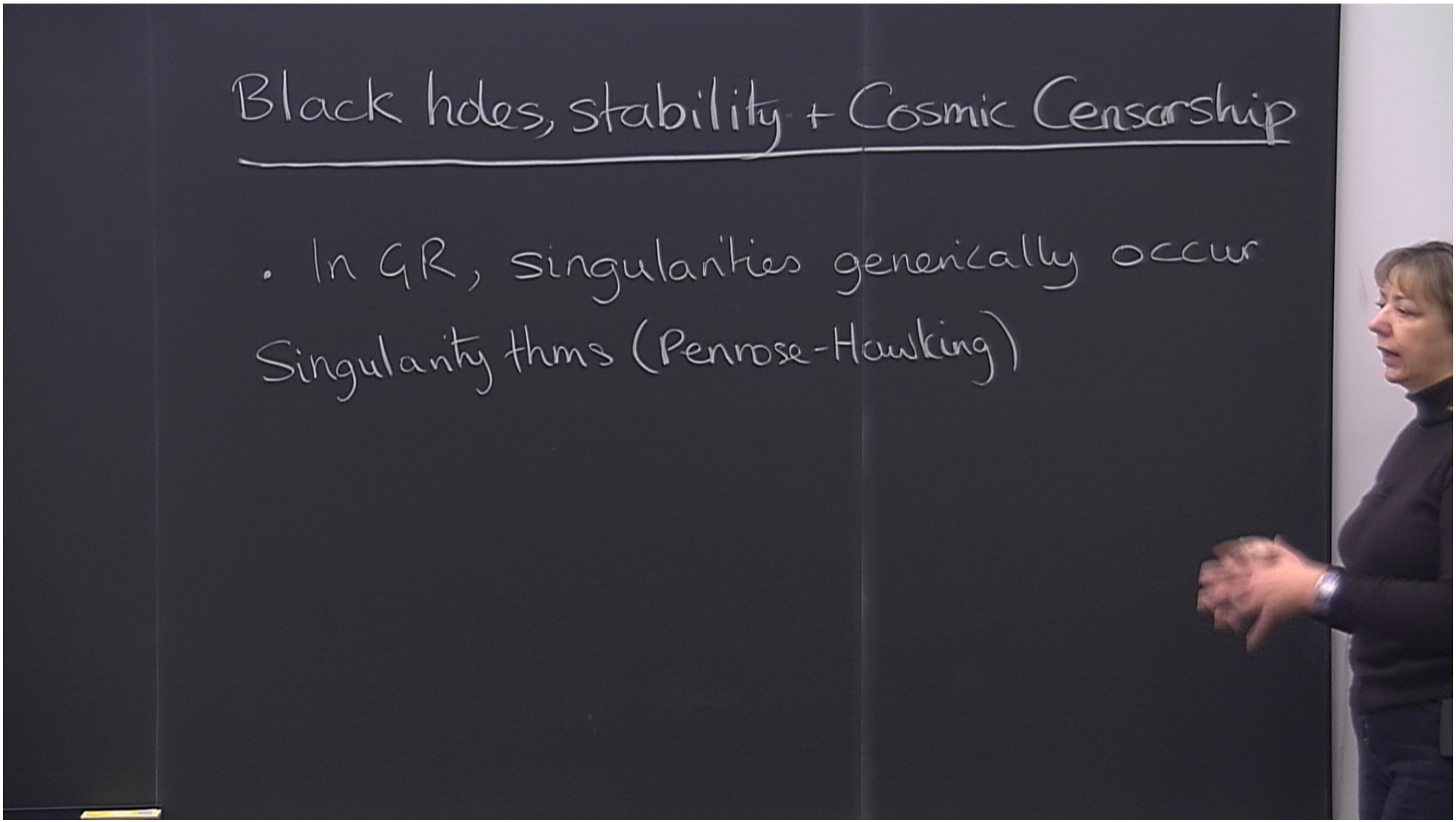
Title: 12/13 PSI - Gravitational Physics Review Lecture 14

Date: Feb 14, 2013 09:00 AM

URL: http://pirsa.org/13020072

Abstract:

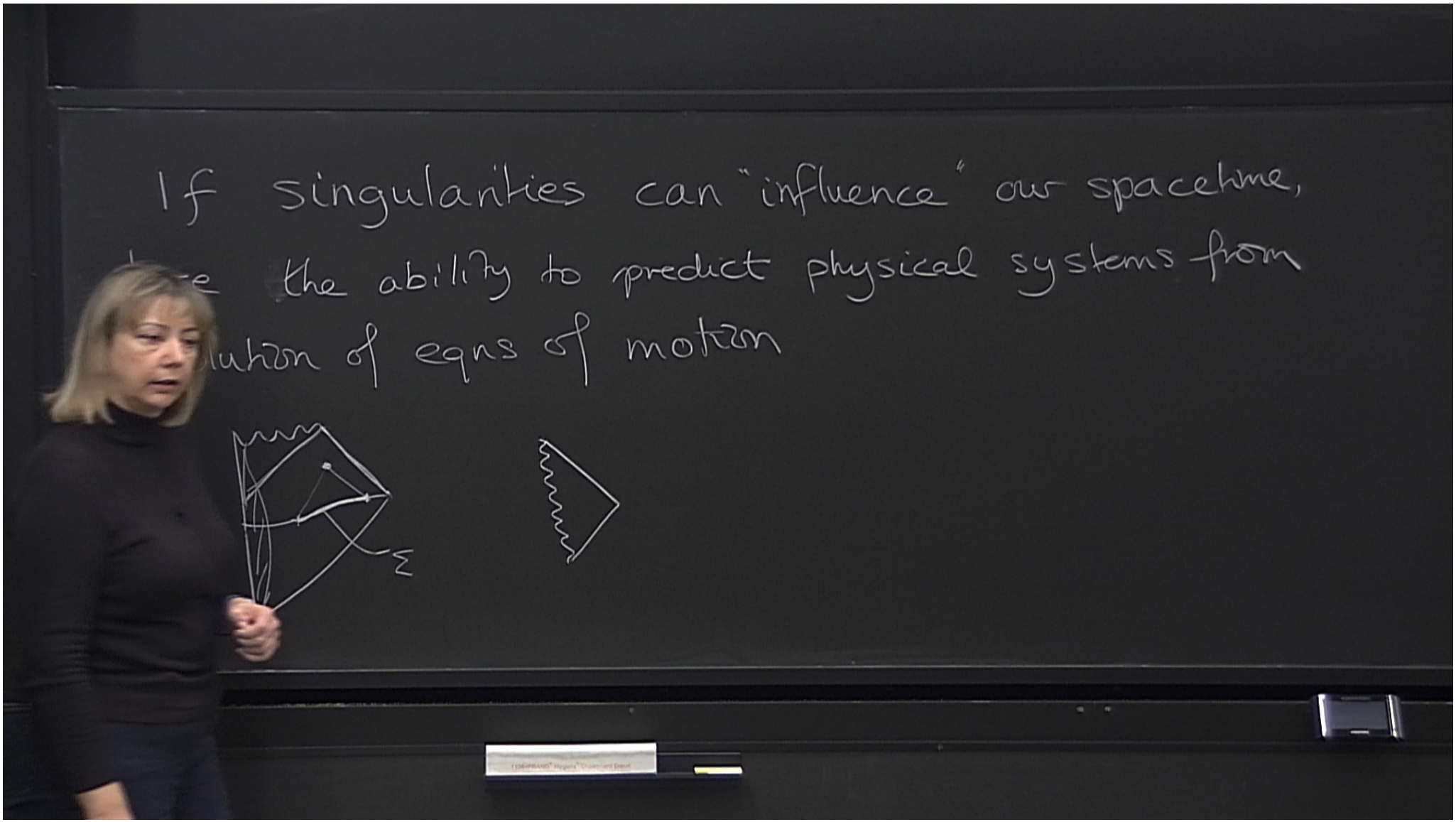
Black holes, stability + Cosmic Censorship


Black holes, stability + Cosmic Censorship
• In GR, singularities generically occur
Singularity thms (Penrose-Hawking)

Black holes, stability + Cosmic Censorship

- In GR, singularities generically occur

Singularity thms (Penrose-Hawking) - if have a trapped surface (null geodesics always converge) + WEC, then $\exists$ singularity in spacetime.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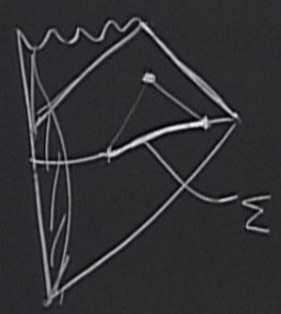

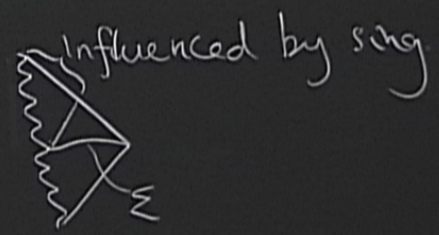

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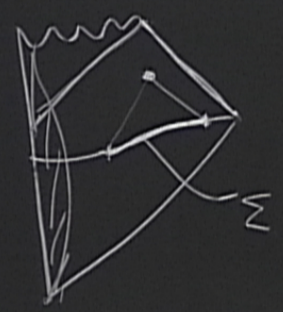

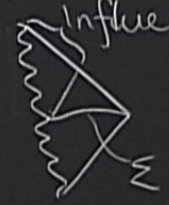

influenced by sing

naked singularity.

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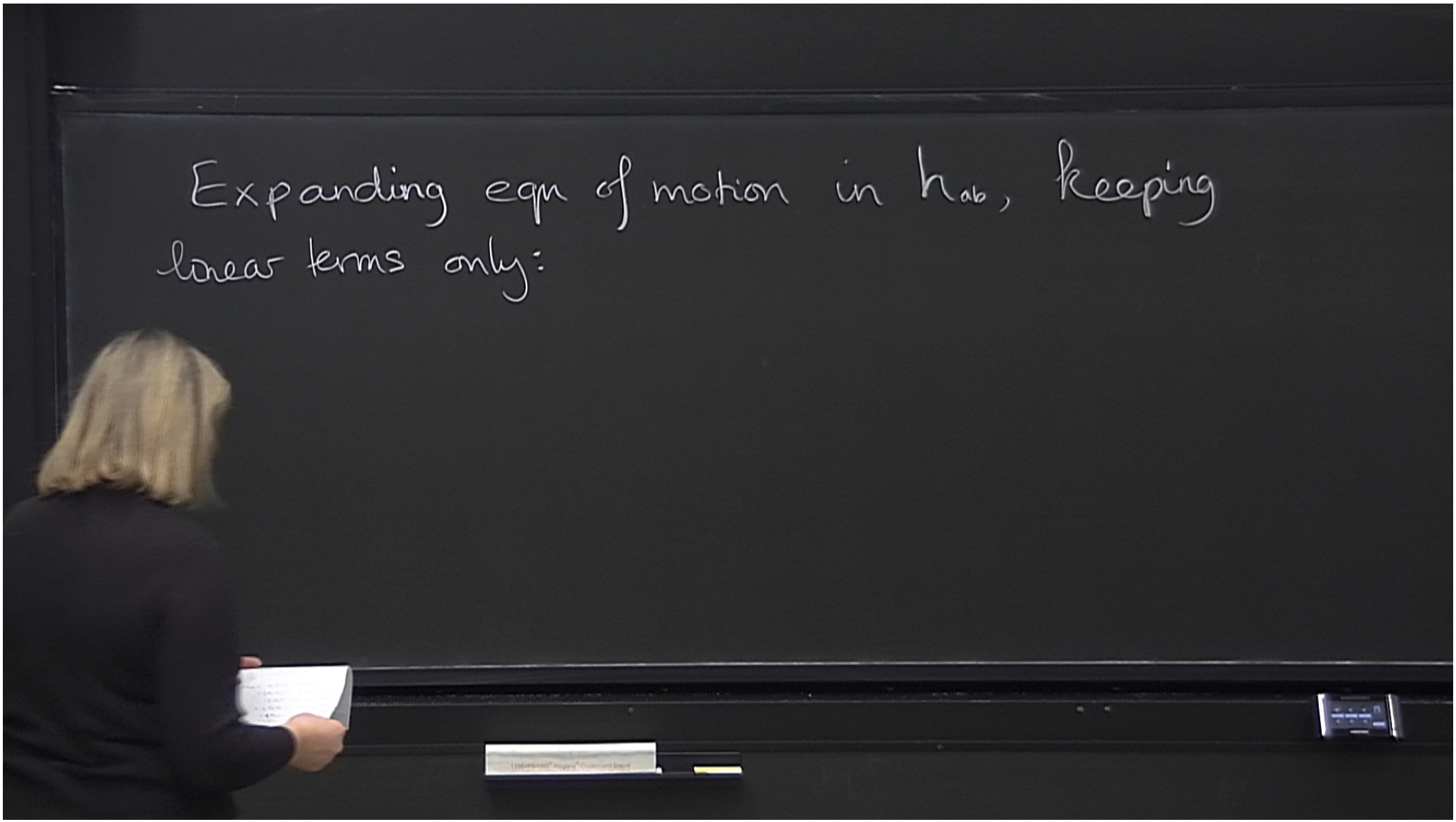
The SCH singularity is cloaked - is it stable?

- Answer via Perturbation Theory.

$$g_{ab} = \underbrace{g_{0ab}}_{\text{KNOWN}} + \underbrace{h_{ab}}_{\text{UNKNOWN "SMALL" PERTURBATION}}$$


Expanding eqn of motion in h_{ab}, keeping
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$$\Gamma^a_{bc} = \frac{1}{2}\left(g_0^{\ ad} - h^{ad}\right)\left(g_{0db,c} + h_{db,c} + \cdots\right)$$

$$= \frac{1}{2}\left(\nabla_b h^a{}_c + \nabla_c h^a{}_b - \nabla^a h_{bc}\right)$$

Expanding eqn of motion in h_{ab}, keeping linear terms only:

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Hence $\delta R_{ab} = \nabla_c \, \delta \Gamma^c_{ab} - \nabla_b \, \delta \Gamma^c_{ac}$

$$= \frac{1}{2} \Big\{$$

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$$= \frac{1}{2}\left\{ \nabla_a \nabla_c h^c{}_b + R^c{}_{dca} h^d{}_b + R_{bdca} h^{dc} - \Box h_{ab} \right.$$
$$\left. + \nabla_b \nabla_c h^c{}_a + R^c{}_{dcb} h^d{}_a + R_{adcb} h^{dc} - \nabla_a \nabla_b h \right\}$$

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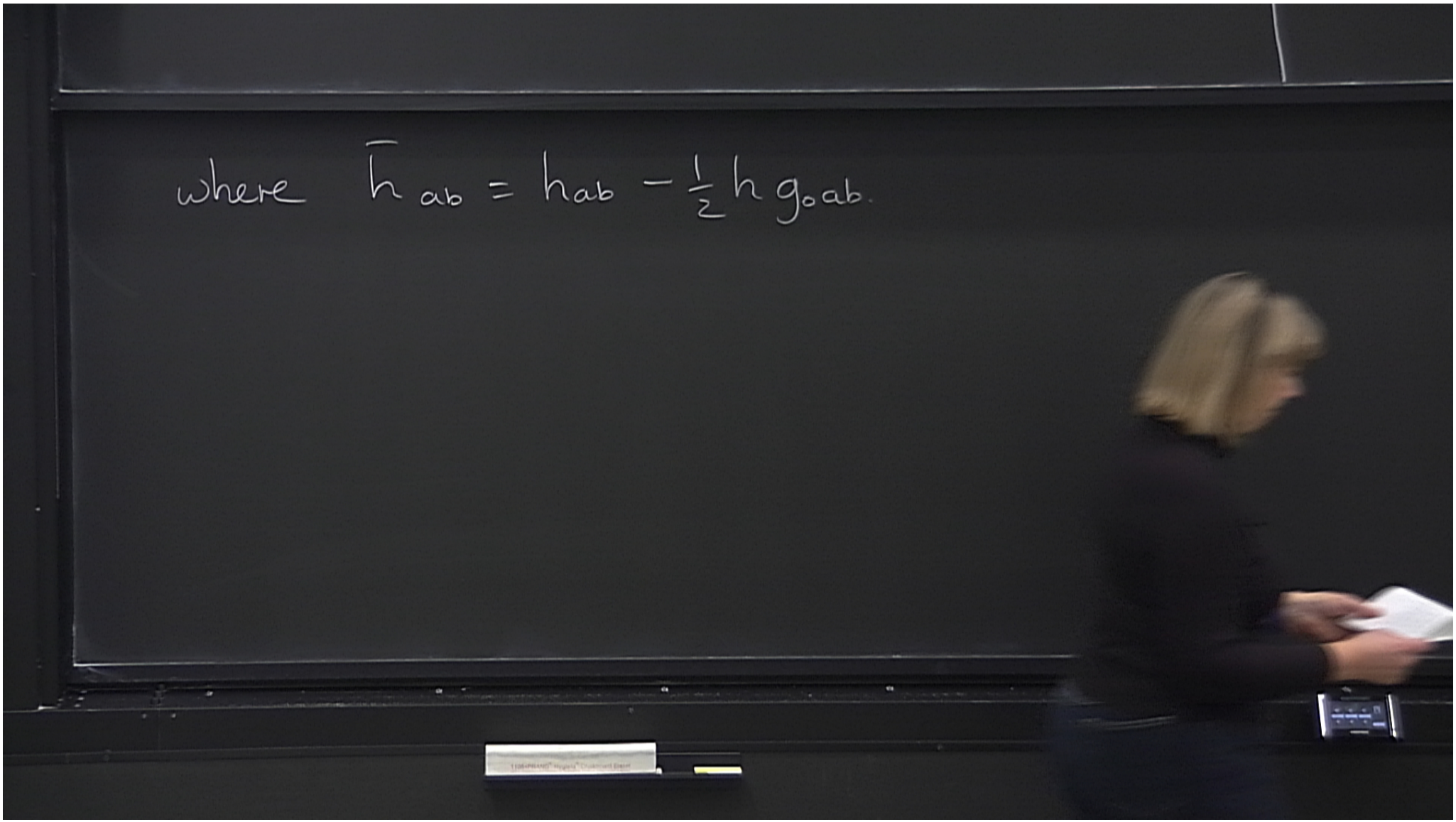
where $\bar{h}_{ab} = h_{ab} - \frac{1}{2} h g_{0ab}$.

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$$\delta R_{ab} = -\frac{1}{2} \Delta_L h_{ab}$$

Lichnerowicz operator.

$= -\frac{1}{2}\{\Box h_{ab} + \Sigma R_{adbc}$

$\frac{1}{2} h g_{0ab}$

Gauge choice is important in GR

$$= -\frac{1}{2}\left\{ \Box h_{ab} + 2R_{adbc}h^{dc} - 2R_{(a}{}^{d}h_{b)d} - 2\nabla_{(a}\nabla^{d}h_{b)d} \right\}$$

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$$X^a \to X^a + \xi^a$$

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- a g.t. generates a perturbation.

$$\bar{h}_{\xi\, ab} = 2\nabla_{(a}\xi_{b)} - \nabla\cdot\xi\, g_{ab}$$

$$\nabla^a \bar{h}_{\xi ab} = \Box\xi_b + \nabla^a\nabla_b\xi_a - \nabla_b(\nabla\cdot\xi)$$

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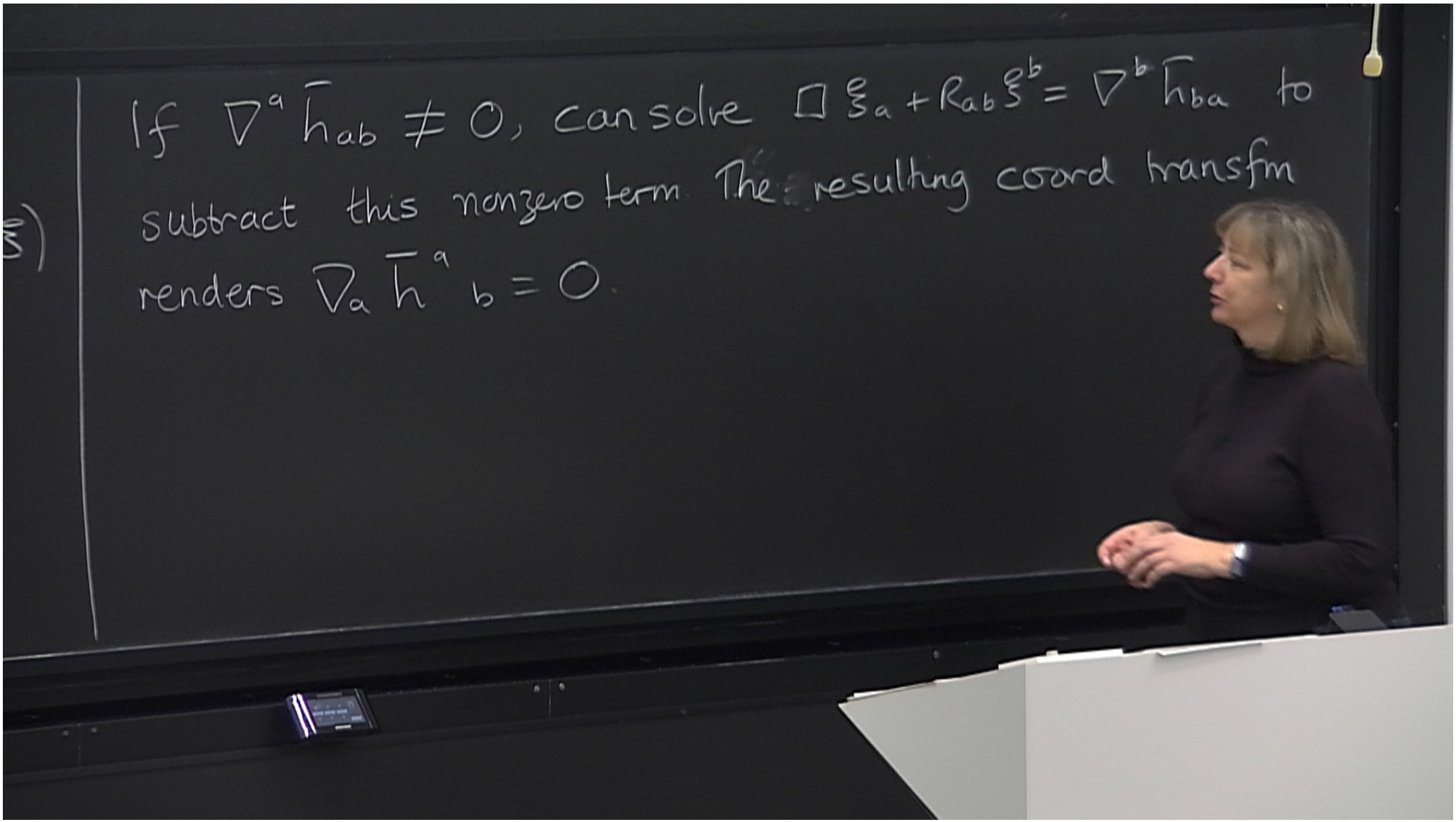
well posed DE.


If $\nabla^a \bar{h}_{ab} \neq 0$, can solve $\Box \xi_a + R_{ab}\xi^b = \nabla^b \bar{h}_{ba}$ to
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renders $\nabla_a \bar{h}^a_{i+\xi\, b} = 0$.

- Remaining gauge freedom? $\quad X^a \to X^a + \chi^a$

$$\Box \chi^a + R^a{}_b \chi^b = 0 \quad \text{required}$$

1 D solutions.

Count degrees of freedom:

h_{ab} - symmetric - $\frac{D(D+1)}{2}$ cpts.

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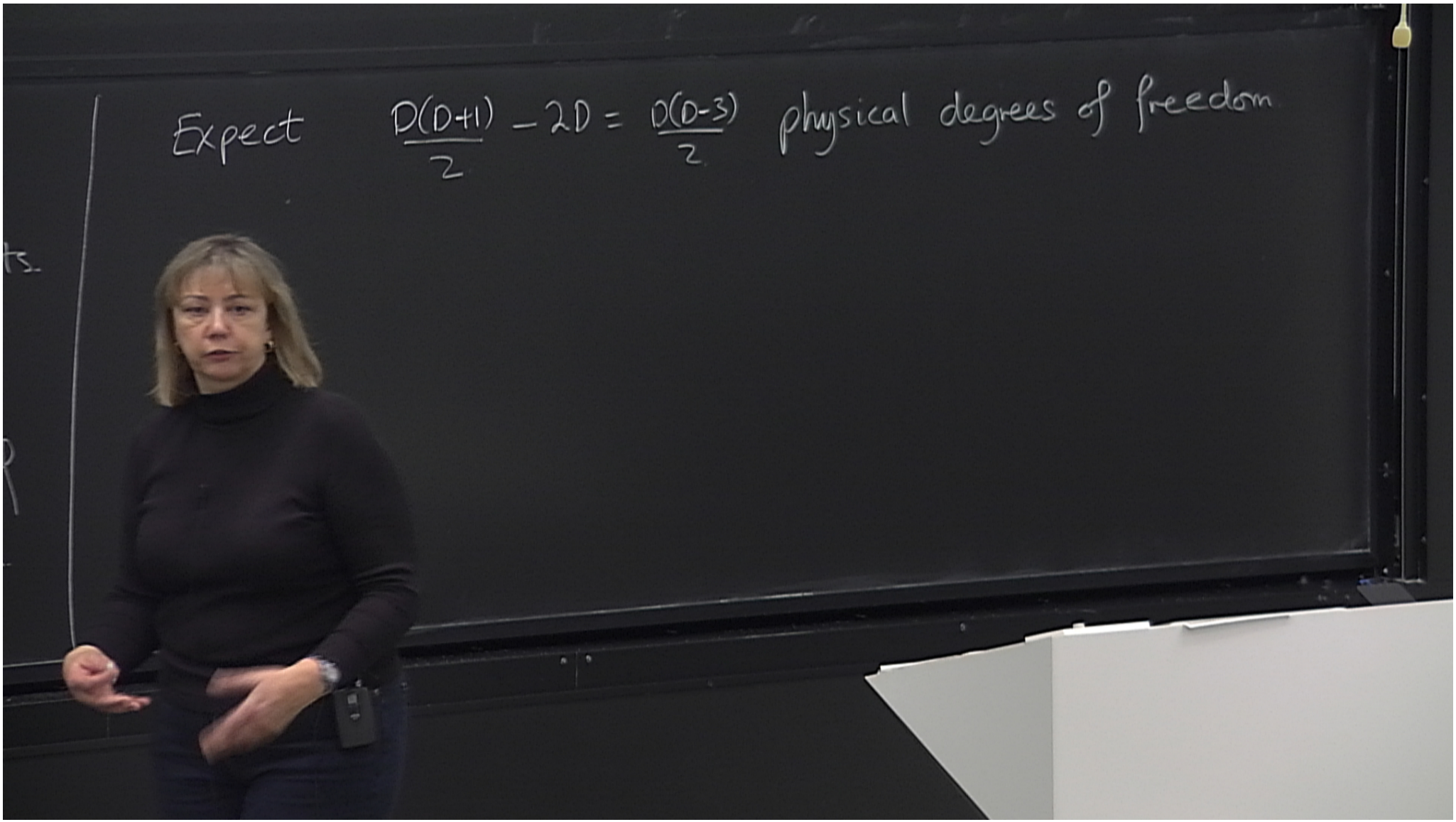
$\nabla_a \bar{h}^{ab} = 0$ - constraint $-D$

X^a - D remaining gauge degrees of freedom.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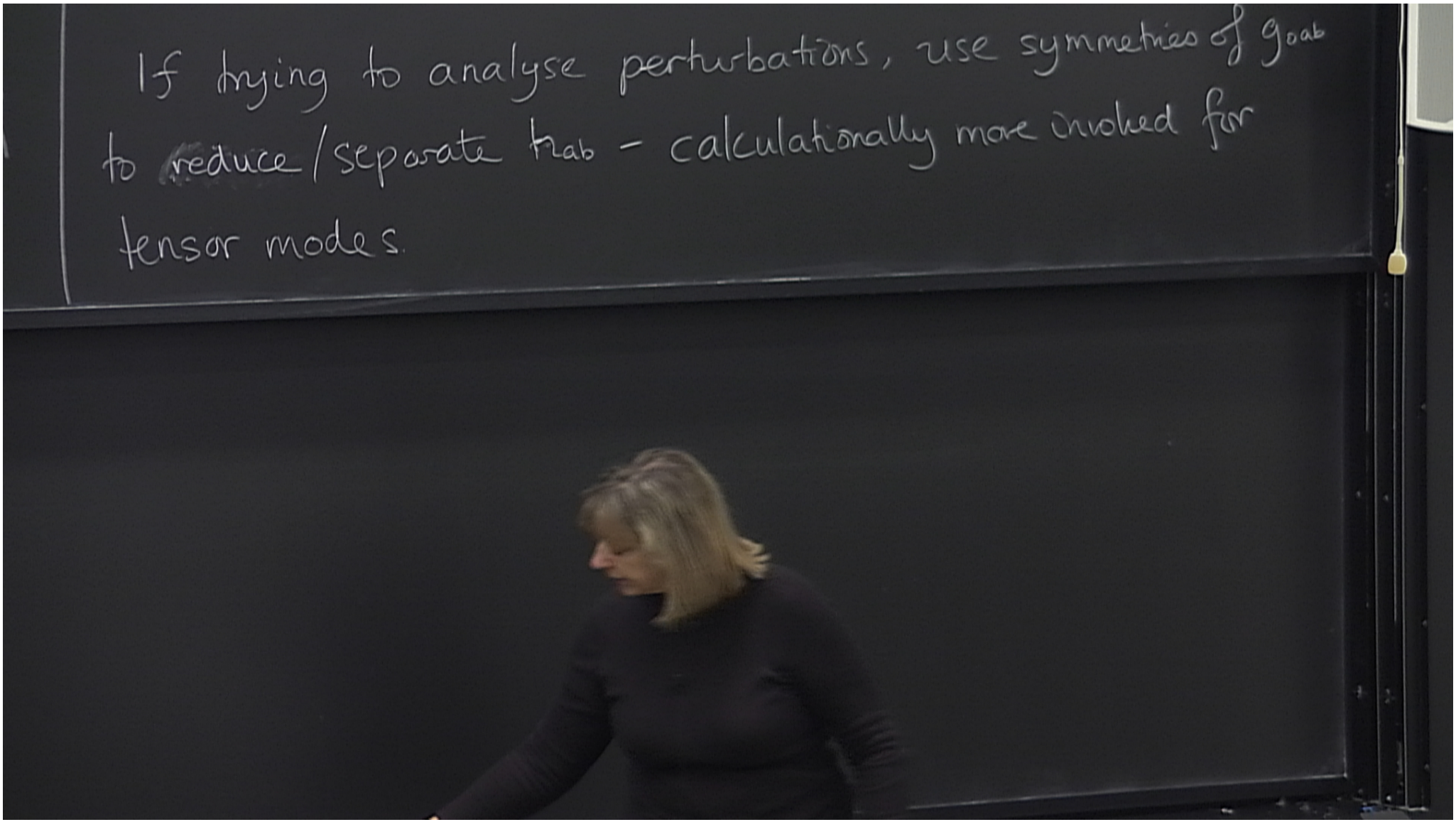
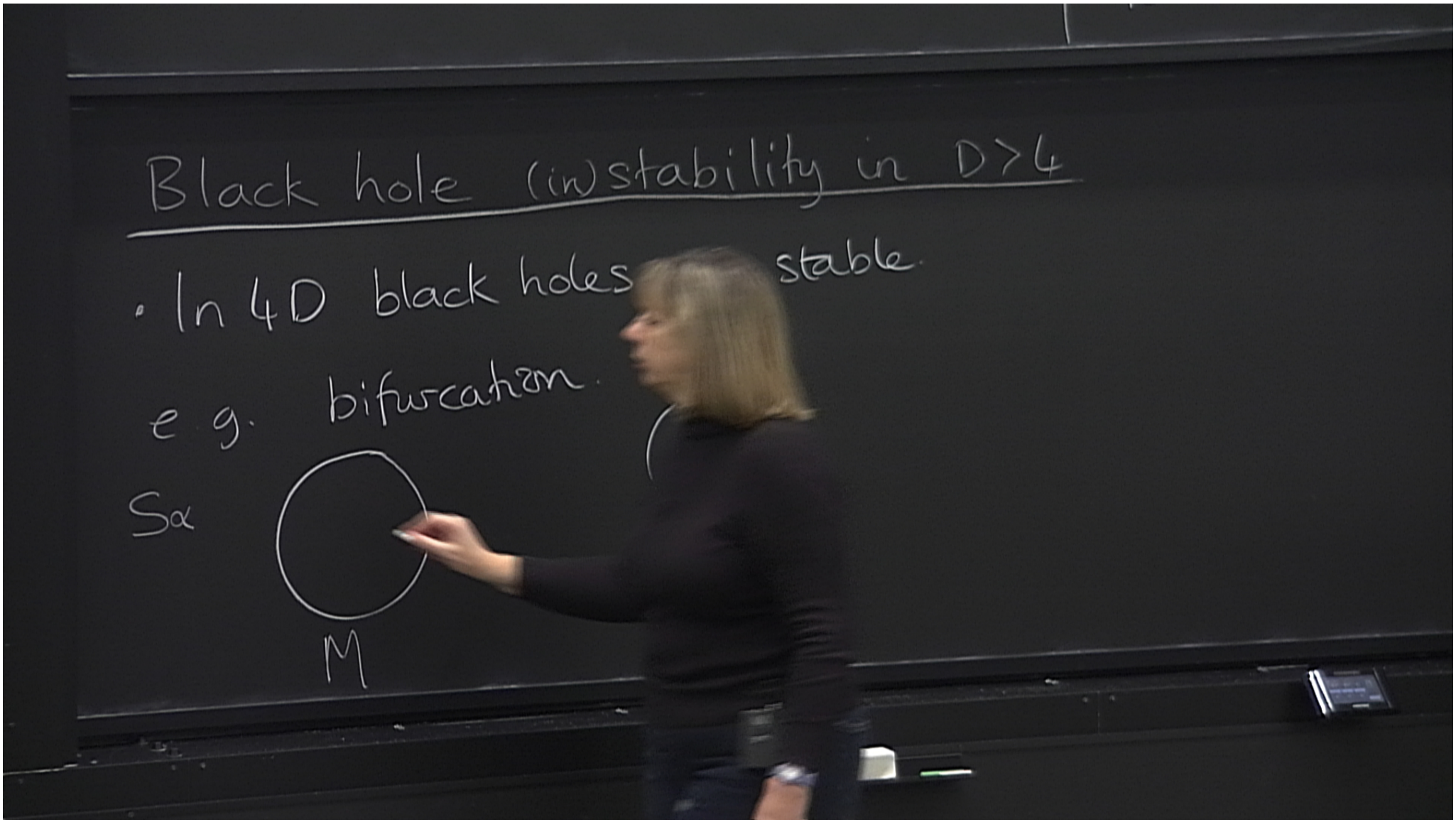
If trying to analyse perturbations, use symmetries of g_{0ab} to reduce / separate h_{ab}


If trying to analyse perturbations, use symmetries of g_{0ab}
to reduce/separate h_{ab} - calculationally more involved for
tensor modes.

Black hole (in)stability in $D>4$

- In 4D black holes are stable.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Sα
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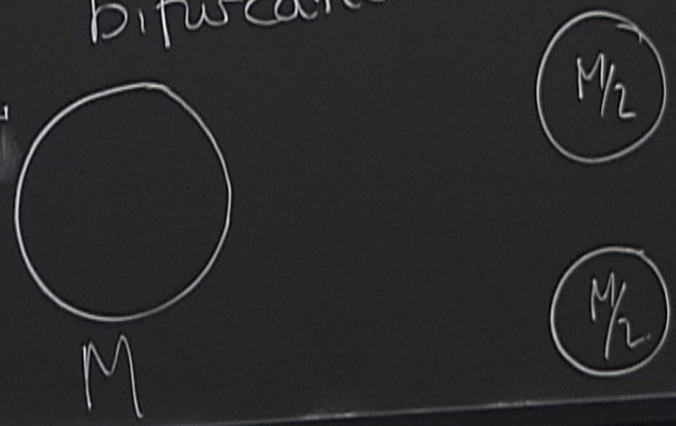
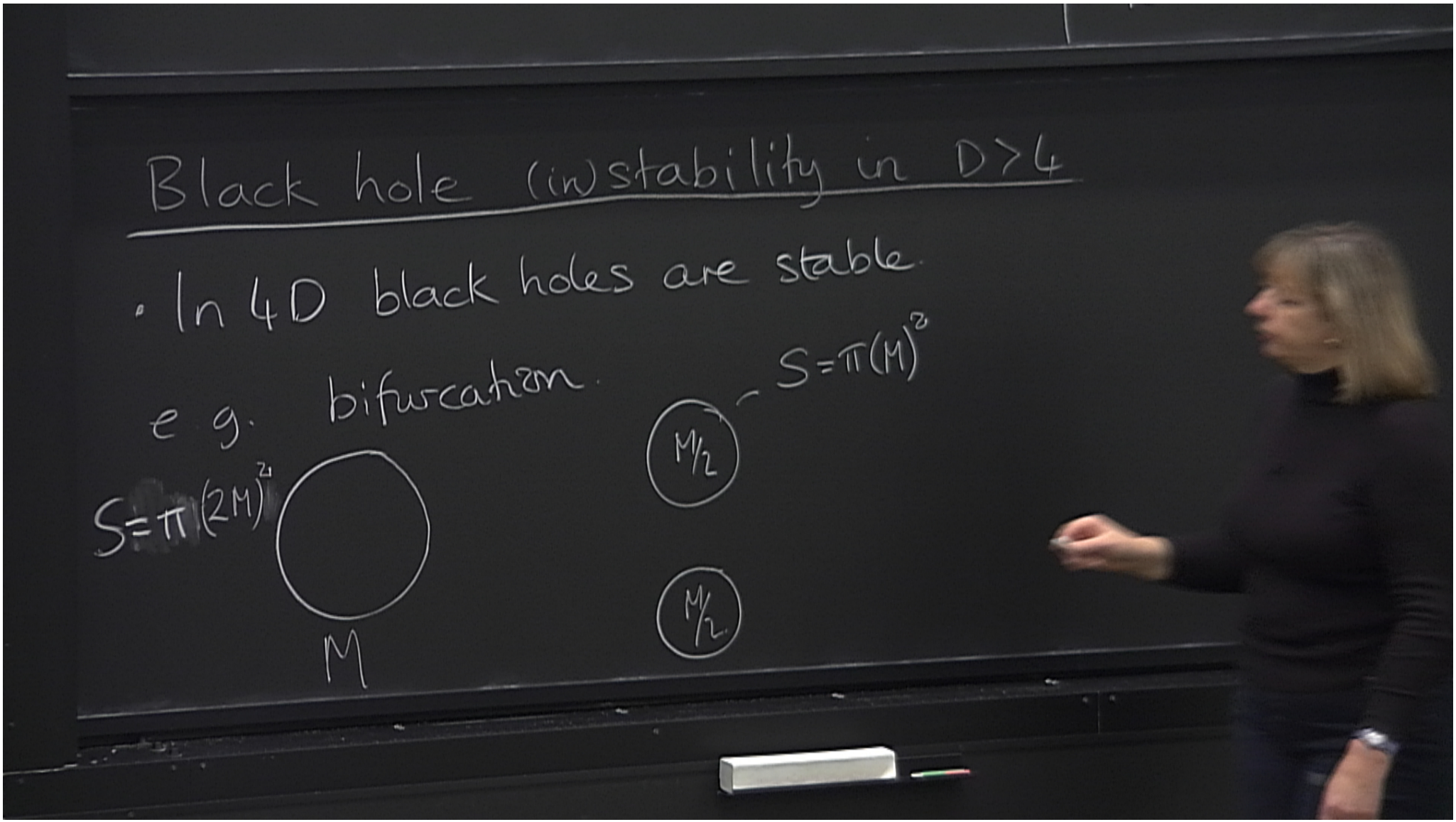
$S=\pi(2M)^2$

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S=π(2M)²
M
M/2
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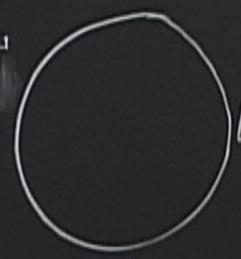


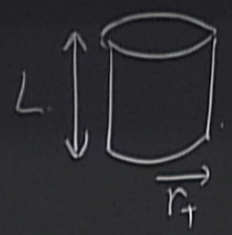
$4\pi M^2$

M

$2\pi M^2$

$S=\pi(M)^2$

> 4



$$\text{MASS} = \frac{r_+ L}{2G_5} = \frac{r_+}{2G_4}$$

cf

$$\text{MASS} = \frac{3\pi r_s^2}{8G_5}$$

$$\text{ENTROPY} = \frac{4\pi r_+^2 L}{4G_5}$$

$$S_{BS} \propto M^2 , \quad S_{BH} \propto M^{3/2}$$

Entropies equal:

$$\frac{4\pi L}{4 g_s} \left(\frac{2 g_s M}{L} \right)^2$$

$$S_{BS} \propto M^2 , \quad S_{BH} \propto M^{3/2}$$

Entropies equal:

$$\frac{4\pi L}{4 g_s}\left(\frac{2 g_s M}{L}\right)^2 = \frac{2\pi^2}{4 g_s}\left(\frac{8 g_s M}{3\pi}\right)^{3/2}$$

$$\frac{2\pi^2}{4G_5}\left(\frac{8G_5M}{3\pi}\right)^{3/2}$$

$$\frac{8}{7\pi} L$$

$$\frac{}{4\pi}$$

Entropy of hole

larger for longer L

Test in pert th.

$$h_{\mu\nu} = e^{\Omega t} e^{i\mu z} \begin{pmatrix} h_{tt} & h_{tr} & & 0 \\ h_{tr} & h_{rr} & & \\ & & K & 0 \\ 0 & & 0 & K\sin^2\theta \end{pmatrix}$$

instability

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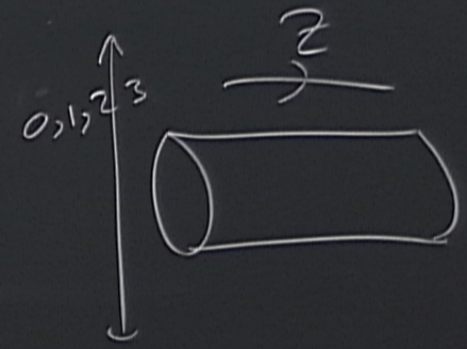

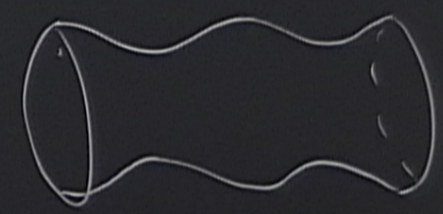

instability

in $\mu = 0,1,2,3$

black 'hole' dim.