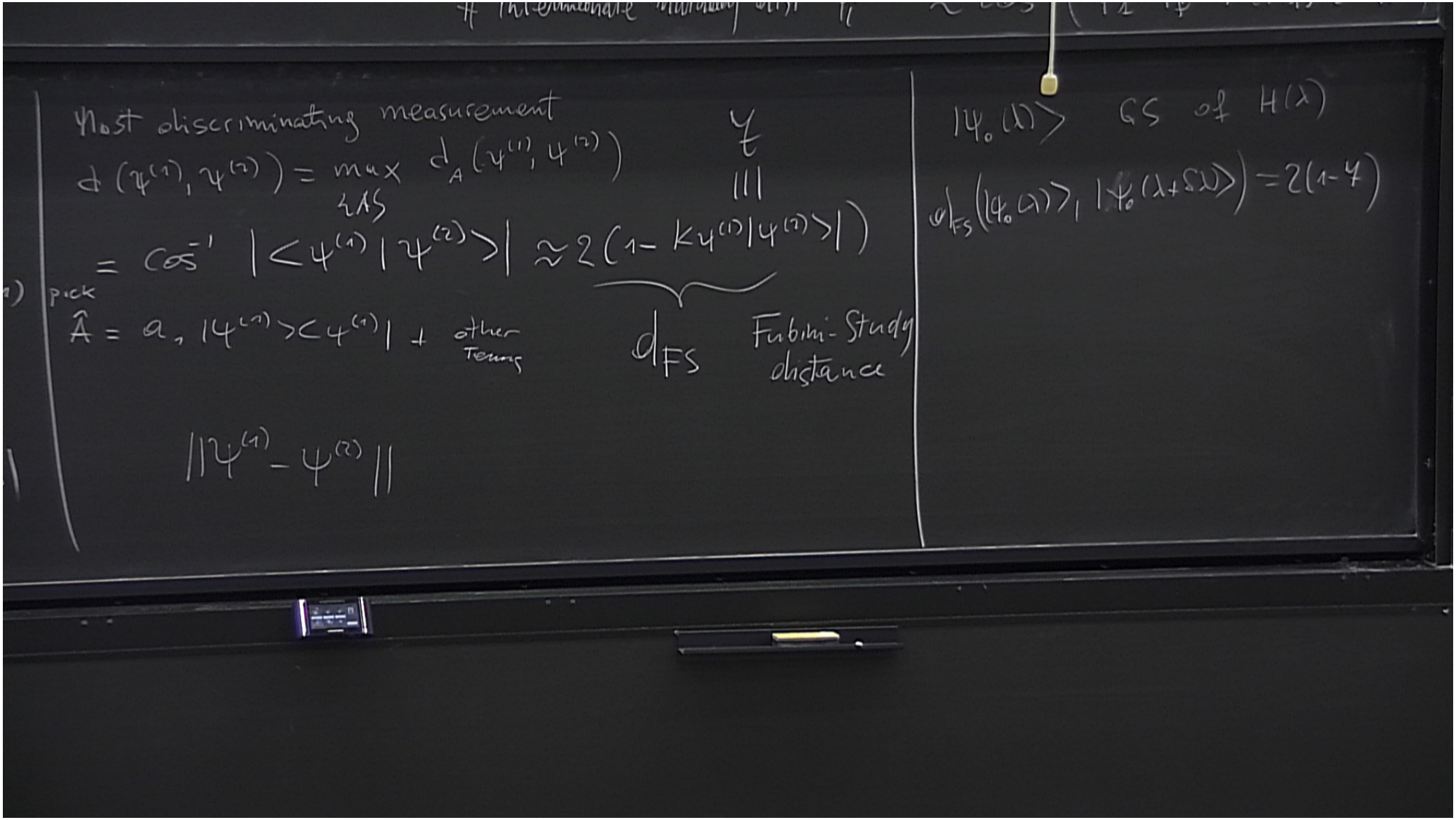
$$d(\psi^{(1)}, \psi^{(2)}) = \max_{\{A\}} d_A(\psi^{(1)}, \psi^{(2)}) \quad \text{(Bures)}$$

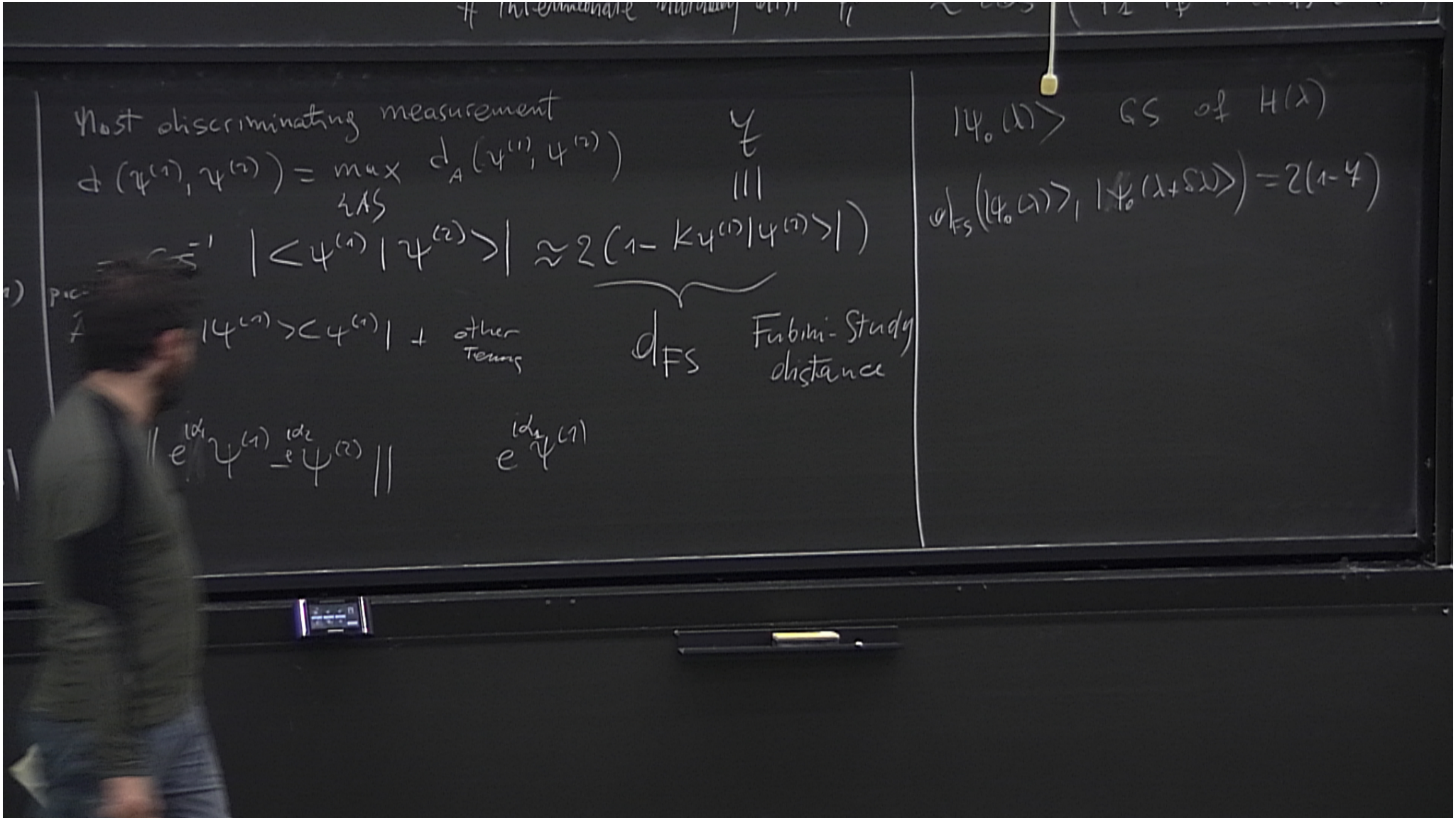
$$= \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle| \approx 2(1 - |\langle \psi^{(1)} | \psi^{(2)} \rangle|)$$

d_{FS} Fubini-Study distance

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Most discriminating measurement
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$\| |\psi^{(1)}\rangle - |\psi^{(2)}\rangle \|$
$|\psi_0(\lambda)\rangle$ GS of $H(\lambda)$
$d_{FS}(|\psi_0(\lambda)\rangle, |\psi_0(\lambda+\delta\lambda)\rangle) = 2(1-\gamma)$


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Most discriminating measurement

$$d(\psi^{(1)}, \psi^{(2)}) = \max_{\{A\}} d_A(\psi^{(1)}, \psi^{(2)}) \qquad \psi \leftrightarrow |\psi\rangle\langle\psi|$$

$$= \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle| \approx 2(1 - |\langle \psi^{(1)} | \psi^{(2)} \rangle|) \qquad d_{FS}$$

Fubini-Study distance

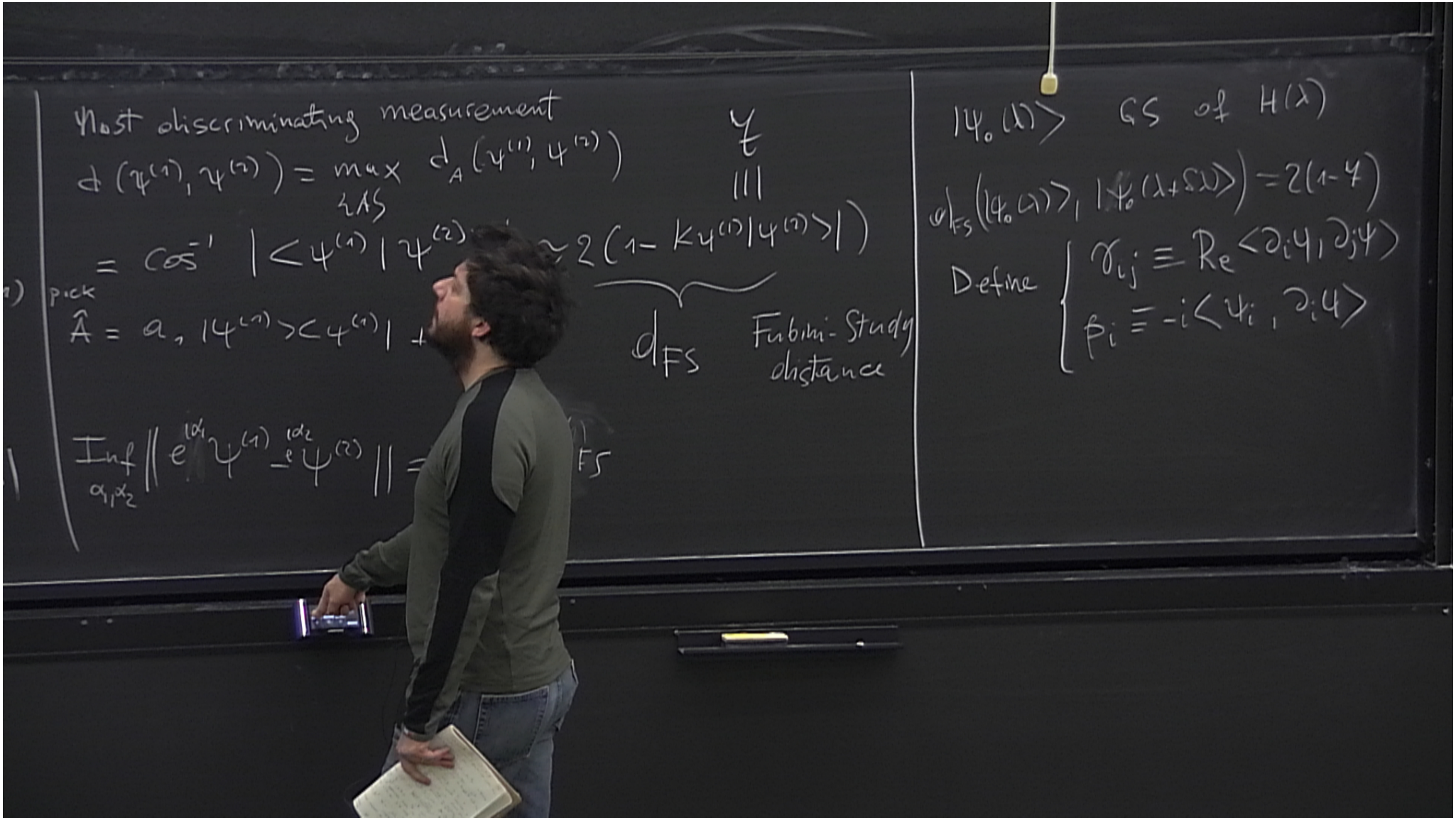
pick $\hat{A} = a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| + \text{other terms}$

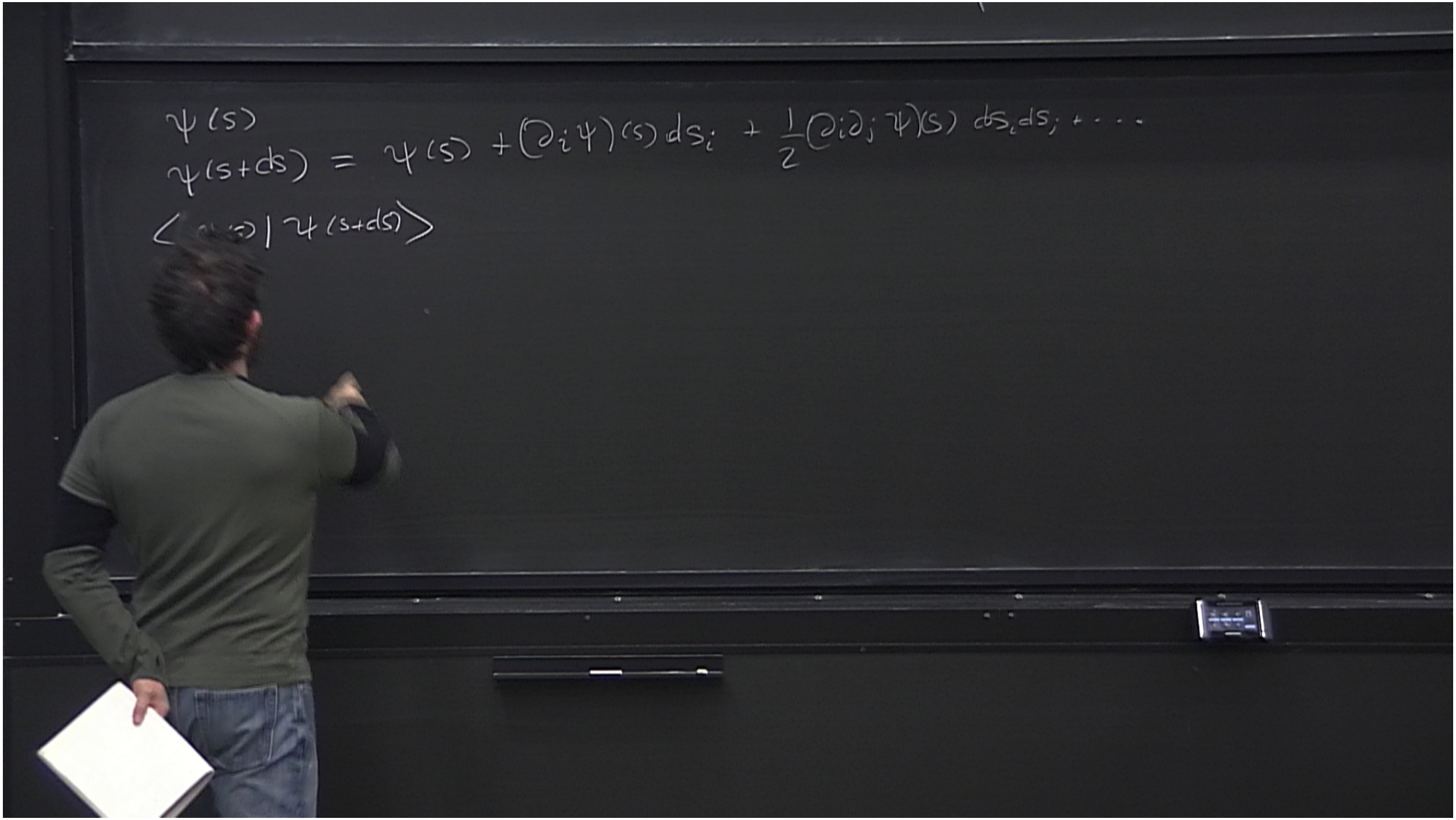
$$\inf_{\alpha_1, \alpha_2} \| e^{i\alpha_1} \psi^{(1)} - e^{i\alpha_2} \psi^{(2)} \| = \ldots = d_{FS}$$

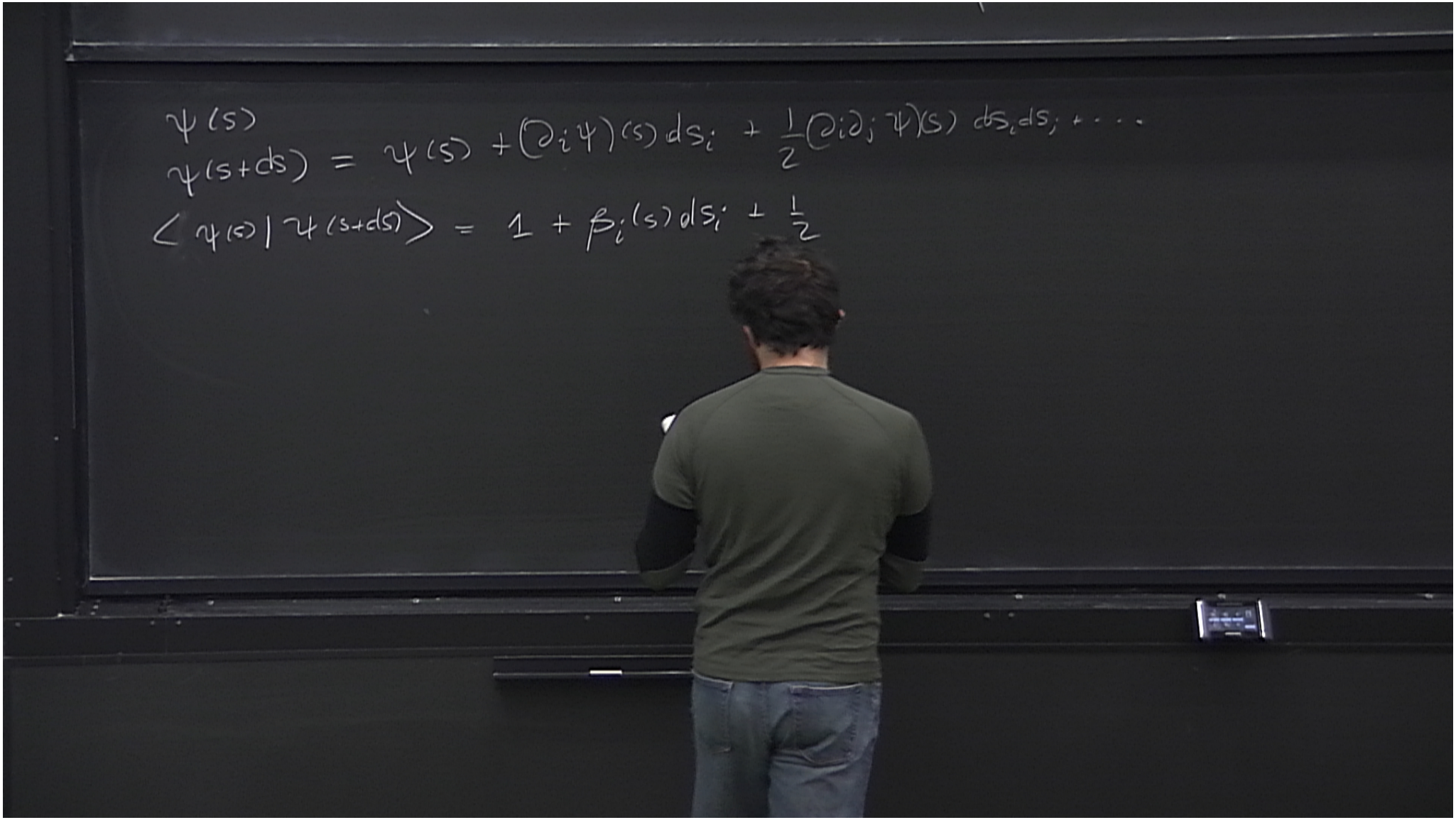
$|\psi_0(\lambda)\rangle$ GS of $H(\lambda)$

$d_{FS}(|\psi_0(\lambda)\rangle, |\psi_0(\lambda+\delta\lambda)\rangle) = 2(1-\gamma)$

Define $\gamma_{ij} = \mathrm{Re}\langle \partial_i \psi | \partial_j \psi \rangle$


Most discriminating measurement
$d(\psi^{(1)}, \psi^{(2)}) = \max_{\{A\}} d_A(\psi^{(1)}, \psi^{(2)})$
$= \cos^{-1} |\langle \psi^{(1)} | \psi^{(2)} \rangle| \approx 2(1 - |\langle \psi^{(1)} | \psi^{(2)} \rangle|)$
d_{FS} Fubini-Study distance
pick
$\hat{A} = a_1 |\psi^{(1)}\rangle\langle\psi^{(1)}| +$
$\inf_{\alpha_1, \alpha_2} \| e^{i\alpha_1} \psi^{(1)} - e^{i\alpha_2} \psi^{(2)} \| =$
$|\psi_0(\lambda)\rangle$ GS of $H(\lambda)$
$d_{FS}(|\psi_0(\lambda)\rangle, |\psi_0(\lambda+\delta\lambda)\rangle) = 2(1-\gamma)$
Define
$\gamma_{ij} \equiv \mathrm{Re}\langle \partial_i \psi, \partial_j \psi \rangle$
$\beta_i \equiv -i\langle \psi, \partial_i \psi \rangle$


$\psi(s)$
$\psi(s+ds) = \psi(s) + (\partial_i \psi)(s)\, ds_i + \frac{1}{2}(\partial_i \partial_j \psi)(s)\, ds_i ds_j + \cdots$
$\langle \quad | \psi(s+ds) \rangle$


$\psi(s)$
$\psi(s+ds) = \psi(s) + (\partial_i \psi)(s)\, ds_i + \frac{1}{2}(\partial_i \partial_j \psi)(s)\, ds_i ds_j + \cdots$
$\langle \psi(s) | \psi(s+ds) \rangle = 1 + \beta_i(s)\, ds_i + \frac{1}{2}$

$$\psi(s)$$

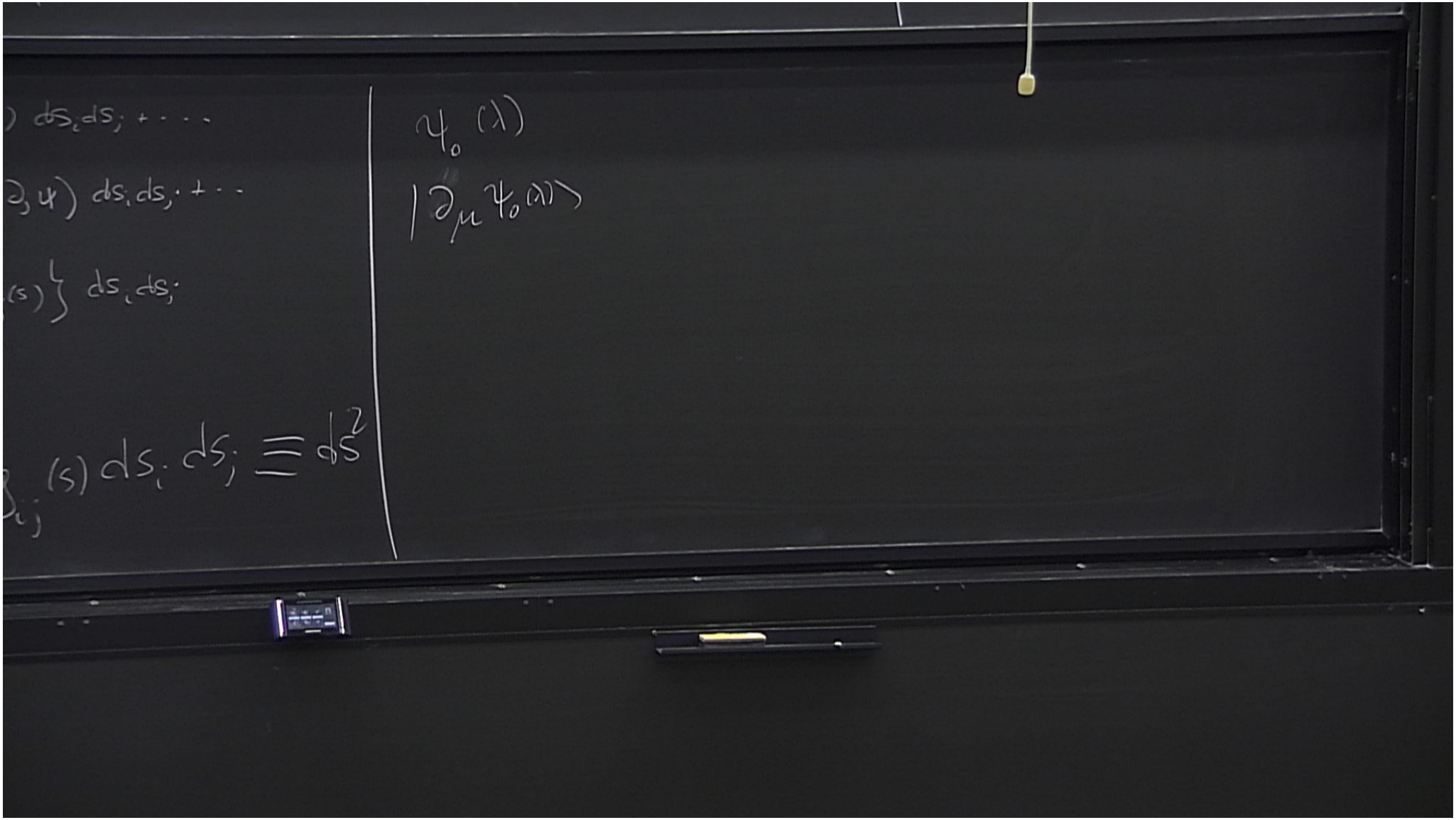
$$\psi(s+ds) = \psi(s) + (\partial_i \psi)(s)\, ds_i + \frac{1}{2}(\partial_i \partial_j \psi)(s)\, ds_i ds_j + \cdots$$

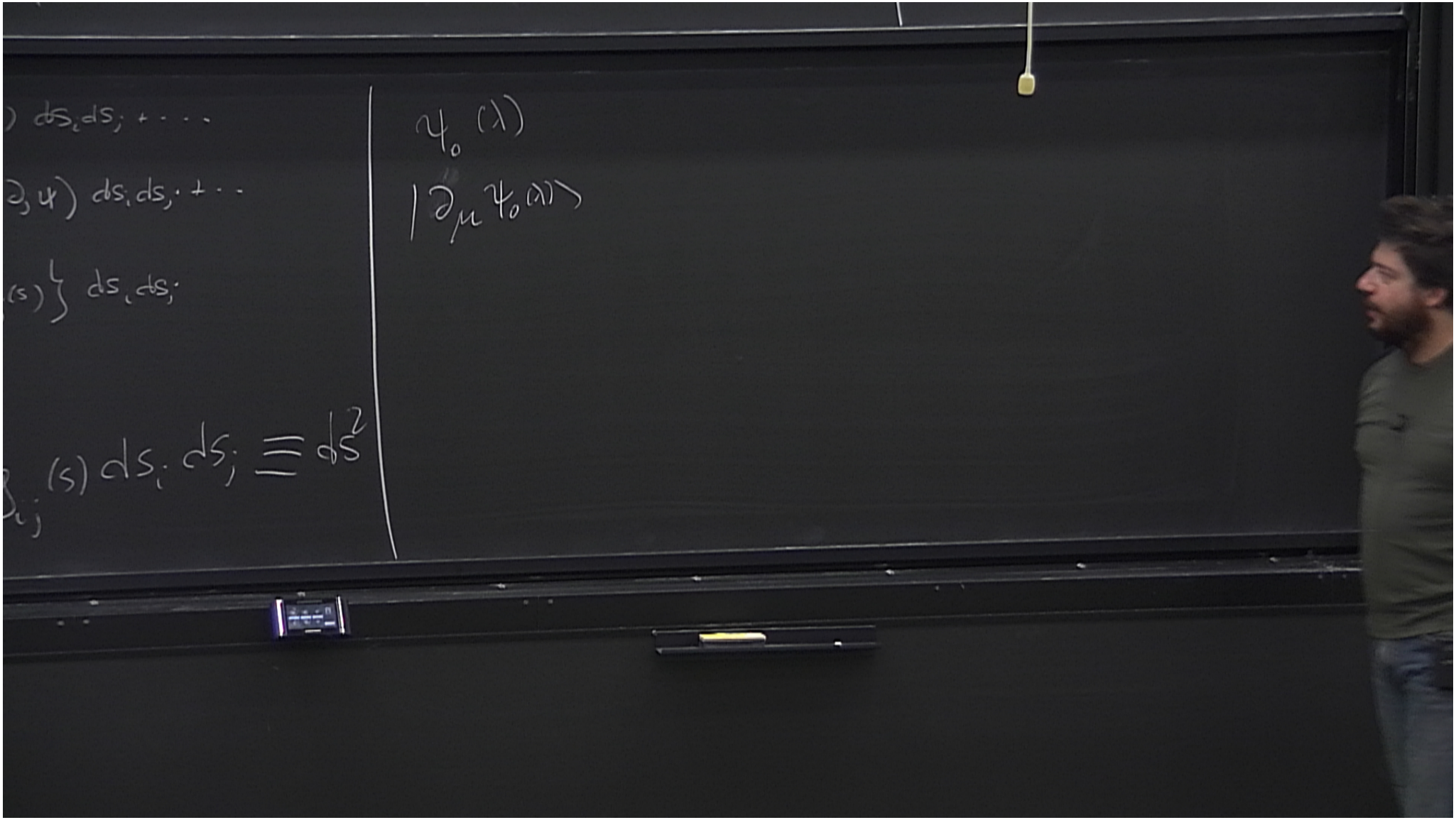
$$\langle \psi(s) | \psi(s+ds) \rangle = 1 + \beta_i(s)\, ds_i + \frac{1}{2}(\psi, \partial_i \partial_j \psi)\, ds_i ds_j + \cdots$$

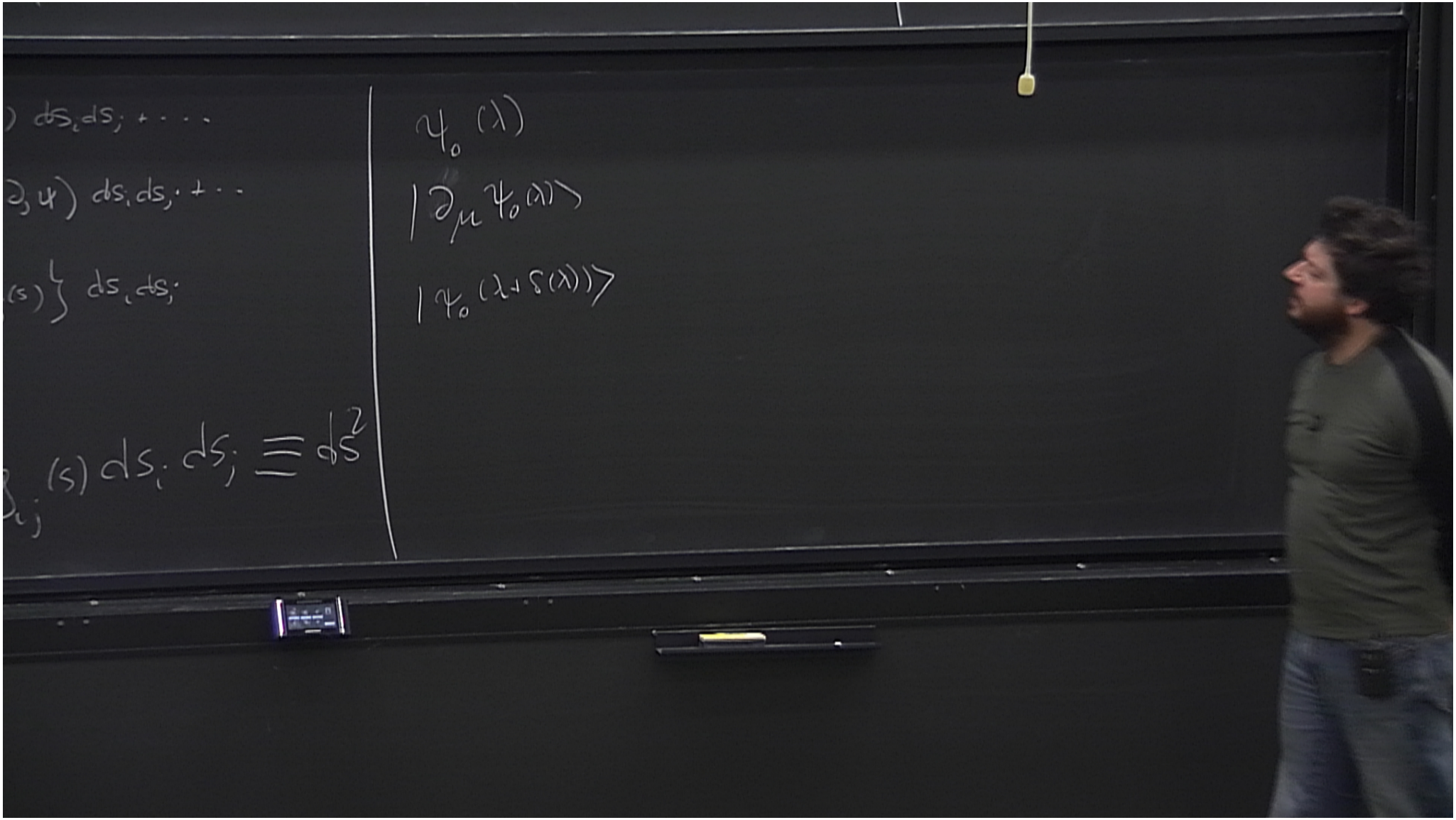
$$\mathcal{F} \equiv |\langle \psi(s) | \psi(s+ds) \rangle| = 1 + \frac{1}{2}\left\{ Re(\psi, \partial_i \partial_j \psi) + \beta_i(s)\beta_j(s) \right\} ds_i ds_j$$

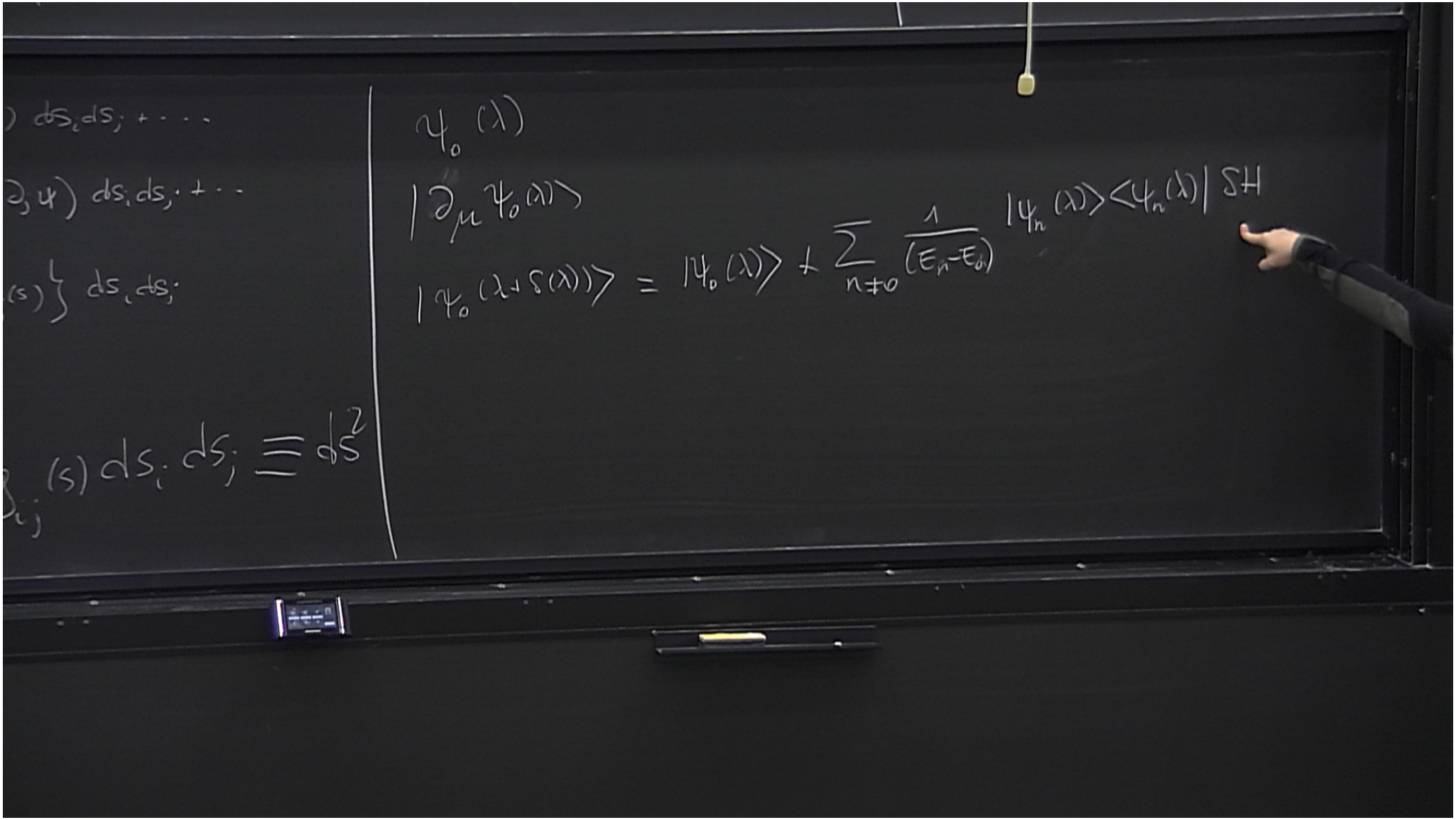
$$-Re(\partial_i \psi, \partial_j \psi) = \gamma_{ij}$$

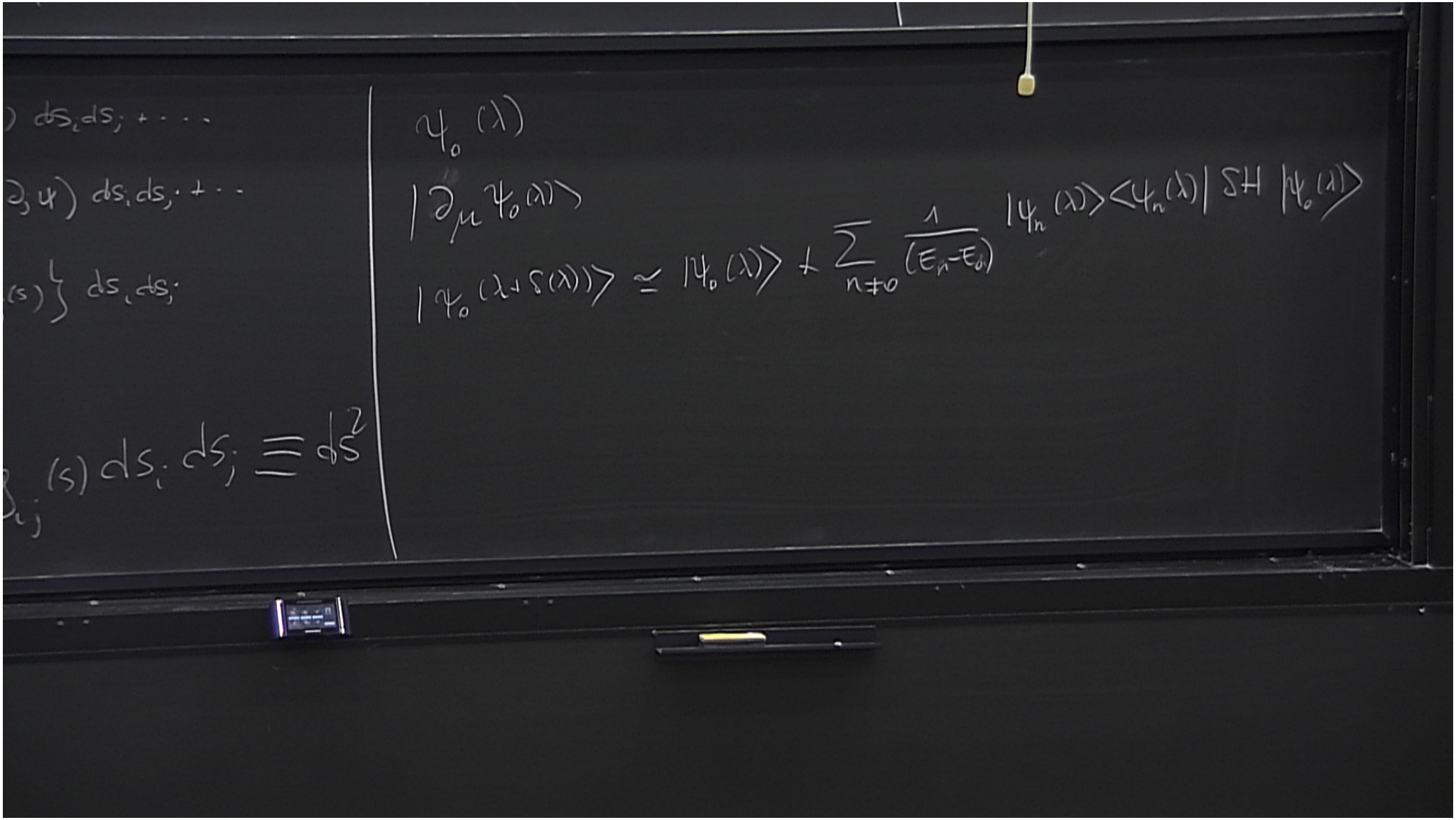
$$d_{SF} = 2(1-\mathcal{F}) = \left(\gamma_{ij} - \beta_i(s)\beta_j(s)\right) ds_i ds_j \equiv g_{ij}(s)\, ds_i ds_j \equiv ds^2$$


$ds_i ds_j + \cdots$
$\psi) ds_i ds_j + \cdots$
$(s)\} ds_i ds_j$
$(s)\, ds_i ds_j \equiv ds^2$
$\psi_0(\lambda)$
$|\partial_\mu \psi_0(\lambda)\rangle$


$ds_i ds_j + \cdots$
$\partial_j \psi)\, ds_i ds_j + \cdots$
$(s)\}\, ds_i ds_j$
$\beta_{ij}(s)\, ds_i ds_j \equiv ds^2$
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$|\partial_\mu \psi_0(\lambda)\rangle$


$ds_i ds_j + \cdots$
$\partial_j \psi) ds_i ds_j + \cdots$
$(s) \} ds_i ds_j$
$(s)\, ds_i ds_j \equiv ds^2$
$\psi_0(\lambda)$
$|\partial_\mu \psi_0(\lambda)\rangle$
$|\psi_0(\lambda + \delta(\lambda))\rangle$


$ds_i ds_j + \cdots$
$\partial_\mu \psi) ds_i ds_j + \cdots$
$(s) \} ds_i ds_j$
$(s)\, ds_i ds_j \equiv ds^2$
$\psi_0(\lambda)$
$|\partial_\mu \psi_0(\lambda)\rangle$
$|\psi_0(\lambda + \delta(\lambda))\rangle = |\psi_0(\lambda)\rangle + \sum_{n \neq 0} \frac{1}{(E_n - E_0)} |\psi_n(\lambda)\rangle\langle\psi_n(\lambda)| \delta H$


$ds_i ds_j + \cdots$
$\partial_\mu \psi) ds_i ds_j + \cdots$
$(s) \} ds_i ds_j$
$(s)\, ds_i ds_j \equiv ds^2$
$\psi_0(\lambda)$
$|\partial_\mu \psi_0(\lambda)\rangle$
$|\psi_0(\lambda + \delta(\lambda))\rangle \simeq |\psi_0(\lambda)\rangle + \sum_{n \neq 0} \frac{1}{(E_n - E_0)} |\psi_n(\lambda)\rangle\langle\psi_n(\lambda)| \delta H |\psi_0(\lambda)\rangle$

$ds_i ds_j + \cdots$

$\partial_\mu \psi) \; ds_i ds_j + \cdots$

$(s)\} \; ds_i ds_j$

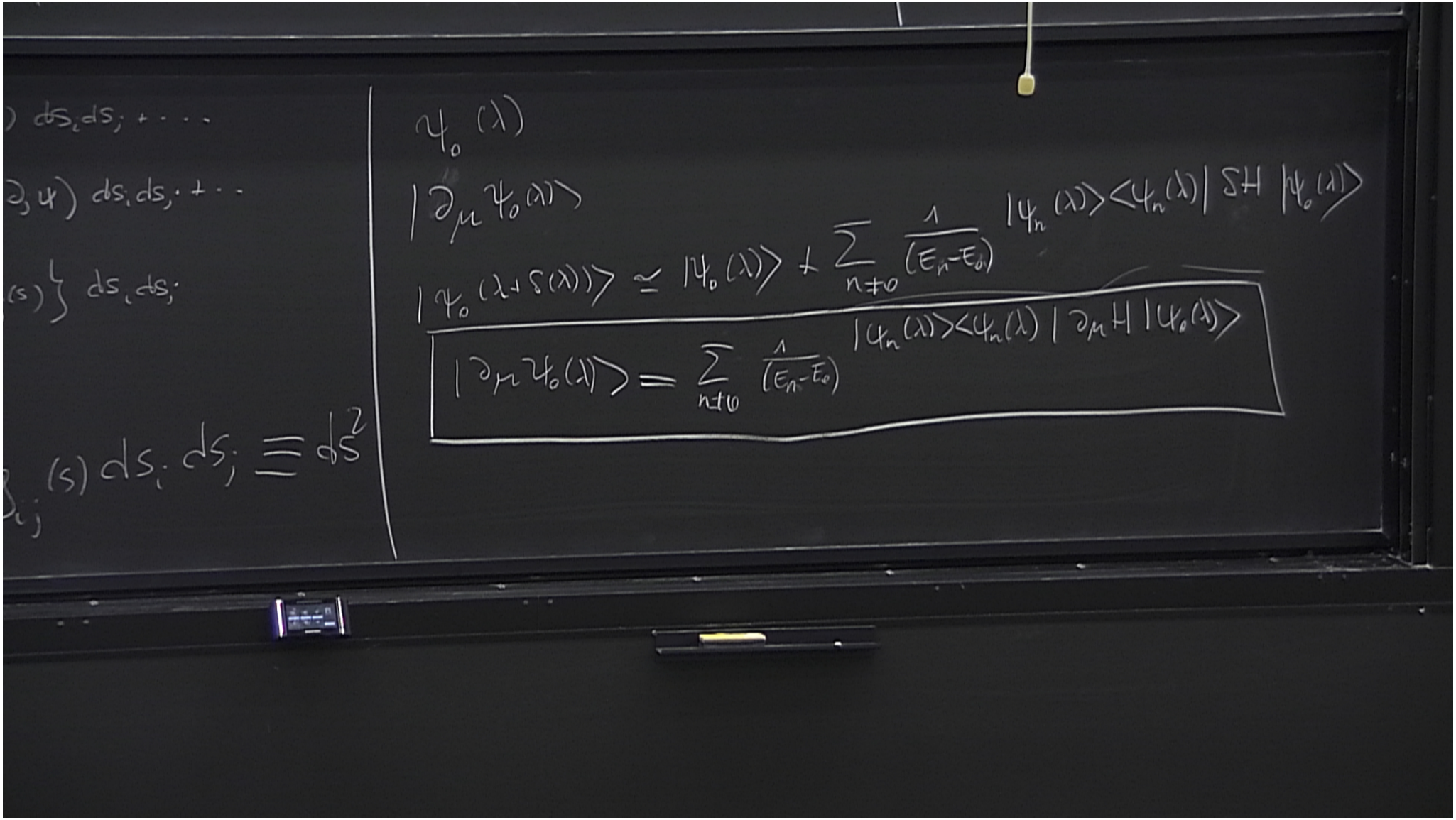
$\beta_{ij}(s)\, ds_i ds_j \equiv ds^2$

$\psi_0(\lambda)$

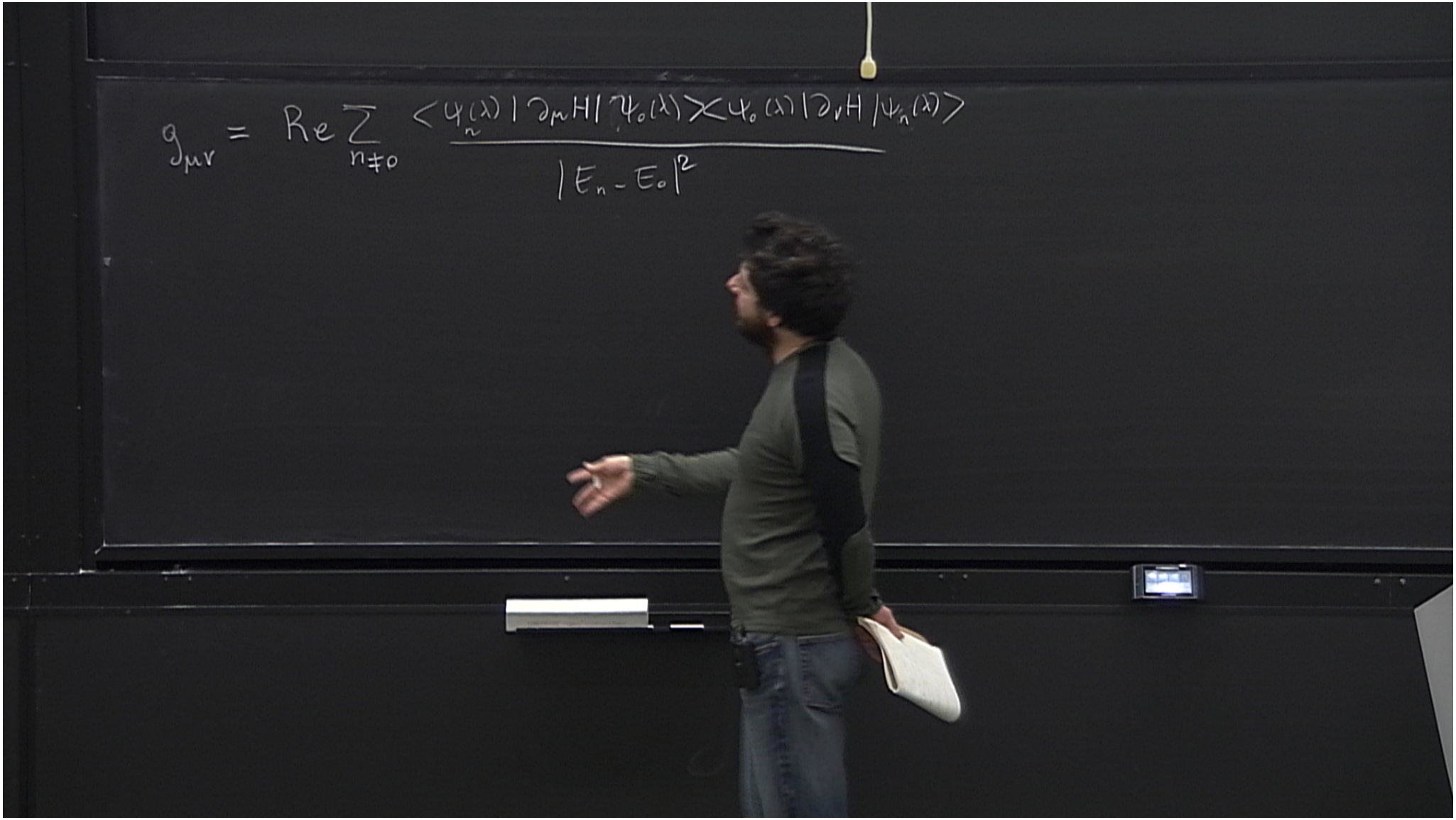
$|\partial_\mu \psi_0(\lambda)\rangle$

$$|\psi_0(\lambda+\delta(\lambda))\rangle \simeq |\psi_0(\lambda)\rangle + \sum_{n\neq 0} \frac{\hat{1}}{(E_n - E_0)} |\psi_n(\lambda)\rangle\langle\psi_n(\lambda)| \delta H |\psi_0(\lambda)\rangle$$

$$|\partial_\mu \psi_0(\lambda)\rangle = \sum_{n\neq 0} \frac{\hat{1}}{(E_n - E_0)} |\psi_n(\lambda)\rangle\langle\psi_n(\lambda)| \partial_\mu H |\psi_0(\lambda)\rangle$$


$\psi_0(\lambda)$
$|\partial_\mu \psi_0(\lambda)\rangle$
$|\psi_0(\lambda+\delta(\lambda))\rangle \simeq |\psi_0(\lambda)\rangle + \sum_{n\neq 0} \frac{1}{(E_n - E_0)} |\psi_n(\lambda)\rangle\langle\psi_n(\lambda)| \delta H |\psi_0(\lambda)\rangle$
$|\partial_\mu \psi_0(\lambda)\rangle = \sum_{n\neq 0} \frac{1}{(E_n - E_0)} |\psi_n(\lambda)\rangle\langle\psi_n(\lambda)| \partial_\mu H |\psi_0(\lambda)\rangle$
$(s)\, ds_i ds_j \equiv ds^2$

$$g_{\mu\nu} = \mathrm{Re} \sum_{n \neq 0} \frac{\langle \psi_n^{(\lambda)} | \partial_\mu H | \psi_0^{(\lambda)} \rangle \langle \psi_0^{(\lambda)} | \partial_\nu H | \psi_n^{(\lambda)} \rangle}{|E_n - E_0|^2}$$


$g_{\mu\nu} = \mathrm{Re} \sum_{n \neq 0} \frac{\langle \psi_n^{(\lambda)} | \partial_\mu H | \psi_0^{(\lambda)} \rangle \langle \psi_0^{(\lambda)} | \partial_\nu H | \psi_n^{(\lambda)} \rangle}{|E_n - E_0|^2}$

$$g_{\mu\nu} = \mathrm{Re} \sum_{n \neq 0} \frac{\langle \psi_n(\lambda) | \partial_\mu H | \psi_0(\lambda) \rangle \langle \psi_0(\lambda) | \partial_\nu H | \psi_n(\lambda) \rangle}{|E_n - E_0|^2}$$

- Application to the Quantum XY model

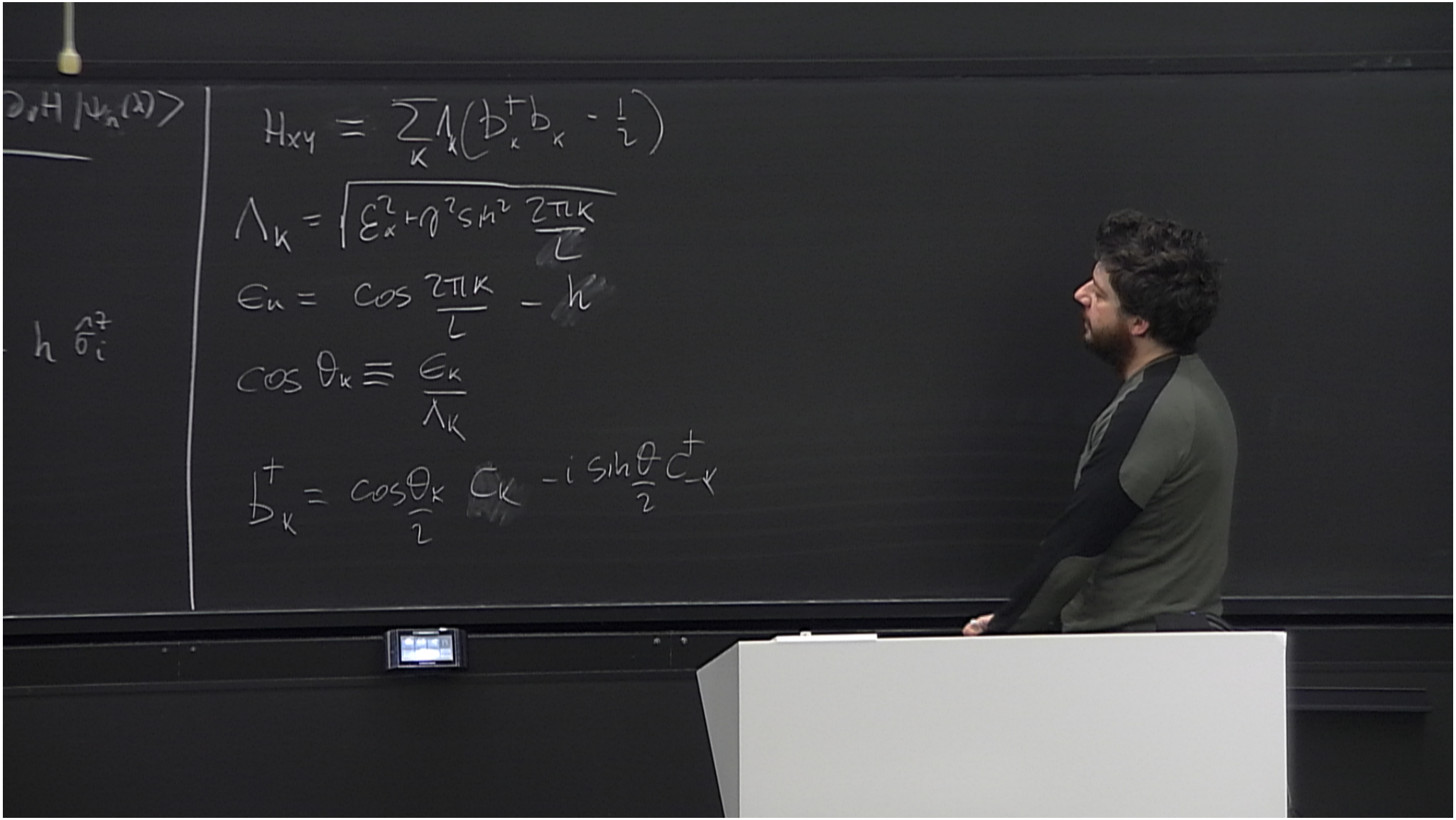
$$H_{XY} = -\sum_{i=-M}^{M} \frac{1-\gamma}{2} \hat{\sigma}_i^x \hat{\sigma}_{i+1}^x + \frac{1+\gamma}{2} \hat{\sigma}_i^y \hat{\sigma}_{i+1}^y + h \hat{\sigma}_i^z$$

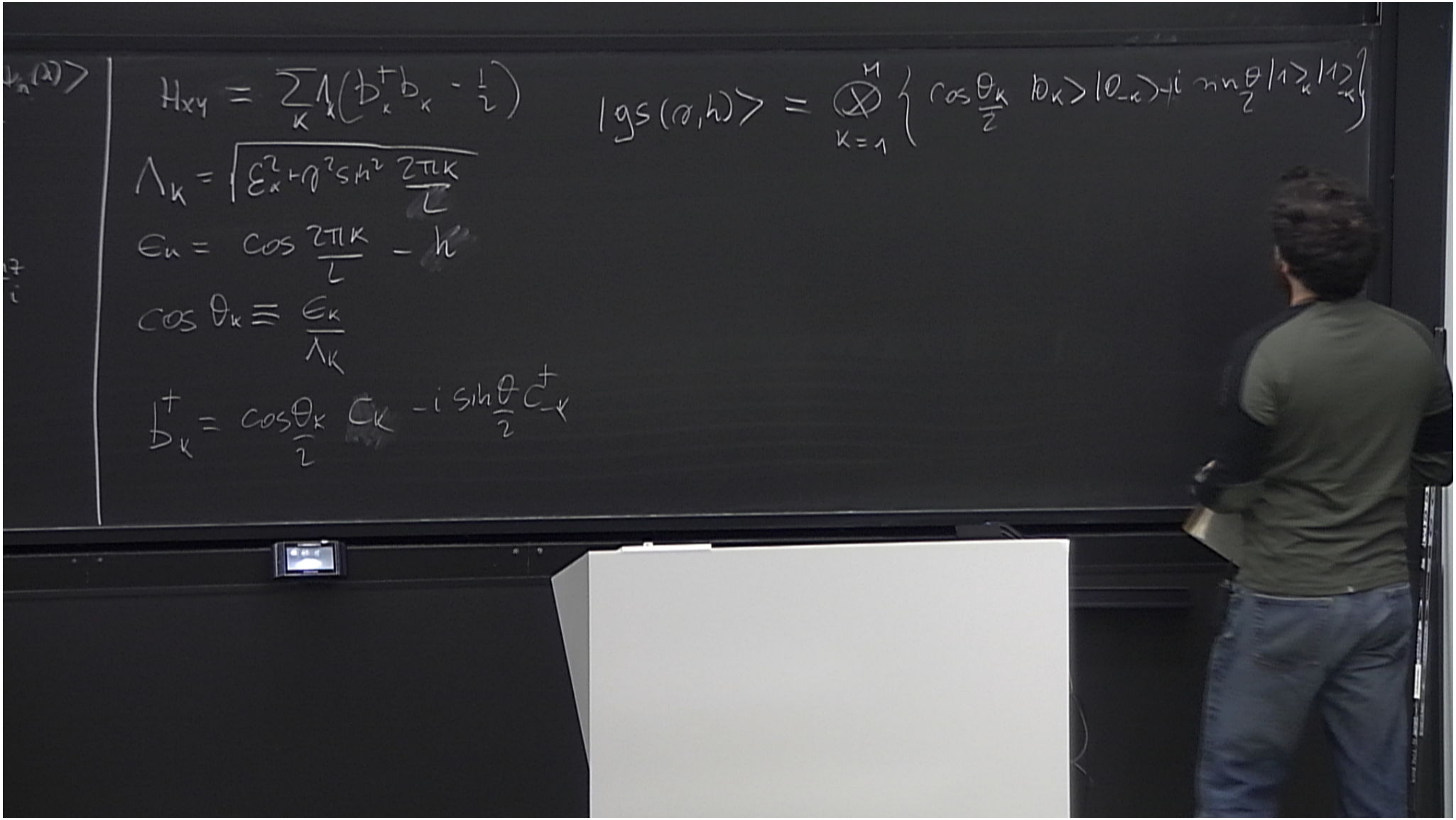
$L = 2M+1$

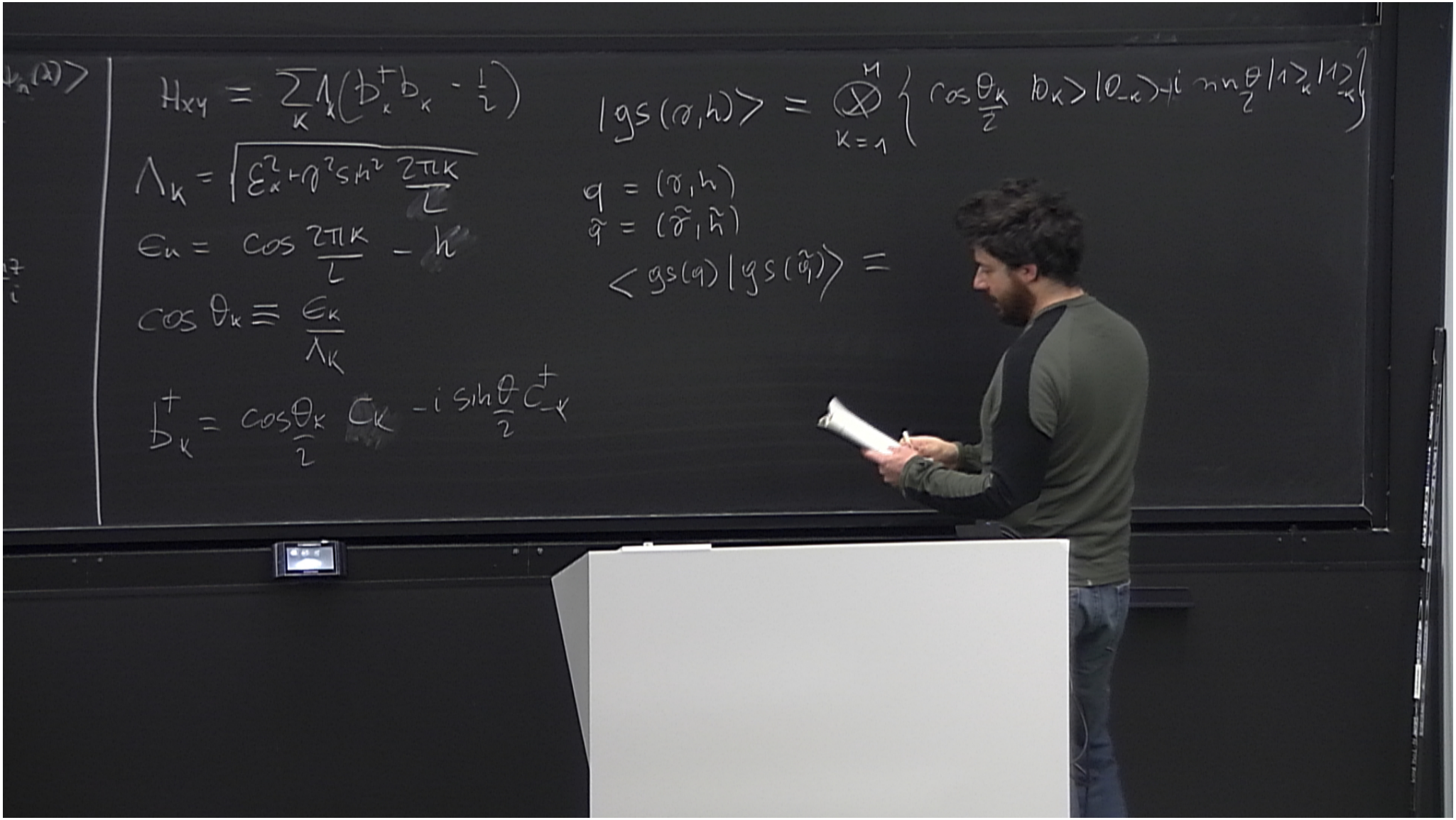
JW → Fourier → Bogoliubov

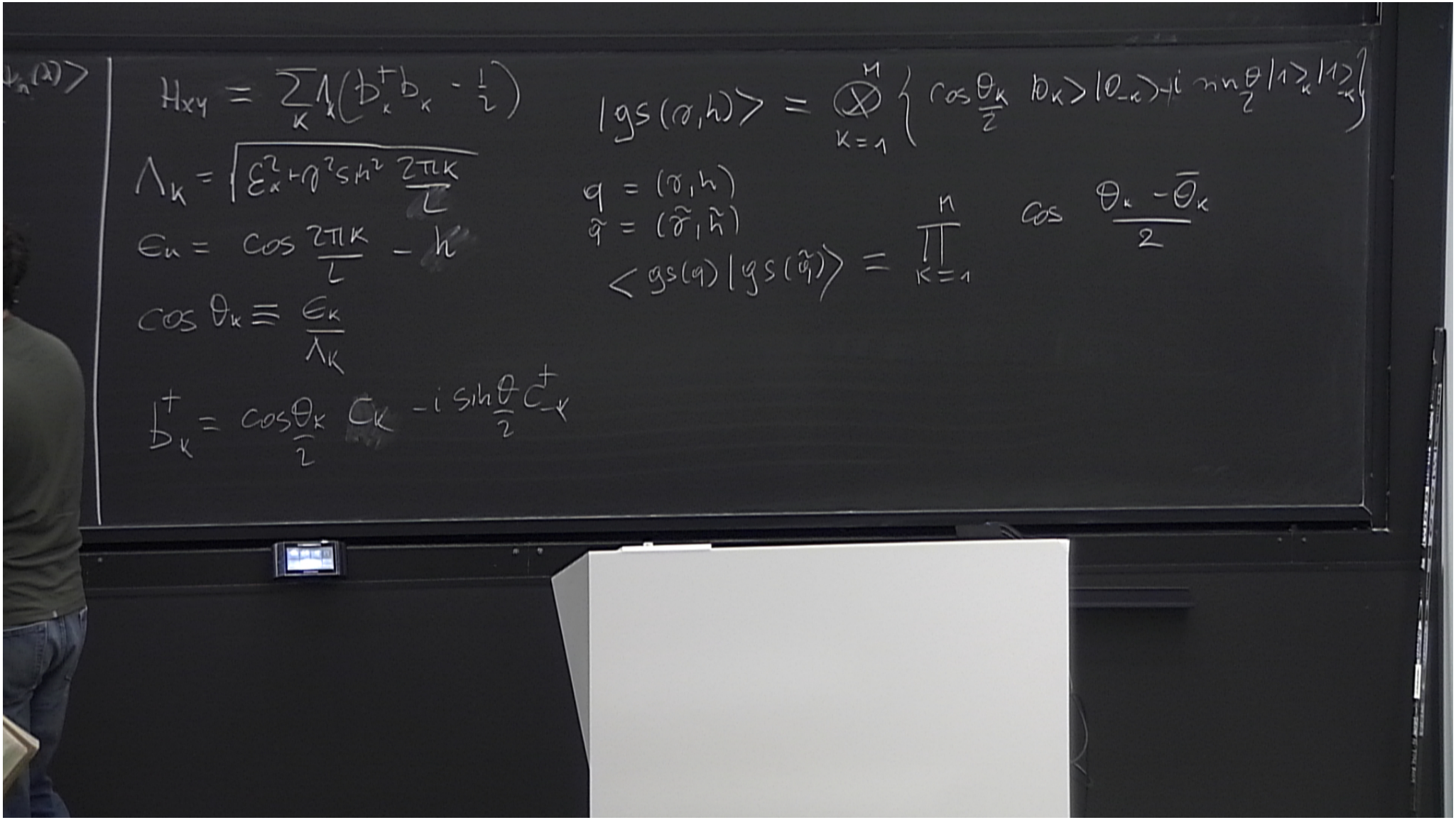
$$H_{XY} = \sum_k \Lambda_k \left(b_k^\dagger b_k - \frac{1}{2} \right)$$

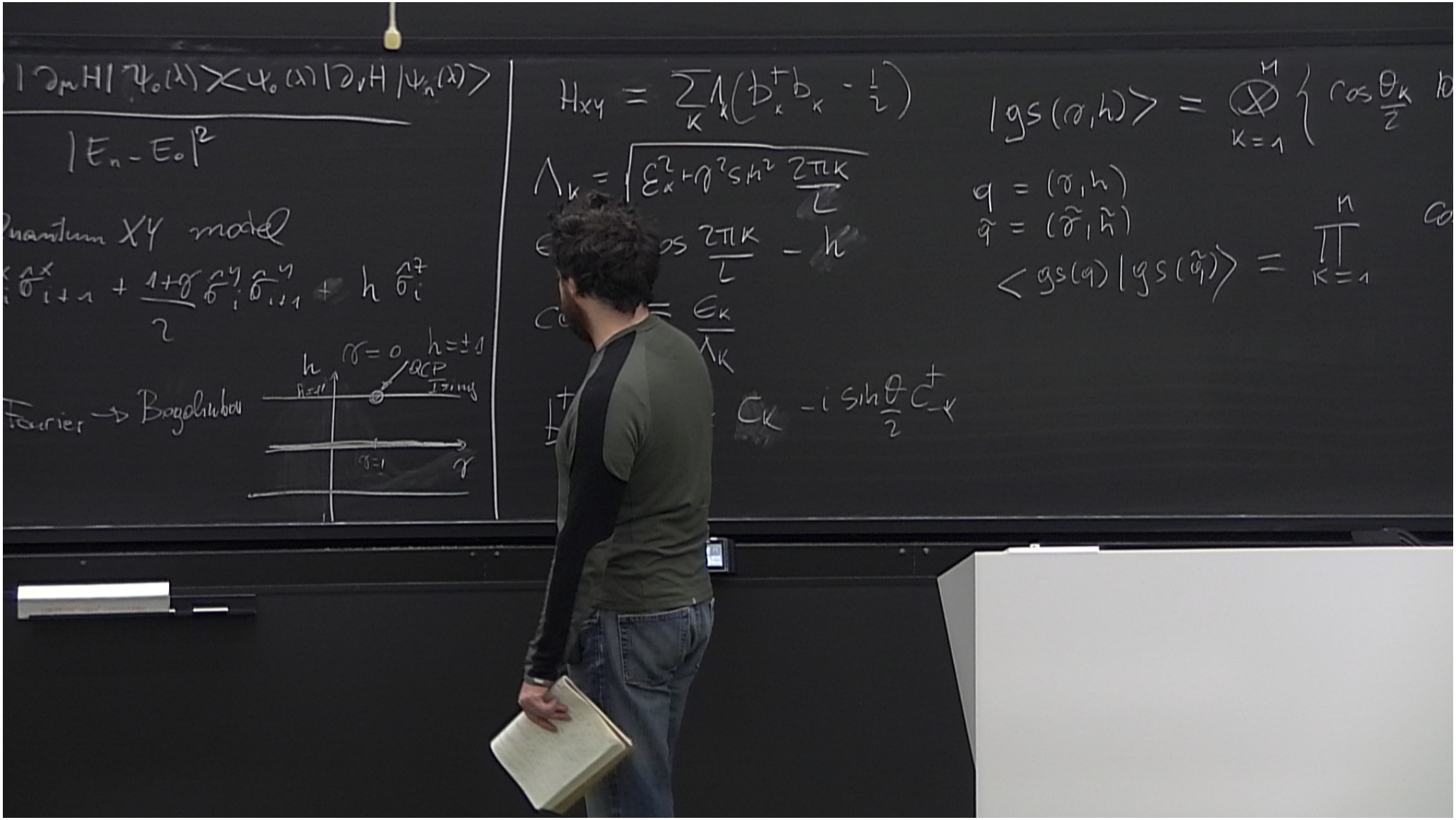
$$\Lambda_k = \sqrt{\varepsilon_k^2 + \gamma^2 \sin^2 \frac{2\pi k}{N}}$$

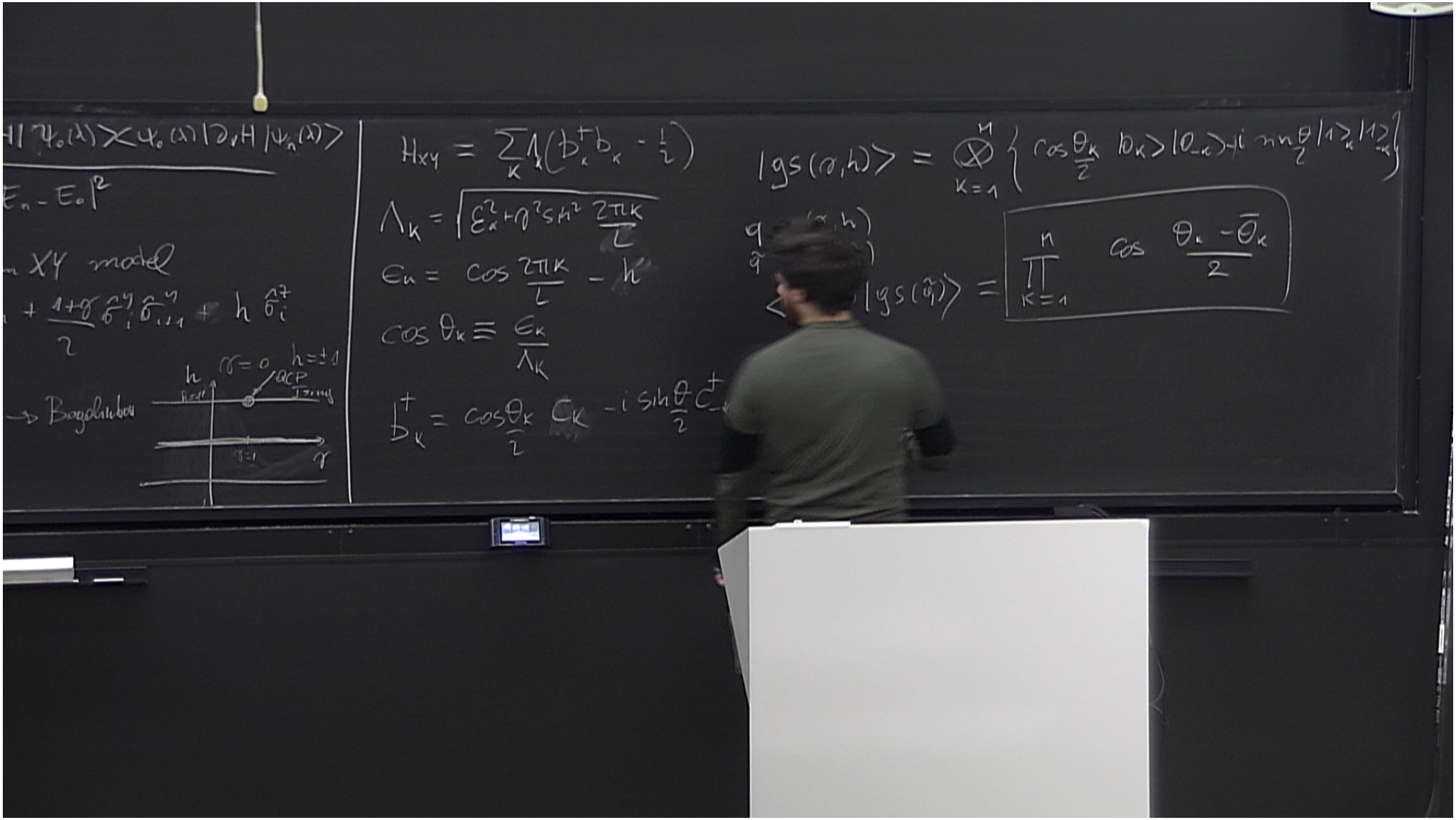

$H_{xy} = \sum_k \Lambda_k \left(b_k^\dagger b_k - \frac{1}{2} \right)$
$\Lambda_k = \sqrt{\varepsilon_k^2 + \eta^2 \sin^2 \frac{2\pi k}{L}}$
$\varepsilon_k = \cos \frac{2\pi k}{L} - h$
$\cos \theta_k \equiv \frac{\varepsilon_k}{\Lambda_k}$
$b_k^\dagger = \cos \frac{\theta_k}{2} c_k - i \sin \frac{\theta}{2} c_{-k}^\dagger$

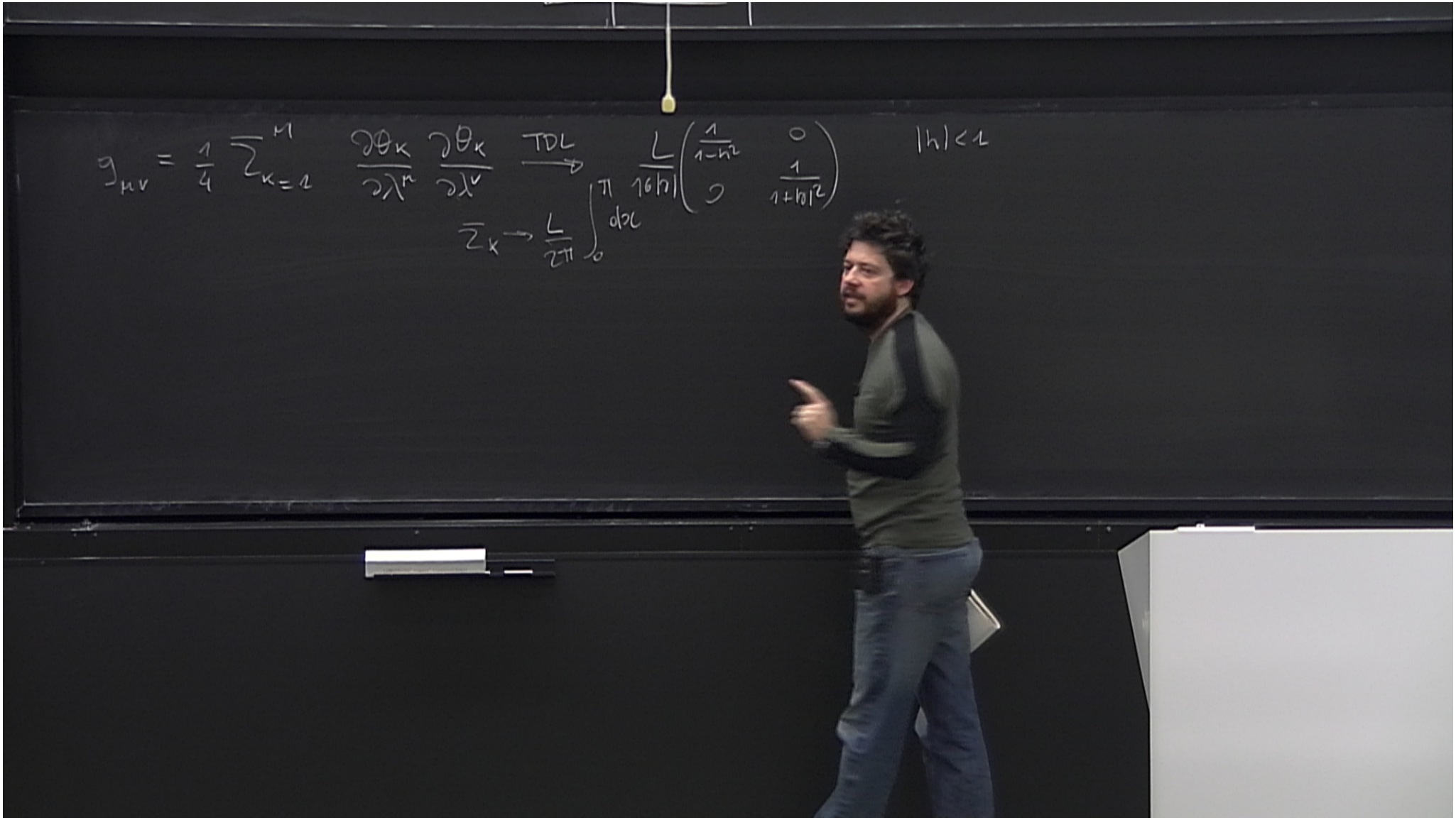

$H_{xy} = \sum_k \Lambda_k \left(b_k^\dagger b_k - \frac{1}{2} \right)$
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$|gs(\eta, h)\rangle = \bigotimes_{k=1}^{M} \left\{ \cos \frac{\theta_k}{2} |0_k\rangle |0_{-k}\rangle + i \sin \frac{\theta}{2} |1_k\rangle |1_{-k}\rangle \right\}$

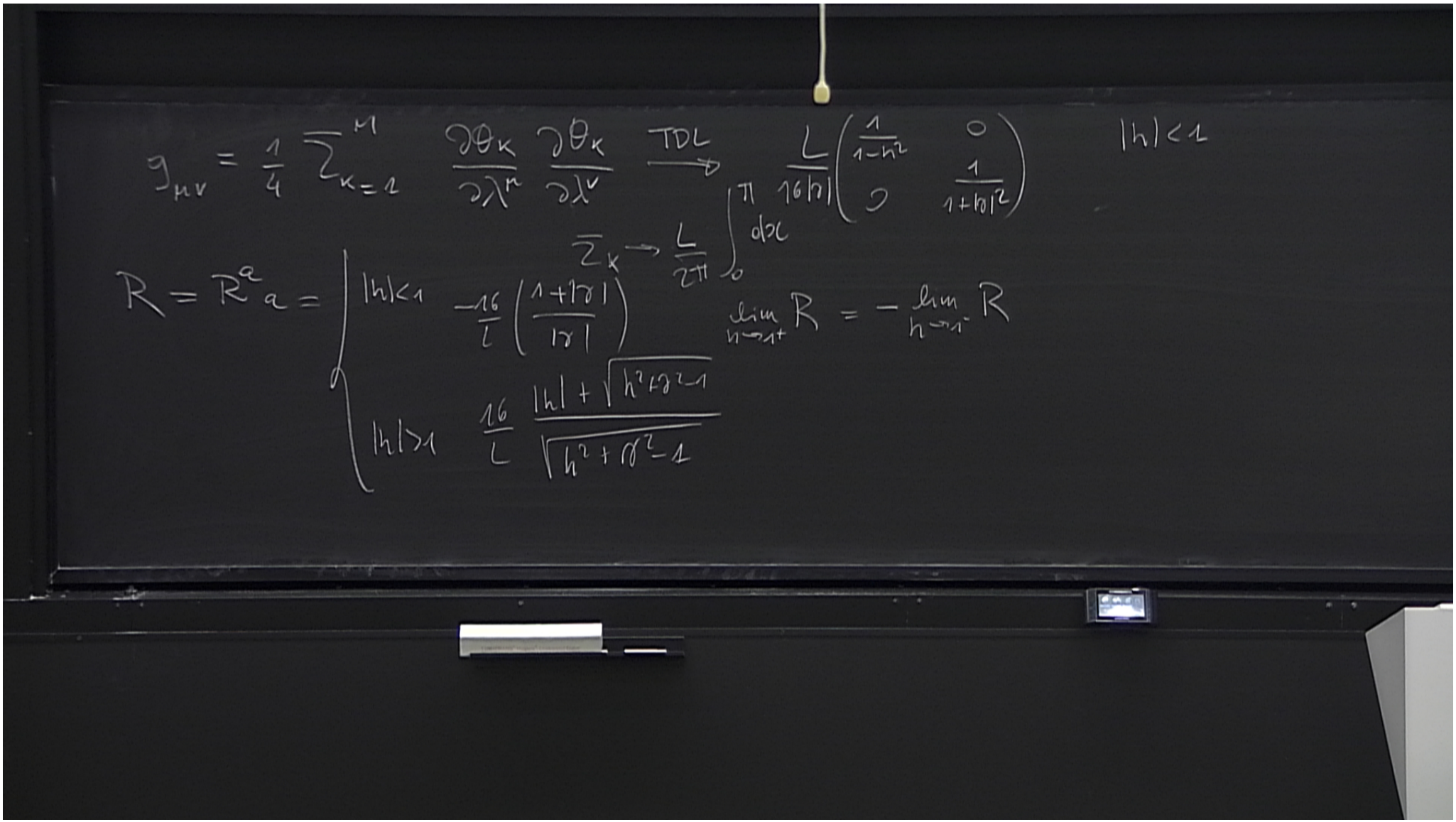

$H_{xy} = \sum_k \Lambda_k \left(b_k^\dagger b_k - \frac{1}{2} \right)$
$\Lambda_k = \sqrt{\epsilon_k^2 + \eta^2 \sin^2 \frac{2\pi k}{L}}$
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$|gs(r,h)\rangle = \bigotimes_{k=1}^{M} \left\{ \cos \frac{\theta_k}{2} |0_k\rangle + i \sin \frac{\theta}{2} |1_k 1_{-k}\rangle \right\}$
$q = (r,h)$
$\tilde{q} = (\tilde{r},\tilde{h})$
$\langle gs(q) | gs(\tilde{q}) \rangle =$

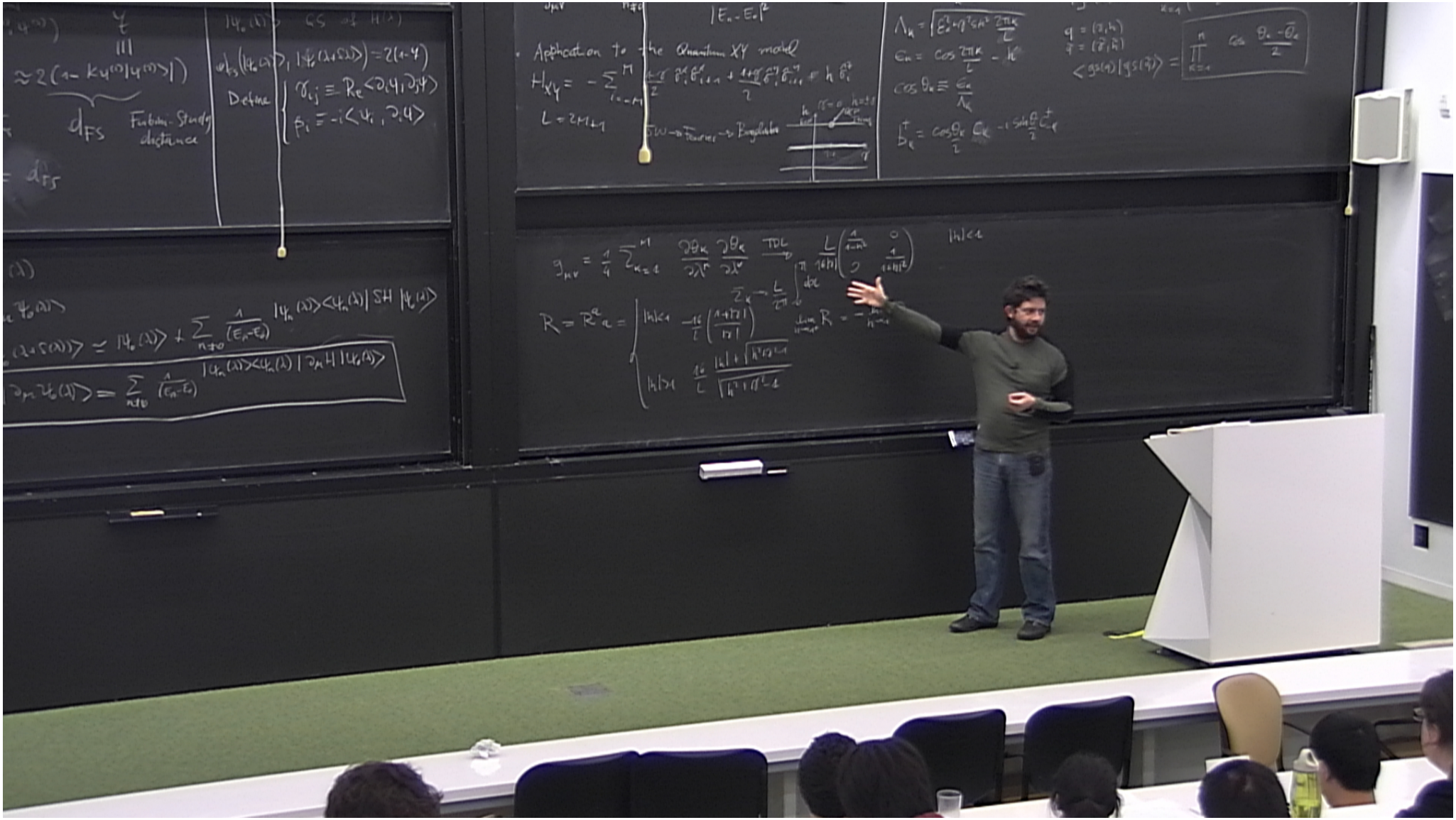

$H_{xy} = \sum_k \Lambda_k \left(b_k^\dagger b_k - \frac{1}{2} \right)$
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$q = (r,h)$
$\tilde{q} = (\tilde{r},\tilde{h})$
$\langle gs(q) | gs(\tilde{q}) \rangle = \prod_{k=1}^{N} \cos \frac{\theta_k - \tilde{\theta}_k}{2}$

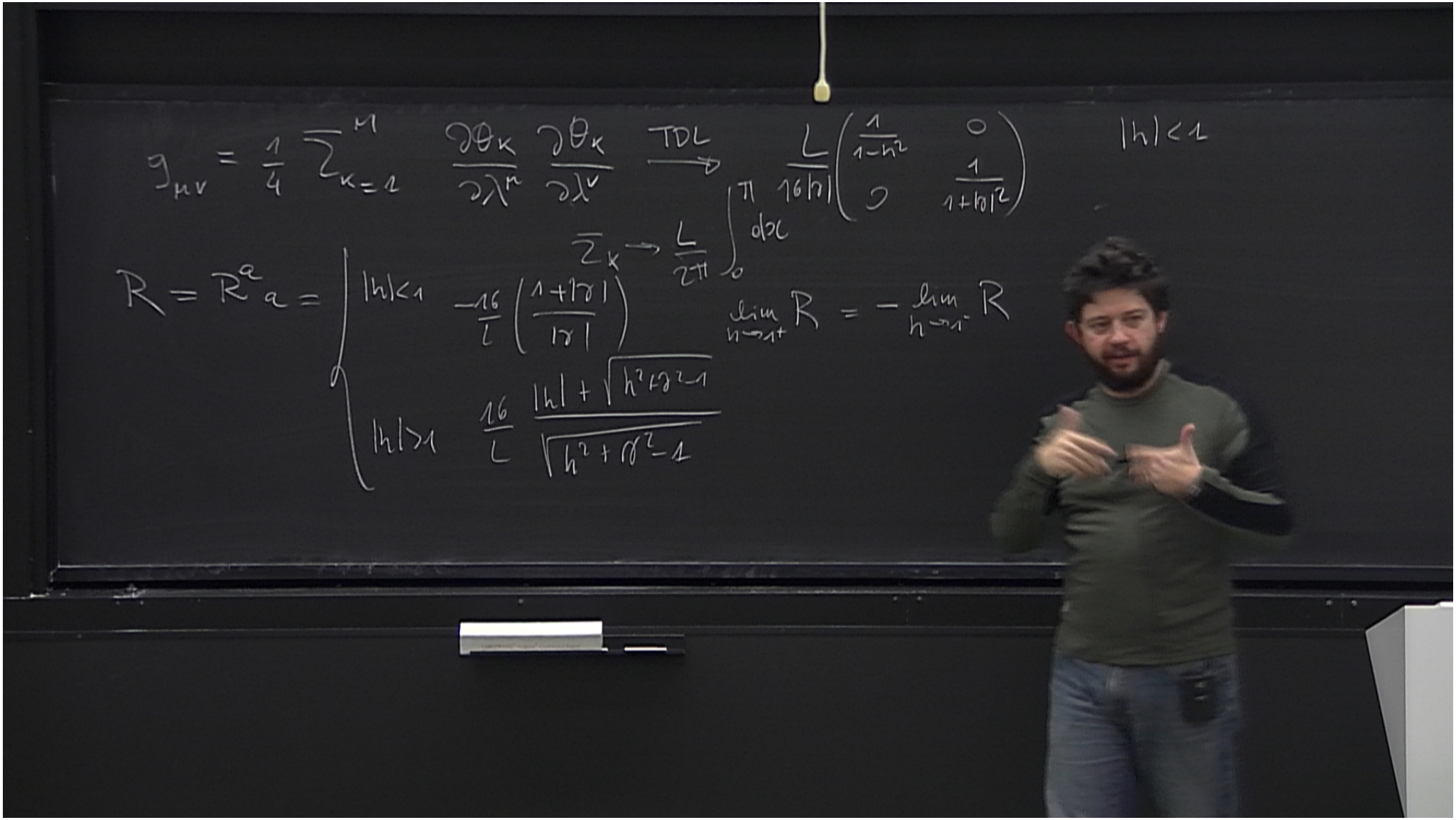

Quantum XY model
Fourier → Bogoliubov
h
γ=0
h=±1
QCP
Ising
γ=1
γ


XY model
Bogoliubov
QCP
Ising
Ferro
h
g
g=1
h=±1


$g_{\mu\nu} = \frac{1}{4} \sum_{k=1}^{M} \frac{\partial\theta_k}{\partial\lambda^\mu} \frac{\partial\theta_k}{\partial\lambda^\nu}$ TDL
$\frac{L}{16|h|} \begin{pmatrix} \frac{1}{1-h^2} & 0 \\ 0 & \frac{1}{1+|h|^2} \end{pmatrix}$ $|h|<1$
$\sum_k \to \frac{L}{2\pi} \int_0^\pi d\alpha$


$g_{\mu\nu} = \frac{1}{4} \sum_{k=1}^{M} \frac{\partial\theta_k}{\partial\lambda^\mu} \frac{\partial\theta_k}{\partial\lambda^\nu} \xrightarrow{TDL} \frac{L}{16|\gamma|} \begin{pmatrix} \frac{1}{1-\eta^2} & 0 \\ 0 & \frac{1}{1+\eta^2} \end{pmatrix}$ $|\eta|<1$
$\sum_k \to \frac{L}{2\pi} \int_0^{\pi} d\alpha$
$R = R^a{}_a = \begin{cases} |\eta|<1 & -\frac{16}{L}\left(\frac{1+|\gamma|}{|\gamma|}\right) \\ |\eta|>1 & \frac{16}{L} \frac{|\eta| + \sqrt{\eta^2+\gamma^2-1}}{\sqrt{\eta^2+\gamma^2-1}} \end{cases}$
$\lim_{\eta\to 1^+} R = -\lim_{\eta\to 1^-} R$


Application to the Quantum XY model
Fubini-Study distance
Define
Fourier
Bogoliubov


$g_{\mu\nu} = \frac{1}{4} \sum_{k=1}^{M} \frac{\partial\theta_k}{\partial\lambda^\mu} \frac{\partial\theta_k}{\partial\lambda^\nu} \xrightarrow{TDL} \frac{L}{16|h|} \begin{pmatrix} \frac{1}{1-h^2} & 0 \\ 0 & \frac{1}{1+h^2} \end{pmatrix}$ $|h|<1$
$\bar{z}_k \to \frac{L}{2\pi} \int_0^{2\pi} dx$
$R = R^a{}_a = \begin{cases} |h|<1 & -\frac{16}{L}\left(\frac{1+|h|}{|h|}\right) \\ |h|>1 & \frac{16}{L} \frac{|h| + \sqrt{h^2+h^2-1}}{\sqrt{h^2+h^2-1}} \end{cases}$
$\lim_{h\to 1^+} R = -\lim_{h\to 1^-} R$