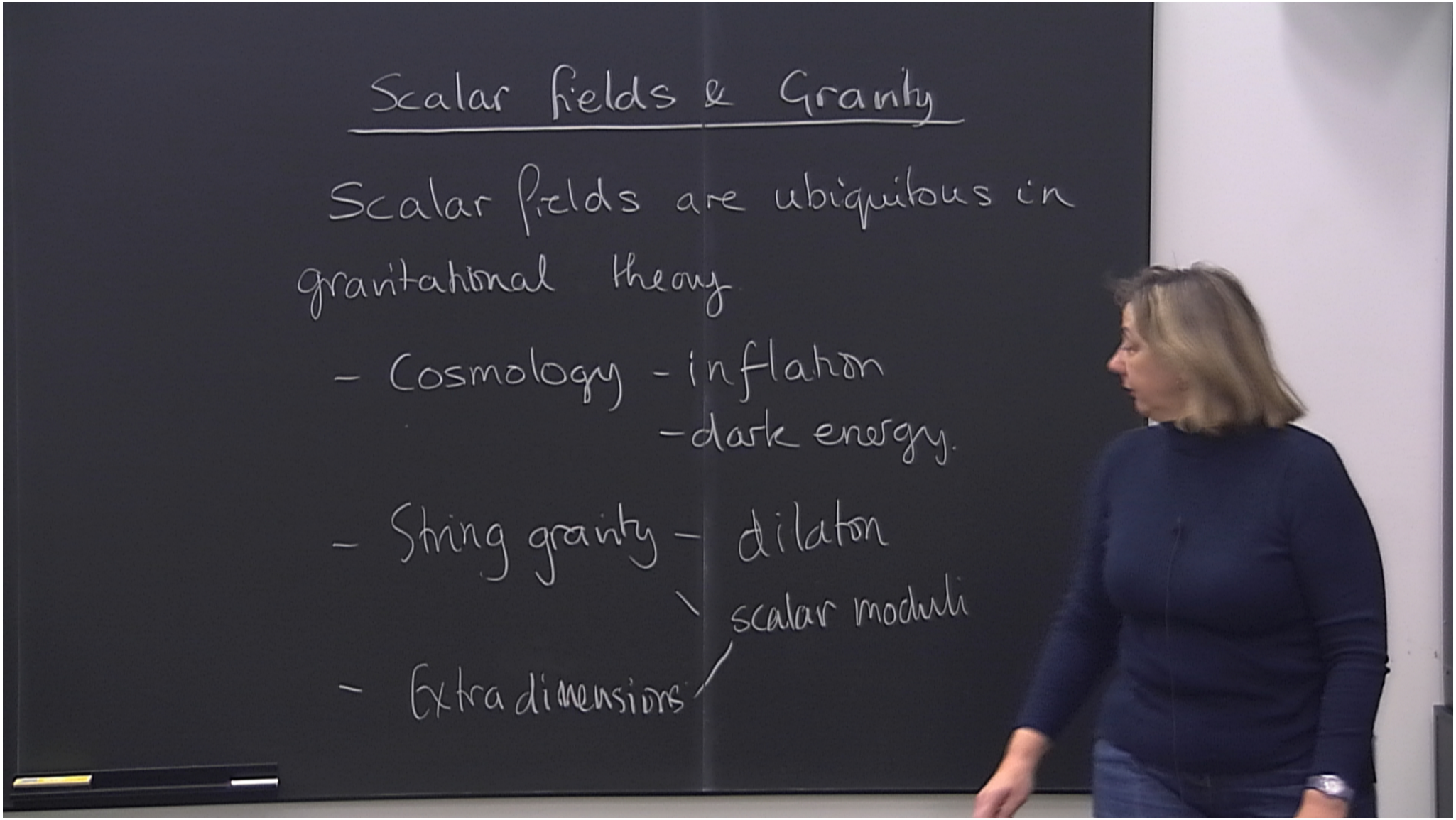
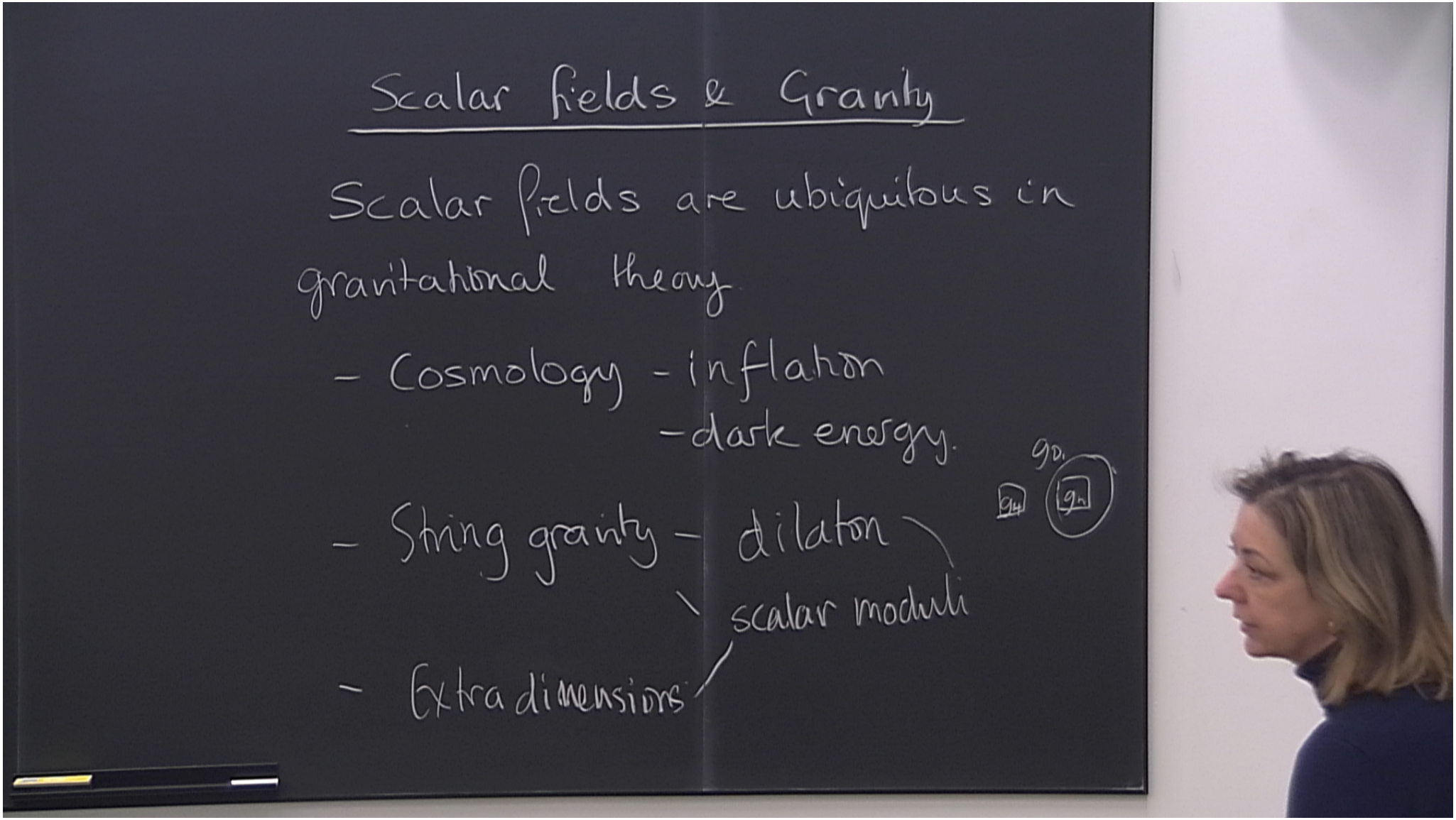
Title: 12/13 PSI - Gravitational Physics Review Lecture 9

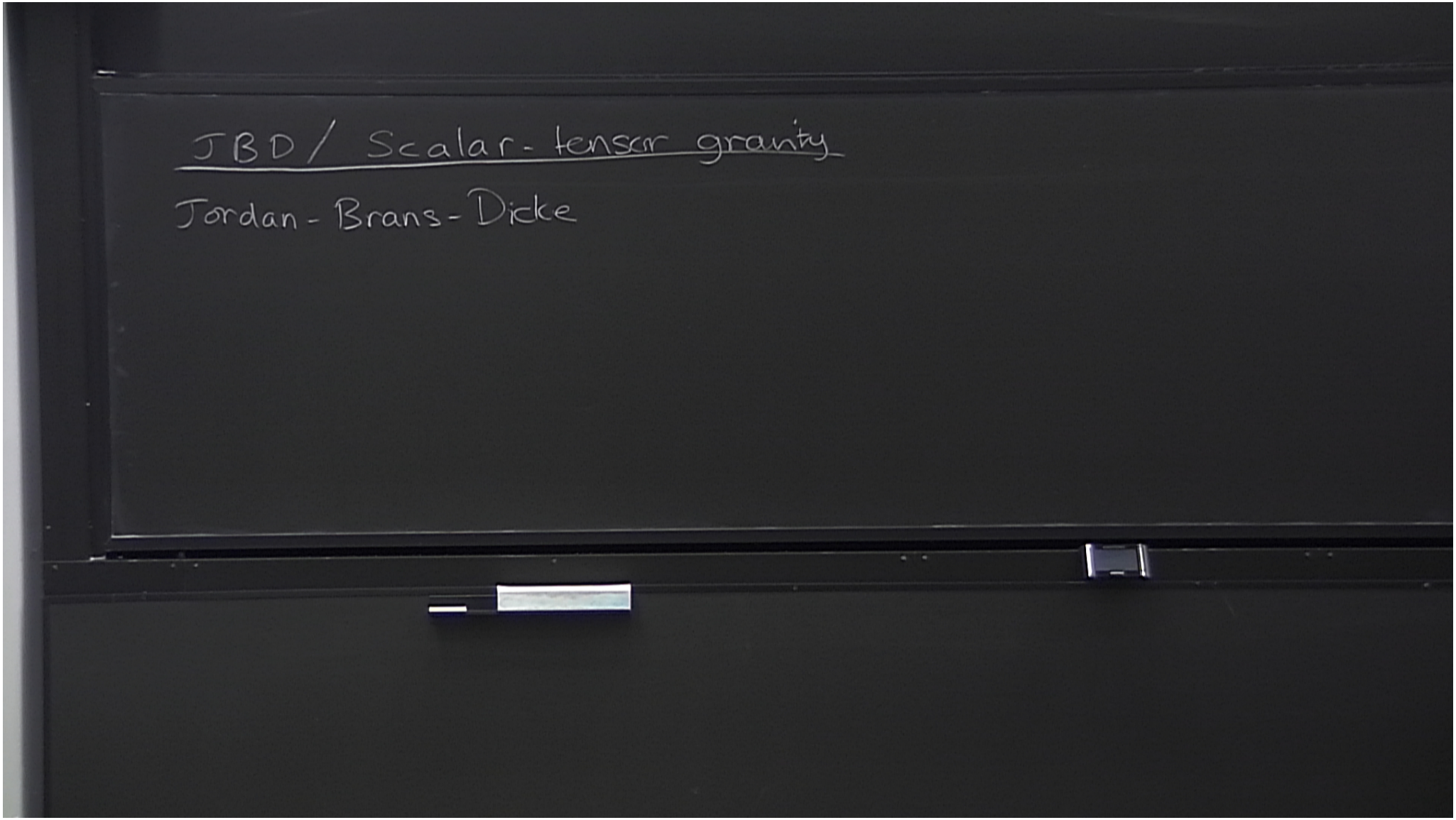
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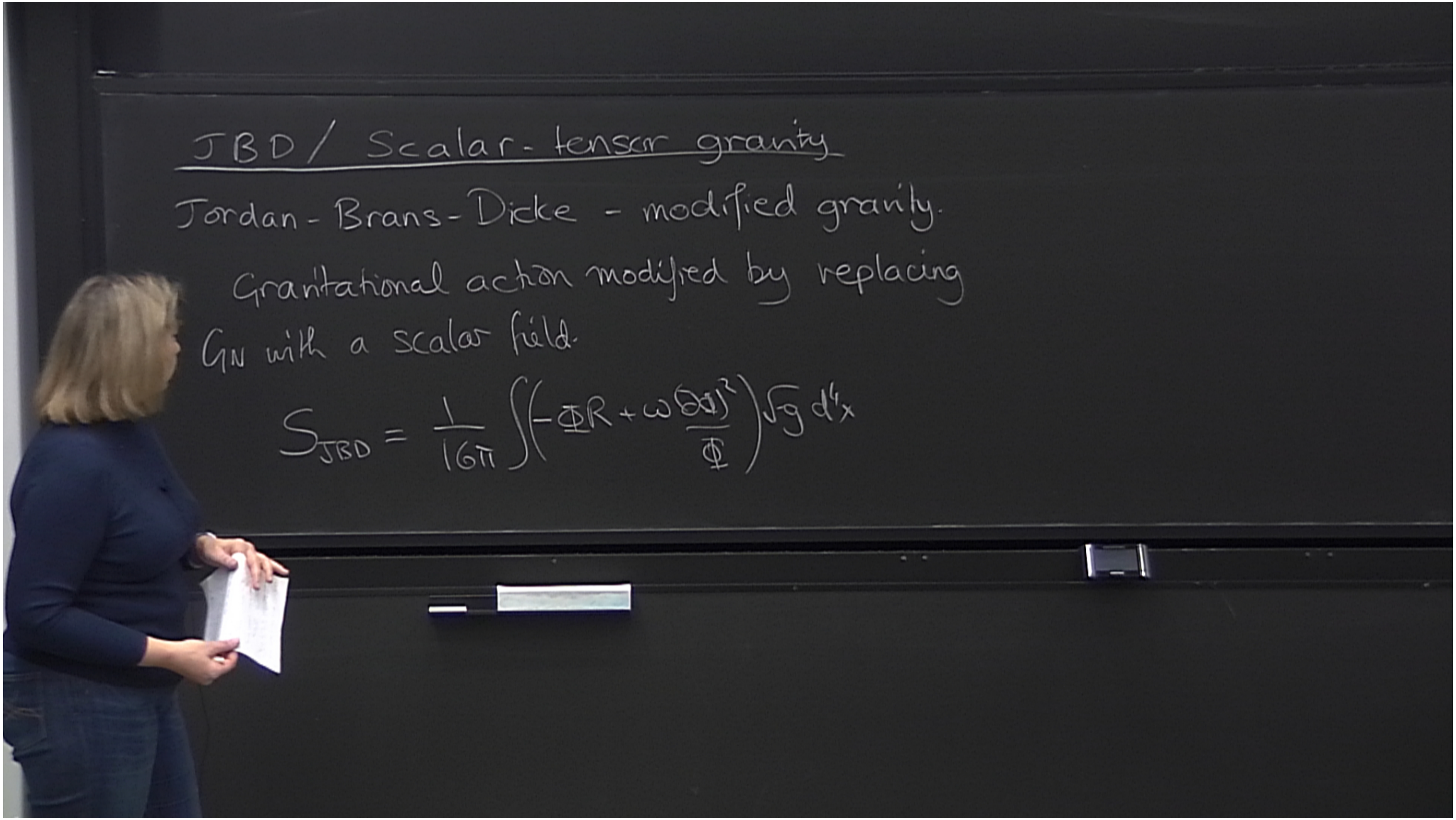
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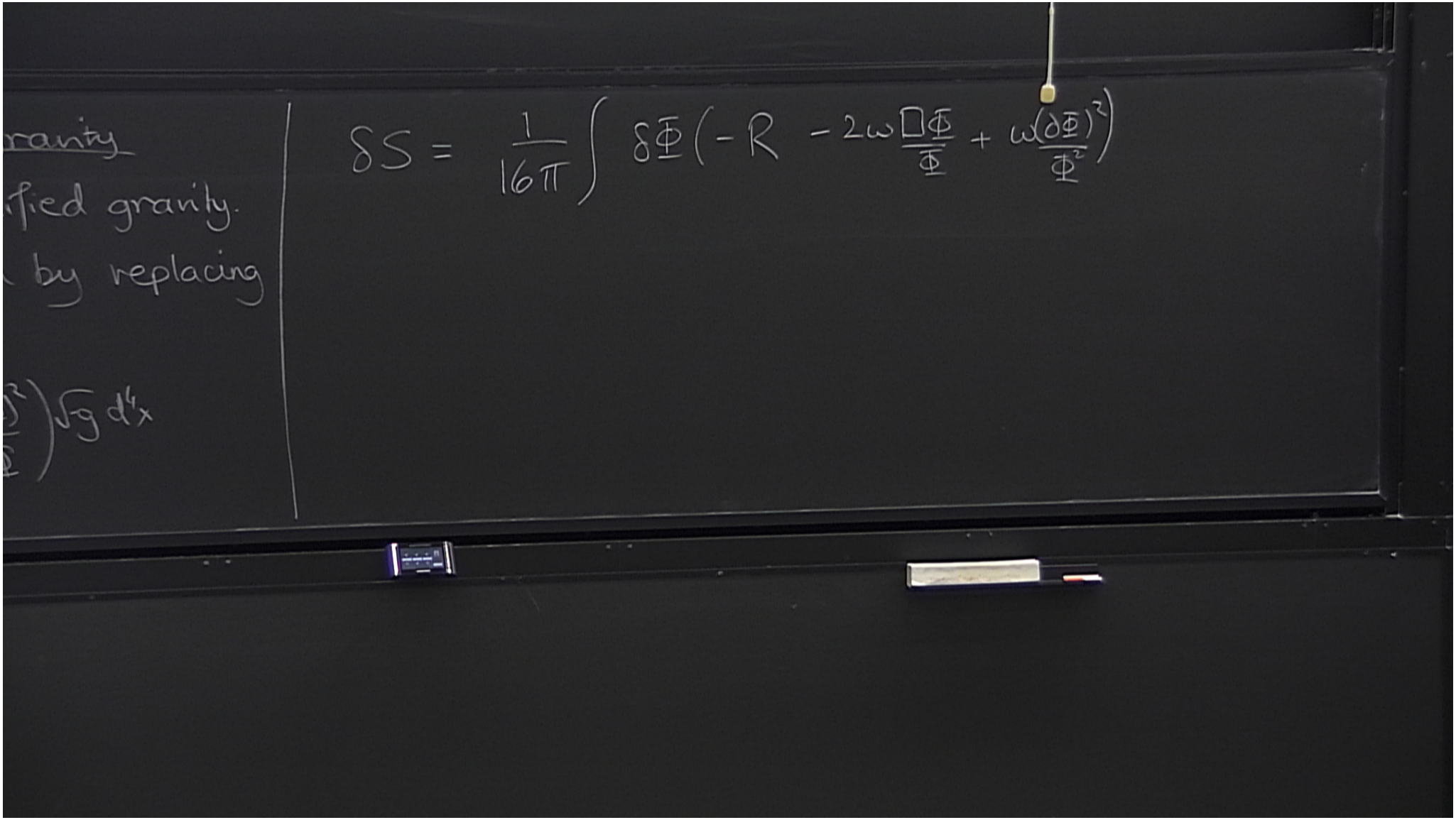
Abstract:

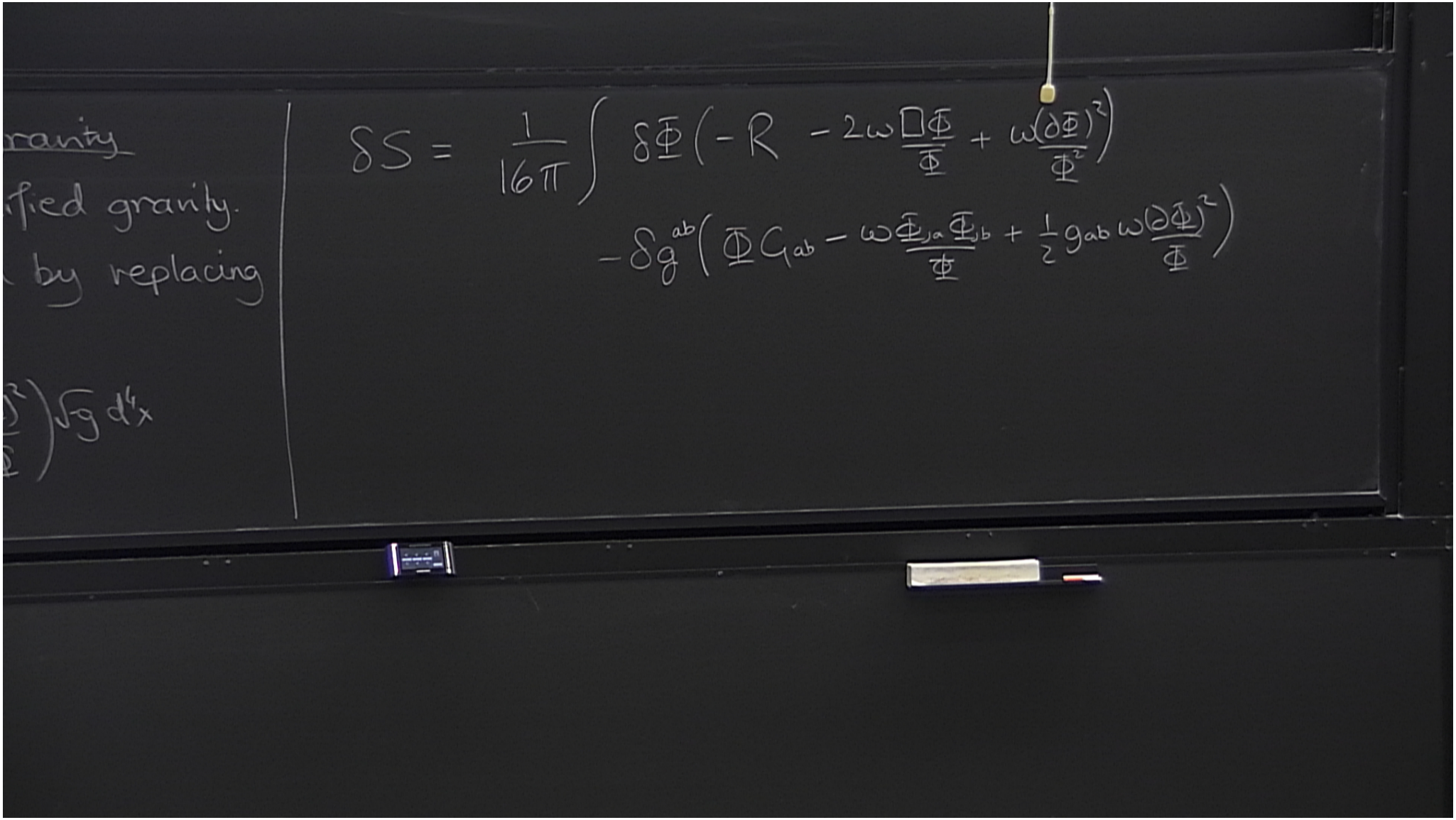

Scalar fields & Gravity
Scalar fields are ubiquitous in gravitational theory
- Cosmology - inflaton
- dark energy.
- String gravity - dilaton
- scalar moduli
- Extra dimensions


Scalar fields & Gravity
Scalar fields are ubiquitous in gravitational theory
- Cosmology - inflaton
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scalar moduli
g00
gμν


JBD / Scalar-tensor gravity
Jordan-Brans-Dicke


JBD / Scalar-tensor gravity
Jordan-Brans-Dicke - modified gravity.
Gravitational action modified by replacing
G_N with a scalar field.
$S_{JBD} = \frac{1}{16\pi}\int\left(-\Phi R + \omega\frac{(\partial\Phi)^2}{\Phi}\right)\sqrt{-g}\,d^4x$


rantty
ified granity.
by replacing
$\delta S = \frac{1}{16\pi}\int \delta\Phi\left(-R - 2\omega\frac{\Box\Phi}{\Phi} + \omega\frac{(\partial\Phi)^2}{\Phi^2}\right)$


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$\delta S = \frac{1}{16\pi}\int \delta\Phi\left(-R - 2\omega\frac{\Box\Phi}{\Phi} + \omega\frac{(\partial\Phi)^2}{\Phi^2}\right)$
$-\delta g^{ab}\left(\Phi G_{ab} - \omega\frac{\Phi_{,a}\Phi_{,b}}{\Phi} + \frac{1}{2}g_{ab}\omega\frac{(\partial\Phi)^2}{\Phi}\right)$
$\sqrt{-g}\,d^4x$

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by replacing

$$\delta S = \frac{1}{16\pi}\int \delta\Phi\left(-R - 2\omega\frac{\Box\Phi}{\Phi} + \omega\frac{(\partial\Phi)^2}{\Phi^2}\right)$$

$$-\delta g^{ab}\left(\Phi G_{ab} - \omega\frac{\Phi_{,a}\Phi_{,b}}{\Phi} + \frac{1}{2}g_{ab}\,\omega\frac{(\partial\Phi)^2}{\Phi}\right)$$

$$+\Phi\left(\nabla_a\nabla_b\delta g^{ab} - \Box g_{ab}\delta g^{ab}\right)$$

$$\left(\frac{(\partial\Phi)^2}{\Phi}\right)\sqrt{-g}\,d^4x$$

Our gravity eqns become:

$$\Rightarrow \bar{\Phi} G_{ab} - \nabla_a \nabla_b \bar{\Phi} + g_{ab} \Box \bar{\Phi} - \frac{\omega}{\bar{\Phi}} \left(\bar{\Phi}_{,a} \bar{\Phi}_{,b} - \frac{1}{2} (\nabla \bar{\Phi})^2 g_{ab} \right) = 8\pi T_{ab}$$

Plus

$$\bar{\Phi} R + 2\omega \Box \bar{\Phi} - \omega \frac{(\nabla \bar{\Phi})^2}{\bar{\Phi}} = 0.$$

← Provided $\bar{\Phi}$ does not couple to matter.

Our gravity eqns become:

$$\Rightarrow \Phi G_{ab} - \nabla_a \nabla_b \Phi + g_{ab} \Box \Phi - \frac{\omega}{\Phi}\left(\Phi_{,a}\Phi_{,b} - \tfrac{1}{2}(\nabla\Phi)^2 g_{ab}\right) = 8\pi T_{ab}$$

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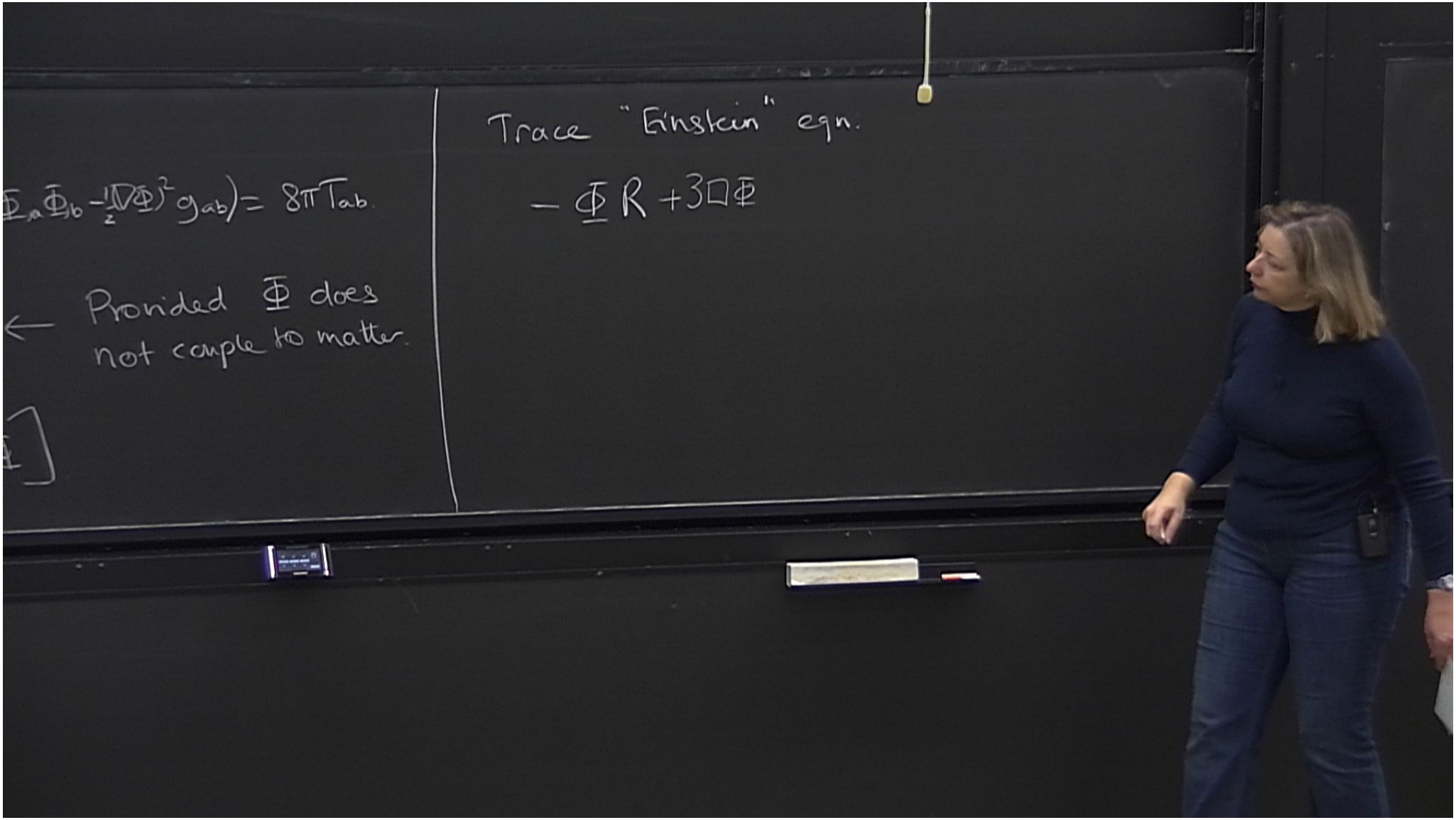
$$\Rightarrow \Phi G_{ab} - \nabla_a \nabla_b \Phi + g_{ab} \Box \Phi - \frac{\omega}{\Phi}\left(\Phi_{,a}\Phi_{,b} - \tfrac{1}{2}(\nabla\Phi)^2 g_{ab}\right) = 8\pi T_{ab}$$

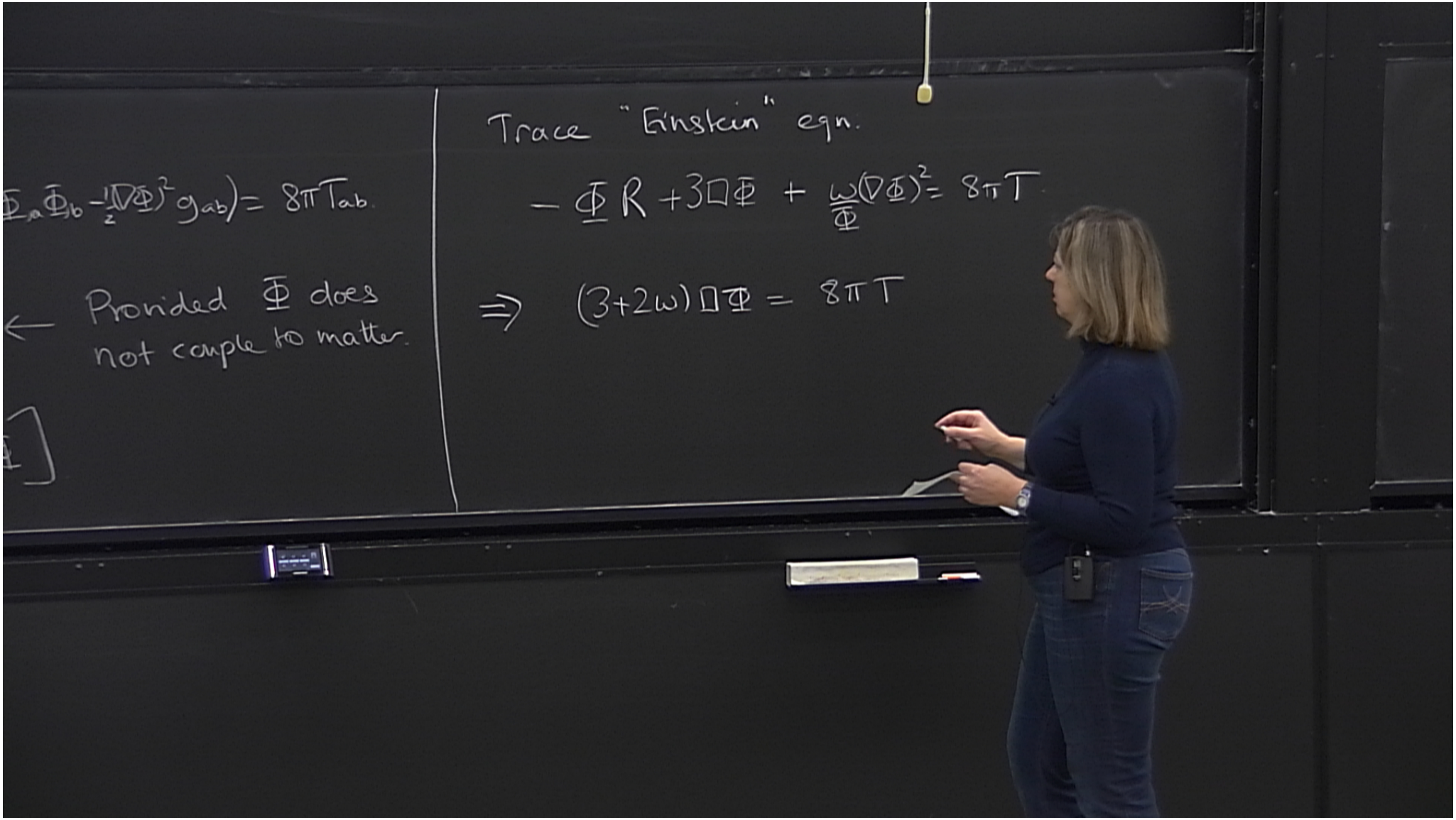
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$\Phi_a\Phi_b - \frac{1}{2}(\nabla\Phi)^2 g_{ab}) = 8\pi T_{ab}$
← Provided Φ does not couple to matter.
Trace "Einstein" eqn.
$-\Phi R + 3\Box\Phi$


$\Phi_a\Phi_b - \frac{1}{2}(\nabla\Phi)^2 g_{ab}) = 8\pi T_{ab}$
Provided Φ does not couple to matter.
Trace "Einstein" eqn.
$-\Phi R + 3\Box\Phi + \frac{\omega}{\Phi}(\nabla\Phi)^2 = 8\pi T$
$\Rightarrow (3+2\omega)\Box\Phi = 8\pi T$

$\Phi_a\Phi_b - \frac{1}{2}(\nabla\Phi)^2 g_{ab}) = 8\pi T_{ab}$

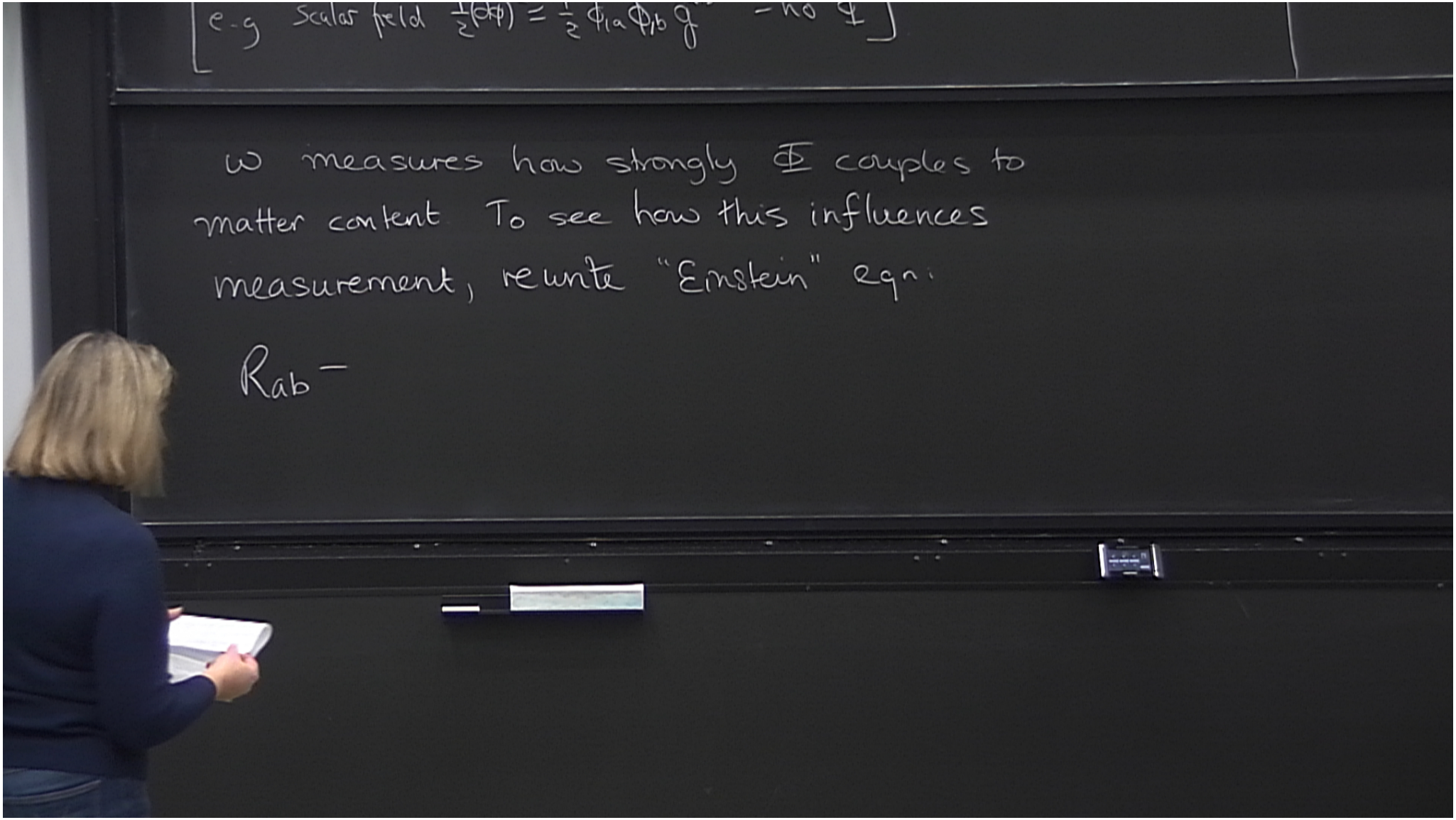
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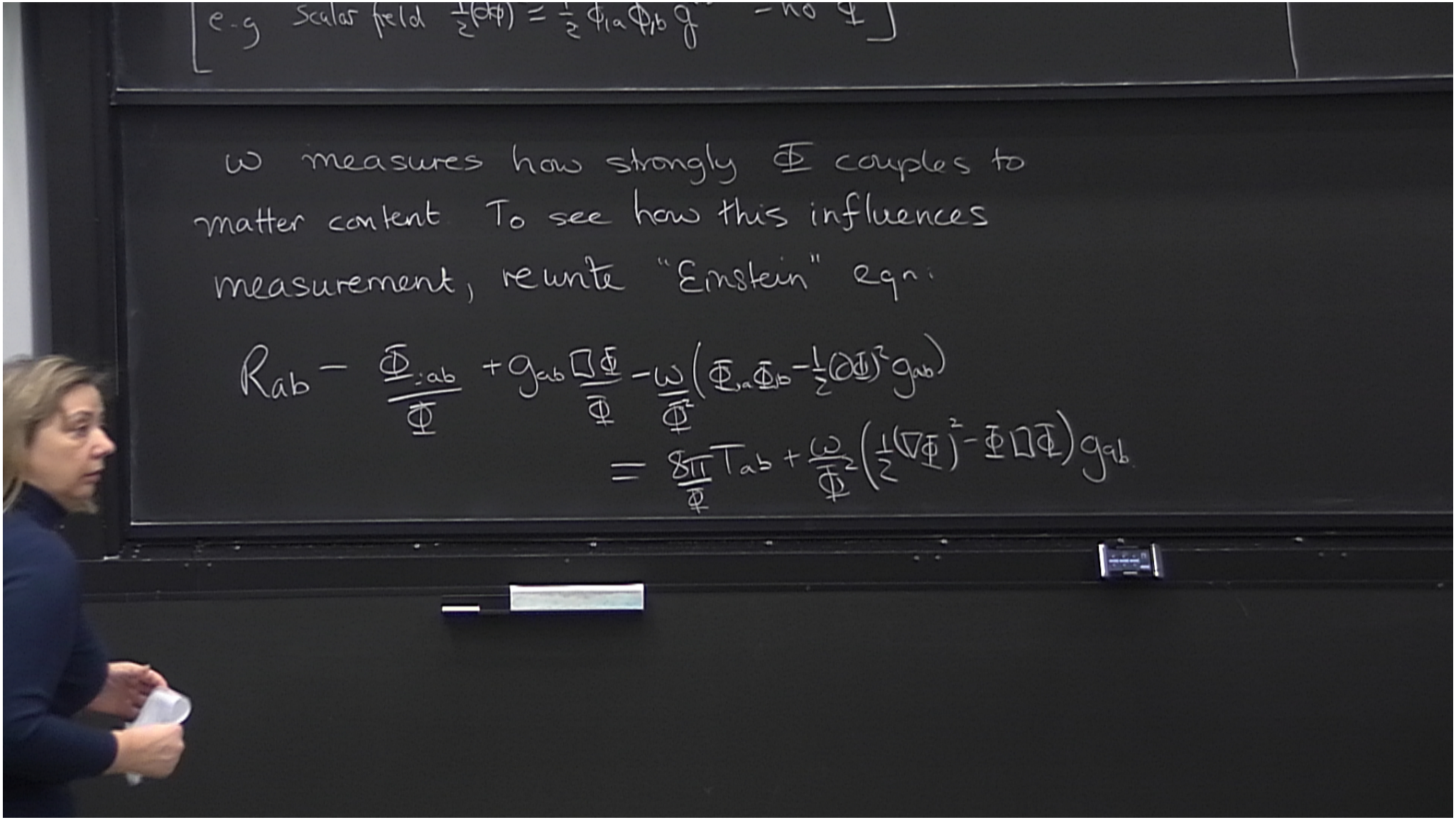
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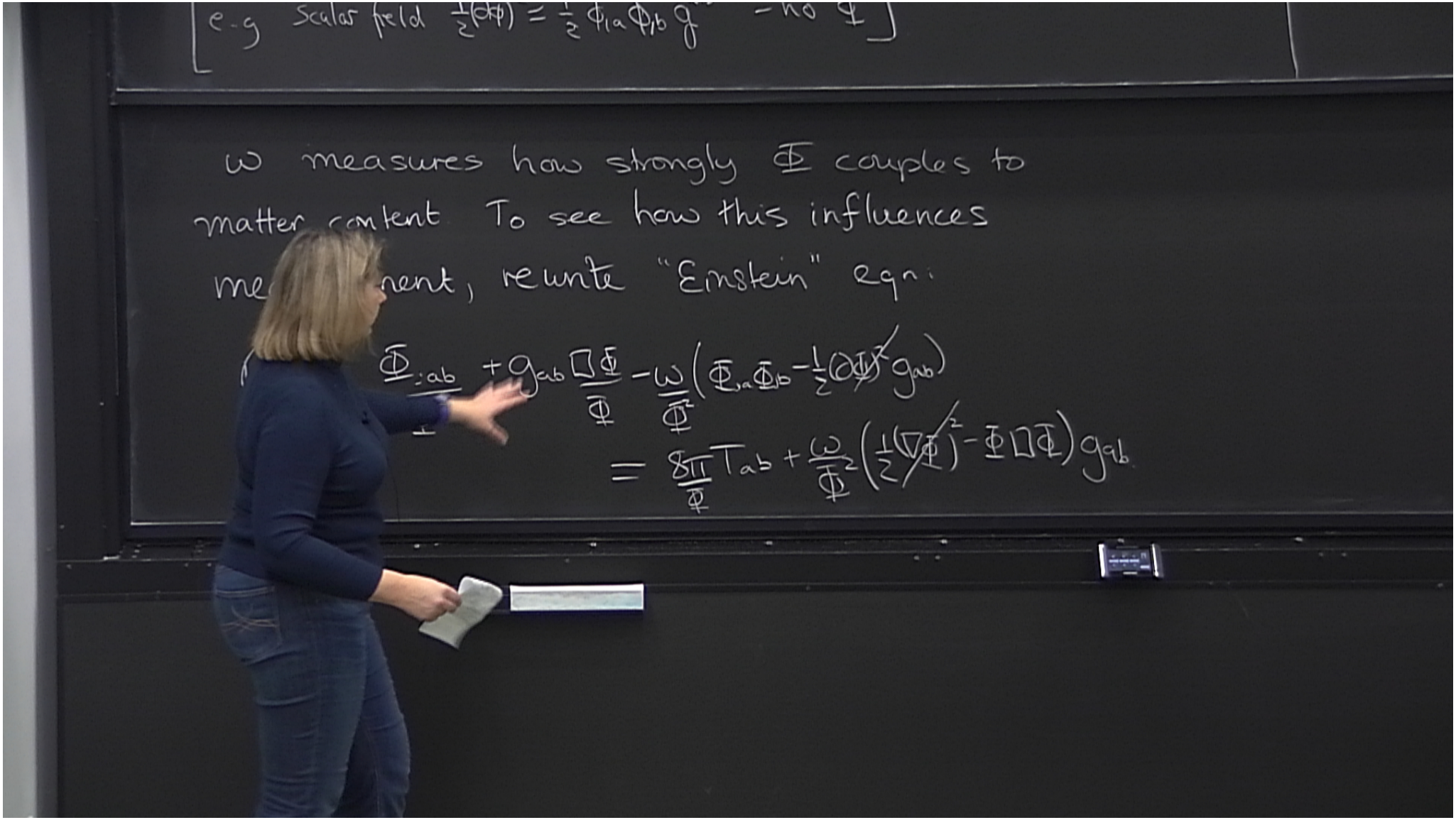
$$-\Phi R + 3\Box\Phi + \frac{\omega}{\Phi}(\nabla\Phi)^2 = 8\pi T$$

$$\Rightarrow \quad (3+2\omega)\Box\Phi = 8\pi T$$

$$\Box\Phi = \frac{8\pi T}{3+2\omega}$$


e.g. scalar field
ω measures how strongly Φ couples to matter content. To see how this influences measurement, rewrite "Einstein" eqn:
R_{ab} −


[e.g scalar field $\frac{1}{2}(\partial\Phi) = \frac{1}{2}\phi_{;a}\phi_{;b}g^{} - \kappa\delta\Phi$]
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$$R_{ab} - \frac{\Phi_{;ab}}{\Phi} + g_{ab}\frac{\Box\Phi}{\Phi} - \frac{\omega}{\Phi^2}\left(\Phi_{,a}\Phi_{,b} - \frac{1}{2}(\partial\Phi)^2 g_{ab}\right)$$
$$= \frac{8\pi}{\Phi}T_{ab} + \frac{\omega}{\Phi^2}\left(\frac{1}{2}(\nabla\Phi)^2 - \Phi\Box\Phi\right)g_{ab}$$


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Φ couples to
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$\Phi_{,a}\Phi_{,b} - \frac{1}{2}(\partial\Phi)^2 g_{ab}$

$T_{ab} + \frac{\omega}{\Phi^2}\left(\frac{1}{2}(\nabla\Phi)^2 - \Phi\Box\Phi\right) g_{ab}$

$$R_{ab} - \frac{\Phi_{,ab}}{\Phi} - \omega\frac{\Phi_{,a}\Phi_{,b}}{\Phi^2} = \frac{8\pi}{\Phi}\left(T_{ab} - \frac{(\omega+1)}{(3+2\omega)} T g_{ab}\right)$$

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Linearized tests of gravity are in good agreement with spin 2 massless graviton.

$$\Box h_{ab} = -8\pi G_0 \left(T_{ab} - \frac{1}{2} T \eta_{ab}\right)$$

$h_{ab} \leftrightarrow \delta g_{ab}$.

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$\frac{\omega+1}{3+2\omega} \to \frac{1}{2}$ as $\omega \to \infty$

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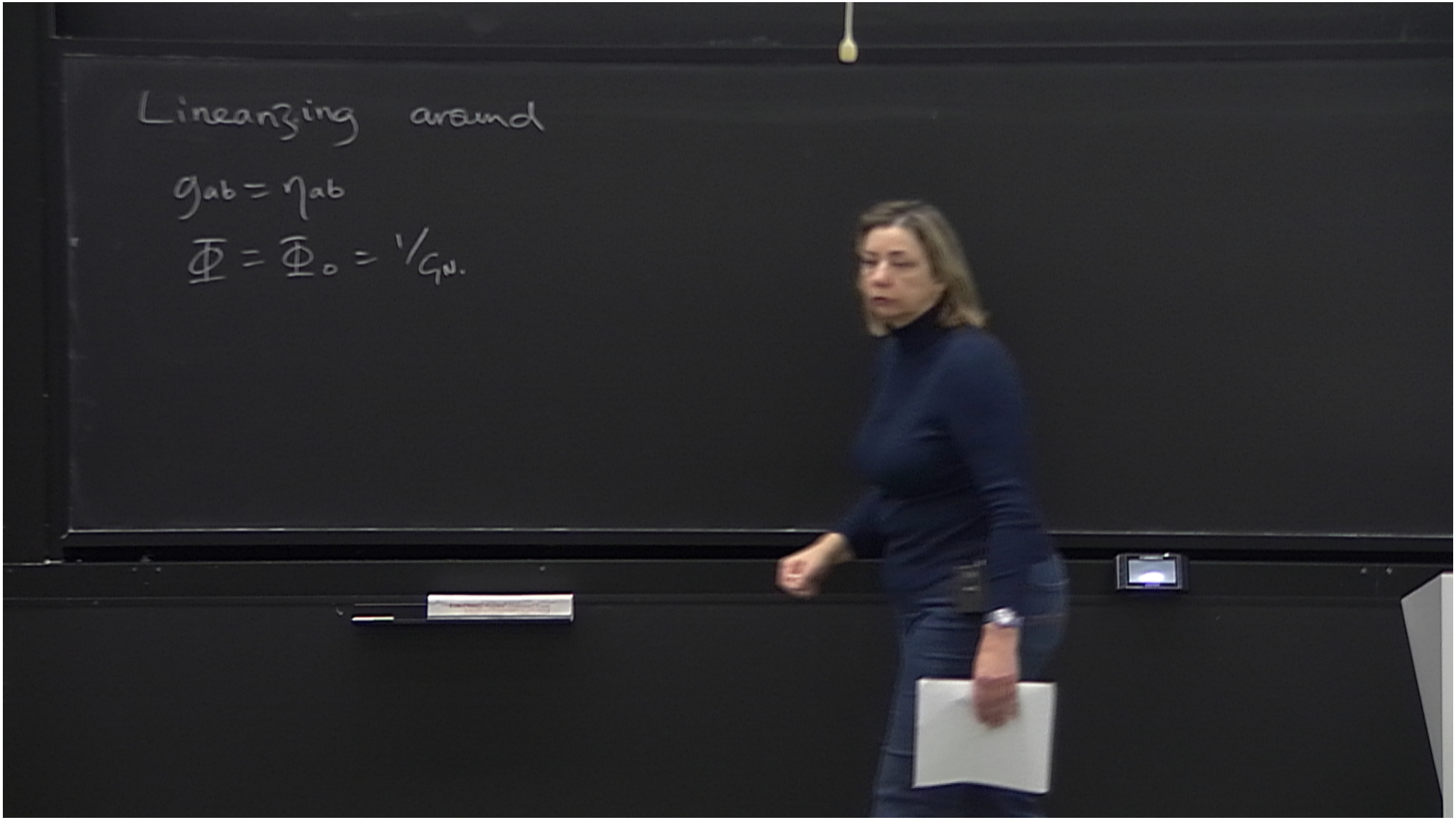
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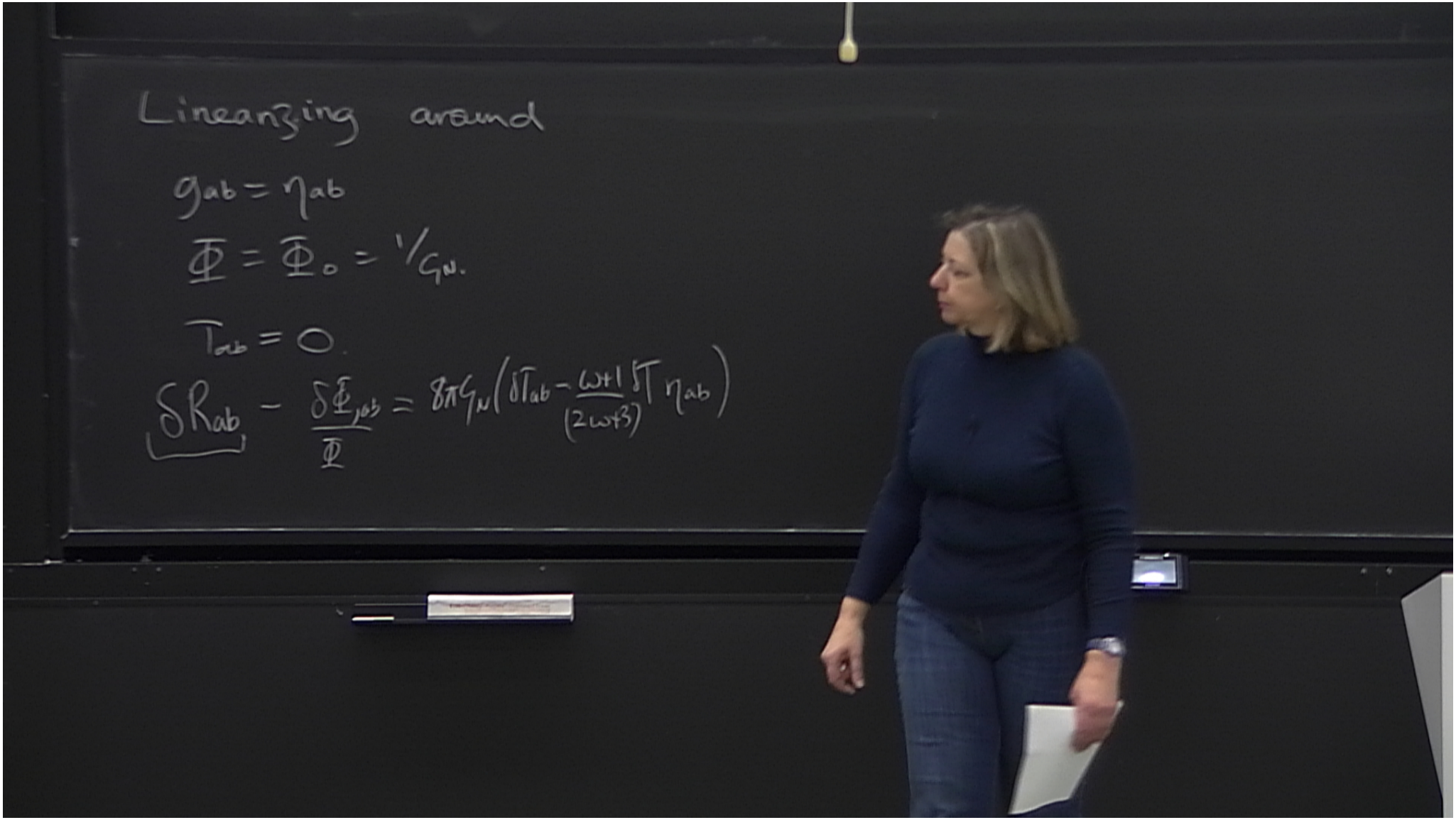
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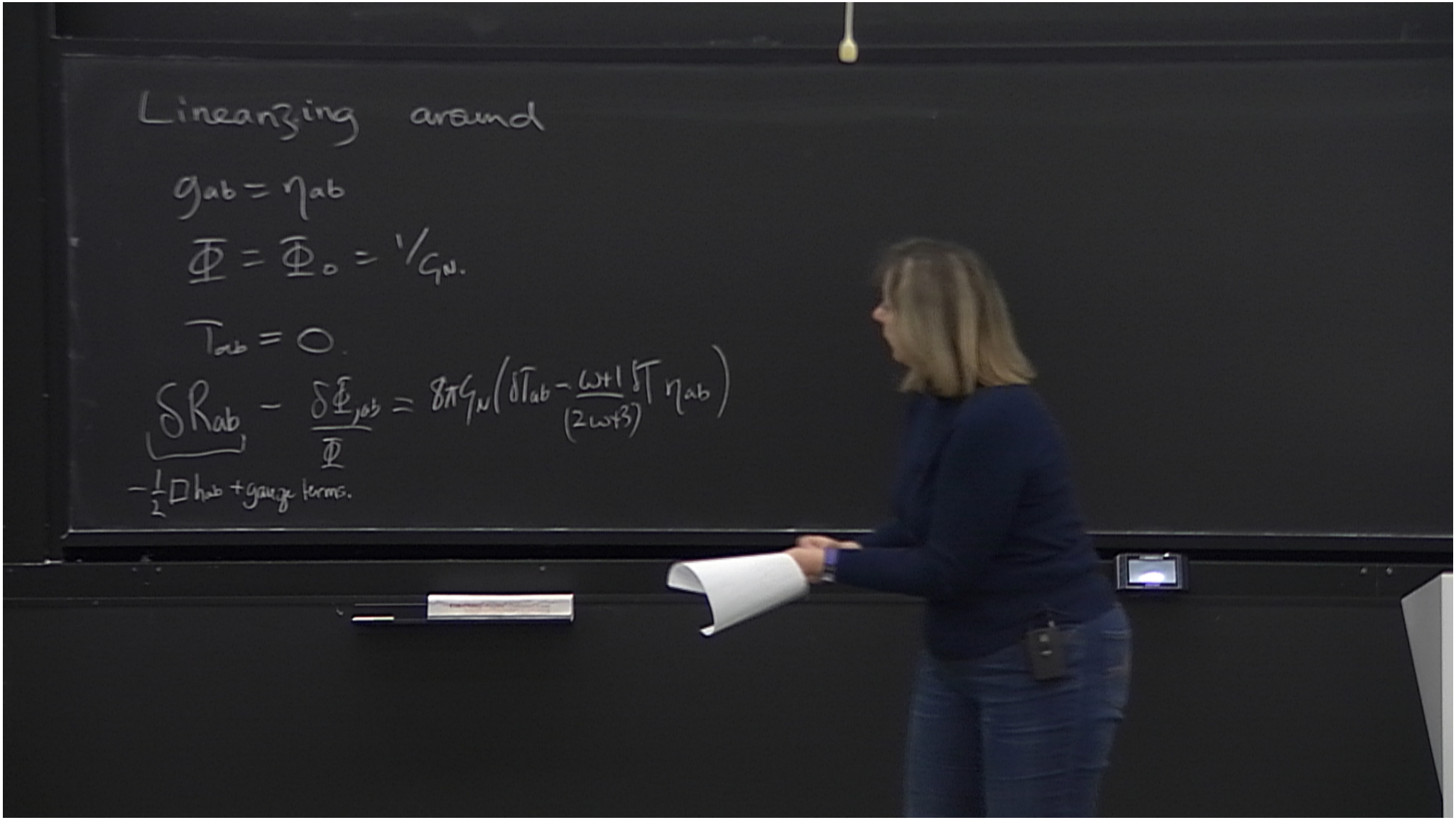
$$\Box h_{ab} = -8\pi G_{\text{eff}}\left(T_{ab} - \frac{1}{2} T \eta_{ab}\right)$$

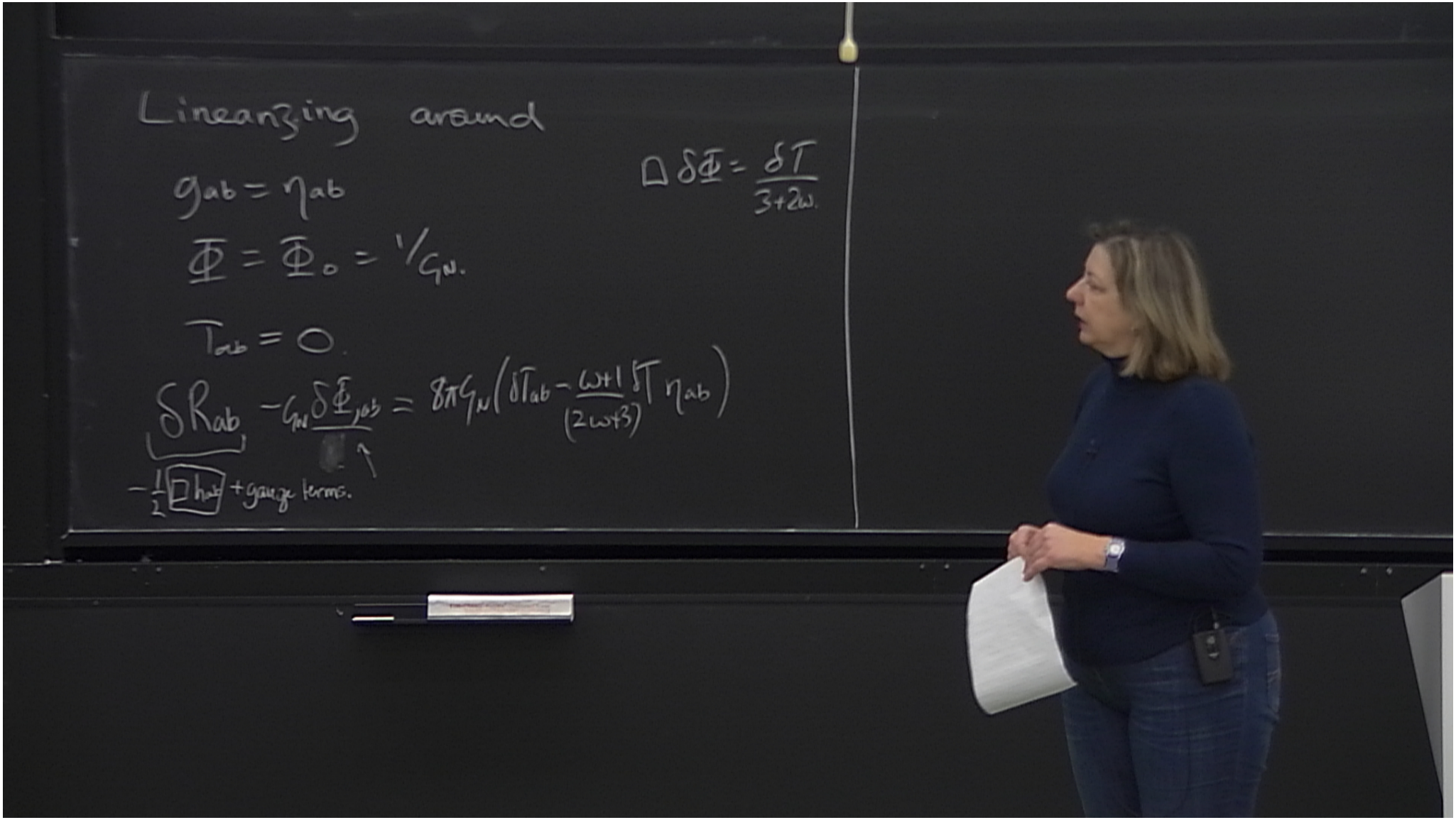
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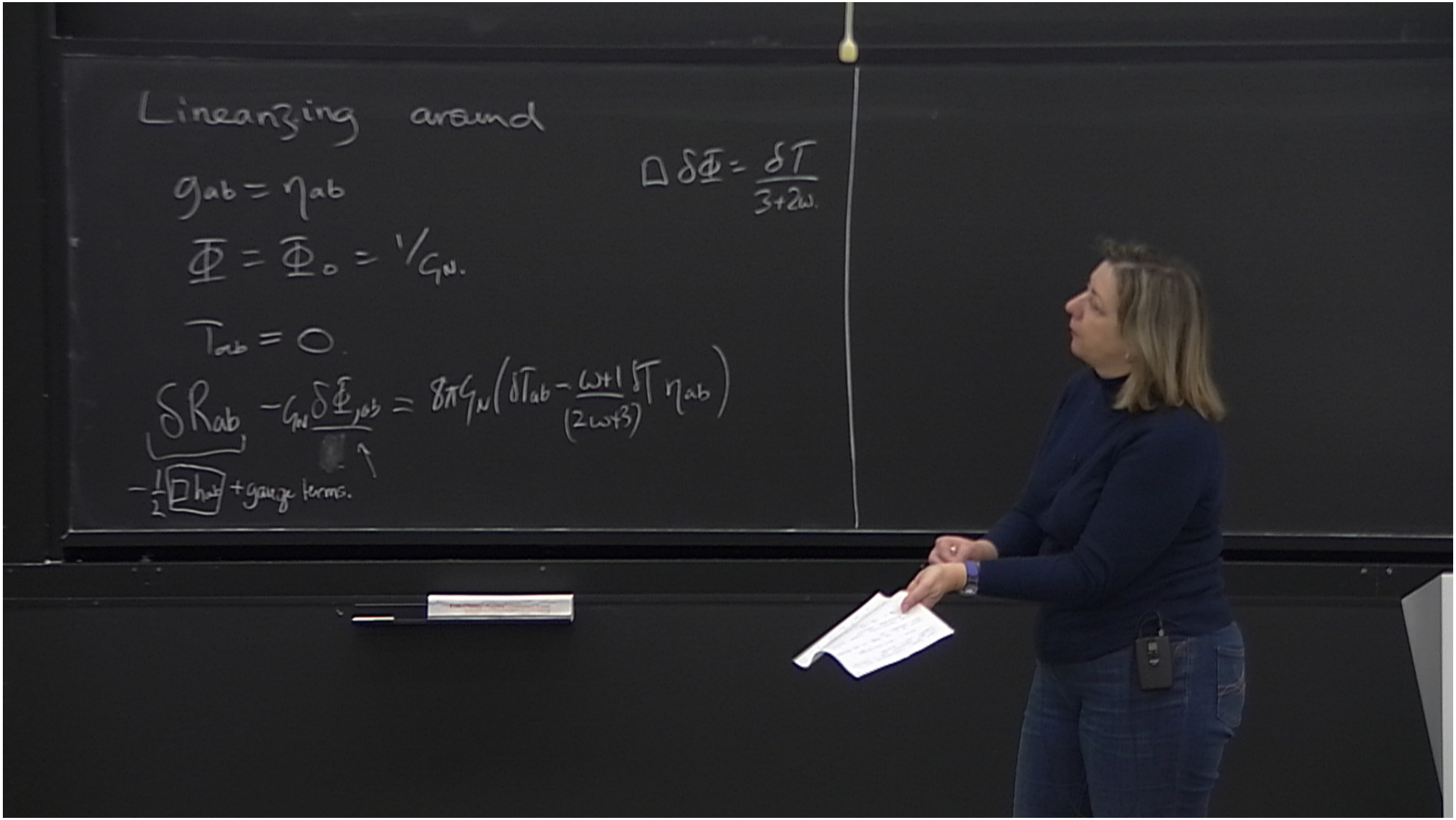
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Linearizing around
$g_{ab} = \eta_{ab}$
$\Phi = \Phi_0 = 1/G_N.$


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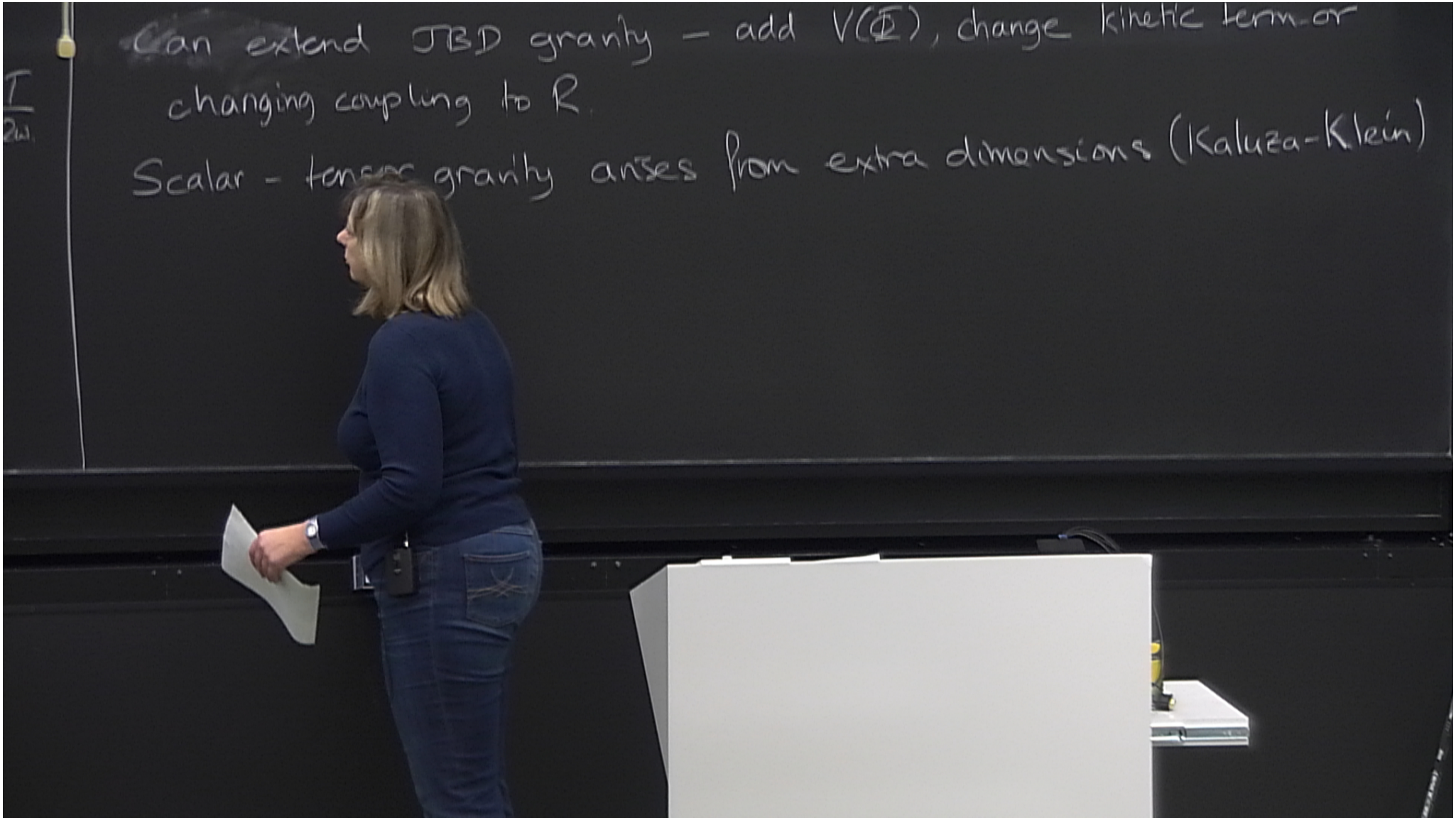
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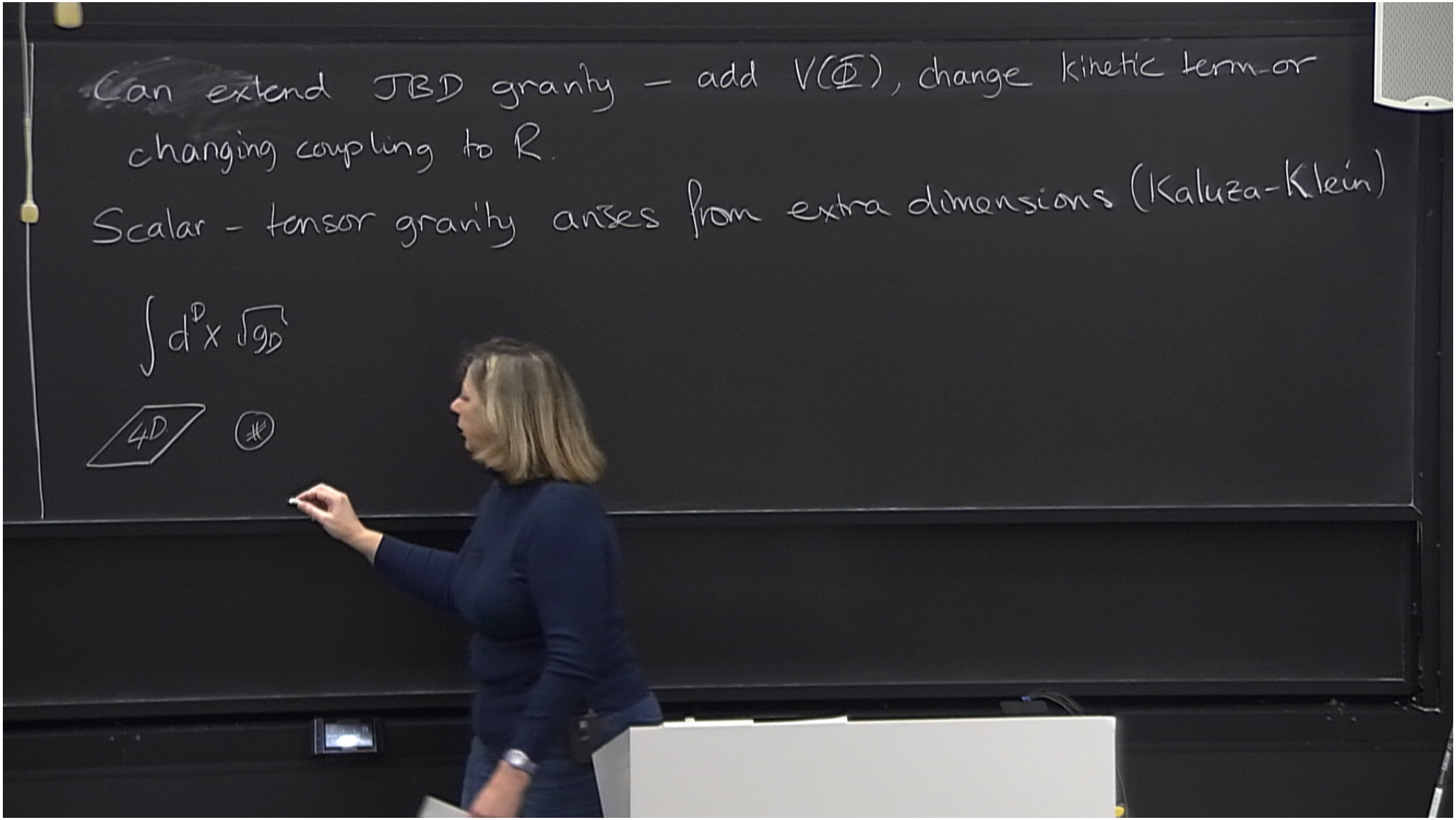
$T_{ab} = 0$

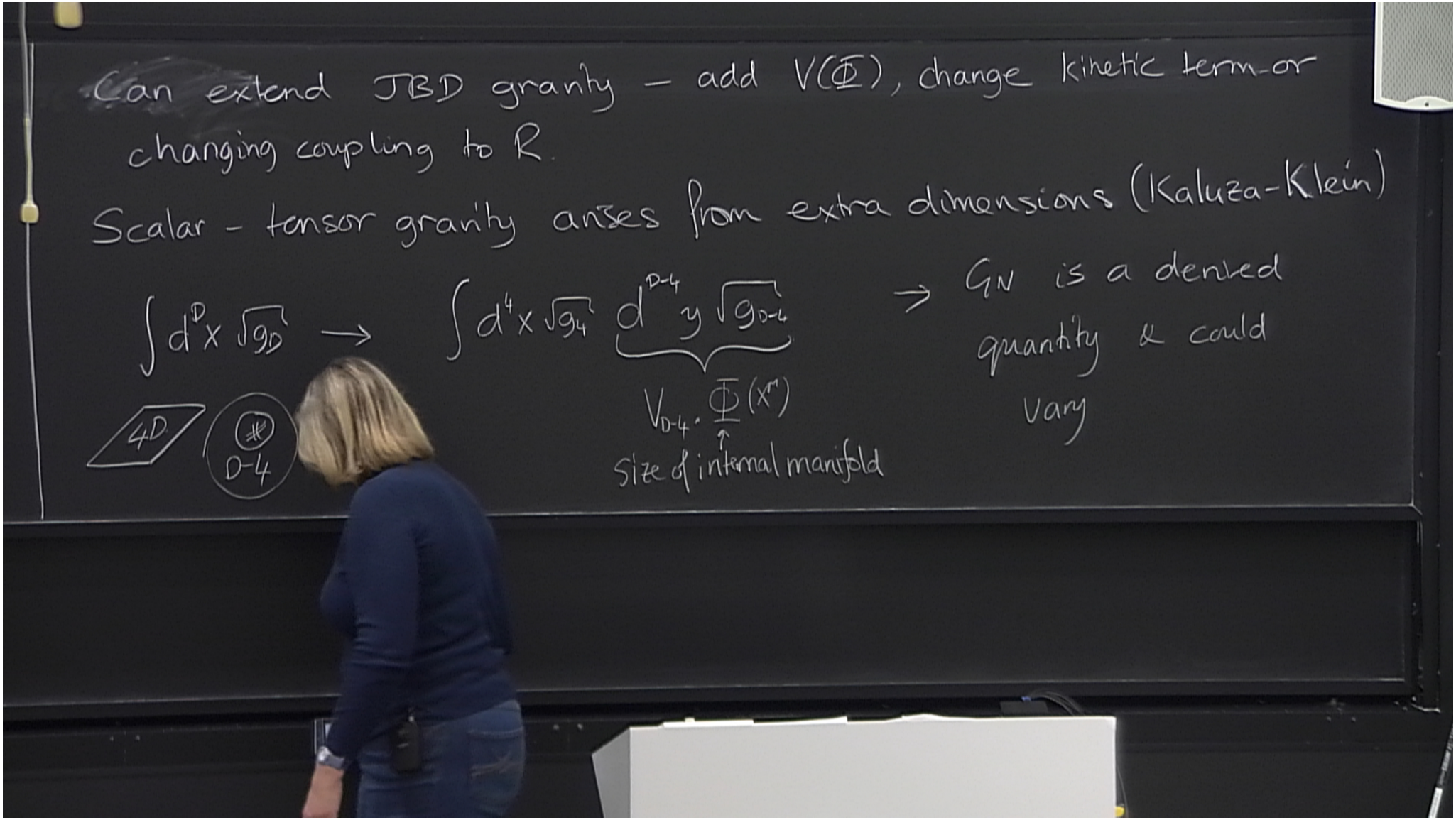
$\Box \delta\Phi = \frac{\delta T}{3+2\omega}$

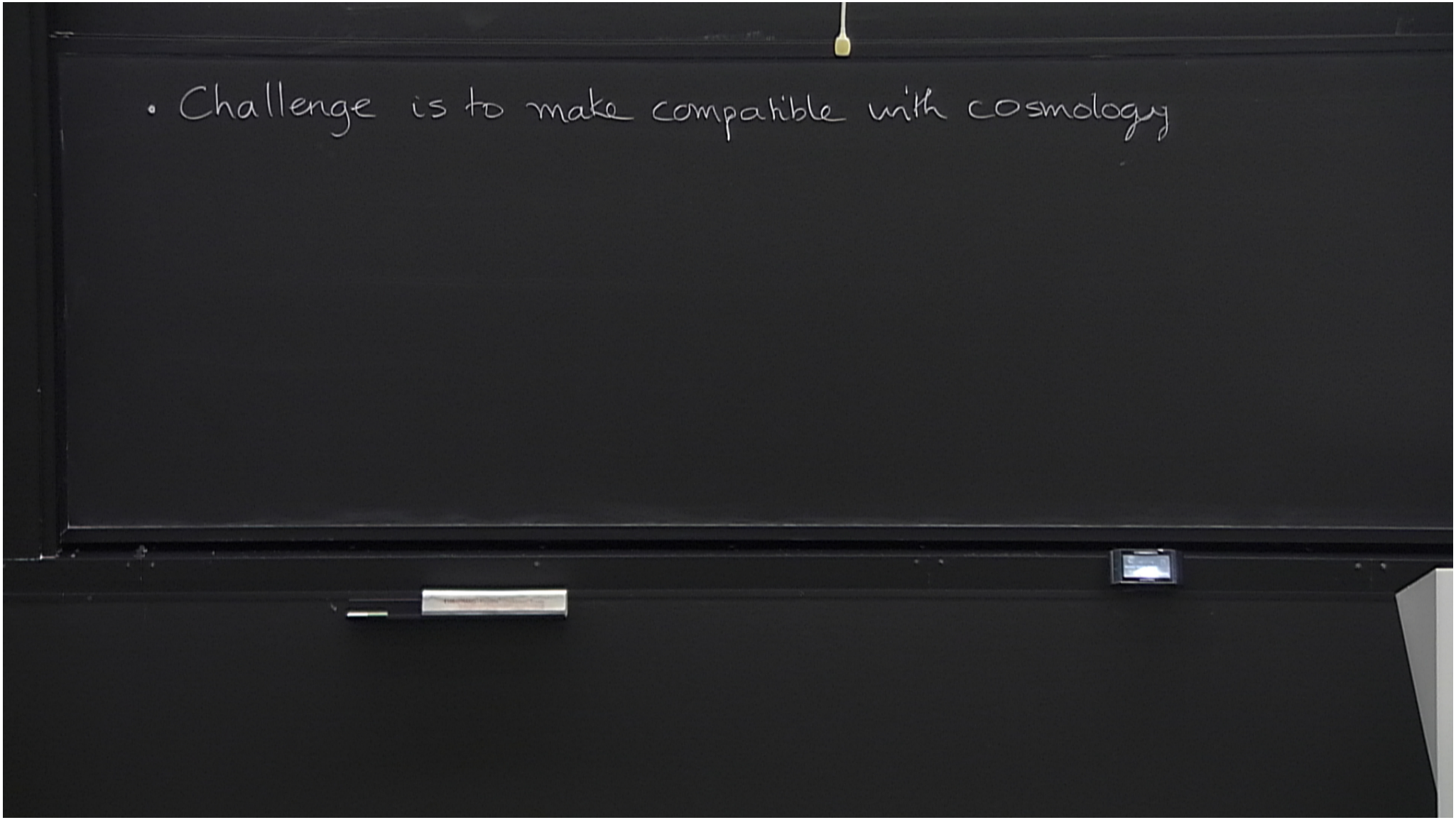
$\delta R_{ab} - G_N \delta\Phi_{ab} = 8\pi G_N \left(\delta T_{ab} - \frac{\omega+1}{(2\omega+3)} \delta T \, \eta_{ab} \right)$

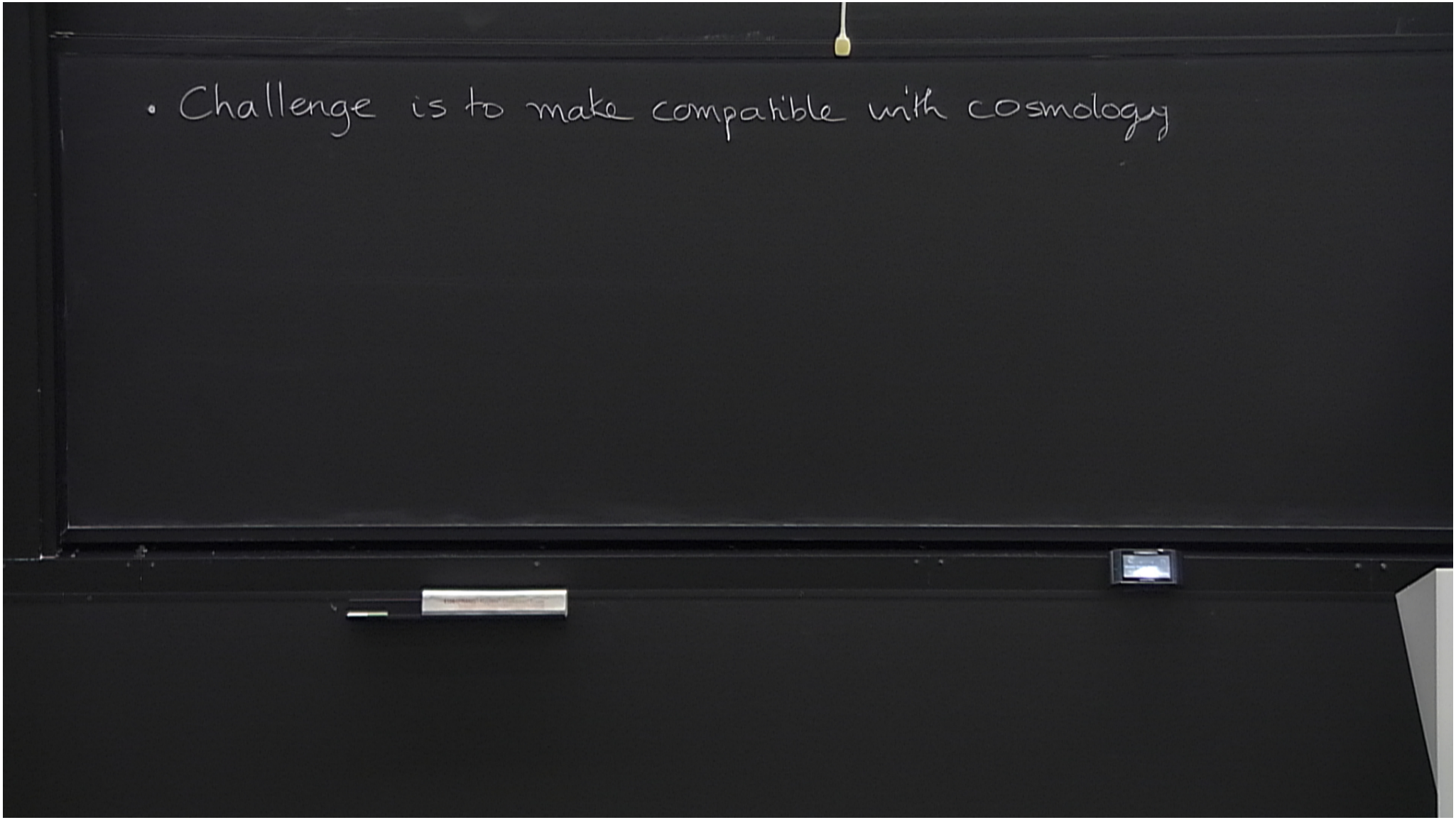
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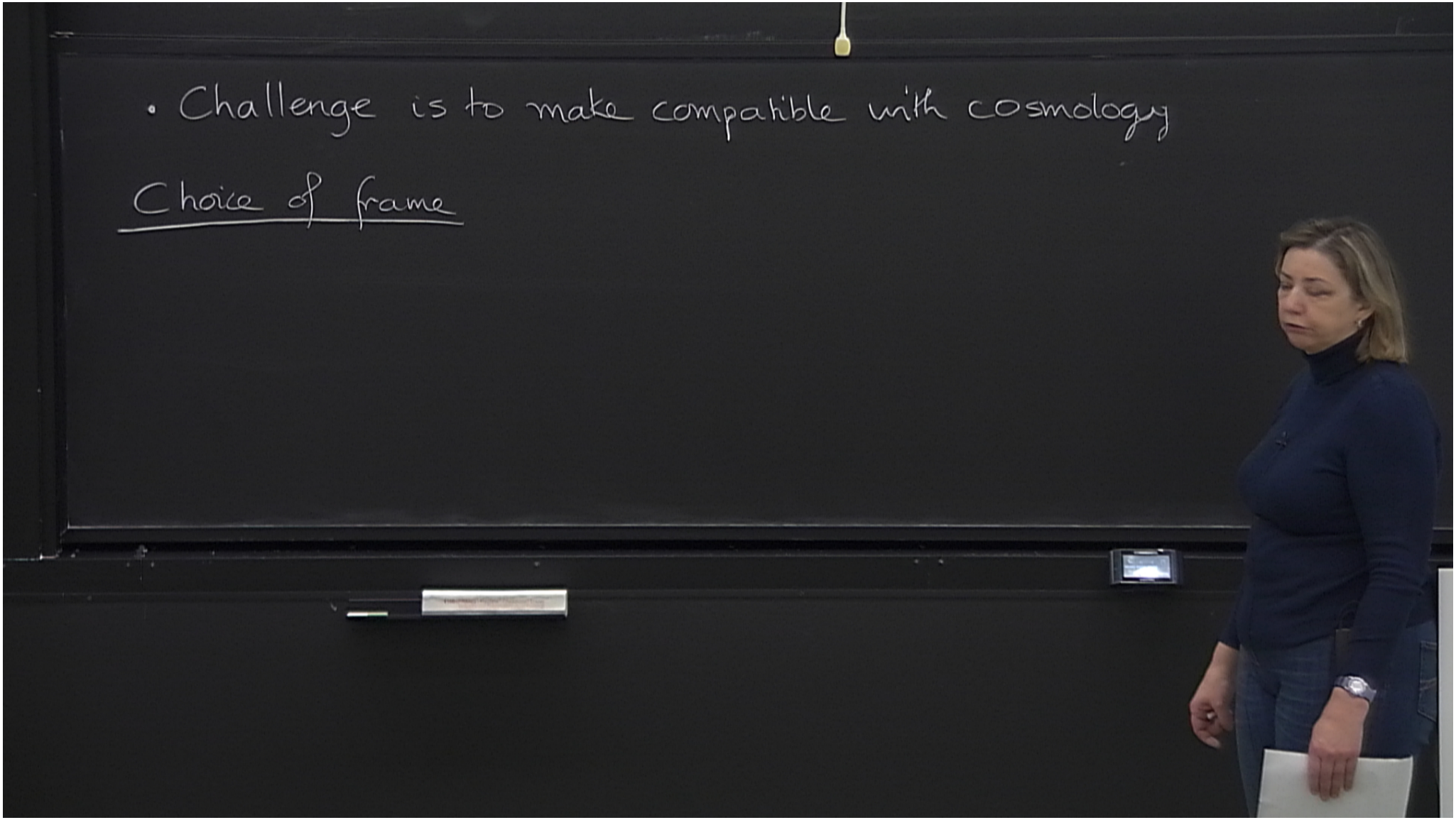

Can extend JBD gravity – add V(Φ), change kinetic term or
changing coupling to R
Scalar – tensor gravity arises from extra dimensions (Kaluza-Klein)

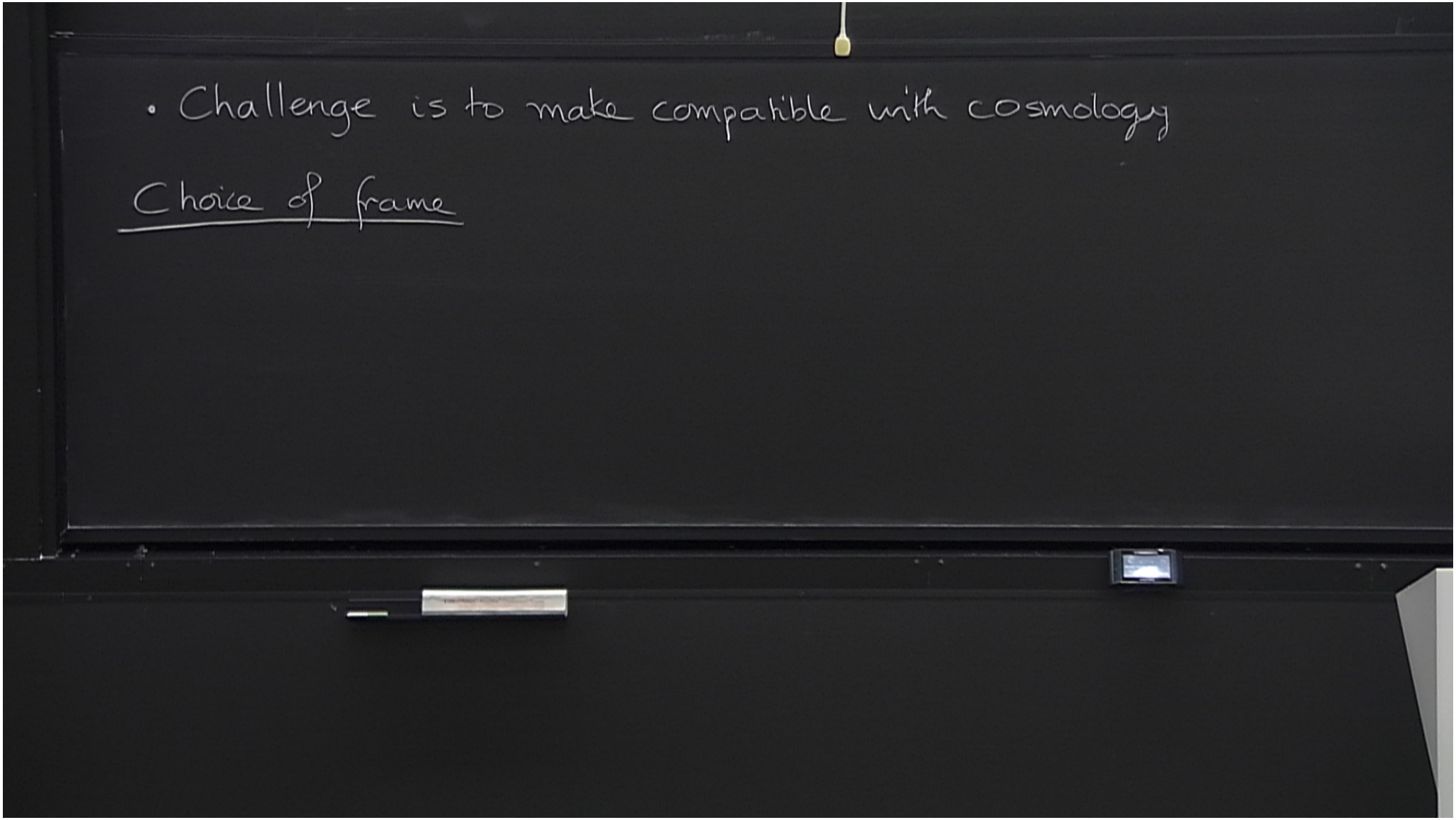

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Scalar - tensor gravity arises from extra dimensions (Kaluza-Klein)
$\int d^D x \sqrt{g_D}$
4D
#

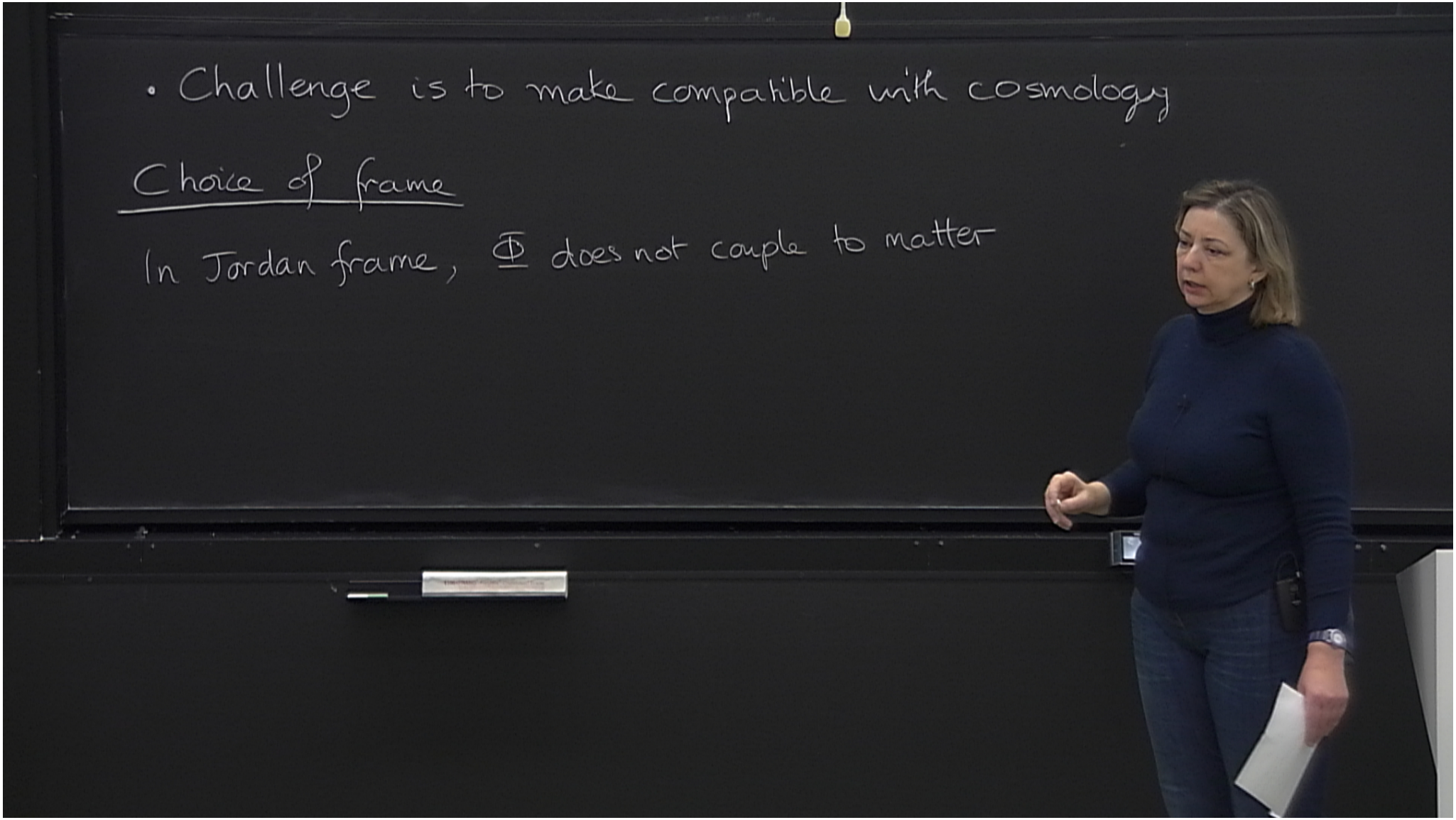

Can extend JBD gravity – add V(Φ), change kinetic term or changing coupling to R.
Scalar-tensor gravity arises from extra dimensions (Kaluza-Klein)
$\int d^D x \sqrt{g_D} \rightarrow \int d^4x \sqrt{g_4} \underbrace{d^{D-4} y \sqrt{g_{D-4}}}$
$V_{D-4} \cdot \Phi(x^\mu)$
size of internal manifold
$\rightarrow G_N$ is a derived quantity & could vary
4D
D-4

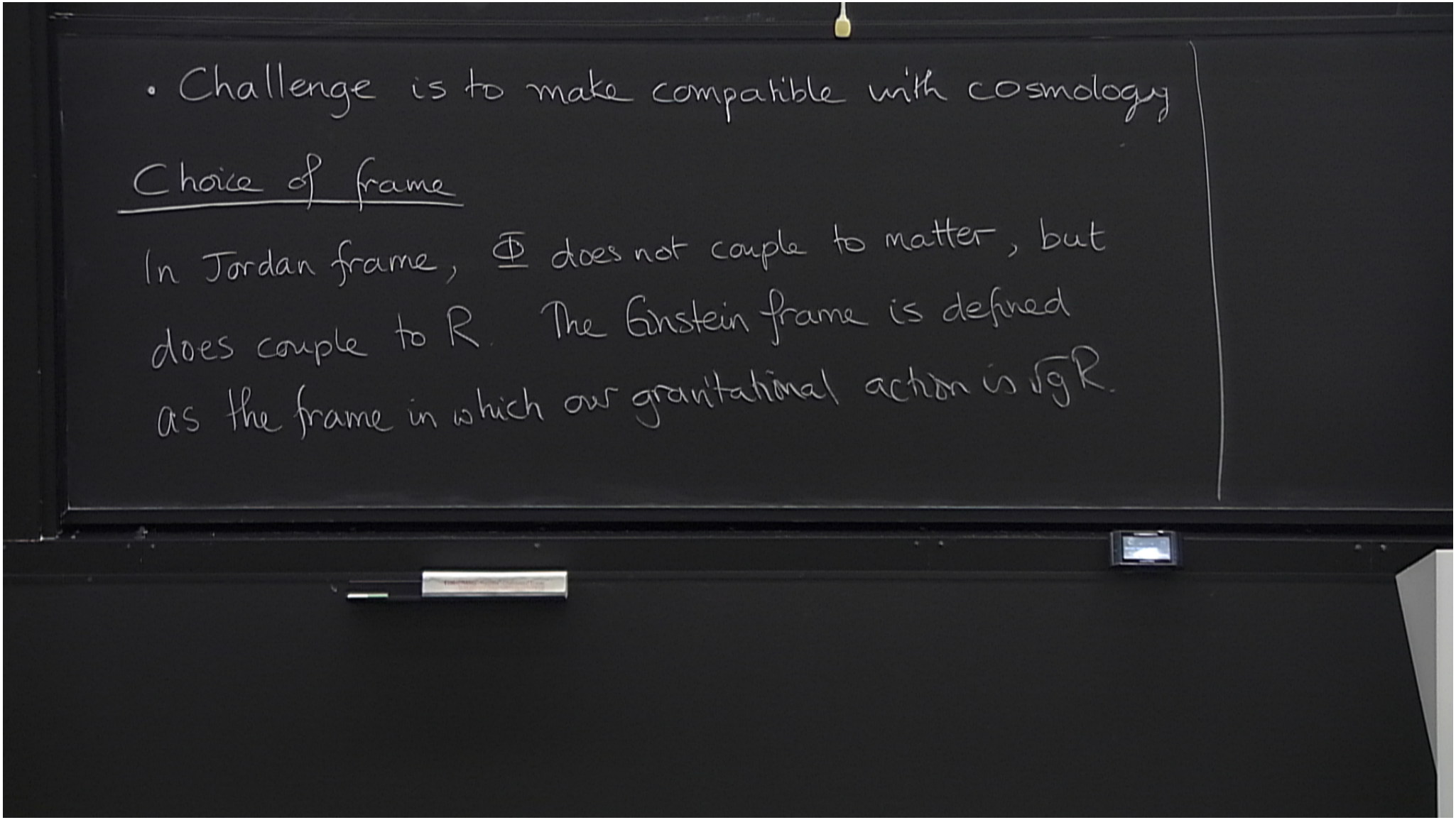

• Challenge is to make compatible with cosmology

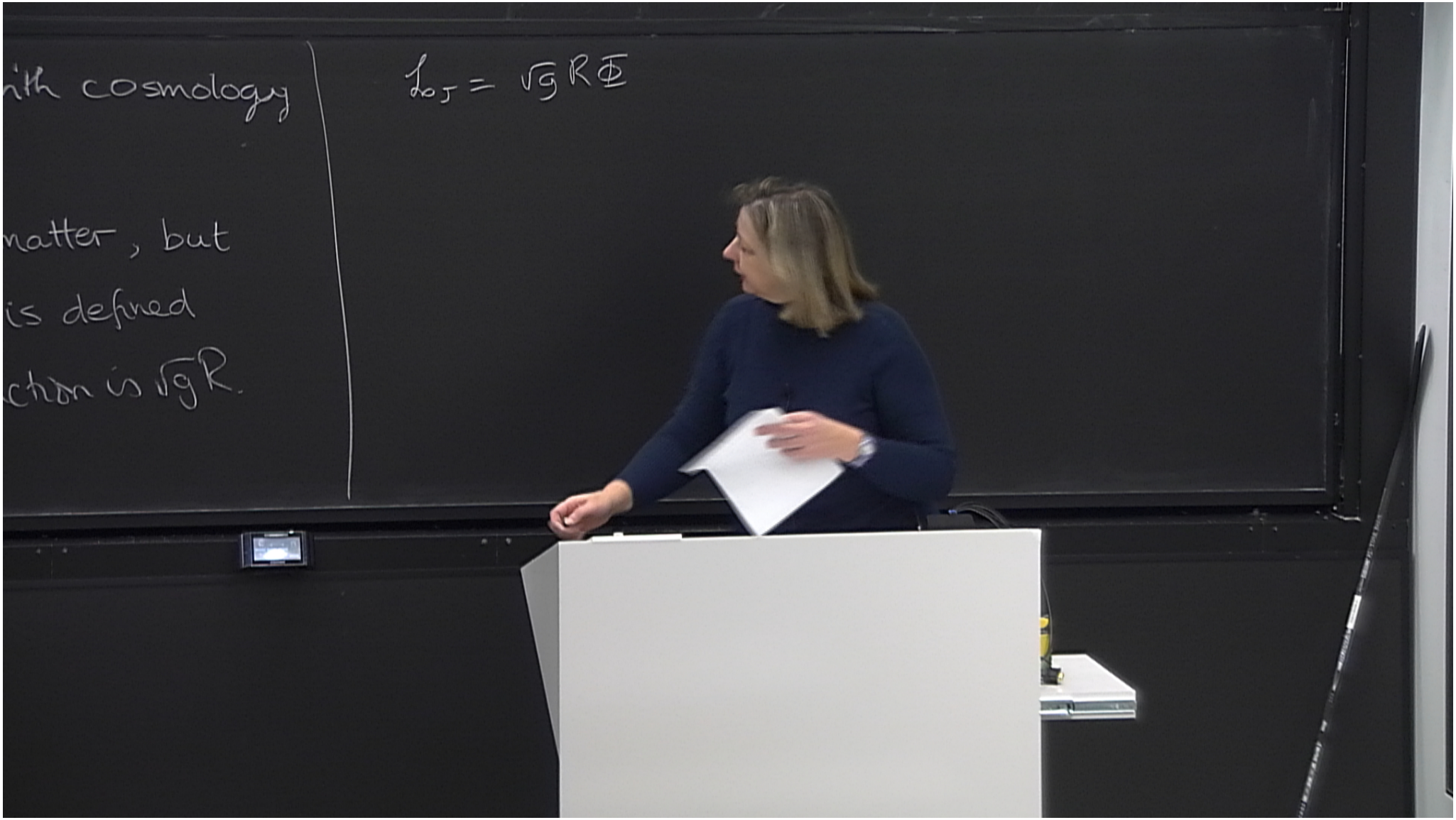

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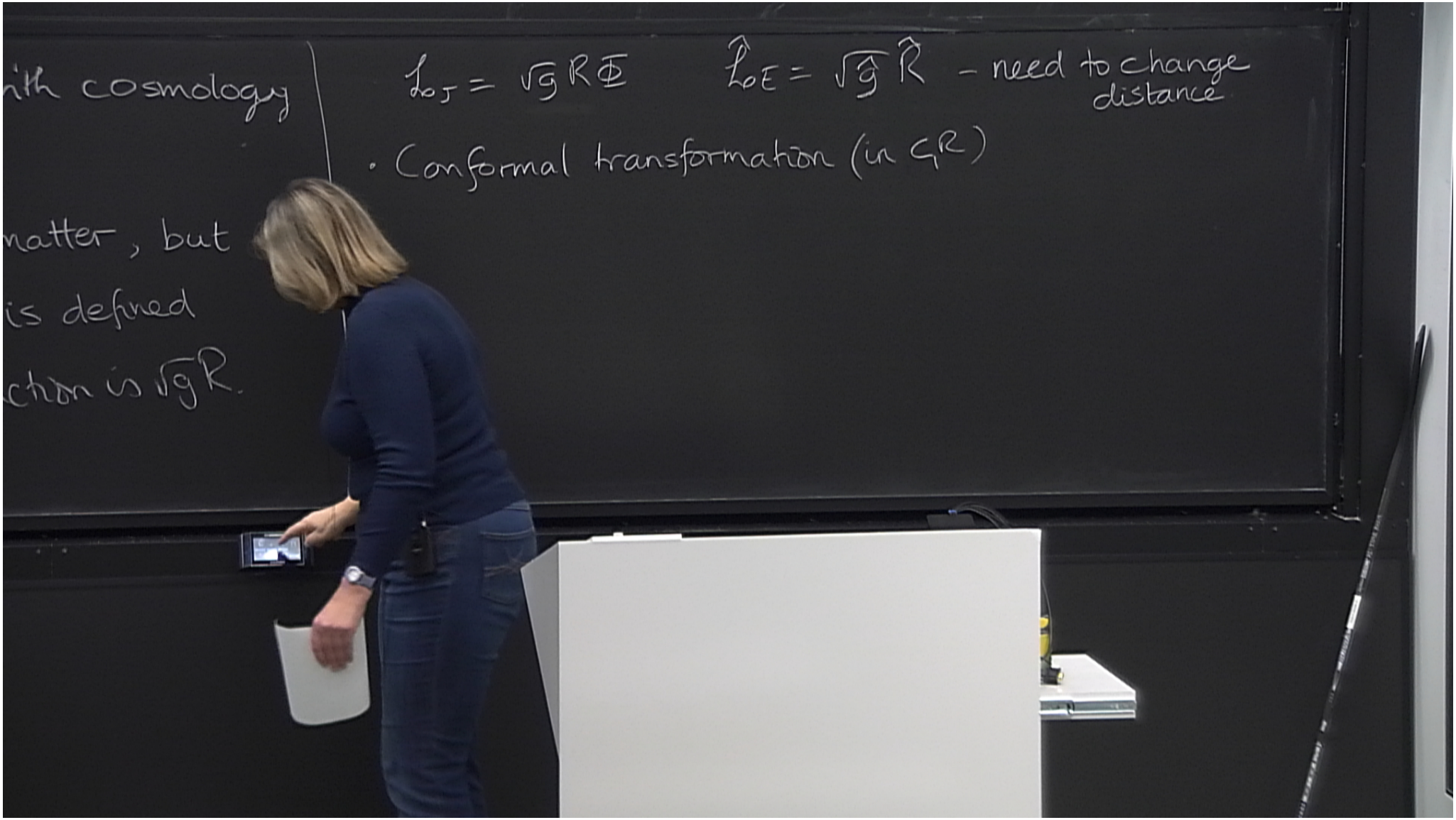

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Choice of frame

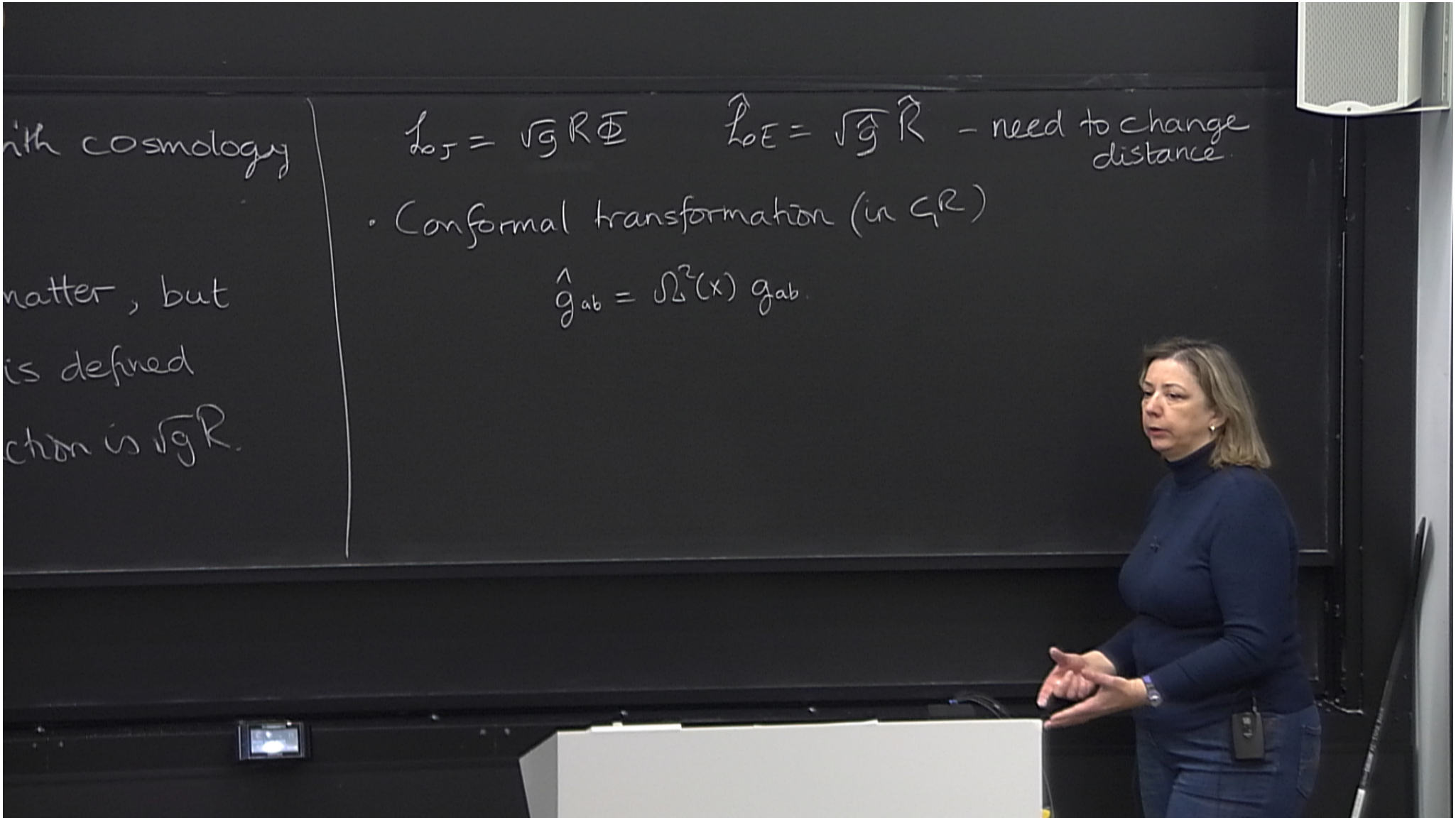

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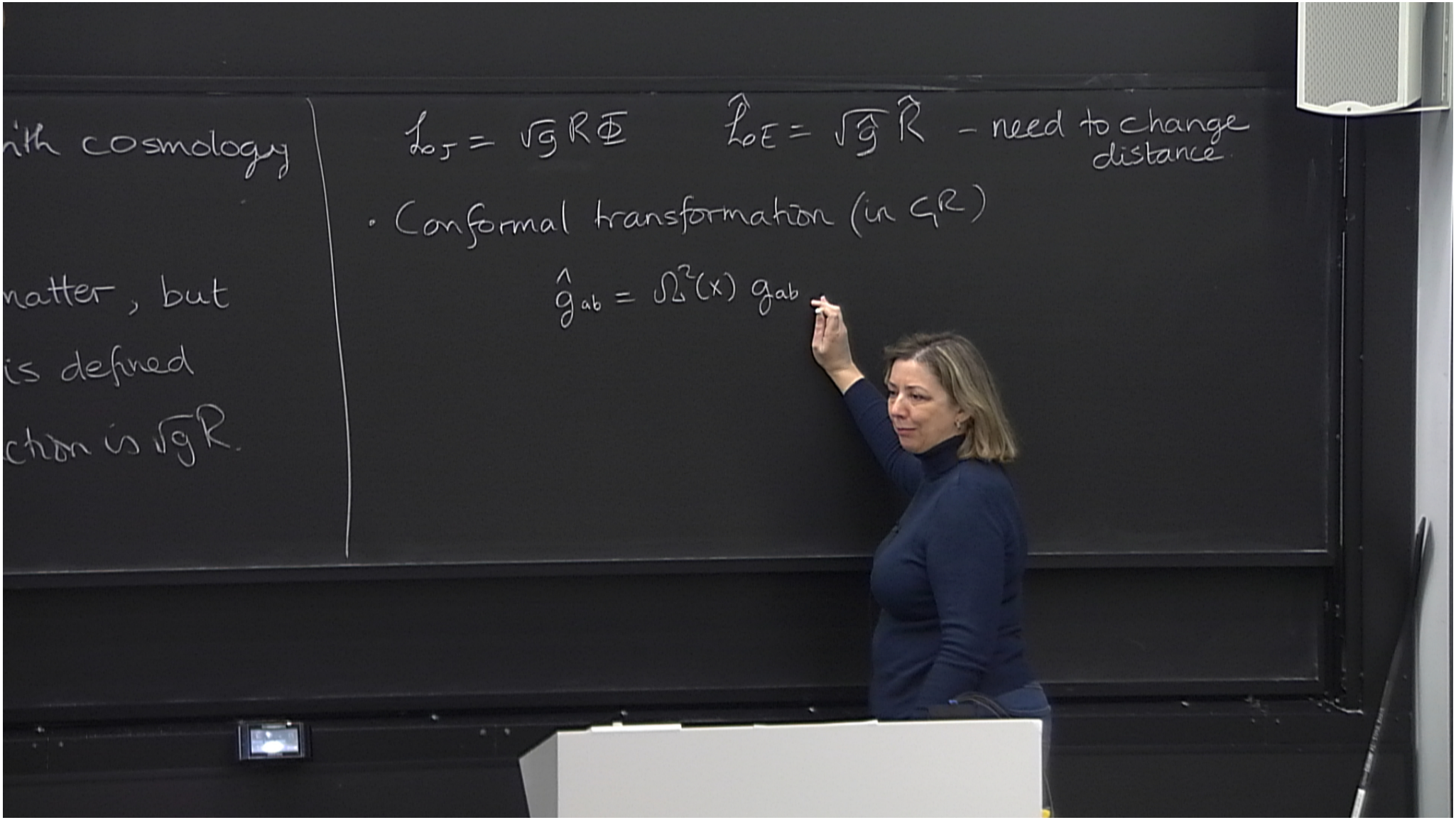

• Challenge is to make compatible with cosmology
Choice of frame
In Jordan frame, Φ does not couple to matter

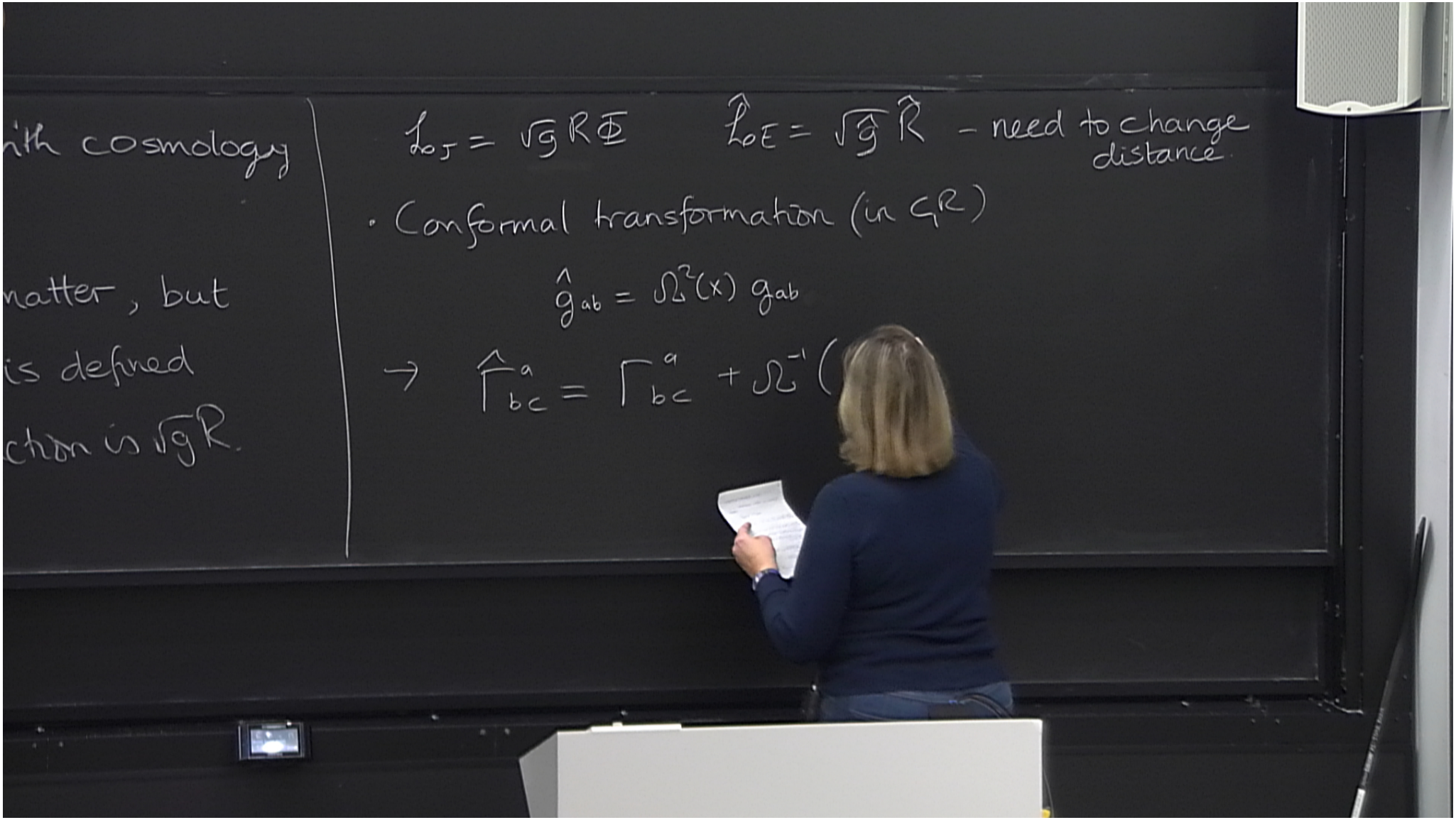

• Challenge is to make compatible with cosmology
Choice of frame
In Jordan frame, Φ does not couple to matter, but
does couple to R. The Einstein frame is defined
as the frame in which our gravitational action is √g R.

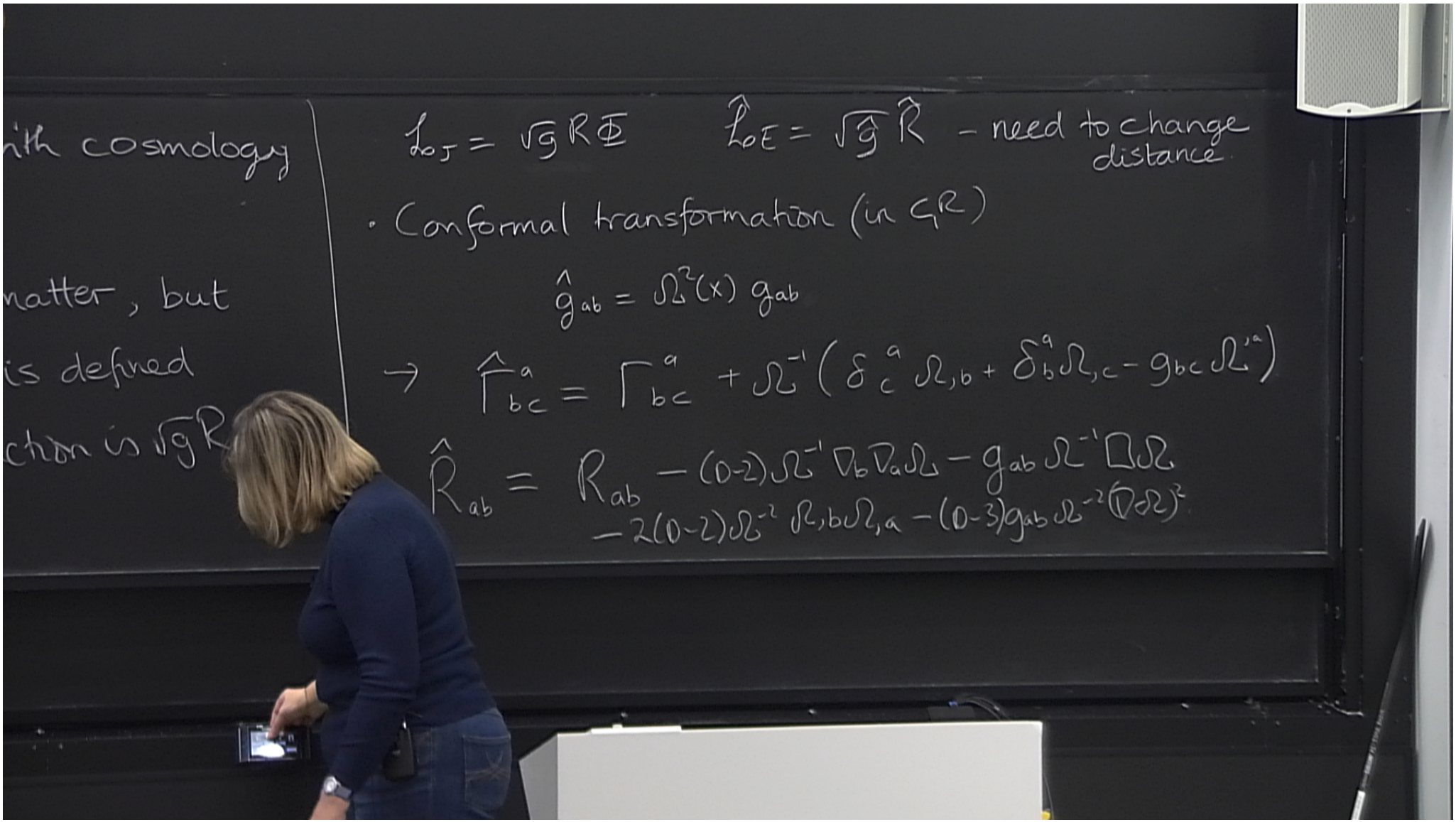

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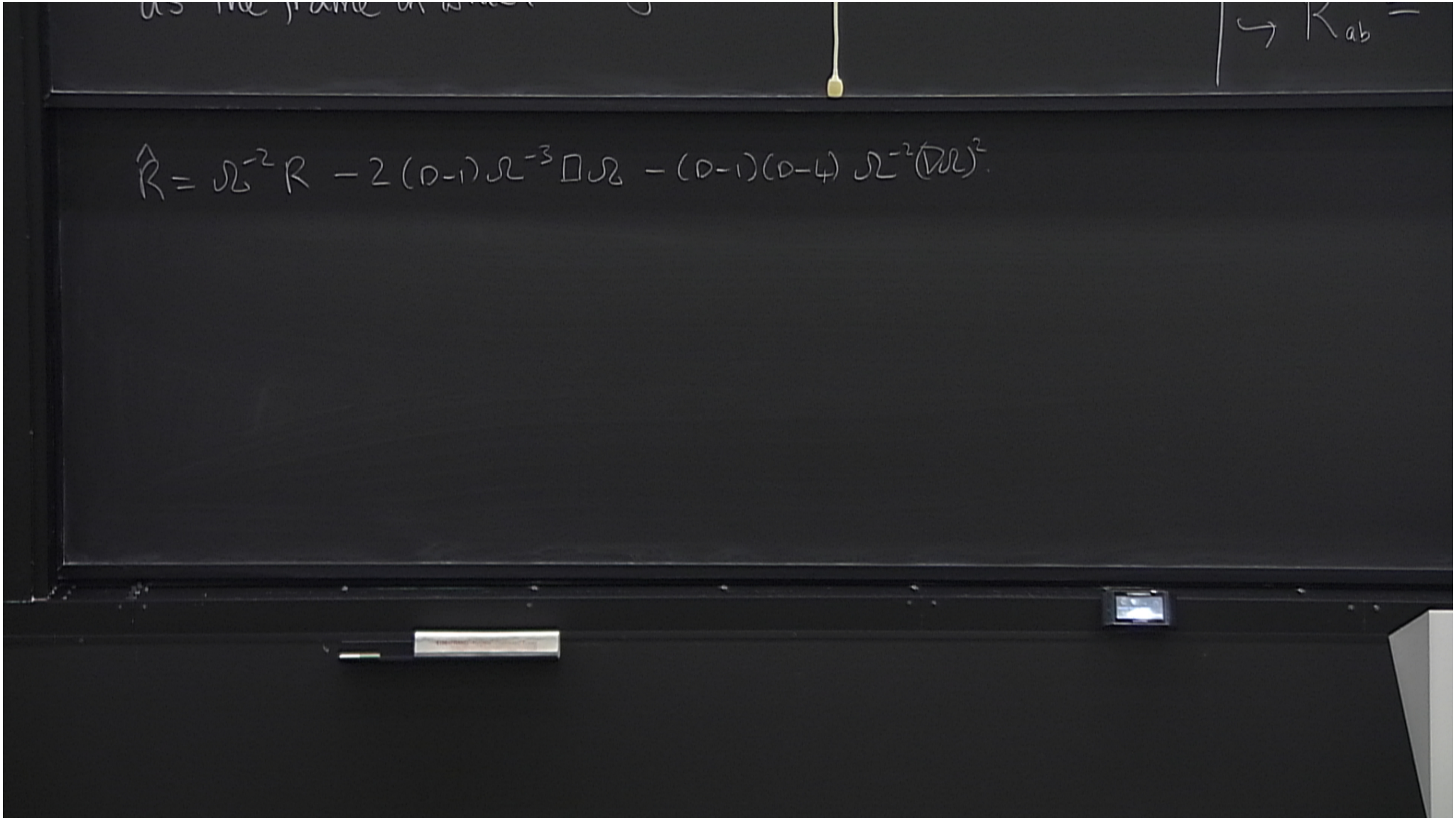

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$\rightarrow \quad \hat{\Gamma}^a_{bc} = \Gamma^a_{bc} + \Omega^{-1}\left(\delta^a_c \Omega_{,b} + \delta^a_b \Omega_{,c} - g_{bc}\Omega^{,a}\right)$
$\hat{R}_{ab} = R_{ab} - (0-2)\Omega^{-1}\nabla_b\nabla_a\Omega - g_{ab}\Omega^{-1}\Box\Omega$
$-2(0-2)\Omega^{-2}\Omega_{,b}\Omega_{,a} - (0-3)g_{ab}\Omega^{-2}(\nabla\Omega)^2$


$\hat{R} = \Omega^{-2} R - 2(D-1)\Omega^{-3}\Box\Omega - (D-1)(D-4)\Omega^{-2}(\nabla\Omega)^2$

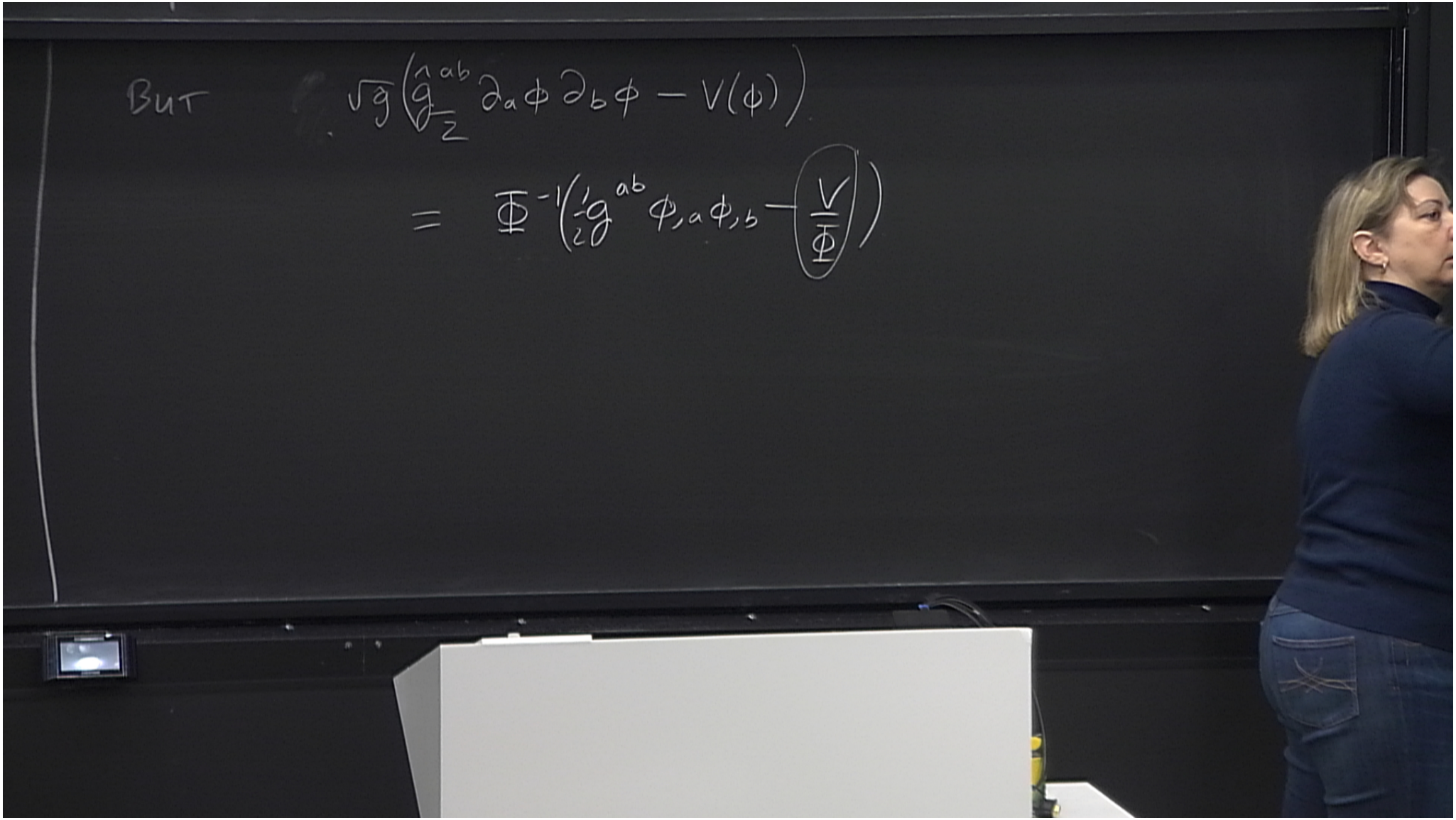
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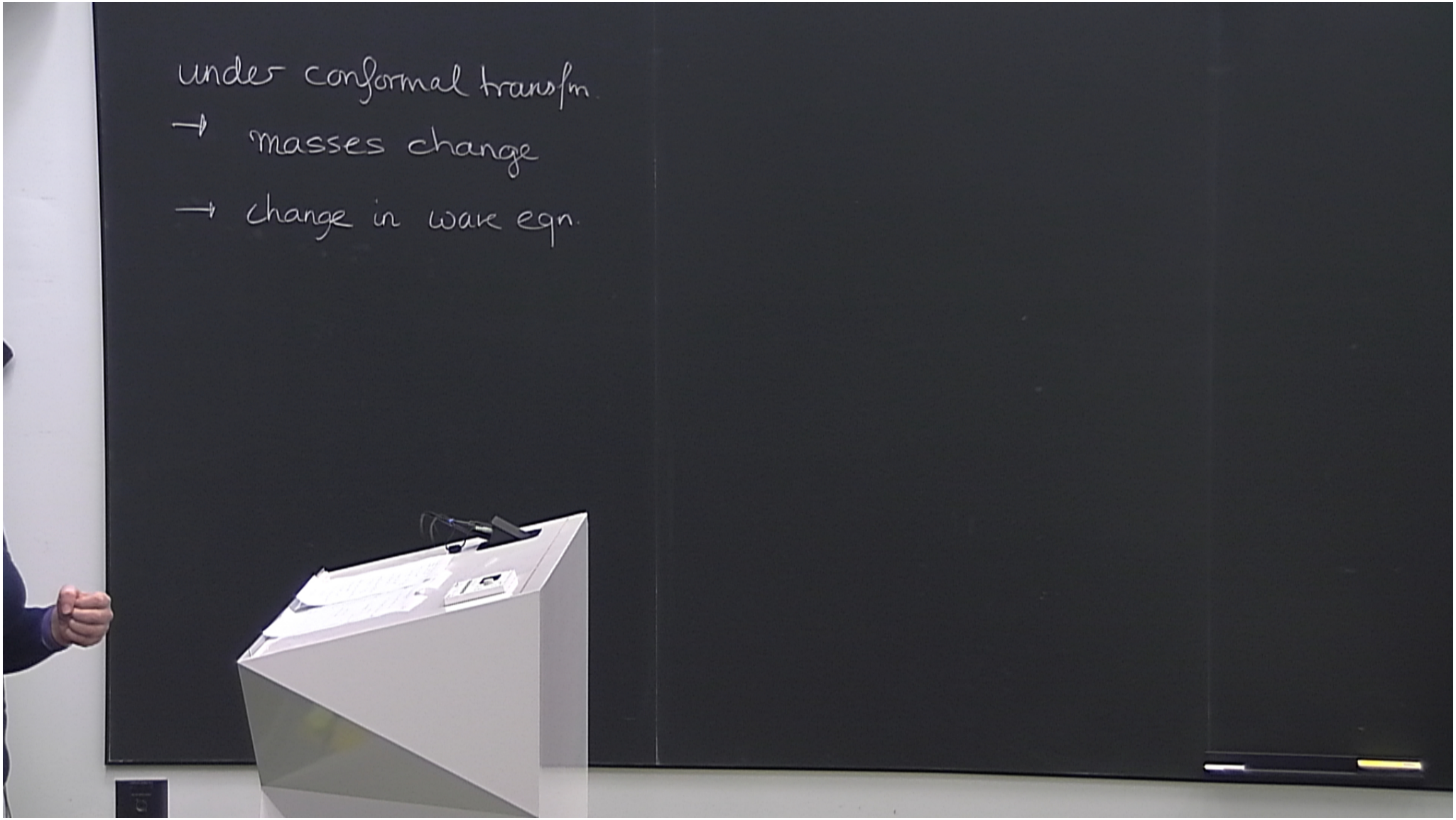
Conformal frame gives simple Einstein variation

BUT coupling to matter changes.

e.g. in 4D. $\hat{g}_{ab} = \frac{g_{ab}}{\Phi} \Rightarrow \sqrt{\hat{g}}\hat{R} = \frac{\sqrt{g}}{\Phi^2} \cdot \Phi R_{\cdot +\cdot -}$

$\Rightarrow \sqrt{g}\hat{R}\Phi = \sqrt{g} R_{\cdot +\cdot -}$


But
$\sqrt{g}\left(\frac{\hat{g}^{ab}}{2}\partial_a\phi\,\partial_b\phi - V(\phi)\right)$
$= \Phi^{-1}\left(\frac{1}{2}g^{ab}\phi_{,a}\phi_{,b} - \frac{V}{\Phi}\right)$


under conformal transfm
→ masses change
→ change in wave eqn.