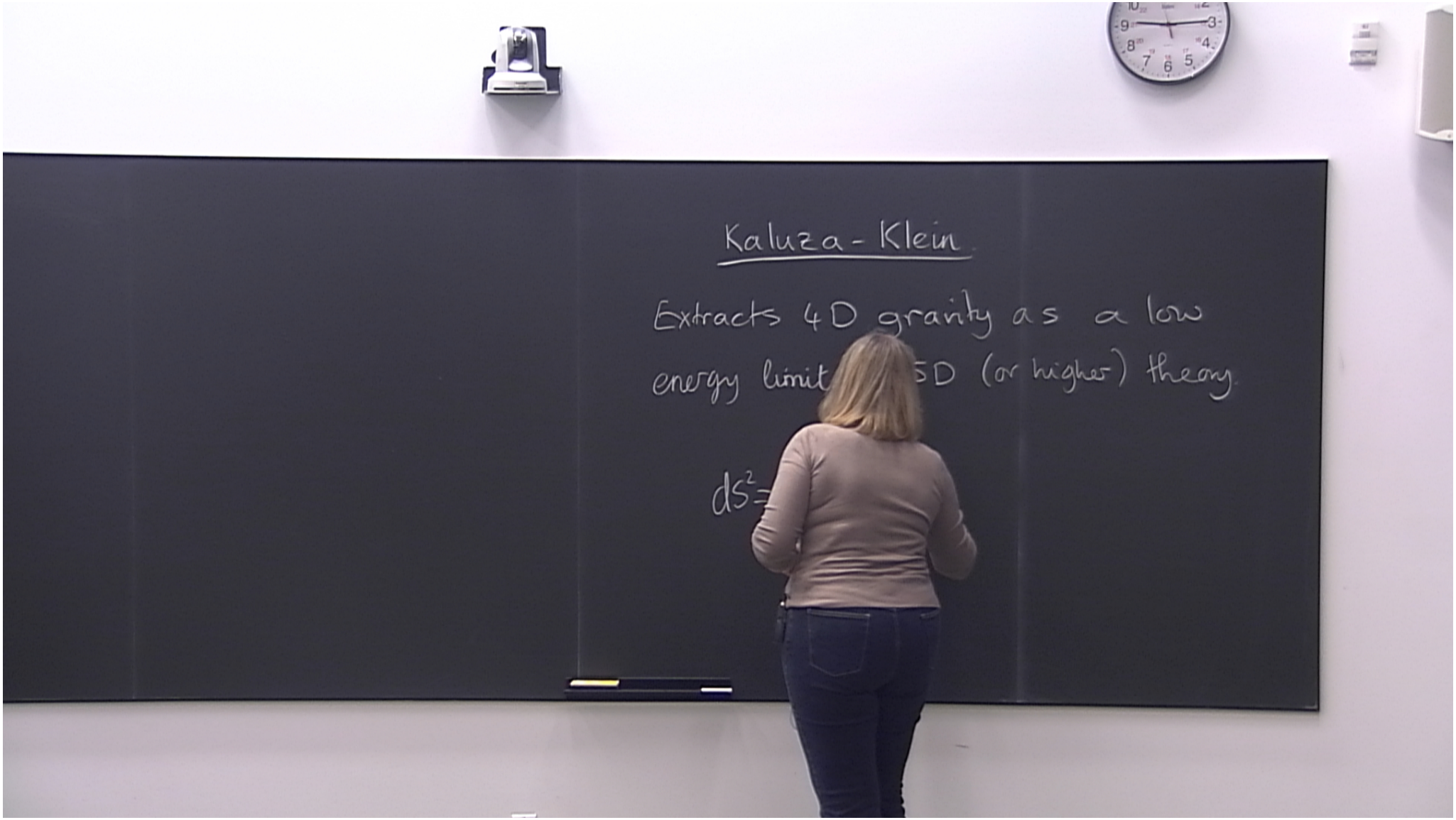
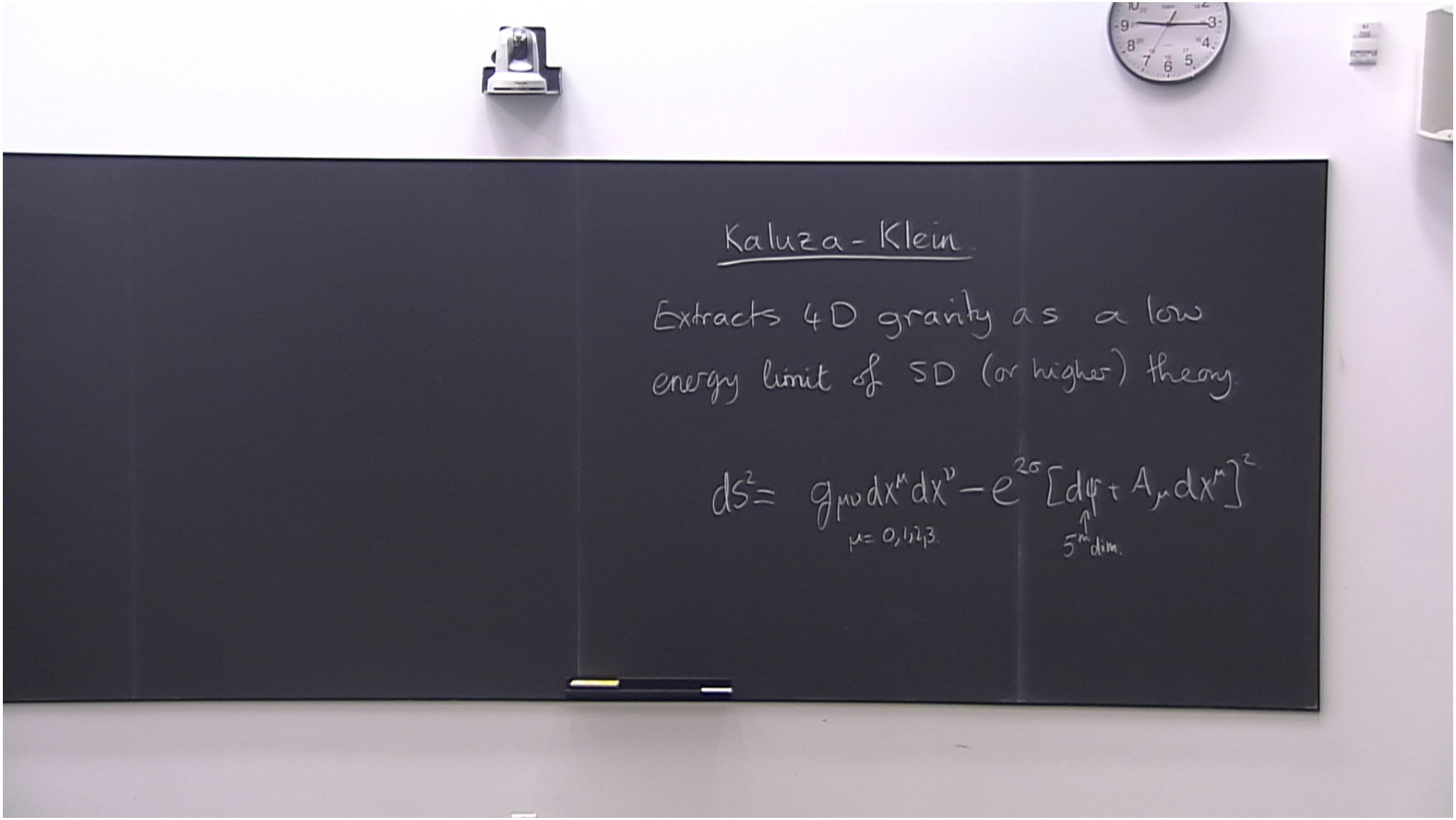
Title: 12/13 PSI - Gravitational Physics Review Lecture 10

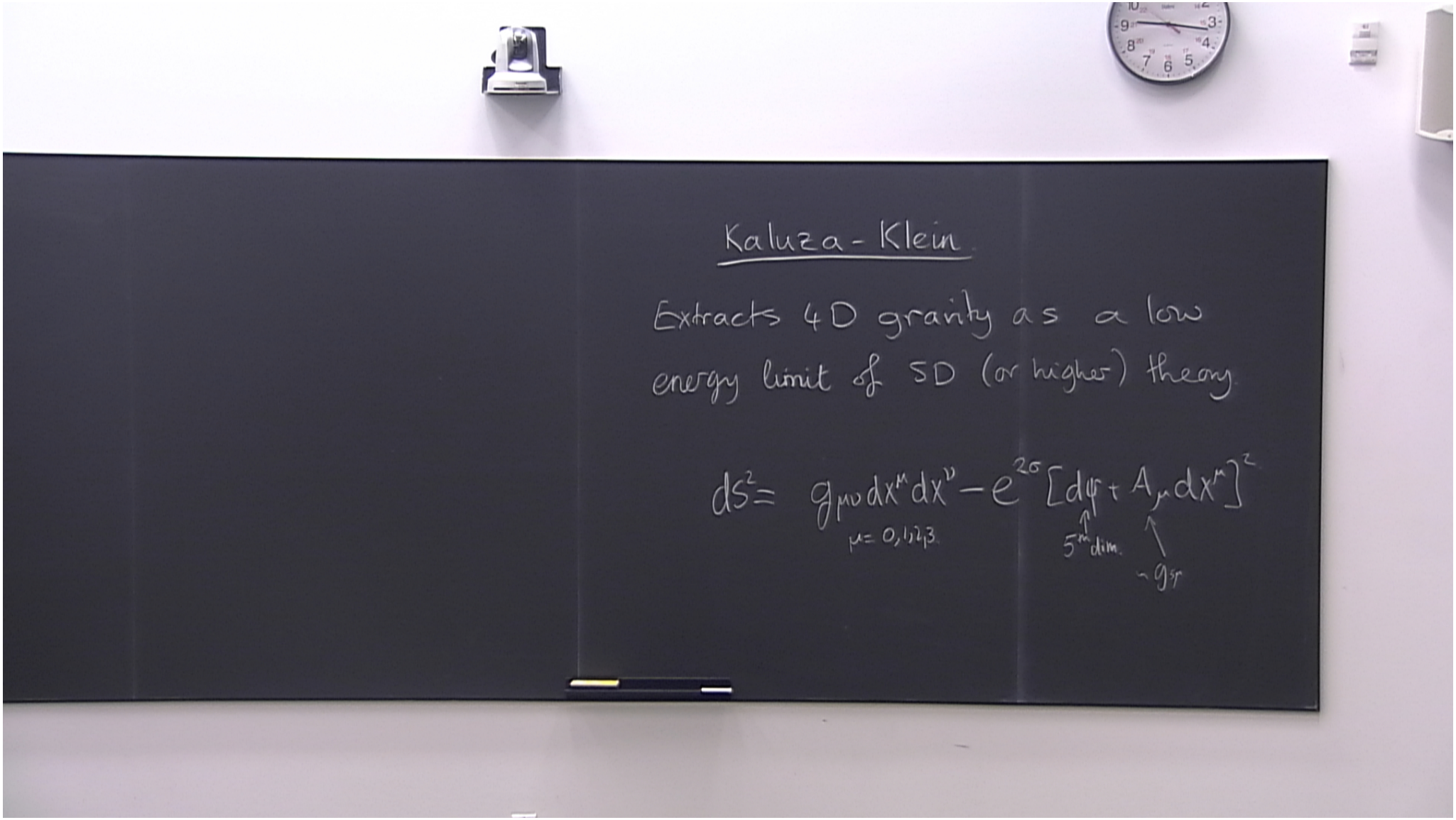
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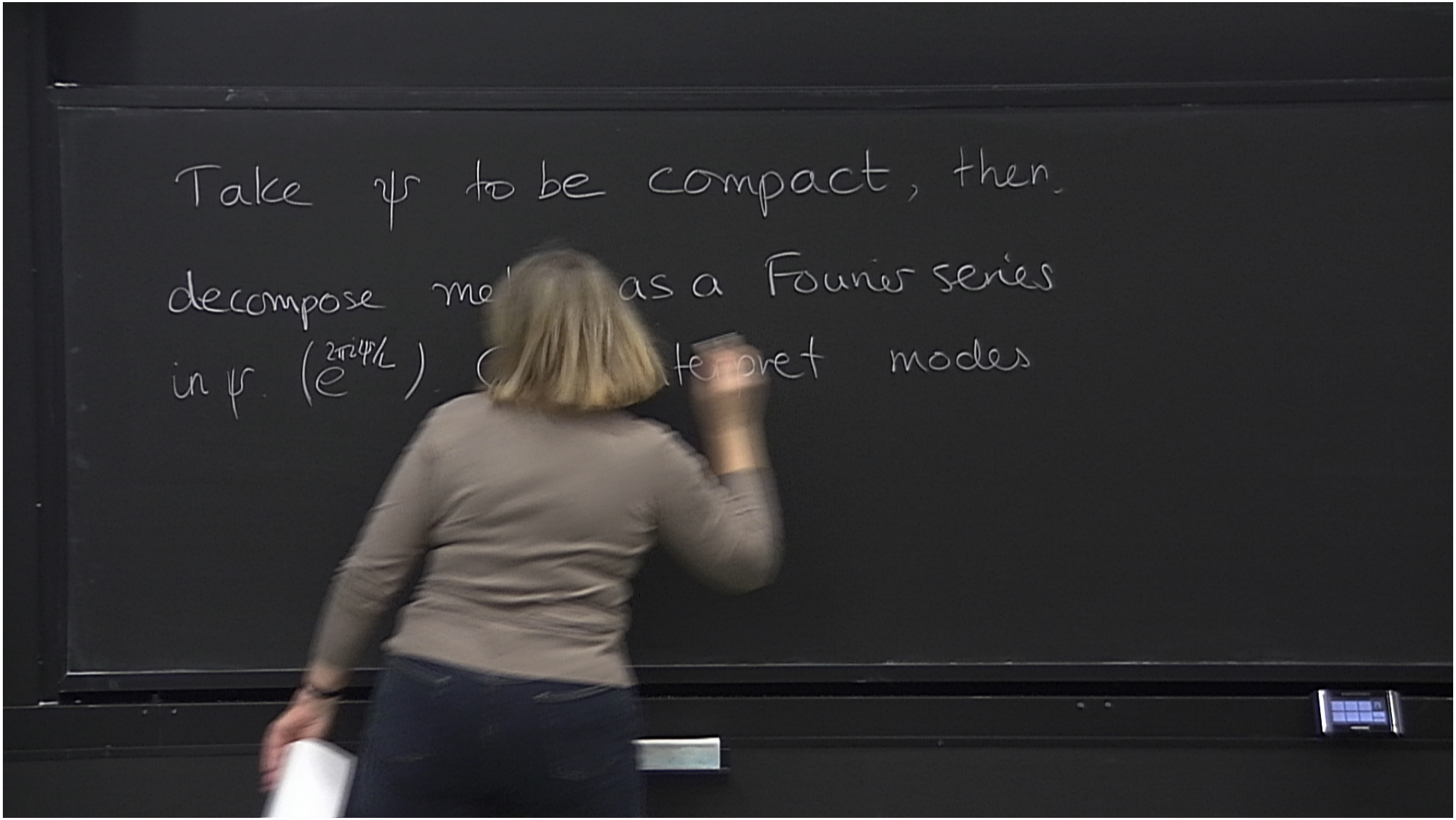
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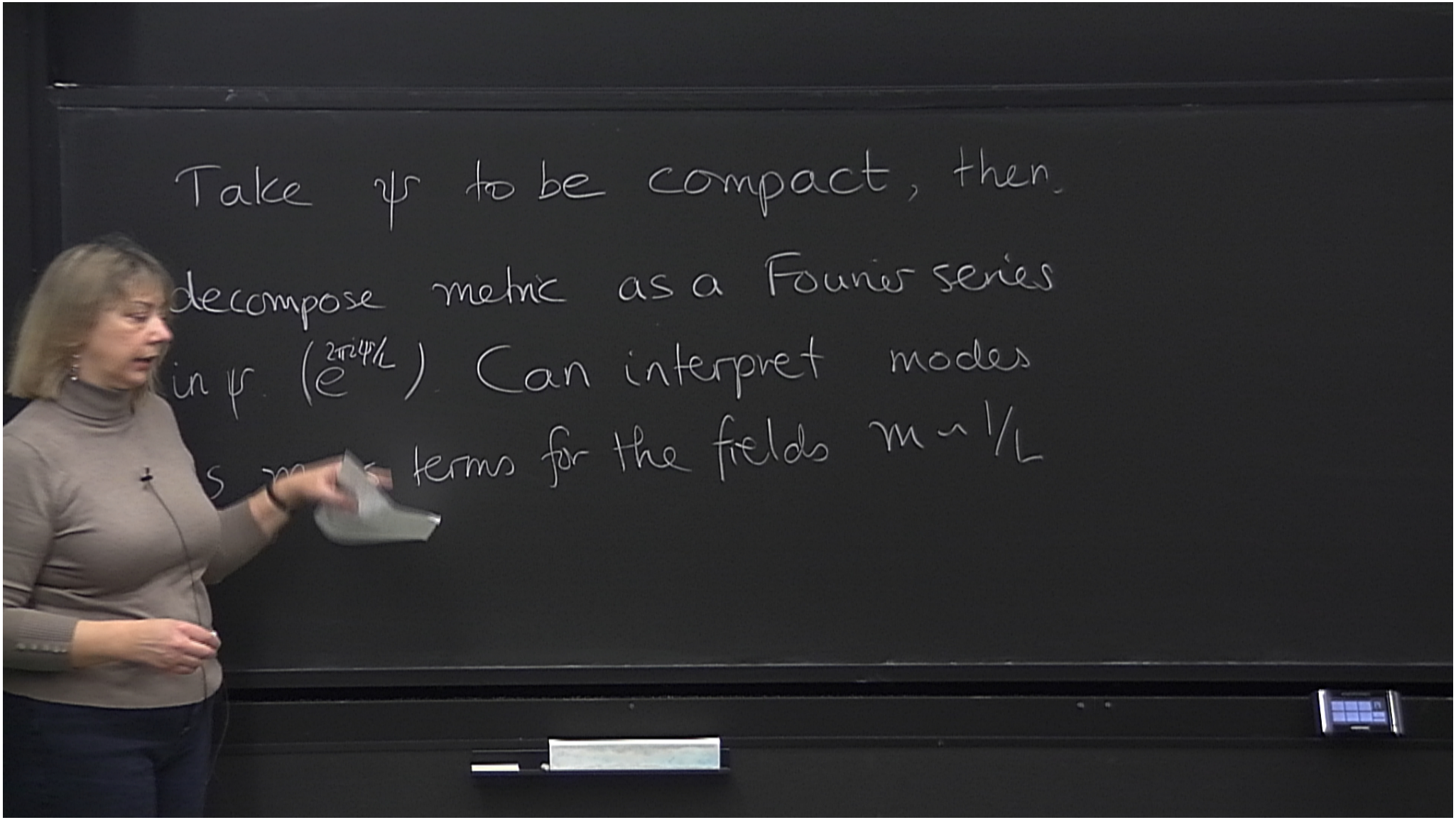
Abstract:


Kaluza-Klein
Extracts 4D gravity as a low
energy limit
5D (or higher) theory
$ds^2 =$

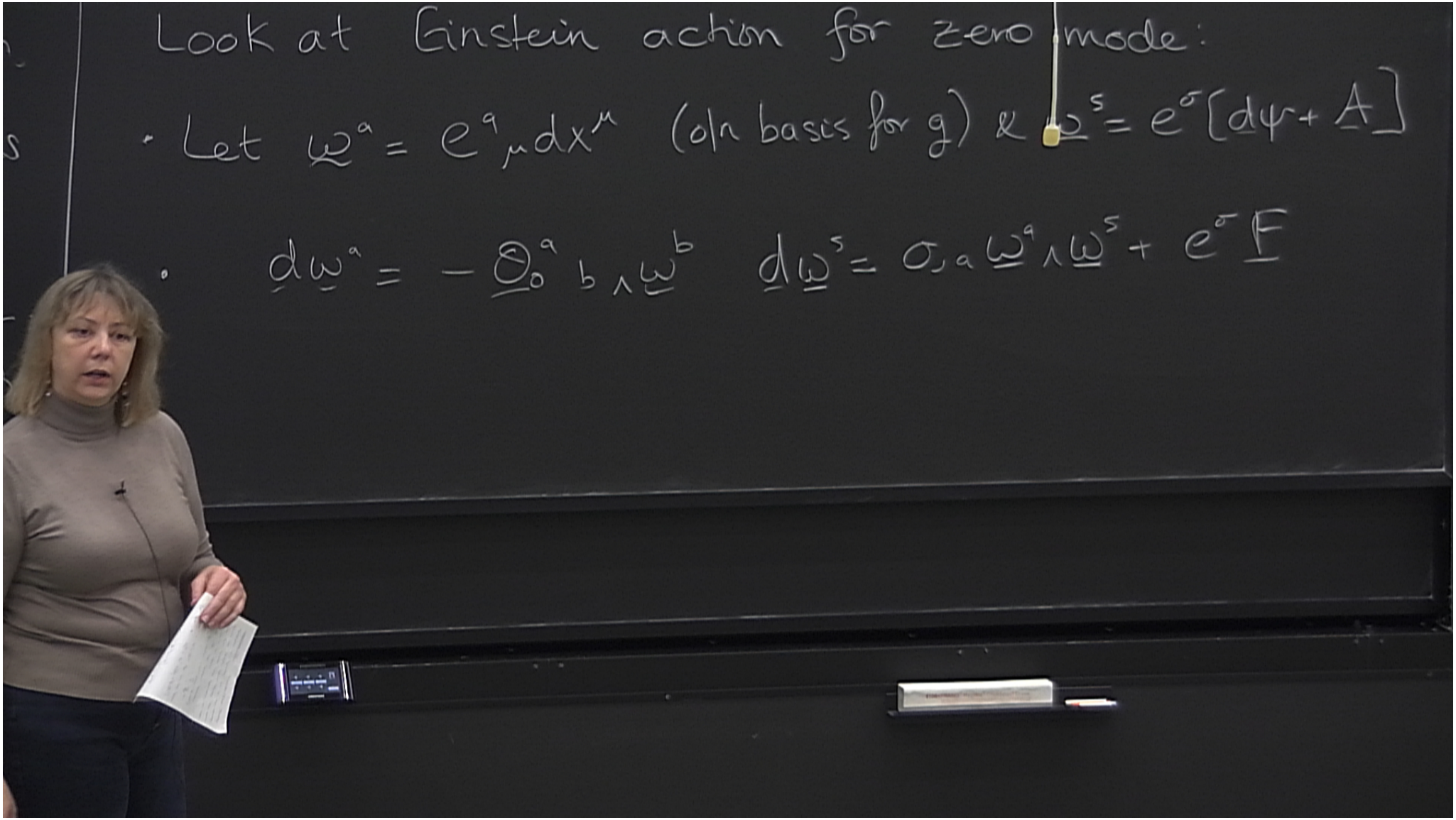

Kaluza-Klein
Extracts 4D gravity as a low energy limit of 5D (or higher) theory
$ds^2 = g_{\mu\nu}dx^\mu dx^\nu - e^{2\sigma}[d\psi + A_\mu dx^\mu]^2$
$\mu = 0,1,2,3$
5^{th} dim.


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5^{th} dim.
$\sim g_{5\mu}$


Take ψ to be compact, then
decompose
as a Fourier series
in ψ
modes


Take ψ to be compact, then
decompose metric as a Fourier series
in ψ ($e^{2\pi i \psi/L}$) Can interpret modes
terms for the fields $m \sim 1/L$

Take φ to be compact, then decompose metric as a Fourier series in φ ($e^{2\pi i \varphi/L}$) Can interpret modes as mass terms for the fields $m \sim 1/L$. The zero mode (no φ-dep) dominates


Look at Einstein action for zero mode:
• Let $\underline{\omega}^a = e^a{}_\mu dx^\mu$ (o/n basis for g) & $\underline{\omega}^5 = e^\sigma[d\psi + \underline{A}]$
• $d\underline{\omega}^a = -\underline{\theta}^a{}_b \wedge \underline{\omega}^b$ $d\underline{\omega}^5 = \sigma_{,a}\underline{\omega}^a \wedge \underline{\omega}^5 + e^\sigma \underline{F}$

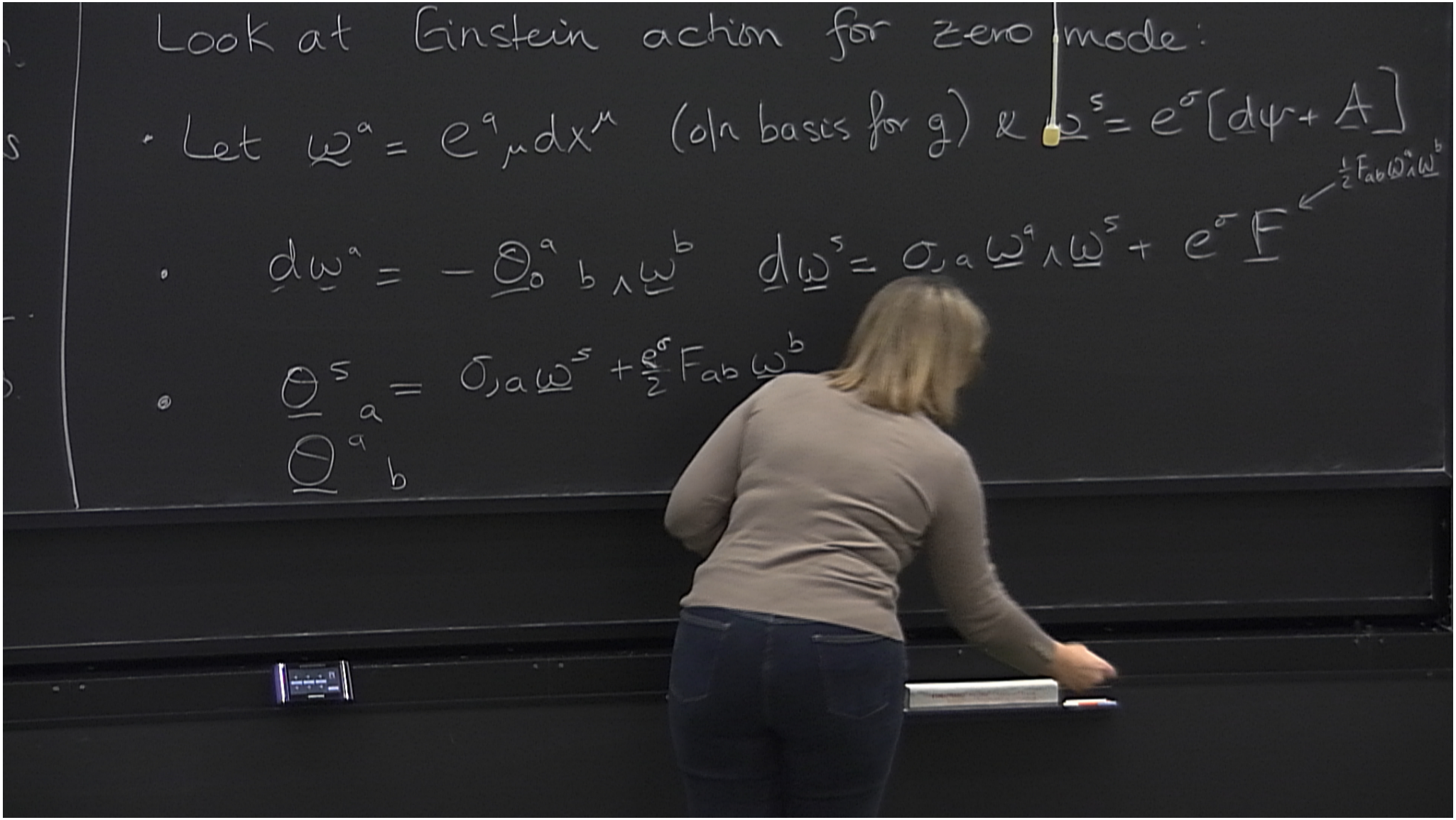
Look at Einstein action for zero mode:

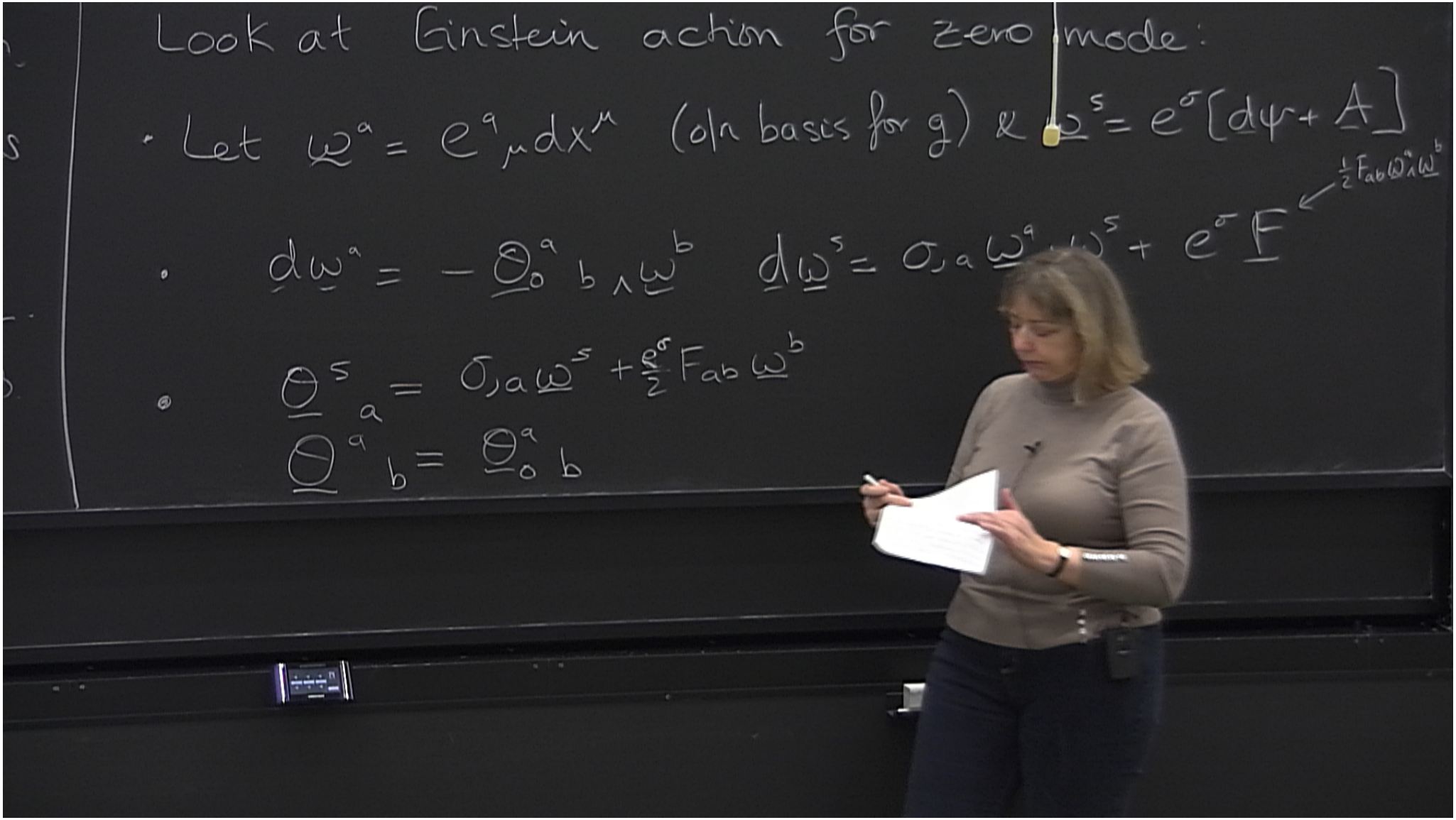
- Let $\underline{\omega}^a = e^a{}_\mu dx^\mu$ (o/n basis for g) & $\underline{\omega}^5 = e^\sigma [d\psi + \underline{A}]$

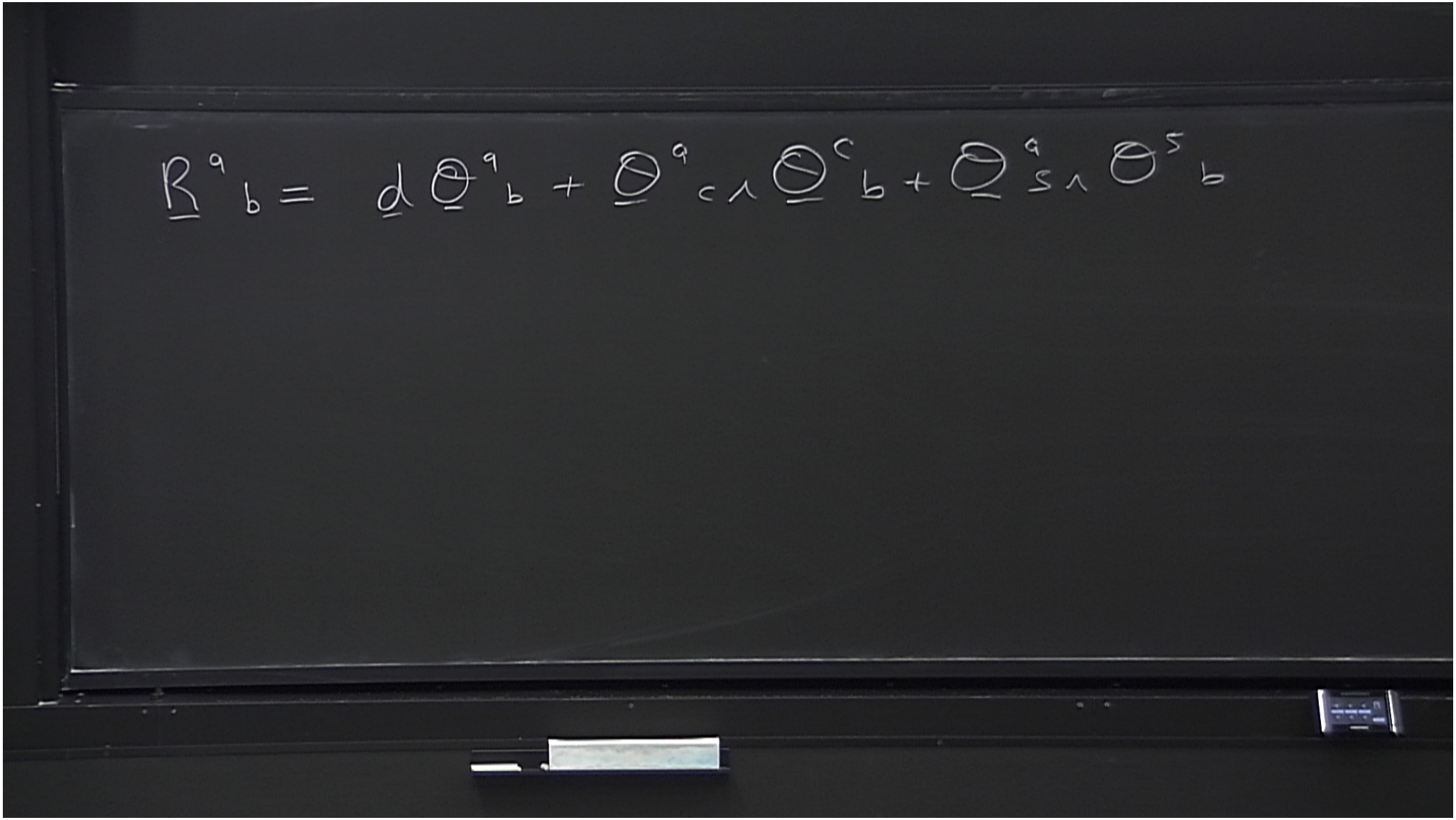
- $\underline{d\omega}^a = -\underline{\theta}^a{}_b \wedge \underline{\omega}^b \qquad \underline{d\omega}^5 = \sigma_{,a}\, \underline{\omega}^a \wedge \underline{\omega}^5 + e^\sigma \underline{F}$ ← $\frac{1}{2} F_{ab}$

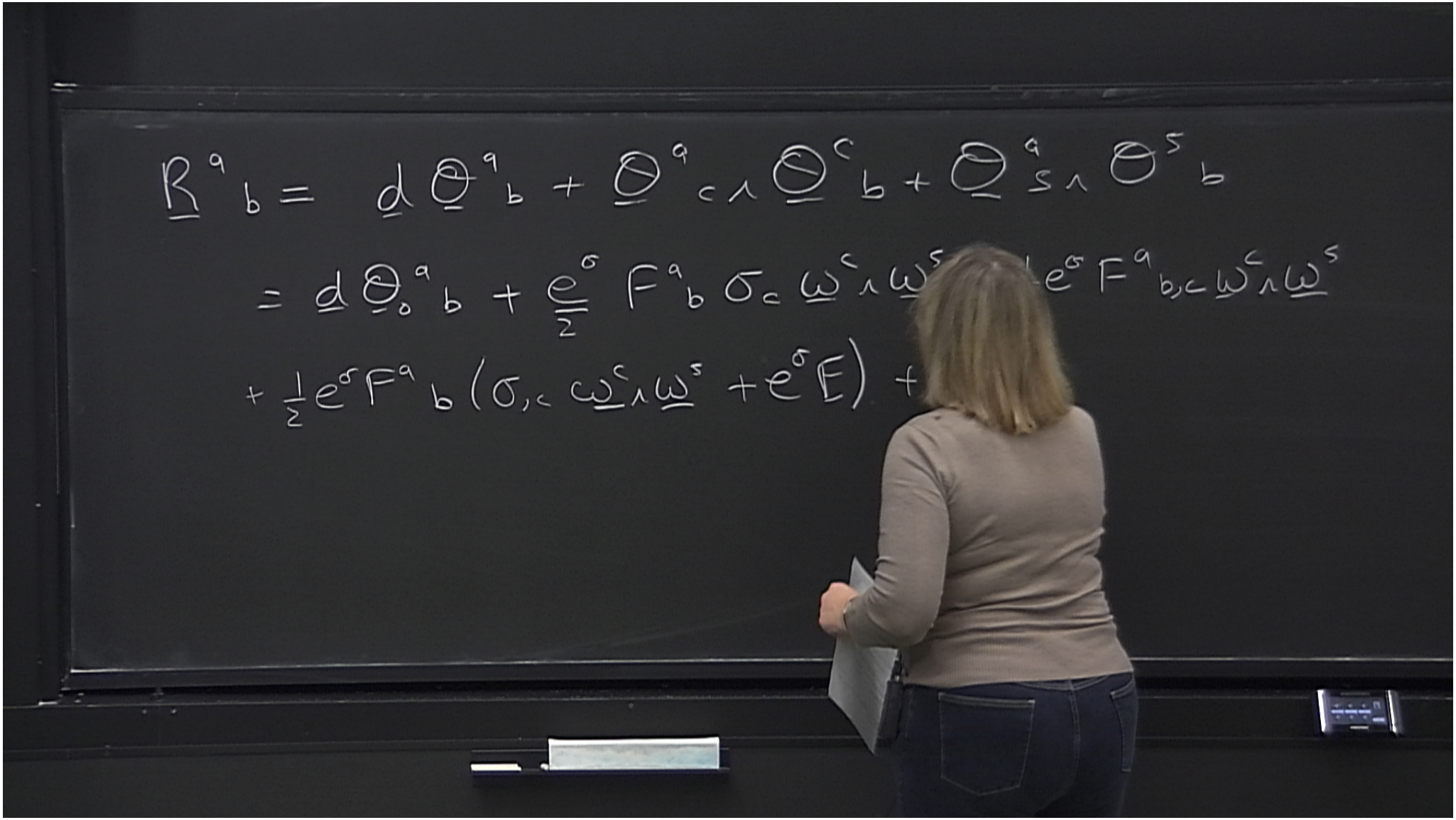
- $\underline{\theta}^5{}_a = \sigma_{,a} \underline{\omega}^5$

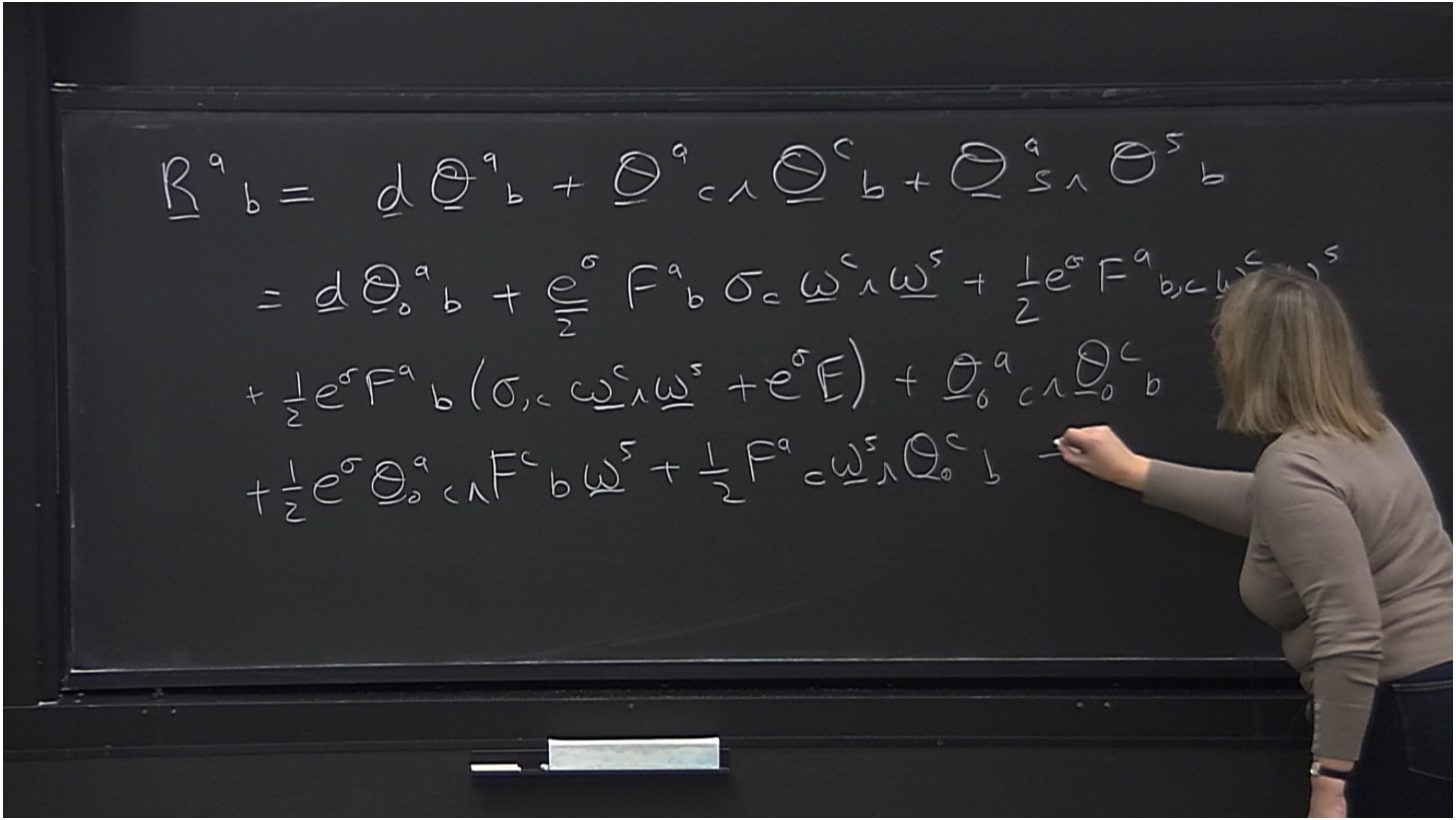
$\underline{\theta}^a{}_b$

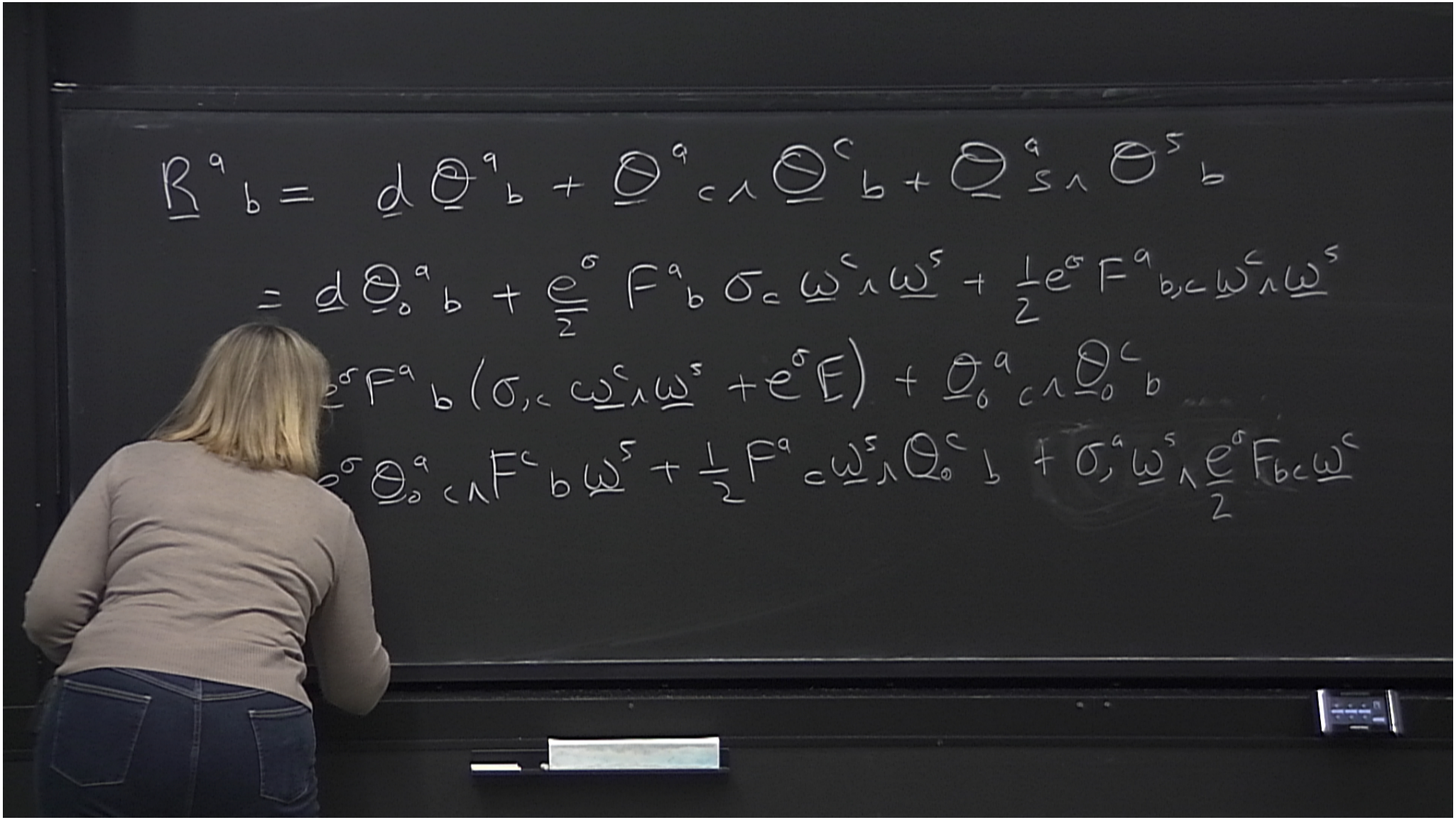

Look at Einstein action for zero mode:
• Let $\underline{\omega}^a = e^a{}_\mu dx^\mu$ (o/n basis for g) & $\underline{\omega}^5 = e^\sigma[d\psi + \underline{A}]$
$\frac{1}{2}F_{ab}\underline{\omega}^a\wedge\underline{\omega}^b$
• $d\underline{\omega}^a = -\underline{\theta}^a{}_b \wedge \underline{\omega}^b$ $d\underline{\omega}^5 = \sigma_{,a}\underline{\omega}^a\wedge\underline{\omega}^5 + e^\sigma \underline{F}$
• $\underline{\theta}^5{}_a = \sigma_{,a}\underline{\omega}^5 + \frac{e^\sigma}{2}F_{ab}\underline{\omega}^b$
$\underline{\theta}^a{}_b$


Look at Einstein action for zero mode:
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$\leftarrow \frac{1}{2}F_{ab}\omega^a \wedge \omega^b$
• $\underline{\theta}^5{}_a = \sigma_{,a}\underline{\omega}^5 + \frac{e^\sigma}{2}F_{ab}\underline{\omega}^b$
$\underline{\theta}^a{}_b = \underline{\theta}^a{}_{0\,b}$


$\underline{R}^{a}{}_{b} = \underline{d}\,\underline{\theta}^{a}{}_{b} + \underline{\theta}^{a}{}_{c} \wedge \underline{\theta}^{c}{}_{b} + \underline{\theta}^{a}{}_{s} \wedge \theta^{s}{}_{b}$


$\underline{R}^{a}{}_{b} = \underline{d}\,\underline{\Theta}^{a}{}_{b} + \underline{\Theta}^{a}{}_{c} \wedge \underline{\Theta}^{c}{}_{b} + \underline{\Theta}^{a}{}_{s} \wedge \underline{\Theta}^{s}{}_{b}$
$= \underline{d}\,\underline{\Theta}_{0}{}^{a}{}_{b} + \frac{e^{\sigma}}{2} F^{a}{}_{b} \sigma_{c} \underline{\omega}^{c} \wedge \underline{\omega}^{s}$
$+ \frac{1}{2} e^{\sigma} F^{a}{}_{b} (\sigma_{,c} \underline{\omega}^{c} \wedge \underline{\omega}^{s} + e^{\sigma} \underline{E})$


$\underline{R}^a{}_b = \underline{d}\underline{\Theta}^a{}_b + \underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b + \underline{\Theta}^a{}_s \wedge \underline{\Theta}^s{}_b$
$= \underline{d}\underline{\Theta}_0{}^a{}_b + \frac{\underline{e}^\sigma}{2} F^a{}_b \sigma_c \underline{\omega}^c \wedge \underline{\omega}^s + \frac{1}{2} e^\sigma F^a{}_{b,c} \underline{\omega}^c \wedge \underline{\omega}^s$
$+ \frac{1}{2} e^\sigma F^a{}_b (\sigma_{,c} \underline{\omega}^c \wedge \underline{\omega}^s + e^\sigma \underline{E}) + \underline{\Theta}_0{}^a{}_c \wedge \underline{\Theta}_0{}^c{}_b$
$+ \frac{1}{2} e^\sigma \underline{\Theta}_0{}^a{}_c \wedge F^c{}_b \underline{\omega}^s + \frac{1}{2} F^a{}_c \underline{\omega}^s \wedge \underline{\Theta}_0{}^c{}_b$


$\underline{R}^a{}_b = \underline{d}\underline{\Theta}^a{}_b + \underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b + \underline{\Theta}^a{}_s \wedge \underline{\Theta}^s{}_b$
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$e^\sigma F^a{}_b (\sigma_{,c} \underline{\omega}^c \wedge \underline{\omega}^s + e^\sigma \underline{E}) + \underline{\Theta}_0{}^a{}_c \wedge \underline{\Theta}_0{}^c{}_b$
$e^\sigma \underline{\Theta}_0{}^a{}_c \wedge F^c{}_b \underline{\omega}^s + \frac{1}{2} F^a{}_c \underline{\omega}^s \wedge \underline{\Theta}_0{}^c{}_b + \sigma_{,a} \underline{\omega}^s \wedge \frac{\underline{e}^\sigma}{2} F_{bc} \underline{\omega}^c$

$$\underline{R}^{a}{}_{b} = \underline{d}\,\underline{\Theta}^{a}{}_{b} + \underline{\Theta}^{a}{}_{c} \wedge \underline{\Theta}^{c}{}_{b} + \underline{\Theta}^{a}{}_{s} \wedge \underline{\Theta}^{s}{}_{b}$$

$$= \underline{d}\,\underline{\Theta}_{0}{}^{a}{}_{b} + \frac{e^{\sigma}}{2} F^{a}{}_{b}\sigma_{c}\,\underline{\omega}^{c} \wedge \underline{\omega}^{s} + \frac{1}{2} e^{\sigma} F^{a}{}_{b,c}\,\underline{\omega}^{c} \wedge \underline{\omega}^{s}$$

$$+ \frac{1}{2} e^{\sigma} F^{a}{}_{b} (\sigma_{,c}\,\underline{\omega}^{c} \wedge \underline{\omega}^{s} + e^{\sigma}\underline{F}) + \underline{\Theta}_{0}{}^{a}{}_{c} \wedge \underline{\Theta}_{0}{}^{c}{}_{b}$$

$$+ \frac{1}{2} e^{\sigma}\,\underline{\Theta}_{0}{}^{a}{}_{c} \wedge F^{c}{}_{b}\,\underline{\omega}^{s} + \frac{1}{2} F^{a}{}_{c}\,\underline{\omega}^{s} \wedge \underline{\Theta}_{0}{}^{c}{}_{b} + \sigma_{,}{}^{a}\,\underline{\omega}^{s} \wedge \frac{e^{\sigma}}{2} F_{bc}\,\underline{\omega}^{c}$$

$$+ \frac{e^{\sigma}}{2} F^{a}{}_{c}\,\underline{\omega}^{c} \wedge \sigma_{,b}\,\underline{\omega}^{r} + \frac{e^{2\sigma}}{4} F^{a}{}_{c}\,\underline{\omega}^{c} \wedge F_{bd}\,\underline{\omega}^{d}$$

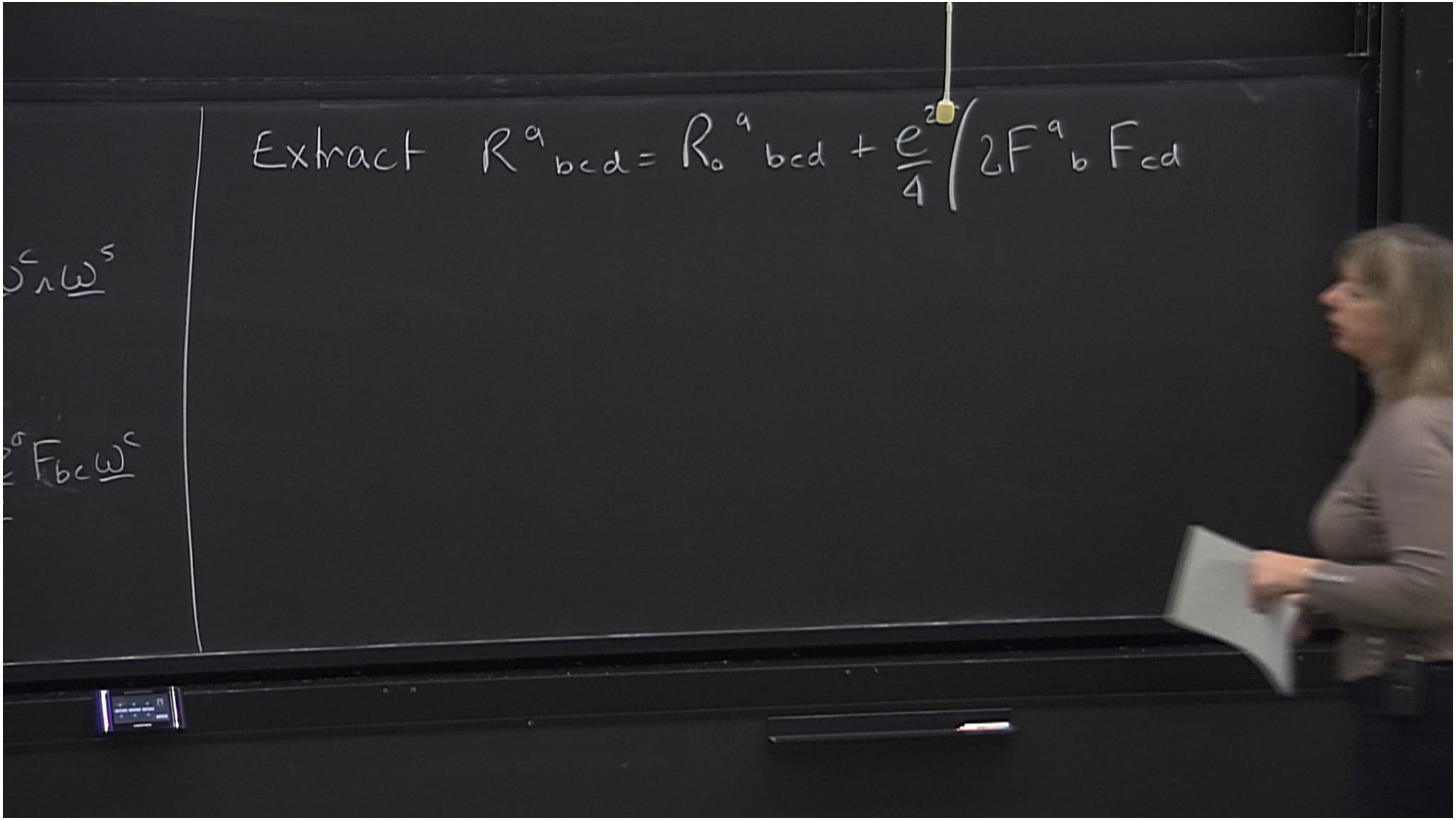
$$\underline{R}^{a}{}_{b} = \underline{d}\,\underline{\Theta}^{a}{}_{b} + \underline{\Theta}^{a}{}_{c} \wedge \underline{\Theta}^{c}{}_{b} + \underline{\Theta}^{a}{}_{s} \wedge \underline{\Theta}^{s}{}_{b}$$

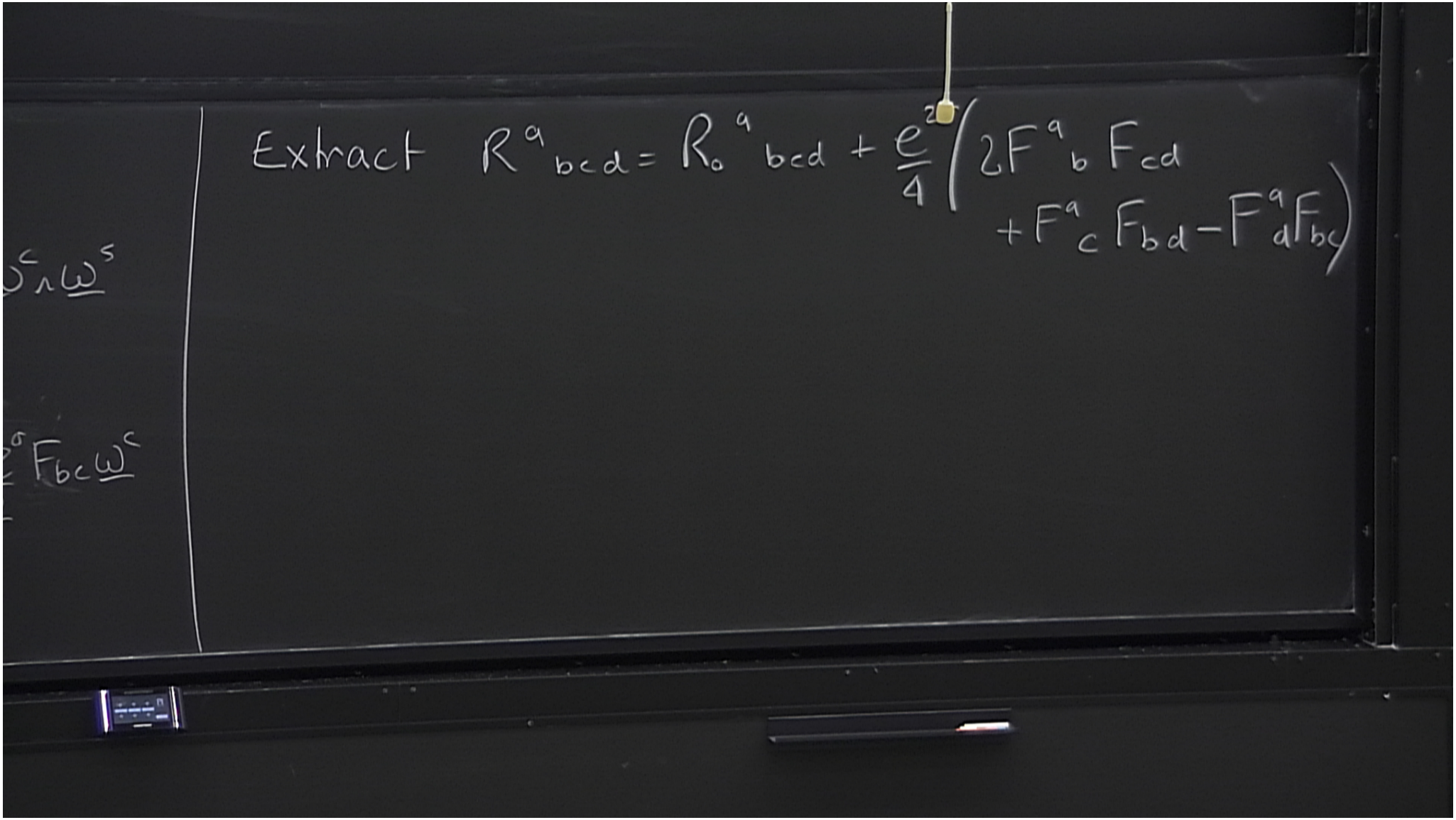
$$= \underline{d\,\Theta_0{}^{a}{}_{b}} + \frac{\underline{e}^{\sigma}}{2} F^{a}{}_{b}\,\sigma_{c}\,\underline{\omega}^{c} \wedge \underline{\omega}^{s} + \frac{1}{2} e^{\sigma} F^{a}{}_{b,c}\,\underline{\omega}^{c} \wedge \underline{\omega}^{s}$$

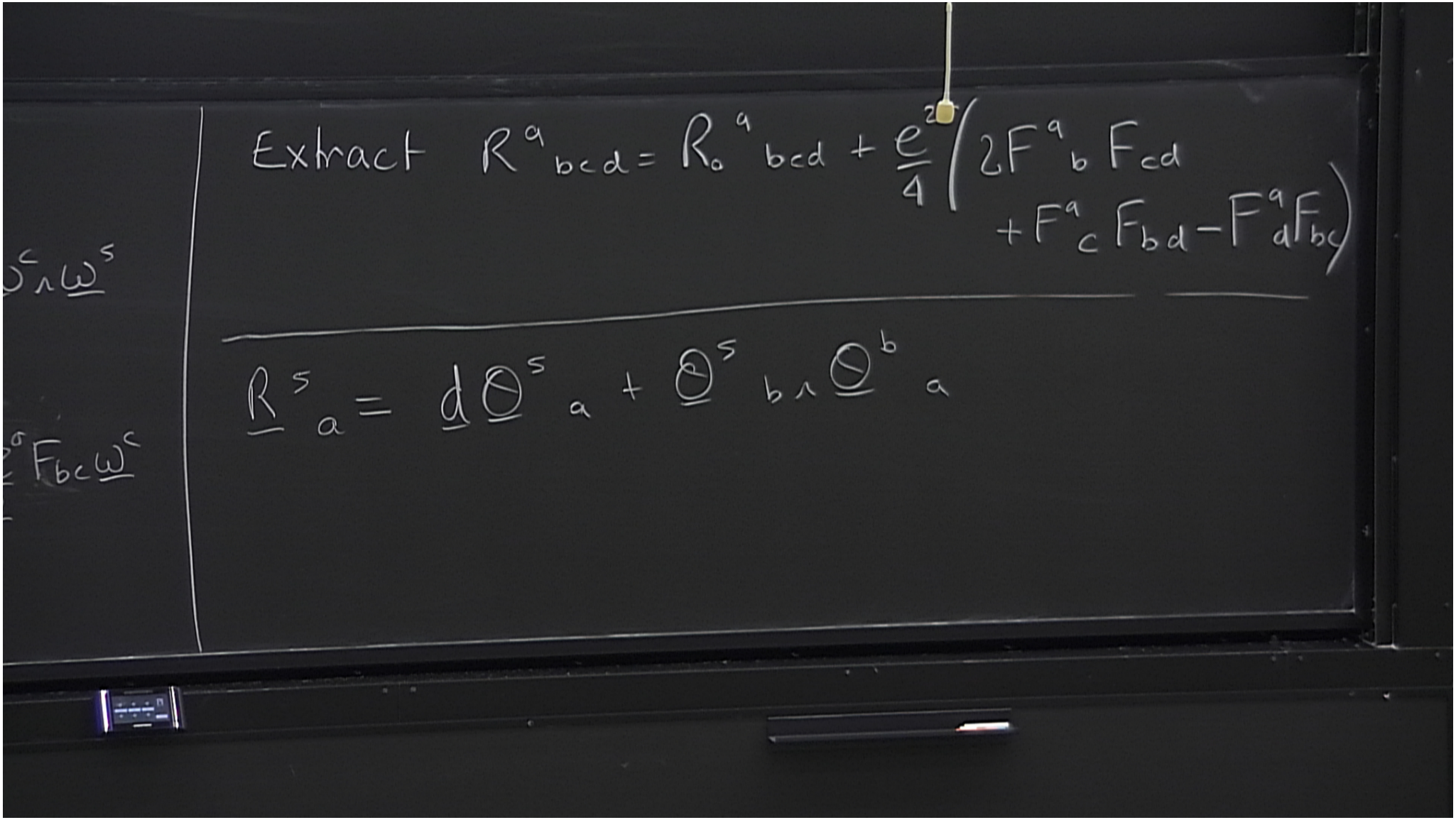
$$+ \frac{1}{2} e^{\sigma} F^{a}{}_{b} \left(\sigma_{,c}\,\underline{\omega}^{c} \wedge \underline{\omega}^{s} + e^{\sigma}\underline{F}\right) + \underline{\Theta_0{}^{a}{}_{c} \wedge \Theta_0{}^{c}{}_{b}}$$

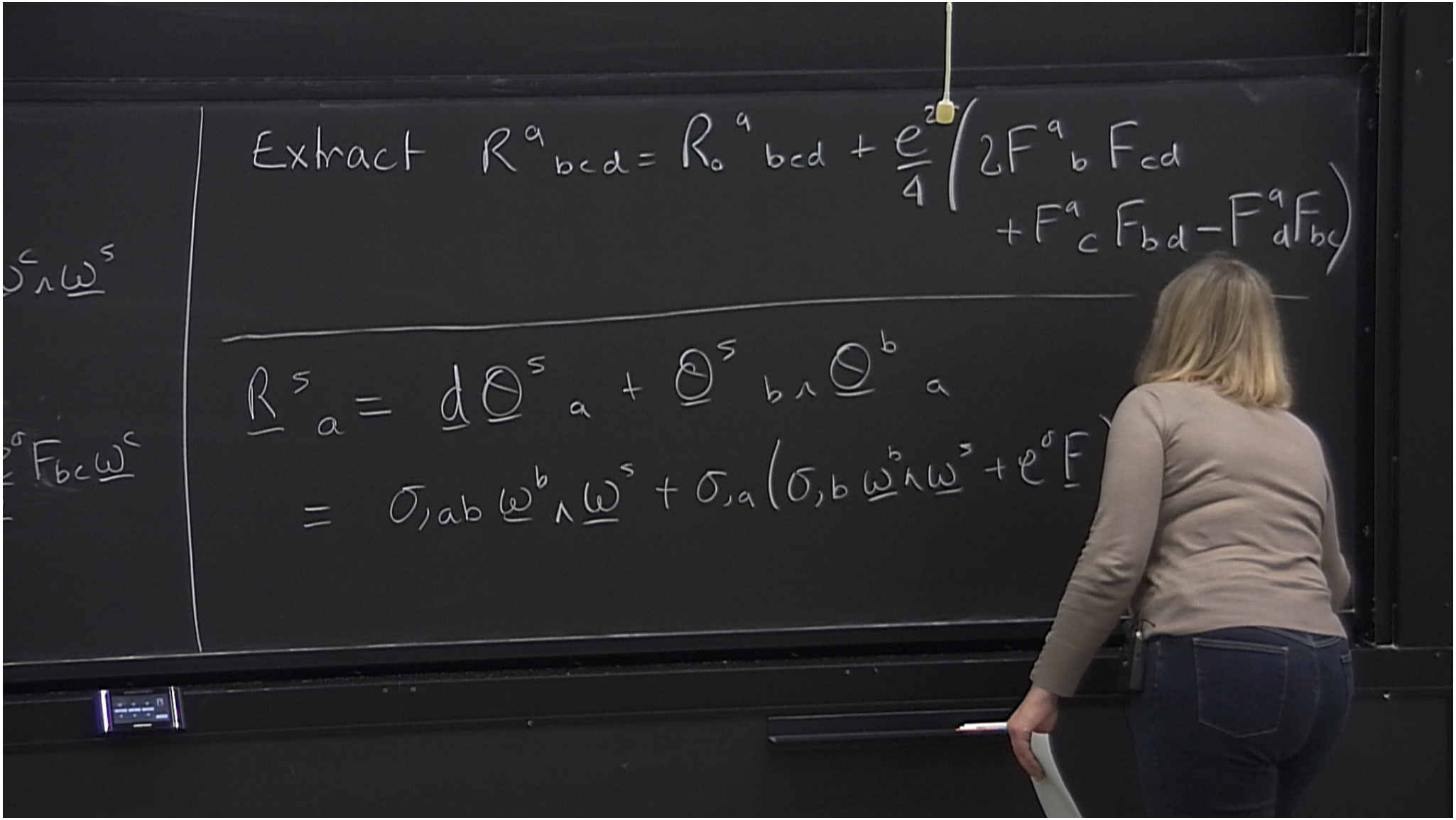
$$+ \frac{1}{2} e^{\sigma}\,\underline{\Theta_0{}^{a}{}_{c}} \wedge F^{c}{}_{b}\,\underline{\omega}^{s} + \frac{1}{2} F^{a}{}_{c}\,\underline{\omega}^{s} \wedge \Theta_0{}^{c}{}_{b} + \sigma_{,}{}^{a}\,\underline{\omega}^{s} \wedge \frac{\underline{e}^{\sigma}}{2} F_{bc}\,\underline{\omega}^{c}$$

$$+ \frac{\underline{e}^{\sigma}}{2} F^{a}{}_{c}\,\underline{\omega}^{c} \wedge \sigma_{,b}\,\underline{\omega}^{r} + \frac{e^{2\sigma}}{4}\,\underline{F^{a}{}_{c}\,\underline{\omega}^{c} \wedge F_{bd}\,\underline{\omega}^{d}}$$


Extract $R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{e^2}{4}(2F^a{}_b F_{cd}$


Extract $R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{e^2}{4}(2F^a{}_b F_{cd} + F^a{}_c F_{bd} - F^a{}_d F_{bc})$
$\omega^c \wedge \omega^s$
$F_{bc}\omega^c$

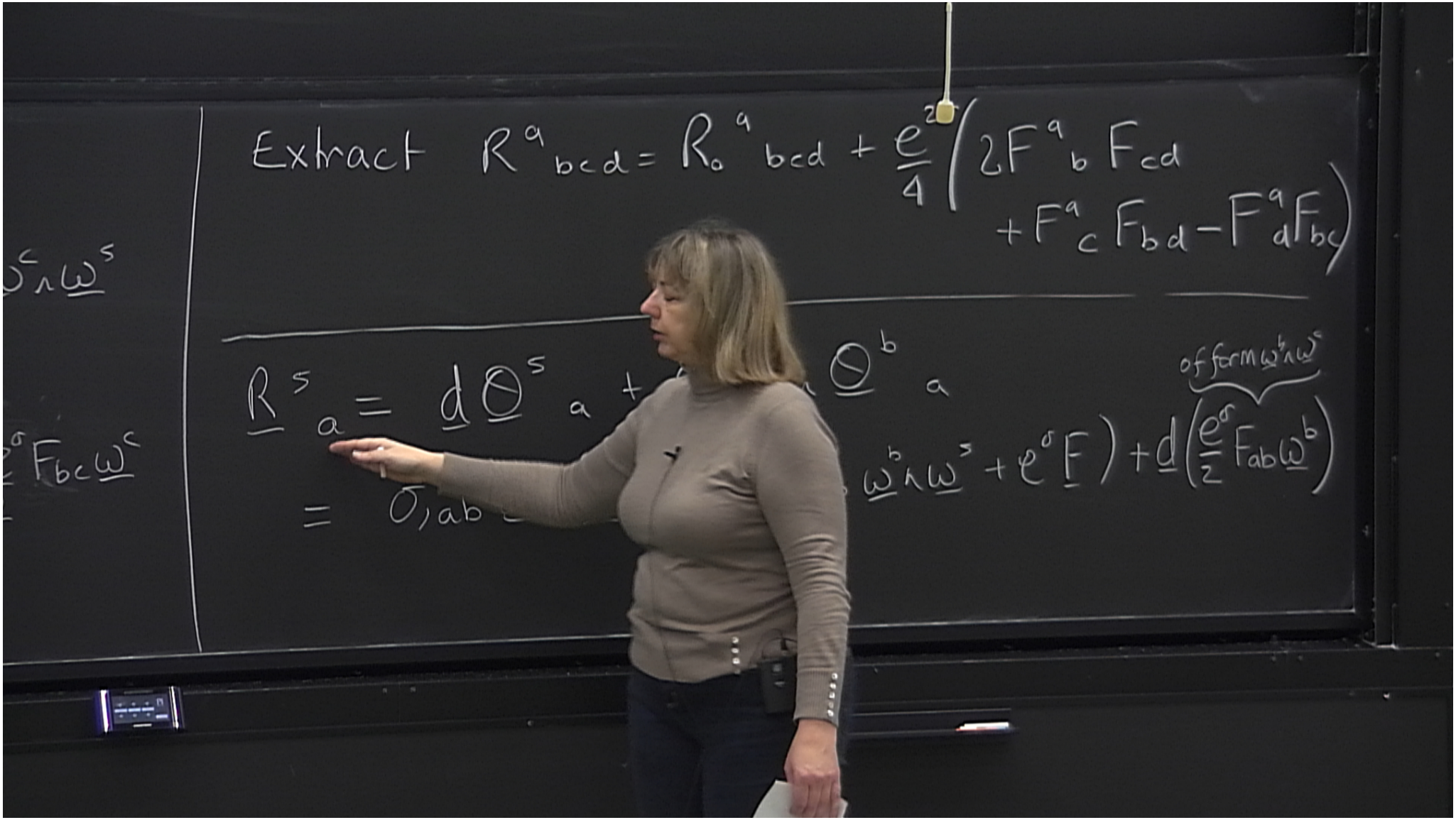

Extract $R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{e^2}{4}(2F^a{}_b F_{cd} + F^a{}_c F_{bd} - F^a{}_d F_{bc})$
$\underline{R}^5{}_a = \underline{d}\underline{\Theta}^5{}_a + \underline{\Theta}^5{}_b \wedge \underline{\Theta}^b{}_a$

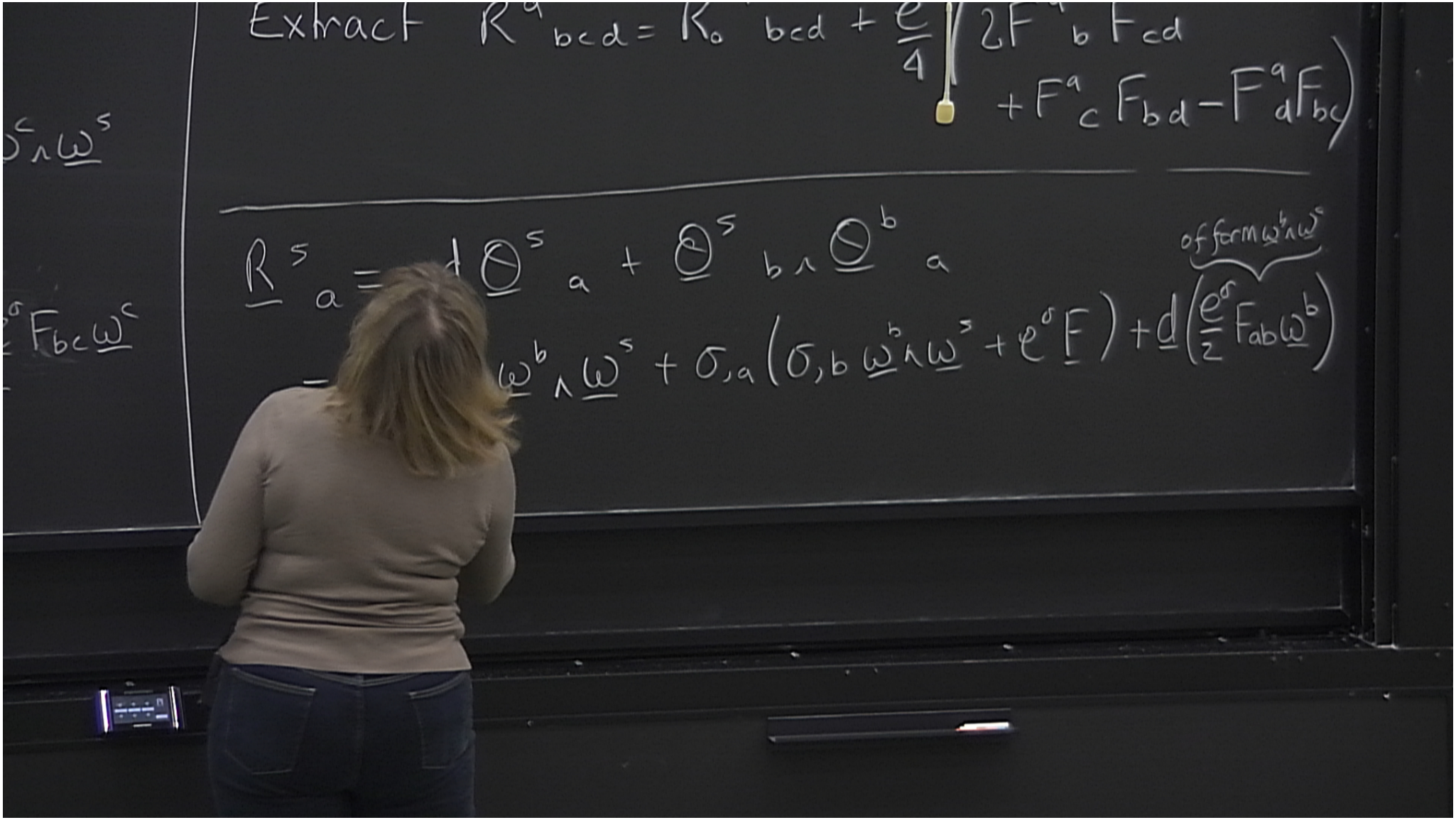

Extract $R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{e^2}{4}(2F^a{}_b F_{cd} + F^a{}_c F_{bd} - F^a{}_d F_{bc})$
$\underline{R}^s{}_a = \underline{d}\underline{\Theta}^s{}_a + \underline{\Theta}^s{}_b \wedge \underline{\Theta}^b{}_a$
$= \sigma_{,ab}\,\underline{\omega}^b \wedge \underline{\omega}^s + \sigma_{,a}(\sigma_{,b}\,\underline{\omega}^b \wedge \underline{\omega}^s + e^\sigma \underline{F}$

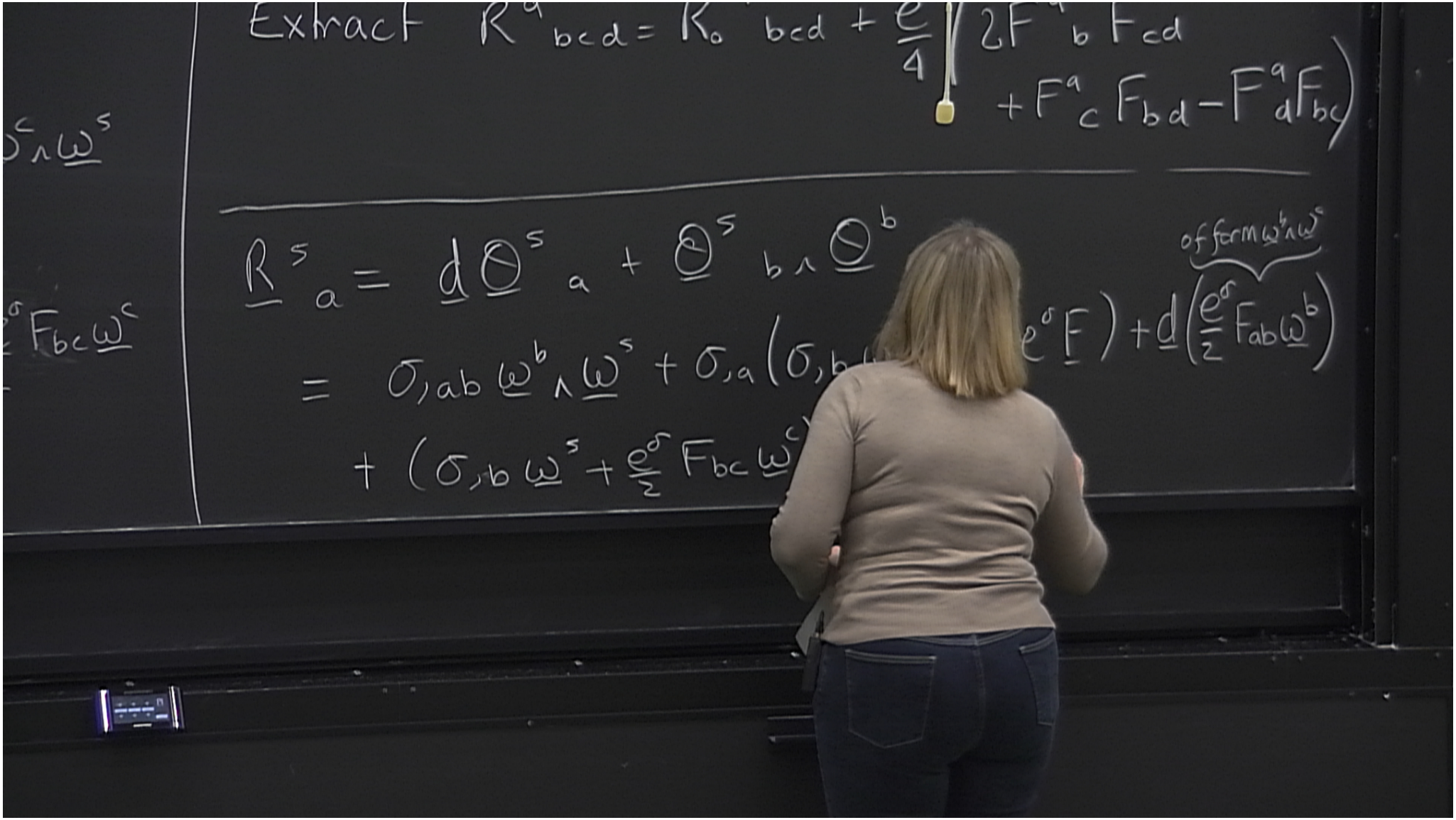
Extract $R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{e^2}{4}\left(2F^a{}_b F_{cd} + F^a{}_c F_{bd} - F^a{}_d F_{bc}\right)$

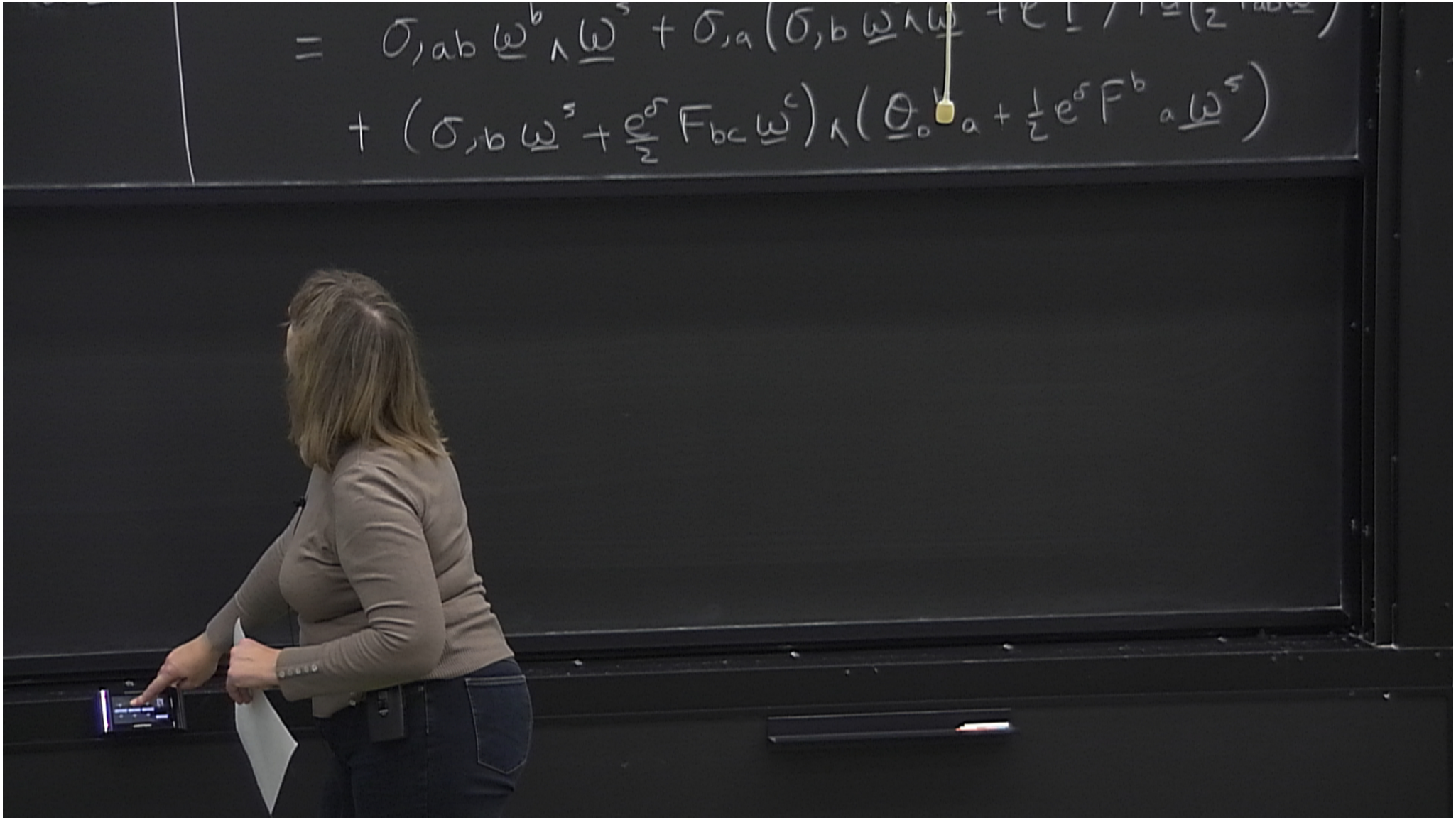
$\underline{R}^s{}_a = \underline{d}\underline{\Theta}^s{}_a + \underline{\Theta}^s{}_b \wedge \underline{\Theta}^b{}_a$

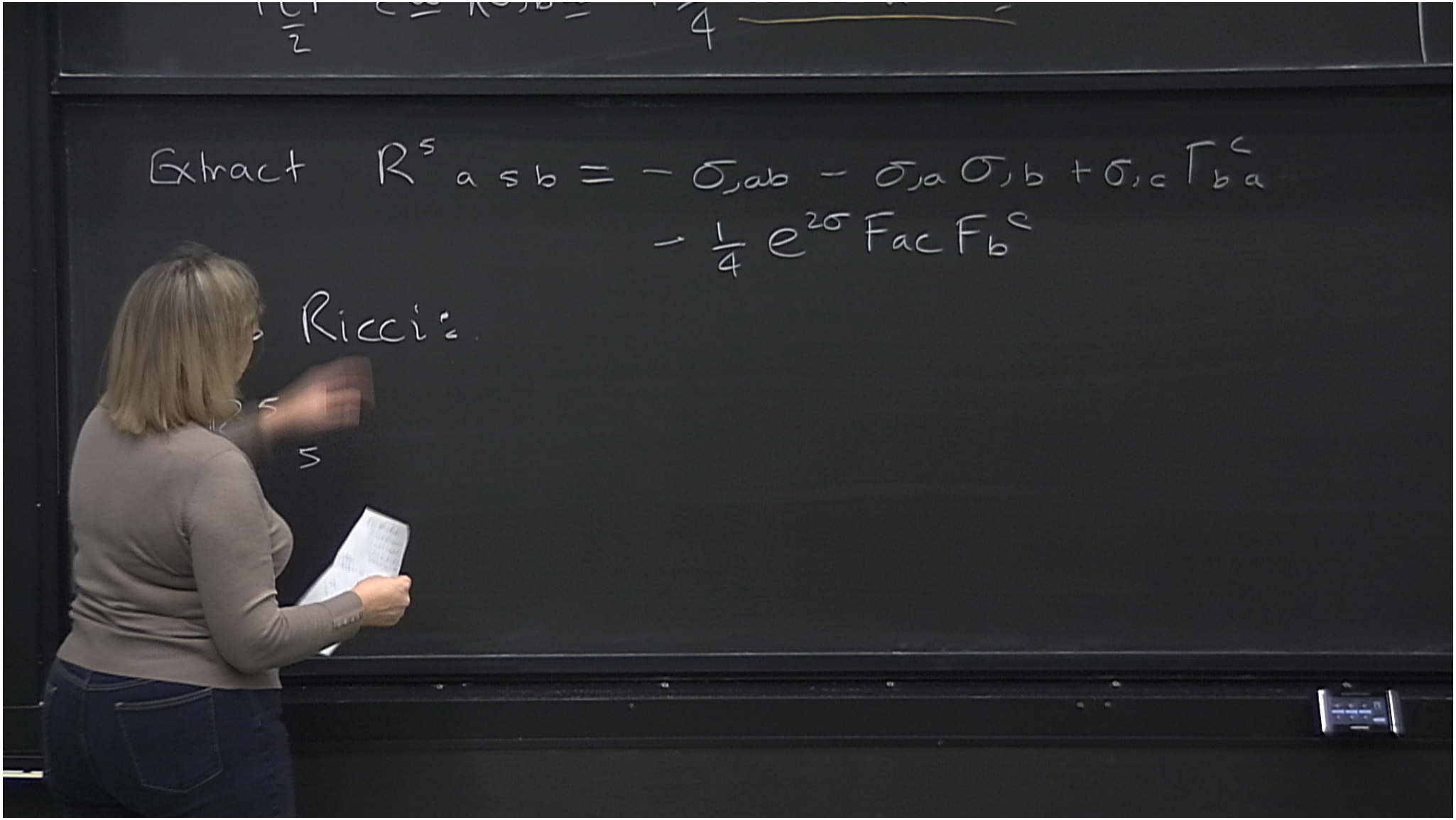
$= \sigma_{,ab}\,\underline{\omega}^b \wedge \underline{\omega}^s + \sigma_{,a}\left(\sigma_{,b}\,\underline{\omega}^b \wedge \underline{\omega}^s + e^\sigma \underline{F}\right) + \underline{d}\left(\frac{e^\sigma}{2} F_{ab}\underline{\omega}^b\right)$


Extract $R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{e^2}{4}\left(2F^a{}_b F_{cd} + F^a{}_c F_{bd} - F^a{}_d F_{bc}\right)$
$\underline{R}^5{}_a = \underline{d}\underline{\Theta}^5{}_a + \ldots \underline{\Theta}^b{}_a$
$= \ldots \underline{\omega}^b \wedge \underline{\omega}^5 + e^\sigma \underline{F}) + \underline{d}\left(\frac{e^\sigma}{2} F_{ab}\underline{\omega}^b\right)$
of form $\underline{\omega}^b \wedge \underline{\omega}^5$


Extract
of form


Extract
$R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{e}{4}(2F^a{}_b F_{cd} + F^a{}_c F_{bd} - F^a{}_d F_{bc})$
$\underline{R}^s{}_a = \underline{d}\underline{\Theta}^s{}_a + \underline{\Theta}^s{}_b \wedge \underline{\Theta}^b$
$= \sigma_{,ab}\underline{\omega}^b \wedge \underline{\omega}^s + \sigma_{,a}(\sigma_{,b}$
$+ (\sigma_{,b}\underline{\omega}^s + \frac{e^\sigma}{2} F_{bc}\underline{\omega}^c$
$e^\sigma \underline{F}) + \underline{d}\left(\frac{e^\sigma}{2} F_{ab}\underline{\omega}^b\right)$
of form $\omega^b \wedge \omega^s$


$= \sigma_{,ab}\,\underline{\omega}^b \wedge \underline{\omega}^a + \sigma_{,a}(\sigma_{,b}\,\underline{\omega} \wedge \underline{\omega}$
$+ (\sigma_{,b}\,\underline{\omega}^b + \frac{e^\sigma}{2} F_{bc}\,\underline{\omega}^c) \wedge (\underline{\theta}_0{}^a{}_a + \frac{1}{2} e^\sigma F^b{}_a\,\underline{\omega}^a)$


Extract $R^s{}_{asb} = -\sigma_{;ab} - \sigma_{;a}\sigma_{;b} + \sigma_{;c}\Gamma^c_{ba}$
$-\frac{1}{4}e^{2\sigma}F_{ac}F_b{}^c$
Ricci:

$\frac{1}{2}$ $= \frac{1}{4}$

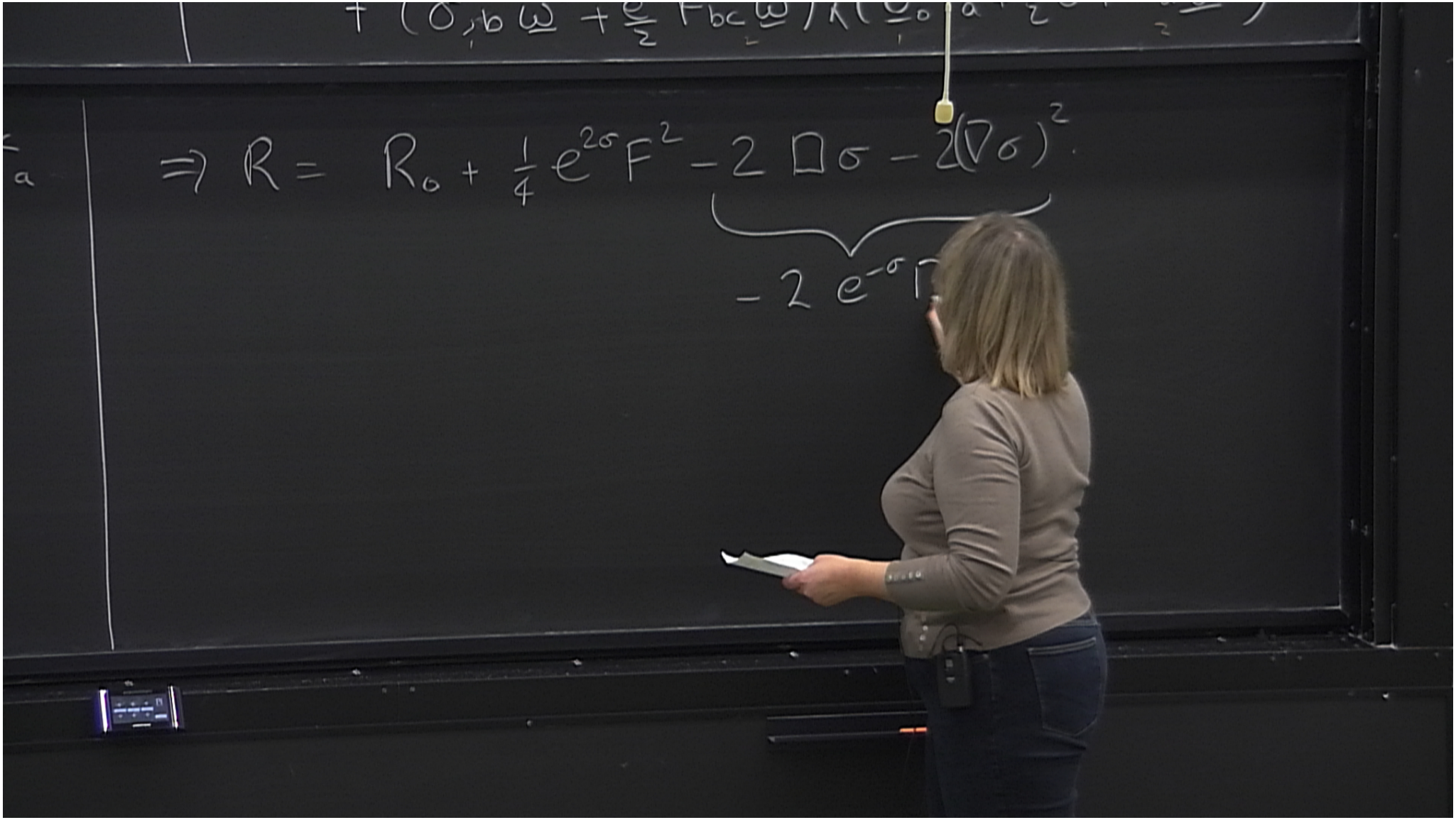
Extract $R^5{}_{a\,5\,b} = -\sigma_{,ab} - \sigma_{,a}\sigma_{,b} + \sigma_{,c}\Gamma^c_{ba}$ $\Rightarrow$

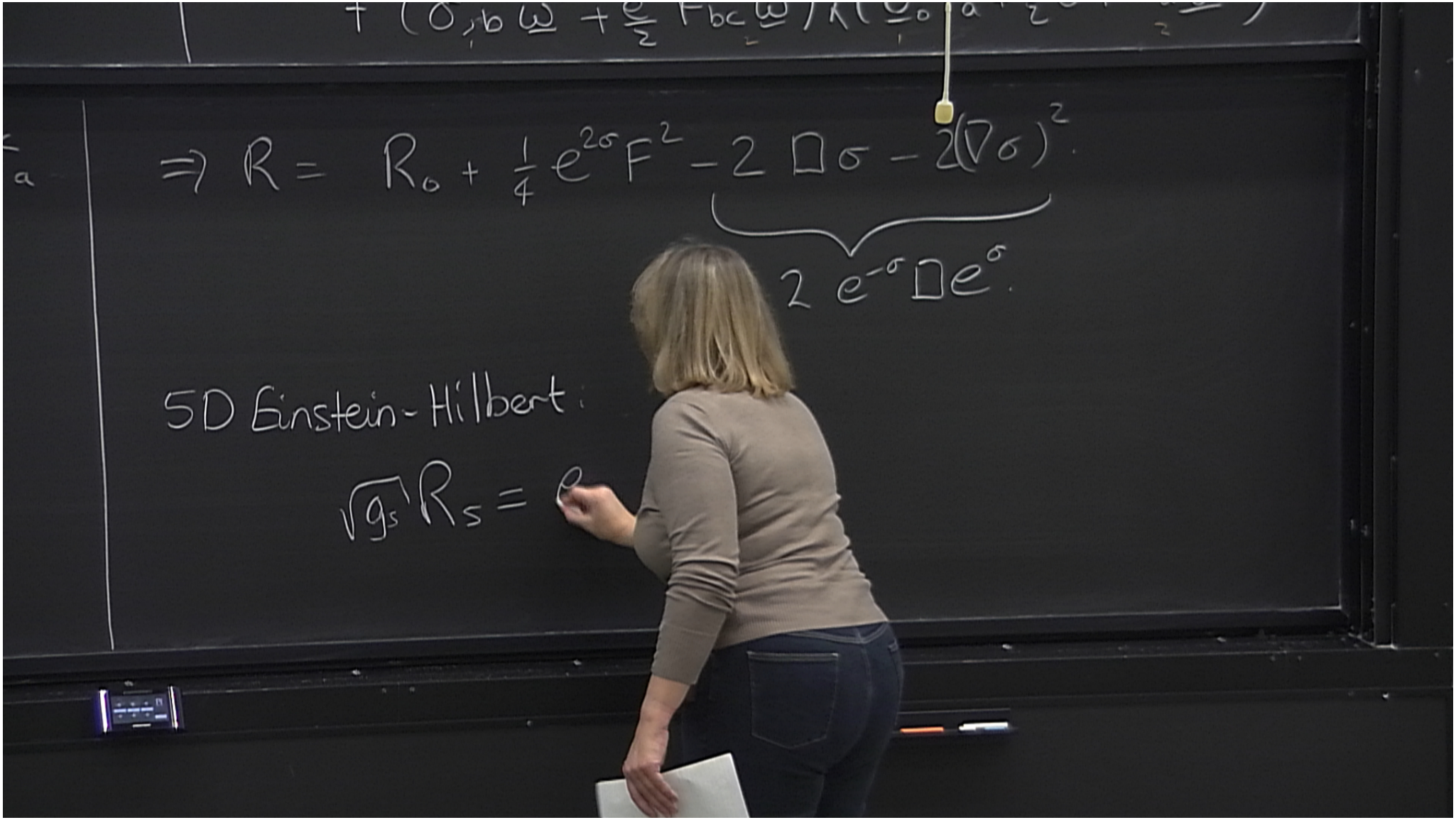
$$- \frac{1}{4} e^{2\sigma} F_{ac} F_b{}^c$$

Gives Ricci:

$$R^5{}_5 = -\Box\sigma - (\nabla\sigma)^2 - \frac{1}{4} e^{2\sigma} F^2$$

$$R_{ab} = R_{0ab} - \nabla_a\nabla_b\sigma - \sigma_{,a}\sigma_{,b} + \frac{e^{2\sigma}}{2} F_{ac} F_b{}^c$$

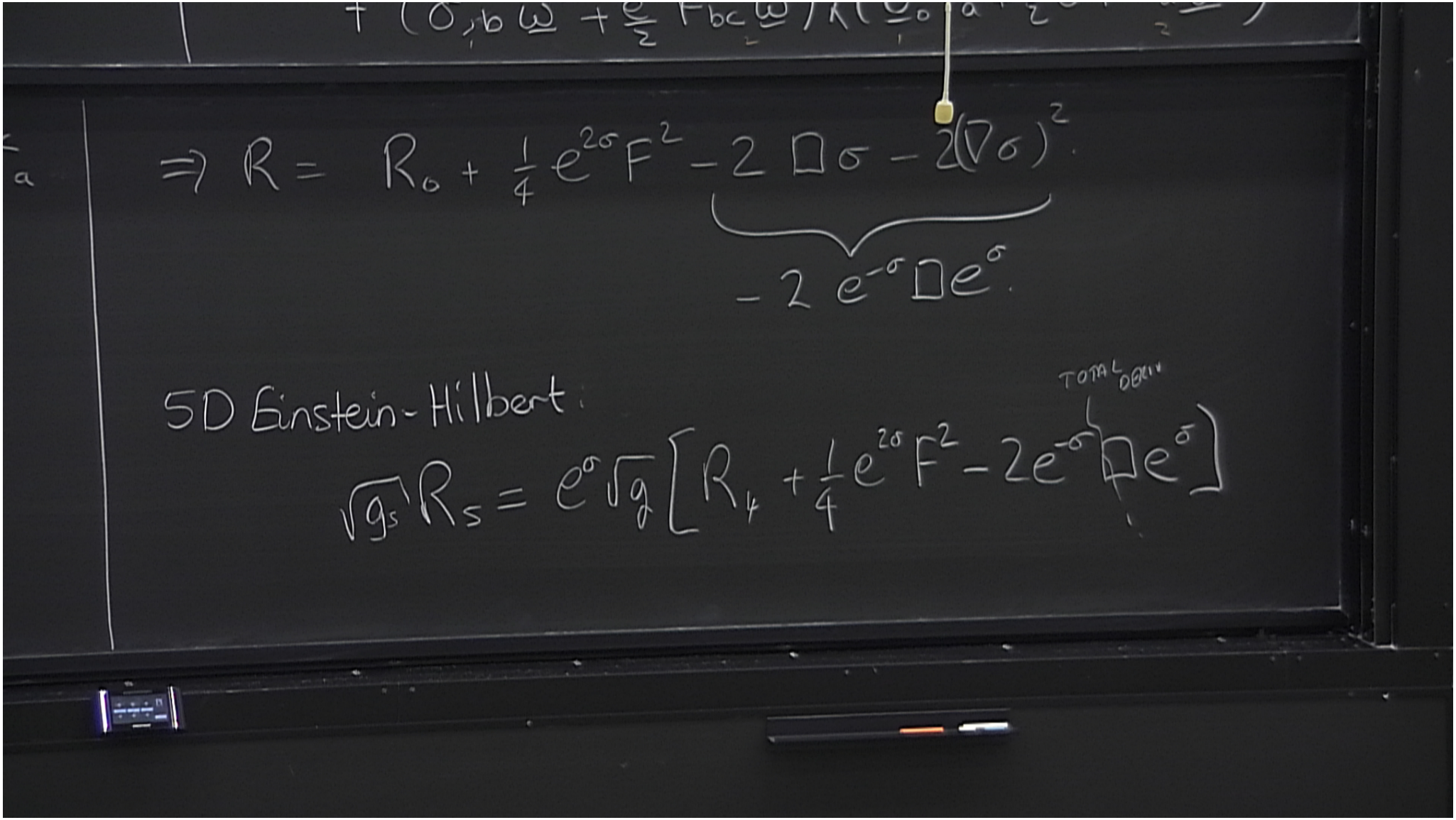

$\Rightarrow R = R_0 + \frac{1}{4} e^{2\sigma} F^2 - 2\Box\sigma - 2(\nabla\sigma)^2$
$- 2 e^{-\sigma}$

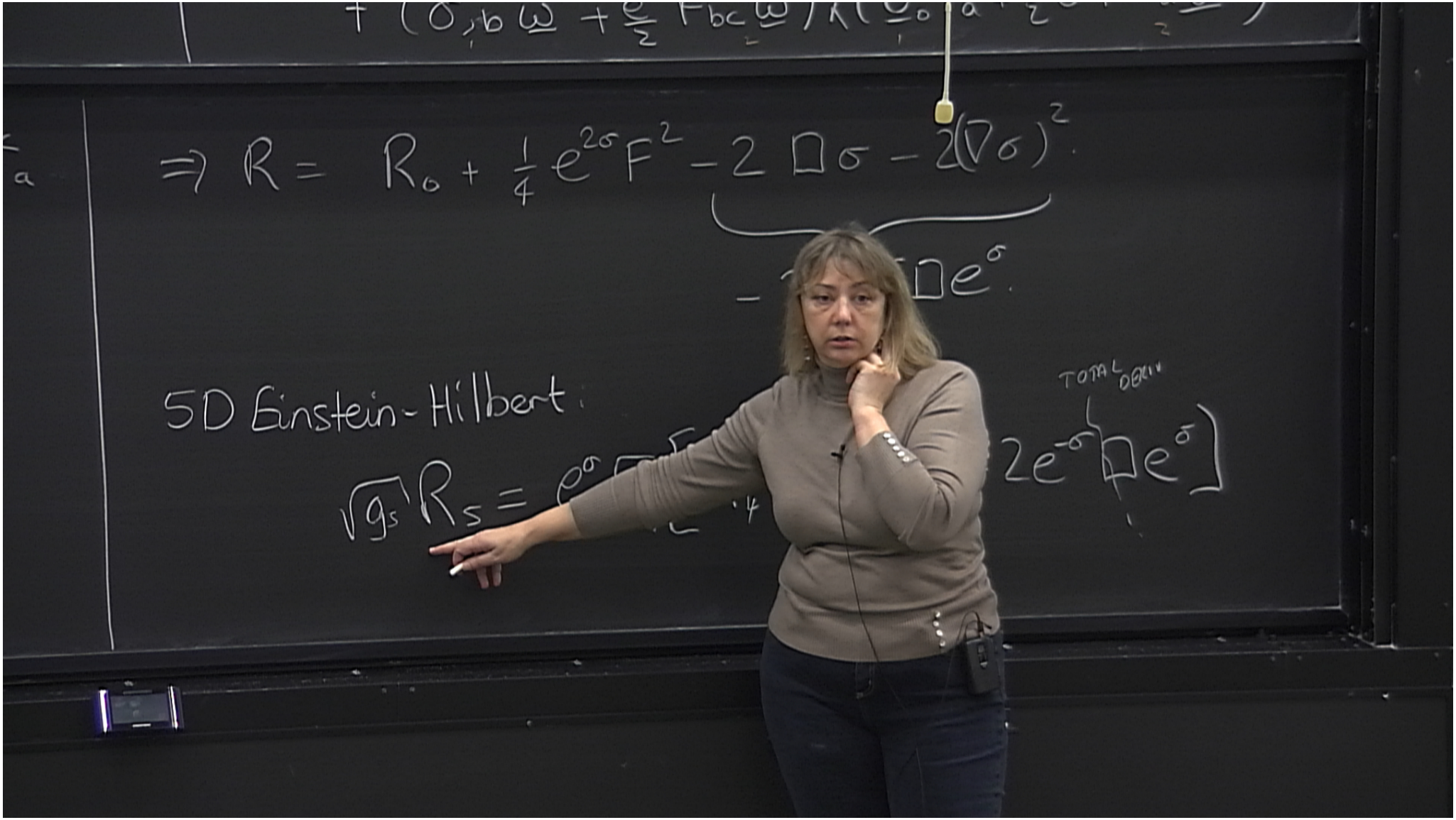

$\Rightarrow R = R_0 + \frac{1}{4} e^{2\sigma} F^2 - 2\,\Box\sigma - 2(\nabla\sigma)^2$
$2\, e^{-\sigma} \Box e^{\sigma}$
5D Einstein-Hilbert:
$\sqrt{g_5} R_5 =$

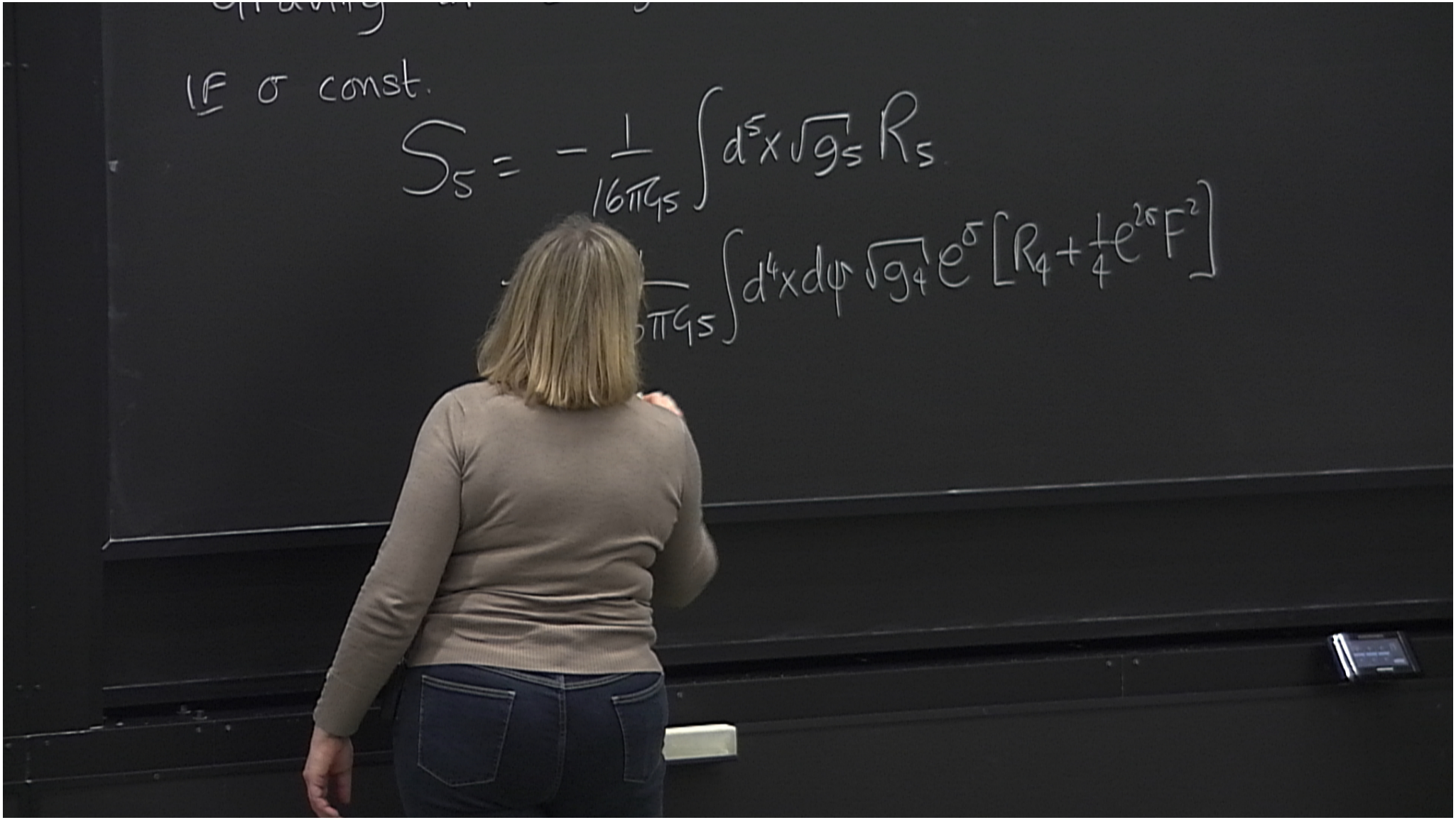
$$\Rightarrow R = R_0 + \frac{1}{4} e^{2\sigma} F^2 \underbrace{- 2\,\Box\sigma - 2(\nabla\sigma)^2}_{-2\, e^{-\sigma} \Box e^{\sigma}}$$

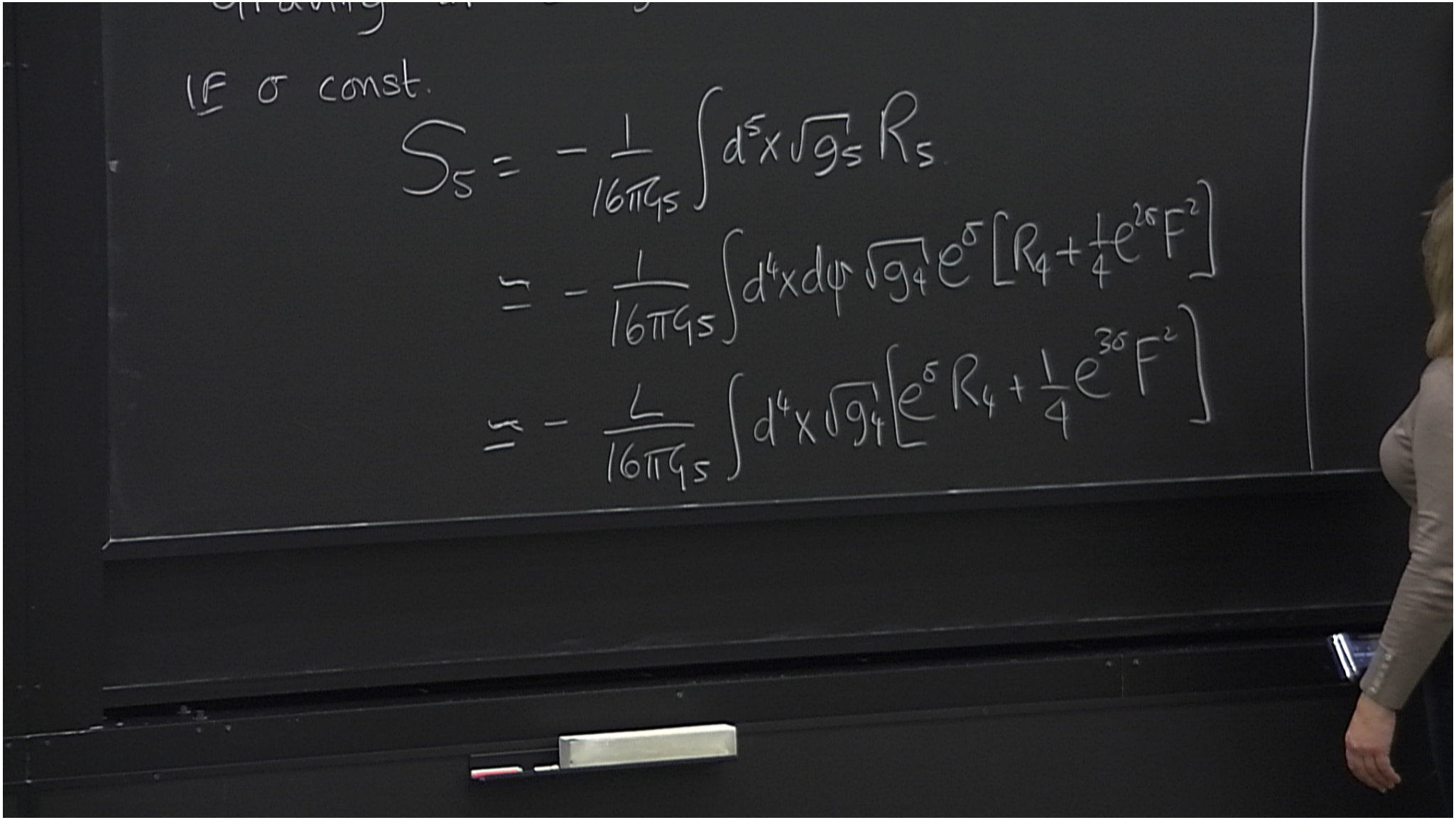
5D Einstein-Hilbert:

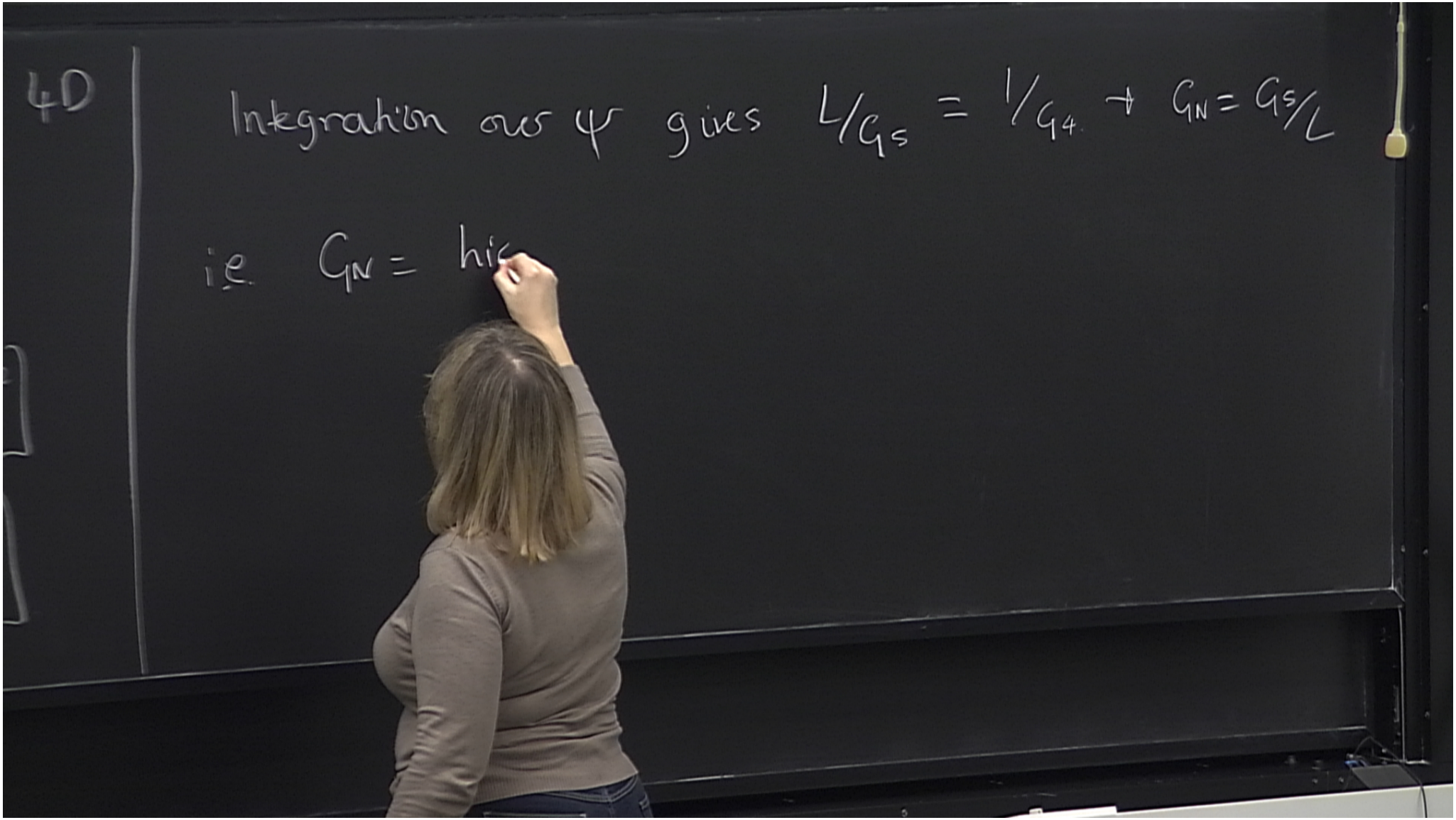
$$\sqrt{g_5}\, R_5 = e^{\sigma}\sqrt{g}\left[R_4 + \frac{1}{4} e^{2\sigma} F^2 - 2 e^{-\sigma} \overbrace{\Box e^{\sigma}}^{\text{TOTAL DERIV}}\right]$$

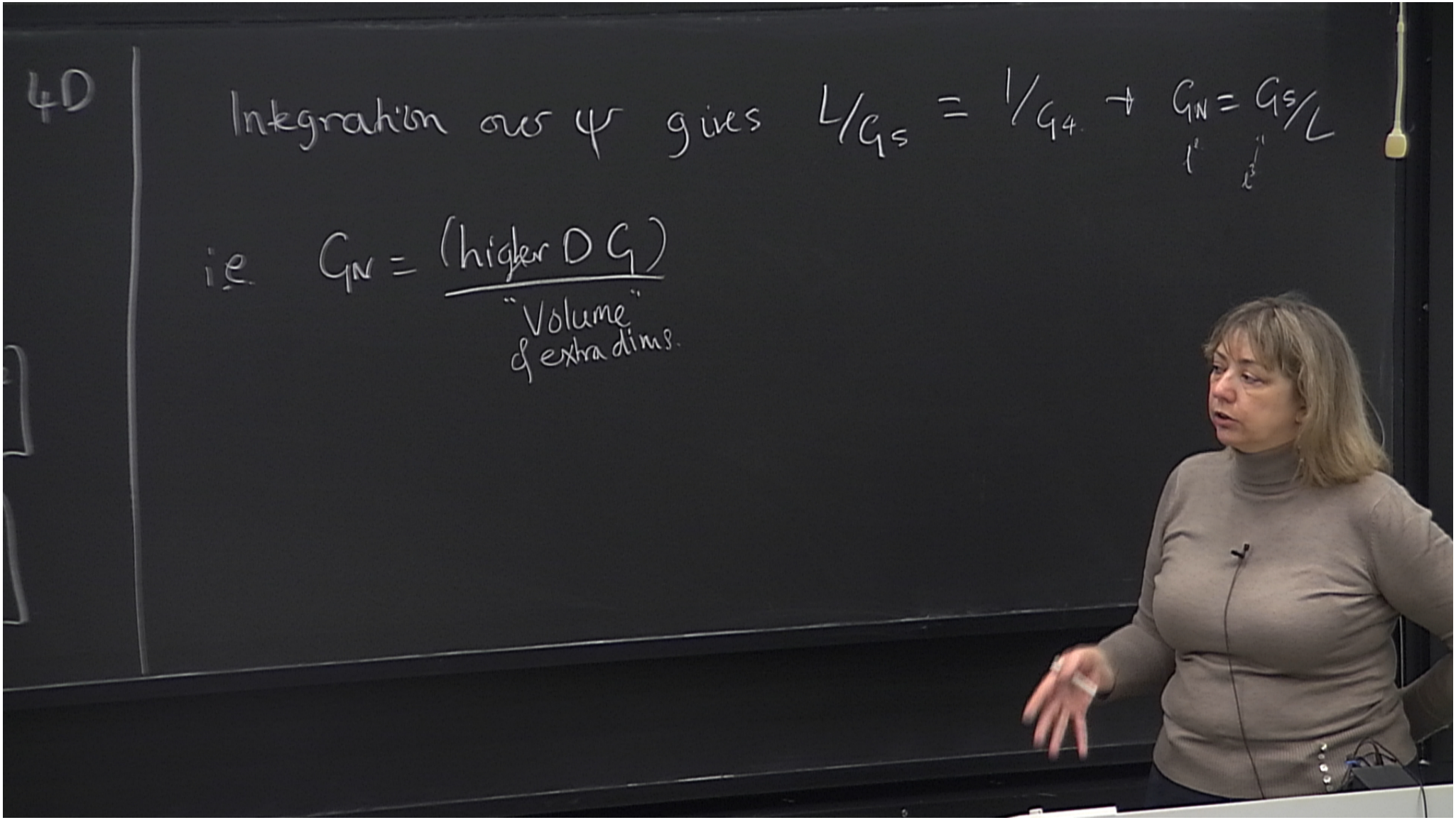

$\Rightarrow R = R_0 + \frac{1}{4} e^{2\sigma} F^2 - 2\Box\sigma - 2(\nabla\sigma)^2$
$-2 e^{-\sigma} \Box e^{\sigma}$
5D Einstein-Hilbert:
TOTAL DERIV
$\sqrt{g_5} R_5 = e^{\sigma}\sqrt{g}\left[R_4 + \frac{1}{4} e^{2\sigma} F^2 - 2 e^{-\sigma} \Box e^{\sigma}\right]$

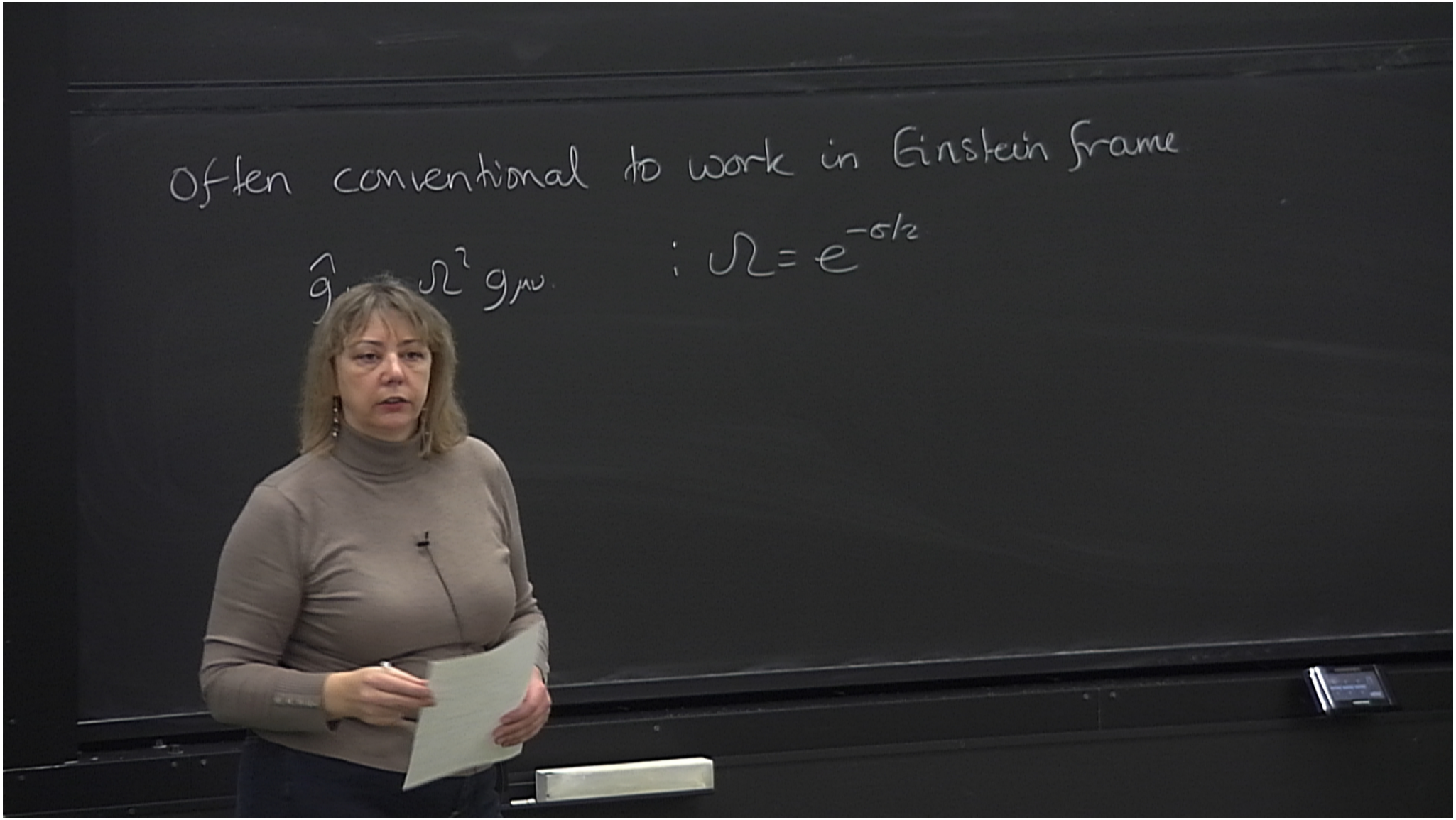

$\Rightarrow R = R_0 + \frac{1}{4} e^{2\sigma} F^2 - 2\Box\sigma - 2(\nabla\sigma)^2$
5D Einstein-Hilbert:
TOTAL DERIV

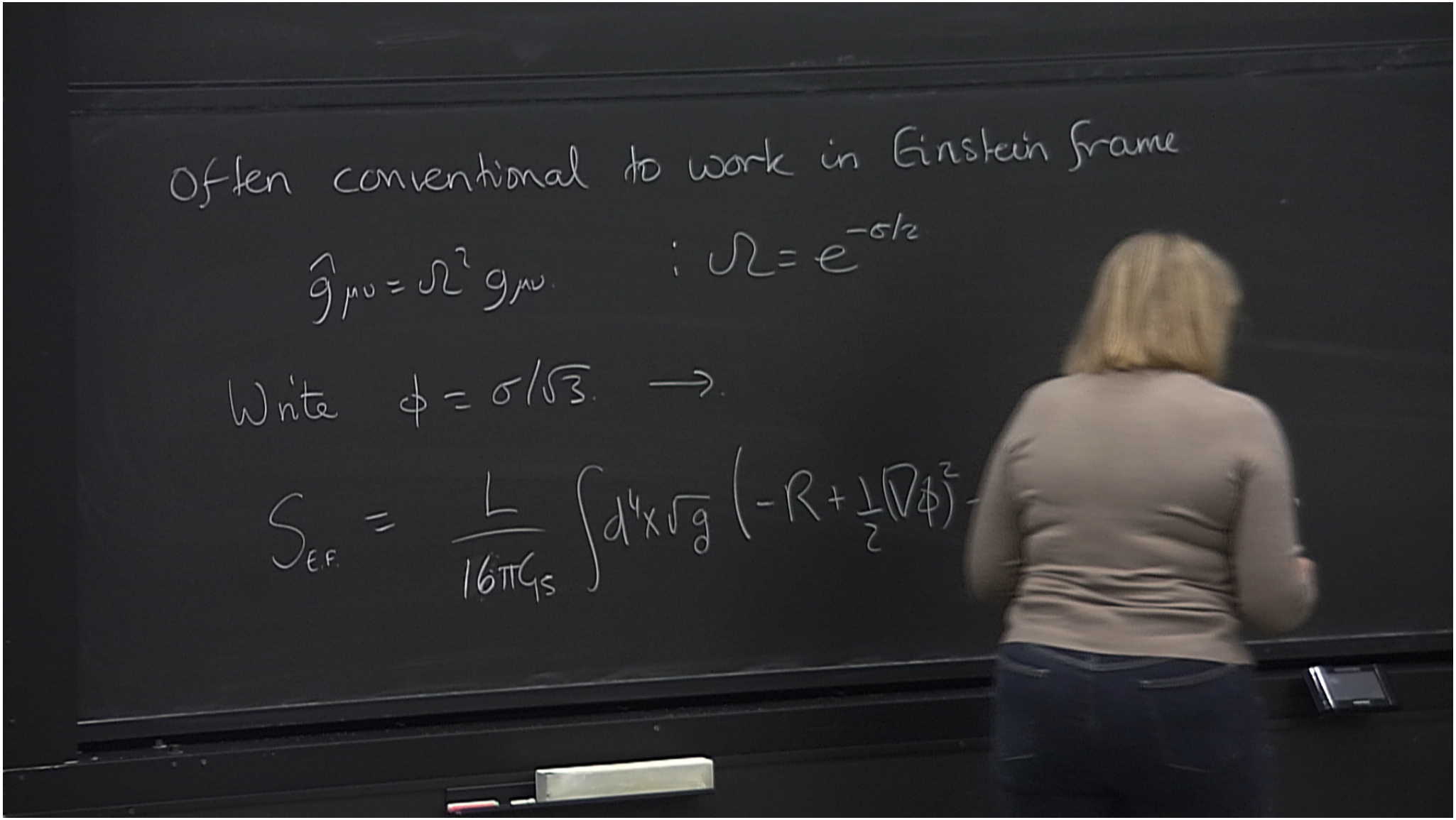

IF σ const.
$S_5 = -\frac{1}{16\pi G_5}\int d^5x\sqrt{g_5}R_5$
$\frac{1}{16\pi G_5}\int d^4x\,d\psi\sqrt{g_4}e^{\sigma}\left[R_4+\frac{1}{4}e^{2\sigma}F^2\right]$

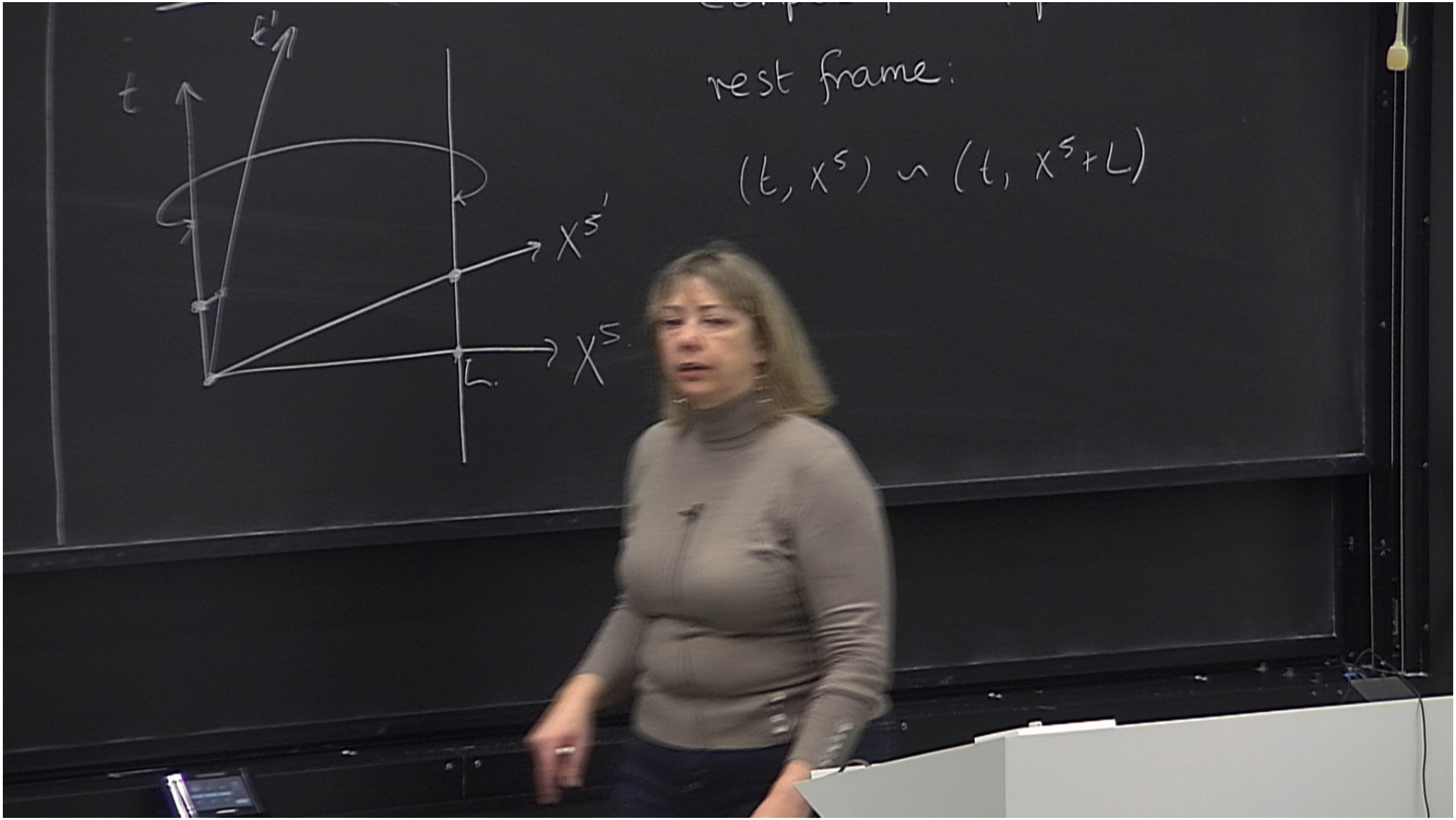

IF σ const.
$S_5 = -\frac{1}{16\pi G_5}\int d^5x\sqrt{g_5}R_5$
$= -\frac{1}{16\pi G_5}\int d^4x\,d\psi\sqrt{g_4}\,e^{\sigma}\left[R_4+\frac{1}{4}e^{2\sigma}F^2\right]$
$= -\frac{L}{16\pi G_5}\int d^4x\sqrt{g_4}\left[e^{\sigma}R_4+\frac{1}{4}e^{3\sigma}F^2\right]$


4D
Integration over ψ gives L/G5 = 1/G4 → GN = G5/L
i.e. GN = hig

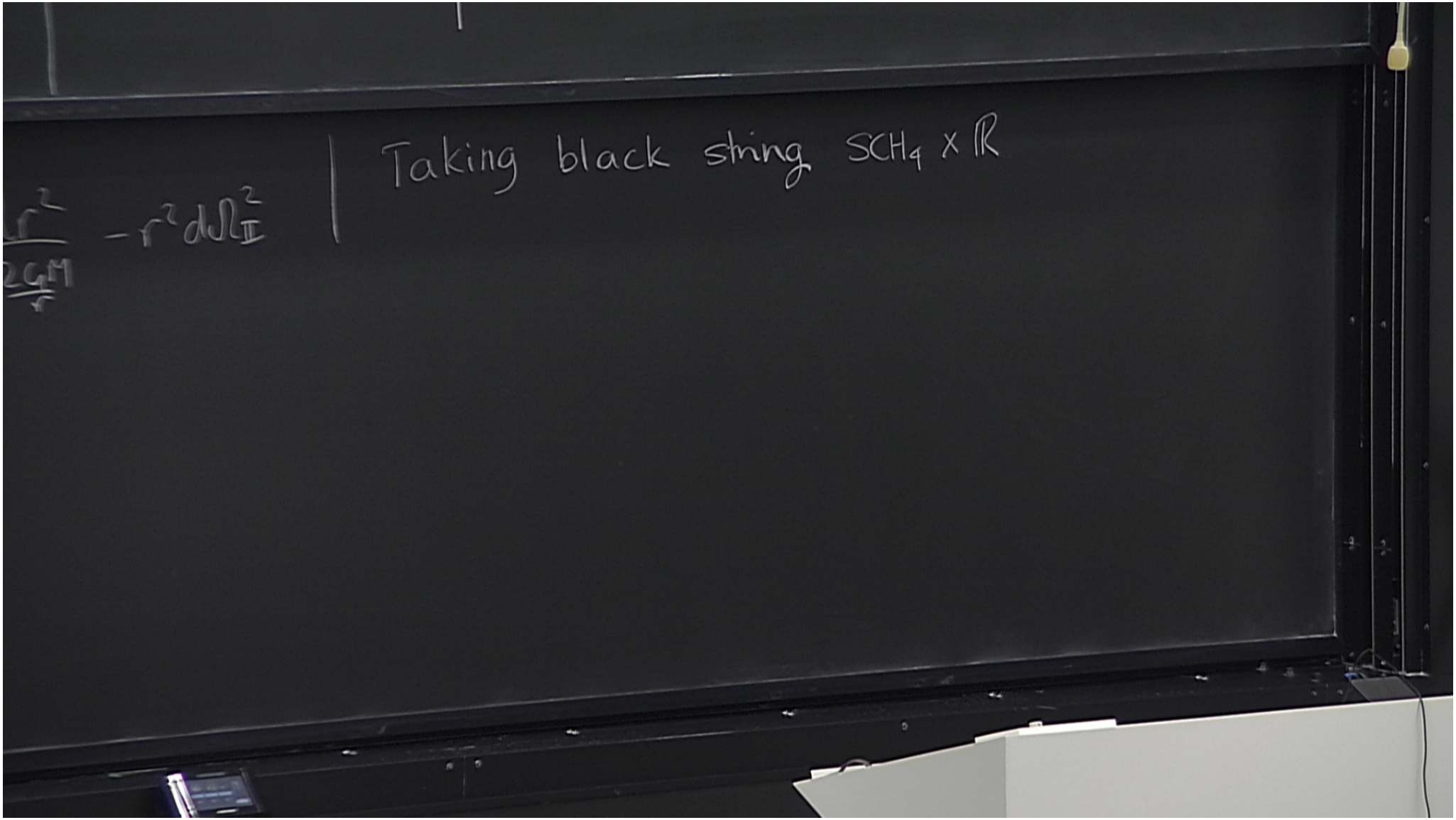

4D
Integration over ψ gives $L/G_5 = 1/G_4 \rightarrow G_N = G_5/L$
i.e. $G_N = \frac{(\text{higher D } G)}{\text{"Volume" of extra dims.}}$

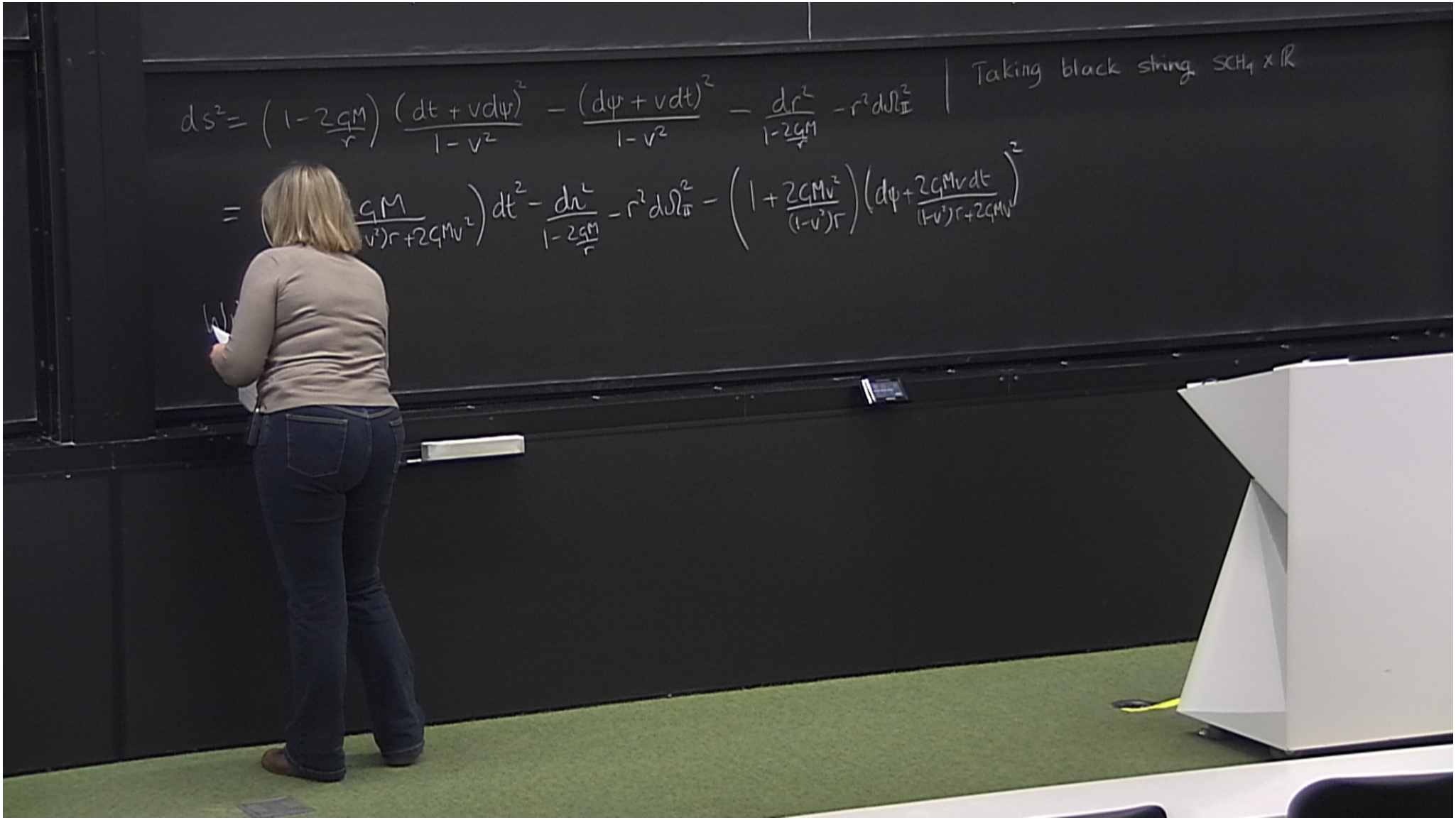

Often conventional to work in Einstein frame
$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$; $\Omega = e^{-\sigma/2}$

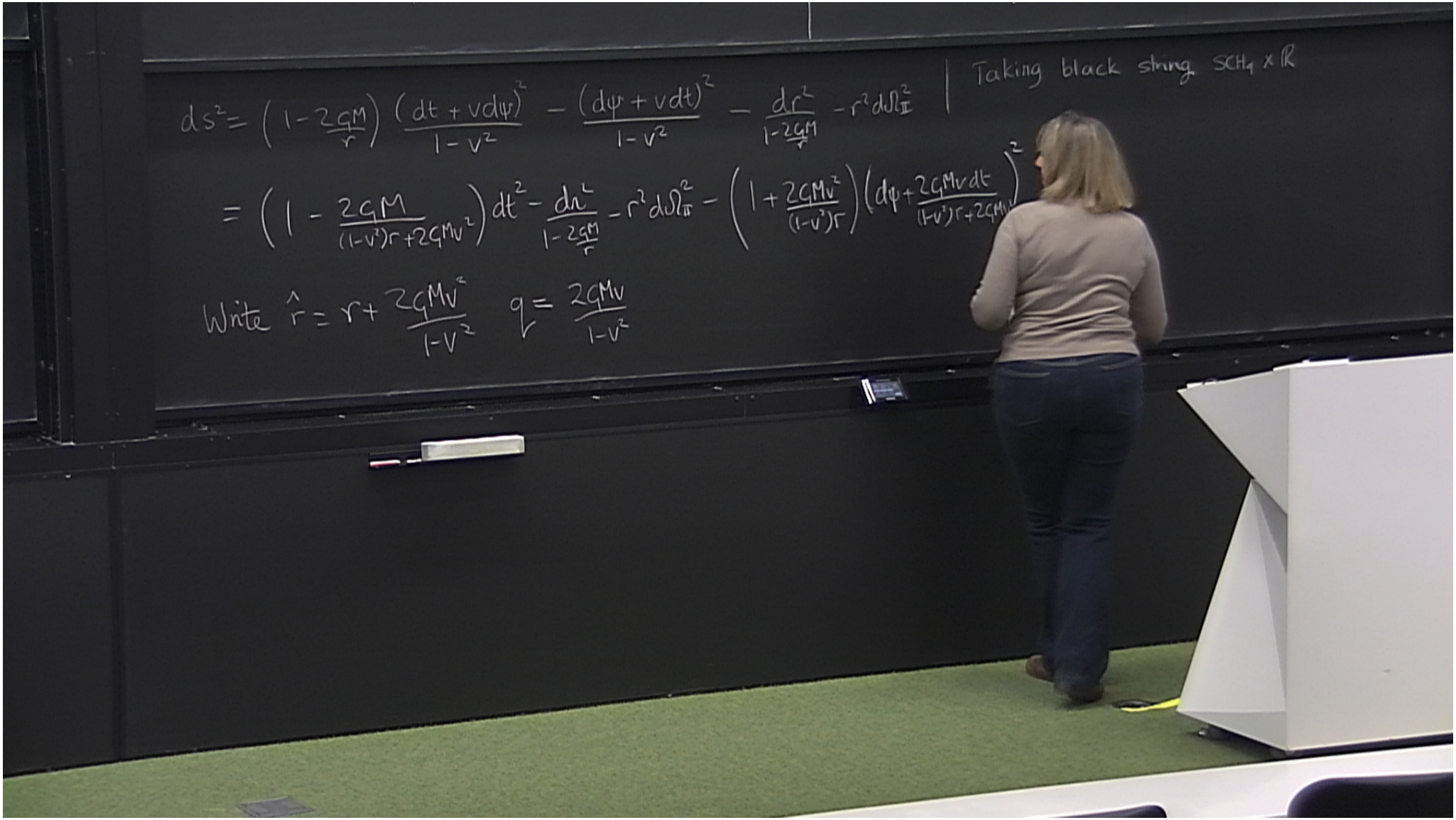

Often conventional to work in Einstein frame
$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$: $\Omega = e^{-\sigma/\sqrt{6}}$
Write $\phi = \sigma/\sqrt{3}$ $\rightarrow$
$S_{E.F.} = \frac{1}{16\pi G_5} \int d^4x \sqrt{g} \left(-R + \frac{1}{2}(\nabla\phi)^2 -$

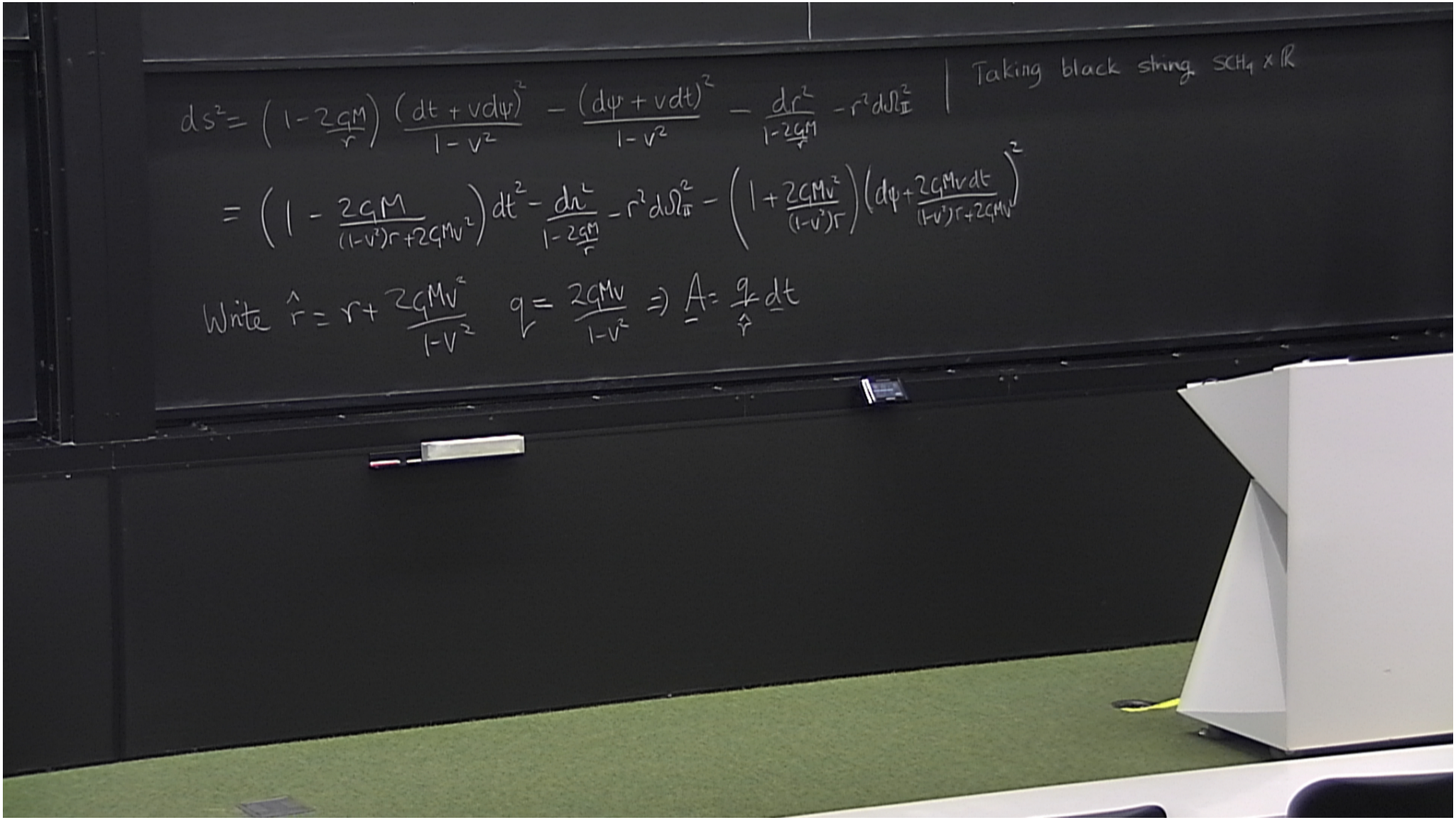

t'
t
$X^{5'}$
X^5
L
rest frame:
$(t, x^5) \sim (t, x^5 + L)$

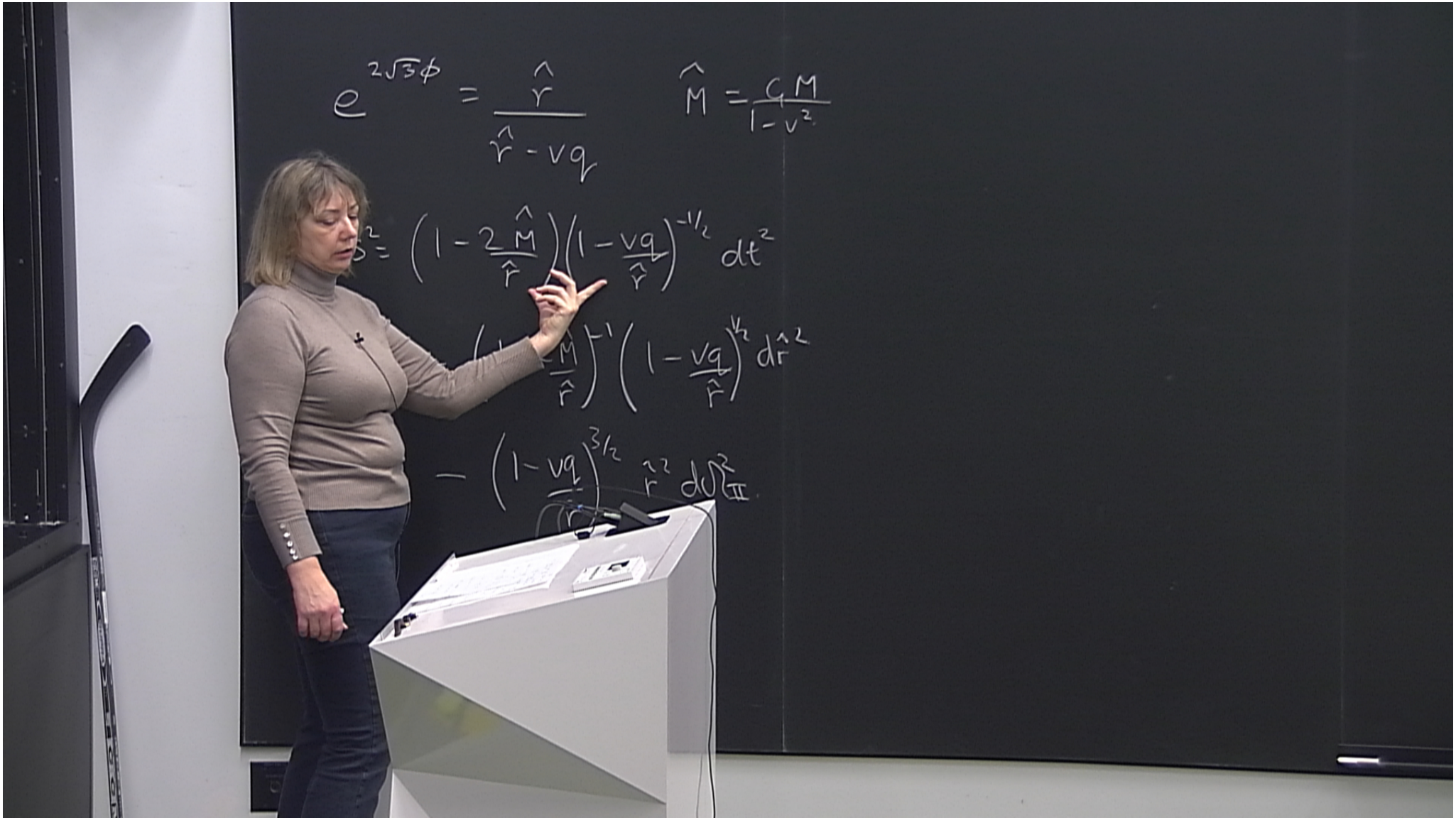
$$ds^2 = \left(1 - \frac{2GM}{r}\right)\frac{(dt + v\,d\psi)^2}{1 - v^2} - \frac{(d\psi + v\,dt)^2}{1 - v^2} - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2 d\Omega_{II}^2$$

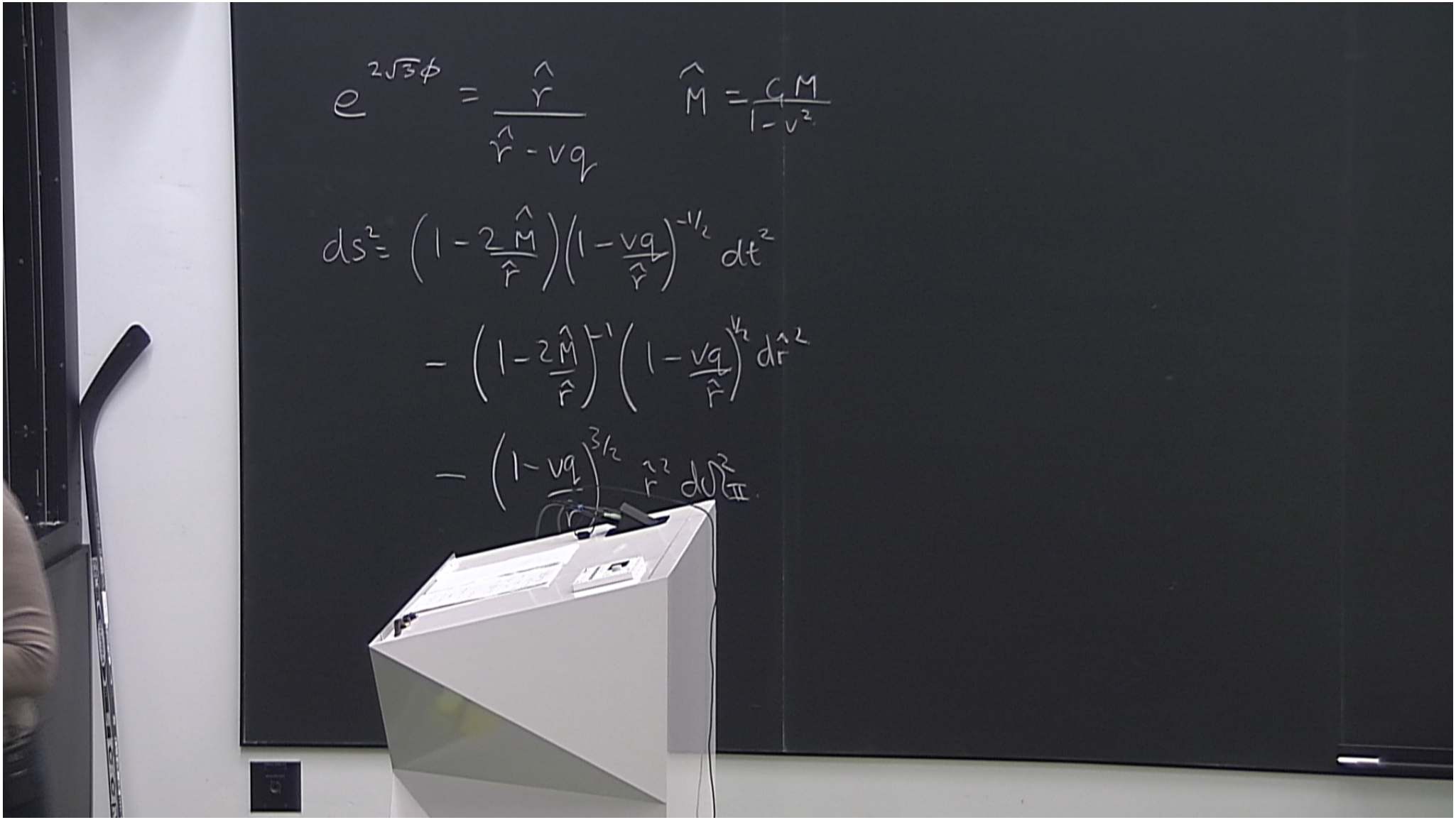

$\frac{dr^2}{1-\frac{2GM}{r}} - r^2 d\Omega_{II}^2$
Taking black string $SCH_4 \times \mathbb{R}$

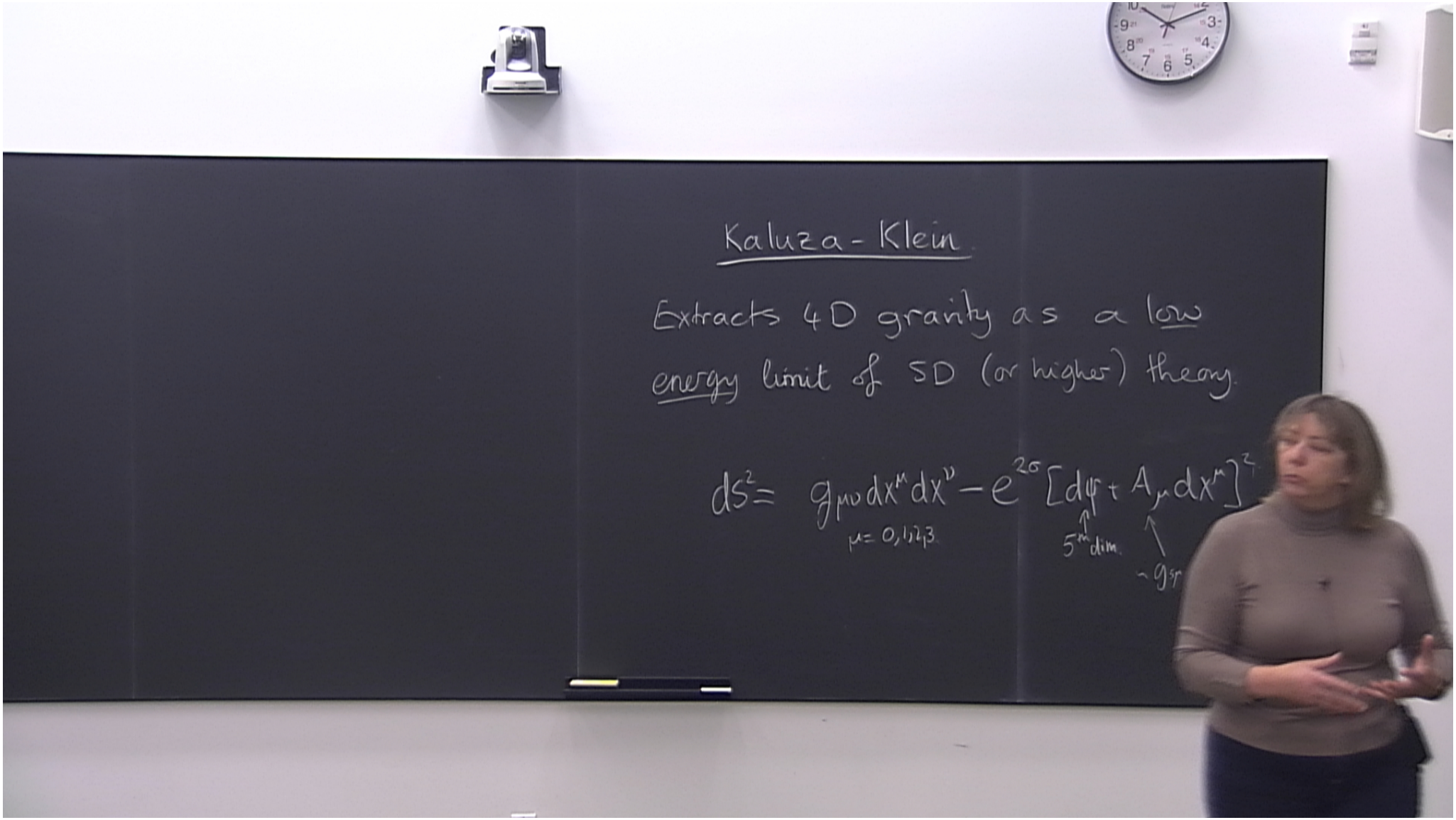

$ds^2 = \left(1 - \frac{2GM}{r}\right)\frac{(dt + v\,d\psi)^2}{1-v^2} - \frac{(d\psi + v\,dt)^2}{1-v^2} - \frac{dr^2}{1-\frac{2GM}{r}} - r^2 d\Omega_{II}^2$
Taking black string $SCH_4 \times \mathbb{R}$
$= \left(\frac{GM}{(1-v^2)r + 2GMv^2}\right) dt^2 - \frac{dr^2}{1-\frac{2GM}{r}} - r^2 d\Omega_{II}^2 - \left(1 + \frac{2GMv^2}{(1-v^2)r}\right)\left(d\psi + \frac{2GMv\,dt}{(1-v^2)r + 2GMv^2}\right)^2$


Taking black string $SCH_4 \times \mathbb{R}$
$ds^2 = \left(1 - \frac{2GM}{r}\right)\frac{(dt + v d\psi)^2}{1-v^2} - \frac{(d\psi + v dt)^2}{1-v^2} - \frac{dr^2}{1-\frac{2GM}{r}} - r^2 d\Omega_{II}^2$
$= \left(1 - \frac{2GM}{(1-v^2)r + 2GMv^2}\right) dt^2 - \frac{dr^2}{1-\frac{2GM}{r}} - r^2 d\Omega_{II}^2 - \left(1 + \frac{2GMv^2}{(1-v^2)r}\right)\left(d\psi + \frac{2GMv\,dt}{(1-v^2)r + 2GMv^2}\right)^2$
Write $\hat{r} = r + \frac{2GMv^2}{1-v^2}$ $q = \frac{2GMv}{1-v^2}$


$ds^2 = \left(1 - \frac{2GM}{r}\right)\frac{(dt + v d\psi)^2}{1-v^2} - \frac{(d\psi + v dt)^2}{1-v^2} - \frac{dr^2}{1-\frac{2GM}{r}} - r^2 d\Omega_{II}^2$
Taking black string $SCH_4 \times \mathbb{R}$
$= \left(1 - \frac{2GM}{(1-v^2)r + 2GMv^2}\right)dt^2 - \frac{dr^2}{1-\frac{2GM}{r}} - r^2 d\Omega_{II}^2 - \left(1 + \frac{2GMv^2}{(1-v^2)r}\right)\left(d\psi + \frac{2GMv\,dt}{(1-v^2)r + 2GMv^2}\right)^2$
Write $\hat{r} = r + \frac{2GMv^2}{1-v^2}$ $q = \frac{2GMv}{1-v^2}$ $\Rightarrow \underline{A} = \frac{q}{\hat{r}}\underline{dt}$


$e^{2\sqrt{3}\phi} = \frac{\hat{r}}{\hat{r} - vq}$
$\hat{M} = \frac{GM}{1-v^2}$
$ds^2 = \left(1 - \frac{2\hat{M}}{\hat{r}}\right)\left(1 - \frac{vq}{\hat{r}}\right)^{-1/2} dt^2$
$\left(1 - \frac{2\hat{M}}{\hat{r}}\right)^{-1}\left(1 - \frac{vq}{\hat{r}}\right)^{1/2} d\hat{r}^2$
$\left(1 - \frac{vq}{\hat{r}}\right)^{3/2} \hat{r}^2 d\Omega_{II}^2$


$e^{2\sqrt{3}\phi} = \frac{\hat{r}}{\hat{r} - vq}$
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$- \left(1 - \frac{2\hat{M}}{\hat{r}}\right)^{-1}\left(1 - \frac{vq}{\hat{r}}\right)^{1/2} d\hat{r}^2$
$- \left(1 - \frac{vq}{\hat{r}}\right)^{3/2} \hat{r}^2 d\Omega_{II}^2$


Kaluza-Klein
Extracts 4D gravity as a low energy limit of 5D (or higher) theory.
$ds^2 = g_{\mu\nu}dx^\mu dx^\nu - e^{2\sigma}[d\psi + A_\mu dx^\mu]^2$
$\mu = 0,1,2,3$
5^{th} dim.