

Title: 12/13 PSI - Gravitational Physics Review Lecture 15

Date: Feb 15, 2013 09:00 AM

URL: <http://pirsa.org/13020073>

Abstract:

Gravity & String Theory

- Have seen in tutorials how to generalize SCH.

$$ds^2 = V_D dt^2 - \frac{dr^2}{V_D} - r^2 d\Omega_{D-2}^2$$

$$V_D = 1 - \left(\frac{r_+}{r}\right)^{D-3}$$

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area of S^{D-2}
↓

$$V_D = 1 - \left(\frac{r_+}{r}\right)^{D-3}$$
$$16\pi G_D M = (D-2) A_{D-2} r_+^{D-3}$$

Temperature: $T = \frac{V_D'}{4\pi} = \frac{D-3}{4\pi r_+} \propto M^{-1/3}$

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We also seen (KK) extended solns.

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$$ds^2 = V_d dt^2 - \frac{dr^2}{V_d} - r^2 d\Omega_{d-2} - dX_{D-d}^2$$

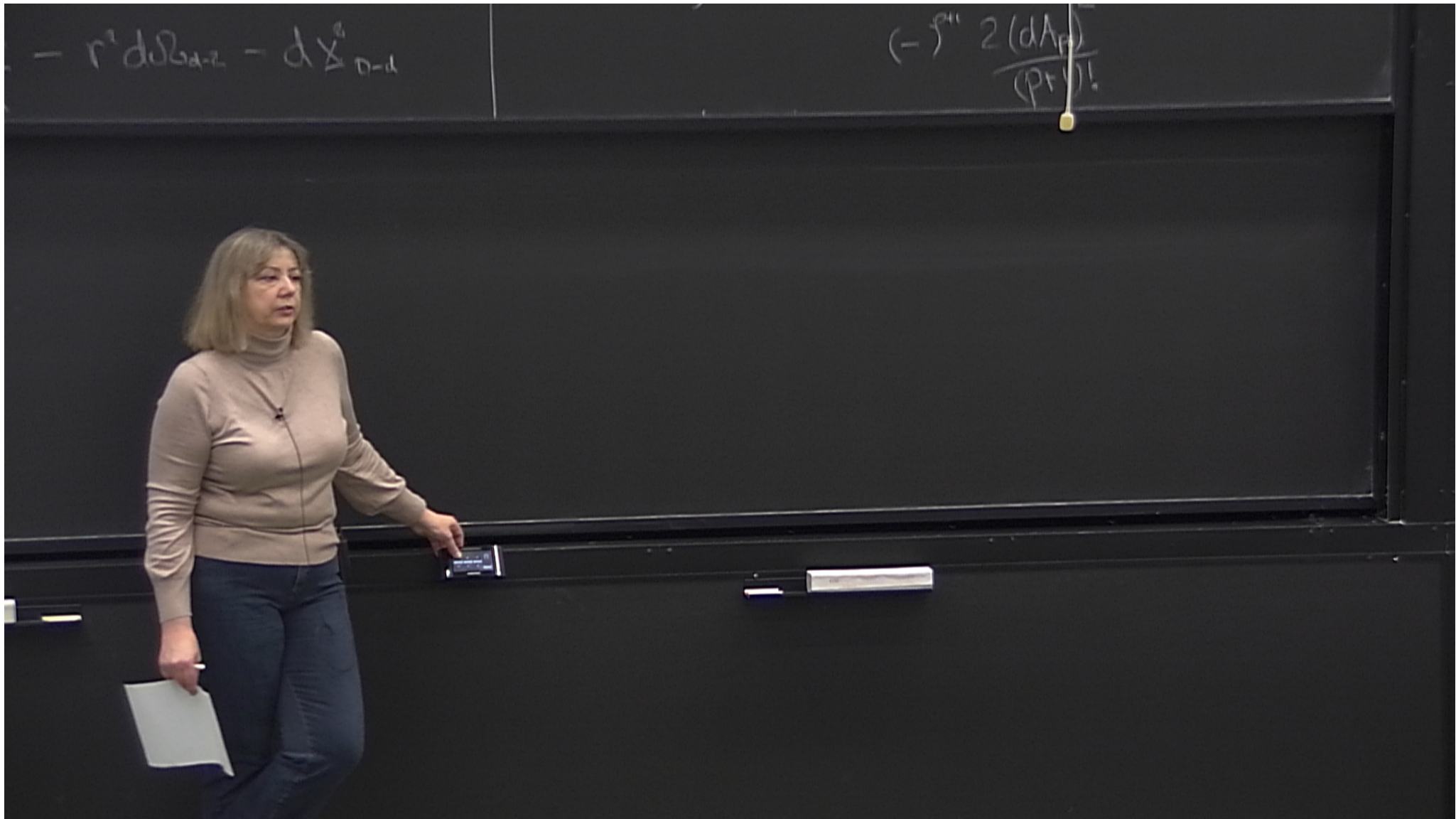
$$= \frac{V_D'}{4\pi} = \frac{D-3}{4\pi r_+} \propto M^{-1/3}$$

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extended solns.

$$- r^2 d\Omega_{D-2} - dX_{D-d}^2$$

- a (D-d)-brane extended black brane.



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In low energy effective string gravity have scalar-tensor theory + charges.

$$S = \int d^{10}x \sqrt{g_{10}} \left\{ e^{-2\phi} \left[-R - \frac{1}{2} (\nabla\phi)^2 + \frac{1}{12} H_{abc}^2 \right] \right. \\ \left. + (-)^{p+1} \frac{2(dA_p)^2}{(p+1)!} \right\}$$

$$H_{abc} = (dB_2)_{abc} = \partial_a B_{bc} + \text{cyclic}$$

$B_2 \leftrightarrow$ NSNS 2-form under which
string is 'charged'

$A_p = p$ -form gauge field

$$A_p \rightarrow A_p + d\Lambda_{p-1}$$

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- Classical solns to the field

B_{bc} + cyclic
under which
charged.

- Classical solns to the field eqns are extended in
general $ds^2 = A^2 dt^2 - B^2 dx_n^2 - \underbrace{dp^2 + c^2 dl^2}_{D-n-2}$

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$$ds^2 = \underbrace{A^2 dt^2 - B^2 dx_n^2}_{\text{Extended in the } \Sigma_n \text{ directions}} - \underbrace{dp^2 - c^2 dl^2}_{\text{transverse to brane}}_{D-n-2}$$

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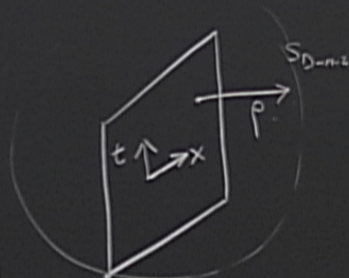
general

$$ds^2 = \underbrace{A^2 dt^2 - B^2 dx_n^2}_{\text{Extended in the } X_n \text{ directions}} - \underbrace{dp^2 - c^2 dl^2}_{\text{transverse to brane}} \quad D=n+2$$

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$$ds^2 = \underbrace{A_p^2 dt^2 - B_p^2 dx_n^2}_{\text{Extended in the } X_n \text{ directions}} - \underbrace{dp^2 + c^2 dl^2}_{\text{transverse to brane } D=n-2}$$

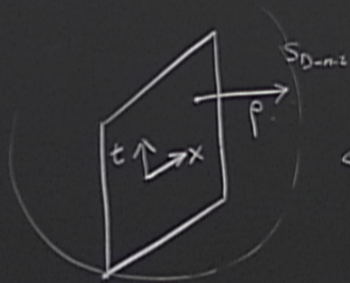


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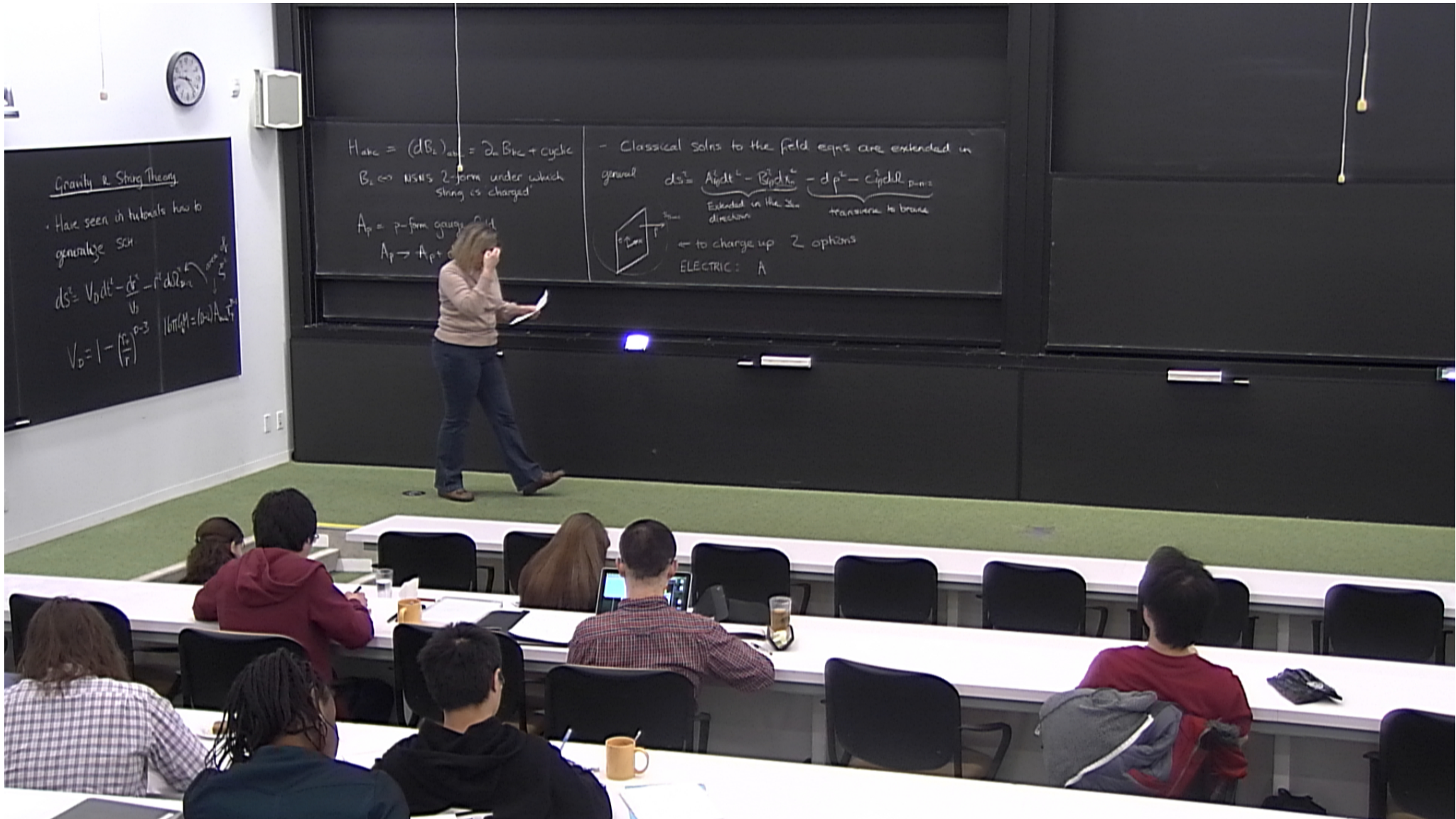
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← to charge up: 2 options

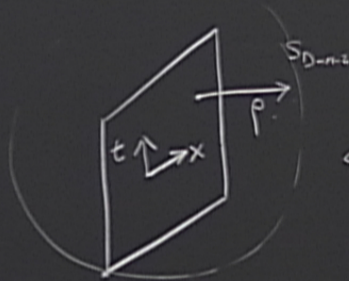




B_{bc} + cyclic
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- Classical solns to the field eqns are extended in general

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← to charge up: 2 options

ELECTRIC: $A_{n+1} \leftrightarrow A_{t, x_n}(p)$ (cf $A_0 = \frac{Q}{r}$ $n=0$)

$$A_p \rightarrow A_p + dA_p$$

$$\text{ELECTRIC: } A_{n+1} \leftrightarrow$$

$$\text{MAGNETIC: } F_{D-2-n} = Q \underbrace{d\ell_{D-2-n}}_{\substack{\text{Volume form} \\ \text{on } S^{D-n-2}}}$$

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e.g. 4D electromagnetism

electric charge $F \sim \frac{Q}{r^2} dt \wedge dr$

magnetic charge $F \sim Q \sin\theta d\theta \wedge d\phi$

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Both have the RN metric as a gravitational soln.

$$V_{RN} = 1 - \frac{2GM}{r} + \frac{Q^2}{r^2} = \frac{(r-r_+)(r-r_-)}{r^2}$$



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The eqns of motion are invariant under this duality.

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For string action, an 'electric' soln with $(p+1)$ -form field strength dual to a 'magnetic' soln with $D-(p+1)$ field strength.

$$A_p \rightarrow A_p + dA_{p-1}$$

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e.g. 4D electromagnetism: $D=4$

$$\text{electric charge } F \sim \frac{Q}{r^2} dt \wedge dr \quad \leftarrow n=0$$

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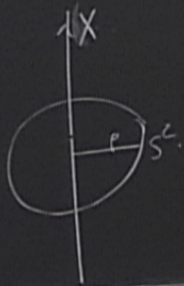
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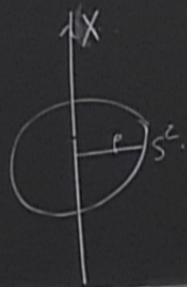
In 4D, have this duality because F and $*F$ are 2-forms.

In 5D, electric soln still a 0-brane,
black hole. The magnetic soln now has
 $n=1$ - a string.



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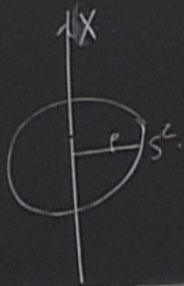
$n=1$ - a string.



In 6D - 2 dimensional
7D - 3 "

In 5D, electric soln still a 0-brane,
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$n=1$ - a string.



In 6D - 2 dimensional (spatial)
7D - 3 "

brane,
has

patial)

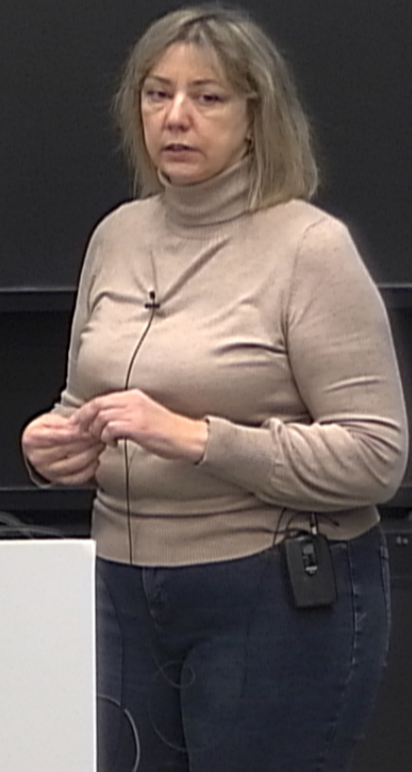
For a soln to be self dual the field strength must
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$$ds^2 = \frac{\left(1 - \frac{r_+^4}{r^4}\right) dt^2}{\left(1 - \frac{r_-^4}{r^4}\right)} - \frac{dr^2}{\left(1 - \frac{r_+^4}{r^4}\right)\left(1 - \frac{r_-^4}{r^4}\right)} - \left(1 - \frac{r_-^4}{r^4}\right) dx_{\text{III}}^2 - r^2 d\Omega_{\text{II}}^2$$



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D3 brane directions

S^5 surrounding
brane in transverse
space (ϵ_5)

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Change co-ords: $p^4 = r^4 - r_+^4$

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$$f = \frac{r^4}{\rho^4} = 1 + \frac{r_+^4}{\rho^4}$$

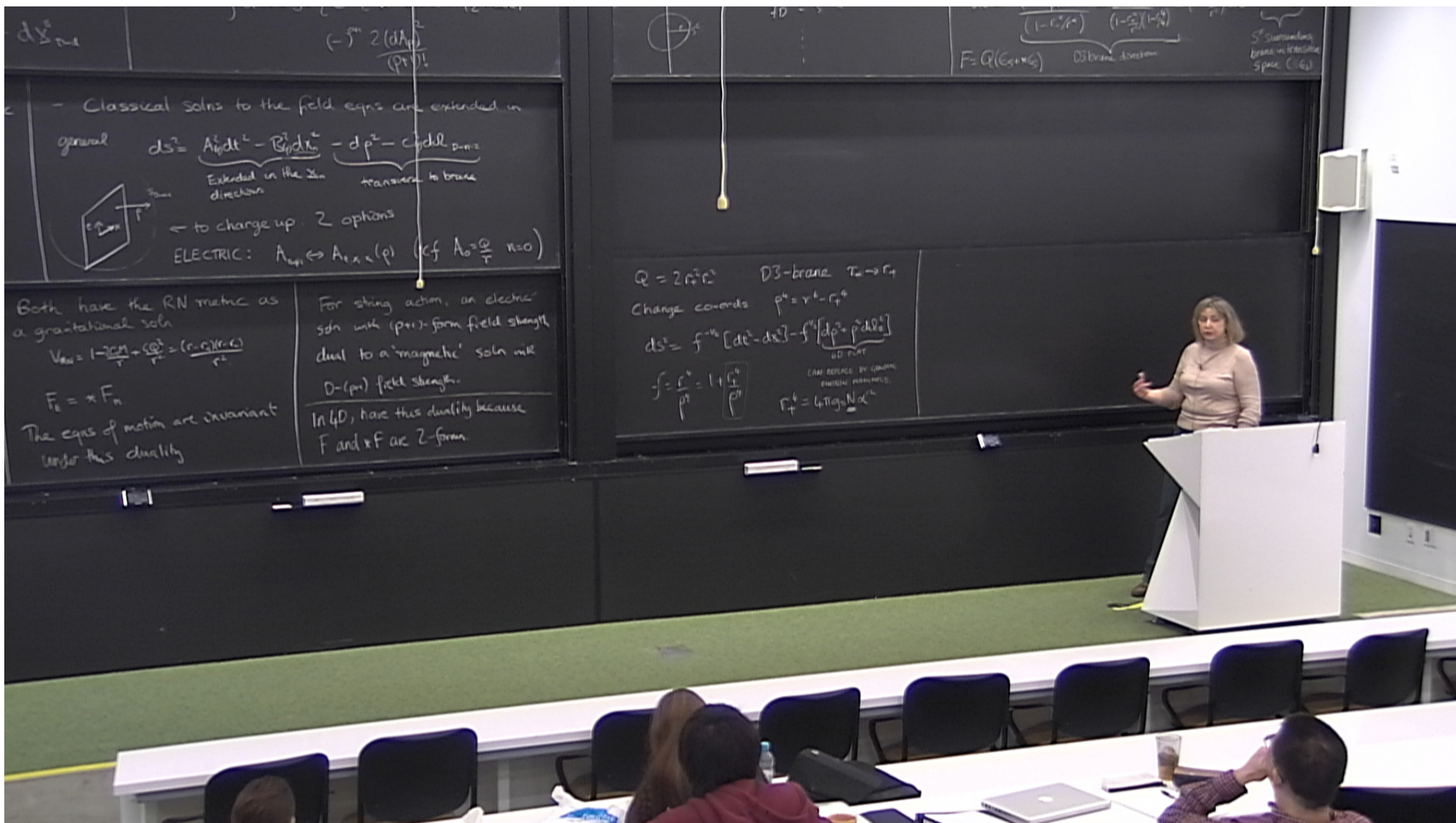
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CAN REPLACE BY GENERAL
EINSTEIN MANIFOLD.



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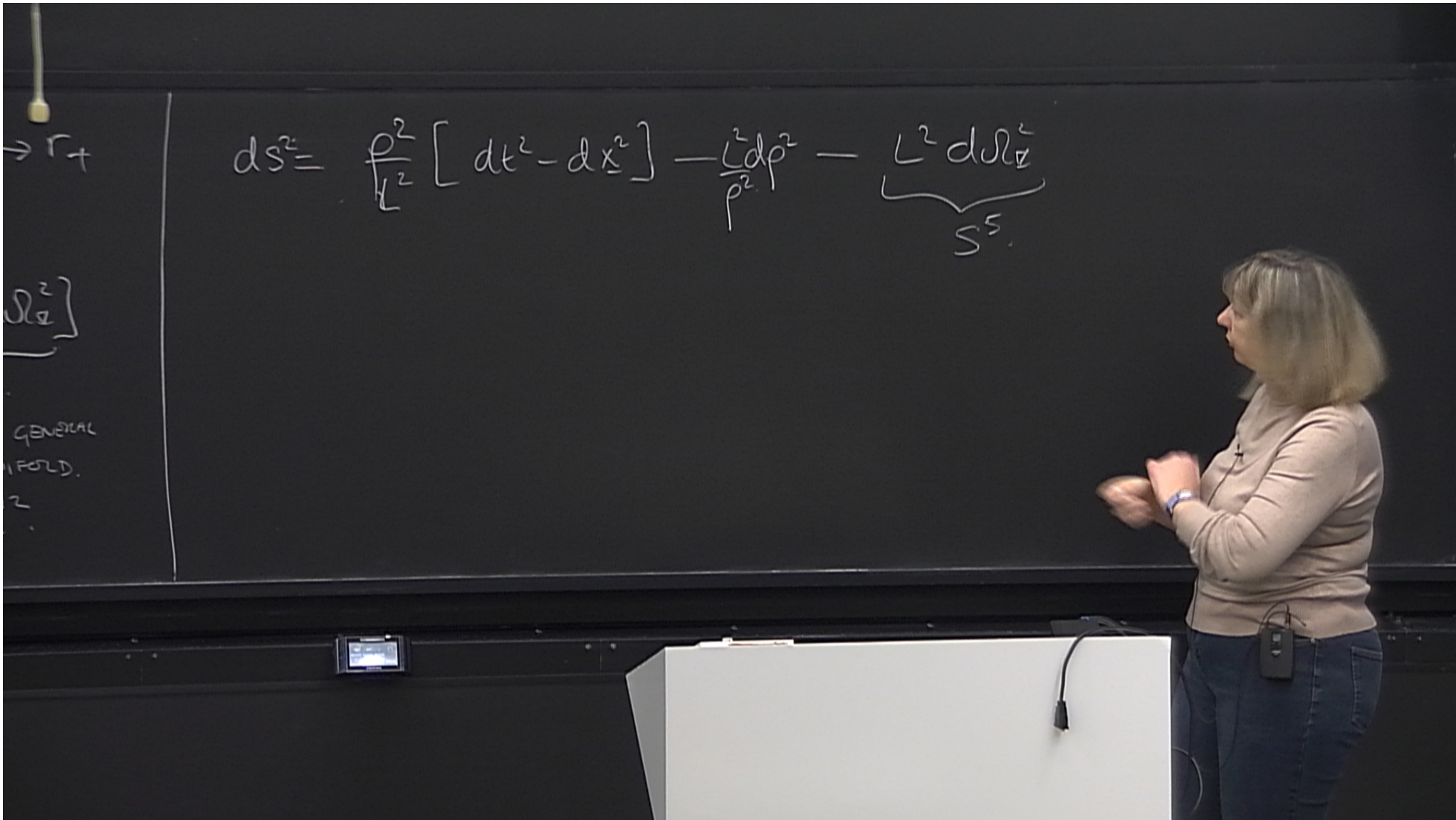
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CAN REPLACE BY GENERAL
EINSTEIN MANIFOLD.

$$r_+^4 = 4\pi g_s N \alpha'^2$$



adS_5

Anti de Sitter space is a soln of Einstein gravity
with a -ve cosmological constant.

Black hole - planar

$$ds^2 = \left(\frac{p^2}{L^2} - \frac{M}{p^2} \right) dt^2 - \frac{p^2}{L^2} dx^2 - \frac{dp^2}{\frac{p^2}{L^2} - \frac{M}{p^2}}$$

Black hole - planar

$$ds^2 = \left(\frac{\rho^2}{L^2} - \frac{M}{\rho^2} \right) dt^2 - \frac{\rho^2}{L^2} dx^2 - \frac{d\rho^2}{\frac{\rho^2}{L^2} - \frac{M}{\rho^2}}$$

horizon at $\rho_+ = (\mu L^2)^{1/4}$

Black hole - planar

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$$T = \frac{1}{4\pi} \left(\frac{2\rho_+}{L^2} + \frac{2\mu}{\rho_+^3} \right)$$

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Black hole - planar

positive specific heat

$$ds^2 = \left(\frac{\rho^2}{L^2} - \frac{M}{\rho^2} \right) dt^2 - \frac{\rho^2}{L^2} dx^2 - \frac{d\rho^2}{\frac{\rho^2}{L^2} - \frac{M}{\rho^2}}$$

horizon at $\rho_+ = (\mu L^2)^{1/4}$

$$T = \frac{1}{4\pi} \left(\frac{2\rho_+}{L^2} + \frac{2M}{\rho_+^3} \right) = \frac{1}{\pi} \frac{\rho_+}{L^2} = \frac{1}{\pi} \frac{M^{1/4}}{L^{3/2}}$$

• Scalars in adS

$$\square \phi + m^2 \phi = 0$$

Change ρ to $x = L^2/\rho$

$$\& \text{ write } \phi = x^{(D-1)/2} \chi$$

$\uparrow$
 $e^{i\omega t - i k x}$

$$\chi_{xx} + \frac{\chi_x}{x} + (\omega^2 - k^2) \chi - \left(m^2 L^2 + \frac{(D-1)^2}{4}\right) \frac{\chi}{x^2} = 0.$$

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BESSEL
EQN

• Scalars in ads

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BESSEL
EQN

Well defined provided

$$m^2 L^2 + \frac{(D-1)^2}{4} \geq 0$$

$$m^2 \geq - \frac{(D-1)^2}{4}$$

$\uparrow$

ads

-0

$$= L^2/p$$

$$X^{(D-1)/2} \chi$$

↑
 $e^{i\omega t - i k x}$

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↑

Breitenlohner
- Freedman
bound