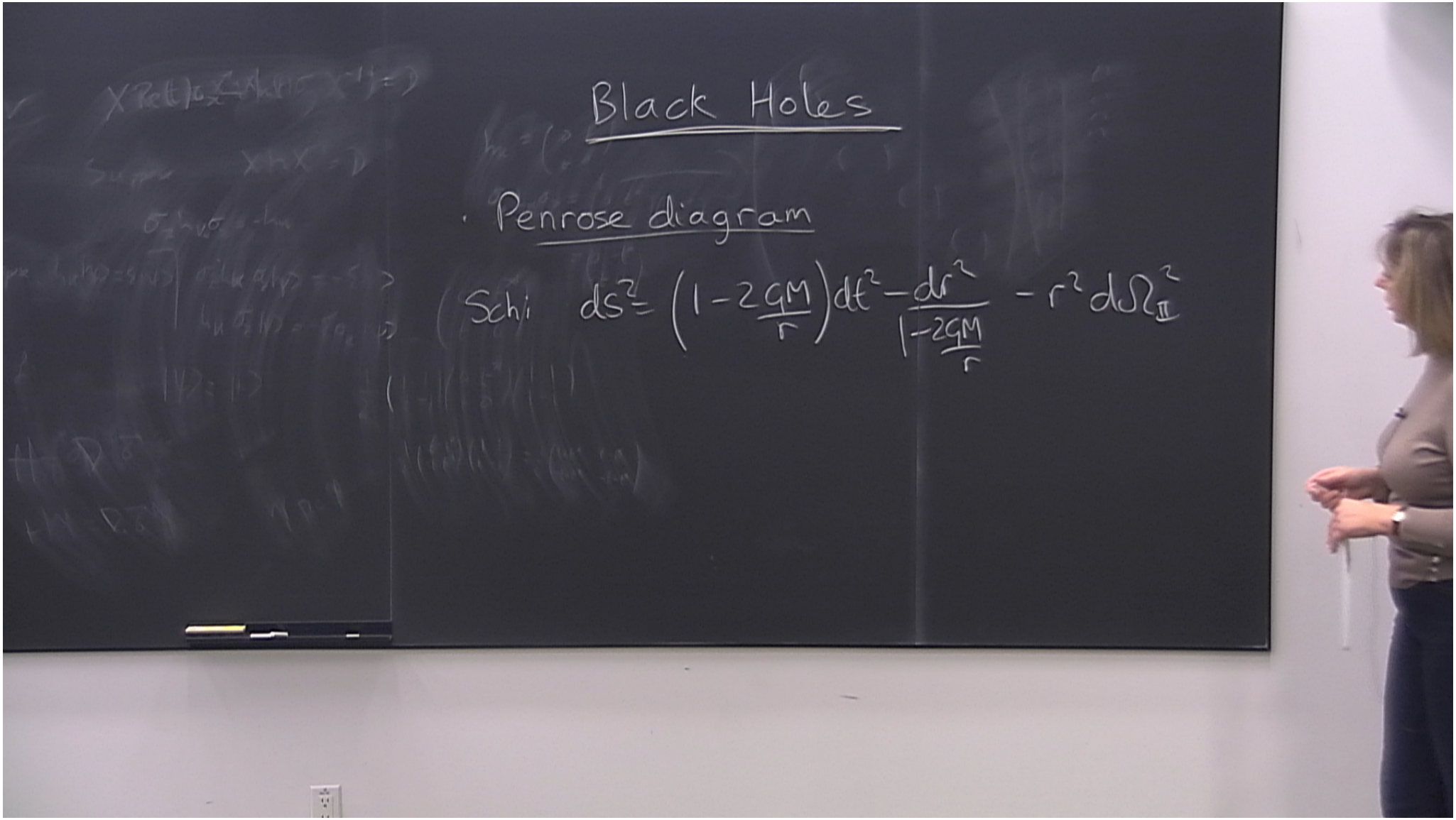
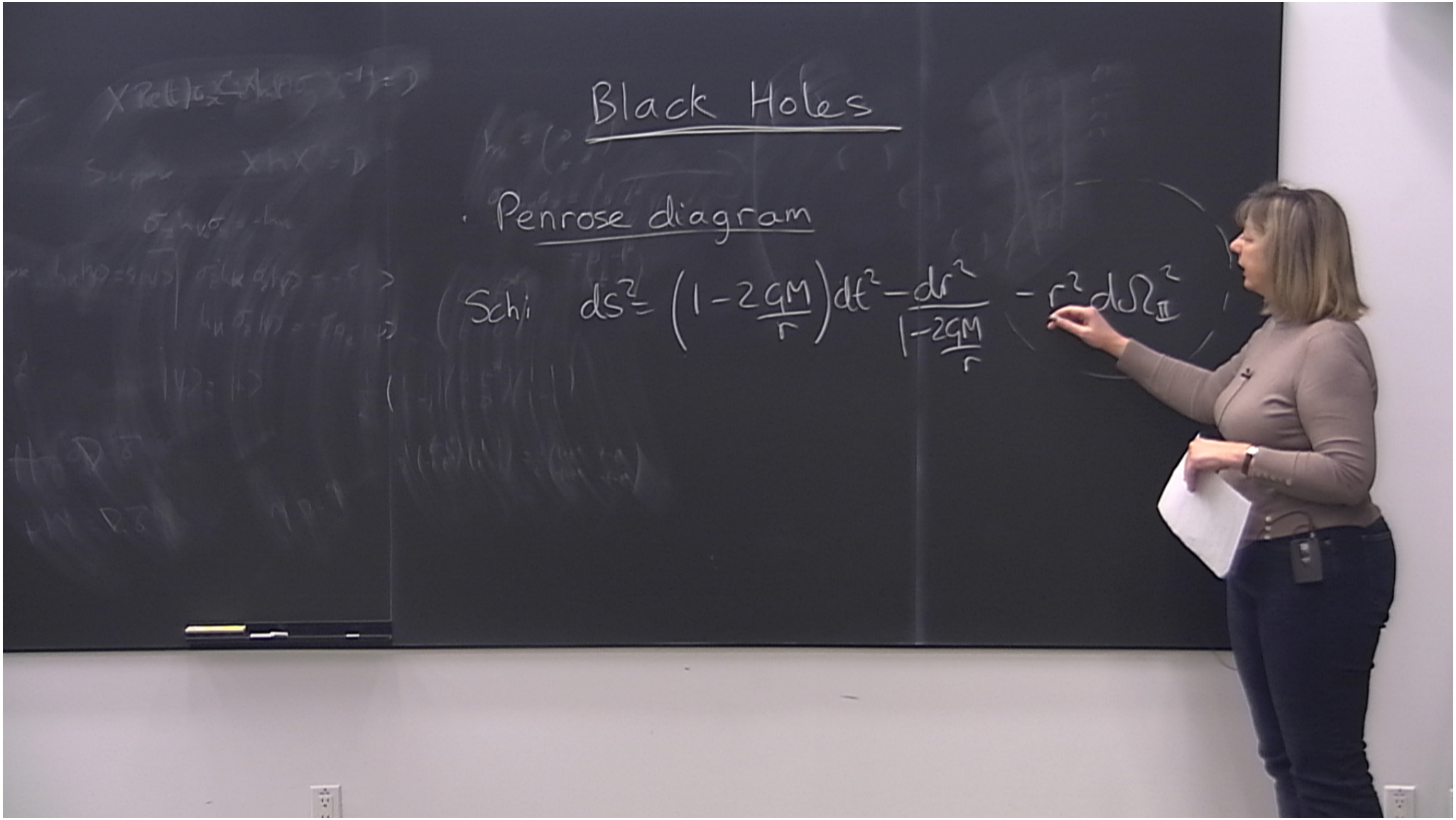
Title: 12/13 PSI - Gravitational Physics Review Lecture 13

Date: Feb 13, 2013 09:00 AM

URL: http://pirsa.org/13020071

Abstract:


Black Holes
· Penrose diagram
Schi $ds^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2 d\Omega_{II}^2$

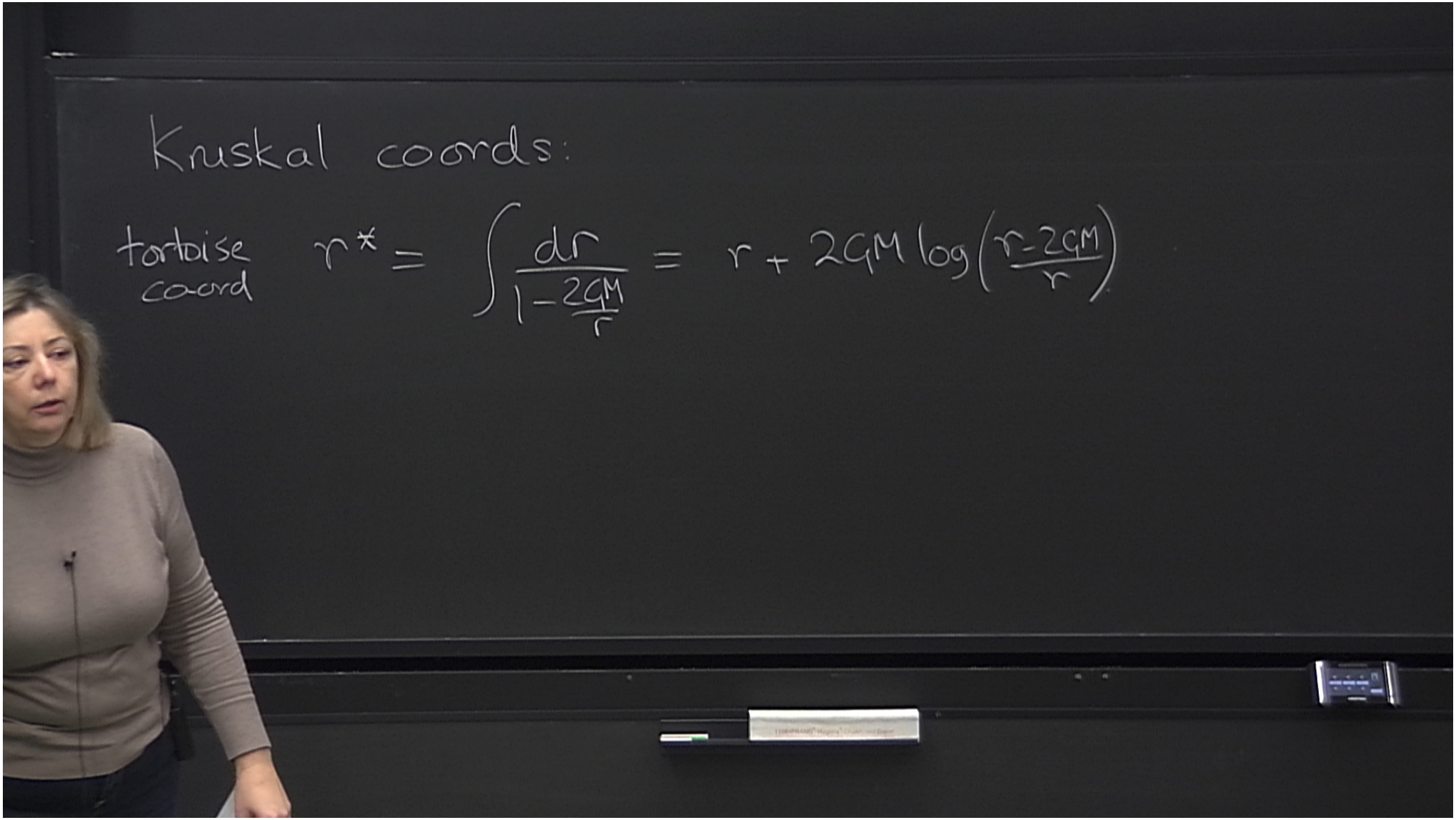

Black Holes
· Penrose diagram
Schi $ds^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2 d\Omega_{II}^2$

# Black Holes

- Penrose diagram

Schi $ds^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2 d\Omega_{II}^2$

Gravity shows up here.


Kruskal coords:
tortoise coord
$r^* = \int \frac{dr}{1-\frac{2GM}{r}} = r + 2GM \log\left(\frac{r-2GM}{r}\right)$

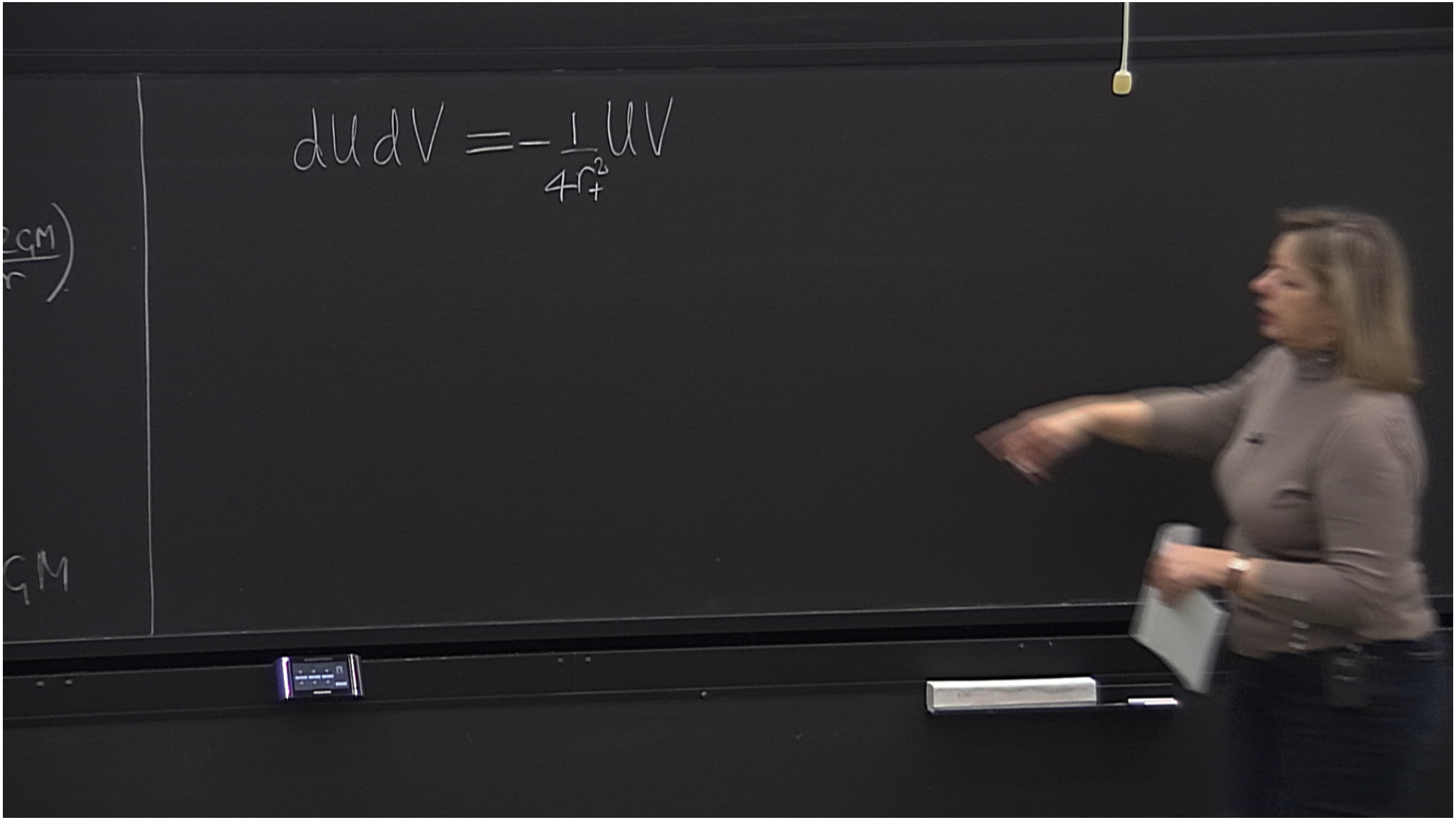
# Kruskal coords:

tortoise coord $r^* = \int \frac{dr}{1-\frac{2GM}{r}} = r + 2GM \log\left(\frac{r-2GM}{r}\right)$

- $U = -r_+ \exp[-(t-r^*)/2r_+]$

  $V = r_+ \exp[(t+r^*)/2r_+]$

  $r_+ = 2GM$


$dUdV = -\frac{1}{4r_+^2}UV$
$\left(\frac{2GM}{r}\right)$
$GM$

$$dU\,dV = -\frac{1}{4r_+^2} UV\,(dt^2 - dr^{*2})$$

$$= \frac{1}{4} e^{r^*/r_+}\,(dt^2 - dr^{*2})$$

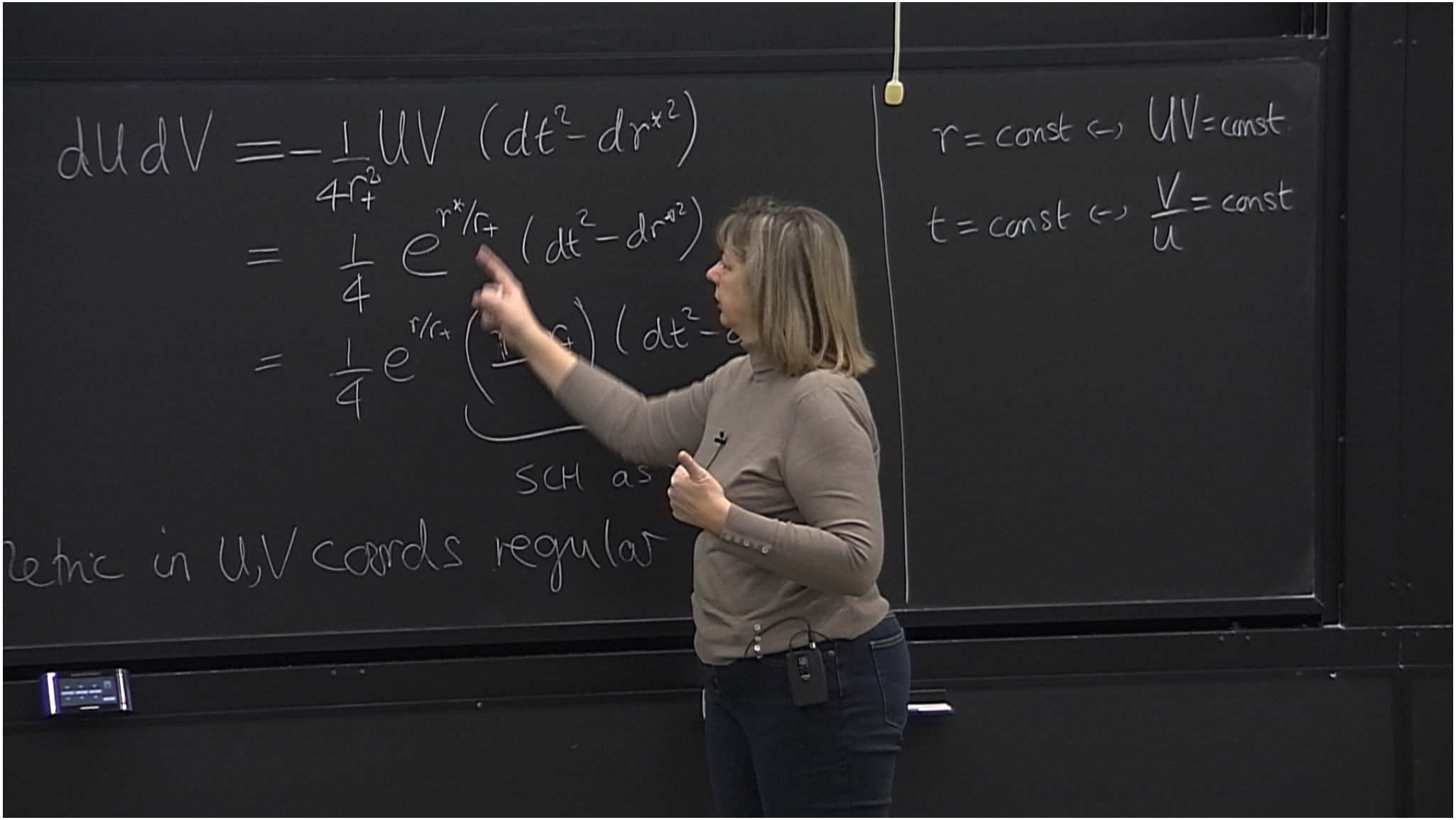
$$dU\,dV = -\frac{1}{4r_+^2} UV\,(dt^2 - dr^{*2})$$

$$= \frac{1}{4} e^{r^*/r_+}\,(dt^2 - dr^{*2})$$

$$= \frac{1}{4} e^{r/r_+} \underbrace{\left(\frac{r - r_+}{r_+}\right)}_{\text{SCH as } r \to r_+} (dt^2 - dr^{*2})$$

metric in U,V coords regular at $r_+$

$r = \text{const}$ ←


$dUdV = -\frac{1}{4r_+^2} UV\,(dt^2 - dr^{*2})$
$= \frac{1}{4} e^{r^*/r_+}\,(dt^2 - dr^{*2})$
$= \frac{1}{4} e^{r/r_+}$
SCH as
metric in U,V coords regular
$r = \text{const} \leftrightarrow UV = \text{const}$
$t = \text{const} \leftrightarrow \frac{V}{U} = \text{const}$

$$dU\,dV = -\frac{1}{4r_+^2} UV\,(dt^2 - dr^{*2})$$

$$= \frac{1}{4} e^{r^*/r_+}\,(dt^2 - dr^{*2})$$

$$= \frac{1}{4} e^{r/r_+} \underbrace{\left(\frac{r - r_+}{r_+}\right)}_{\text{SCH as } r \to r_+} (dt^2 - dr^{*2})$$

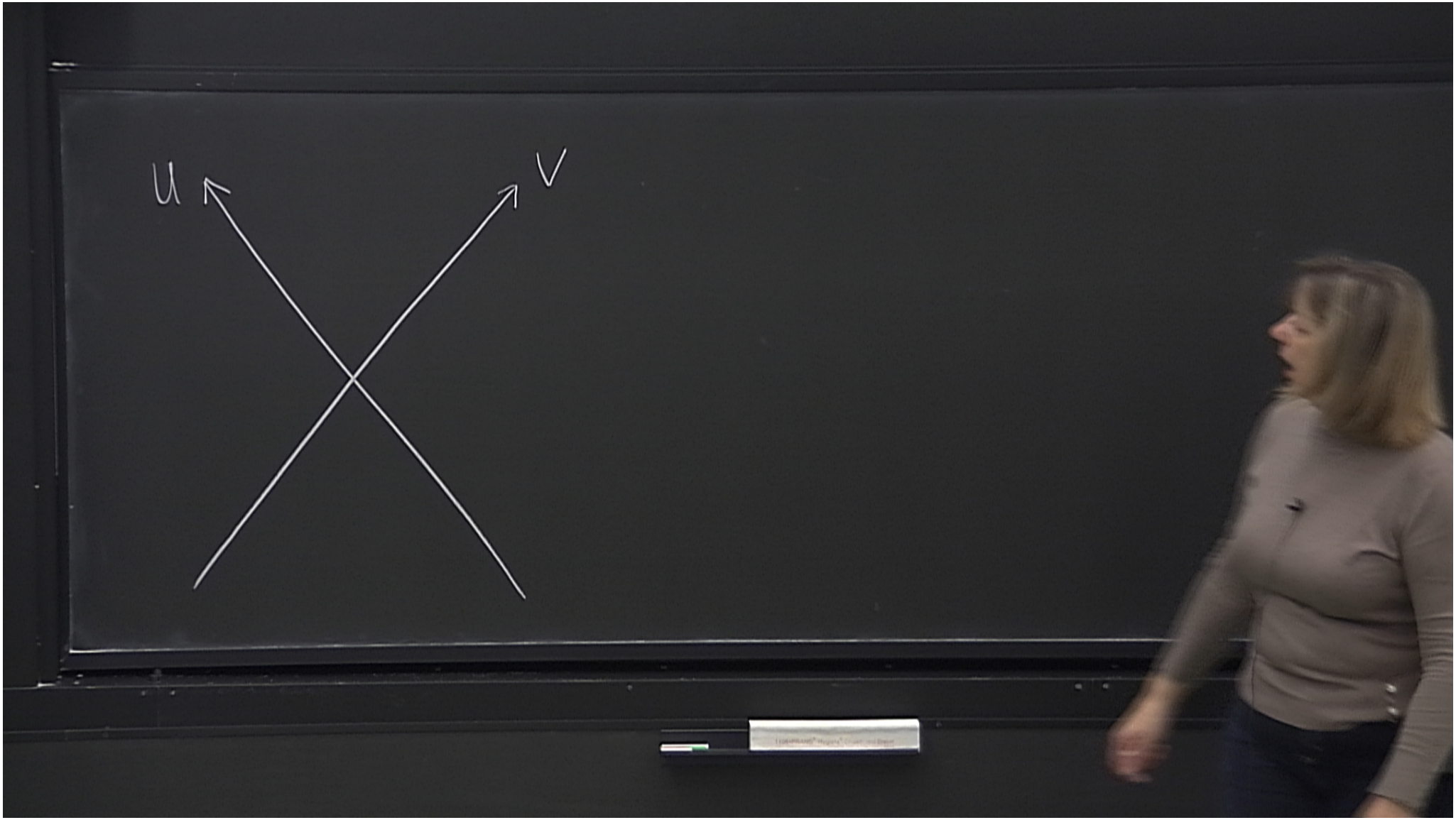
Metric in U,V coords regular at $r_+$

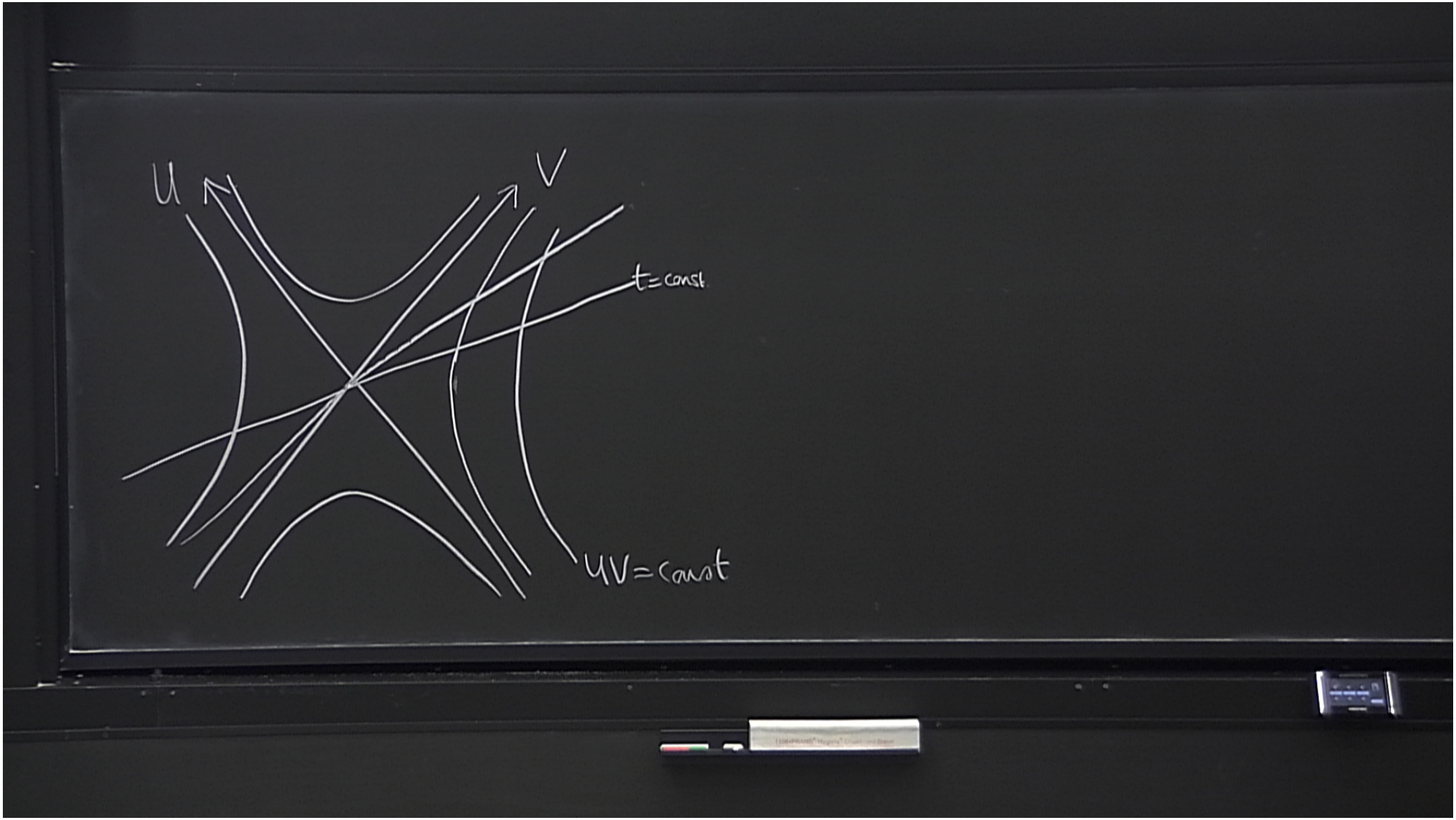
$r = \text{const} \leftrightarrow UV = \text{const}$

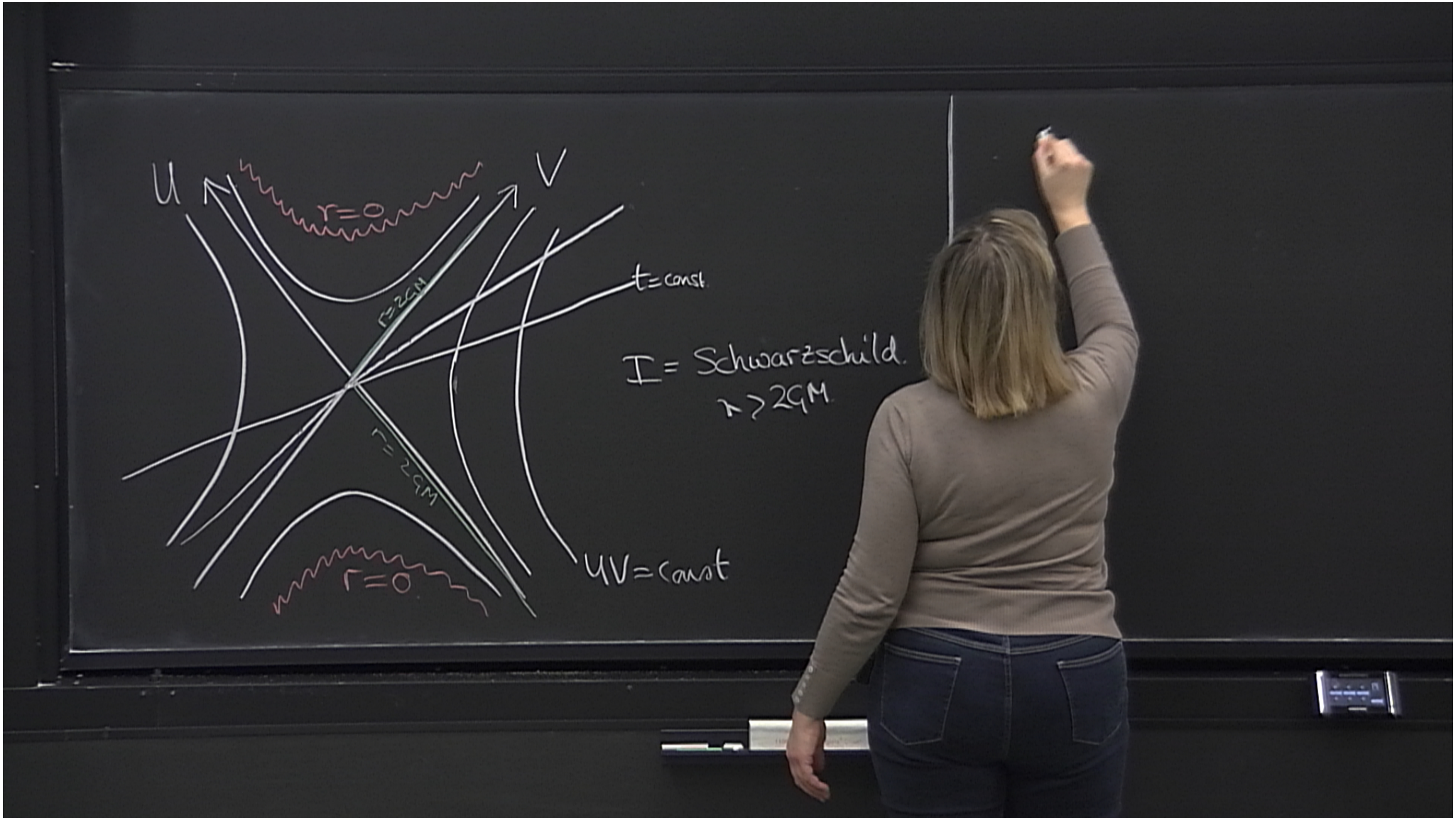
$t = \text{const} \leftrightarrow \frac{V}{U} = \text{const}$

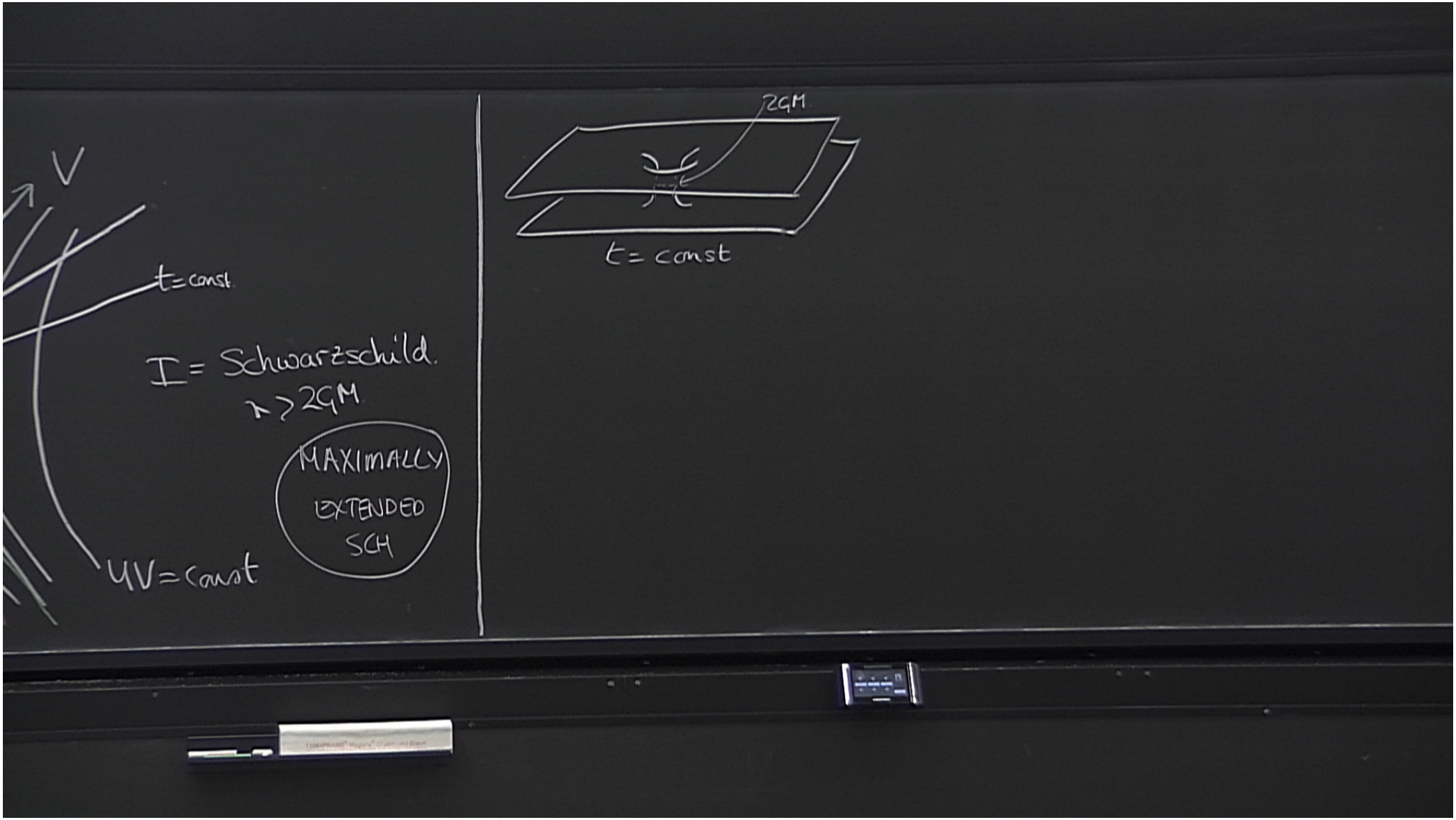
$r = r_+ \leftrightarrow UV = 0$

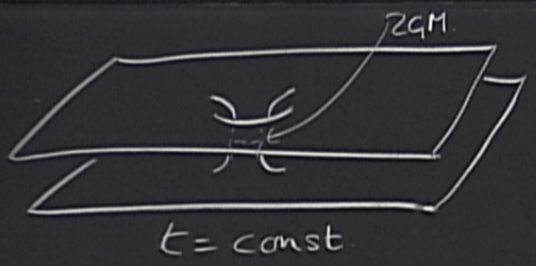
$r = 0 \leftrightarrow UV = 1/4$


U
V


U
V
t=const
UV=const


U
V
r=0
r=2M
t=const.
I= Schwarzschild.
r>2M
r=2M
UV=const
r=0


V
t=const
I = Schwarzschild.
r > 2GM
MAXIMALLY
EXTENDED
SCH
UV=const
2GM
t= const




$UV = r_+^2 \leftrightarrow \tan(p+q) =$

$p + q = \pm \pi/2$

Define $\left.\begin{aligned} p &= \arctan V/r_+ \\ q &= \arctan U/r_+ \end{aligned}\right\} \in (-\pi/2, \pi/2)$

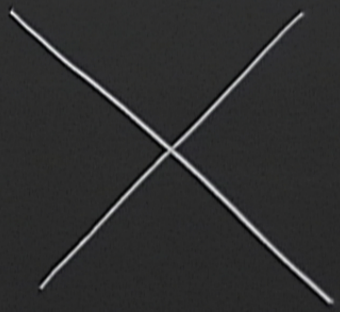
$V = 0 \leftrightarrow p = 0$

$V = r_+ \leftrightarrow p = \pi/4$

$UV = r_+^2 \leftrightarrow \tan(p+q) \to \infty$

$p+q = \pm \pi/2$.

$p = \arctan V/r_+$
$q = \arctan U/r_+$
$\Big\} \in (-\pi/2, \pi/2)$



$\leftrightarrow p = 0$.

$\leftrightarrow p = \pi/4$

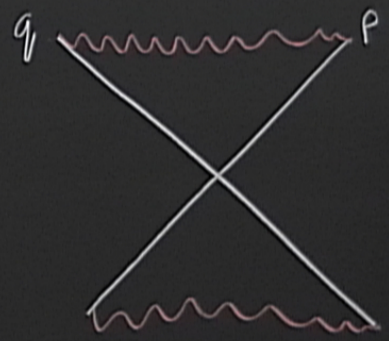
$UV = r_+^2 \leftrightarrow \tan(p+q) \to \infty$

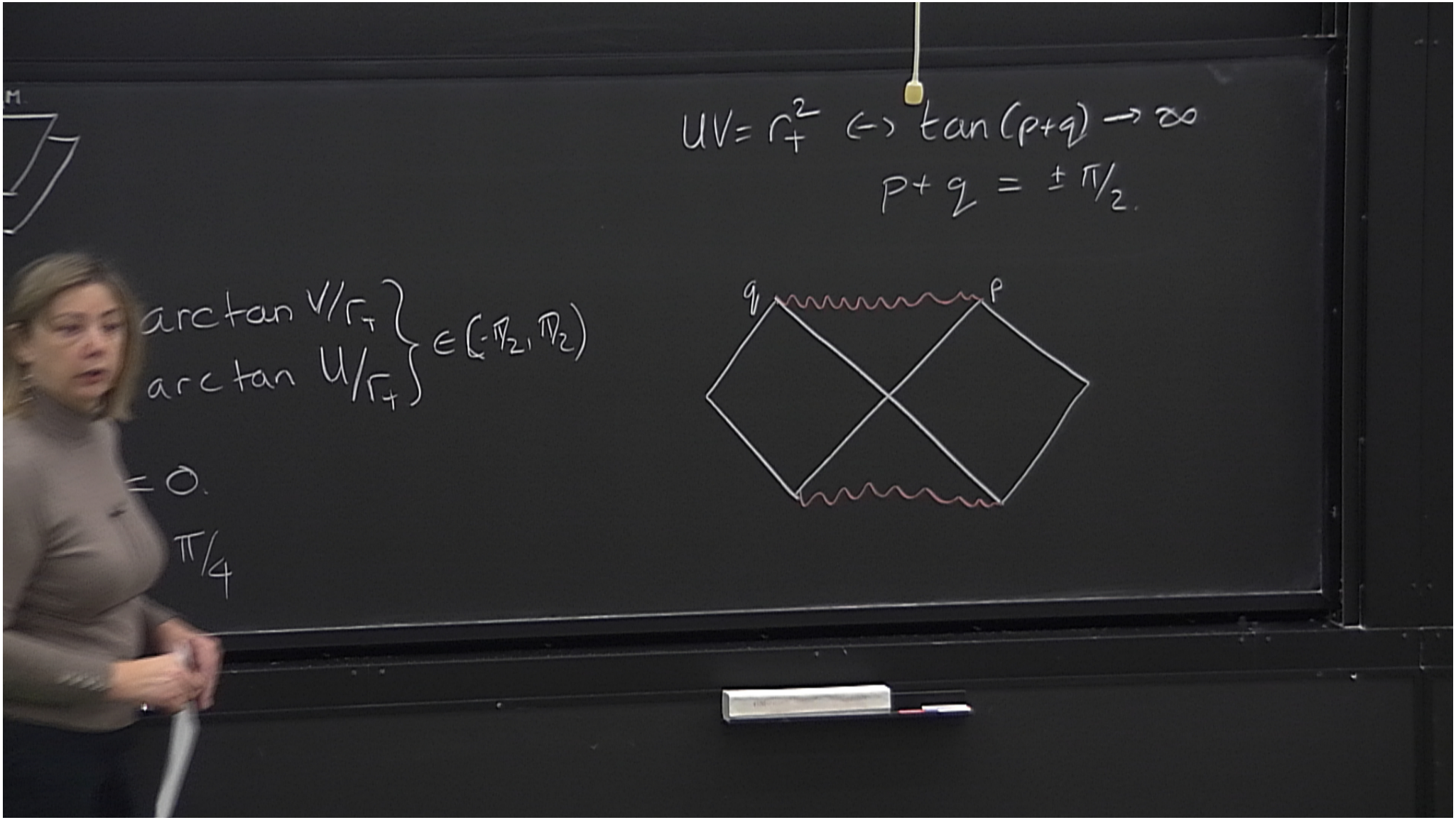
$p+q = \pm \pi/2$

$p = \arctan V/r_+$
$q = \arctan U/r_+$
$\in (-\pi/2, \pi/2)$

$\leftrightarrow p = 0$

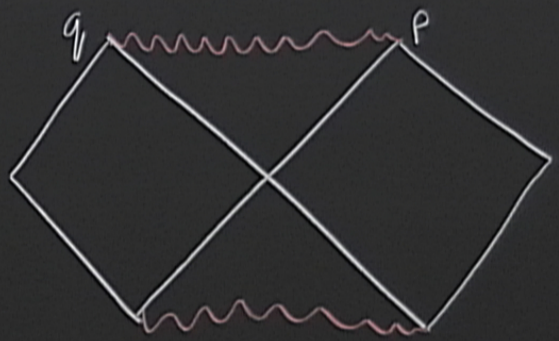
$\leftrightarrow p = \pi/4$




$UV = r_+^2 \iff \tan(p+q) \to \infty$
$p+q = \pm \pi/2$.
$\left.\begin{matrix} \arctan V/r_+ \\ \arctan U/r_+ \end{matrix}\right\} \in (-\pi/2, \pi/2)$
$= 0.$
$\pi/4$
q
p

$p + q = \pm \pi/2$.

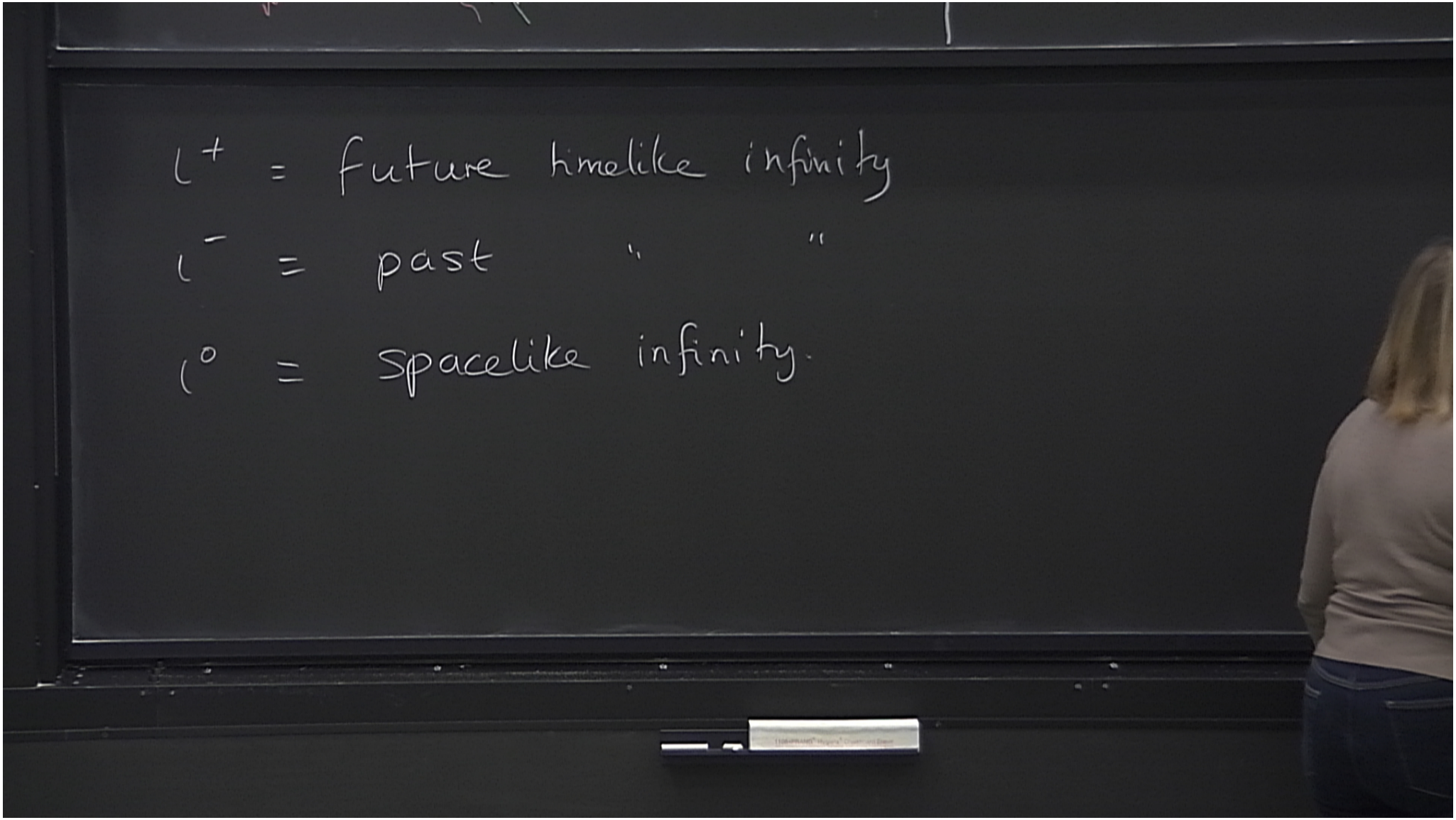
$$\left.\begin{aligned} p &= \arctan V/r_+ \\ q &= \arctan U/r_+ \end{aligned}\right\} \in (-\pi/2, \pi/2)$$

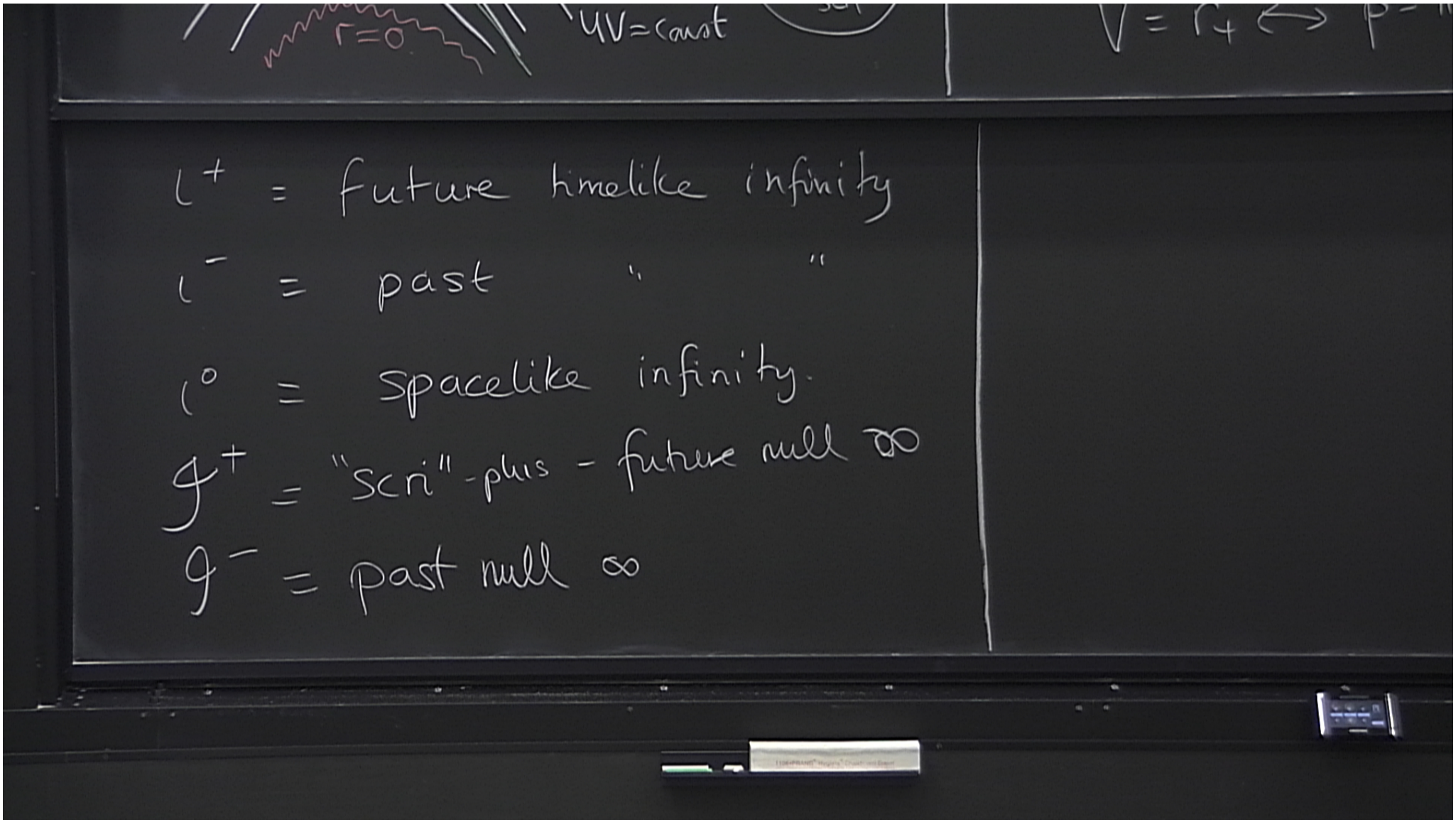



$\Rightarrow p = 0$.

$\Leftrightarrow p = \pi/4$

Penrose (-Carter) diagram for SCH.


$\mathscr{I}^+$ = future timelike infinity
$\mathscr{I}^-$ = past " "
$\mathscr{I}^0$ = spacelike infinity.


r=0
UV=const
$V = r_+ \Leftrightarrow$
$i^+$ = future timelike infinity
$i^-$ = past " "
$i^0$ = spacelike infinity.
$\mathcal{I}^+$ = "scri"-plus - future null $\infty$
$\mathcal{I}^-$ = past null $\infty$

$r = r_+ \leftrightarrow \rho = \pi/4$ Penrose (-Carter) d

nfinity

"

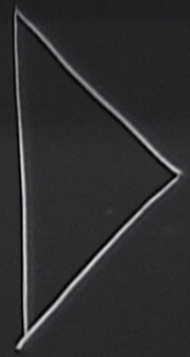
ty.

null $\infty$

$H^+$ = future event horizon

$H^-$ = past " "

Minkowski: $\underbrace{dt^2 - dr^2}_{-dudv} - r^2 d\Omega_{II}^2$



$u = t - r$ $\quad r > 0 \Rightarrow v > u$

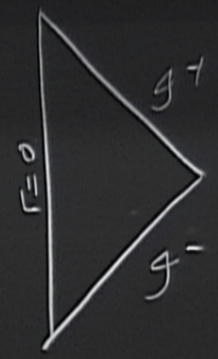
$v = t + r$

$V = r_+ \leftrightarrow \rho = \pi/4$

Penrose (= Carter) diagram

$H^+$ = future event horizon

$H^-$ = past " "




Minkowski: $\underbrace{dt^2 - dr^2}_{-dudv} - r^2 d\Omega_{\mathbb{E}}^2$

$u = t - r$ $\quad r > 0 \Rightarrow v > u$

$v = t + r$

$V = r_+ \Leftrightarrow p = 1/4$

Penrose (-Carter) diagram for

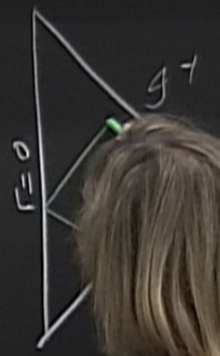
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Minkowski: $\underbrace{dt^2 - dr^2}_{-dudv} = r^2 d\Omega_{II}^2$

$u = t - r$ $\quad r > 0 \Rightarrow v > u$

$v = t + r$



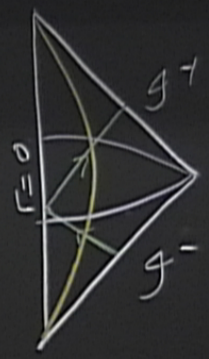
$V = r_+ \Leftrightarrow p = 1/4$

Penrose (-Carter) diagram for

A black hole form

$H^+$ = future event horizon

$H^-$ = past " "




Minkowski: $\underbrace{dt^2 - dr^2}_{-dudv} - r^2 d\Omega_{II}^2$

$u = t - r \quad r > 0 \Rightarrow v > u$

$v = t + r$

→ $\rho = \pi/4$

Penrose (-Carter) diagram for SCH

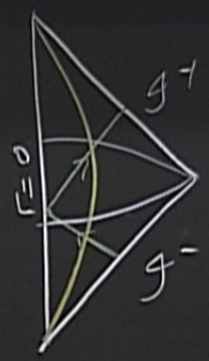
ure event horizon

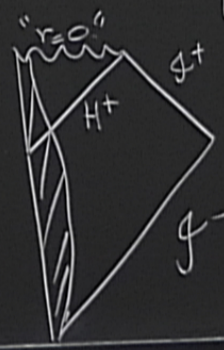
ast " "

ski: $\underbrace{dt^2 - dr^2}_{-dudv} - r^2 d\Omega_{II}^2$

r $r > 0 \Rightarrow v > u$

r




A black hole formed by (spherical) collapse will be more like MST in past & SCH in future:

Penrose (-Carter) diagram for SCH

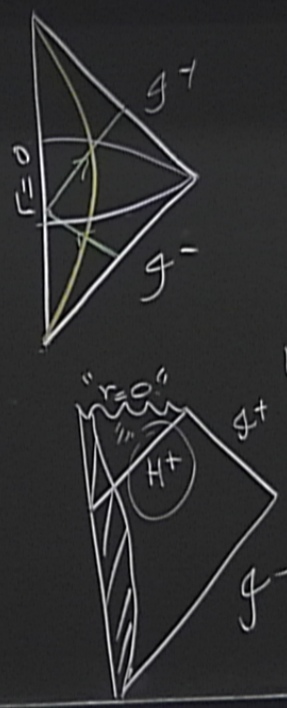
ure event horizon

ast " "

ski: $\underbrace{dt^2 - dr^2}_{-dudv} - r^2 d\Omega_{II}^2$

r $r > 0 \Rightarrow v > u$

r

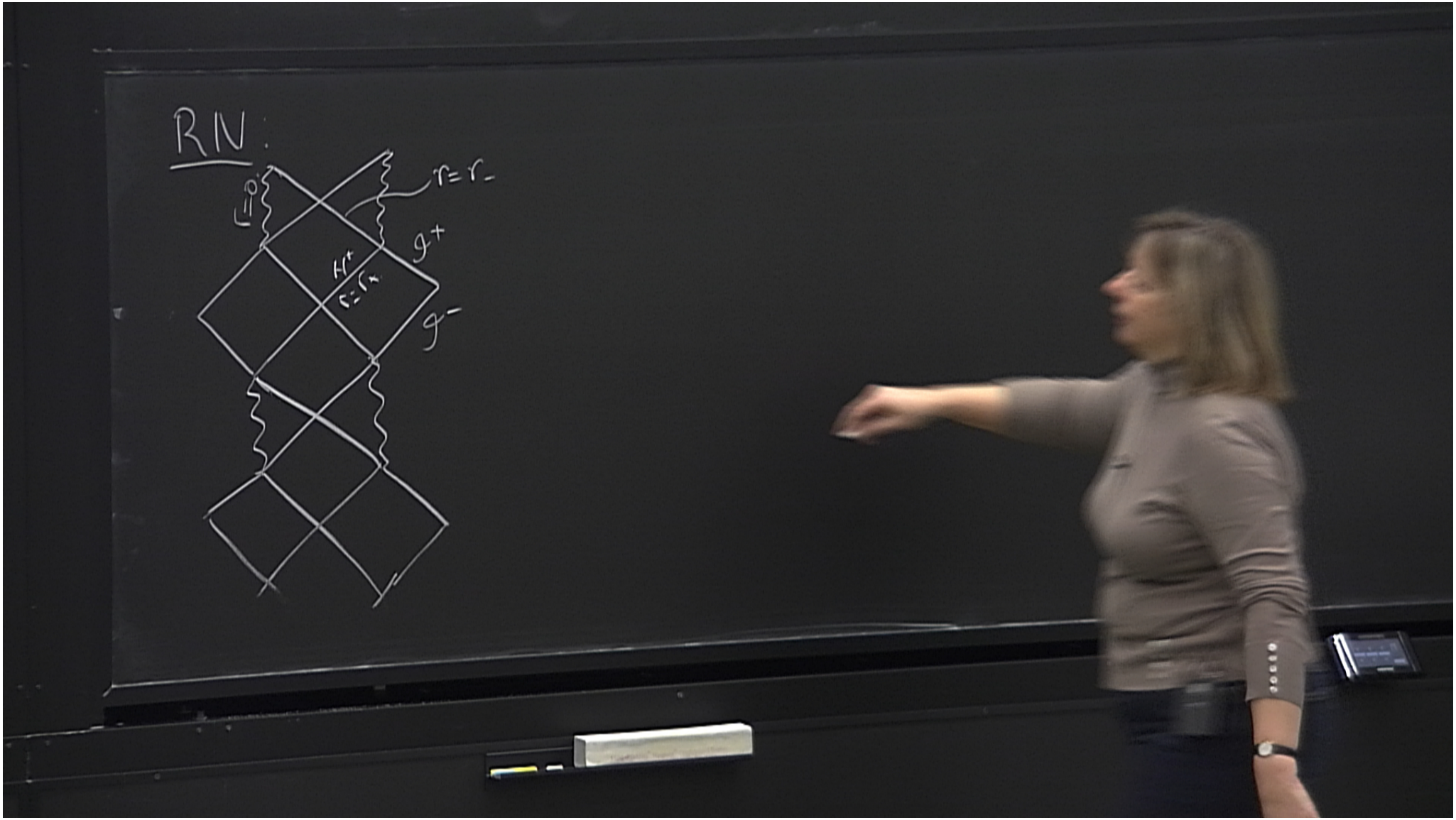




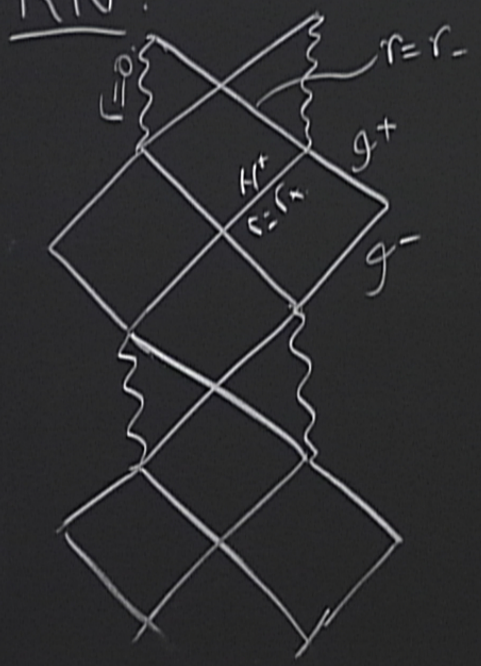


A black hole formed by (spherical) collapse will be more like MST in past & SCH in future:

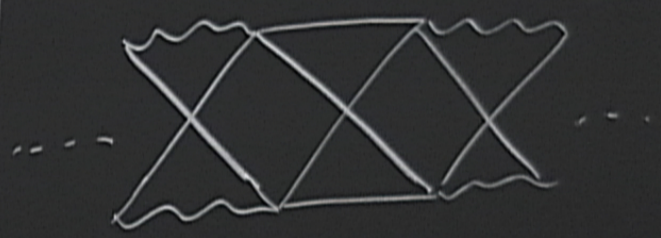
$H^+$ is the boundary of causal past of future timelike infinity


RN
r=r-
g+
H+
r=r+
g-

RN:




SCH - dS



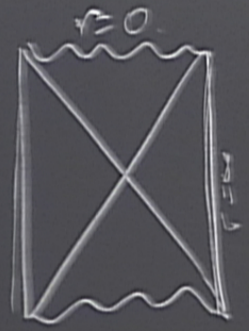
$$ds^2 = \left(1 - H^2 r^2 - \frac{r_s}{r}\right) dt^2 - \frac{dr^2}{\left(1 - H^2 r^2 - \frac{r_s}{r}\right)} - r^2 d\Omega_{II}^2$$

SCH - a

$ds^2 = ($

SCH - ads

$$ds^2 = \left(1 + H^2 r^2 - \frac{r_s}{r}\right) dt^2 - \frac{dr^2}{\left(1 + H^2 r^2 - \frac{r_s}{r}\right)} - r^2 d\Omega_{II}^2$$




What properties can a black hole have?

- Topology of event horizon
  - spherical in 4D
  - higher dims; cylindrical, rings, Saturn...
  - negative $\Lambda$ can also have planar/hyperbolic horizon in 4D

Black hole characterized by few charges.

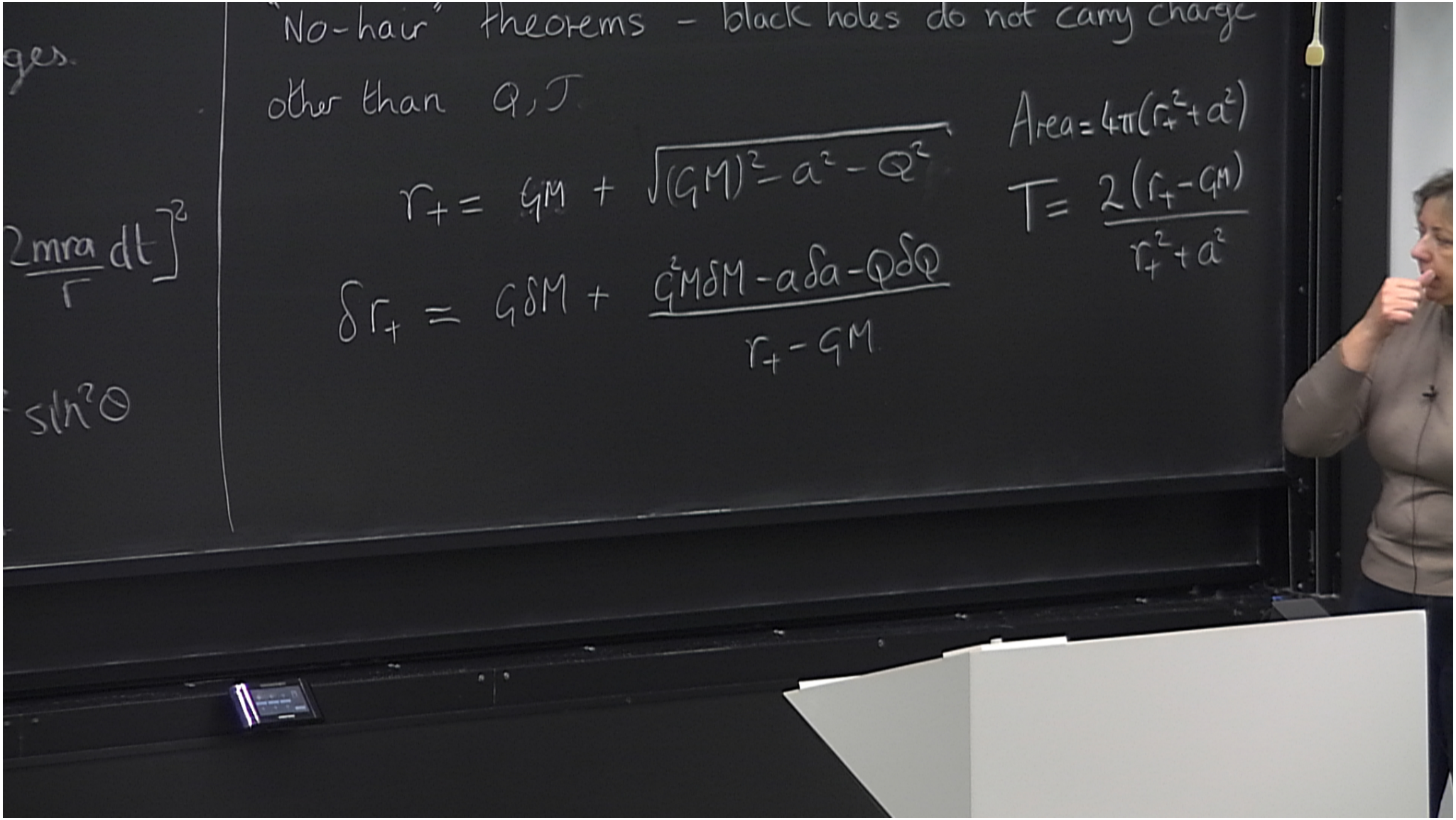
M, Q, J.

$$ds^2 = \frac{\Delta\Sigma}{\Gamma}dt^2 - \frac{\Sigma}{\Delta}dr^2 - \Sigma\, d\theta^2 - \frac{\Gamma\sin^2\theta}{\Sigma}\left[d\varphi - \frac{2mra}{\Gamma}dt\right]^2$$

$$\Delta = r^2 + a^2 + Q^2 - 2GMr \qquad \Gamma = (r^2+a^2)^2 - \Delta a^2 \sin^2\theta$$

$$\Sigma = r^2 + a^2\cos^2\theta. \qquad a = J/M$$

"No-


"No-hair" theorems - black holes do not carry charge
other than Q, J
$r_+ = GM + \sqrt{(GM)^2 - a^2 - Q^2}$
$\delta r_+ = G\delta M + \frac{G^2 M\delta M - a\delta a - Q\delta Q}{r_+ - GM}$
$\text{Area} = 4\pi(r_+^2 + a^2)$
$T = \frac{2(r_+ - GM)}{r_+^2 + a^2}$

- higher dims; cylindrical, rings, Saturn...
- negative $\Lambda$ can also have planar/hyperbolic horizon in 4D

ges.

$$\frac{2mra}{r}dt\Big]^2$$

$$\sin^2\theta$$

"No-hair" theorems - black holes do not carry charge other than $Q, J$

$$r_+ = GM + \sqrt{(GM)^2 - a^2 - Q^2}$$

$$\text{Area} = 4\pi(r_+^2 + a^2)$$

$$T = \frac{2(r_+ - GM)}{r_+^2 + a^2}$$

$$\delta r_+ = G\delta M + \frac{G^2 M\delta M - a\delta a - Q\delta Q}{r_+ - GM}$$

$$\downarrow$$

$$\delta A = 8\pi(r_+\delta r_+ + a\delta a)$$

$\Sigma = r^2 + a^2 \cos^2\theta$.  $a = J/M$

$$\delta M = T\,\delta A + \Omega\,\delta J + \Phi\,\delta Q$$

$A$

$\Omega$: $\dfrac{a}{r_+^2 + a^2}$

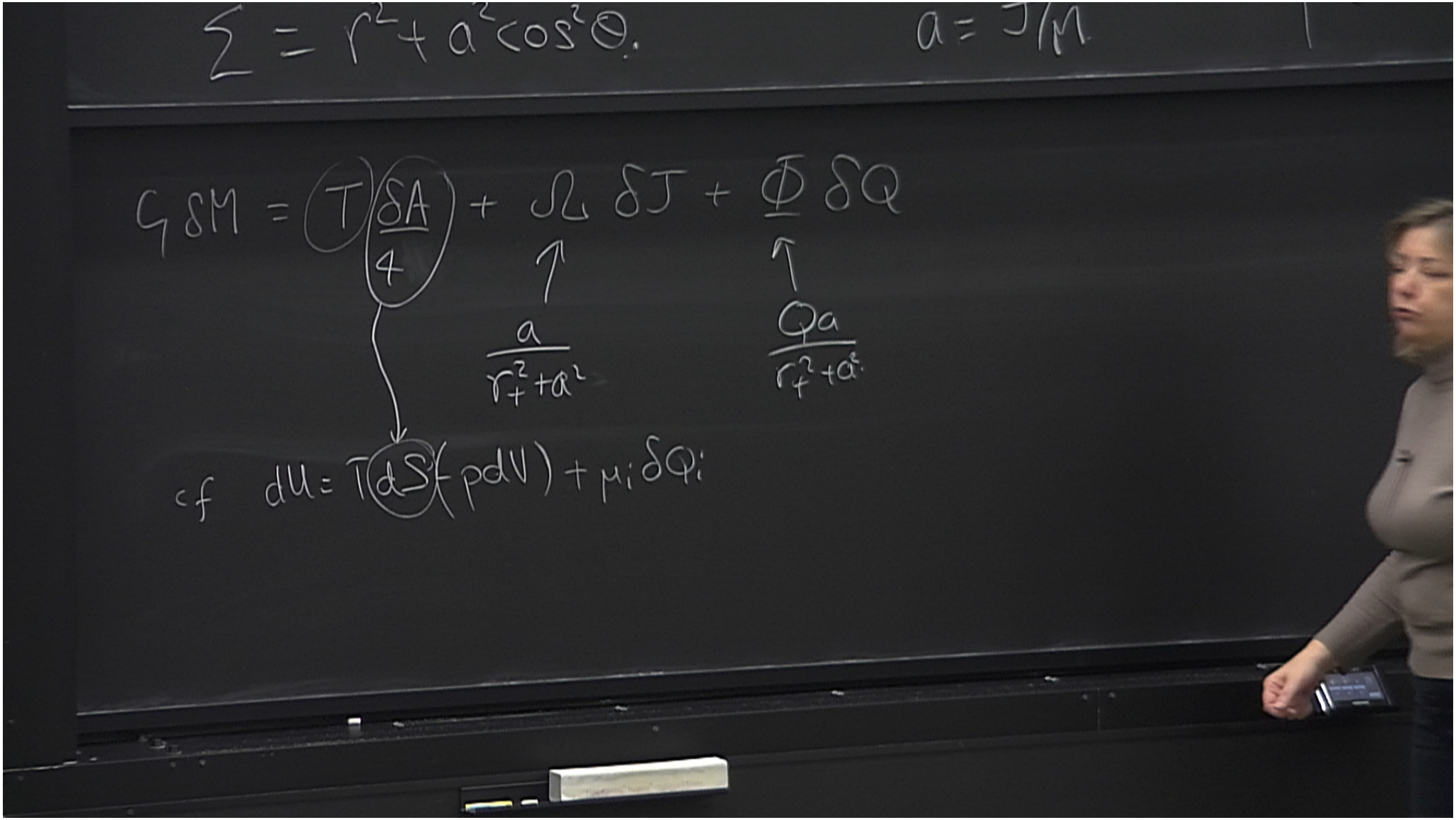
$\Phi$: $\dfrac{Qa}{r_+^2 + a^2}$

$\Sigma = r^2 + a^2 \cos^2\theta$.  $a = J/M$

$$\delta M = T\,\frac{\delta A}{4} + \Omega\,\delta J + \Phi\,\delta Q$$

$$\Omega = \frac{a}{r_+^2 + a^2} \qquad \Phi = \frac{Qa}{r_+^2 + a^2}$$

cf $dU = TdS + (-pdV) + \mu_i \delta Q_i$

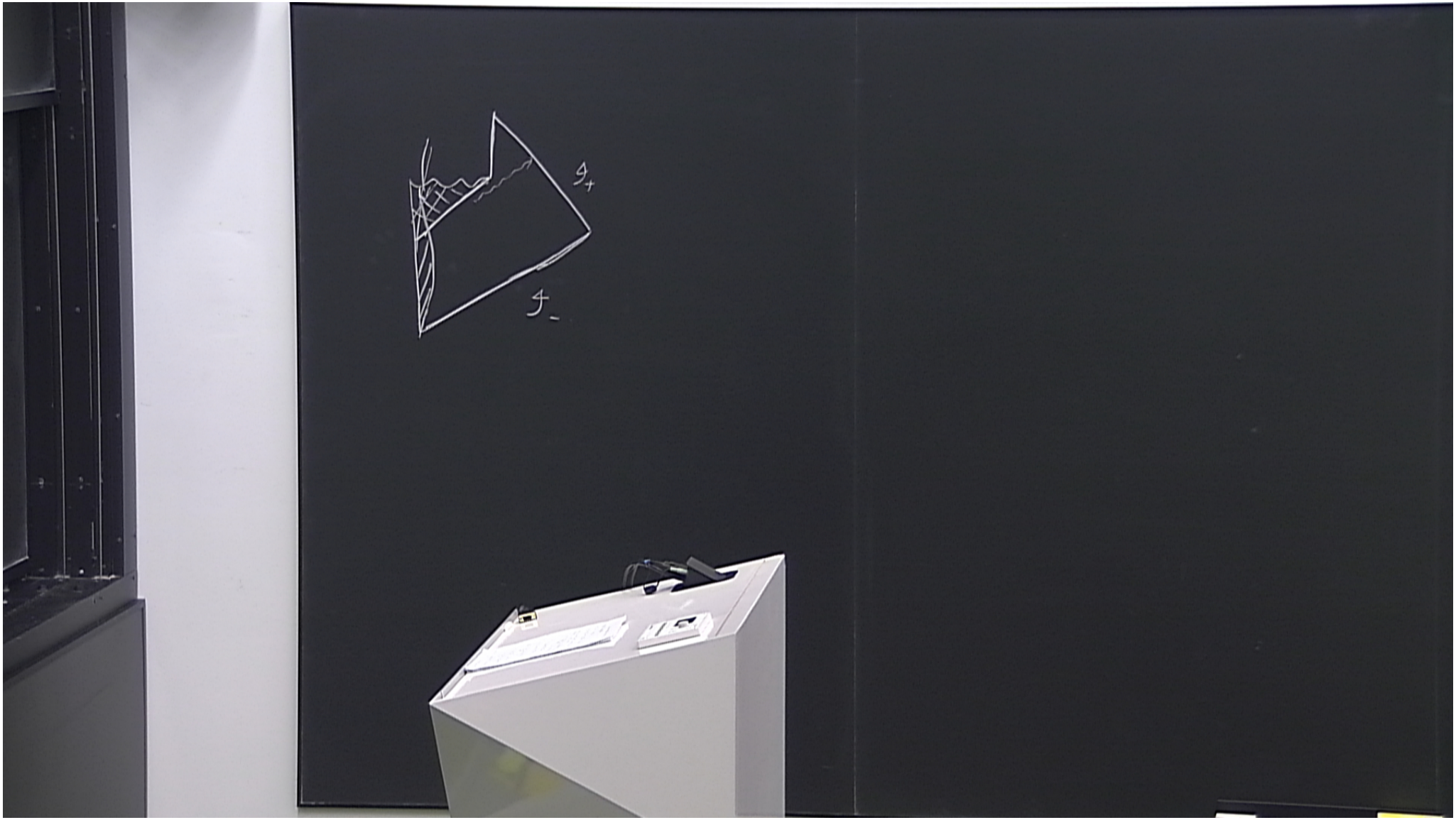

$\Sigma = r^2 + a^2 \cos^2\theta$.
$a = J/M$
$\delta M = T \frac{\delta A}{4} + \Omega\, \delta J + \Phi\, \delta Q$
$\frac{a}{r_+^2 + a^2}$
$\frac{Qa}{r_+^2 + a^2}$
cf $dU = T(dS) - pdV) + \mu_i \delta Q_i$

$\Sigma = r^2 + a^2 \cos^2\theta$.  $a = J/M$

$$\delta M = T\frac{\delta A}{4} + \Omega\,\delta J + \Phi\,\delta Q$$

$\Omega$: $\frac{a}{r_+^2 + a^2}$  $\Phi$: $\frac{Qa}{r_+^2 + a^2}$

cf $dU = T(dS) - pdV) + \mu_i \delta Q_i$


$g_+$
$f_-$